- Homework 6

The Soyuz capsule orbits the Earth at an altitude of 400 km (in circular, equatorial orbit). Its main data are

$m = 2400$ Kg $\quad S = 3.8\ m^2$ (aerodynamic surface)

$C_D = 1.341$ (average value of $C_D$ along the entire flight)

$\tilde{L} = 0$ (no lift)

$R_c = 2.235$ m (radius of curvature at stagnation point i.e. at the nose)

De-orbit follows a tangential braking $\vec{\Delta v}$. Two cases are considered: (1) $\Delta v = 0.2\ \frac{Km}{sec}$ (2) $\Delta v = 0.4\ \frac{Km}{sec}$

For each of these two cases

(a) Find $v_{I,ini}$ and $\gamma_{I,ini}$ at $h_{EI} = 100$ km, i.e. the inertial velocity magnitude and the related flight path angle at the entry interface

(b) Find $v_{R,ini}$ and $\gamma_{R,ini}$ associated with $v_{I,ini}$ and $\gamma_{I,ini}$ using the relation $\vec{v}_I = \vec{v}_R + \vec{\omega}_E \times \vec{r}$ ; the two values $v_{R,ini}$ and $\gamma_{R,ini}$ coincide with $v_{EI}$ and $\gamma_{EI}$, used as initial conditions for reentry.

(c) Propagate numerically the reentry trajectory up to touchdown (altitude = 0) using equation set (B1); portray the time histories of $h(t)$, $\gamma_R(t)$, $v_R(t)$, and the reentry trajectory.

(d) Plot the time histories of the quaternions associated with $\underset{C \leftarrow N}{R}$, where the inertial frame $N$ is ECI and the (commanded) body axes are BA (see notes)

Use the initial conditions $\theta_{G0} = 60$ deg and $\lambda_{g0} = 30$ deg and zero control angles (at all times).

Comment and explain the symmetries in the time histories of the quaternions

(e) Report the value of the time of flight and the values of $a_d^{(max)}$, $q_s^{(max)}$, $p_d^{(max)}$ and the times at which they occur.

For $a_d$, use $\quad a_d = -\dot{v}_I = -\frac{d}{dt}\sqrt{v_R^2 + w_E^2 r^2 + 2 w_E r v_R c_{\gamma_R}}$

(f) Portray $\gamma_R(h)$, $v_R(h)$, $q_s(h)$, $a_d(h)$, $p_d(h)$ for the numerically integrated trajectory found at previous points and (in the same figures) the respective plots obtained with the use of the Allen-Eggers analytical solution.

For the evaluation of $q_s$, use the following constant:

$$\frac{K}{v_0^3 \sqrt{\rho_0}} = 1.762 \cdot 10^{-4}\ Kg^{1/2}\ m^{-1}$$