

## 0.1 Assumptions

1. Transaction costs discarded.
2. Lognormal price distribution.
3. Simplified fees calculations.
4. Slippage is not considered.
5. Continuous spaces, no discretization.

## 0.2 Supplementary

$$xy = L^2$$

$$(x_{real} + \frac{L}{\sqrt{p_+}})(y_{real} + L\sqrt{p_-}) = L^2 \quad (1)$$

$$\text{IL} = 1 - \frac{\text{TV}(p_t)}{\text{TV}(p_0)},$$

where is  $\text{TV}(p) = px_{real} + y_{real}$  (2)

$$\text{IL} = 1 - \frac{2\sqrt{\frac{p}{p_0}}}{\frac{p}{p_0} + 1}$$

$$\begin{aligned} \dot{p} &= \alpha p + \sigma p \eta \\ p(t_0) &= 1 \\ \eta &\sim \rho[\eta] \end{aligned} \quad (3)$$

$$P_{\text{IL}}(u, t) = \sum_{p: p=\text{IL}^{-1}(u)} \frac{\sqrt{p}(p+1)^2}{|p-1|} \cdot P_{p(t)}(p, t)$$

$$P_{p(t)}(p, t) = \langle \delta(p - p[\eta]) \rangle_\eta = \frac{1}{\sigma p} \cdot \rho_\eta \left( \frac{\log p - (\alpha - \frac{\sigma^2}{2})t}{\sigma} \right) \quad (4)$$

$$P_{\text{IL}}(u, t) = \sum_{p: p=\text{IL}^{-1}(u)} \frac{\sqrt{p}(p+1)^2}{|p-1|} \frac{1}{\sqrt{2\pi t}} \frac{1}{\sigma p} \exp \left\{ -\frac{(\log p - (\alpha - \frac{\sigma^2}{2})t)^2}{2\sigma^2 t} \right\} \quad (5)$$

$$Fees = \gamma \int dt e^{-rt} V(t) \int dp P_{p(t)}(p, t) \frac{L(p(t))}{L_{total}(p(t))}$$

$$\begin{aligned} &\max_{p_-, p_+} Fees(T) \\ &\text{s. t. } P(\text{IL}(T) < \Delta_1) \geq q_1 \\ &\quad P(\max_{t \leq T} \text{IL} < \Delta_2) \geq q_2 \end{aligned} \quad (6)$$

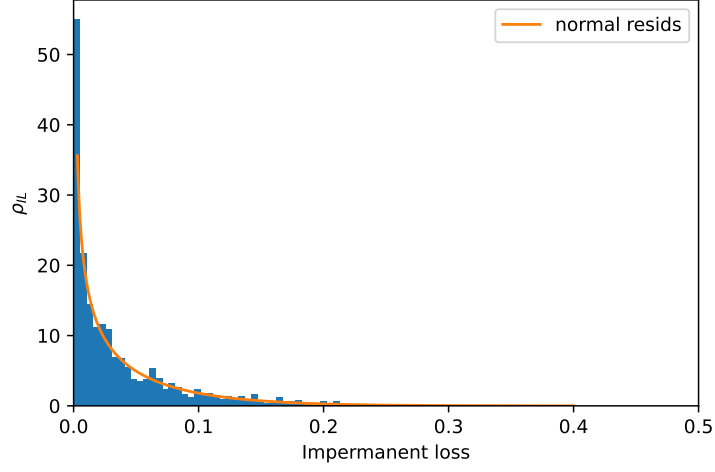


Figure 1: Impermanent loss dist.

$$\begin{aligned}
 \text{TV}(p) &= px_{real} + y_{real} = L \left( p \left( \frac{1}{\sqrt{p_{tr}}} - \frac{1}{\sqrt{p_+}} \right) + \sqrt{p_{tr}} - \sqrt{p_-} \right) \\
 p_{tr} &= \text{clip}(p, \min = p_-, \max = p_+) \\
 \frac{\partial \text{TV}}{\partial p} &= x_{real} + \left( p \frac{\partial x}{\partial y} + 1 \right) \frac{\partial y}{\partial p} = x_{real}
 \end{aligned} \tag{7}$$

$$\text{TV}(p) = 2L\sqrt{p} \left( 1 - \frac{\sqrt{\frac{p}{p_+}} + \sqrt{\frac{p_-}{p}}}{2} \right), \text{ when } p_- < p < p_+ \tag{8}$$

Now  $\left( 1 - \frac{\sqrt{\frac{p}{p_+}} + \sqrt{\frac{p_-}{p}}}{2} \right)^{-1}$  can be interpreted as liquidity efficiency coefficient

### 0.3 Speculative

How would liquidity be distributed in order to maximize fees?

$$\begin{aligned}
 J[L_{total}] &= \int dp \phi(p, t) \frac{1}{L_{total}(p)} \\
 G[L_{total}] &= \int dp L_{total} = \Sigma = \text{const}
 \end{aligned} \tag{9}$$

$$L = \frac{\sqrt{\phi}}{\left\langle \frac{1}{\sqrt{\phi}} \right\rangle} \Sigma \sim \sqrt{\phi} \tag{10}$$

If we were maximizing TV instead we would obtain (which is totally not true since we must infer  $\rho(p_-, p_+)$  and keep in mind transaction costs):

$$L \sim \frac{\sqrt{\phi}}{p^{\frac{1}{4}}} \quad (11)$$

If our view differs from global  $\rho(p_-, p_+)$ , we would place  $\delta L(p)$  corresponding to ours view

Can liquidity shaping help with hedging?

$$TV = \int dTV(L) = \int dL \left[ p \left( \frac{1}{\sqrt{p}} - \frac{1}{\sqrt{p_+(L)}} \right) + \sqrt{p} - \sqrt{p_-(L)} \right] \quad (12)$$

$$\Pi_L = 1 - \frac{TV(p_t)}{p \cdot x_{real}(t_0) + y_{real}(t_0)} \stackrel{(p_-, p_+) = (0, \infty)}{=} 1 - \frac{2\sqrt{\frac{p}{p_0}}}{\frac{p}{p_0} + 1} \quad (13)$$