0.1 Assumptions

1. Lognormal price distribution.
2. Continuous spaces, no discretization.
3. Constant transaction fees.
4. Simplified fees calculations.
5. Slippage is not considered.

0.2 Supplementary

$$\begin{gathered} xy = L^2 \\ (x_{real} + \frac{L}{\sqrt{p_+}})(y_{real} + L\sqrt{p_-}) = L^2 \end{gathered} \tag{1}$$

$$\begin{gathered} \mathrm{IL} = 1 - \frac{\mathrm{TV}(p_t)}{\mathrm{TV}(p_0)}, \\ \text{where } \mathrm{TV}(p_t) = px_{real} + y_{real} \\ \mathrm{IL} = 1 - \frac{2\sqrt{\frac{p}{p_0}}}{\frac{p}{p_0} + 1} \end{gathered} \tag{2}$$

$$\begin{gathered} \dot{p} = \alpha p + \sigma p \eta \\ p(t_0) = 1 \\ \eta \sim P[\eta] \end{gathered} \tag{3}$$

$$\begin{gathered} P_{\mathrm{IL}}(u,t) = \sum_{p:p=\mathrm{IL}^{-1}(u)} \frac{\sqrt{p}(p+1)^2}{|p-1|} * P_{p(t)}(p,t) \\ P_{p(t)}(p,t) = \langle \delta(p - p[\eta]) \rangle_\eta = \frac{1}{\sigma p} * \rho_\eta \left(\frac{\log p - (\alpha - \frac{\sigma^2}{2})t}{\sigma} \right) \end{gathered} \tag{4}$$

$$P_{\mathrm{IL}}(u,t) = \sum_{p:p=\mathrm{IL}^{-1}(u)} \frac{\sqrt{p}(p+1)^2}{|p-1|} \frac{1}{\sqrt{2\pi t}} \frac{1}{\sigma p} \exp\left\{ -\frac{(\log p - (\alpha - \frac{\sigma^2}{2})t)^2}{2\sigma^2 t} \right\} \tag{5}$$

$$Fees = \gamma \int dt\, V(t) \int dp\, \delta(p - p(t)) \frac{L(p(t))}{L_{total}(p(t))}$$

$$\begin{aligned} &\max_{p_-,p_+}\ Fees(t) \\ &\text{s. t. } P(\mathrm{IL}(T) < \Delta_1) \geq q_1 \\ &\qquad P(\max_{t \leq T} \mathrm{IL} < \Delta_2) \geq q_2 \end{aligned} \tag{6}$$

$$
\begin{aligned}
\mathrm{TV}(p_t) = px_{real} + y_{real} = L\left(p(\frac{1}{\sqrt{p}_{tr}} - \frac{1}{\sqrt{p_-}}) + \sqrt{p_{tr}} - \sqrt{p_-}\right) \\
p_{tr} = clip(p, min = p_-, max = p_+) \\
\frac{\partial \mathrm{TV}}{\partial p} = x_{real} + \left(p\frac{\partial x}{\partial y} + 1\right)\frac{\partial y}{\partial p} = x_{real}
\end{aligned} \tag{7}
$$

$$
TV(p) = 2L\sqrt{p}\left(1 - \frac{\sqrt{\frac{p}{p_+}} + \sqrt{\frac{p_-}{p}}}{2}\right)
$$