0.1 Assumptions

1. Transaction costs discarded.
2. Lognormal price distribution.
3. Simplified fees calculations.
4. Slippage is not considered.
5. Continuous spaces, no discretization.

0.2 Supplementary

$$\begin{gathered} xy = L^2 \\ (x_{real} + \frac{L}{\sqrt{p_+}})(y_{real} + L\sqrt{p_-}) = L^2 \end{gathered} \tag{1}$$

$$\begin{gathered} \mathrm{IL} = 1 - \frac{\mathrm{TV}(p_t)}{\mathrm{TV}(p_0)}, \\ \text{where is } \mathrm{TV}(p) = px_{real} + y_{real} \\ \mathrm{IL} = 1 - \frac{2\sqrt{\frac{p}{p_0}}}{\frac{p}{p_0} + 1} \end{gathered} \tag{2}$$

$$\begin{gathered} \dot{p} = \alpha p + \sigma p \eta \\ p(t_0) = 1 \\ \eta \sim \rho[\eta] \end{gathered} \tag{3}$$

$$\begin{gathered} P_{\mathrm{IL}}(u,t) = \sum_{p:p=\mathrm{IL}^{-1}(u)} \frac{\sqrt{p}(p+1)^2}{|p-1|} \cdot P_{p(t)}(p,t) \\ P_{p(t)}(p,t) = \langle \delta(p - p[\eta]) \rangle_\eta = \frac{1}{\sigma p} \cdot \rho_\eta \left(\frac{\log p - (\alpha - \frac{\sigma^2}{2})t}{\sigma} \right) \end{gathered} \tag{4}$$

$$P_{\mathrm{IL}}(u,t) = \sum_{p:p=\mathrm{IL}^{-1}(u)} \frac{\sqrt{p}(p+1)^2}{|p-1|} \frac{1}{\sqrt{2\pi t}} \frac{1}{\sigma p} \exp\left\{ -\frac{(\log p - (\alpha - \frac{\sigma^2}{2})t)^2}{2\sigma^2 t} \right\} \tag{5}$$

$$Fees = \gamma \int dt\, e^{-rt} V(t) \int dp\, P_{p(t)}(p,t) \frac{L(p(t))}{L_{total}(p(t))}$$

$$\begin{aligned} &\max_{p_-,p_+} Fees(T) \\ &\text{s. t. } P(\mathrm{IL}(T) < \Delta_1) \geq q_1 \\ &\qquad P(\max_{t \leq T} \mathrm{IL} < \Delta_2) \geq q_2 \end{aligned} \tag{6}$$

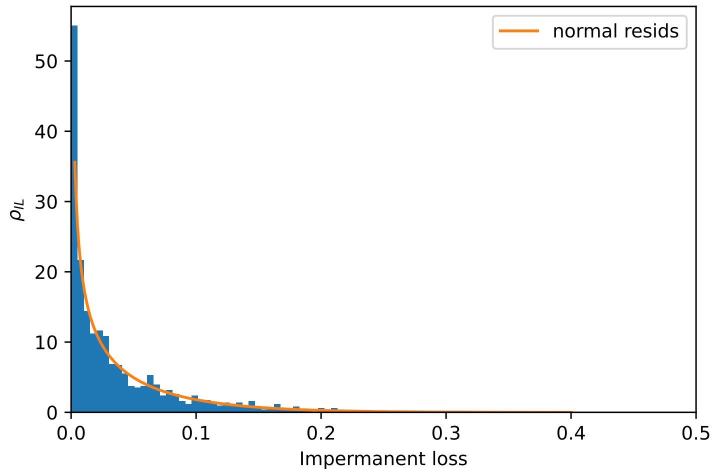

Figure 1: Impermanent loss dist.

$$\begin{aligned} \mathrm{TV}(p) = px_{real} + y_{real} = L\left(p(\frac{1}{\sqrt{p}_{tr}} - \frac{1}{\sqrt{p_+}}) + \sqrt{p_{tr}} - \sqrt{p_-}\right) \\ p_{tr} = clip(p, min = p_-, max = p_+) \\ \frac{\partial \mathrm{TV}}{\partial p} = x_{real} + \left(p\frac{\partial x}{\partial y} + 1\right)\frac{\partial y}{\partial p} = x_{real} \end{aligned} \tag{7}$$

$$\mathrm{TV}(p) = 2L\sqrt{p}\left(1 - \frac{\sqrt{\frac{p}{p_+}} + \sqrt{\frac{p_-}{p}}}{2}\right), \text{when } p_- < p < p_+ \tag{8}$$

Now $\left(1 - \frac{\sqrt{\frac{p}{p_+}} + \sqrt{\frac{p_-}{p}}}{2}\right)^{-1}$ can be interpreted as liquidity efficiency coefficient

0.3 Speculative

How would liquidity be distributed in order to maximize fees?

$$\begin{aligned} J[L_{total}] = \int dp\ \phi(p,t)\frac{1}{L_{total}(p)} \\ G[L_{total}] = \int dp\ L_{total} = \Sigma = const \end{aligned} \tag{9}$$

$$L = \frac{\sqrt{\phi}}{\left\langle \frac{1}{\sqrt{\phi}} \right\rangle}\Sigma \sim \sqrt{\phi} \tag{10}$$

If we were maximizing TV instead we would obtain (which is totally not true since we must infer $\rho(p_-, p_+)$ and keep in mind transaction costs):

$$L \sim \frac{\sqrt{\phi}}{p^{\frac{1}{4}}} \tag{11}$$

If our view differs from global $\rho(p_-, p_+)$, we would place $\delta L(p)$ corresponding to ours view

Can liquidity shaping help with hedging?

$$TV = \int d\mathrm{TV}(L) = \int dL \left[p(\frac{1}{\sqrt{p}} - \frac{1}{\sqrt{p_+(L)}}) + \sqrt{p} - \sqrt{p_-(L)} \right] \tag{12}$$

$$\begin{aligned} \frac{\partial \mathrm{TV}}{\partial p} &= \int dL \left(\frac{1}{\sqrt{p_{tr}(L)}} - \frac{1}{\sqrt{p_+(L)}} \right) \\ \frac{\partial^2 \mathrm{TV}}{\partial p^2} &= -\frac{1}{2} p^{-1.5} \int dL \; \mathbf{1}_{[p_-(L) \leq L \leq p_+(L)]} \end{aligned} \tag{13}$$