

CS 477/677 Analysis of Algorithms

Homework 2

Due February 17, 2014

1. (U & G-required) [30 points] Consider the following algorithm.

```
ALGORITHM Enigma(A[0..n - 1, 0..n - 1])
//Input: A matrix A[0..n - 1, 0..n - 1] of integer numbers
for  $i \leftarrow 0$  to  $n - 2$  do
    for  $j \leftarrow i + 1$  to  $n - 1$  do
        if  $A[i, j] \neq A[j, i]$ 
            return false
return true
```

- a) [5 points] What does this algorithm do?
- b) [25 points] Compute the running time of this algorithm.

2. (U & G-required) [30 points]

Solve the following recurrences using the method of your choice.

- a) [15 points] $T(n) = 4T\left(\frac{n}{3}\right) + n^2$
- b) [15 points] $T(n) = T(n - 1) + 5$

3. (U & G-required) [40 points]

Consider the following recursive algorithm for computing the sum of the first n cubes:

$$S(n) = 1^3 + 2^3 + \dots + n^3$$

```
ALGORITHM S(n)
// Input: A positive integer n
// Output: The sum of the first n cubes
if  $n = 1$ 
    return 1
else
    return  $S(n - 1) + n * n * n$ 
```

- a) [20 points] Write and solve a recurrence relation for the number of multiplications made by this algorithm and solve it.
- b) [20 points] How does this algorithm compare with the straightforward non-recursive algorithm for computing this function?

4. (G-Required) [20 points]

Consider the following recursive algorithm:

ALGORITHM *Min* (**A**, **l**, **r**)

// Input: An array $A[0..n - 1]$ of integer numbers

// The initial call is *Min* (*A*, 0, $n - 1$)

if $l = r$

return $A[l]$

else $temp1 \leftarrow Min(A, l, \lfloor (l + r)/2 \rfloor)$

$temp2 \leftarrow Min(A, \lfloor (l + r)/2 \rfloor + 1, r)$

if $temp1 \leq temp2$

return $temp1$

else

return $temp2$

- a) [10 points] Write the recurrence relation for the above algorithm.
- b) [10 points] Solve the recurrence obtained in part a).

Extra credit

5. [20 points] Consider the following algorithm.

ALGORITHM *Mystery*(**n**)

//Input: A nonnegative integer n

$S \leftarrow 0$

for $i \leftarrow 1$ **to** n **do**

$S \leftarrow S + i * i$

return S

- a) [5 points] What does this algorithm compute?
- b) [15 points] Compute the running time of this algorithm.