Name: Solutions

Please show your work
(10 pts.)

Math 330 – Quiz 5
Tuesday, 9 Oct 2012

1. Use elimination to find the reduced row echelon form and the rank of the following matrix:

$$A = \begin{bmatrix} 1 & 2 & 1 & 0 \\ 2 & 3 & 5 & 1 \\ 1 & 0 & 7 & 2 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 2 & 1 & 0 \\ 2 & 3 & 5 & 1 \\ 1 & 0 & 7 & 2 \end{bmatrix} \xrightarrow[R_3 - R_1]{R_2 - 2R_1} \begin{bmatrix} 1 & 2 & 1 & 0 \\ 0 & -1 & 3 & 1 \\ 0 & -2 & 6 & 2 \end{bmatrix} \xrightarrow{R_3 - 2R_2} \begin{bmatrix} 1 & 2 & 1 & 0 \\ 0 & -1 & 3 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix} \xrightarrow{R_1 + 2R_2}$$

$$\begin{bmatrix} 1 & 0 & 7 & 2 \\ 0 & -1 & 3 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix} \xrightarrow{-1 \times R_2} \begin{bmatrix} 1 & 0 & 7 & 2 \\ 0 & 1 & -3 & -1 \\ 0 & 0 & 0 & 0 \end{bmatrix} \leftarrow \text{rref}(A)$$

Two pivots so rank(A) = 2.

2. Let A and $\mathbf{b}$ be given below. First find a particular solution $\mathbf{x}_p$ satisfying $A\mathbf{x}_p = \mathbf{b}$ by setting all free variables equal to zero. Next find all solutions to the equation $A\mathbf{x}_n = \mathbf{0}$ (take all linear combinations of the special solutions). Finally find the complete solution to $A\mathbf{x} = \mathbf{b}$ by combining these.

$$A = \begin{bmatrix} 1 & -2 & 0 & 4 \\ 0 & 0 & 1 & -3 \\ 0 & 0 & 0 & 0 \end{bmatrix}, \qquad \mathbf{b} = \begin{bmatrix} 3 \\ 5 \\ 0 \end{bmatrix}.$$

$\vec{x}_p$

$x_1 - 2x_2 + 4x_4 = 3$
$x_3 - 3x_4 = 5$

setting $x_2 = x_4 = 0$ we get
$x_1 = 3$, $x_3 = 5$
so $\vec{x}_p = (3, 0, 5, 0)$

Special solutions

$x_1 - 2x_2 + 4x_4 = 0$
$x_3 - 3x_4 = 0$

i) $x_2 = 1$, $x_4 = 0$ gives
$x_1 - 2 = 0$, $x_3 = 0$ so
$x_1 = 2$, $x_3 = 3$ and $\vec{s}_1 = (2, 1, 0, 0)$

ii) $x_2 = 0$, $x_4 = 1$ gives
$x_1 + 4 = 0$, $x_3 - 3 = 0$ so
$x_1 = -4$, $x_3 = 3$ and $\vec{s}_2 = (-4, 0, 3, 1)$

$$\vec{x}_n = x_2\vec{s}_1 + x_4\vec{s}_2 = x_2\begin{bmatrix} 2 \\ 1 \\ 0 \\ 0 \end{bmatrix} + x_4\begin{bmatrix} -4 \\ 0 \\ 3 \\ 1 \end{bmatrix} = \begin{bmatrix} 2x_2 - 4x_4 \\ x_2 \\ 3x_4 \\ x_4 \end{bmatrix}$$

Complete solution to $A\vec{x} = \vec{b}$

$$\vec{x} = \vec{x}_p + x_2\vec{s}_1 + x_4\vec{s}_2 = \begin{bmatrix} 3 \\ 0 \\ 5 \\ 0 \end{bmatrix} + x_2\begin{bmatrix} 2 \\ 1 \\ 0 \\ 0 \end{bmatrix} + x_4\begin{bmatrix} -4 \\ 0 \\ 3 \\ 1 \end{bmatrix} = \begin{bmatrix} 3 + 2x_2 - 4x_4 \\ x_2 \\ 5 + 3x_4 \\ x_4 \end{bmatrix}$$