THE SCALAR PRODUCT

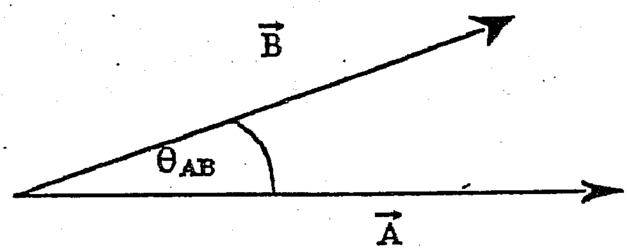
__Object:__ To study the scalar product of two vectors, and the concept of work.

__Apparatus:__ Rod mounted vertically on the table, spring balance, additional spring and pointer, protractor, meter stick, wooden block, kg weights, masking tape, and copier paper.

__Theory:__ We have seen that vector quantities are quantities which are characterized by **both** magnitude and direction. In three-dimensional space three numbers are required to specify a vector: in Cartesian coordinates these numbers would be the components along the coordinate directions. (A magnitude and two angles, say azimuth and the dihedral angle would do as well.)

We should like to study one type of product involving vectors: the so-called dot product, or **scalar product**. Though the multiplier and multiplicand are both vectors, the product is, by definition, a scalar. The operation is defined as follows:

$$C = \vec{A} \cdot \vec{B} = AB \cos(\theta_{AB}) \qquad (1)$$



In terms of Cartesian coordinates, if the force is given by ($\hat{e}$'s are unit vectors along axes)

$$\vec{F} = F_x\hat{e}_x + F_y\hat{e}_y + F_z\hat{e}_z \qquad (2)$$

and if the displacement is given by

$$\Delta\vec{s} = \Delta x\hat{e}_x + \Delta y\hat{e}_y + \Delta z\hat{e}_z \qquad (3)$$

then the work done, ΔW, is

$$\Delta W = F_x\Delta x + F_y\Delta y + F_z\Delta z \qquad (4)$$

[N.B. Please satisfy yourself that Eq. (4) is correct by carrying out the various steps in taking the dot product of the right-hand sides of the two previous equations. Remember

$$\hat{e}_x \; \hat{e}_x = \hat{e}_y \cdot \hat{e}_y = \hat{e}_z \; \hat{e}_z = 1 \qquad (5)$$

and

$$\hat{e}_x \; \hat{e}_y = \hat{e}_x \cdot \hat{e}_z = \hat{e}_y \; \hat{e}_z = 0. \qquad (6)$$

Since work involves a force acting along a path, the idea of a small increment of work

$$dW = \vec{F} \cdot \vec{ds} \tag{7}$$

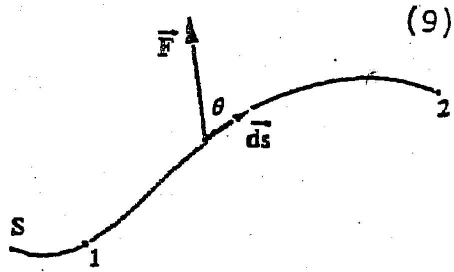
presents itself. For forces and displacements confined to a plane, Eq. (7) becomes

$$dW = F_x dx + F_y dy \tag{8}$$

In particular, if one wished to find the work necessary to go from point 1 to point 2 along a certain path S, he could find the integral

$$W_{12} = \int_1^2 \vec{F} \cdot \vec{ds} \tag{9}$$

along the path S. Here the force $\vec{F}$ is presumed to be known at each point along the path; the component of the force along the direction of path element $\vec{ds}$ is obtained, and the product of force component and path element evaluated.



The sum of these along the path constitutes the evaluation of the integral along the path from point 1 to point 2. Such an integral is known as a **line integral** (since it is evaluated along a line), and it will prove to be a useful tool in physics.

Experimental Work

1. First Line Integral
   (a) With the aid of Scotch tape fasten a piece of paper securely to the laboratory table conveniently near a vertical rod (see photograph).

(b) With protractor or compass sketch a suitable closed semicircular path on the paper.

(c) The available spring balances by themselves do not allow sufficient play to enable the student to trace out the circle with a pointer attached to the balance. Therefore, use the auxiliary spring. Slip the loop of the balance over the vertical rod attached to the table, attach the auxiliary spring and pointer to the balance, and check to see that the pointer will trace out the curve which you have drawn on the paper. Convince yourself that, in spite of the presence of the auxiliary spring the balance still gives the tension in the system.

(d) Divide the semi-circle into suitable segments which can be approximated by straight lines (~ 10). At each end of the (many) segments of the semi-circle, place the pointer and note down near each point the reading of the balance.

(e) Without removing the paper, unscrew the vertical rod from its place in the table. With the aid of a ruler sketch a line from each point on the paper to the center of the hole where the rod was located.

(f) Use a protractor to obtain the angle between line segments.

2. Second line integral

(a) A chalk circle (or square) may be available on the laboratory floor. If not, sketch a suitable closed loop.

(b) Weight down a wooden block with the large weights provided.

(c) Using a spring balance, drag the block around the loop, taking the reading of the balance at closely-spaced points all the way around the loop.

<u>Report</u>

1. Consider one of the force-segment pairs in the first integral problem :
   (a) Calculate the work, dW, for this segment directly.
   (b) Set up an arbitrary rectangular coordinate system.
      i. Obtain the components of the force and of the segment in this coordinate system.

      ii. Using the expression (9), calculate the work done.

      iii. Compare the values found in the above two calculations (parts a and b.).

(c) For the spring balance-rod arrangement, determine the value of the line integral around the loop on the paper. $W = \int_s \vec{F} \bullet \vec{ds}$. If all has gone well, the value of W should be zero.

(d) Obtain the value of the second line integral, namely the work done in dragging the block around the loop on the floor.

(e) Think about the two line integrals carefully, and explain why the values should be so very different.