Name: Solutions

Please show your work
(10 pts.)

Math 330 – Quiz 4
Tuesday, 25 Sept 2012

1. For which $\mathbf{b} = (b_1, b_2, b_3)$ does the equation $A\mathbf{x} = \mathbf{b}$ have a solution where

$$A = \begin{bmatrix} 1 & 1 & 3 \\ 2 & 4 & 5 \\ 3 & 7 & 7 \end{bmatrix}.$$

$$[A\ \vec{b}] = \begin{bmatrix} 1 & 1 & 3 & b_1 \\ 2 & 4 & 5 & b_2 \\ 3 & 7 & 7 & b_3 \end{bmatrix} \xrightarrow[R_3-3R_1]{R_2-2R_1} \begin{bmatrix} 1 & 1 & 3 & b_1 \\ 0 & 2 & -1 & b_2-2b_1 \\ 0 & 4 & -2 & b_3-3b_1 \end{bmatrix}$$

$$\xrightarrow[R_3-2R_2]{} \begin{bmatrix} 1 & 1 & 3 & b_1 \\ 0 & 2 & -1 & b_2-2b_1 \\ 0 & 0 & 0 & b_3-2b_2+b_1 \end{bmatrix}$$

So $A\vec{x} = \vec{b}$ has a solution if

$$b_3 - 2b_2 + b_1 = 0$$

2. Let A be given below. For each free variable find a special solution to $A\mathbf{x} = \mathbf{0}$. Then find the complete solution.

$$A = \begin{bmatrix} 1 & 0 & 4 & -3 \\ 0 & 1 & -5 & 2 \end{bmatrix}.$$

$A\vec{x} = 0$ iff $\quad x_1 + 4x_3 - 3x_4 = 0$
$\quad x_2 - 5x_3 + 2x_4 = 0$

Special solutions: free variables x_3, x_4

a) $x_3 = 1,\ x_4 = 0,\quad x_1 + 4 = 0 \quad x_1 = -4$
$\quad x_2 - 5 = 0 \quad x_2 = 5$

$\vec{s}_1 = (-4, 5, 1, 0)$

b) $x_4 = 1,\ x_3 = 0,\quad x_1 - 3 = 0 \quad x_1 = 3$
$\quad x_2 + 2 = 0 \quad x_2 = -2$

$\vec{s}_2 = (3, -2, 0, 1)$

So $\vec{s}_1, \vec{s}_2$ are the two special solutions

Complete solution:

$$\vec{x} = x_3\vec{s}_1 + x_4\vec{s}_4 = x_3\begin{bmatrix} -4 \\ 5 \\ 1 \\ 0 \end{bmatrix} + x_4\begin{bmatrix} 3 \\ -2 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} -4x_3 + 3x_4 \\ 5x_3 - 2x_4 \\ x_3 \\ x_4 \end{bmatrix}$$