Name: Solutions

Please show your work

Math 330: Quiz 2 (10 pts)
Tuesday, 11 Sept 2012

1. Use elimination and back substitution to solve the following system. Indicate which elimination steps you use.

$$x + 2y - 4z = 3$$
$$2x + 3y - 2z = 7$$
$$4x + 5y + 4z = 17$$

$$\begin{aligned} x+2y-4z&=3\\ 2x+3y-2z&=7\\ 4x+5y+4z&=17 \end{aligned} \xrightarrow[\substack{(2)\to(2)-2(1)\\(3)\to(3)-4(1)}]{} \begin{aligned} x+2y-4z&=3\\ -y+6z&=1\\ -3y+20z&=5 \end{aligned}$$

$$\xrightarrow[(3)\to(3)-3(2).]{} \begin{aligned} x+2y-4z&=3\\ -y+6z&=1\\ 2z&=2 \end{aligned}$$

Back substitution: (3) $2z=2 \Rightarrow z=1$

(2) $-y+6\cdot 1=1 \Rightarrow y=5$

(1) $x+2\cdot 5-4\cdot 1=3$

$\Rightarrow x=-3$

Solution $x=-3,\ y=5,\ z=1$

2. Apply elimination to the augmented matrix $[A\ \mathbf{b}]$ associated to the following matrix equation (indicate the operations you use) and then solve it using back substitution; find the corresponding elimination matrices, E_{21}, E_{31}, E_{32}.

$$\begin{bmatrix} 1 & 1 & 0 \\ 3 & 4 & 2 \\ -2 & -1 & 4 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 2 \\ 9 \\ 3 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 1 & 0 & 2 \\ 3 & 4 & 2 & 9 \\ -2 & -1 & 4 & 3 \end{bmatrix} \xrightarrow[\substack{R_2-3R_1\\R_3+2R_1}]{} \begin{bmatrix} 1 & 1 & 0 & 2 \\ 0 & 1 & 2 & 3 \\ 0 & 1 & 4 & 7 \end{bmatrix} \xrightarrow[R_3-R_2]{} \begin{bmatrix} 1 & 1 & 0 & 2 \\ 0 & 1 & 2 & 3 \\ 0 & 0 & 2 & 4 \end{bmatrix}$$

Back sub:
$$\begin{aligned} x+y\quad&=2\\ y+2z&=3\\ 2z&=4 \end{aligned}$$

(3) $2z=4 \Rightarrow z=2$

(2) $y+2\cdot 2=3 \Rightarrow y=-1$

(1) $x+(-1)=2 \Rightarrow x=3$

Solution $\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 3 \\ -1 \\ 2 \end{bmatrix}$

$E_{21} = \begin{bmatrix} 1 & 0 & 0 \\ -3 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ $E_{31} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 2 & 0 & 1 \end{bmatrix}$ $E_{32} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & -1 & 1 \end{bmatrix}$