Name: Solutions

Please show your work
(10 pts.)

Math 330 – Quiz 7
Tuesday, 23 Oct 2012

1. Let V be the subspace of $\mathbb{R}^4$ spanned by the vectors $\mathbf{v} = (1, 0, 2, 3)$ and $\mathbf{w} = (0, 1, -3, 4)$. Find a basis for $V^\perp$. (*Hint:* Find a matrix A with row space V.)

Let $A = \begin{bmatrix} 1 & 0 & 2 & 3 \\ 0 & 1 & -3 & 4 \end{bmatrix}$. then $V = C(A^T)$.

So $V^\perp = C(A^T)^\perp = N(A)$.

Basis for $V^\perp$ = special solutions to $A\vec{x} = \vec{0}$

$A\vec{x} = 0$

$$\begin{aligned} x_1 \quad + 2x_3 + 3x_4 &= 0 \\ x_2 - 3x_3 + 4x_4 &= 0 \end{aligned}$$

If $x_3 = 1, x_4 = 0$, $x_1 = -2$, $x_2 = 3$, so $\vec{s}_1 = (-2, 3, 1, 0)$

If $x_3 = 0, x_4 = 1$, $x_1 = -3$, $x_2 = -4$, so $\vec{s}_2 = (-3, -4, 0, 1)$

$V^\perp$ basis $= \{(-2, 3, 1, 0), (-3, -4, 0, 1)\}$

2. Find the projection matrix P onto the line through $\mathbf{a} = (2, -1, 1)$ (that is, the subspace spanned by $\mathbf{a}$). Use P to find the projection $\mathbf{p}$ of the vector $\mathbf{b} = (1, 2, 3)$ onto this subspace.

$\|\vec{a}\|^2 = 2^2 + (-1)^2 + 1^2 = 6$

$$P = \frac{1}{\|\vec{a}\|^2} \vec{a}\,\vec{a}^T = \frac{1}{6} \begin{bmatrix} 2 \\ -1 \\ 1 \end{bmatrix} \begin{bmatrix} 2 & -1 & 1 \end{bmatrix}$$

$$= \frac{1}{6} \begin{bmatrix} 4 & -2 & 2 \\ -2 & 1 & -1 \\ 2 & -1 & 1 \end{bmatrix}$$

$$\vec{p} = P\vec{b} = \frac{1}{6} \begin{bmatrix} 4 & -2 & 2 \\ -2 & 1 & -1 \\ 2 & -1 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} = \frac{1}{6} \begin{bmatrix} 6 \\ -3 \\ 3 \end{bmatrix}$$

$$= \frac{1}{2} \begin{bmatrix} 2 \\ -1 \\ 1 \end{bmatrix}$$