HW 1.3

5| Write a system of equations equivalent to the given vector equation:

$$x_1 \begin{bmatrix} 3 \\ -2 \\ 8 \end{bmatrix} + x_2 \begin{bmatrix} 5 \\ 0 \\ 9 \end{bmatrix} = \begin{bmatrix} 2 \\ -3 \\ 8 \end{bmatrix} \longrightarrow \boxed{\begin{cases} 3x_1 + 5x_2 = 2 \\ -2x_1 = -3 \\ 8x_1 + 9x_2 = 8 \end{cases}}$$

10| Write a vector equation that is equivalent to the given system

$$\begin{cases} 3x_1 - 2x_2 + 4x_3 = 3 \\ -2x_1 - 7x_2 + 5x_3 = 1 \\ 5x_1 + 4x_2 - 3x_3 = 2 \end{cases} \longrightarrow \boxed{x_1 \begin{bmatrix} 3 \\ -2 \\ 5 \end{bmatrix} + x_2 \begin{bmatrix} -2 \\ -7 \\ 4 \end{bmatrix} + x_3 \begin{bmatrix} 4 \\ 5 \\ -3 \end{bmatrix} = \begin{bmatrix} 3 \\ 1 \\ 2 \end{bmatrix}}$$

11| Determine if $\vec{b}$ is a linear combination of $\vec{a}_1$, $\vec{a}_2$, and $\vec{a}_3$

$$\vec{a}_1 = \begin{bmatrix} 1 \\ -2 \\ 0 \end{bmatrix}, \quad \vec{a}_2 = \begin{bmatrix} 0 \\ 1 \\ 2 \end{bmatrix}, \quad \vec{a}_3 = \begin{bmatrix} 5 \\ -6 \\ 8 \end{bmatrix}, \quad \vec{b} = \begin{bmatrix} 2 \\ -1 \\ 6 \end{bmatrix}$$

$$\left[\begin{array}{ccc|c} 1 & 0 & 5 & 2 \\ -2 & 1 & -6 & -1 \\ 0 & 2 & 8 & 6 \end{array}\right] \xrightarrow{R2=R2+2R1} \begin{bmatrix} 1 & 0 & 5 & 2 \\ 0 & 1 & 4 & 3 \\ 0 & 2 & 8 & 6 \end{bmatrix} \xrightarrow{R3=R3-2R2} \begin{bmatrix} 1 & 0 & 5 & 2 \\ 0 & 1 & 4 & 3 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

This augmented matrix shows that the solution to the system is $\begin{cases} x_1 + 5x_3 = 2 \\ x_2 + 4x_3 = 3 \\ x_3 \text{ is free} \end{cases}$

indicating that there are infinitely many solutions, meaning that

$\boxed{\vec{b} \text{ is a linear combination of } \vec{a}_1, \vec{a}_2, \text{ and } \vec{a}_3.}$

14| See #11

(same problem)

16| Let $\vec{v}_1 = \begin{bmatrix} 1 \\ 0 \\ -2 \end{bmatrix}$, $\vec{v}_2 = \begin{bmatrix} -2 \\ 1 \\ 7 \end{bmatrix}$, and $\vec{y} = \begin{bmatrix} h \\ -3 \\ 5 \end{bmatrix}$. For what value(s)

of h is $\vec{y}$ in the plane generated by v_1 and v_2?

$$\begin{bmatrix} 1 & -2 & h \\ 0 & 1 & -3 \\ -2 & 7 & 5 \end{bmatrix} \xrightarrow{R3 = R3 + 2R1} \begin{bmatrix} 1 & -2 & h \\ 0 & 1 & -3 \\ 0 & 3 & 2h+5 \end{bmatrix} \xrightarrow{R3 = R3 - 3R2} \begin{bmatrix} 1 & -2 & h \\ 0 & 1 & -3 \\ 0 & 0 & 2h+14 \end{bmatrix}$$

$$\xrightarrow{R1 = R1 + 2R2} \begin{bmatrix} 1 & 0 & h-6 \\ 0 & 1 & -3 \\ 0 & 0 & 2h+14 \end{bmatrix}$$

$\rightarrow$ for $\vec{y}$ to be in the plane generated by $\vec{v}_1$ and $\vec{v}_2$, the system must be consistent, therefore R3 must remain true

$0x_1 + 0x_2 = 2h + 14 \rightarrow 0 = 2h + 14 \rightarrow 2h = -14 \rightarrow \boxed{h = -7}$ answer

* This answer can be verified by substituting h back into R1 and evaluating the system.

24| Mark each statement True or False, use justification.

b) Any list of 5 real numbers is a vector in $\mathbb{R}^5$.

<u>True</u>, a vector is an "ordered list" of numbers (ordered by the dimension they correspond to), the $\mathbb{R}$ refers to the fact that the list contains real numbers, and the exponent 5 indicates that the vector will have 5 entries. This statement is more or less the definition of $\mathbb{R}^5$.

d) The vector $\vec{v}$ results when a vector $\vec{u} - \vec{v}$ is added to the vector $\vec{v}$.

<u>False</u>, because vectors can be manipulated using algebra, the above statement results as:

$(\vec{u} - \vec{v}) + \vec{v} = \vec{v}$ or $\vec{u} - \vec{v} + \vec{v} = \vec{v}$, which is false; really: $\vec{u} - \vec{v} + \vec{v} = \vec{u}$.

Therefore, the statement is false, provided that $\vec{u}$ is not equal to $\vec{v}$