Name: Solutions

Please show your work
(10 pts)

Math 330 – Quiz 6
Tuesday, 16 Oct 2012

1. (4 pts) Determine whether the following vectors are linearly independent. Why? Do they form a basis for $\mathbb{R}^3$?

$$(1,2,3), \quad (1,3,5), \quad (0,1,4)$$

$$A = \begin{bmatrix} 1 & 1 & 0 \\ 2 & 3 & 1 \\ 3 & 5 & 4 \end{bmatrix} \xrightarrow[R_3-3R_1]{R_2-2R_1} \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 0 & 2 & 4 \end{bmatrix} \xrightarrow[R_3-2R_2]{} \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & 2 \end{bmatrix}$$

three pivots!

The given vectors are the columns of A.
Since there is a pivot in each column, the columns are linearly independent.
So the given vectors are linearly independent and form a basis for $\mathbb{R}^3$.

2. (6 pts) Find a basis for each of the four fundamental subspaces of the following matrix

$$A = \begin{bmatrix} 1 & 0 & 2 & 3 \\ 2 & 1 & 3 & 7 \\ 0 & 3 & -3 & 3 \end{bmatrix}$$

$$[A \mid \vec{b}] = \left[\begin{array}{cccc|c} 1 & 0 & 2 & 3 & b_1 \\ 2 & 1 & 3 & 7 & b_2 \\ 0 & 3 & -3 & 3 & b_3 \end{array}\right] \xrightarrow{R_2-2R_1} \left[\begin{array}{cccc|c} 1 & 0 & 2 & 3 & b_1 \\ 0 & 1 & -1 & 1 & b_2-2b_1 \\ 0 & 3 & -3 & 3 & b_3 \end{array}\right] \xrightarrow{R_3-3R_2} \left[\begin{array}{cccc|c} 1 & 0 & 2 & 3 & b_1 \\ 0 & 1 & -1 & 1 & b_2-2b_1 \\ 0 & 0 & 0 & 0 & b_3-3b_2+6b_1 \end{array}\right]$$

$C(A)$ basis $= (1,2,0), (0,1,3)$ pivot cols of A

$N(A)$ basis $= (-2,1,1,0), (-3,-1,0,1)$ special solutions to $A\vec{x} = \vec{0}$

$C(A^T)$ basis $= (1,0,2,3), (0,1,-1,1)$ pivot rows of U

$N(A^T)$ basis $= (6,-3,1)$ from equations in the reduced aug. matrix.