$\bar{x} = \frac{\Sigma x}{n}$

median → if n is odd → the value in position $\frac{n+1}{2}$
→ if n is even → the mean of the values in positions $\frac{n}{2}$ and $\frac{n}{2}+1$

$s = \sqrt{\frac{\Sigma[(x-\bar{x})^2]}{n-1}}$

percentile = the value in the $(\frac{p}{100}) \cdot n$ position

HW 1.2

10) 1000 babies are assigned an Apgar score using integers 0 to 10. The score distribution of the sample is as follows:

Score	0	1	2	3	4	5	6	7	8	9	10
# of Babies	1	3	2	4	25	35	198	367	216	131	18

a) Find the sample mean Apgar score.

$$\bar{x} = \frac{\sum x}{n} = \frac{1\cdot 0 + 3\cdot 1 + 2\cdot 2 + 4\cdot 3 + 25\cdot 4 + 35\cdot 5 + 198\cdot 6 + 367\cdot 7 + 216\cdot 8 + 131\cdot 9 + 10\cdot 18}{1000}$$

$$= \frac{7138}{1000} \boxed{= 7.138}$$

b) Find the sample standard deviation of the Apgar scores.

$$s = \sqrt{\frac{\Sigma[x-\bar{x}]^2}{n-1}} = \sqrt{\frac{[1(0-7.138)^2 + 3(1-7.138)^2 + 2(2-7.138)^2 + 4(3-7.138)^2 + 25(5-7.138)^2 + 198(6-7.138)^2 + 367(7-7.138)^2 + 216(8-7.138)^2 + 131(9-7.138)^2 + 18(10-7.138)^2]}{(1000)-1}}$$

$$= \sqrt{\frac{50.95 + 113.025 + 52.798 + 68.492 + 246.176 + 159.986 + 256.418 + 6.989 + 160.498 + 454.183 + 147.439}{999}}$$

$$= \sqrt{\frac{1716.956}{999}} \boxed{= 1.311}$$

c) Find the sample median of the Apgar scores.

median → n is even → = mean of the values in positions $\frac{n}{2}$ and $\frac{n}{2}+1$

median = values in positions $\frac{(1000)}{2}$ and $\frac{(1000)}{2}+1$ = positions 500 and 501

Position	1	(1+3)	(4+2)	(6+25)	(31+35)=66	(66+198)=264	(264+367)=631	...
Value	0	1	2	3	4	5	6	

median $= \frac{6+6}{2} \boxed{=6}$

d) What is the first quartile of the scores?

percentile = the value in the $(\frac{p}{100})\cdot n$ position → $(\frac{(25)}{100})\cdot(1000) = 250$

→ *use above table to find reference position → 25th percentile $\boxed{=5}$

e) What proportion of scores is greater than the mean?

$P = \frac{\#\text{ of } x > \bar{x}}{n}$ → $\bar{x} = 7.138$ → $x \geq 8$ are $> \bar{x}$ → # of $x > \bar{x} = 216 + 131 + 18 = 365$

$P = \frac{365}{1000} \boxed{= .365}$

f) What proportion of the scores is more than one standard deviation from the mean?

$P = \frac{\#\text{ of } x > \text{or} < \bar{x} \pm s}{n}$ → $\bar{x} \pm s = 7.138 \pm 1.311 = 8.449, 5.827$ → all values ≥ 9 and ≤ 5

$= (131+18) + (1+3+2+4+25+35) = 219$ → $P = \frac{219}{1000} \boxed{= .219}$

$p\%$ trimmed mean $= \dfrac{\Sigma x \text{ excluding all } x \text{ in positions } > \frac{100-p}{100}\cdot n \text{ and } < \frac{p}{100}\cdot n}{n_{trimmed}}$

g) What proportion of the scores is within one standard deviation of the mean?

$P = \dfrac{\# \text{ of } x > \text{ or } < \bar{x} \pm s}{n} \rightarrow \bar{x} \pm s = 7.138 \pm 1.311 = 8.449,\ 5.827 \rightarrow$ all values ≥ 6 and ≤ 8

$\rightarrow = 198 + 367 + 216 = 781 \rightarrow P = \frac{(781)}{1000} \boxed{= .781}$

11 | In a class of 30 students, the mean score was 72. In another class of 40, the mean score was 79. What was the mean for both classes combined?

$\bar{X} = \frac{\Sigma x}{n} \quad \bar{X}_c = \frac{\Sigma x_1 + \Sigma x_2}{n_1 + n_2} = \frac{30 \cdot 72 + 40 \cdot 79}{30 + 40} = \frac{2160 + 3160}{70} = \frac{5320}{70} \boxed{= 76}$

12 | 16 students measure the circumference (in cm) of a ball using four methods:

A) by eye: 18.0, 18.0, 18.0, 20.0, 22.0, 22.0, 22.5, 23.0, 24.0, 24.0, 25.0, 25.0, 25.0, 25.0, 26.0, 26.4

B) measure diameter → calculate circumference: 18.8, 18.9, 18.9, 19.6, 20.1, 20.4, 20.4, 20.4, 20.4, 20.5, 21.2, 22.0, 22.0, 22.0, 22.0, 23.6

C) wrap ball with string → measure string length: 20.2, 20.5, 20.5, 20.7, 20.8, 20.9, 21.0, 21.0, 21.0, 21.0, 21.0, 21.5, 21.5, 21.5, 21.5, 21.6

D) roll ball along ruler: 20.0, 20.0, 20.0, 20.0, 20.2, 20.5, 20.5, 20.7, 20.7, 20.7, 21.0, 21.1, 21.5, 21.6, 22.1, 22.3

a) Find the mean of each method. *Computations were done using technology

$\bar{X} = \frac{\Sigma x}{n} \quad \bar{X}_A = \frac{363.9}{16} \boxed{= 22.744\text{cm}} \quad \bar{X}_B = \frac{331.2}{16} \boxed{= 20.7\text{cm}} \quad \bar{X}_C = \frac{336.2}{16} \boxed{= 21.013\text{cm}} \quad \bar{X}_D = \frac{332.7}{16} \boxed{= 20.794\text{ cm}}$

b) Find the median values.

median $= \rightarrow n$ is even $\rightarrow$ = mean of values in positions $\frac{n}{2}$ and $\frac{n}{2}+1 \rightarrow$ positions $\frac{(16)}{2} + \frac{(16)}{2}+1$ = positions 8 and 9

$\text{median}_A = \frac{23.0\text{cm} + 24.0\text{cm}}{2} \boxed{= 23.5\text{ cm}} \quad \text{median}_B = \frac{20.4\text{cm} + 20.4\text{cm}}{2} \boxed{= 20.4\text{cm}}$

$\text{median}_C = \frac{21.0\text{cm} + 21.0\text{cm}}{2} \boxed{= 21.0\text{cm}} \quad \text{median}_D = \frac{20.7\text{cm} + 20.7\text{cm}}{2} \boxed{= 20.7\text{cm}}$

c) Compute the 20% trimmed mean for each measurement.

"Trimmed" values = x in positions $> \frac{100-p}{100}\cdot n$ and $< \frac{p}{100}\cdot n \Rightarrow \frac{100-(20)}{100}\cdot(16)$ and $< \frac{(20)}{100}\cdot(16) = > 12.8$ and < 3.2

→ only use values in positions 4 through 12. *Computations done using technology.

25% Trimmed mean $= \frac{\Sigma x_4 \dots x_{12}}{n_{trimmed}} \quad 25\%TM_A = \frac{207.5}{9} \boxed{= 20.056\text{cm}} \quad 25\%\ TM_B = \frac{185}{9} \boxed{= 20.556\text{cm}}$

$25\%TM_C = \frac{188.9}{9} \boxed{= 20.989\text{cm}} \quad 25\%\ TM_D = \frac{185.4}{9} \boxed{= 20.6\text{ cm}}$

d) Compute the first and third quartiles for each method.

"p" quartile = the value in position $\frac{p}{100}\cdot n$

1st quartile (25th percentile) = the value in position $\frac{(25)}{100}\cdot(16)$ = value in position 4

1st quartile A $\boxed{= 20.0\text{cm}}$ 1st quartile B $\boxed{= 19.6\text{ cm}}$ 1st quartile C $\boxed{= 20.7\text{cm}}$ 1st quartile D $\boxed{= 20.0\text{ cm}}$

3rd quartile (75th percentile) = the value in position $\frac{(75)}{100}\cdot(16)$ = the value in position 12

3rd quartile A $\boxed{= 25.0\text{cm}}$ 3rd quartile B $\boxed{= 22.0\text{cm}}$ 3rd quartile C $\boxed{= 21.5\text{cm}}$ 3rd quartile D $\boxed{= 21.1\text{ cm}}$

$s = \sqrt{\frac{\Sigma[(x-\bar{x})^2]}{n-1}}$

e) Compute the standard deviation of the measurements of each method.

$s = \sqrt{\frac{\Sigma[(x-\bar{x})^2]}{n-1}}$ *Computed using technology

$s_A = \sqrt{\frac{123.7594}{16-1}} \boxed{= 2.872 \text{ cm}}$ $s_B = \sqrt{\frac{27.48}{16-1}} \boxed{= 1.354 \text{ cm}}$ $s_C = \sqrt{\frac{2.6375}{16-1}} \boxed{= 0.420 \text{ cm}}$ $s_D = \sqrt{\frac{7.769375}{16-1}} \boxed{= 0.720 \text{ cm}}$

f) For which method is the standard deviation the largest? Why is this to be expected?

Method A has the largest standard deviation. This is as expected because it is the least precise method. "Eyeballing" the circumference is not a careful or efficable method.

g) Other things being equal, is it better for a method of measurement to have a larger or smaller standard deviation? Or does it matter? Explain.

It is better for a method to have a smaller standard deviation because then the method is more likely to produce a value closer to the "correct" value, and it indicates that the method probably has good reproducibility.

14| A list of 10 numbers has a mean of 20, a median of 18, and a standard deviation of 5. The largest number on the list is 39.27. By accident, this number is changed to 392.7.

a) What is the value of the mean after the change?

pre change: $\bar{x} = \frac{\sum_1^9 x + 39.27}{10} \rightarrow$ *Solve for $\Sigma x \rightarrow \Sigma x = (\bar{x} \cdot n) - 39.27$

$= ((20)\cdot(10)) - 39.27 = 160.73 \rightarrow$ *Re-compute $\bar{x}$ with 392.7 $\rightarrow \bar{x} = \frac{\Sigma x + 392.7}{n} = \frac{160.73 + 392.7}{10} \boxed{= 55.343}$

b) What is the median after the change?

List of $x = x_1 \; x_2 \; x_3 \; x_4 \; x_5 \; x_6 \; x_7 \; x_8 \; x_9 \; 392.7$

$\rightarrow$ The values in the central positions has not changed, ∴ the median remains $\boxed{= 18}$

c) What is the standard deviation after the change?

pre-change $s = \sqrt{\frac{\Sigma[(x-\bar{x})^2] + (39.27-\bar{x})^2}{9}} \rightarrow$ *Solve for $\Sigma[(x-\bar{x})^2]$ $\rightarrow \Sigma[(x-\bar{x})^2] = (n-1)s^2 - (39.27-\bar{x})^2$

$= ((10)-1)(5)^2 - (39.27-(20))^2 = 225 - 371.3329 = -127.0629$

$\rightarrow$ *Recompute s using 392.7 $\rightarrow s = \sqrt{\frac{\Sigma[(x-\bar{x})^2] + (392.7-\bar{x})^2}{n-1}} = \sqrt{\frac{-127.0629 + (392.7-(20))^2}{(10)-1}} \boxed{= 124.176}$

$S = \{$ All possible outcomes $\}$

For Mutually Exclusive Events

$P(A \cup B) = P(A) + P(B)$

$P(A \cap B) = 0$

$P(A^c) = 1 - P(A)$

HW 2.1

2| An octahedral die has the number 1 painted on two faces, 2 on three faces, 3 on two, and 4 on the last. When rolled, assume each face is equally likely to come up.

a) Find the sample space for rolling the die once.

$S = \boxed{\{1, 1, 2, 2, 2, 3, 3, 4\}}$

b) Find P(even number)

$P(\text{even number}) = P(2 \cup 4) = P(2) + P(4) = \frac{3}{8} + \frac{1}{8} = \frac{4}{8} \boxed{= \frac{1}{2}}$

c) If the die were loaded so that the 4 face were twice as likely to come up as the other 7 faces, would the sample space change? Explain.

Yes, loading one face is like adding another face similar to the loaded one, changing the sample space to that of a 9-sided die with $S = \{1, 1, 2, 2, 2, 3, 3, 4, 4\}$

d) If the die is loaded like the above situation, does this change the value of P(even)? Explain.

Yes. The loading makes P(4) twice as likely $\rightarrow 2 \cdot (\frac{1}{8}) = \frac{2}{8}$. If we re-compute P(even), the result is: $P(\text{even}) = P(2 \cup 4) = P(2) + P(4) = \frac{3}{8} + \frac{2}{8} \boxed{= \frac{5}{8}}$

3| A section of an exam includes 4 True-False questions. A completed exam is selected at random and the 4 answers are recorded.

a) List the 16 outcomes in the sample space.

S = { TTTT TTTF TTFT TFTT FTTT TTFF TFTF FTTF
TFFT FTFT FFTT TFFF FTFF FFTF FFFT FFFF }

b) Assuming all outcomes are equally likely, find P(all responses are the same).

$P(\text{all responses same}) = P(\text{all T} \cup \text{all F}) = P(\text{all T}) + P(\text{all F}) = \frac{1}{16} + \frac{1}{16} = \frac{2}{16} \boxed{= \frac{1}{8}}$

c) Assuming all outcomes are equally likely, Find P(exactly one T).

$P(\text{one T}) = \frac{4}{16} \boxed{= \frac{1}{4}}$

d) Assuming all outcomes are equally likely, find P(at most one T).

$P(\text{at most one T}) = P(\text{no T} \cup \text{one T}) = P(\text{no T}) + P(\text{one T}) = \frac{1}{16} + \frac{4}{16} \boxed{= \frac{5}{16}}$

7| 60% of purchases made are a store are personal computers, 30% are laptops, and 10% are peripheral devices. One purchase record is sampled at random.

a) What is the probability the purchase is a personal computer?

$P(PC) \boxed{= .6}$

b) What is the probability the purchase is either a laptop or personal computer?

$P(\text{PC or laptop}) = P(\text{PC} \cup \text{laptop}) = P(\text{PC}) + P(\text{laptop}) = .6 + .3 \boxed{= .9}$

For non-exclusive events

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$P(A \cap B)$ must be either given or solved for.

$P(A^c) = 1 - P(A)$

de Morgan Property

$(A \cup B)^c = A^c \cap B^c$

8| A certain manufacturing process has a probability of producing a defective of 0.10. True or False:

a) If a sample of 100 items is drawn, exactly 10 will be defective.

False, while it is likely 10 will be defective, probability does not dictate what will happen, only what will likely happen

b) If a sample of 100 items is drawn, the number of defectives is likely to be near but not equal to 10.

False, while it is true that the number of defectives is likely near 10, it could also be 10

c) As more and more items are sampled, the proportion of defectives will approach 10%.

True

13| In a semester, 500 students are enrolled in Calculus 1 and Physics 1. Of these students 82 got an A in Calculus, 73 got an A in Physics, and 42 got an A in both.

a) Find the probability that a randomly chosen student got an A in at least one course.

$P(\text{at least one } A) = P(C \cup P) = P(C) + P(P) - P(C \cap P) = \frac{82}{500} + \frac{73}{500} - \frac{42}{500} \boxed{= \frac{113}{500}}$

b) Find the probability that a randomly chosen student got less than an A in at least one course.

$P(\text{at least one } \cancel{A}) = P([C \cup P]^c) = (1 - P(C \cup P)) = 1 - \frac{113}{500} \boxed{= \frac{387}{500}}$

c) Find the probability that a random student got an A in Calculus, but not Physics.

$P(C \text{ but not } P) = P(C) - P(C \cap P) = \frac{82}{500} - \frac{42}{500} \boxed{= \frac{40}{500}}$

d) Find the probability that a randomly chosen student got an A in Physics and not Calculus.

$P(P \text{ but not } C) = P(P) - P(C \cap P) = \frac{73}{500} - \frac{42}{500} \boxed{= \frac{31}{500}}$

19| True or False: If A and B are mutually exclusive,

a) $P(A \cup B) = 0$

False, $P(A \cup B) = P(A) + P(B)$

b) $P(A \cap B) = 0$

True

c) $P(A \cup B) = P(A \cap B)$

False

d) $P(A \cup B) = P(A) + P(B)$

True

Practice "Lottery" Questions

Order doesn't matter
$\binom{n}{r}C = \frac{n!}{r!(n-r)!}$

Order does matter
$\binom{n}{r}P = \frac{n!}{(n-r)!}$

HW 2.2

4| A group of 10 people gather to play basketball. How many ways can they make 2 teams of 5?

$n=10 \quad r=5 \quad \binom{10}{5}C = \frac{10!}{5!(10-5)!} = \frac{10!}{5!5!} \boxed{= 252}$

6| A committee of 8 must choose a president, VP, and Secretary. How many ways can this be done ran

$n=8 \quad r=3$, order matters $\binom{8}{3}P = \frac{8!}{(8-3)!} = \frac{8!}{5!} \boxed{= 336}$

7| A test consists of 15 questions, 10 T/F and 5 four option multiple choice. How many ways can a response for every question be randomly chosen?

$\#$ Response Combinations $= 2^{10} \cdot 4^5 \boxed{= 1048576}$ By Fundamental Counting Principle

8| In a certain state, license plates consist of three letters followed by 3 numbers.

a) How many different license plates can be made?

$\#$ unique plates $= 26^3 \cdot 10^3 \boxed{= 17,576,000}$

b) How many different plates can be made in which no letter or number is repeated?

$\#$ unique, non-repeating plates $= 26 \cdot 25 \cdot 24 \cdot 10 \cdot 9 \cdot 8 \boxed{= 11,232,000}$

c) If a plate is chosen at random, what is the probability it has no repeating characters?

$P(\text{non-repeating plate}) = \frac{11,232,000}{17,576,000} \boxed{= 0.639}$

11| One drawer in a dresser contains 8 blue socks and 6 white ones. A second drawer contains 4 blue socks and 2 white socks. If one sock is chosen from each drawer, what is the probability the

$P(\text{matched pair}) = \frac{\#\text{ matching pairs}}{\text{total }\#\text{ pairs}} = \frac{(8\cdot4)+(6\cdot2)}{14\cdot6} = \frac{32+12}{84} = \frac{44}{84} \boxed{= \frac{11}{21}}$

12| A drawer contains 6 red socks, 4 green ones, and 2 black ones. If two socks are chosen at random, what is the probability that they match?

$P(\text{matched pair}) = \frac{\#\text{ matching pairs}}{\#\text{ total pairs}} = \frac{\binom{6}{2}C + \binom{4}{2}C + \binom{2}{2}C}{\binom{12}{2}C} = \frac{\frac{6!}{2!(6-2)!} + \frac{4!}{2!(4-2)!} + \frac{2!}{2!(2-2)!}}{\frac{12!}{2!(12-2)!}}$

$= \frac{15+6+1}{66} = \frac{22}{66} \boxed{= \frac{1}{3}}$

EXTRA CREDIT 1 DUE 7/20/2012

What is the probability of picking a number that is a multiple of 2, 3, or 7 from the set $\{1 \leq x \leq 10{,}000\}$?

# of multiples of 2 in 10,000 = $\frac{10{,}000}{2} = 5{,}000 \longrightarrow P(2) = \frac{5\,000}{10\,000} = .5$

# of multiples of 3 in 10,000 = $\frac{10{,}000}{3} = 3{,}333.\overline{3} \rightarrow 3{,}333 \rightarrow P(3) = \frac{3\,333}{10\,000} = .3333$

# of multiples of 7 in 10,000 = $\frac{10{,}000}{7} = 1{,}428.57 \rightarrow 1{,}428 \rightarrow P(7) = \frac{1\,428}{10\,000} = .1428$

# of multiples of 2 and 3 in 10,000 = $\frac{10{,}000}{2\cdot 3} = 1{,}666.\overline{6} \rightarrow 1{,}666 \rightarrow P(2\cap 3) = \frac{1666}{10\,000} = .1666$

# of multiples of 2 and 7 in 10,000 = $\frac{10000}{2\cdot 7} = 714.29 \rightarrow 714 \rightarrow P(2\cap 7) = \frac{714}{10000} = .0714$

# of multiples of 3 and 7 in 10,000 = $\frac{10000}{3\cdot 7} = 476.19 \rightarrow 476 \rightarrow P(3\cap 7) = \frac{476}{10\,000} = .0476$

# of multiples of 2,3, and 7 in 10,000 = $\frac{10000}{2\cdot 3\cdot 7} = 238.09 \rightarrow 238 \rightarrow P(2\cap 3\cap 7) = \frac{238}{10000} = .0238$

$$P(2\cup 3\cup 7) = P(2) + P(3) + P(7) - P(2\cap 3) - P(2\cap 7) - P(3\cap 7) + P(2\cap 3\cap 7)$$
$$= .5 + .3333 + .1428 - .1666 - .0714 - .0476 + .0238 \boxed{= .7143}$$