Math 330 – Review sheet **Test I:** Thursday, 4 Oct

1 Oct / A Kumjian

Test I will cover all sections of the text through section 3.2. You may bring a formula sheet to the test (please do not include worked examples). There will be a review Tuesday and a supplementary review session Wednesday afternoon if there is interest (DMS 315, 2:30 to 4:00 pm). Go over homework and quizzes (solutions available at http://wolfweb.unr.edu/homepage/alex/330/). Here is a list of sample questions.

(1) Show that $\mathbf{u}$ is perpendicular to $\mathbf{v}$ but not $\mathbf{w}$. Find the angle (in radians) between $\mathbf{v}$ and $\mathbf{w}$.

$$\mathbf{u} = (1, 2, 1), \quad \mathbf{v} = (1, -2, 3), \quad \mathbf{w} = (2, -1, -1).$$

Find if possible a nonzero vector perpendicular to both $\mathbf{v}$ and $\mathbf{w}$. $\theta \approx 1.46$, $(5, 7, 3)$

(2) Use elimination and back substitution to solve each of the following systems. Indicate which elimination steps you use and find the pivots.

$$\begin{aligned} 2x + 3y - 2z &= 4 \\ x + 2y - 4z &= 1 \\ 4x + 5y + 2z &= 10 \end{aligned} \qquad \begin{aligned} x + y + z + w &= 1 \\ 2x - 2y + z + 2w &= 3 \\ 5x - 3y + 3z + 5w &= 8 \end{aligned}$$

(3) Give the augmented matrix $[\,A\ \mathbf{b}\,]$ of each of the above systems. For the first system, find a permutation matrix that simplifies the elimination process. For the second system, find the elimination matrices, E_{21}, E_{31}, E_{32}, corresponding to the elimination steps you used above. Compute $E_{32}E_{31}E_{21}[\,A\ \mathbf{b}\,]$.

(4) Find a 3×3 permutation matrix $P \neq I$ so that $P^T = P^2$.

(5) Can you find a nonzero 3×2 matrix so that $A^T A = 0$? no

(6) Find the factorization $A = LU$ for the following matrix A (check that this works!). Use this factorization to solve $A\mathbf{x} = \mathbf{b}$ where

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 3 & 8 & 8 \\ 2 & 2 & 5 \end{bmatrix}, \qquad \mathbf{b} = \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix}.$$

Express $\mathbf{b}$ as a linear combination of the columns of A. $\mathbf{x} = (-2, 0, 1)$, $\mathbf{b} = -2A_1 + A_3$

(7) Determine whether the above matrix A is invertible. If so find its inverse A^{-1} using Gauss-Jordan elimination and check that it is the inverse. Use it to solve $A\mathbf{x} = \mathbf{b}$.

$$\frac{1}{4}\begin{bmatrix} -24 & 4 & 8 \\ -1 & 1 & -1 \\ 10 & -2 & -2 \end{bmatrix}$$

(8) Find the factorization $A = LDL^T$ for the following matrix A where L is lower triangular and D is diagonal.

$$A = \begin{bmatrix} 1 & 2 & 0 \\ 2 & 2 & 4 \\ 0 & 4 & 4 \end{bmatrix},$$

Check that this works! Use this factorization to solve $A\mathbf{x} = (7, 0, 4)$. $\mathbf{x} = (1, 3, -2)$

(9) Which of the following subsets of $\mathbb{R}^4$ are subspaces? Give a brief reason.

a. All vectors $\mathbf{b} = (b_1, b_2, b_3, b_4)$ satisfying $b_1 = b_2 b_3$. no

b. All linear combinations of $\mathbf{v} = (1, 2, 3, 4)$ and $\mathbf{w} = (1, -1, 1, -1)$. yes

c. All vectors $\mathbf{b} = (b_1, b_2, b_3, b_4)$ satisfying $b_1 - 2b_2 - b_3 + 3b_4 = 0$. yes

d. All vectors $\mathbf{b} = (b_1, b_2, b_3, b_4)$ satisfying $b_1 - 2b_2 - b_3 + 3b_4 = 5$. no

(10) Apply elimination to the following matrix. How may pivots are there in the resulting upper triangular matrix? Find all special solutions and then find the complete solution to $A\mathbf{x} = \mathbf{0}$.

$$A = \begin{bmatrix} 1 & 2 & 0 & 1 & -1 \\ 1 & 2 & 1 & 2 & 1 \\ 0 & 0 & 1 & 1 & 2 \\ 0 & 0 & 0 & 1 & 1 \end{bmatrix}$$

3 pivots, $(-2, 1, 0, 0, 0)$, $(2, 0, -1, -1, 1)$

(11) Find the nullspace of A. When is $\mathbf{b} = (b_1, b_2, b_3, b_4)$ in the column space of $\mathbf{C}(A)$? $11b_1 - 4b_2 - 2b_3 + b_4 = 0$

(12) Find if possible a 3×4 matrix B so that $B\mathbf{x} = \mathbf{b}$ has a solution if and only if the components of $\mathbf{b} = (b_1, b_2, b_3)$ satisfy $b_1 - b_2 + 2b_3 = 0$. Is $(1, -1, -1)$ in $\mathbf{C}(B)$?

(13) Find if possible a 3×4 matrix B so that the vectors, $\mathbf{v} = (1, 2, 1, 0)$ and $\mathbf{w} = (2, -3, 0, 1)$ are in its nullspace.