Name: Solutions

Please show your work
(10 pts.)

Math 330 – Quiz 8
Tuesday, 30 Oct 2012

1. Use the least squares method to fit a line $y = c + dt$ to the following data.

t	-1	0	1	2
y	3	3	2	0

Take $A = \begin{bmatrix} 1 & -1 \\ 1 & 0 \\ 1 & 1 \\ 1 & 2 \end{bmatrix}$, $\vec{b} = \begin{bmatrix} 3 \\ 3 \\ 2 \\ 0 \end{bmatrix}$; solve $A^T A \hat{x} = A^T \vec{b}$.

$A^T A = \begin{bmatrix} 1 & 1 & 1 & 1 \\ -1 & 0 & 1 & 2 \end{bmatrix} \begin{bmatrix} 1 & -1 \\ 1 & 0 \\ 1 & 1 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} 4 & 2 \\ 2 & 6 \end{bmatrix}$, $(A^T A)^{-1} = \frac{1}{20}\begin{bmatrix} 6 & -2 \\ -2 & 4 \end{bmatrix} = \frac{1}{10}\begin{bmatrix} 3 & -1 \\ -1 & 2 \end{bmatrix}$

$A^T \vec{b} = \begin{bmatrix} 1 & 1 & 1 & 1 \\ -1 & 0 & 1 & 2 \end{bmatrix} \begin{bmatrix} 3 \\ 3 \\ 2 \\ 0 \end{bmatrix} = \begin{bmatrix} 8 \\ -1 \end{bmatrix}$

Then $\hat{x} = (A^T A)^{-1} A^T \vec{b} = \frac{1}{10}\begin{bmatrix} 3 & -1 \\ -1 & 2 \end{bmatrix}\begin{bmatrix} 8 \\ -1 \end{bmatrix} = \frac{1}{10}\begin{bmatrix} 25 \\ -10 \end{bmatrix} = \frac{1}{2}\begin{bmatrix} 5 \\ -2 \end{bmatrix}$

So the least squares line is $y = \frac{5}{2} - t$.

2. Use Gram-Schmidt to find two orthonormal vectors $\mathbf{q}_1$, $\mathbf{q}_2$ in the subspace V spanned by $(2, 2, -1)$, $(3, 6, 0)$. Use $\mathbf{q}_1$ and $\mathbf{q}_2$ to find the projection of $\mathbf{b} = (1, 1, 1)$ onto V.

$\vec{a}_1 = (2, 2, -1)$, $\vec{a}_2 = (3, 6, 0)$

$\|\vec{v}_1\| = 3$

$\vec{v}_1 = \vec{a}_1 = (2, 2, -1)$

$\vec{v}_2 = \vec{a}_2 - \frac{\vec{v}_1^T \vec{a}_2}{\vec{v}_1^T \vec{v}_1} \vec{v}_1 = \begin{bmatrix} 3 \\ 6 \\ 0 \end{bmatrix} - \frac{18}{9}\begin{bmatrix} 2 \\ 2 \\ -1 \end{bmatrix} = \begin{bmatrix} -1 \\ 2 \\ 2 \end{bmatrix}$ $\qquad \|\vec{v}_2\| = 3$

then $\vec{q}_1 = \frac{\vec{v}_1}{\|\vec{v}_1\|} = \frac{1}{3}\begin{bmatrix} 2 \\ 2 \\ -1 \end{bmatrix}$, $\vec{q}_2 = \frac{\vec{v}_2}{\|\vec{v}_2\|} = \frac{1}{3}\begin{bmatrix} -1 \\ 2 \\ 2 \end{bmatrix}$

$\vec{p} = (\vec{q}_1^T \vec{b})\vec{q}_1 + (\vec{q}_2^T \vec{b})\vec{q}_2$ $\qquad \vec{q}_1^T \vec{b} = \vec{q}_2 \vec{b} = 1$

$= 1 \cdot \vec{q}_1 + 1 \cdot \vec{q}_2 = \frac{1}{3}\begin{bmatrix} 1 \\ 4 \\ 1 \end{bmatrix}$.