26 A X² variate with degrees of freedom equal to df has representation:

Z,²+...+ Z,g, where Z, ..., Z,g are independent standard normal variates.

a) If Z has a standard normal distribution, what distribution does Z² have?

Z² has a X² distribution with df degrees of freedom, where df is the number of variates/terms represented by Z

b) Show that, if Y, and Yz are independent X² variates with degrees of freedom If, and Ifz, then Y, + Yz has a X² distribution with If = If, + Ifz.

Y, is represented by: Y12 + Y22 ... + Y28fz

then

Y, + Yz is represented by: Y, 2 + Yz, 2 + Y, 2 + Yz, 2 ... + Y, df, + Yz dfz

therefore

if df, and dfz represent the counts of Y, terms and Yz terms respectively, then the df for the X² distribution of Y, + Yz will be equal to the total number of terms in the X² distribution, i.e. df=df, + dfz

33 In murder trials in 20 Florida Counties during 1976 and 1977, the death penalty was given in 19 out of 151 cases in which a white killed a white, in o out of 9 cases in which a white killed a black, in 11 out of 63 cases when a black killed a white, and in 6 out of 103 cases when a black killed a black. a) Exhibit the data in a 3-way contingency table

Decision (DP V/N)

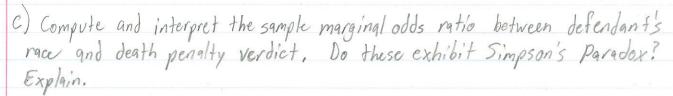
		pecis	ion (UP Y/N)		
Murderer	Victim	У	N	Total	1. Received DP
White	White	19	132	151	12.583%
WMIIO	Black	0	9	9	0%
Black	White	11	52	63	17.46%
DIACK	Blach	6	97	103	5.825%
Total	White	30	184	214	14.019%
	Black	6	106	112	5.357%

b) Construct the partial tables necessary to study the conditional association between defendant's race and death penalty verdict. Find and interpret the sample conditional odds ratios, adding 0.5 to each cell to reduce the impact of the O cell count.

X = Murderer's Race Y = Death Penalty Decision Z = Victim's Race Z = White Victim Z = Black Victim

1	Decision							Pecision		
l by		Y	N					Y	N	
Murderer	White	19.5	132,5	1.7	murderur	White	0.5	9.5		
	Black	11,5	52.5	M L.J. K. KI		MAGGAN	Black	6,5	97.5	
$\hat{\Theta}_{z=\text{White}} = 19.5 \cdot 57.5 = 0.67186$					Bz=Black = 0.5 . 97.5 = 0.78947					
137.5 • 11 • 5					6.5.9.5					

In both cases, the odds ratios show that the odds of a white murderer receiving the death penalty are lower than those of a black murderer receiving the death penalty, leading us to believe that black murderers receive the death penalty more often.



Decision  $\frac{1}{1}$   $\frac{1}{$ 

This odds ratio indicates that the odds that a white murderer receives the death penalty are higher than those of a black murderer. These results do exhibit Simpson's Paradox: inference made using conditional data will likely contradict inference made using marginal data.

X vs. Z

Victim White Black

Norderer White 151 9  $\hat{\theta} = 151$ , 103 = 27.4Black 63 103 63.9Z vs. Y

Norderer White 30 184  $\hat{\theta} = \frac{30.106}{6.184}$ Black 6 106 = 2.88

Overall: By examining other partial tables, we can see why Simpson's paradox grose: because white victim crimes are more likely to receive the death penalty, and whites are much more likely to kill whites, making it appear that whites receive the death penalty more, as opposed to the more refined conclusion that black murdurers are more likely to receive the death penalty.

At each age level, the death rate is higher in South Carolina than in Maine, but overall the death rate is higher in Maine. Explain how this could be possible. It is typical for older age groups to have higher death rates than younger ones, so it is possible that Maine has an 'older' population than South Carolina, creating a confound and causing Maine to appear to be a more deadly place to live than South Carolina.



Based on murder rates in the U.S., the Associated Press reported that the probability of a newborn child has of eventually being a murder victim is 0,0263. for non-white males, 0.0049 for white males, 0.0072 for non-white females, and 0.0023 for white females.

a) Find the conditional odds ratios between race and victim status, given gender. Interpret.  $X = race \ Y/N \qquad Y = victim \ status \ Y/N \qquad Z = gender \ m/F$ 

The odds ratios indicate that whites are much less likely to be murder victims than non-whites. Also, the difference is less drastic for females.

b) If half the newborns are of each genter, for each race, find the marginal odds ratio between race and whether a murder victim.

In this case, Simpson's paradox fails to emerge. Gender is not a confound in this example because there was no disparity between number of make cases and number of female cases in general or for either race.

