

1. For A given below compute $\det A$ using row operations and properties of the determinant.

$$A = \begin{bmatrix} 1 & 2 & 3 & 4 \\ 0 & 0 & 2 & 8 \\ 0 & 3 & 7 & 9 \\ 2 & 4 & 6 & 7 \end{bmatrix}$$

$$A = \begin{bmatrix} 1 & 2 & 3 & 4 \\ 0 & 0 & 2 & 8 \\ 0 & 3 & 7 & 9 \\ 2 & 4 & 6 & 7 \end{bmatrix} \xrightarrow{R_2 \leftrightarrow R_3} \begin{bmatrix} 1 & 2 & 3 & 4 \\ 0 & 3 & 7 & 9 \\ 0 & 0 & 2 & 8 \\ 2 & 4 & 6 & 7 \end{bmatrix} \xrightarrow{R_4 - 2R_1} \begin{bmatrix} 1 & 2 & 3 & 4 \\ 0 & 3 & 7 & 9 \\ 0 & 0 & 2 & 8 \\ 0 & 0 & 0 & -1 \end{bmatrix} = U$$

Since there was 1 row swap we have

$$\det A = (-1)^1 \det U = (-1) \cdot 1 \cdot 3 \cdot 2 \cdot (-1) = 6$$

2. Use the cofactor formula to compute the following determinant; indicate which row (or column) you are using.

$$|A| = \begin{vmatrix} 1 & 2 & 3 & 4 \\ 0 & 0 & 2 & 0 \\ 2 & 5 & 6 & 0 \\ 0 & 1 & 9 & 3 \end{vmatrix}$$

Expand along 2nd row.

$$|A| = 2(-1)^{2+3} \begin{vmatrix} 1 & 2 & 4 \\ 2 & 5 & 0 \\ 0 & 1 & 3 \end{vmatrix} \leftarrow \text{Now expand along 3rd row.}$$

$$= -2 \left(1(-1)^{3+2} \begin{vmatrix} 1 & 4 \\ 2 & 0 \end{vmatrix} + 3(-1)^{3+3} \begin{vmatrix} 1 & 2 \\ 2 & 5 \end{vmatrix} \right)$$

$$= -2 \left(-1(-8) + 3(5-4) \right)$$

$$= -22$$