

## Mapping Electric Fields and Equipotential Surfaces in Two Dimensions

### Object:

To determine Equipotential, and electric field lines in two dimensions using electrically conducting paper.

### Apparatus:

Potential Demonstrator, electrically conducting paper, 4 volt dc power supply, digital voltmeter, carbon paper, white paper, connection wires: 1cm probe, banana jacks, safety scissors.

### Theory:

The electric field at a point in the vicinity of electric charges is the force per unit charge on a test charge (small enough that it does not disturb the field appreciably) placed at the point in question. Since force  $\vec{F}$  is a vector and charge  $Q$  is a scalar, the electric field which is  $\vec{F}/Q$  is also a vector quantity, its direction being the direction of the force on charge  $Q$ , i.e.,

$$\vec{E} = \frac{\vec{F}}{Q} \quad (1)$$

The potential difference( $\Delta V$ ) between two points in an electric field is the work per unit charge  $\Delta W$  done on a charge  $Q$  in moving it from one point to another against the electric field. Since both  $W$  and  $Q$  are scalar quantities, the potential difference is also a scalar. If a small distance ( $\Delta \vec{s}$ ), separates the two points the work per unit charge, using equation (1) is:

$$\Delta V = -\frac{\Delta W}{Q} = -\left(\frac{\vec{F}}{Q}\right) \bullet \Delta \vec{s} = -\vec{E} \bullet \Delta \vec{s} \quad (2)$$

The minus sign is necessary because  $\Delta W$  is the work done against the electric field. Using this terminology, and the corresponding vector notation, this can be applied to both two and three dimensional situations,

$$\vec{E} = -gradV = -\nabla V \quad (3)$$

Another way of saying this is that the electric field has the opposite direction of the greatest rate of change of potential with distance away from the point in question, and its magnitude is equal to that rate of change.

The units of electric field in the mks system are Newtons per coulomb (N/Coul), see Eq. 1, and those of potential difference are joules per coulomb (J/Coul), see Eq. 2. Potential differences as read on a voltmeter are in volts where the volt is defined as:

$$1volt = 1 \frac{Joule}{Coulomb}$$

An equivalent set of units for electric field can be obtained as follows: Since

$$\vec{E} = \frac{\vec{F}}{Q}$$

Its units are:

$$\frac{\text{Newtons}}{\text{coul}} = \frac{\text{Newtons} \cdot \text{m}}{\text{coul} \cdot \text{m}} = \frac{\text{joules}}{\text{coul} \cdot \text{m}} = \frac{\text{volts}}{\text{m}}$$

If no current is flowing throughout a conductor, every point on the surface of the conductor is at the same potential. Such a surface is called an equipotential surface. A charged conductor is surrounded by an electric field. At a very small distance away from a conductor, there exists another surface (whose shape may be slightly different from that of the conductor itself) such that the potential difference between the new surface and the conductor is the same at all points on the new surface. This is also called an equipotential surface, but, of course, it is not a surface in the sense that a conductor is because it does not represent the boundary of a solid and it cannot conduct electricity. Other equipotentials exist with greater potential difference with respect to the conductor. If another charged conductor (equipotential surface) is brought near, the electric field is changed, but equipotential surfaces can still be imagined in the vicinity of two or more conductors.

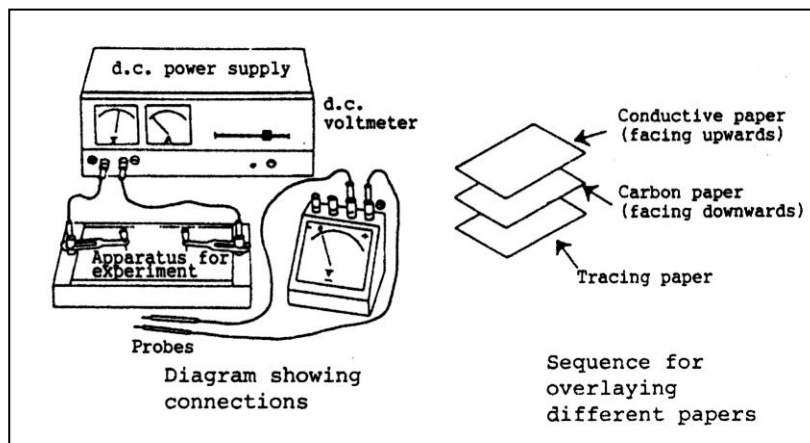
In order to determine the potential difference between a point in the electric field and the conductor itself, it would be necessary to place one test lead or probe of a voltmeter on the conductor and one at the point in question. Since even the most sensitive probe and the associated meter placed at a point in the field would disturb the static field and equipotential lines (because a voltmeter always draws some current), a slight modification of some of the above ideas is necessary in order to “map” equipotentials or determine electric fields. If one deals with a dynamic rather than an electrostatic situation, i.e., if a current flows continuously between the electrodes, more charge is available at any given point than would be, especially in a vacuum, if no current were flowing, and the effect of the probe can be made relatively very small. In order to do this, it is necessary to have the space surrounding a pair of electrodes filled with a material, which conducts, but, of course, it must not conduct nearly as well as the electrodes themselves. One such scheme is to immerse the electrodes in a tank of dilute saltwater or other mildly conducting solution. One probe of the meter is connected to one electrode and the other measures potentials at various points in the solution without disturbing the electric field appreciably, provided that the current drawn by the voltmeter is small compared to the total current flowing in the vicinity of the point in question. Meters are available which draw extremely small currents or, to say it another way, which have very high probe or input resistances. Many voltmeters have input resistances of the order of 10 megaohms ( $10 \times 10^6$ ) or more.

**Question: compute the current drawn by such a meter while measuring a potential difference of 10 volts.** The scheme described for determining equipotential surfaces using an electrolyte is called the electrolytic tank method.

In the experiment you will do today, a two-dimensional modification of the electrolytic tank will be used. You will be using paper that has been coated with a black pigment made from fine particles of carbon black. This allows the paper to conduct electricity. Therefore, a two-dimensional electric field will be created when a dc voltage is applied across two metal electrodes that touch the paper. If the ground terminal of the voltmeter is connected to the negative electrode, and the positive meter probe to the other, the meter will read the applied voltage, but if the probe is touched to the paper surrounding the electrodes, a lower potential difference will be read. By moving the probe around over the paper, a series of points all having the same potential relative to the grounded electrode can be found. These points, when connected together by a line drawn with an ordinary pencil, form an equipotential line (in three dimensions this would be a surface).

Electric field gradients can be measured by touching the conducting paper with both probes of the meter together. If the distance between the probes is held at 1cm, and they are touched to the paper, the meter reads the number of volts per cm of spacing. This is the electric field. Volts/cm is often used instead of volts/meter because the latter is rather clumsy as a practical unit.

**Procedure:**



1. Using the scissors cut a piece of white paper to a rectangle 8 inches long by 6 inches wide. Cut a piece of electrically conducting paper and carbon paper to the same size. Loosen the electrode connecting pins by unscrewing the red and black knobs on the connecting posts. Remove the white magnetic strips from the potential demonstrator device. Place the sheet of white paper on the potential demonstrator device between the red and black connecting posts. Place a sheet of carbon paper on top of this with the ink side down. Place the conducting paper over the first two sheets and secure all three sheets together with the white magnetic strips.
2. Place the rectangular metal electrodes over the three layers of paper near the red and black connecting posts. Place the red and black connecting pins in the small dimple at the center of each electrode. Place a tension on the electrodes by slightly tightening the red and black knobs on the connecting posts. **Do not over tighten!** Connect the electrode to the dc power supply using wires with banana jacks that fit into the red and black connecting posts.
3. Set the digital voltmeter to the dc position and connect the ground lead to the negative electrode. Starting at some point between the two electrodes, LIGHTLY touch the positive probe of the voltmeter to the conducting paper and note the voltage. Do not poke holes in the conducting paper or write on it. With a light pressure, and preferable a dull point like a worn pencil tip, press lightly on the conduction paper just enough to make a mark on the white paper with the carbon paper. Remove one of the white magnetic strips so you can check the bottom white sheet to see if a clear mark has been made. Darken the mark on the white sheet if necessary. Move the probe to a new spot nearby where the voltage reading is the same as before and make another mark. Continue this process until you have enough points to "imagine" the equipotential surface. Once you have enough points, this surface can be drawn on the white paper. Repeat this process for at least five different voltages "imaging" equipotential surfaces both near the electrodes and at points in between. While you are doing this, be sure to keep your body away from the conduction paper because currents, which pass through you, are large enough to modify the electric field, especially during the gradient part of the experiment.

4. Do not poke holes in the conducting paper or write on the conduction paper. All your marks and work is to be done on the white sheet of paper. You will turn this sheet in to your lab instructor. It is probably easiest to make the marks on the paper first, labeling the voltage, if necessary, to avoid confusion (you will have a lot of points), then when you are finished making marks, remove the top two sheets and draw in the equipotential surfaces. However, as a first step before you make any marks; lightly trace the outline of your electrodes through to the white paper labeling them with the correct voltage. (Check the voltage of the power supply with your voltmeter).
5. In order to determine potential gradients, you will use the special two-pronged probe provided. Remove the leads from the voltmeter and connect the two-pronged probe to its input. The separation of the two prongs is approximately 1cm. Using the negative side of the probe as a fixed point, move the positive prong of the probe around until a maximum reading on the voltmeter is obtained. Using light pressure make marks for the two points where the probe touches the paper. Lift the upper sheets and draw an arrow between these marks that point from the negative probe to the positive probe. This is the direction of the field gradient, which is in the opposite direction to the electric field,  $E$ . The voltmeter reading divided by the probe separation is the field strength. Write this value in volts/cm next to the arrow and with a different colored ink or pencil draw the electric field vector  $\vec{E}$ . Repeat this for at least five different points both near the electrodes and in between.
6. According to Eq. # 3,  $\Delta V$  is a maximum and hence the gradient is a maximum when  $\Delta s$  is parallel to  $E$ , i.e. when  $\Delta s$  is perpendicular to an equipotential surface. In other words the electric field lines begin on the positive electrode and end on the negative electrode following paths that cross the equipotential surfaces at right angles everywhere between the electrodes. After you have drawn the equipotential surfaces on the white sheet of paper, draw in 5 or 6 field lines according to the previous statement. Do the field lines as determined by the gradient technique conform to the general “flow” of these lines?

#### **QUESTIONS:**

- 1) What are the directions of the electric field lines of the experimental configuration?
- 2) Sketch the electric field for a configuration of two positive point charges.
- 3) According to Ohm’s law, how does
  - a) Current vary with voltage?
  - b) Voltage vary with resistance?
  - c) Current vary with resistance?
- 4) Assume a typical voltmeter which has an input resistance on the order of 10 M $\Omega$ . Compute the current drawn by the meter while measuring a potential difference of 10 volts.