CS 682: AI Quiz 4

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October 6, 2014

Consider the following deterministic scheduling problem: There are 4 operations that need to be performed in a certain order. Call these operations A, B, C, and D. There are constraints on the order in which the operations must be performed. Operations A must come before C and operation D must be performed before operation B.

Each operation x incurs a startup cost S_x in case it is chosen as the first operation: $S_A = 4$, $S_B = 2$, $S_C = 7$, $S_D = 1$. For every pair of actions, there is a cost C_{xy} for performing the sequence of actions x followed by y. These costs are represented by the following matrix:

```
    A B C D (y)
A 1 2 3 4
B 5 1 2 7
C 3 2 1 1
D 9 2 8 1
(x)
```

Using the information above, answer the following questions:

1. Some of the costs S_x above are irrelevant because they describe impossible initial actions. Write down the startup costs that are irrelevant.
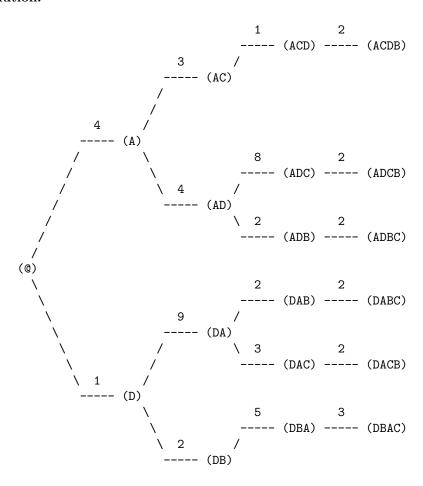
   Solution: The startup costs $S_B = 2$ and $S_C = 7$ are irrelevant because they have an operation they are dependent on preceding them.

2. Some of the step costs in the matrix above are irrelevant because the implied sequence of actions is impossible. Rewrite the matrix, writing an in place of the cost C_{xy} when the sequence x, y is impossible.

   Solution: The problem constraints demand that you cannot go from C to A, nor can you go from B to D, further, it doesn't make sense to schedule an action before itself:

```
    A B C D
A X 2 3 4
B 5 X 2 X
C X 2 X 1
D 9 2 8 X
```

3. Using the example from class as a guide, draw a tree that shows the possible schedules that can be chosen. Along each edge, write the appropriate cost number. On each node, write the appropriate partial schedule.

Solution:

```
                                          1               2
                                    ----- (ACD) ----- (ACDB)
                          3       /
                    ----- (AC)
                   /
                  /
            4    /
        ----- (A)
        /        \              8               2
       /          \       ----- (ADC) ----- (ADCB)
      /          \ 4    /
     /            ----- (AD)
    /                  \  2               2
   /                    ----- (ADB) ----- (ADBC)
  /
(@)
  \                      2               2
   \                ----- (DAB) ----- (DABC)
    \             9    /
     \          ----- (DA)
      \        /       \  3               2
       \      /         ----- (DAC) ----- (DACB)
        \  1 /
        ----- (D)
              \          5               3
               \   ----- (DBA) ----- (DBAC)
                \ 2    /
                ----- (DB)
```

4. Rewrite the tree from the previous problem, but instead of writing partial schedules in the nodes, write the appropriate cost to go that would result from applying the dynamic programming algorithm to find the optimal schedule (that is, work backwards to get the optimal cost to go for all of the possible partial schedules in the problem).

   Solution:

```
                                            1           2
                                          ----- ( 2) ----- ( 0)
                                3       /
                              ----- ( 3)
                             /
                            /
                 4         /
               ----- ( 6)
              /          \              8           2
             /            \           ----- ( 2) ----- ( 0)
            /              \   4     /
           /              ----- ( 4)
          /                        \  2           2
         /                          ----- ( 2) ----- ( 0)
        /
     (10)
        \                               2           2
         \                            ----- ( 2) ----- ( 0)
          \                 9       /
           \              ----- ( 4)
            \            /         \  3           2
             \          /           ----- ( 2) ----- ( 0)
              \   1    /
              ----- (10)
                    \              5           3
                     \           ----- ( 3) ----- ( 0)
                      \  2     /
                      ----- ( 8)
```

5. What is the cost of the optimal solution?

   Solution:
   The optimal scheduling cost is 10.

6. What is the optimal schedule?

   Solution:
   $A \Rightarrow C \Rightarrow D \Rightarrow B$