Name: Solutions

Please show your work
(10 pts.)

Math 330 – Quiz 1
Tuesday, 4 Sept 2012

1. (5 pts) Determine whether the given vectors are perpendicular; if not find the angle between them:

$$\mathbf{v} = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}, \quad \mathbf{w} = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}.$$

$\vec{v} \cdot \vec{w} = 1 + 0 + 0 = 1 \neq 0$

So the vectors are not perpendicular.

Use formula $\theta = \cos^{-1} \dfrac{\vec{v} \cdot \vec{w}}{\|\vec{v}\| \, \|\vec{w}\|}$

$\|\vec{v}\| = \sqrt{1^2+0^2+1^2} = \sqrt{2}$, $\|\vec{w}\| = \sqrt{1^2+1^2+0^2} = \sqrt{2}$ $\vec{v} \cdot \vec{w} = 1$

So

$$\theta = \cos^{-1} \frac{1}{\sqrt{2}\sqrt{2}} = \cos^{-1} \frac{1}{2} = \frac{\pi}{3}$$

2. (5 pts) Find the coefficient matrix A of the following linear system and verify that $\mathbf{x} = (2, -3, 1)$ is a solution of the matrix form of the system. Express $\mathbf{b} = (-1, 3, -7)$ as a linear combination of the columns of A (if possible).

$$\begin{aligned} x_1 + 2x_2 + 3x_3 &= -1 \\ 2x_1 + 3x_2 + 8x_3 &= 3 \\ x_2 - 4x_3 &= -7 \end{aligned}$$

$A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 8 \\ 0 & 1 & -4 \end{bmatrix}$ coefficient matrix

We verify that $\vec{x} = (2, -3, 1)$ is a solution to the matrix form of the system: $A\vec{x} = \vec{b}$

$$\begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 8 \\ 0 & 1 & -4 \end{bmatrix} \begin{bmatrix} 2 \\ -3 \\ 1 \end{bmatrix} = \begin{bmatrix} 2-6+3 \\ 4-9+8 \\ 0-3-4 \end{bmatrix} = \begin{bmatrix} -1 \\ 3 \\ -7 \end{bmatrix}$$

Use column form of above to get

$$\begin{bmatrix} -1 \\ 3 \\ -7 \end{bmatrix} = (-2) \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix} + (-3) \begin{bmatrix} 2 \\ 3 \\ 1 \end{bmatrix} + 1 \begin{bmatrix} 3 \\ 8 \\ -4 \end{bmatrix}$$