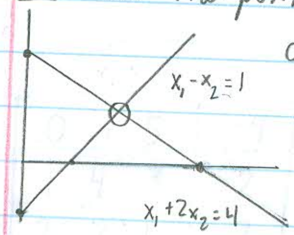
HW 1.1

3) Find the point (x_1, x_2) that lies on the line $x_1 + 2x_2 = 4$ and on the line $x_1 - x_2 = 1$.



$$\left[\begin{array}{cc|c} 1 & 2 & 4 \\ 1 & -1 & 1 \end{array}\right] \xrightarrow[R2 = R2 + -1R1]{} \left[\begin{array}{cc|c} 1 & 2 & 4 \\ 0 & -3 & -3 \end{array}\right]$$

$$\xrightarrow[R2 = -\frac{1}{3}R2]{} \left[\begin{array}{cc|c} 1 & 2 & 4 \\ 0 & 1 & 1 \end{array}\right] \rightarrow x_2 = 1 \rightarrow x_1 + 2(1) = 4 \rightarrow \begin{array}{l} x_1 = 2 \\ x_2 = 1 \end{array} \boxed{\rightarrow (2,1)}$$

7) This augmented matrix has been reduced by elementary row operations. Continue the appropriate row operations and describe the solution set of the original system.

$$\left[\begin{array}{ccc|c} 1 & 7 & 3 & -4 \\ 0 & 1 & -1 & 3 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & -2 \end{array}\right] \xrightarrow[R_3 \leftrightarrow R_4]{} \left[\begin{array}{ccc|c} 1 & 7 & 3 & -4 \\ 0 & 1 & -1 & 3 \\ 0 & 0 & 1 & -2 \\ 0 & 0 & 0 & 1 \end{array}\right]$$

→ R4 indicates that the system is inconsistent and has no solution

11 16 22 28

11 Solve the system
$$\begin{cases} \quad\quad x_2 + 5x_3 = -4 \\ x_1 + 4x_2 + 3x_3 = -2 \\ 2x_1 + 7x_2 + x_3 = -2 \end{cases}$$

$$\left[\begin{array}{ccc|c} 0 & 1 & 5 & -4 \\ 1 & 4 & 3 & -2 \\ 2 & 7 & 1 & -2 \end{array}\right] \xrightarrow{R_1 \leftrightarrow R_2} \left[\begin{array}{ccc|c} 1 & 4 & 3 & -2 \\ 0 & 1 & 5 & -4 \\ 2 & 7 & 1 & -2 \end{array}\right] \xrightarrow{R3 = R3 + -2R1} \left[\begin{array}{ccc|c} 1 & 4 & 3 & -2 \\ 0 & 1 & 5 & -4 \\ 0 & -1 & -5 & 2 \end{array}\right]$$

$$\xrightarrow{R3 = R3 + R2} \left[\begin{array}{ccc|c} 1 & 4 & 3 & -2 \\ 0 & 1 & 5 & -4 \\ 0 & 0 & 0 & 2 \end{array}\right]$$

→ R3 indicates that this system is inconsistent and has no solution

16 Determine if the system is consistent or not (Do not solve completely)
$$\begin{cases} 2x_1 \quad\quad - 4x_4 = -10 \\ 3x_2 + 3x_3 = 0 \\ x_3 + 4x_4 = -1 \\ -3x_1 + 2x_2 + 3x_3 + x_4 = 5 \end{cases}$$

$$\begin{bmatrix} 2 & 0 & 0 & -4 & -10 \\ 0 & 3 & 3 & 0 & 0 \\ 0 & 0 & 1 & 4 & -1 \\ -3 & 2 & 3 & 1 & 5 \end{bmatrix} \xrightarrow{R1 \leftrightarrow R4} \begin{bmatrix} -3 & 2 & 3 & 1 & 5 \\ 0 & 3 & 3 & 0 & 0 \\ 0 & 0 & 1 & 4 & -1 \\ 2 & 0 & 0 & -4 & -10 \end{bmatrix} \xrightarrow{R1 = R1 + 2R4} \begin{bmatrix} 1 & 2 & 3 & -7 & -15 \\ 0 & 3 & 3 & 0 & 0 \\ 0 & 0 & 1 & 4 & -1 \\ 2 & 0 & 0 & -4 & -10 \end{bmatrix}$$

$$\xrightarrow{R2 = \frac{1}{3}R2} \begin{bmatrix} 1 & 2 & 3 & -7 & -15 \\ 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 4 & -1 \\ 2 & 0 & 0 & -4 & -10 \end{bmatrix} \xrightarrow{R4 = R4 - 2R1} \begin{bmatrix} 1 & 2 & 3 & -7 & -15 \\ 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 4 & -1 \\ 0 & -4 & -6 & 10 & 20 \end{bmatrix}$$

$$\xrightarrow{R4 = R4 + 4R2} \begin{bmatrix} 1 & 2 & 3 & -7 & -15 \\ 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 4 & -1 \\ 0 & 0 & -2 & 10 & 20 \end{bmatrix} \xrightarrow{R4 = R4 + 2R3} \begin{bmatrix} 1 & 2 & 3 & -7 & -15 \\ 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 4 & -1 \\ 0 & 0 & 0 & 18 & 18 \end{bmatrix} \xrightarrow{R4 = \frac{1}{18}R4} \left[\begin{array}{cccc|c} 1 & 2 & 3 & -7 & -15 \\ 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 4 & -1 \\ 0 & 0 & 0 & 1 & 1 \end{array}\right]$$

The system is consistent

22 Determine the value(s) of h such that the matrix is the augmented matrix of a consistent linear system.

$$\begin{bmatrix} -4 & 12 & h \\ 2 & -6 & -3 \end{bmatrix} \xrightarrow{R1 = R1 + 2R2} \begin{bmatrix} 0 & 0 & h-6 \\ 2 & -6 & -3 \end{bmatrix} \rightarrow 0x_1 + 0x_2 = h-6$$

$$0 = h - 6 \rightarrow \boxed{h = 6 \qquad \text{Verification: } \begin{bmatrix} -4 & 12 & 6 \\ 2 & -6 & -3 \end{bmatrix} \xrightarrow{R1 = -\frac{1}{2}R1} \begin{bmatrix} 2 & -6 & -3 \\ 2 & -6 & -3 \end{bmatrix}}$$

28 Construct 3 different augmented matrices for the linear system with solution set $x_1 = 3,\ x_2 = -2,\ x_3 = -1$

$$\begin{bmatrix} 1 & 0 & 0 & 3 \\ 0 & 1 & 0 & -2 \\ 0 & 0 & 1 & -1 \end{bmatrix} \xrightarrow{R1 = R1 + R2} \begin{bmatrix} 1 & 1 & 0 & 1 \\ 0 & 1 & 0 & -2 \\ 0 & 0 & 1 & -1 \end{bmatrix} \xrightarrow{R1 \leftrightarrow R3} \begin{bmatrix} 0 & 0 & 1 & -1 \\ 0 & 1 & 0 & -2 \\ 1 & 1 & 0 & 1 \end{bmatrix} \text{①}$$

$$\xrightarrow{R2 = R2 + R1} \begin{bmatrix} 1 & 0 & 0 & 3 \\ 1 & 1 & 0 & 1 \\ 0 & 0 & 1 & -1 \end{bmatrix} \xrightarrow{R3 = 4R3} \begin{bmatrix} 1 & 0 & 0 & 3 \\ 1 & 1 & 0 & 1 \\ 0 & 0 & 4 & -4 \end{bmatrix} \xrightarrow{R3 = R3 + R2} \begin{bmatrix} 1 & 0 & 0 & 3 \\ 1 & 1 & 0 & 1 \\ 1 & 1 & 4 & -3 \end{bmatrix} \text{②}$$

$$\xrightarrow{R1 = R1 + R2} \begin{bmatrix} 2 & 1 & 0 & 4 \\ 1 & 1 & 0 & 1 \\ 0 & 0 & 1 & -1 \end{bmatrix} \xrightarrow{R2 = 5R2} \begin{bmatrix} 2 & 1 & 0 & 4 \\ 5 & 5 & 0 & 5 \\ 0 & 0 & 1 & -1 \end{bmatrix} \xrightarrow{R1 = R1 + R3} \begin{bmatrix} 2 & 1 & 1 & 3 \\ 5 & 5 & 0 & 5 \\ 0 & 0 & 1 & -1 \end{bmatrix} \text{③}$$