Math 330: Review sheet **Final Exam:** Thursday, 13 December, 8 am

10 December 2012

The final exam will be comprehensive with some emphasis on the material not covered by the first two exams: 5.3, 6.1-6.4, 7.1-7.2. You may bring a formula sheet (front and back) to the test. Please do not include worked examples. There will be a review Tuesday and a supplementary review session Wednesday 9:30 to 11 am DMS 315. Go over past review sheets, tests, quizzes and homework. The solutions to the tests and quizzes are available on the class web page: http://wolfweb.unr.edu/homepage/alex/330/. Here is a list of sample questions relevant to the latest material.

(1) With A given below, find the volume of the skew box formed from the rows of A. Find the cofactor matrix C and the inverse $A^{-1} = \frac{C^T}{\det A}$ where $V = 2$

$$A = \begin{bmatrix} 2 & 1 & 3 \\ 0 & 1 & 0 \\ 4 & 0 & 5 \end{bmatrix}.$$

(2) Find the eigenvalues and a basis of eigenvectors (if possible) for each of the following matrices. If the matrix is diagonalizable, find a diagonal matrix Λ and an invertible matrix S so that the given matrix is equal to $S\Lambda S^{-1}$. $\lambda = 2, 0, -1$, $\lambda = 2 \pm 3i$, $\lambda = 3$

$$A = \begin{bmatrix} 0 & -1 & 0 \\ -1 & 1 & -1 \\ 0 & -1 & 0 \end{bmatrix}, \quad B = \begin{bmatrix} 6 & 5 \\ -5 & -2 \end{bmatrix}, \quad C = \begin{bmatrix} 5 & -2 \\ 2 & 1 \end{bmatrix}.$$

(3) Use the eigenvalues and eigenvectors of A above to compute $A^{10}\mathbf{x}$ where $\mathbf{x} = (0, 0, 1)$. $(171, -341, 171)$

(4) For D given below find the matrix exponential e^{Dt} and use it to solve $\mathbf{u}' = D\mathbf{u}$ starting with $\mathbf{u}(0) = (1, 1)$.

$$D = \begin{bmatrix} 1 & 2 \\ 1 & 0 \end{bmatrix}$$

$$e^{Dt} = \frac{1}{3}\begin{bmatrix} 2e^{2t} + e^{-t} & 2e^{2t} - 2e^{-t} \\ e^{2t} - e^{-t} & e^{2t} + 2e^{-t} \end{bmatrix}, \ \mathbf{u} = \frac{1}{3}(4e^{2t} - e^{-t}, 2e^{2t} + e^{-t})$$

(5) Let $a_0, a_1, a_2, \ldots$ be the sequence defined by $a_{n+1} = 5a_n - 6a_{n-1}$ and $a_0 = -1$, $a_1 = 0$. Find a formula for a_n. Hint: find a matrix A so that $(a_n, a_{n+1}) = A(a_{n-1}, a_n)$ and then compute $A^n(a_0, a_1)$. $a_n = 2 \cdot 3^n - 3 \cdot 2^n$

(6) Suppose that A is an $n \times n$ matrix with $A^2 = I$. Show that if λ is an eigenvalue of A, then $\lambda = \pm 1$.

(7) Find if possible a 3×3 matrix with eigenvalues $3, 1, -2$ and corresponding eigenvectors $(1, 1, 0)$, $(1, 0, 1)$, $(0, 1, 1)$.

(8) Two $n \times n$ matrices A and B are said to be similar if there is an invertible matrix C so that $B = CAC^{-1}$. Show that if A and B are similar, then A and B have the same eigenvalues.

(9) Find the eigenvalues of the matrix B given below. Is B symmetric? If so find an orthogonal matrix Q which diagonalizes it: $B = Q\Lambda Q^T$. yes, $\lambda = 4, 7, 1$

$$B = \begin{bmatrix} 5 & 0 & 2 \\ 0 & 3 & -2 \\ 2 & -2 & 4 \end{bmatrix}$$

(10) Suppose a linear transformation T transforms $(1, 1)$ to $(1, 7)$ and $(1, -1)$ to $(3, 5)$. Find $T(\mathbf{v})$ for $\mathbf{v} = (1, 0)$ and $\mathbf{v} = (0, 1)$. Find a 2×2 matrix A so that $T(\mathbf{v}) = A\mathbf{v}$. $A = \begin{bmatrix} 2 & -1 \\ 6 & 1 \end{bmatrix}$

(11) Suppose that T is the transformation from $\mathbb{R}^2$ to $\mathbb{R}^2$ given by reflection across the x-axis and that S is the transformation given by reflection across the line $x = y$. Find matrices A, B which represent these transformations. Show that both compositions ST and TS are rotations (find the angles) but that $ST \neq TS$.