Name: Solutions

Please show your work
(10 pts)

Math 330 – Quiz 10
Tuesday, 20 Nov 2012

1. Find the eigenvalues and eigenvectors of the matrix $\begin{bmatrix} 2 & -2 \\ 2 & -3 \end{bmatrix}$.

$$|A-\lambda I| = \begin{vmatrix} 2-\lambda & -2 \\ 2 & -3-\lambda \end{vmatrix} = (2-\lambda)(-3-\lambda)+4 = \lambda^2+\lambda-2$$
$$= (\lambda+2)(\lambda-1) = 0 \quad \text{if } \lambda = -2, 1$$

$\lambda_1 = -2$ $\quad A+2I = \begin{bmatrix} 4 & -2 \\ 2 & -1 \end{bmatrix} \rightsquigarrow \begin{bmatrix} 2 & -1 \\ 0 & 0 \end{bmatrix}$ $\quad 2x - y = 0$, $x = \frac{1}{2}y$ $\quad \vec{x}_1 = \begin{bmatrix} 1/2 \\ 1 \end{bmatrix}$

$\lambda_2 = 1$ $\quad A - I = \begin{bmatrix} 1 & -2 \\ 2 & -4 \end{bmatrix} \rightsquigarrow \begin{bmatrix} 1 & -2 \\ 0 & 0 \end{bmatrix}$ $\quad x - 2y = 0$, $x = 2y$ $\quad \vec{x}_2 = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$

2. Find the cofactor matrix C and the inverse $A^{-1} = \frac{C^T}{\det A}$ where $A = \begin{bmatrix} 3 & 1 & 0 \\ 2 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$.

$c_{11} = (-1)^{1+1}\begin{vmatrix} 0 & 0 \\ 0 & 1 \end{vmatrix} = 0$, $\quad c_{12} = (-1)^{1+2}\begin{vmatrix} 2 & 0 \\ 0 & 1 \end{vmatrix} = -2$, $\quad c_{13} = (-1)^{1+3}\begin{vmatrix} 2 & 0 \\ 0 & 0 \end{vmatrix} = 0$

$c_{21} = (-1)^{2+1}\begin{vmatrix} 1 & 0 \\ 0 & 1 \end{vmatrix} = -1$, $\quad c_{22} = (-1)^{2+2}\begin{vmatrix} 3 & 0 \\ 0 & 1 \end{vmatrix} = 3$, $\quad c_{23} = (-1)^{2+3}\begin{vmatrix} 3 & 1 \\ 0 & 0 \end{vmatrix} = 0$

$c_{31} = (-1)^{3+1}\begin{vmatrix} 1 & 0 \\ 0 & 0 \end{vmatrix} = 0$, $\quad c_{32} = (-1)^{3+2}\begin{vmatrix} 3 & 0 \\ 2 & 0 \end{vmatrix} = 0$, $\quad c_{33} = (-1)^{3+3}\begin{vmatrix} 3 & 1 \\ 2 & 0 \end{vmatrix} = -2$

So $\quad C = \begin{bmatrix} 0 & -2 & 0 \\ -1 & 3 & 0 \\ 0 & 0 & -2 \end{bmatrix}$

Use row 1 cofactor expansion to find det A

$$\det A = a_{11}c_{11} + a_{12}c_{12} + a_{13}c_{13} = 3\cdot 0 + 1(-2) + 0\cdot 0 = -2$$

So $A^{-1} = \frac{1}{\det A} C^T = \frac{1}{2}\begin{bmatrix} 0 & 1 & 0 \\ 2 & -3 & 0 \\ 0 & 0 & 2 \end{bmatrix}$