Name: solutions

Please show your work
(10 pts.)

Math 330 – Quiz 11
Tuesday, 27 Nov 2012

1. Show that the following matrix A is diagonalizable by finding an eigenvalue matrix Λ and an eigenvector matrix S so that $AS = S\Lambda$.

$$A = \begin{bmatrix} 1 & 2 \\ 3 & 0 \end{bmatrix}$$

$\det(A-\lambda I) = \begin{vmatrix} 1-\lambda & 2 \\ 3 & -\lambda \end{vmatrix} = -\lambda(1-\lambda) - 6 = \lambda^2 - \lambda - 6 = (\lambda-3)(\lambda+2) = 0$
if $\lambda = 3, -2$

$\lambda_1 = 3$ $\quad A - 3I = \begin{bmatrix} -2 & 2 \\ 3 & -3 \end{bmatrix}$ $\quad -2x + 2y = 0$ if $y = 1, x = 1$ $\quad$ so $\vec{x}_1 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$

$\lambda_2 = -2$ $\quad A + 2I = \begin{bmatrix} 3 & 2 \\ 3 & 2 \end{bmatrix}$ $\quad 3x + 2y = 0$ If $y = 3, x = -2$ $\quad$ so $\vec{x}_2 = \begin{bmatrix} -2 \\ 3 \end{bmatrix}$

So $\Lambda = \begin{bmatrix} 3 & 0 \\ 0 & -2 \end{bmatrix}$ and $S = \begin{bmatrix} 1 & -2 \\ 1 & 3 \end{bmatrix}$

Check: $\quad AS = \begin{bmatrix} 1 & 2 \\ 3 & 0 \end{bmatrix}\begin{bmatrix} 1 & -2 \\ 1 & 3 \end{bmatrix} = \begin{bmatrix} 3 & 4 \\ 3 & -6 \end{bmatrix} = \begin{bmatrix} 1 & -2 \\ 1 & 3 \end{bmatrix}\begin{bmatrix} 3 & 0 \\ 0 & -2 \end{bmatrix} = S\Lambda$

2. Find a formula for A^n given that A is diagonalizable with $A = S\Lambda S^{-1}$ where

$$A = \begin{bmatrix} -1 & 2 \\ -4 & 5 \end{bmatrix}, \qquad \Lambda = \begin{bmatrix} 1 & 0 \\ 0 & 3 \end{bmatrix}, \qquad S = \begin{bmatrix} 1 & 1 \\ 1 & 2 \end{bmatrix}.$$

$S^{-1} = \begin{bmatrix} 2 & -1 \\ -1 & 1 \end{bmatrix}$ $\qquad \Lambda^n = \begin{bmatrix} 1 & 0 \\ 0 & 3^n \end{bmatrix}$

$A^n = S\Lambda^n S^{-1} = \begin{bmatrix} 1 & 1 \\ 1 & 2 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ 0 & 3^n \end{bmatrix}\begin{bmatrix} 2 & -1 \\ -1 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 3^n \\ 1 & 2\cdot 3^n \end{bmatrix}\begin{bmatrix} 2 & -1 \\ -1 & 1 \end{bmatrix}$
$= \begin{bmatrix} 2-3^n & -1+3^n \\ 2-2\cdot 3^n & -1+2\cdot 3^n \end{bmatrix}$