Name: Solutions

Please show your work
(10 pts.)

Math 330 – Quiz 3
Tuesday, 18 Sept 2012

1. Use Gauss-Jordan elimination to find the inverse of the following matrix A; check your work by showing that $AA^{-1} = I$.

$$\begin{bmatrix} 1 & 2 \\ 2 & 6 \end{bmatrix}$$

$$[A \mid I] = \left[\begin{array}{cc|cc} 1 & 2 & 1 & 0 \\ 2 & 6 & 0 & 1 \end{array}\right] \xrightarrow[R_2-2R_1]{} \left[\begin{array}{cc|cc} 1 & 2 & 1 & 0 \\ 0 & 2 & -2 & 1 \end{array}\right] \xrightarrow[R_1-R_2]{} \left[\begin{array}{cc|cc} 1 & 0 & 3 & -1 \\ 0 & 2 & -2 & 1 \end{array}\right]$$

$$\xrightarrow[\frac{1}{2}*R_2]{} \left[\begin{array}{cc|cc} 1 & 0 & 3 & -1 \\ 0 & 1 & -1 & 1/2 \end{array}\right] = [I \mid A^{-1}]$$

So $A^{-1} = \begin{bmatrix} 3 & -1 \\ -1 & 1/2 \end{bmatrix}$

Check: $AA^{-1} = \begin{bmatrix} 1 & 2 \\ 2 & 6 \end{bmatrix}\begin{bmatrix} 3 & -1 \\ -1 & 1/2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = I$ ✓

2. Find the factorization $A = LU$ for the following matrix A (check that this works!):

$$\begin{bmatrix} 1 & 2 & 3 \\ 2 & 6 & 5 \\ 3 & 4 & 7 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 2 & 3 \\ 2 & 6 & 5 \\ 3 & 4 & 7 \end{bmatrix} \xrightarrow[\substack{R_2-2R_1 \\ R_3-3R_1}]{} \begin{bmatrix} 1 & 2 & 3 \\ 0 & 2 & -1 \\ 0 & -2 & -2 \end{bmatrix} \xrightarrow[R_3+R_1]{} \begin{bmatrix} 1 & 2 & 3 \\ 0 & 2 & -1 \\ 0 & 0 & -3 \end{bmatrix} = U$$

$$L = E_{21}^{-1} E_{31}^{-1} E_{21}^{-1} = \begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 3 & -1 & 1 \end{bmatrix}$$

Check:

$$LU = \begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 3 & -1 & 1 \end{bmatrix}\begin{bmatrix} 1 & 2 & 3 \\ 0 & 2 & -1 \\ 0 & 0 & -3 \end{bmatrix} = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 6 & 5 \\ 3 & 4 & 7 \end{bmatrix} = A$$ ✓