**Equilibrium of Concurrent Forces**

Terence Henriod

TA: Marshall Liddle

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**Abstract**

Forces were balanced using a force table to achieve equilibrium. The forces were then measured and processed to verify that when the sum of all forces is zero, no acceleration results. The forces were processed in terms of magnitude and direction (i.e., as vectors) to verify both Newton’s first law and the properties of vectors. The resulting vectors of the trials in this experiment did in fact have near zero values, thus supporting the theories discussed in the experiment. These resulting vectors were found to be in magnitude and orientation of 0.00 N and 276˚, and 0.07 N and 353˚, respectively. The resulting vectors were within the sensitivity thresholds of the apparatus, which were 0.02 N and 0.10 N respectively, also demonstrating the utility of a test criterion. Because the resulting vectors did not break the sensitivity thresholds, equilibrium was maintained due to the sum of the forces being near zero.

**Theory**

This experiment facilitated the testing of two theories related to Physics: first, that when no net force is applied to an object, no acceleration of said object will result; and second, that some quantities are vector quantities, meaning that they have both a magnitude and direction, and it is imperative to recognize both components to fully describe certain phenomena.

According to Newton’s first law, if the sum of all forces is zero, no acceleration occurs:

[ units: force: Newtons (N), acceleration: meters per second2 (m/ss) ]

By Newton’s Second law, force is equal to mass multiplied by acceleration:

[ units: force: Newtons (N), mass: kilograms (kg), acceleration: meters per second2 ]

Forces are vector quantities, having both magnitude and direction, and as such can be utilized and manipulated and processed both graphically and analytically. Often it is easier to break vectors into their component parts (generally parallel to the axes of the graph) to process them. Sometimes the components are referred to x and y components, others they are labeled as *i* and *j* components. In this manner, vectors can be added:

Analytically, this can be done by finding the component vectors using trigonometry:

and

[ units: magnitude: Newtons (N), angle: degrees (˚) ]

The magnitude of any vector can be found by taking the square root of the sum of its components squared:

[ unit: Newtons (N) ]

The direction of any vector can be found by taking the invers tangent of the ratio of its component vectors:

[ unit: degrees (˚) ]

When manipulated graphically, they are drawn so that their length is drawn at a scale representative of their magnitude, and they are oriented appropriately on the graph to represent their relative direction. Then they can be added or subtracted. This can be done in two ways: the Triangle method and the Parallelogram Method.

The Triangle method simply places the first vector on the graph with its tail at the origin. The next vector is placed with its tail at the head of the first, and the “head to tail” addition process is continued for all vectors to be added. Then the resultant vector is drawn, starting with its tail at the origin and its head placed at the head of the final added vector.

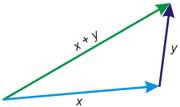


Figure 1: Vector summation by Triangle method, or head to tail addition.

(Image courtesy of: http://illuminations.nctm.org/LessonDetail.aspx?id=L681, 10/3/2012)

In the parallelogram method, two vectors at a time are added by placing both vectors with tails at the origin, and then lines parallel to each vector are drawn starting at the other vector’s head. Where these lines meet is where the head of the resultant vector lies, with its tail at the origin. This can be repeated until all desired vectors have been summed.

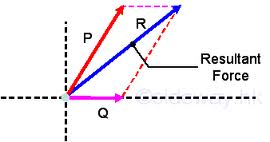


Figure 3: Vector summation by the parallelogram method.

(Image courtesy of: https://encrypted-tbn1.gstatic.com/images?q=tbn:ANd9GcQfHiDraB0AbGSbqzyn4npuc83UdkXkFyE1gWYxBEwn3-Ro6cKB, 10/3/2012)

Vectors can also be multiplied, but this is a topic beyond the scope of this discussion.

Finally, once again, experimental values will be compared to accepted or expected ones with a percent difference approach.

[unit: percent]

It would also be beneficial to discuss the concept of the sensitivity of measuring apparatuses. In this lab the sensitivity was referred to as the Departure From Equilibrium (DFE) threshold, and it was a useful tool to illustrate how measuring instruments are limited in their ability to truly measure a value due to the lack of their precision. This was simply a quantity found by placing weight in small increments to the hanger until it equilibrium of the system was observably disrupted, giving us a sort of maximum error threshold.

*Procedure*

This experiment was performed using a force table, and large disk supported only in the center so that it will tip if the weights it supports are unbalanced. The disc is marked on the rim in degrees so that the position of weights can be determined. Weights are then hung on the table using a system of pulleys and strings, all of which are connected in the center to a ring. This ring acts as a mediator between the strings of different directions, and will move in the direction of the resultant force. The center of the force table has a small point to act as a point of reference for the movement of the central ring. The weights are hung from hangers attached to the strings that run through the pulleys on the outside of the tables.

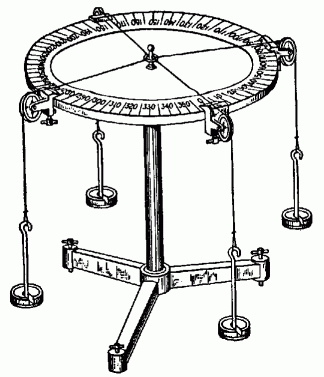


Figure 3: A Force Table.

(Image courtesy of: http://www.lhup.edu/~dsimanek/scenario/labman1/forctabl.htm, 10/3/2012)

Various weights were hung from the hangers, and then the positions of the pulleys were adjusted so that the center of the ring was at the center point of the table.

Two trials were performed, the first following this procedure using three forces (hanger, pulley, & string sets); and again with four forces. Data was recorded, and computations were performed graphically and analytically.

The DFE threshold was found simply by testing the apparatus to see how much weight it took to disturb the equilibrium of the system by placing small increments of weight on one of the hangers until the ring moved from the equilibrium position about 3 mm.

**Data**

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
|  | Magnitude (N) | Direction (degrees) | X-component (N) | Y-component (N) |
| Force A | 2.45 | 0 | 2.45 | 0.00 |
| Force B | 1.96 | 143 | -1.57 | 1.18 |
| Force C | 1.47 | 233 | -0.89 | -1.18 |
| Net Component | | | 0.00 | 0.00 |
| Result | 0.00 | 276 | DFE (N) = | 0.02 |

Table 1: Displays the results of Trial 1, including the magnitude and orientation of the forces that were hung, and their respective component vectors. Also included is the DFE. Note that the resultant forces in either direction overcome the DFE to upset the equilibrium of the system.

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
|  | Magnitude (N) | Direction (degrees) | X-component (N) | Y-component (N) |
| Force A | 2.94 | 0 | 2.94 | 0.00 |
| Force B | 1.96 | 101 | -0.37 | 1.93 |
| Force C | 1.96 | 191 | -1.93 | -0.37 |
| Force D | 1.67 | 250 | -0.57 | -1.57 |
| Net Component | | | 0.08 | -0.01 |
| Result | 0.07 | 353 | DFE (N) = | 0.10 |

Table 2: Displays the results of Trial 2, including the magnitude and orientation of the forces that were hung, and their respective component vectors. Also included is the DFE. Note that the resultant forces in either direction overcome the DFE to upset the equilibrium of the system.

**Computations**

Computations were largely performed by inputting data in Excel. Attached are the results of vector computation performed graphically.

Because the force table system didn’t move when the forces were balanced, it was also assumed that the forces were balanced because the system did not accelerate:

The force (weight) on each hanger was calculated using the total mass of the weight and hanger and the accepted constant of Earth’s gravity:

Vector components were found using the reference angle relative to the positive x axis, opening in the positive direction. The resulting vectors were parallel to the x and y axes respectively.

and

The resultant vector’s magnitude was calculated by summing the component vectors of the system, the full summation computation will not be shown here, rather the results of these sums will be input:

The resulting vector’s angle was found by taking the inverse tangent of the resulting ratios:

Finally, percent difference computed using the DFE value as the “accepted level of error,” and the magnitude of a resultant vector is the “observed value (I will make it a point though that this was a case when it was good to have a percent difference error because it means that the resultant vector is on the favorable side of discrepancy from the DFE value):

*Uncertainty*

This section will appear in future lab reports starting next week, but not this one.

**Results**

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
|  | Magnitude (N) | Direction (degrees) | Net X-Component (N) | Net Y-Component (N) | DFE Threshold (N) | Threshold Exceeded? (Yes/No) |
| Trial 1 | 0.00 | 276 | 0.00 | 0.00 | 0.02 | N |
| Trial 2 | 0.07 | 353 | 0.08 | -0.01 | 0.10 | N |

Table 3: A reiteration of the results.

Figures 1- 5 feature graphical processing of the vectors and are attached at the end of the document.

The results found in this experiment supported the theories that when there is no net force, no acceleration results, and that vectors are quantities of both direction and magnitude. The numerical results can be seen in Table 3, and graphical results are found in Figures 4-7 (which are attached at the end of this document). The sums of the forces were balanced enough (summing to near zero) to maintain equilibrium, with the resultant vector in Trial 1 having a calculated magnitude and orientation of 0.00 N and 276˚, while the resultant vector found in Trial 2 had a calculated magnitude and orientation of 0.07 N and 353˚. The resulting vectors were within the DFE threshold values, which were 0.02 N and 0.10 N respectively. It should be noted that the directions of the analytically found resultant vectors do not necessarily agree with the resultant vectors found by graphing. The error found by graphing was consistent throughout the graphs, but of different orientation than the error found analytically. There were no percent difference values important enough to mention.

*Questions*

There were no questions for this lab.

*Discussion*

The concepts of Newton’s 1st and, to a lesser extent, 2nd laws were tested. This was done by balancing forces using a force table, and collecting the data of the forces’ magnitudes and orientations. The sensitivity of the force table apparatus was found to “give us a test criterion” (Lab, 9/27/2012) to determine if the forces were reasonably balanced. This all occurred in the experiment, as expected: resultant vectors were less in magnitude than the DFEs, with the Trial 1 vector being 0.00 N in magnitude and pointed at 276˚ and the Trial 2 vector being 0.07 N in magnitude and pointed at 353˚, meaning equilibrium of the system was reached through proper vector summation.

These results did confirm the theories in question. The resultant vectors were nearly 0, indicating that when the net force is zero, no acceleration occurs. The resultant vectors were able to be summed to near 0 values, demonstrating that vectors do have both magnitude and directional components, and can be processed using these components. This was verified both graphically and analytically. If the force table apparatus were even more sensitive, i.e. allowed for lower DFE thresholds, even smaller resultant vectors could have been found, allowing for even stronger support of the theories tested.

Sources of error in this experiment could be collectively represented by the DFE values, which in fact were a sort of measure of the maximum error allowed by this experiment. These values were certainly due to variations of friction within the pulleys, and not accounting for the weight of the pulleys and strings in our measurements of the forces applied. While the results were reasonably close, using better strings (smoother, single stranded ones) and accounting for the weights of pulleys and strings would reduce the already minor amount of error observed in this experiment. Error in summing the vectors graphically can be identified by the imperfect capabilities of a human to draw vectors at perfect angles and to perfect scales. Graphing is not an ideal method for vector calculation for this very reason.