**The Scalar Product**

Terence Henriod

TA: Marshall Liddle

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**Abstract**

Two theories were tested in this week’s lab. First, a spring scale was stretched at equally spaced points along an arc, I measured the force, and calculated the work done across each distance to demonstrate the theory that the work done by conservative forces sums to zero when the work is done over a closed path. Second, a block was dragged on a closed path by a spring scale, the force was measured by another student, and the work was calculated to verify that the work done by a non-conservative force simply accumulates, even when the work is done over a closed path.

The results of these experiments did support the theories in question: the total work done in the conservative force portion summed to -0.012 joules, only slightly differing from zero, and on a scale only slightly less precise than the most fine measurements used; the work in the non-conservative force experiment summed to 10.6 joules, only a meager 0.03 joules away from the expected 10.63 joules (found from the average force applied to the block multiplied by the length of the perimeter of the path), which was an acceptable result, given the imprecision of some measurements.

**Theory**

This experiment allowed us to test the theories that the work done by conservative forces on closed paths sums to zero, while the work simply accumulates when done by a non-conservative force on a closed path.

It should be noted that a *conservative force* is a force that does only a certain amount of net work moving an object from one point to another, regardless of the path that object takes. This is why the work done moving an object on a closed path is zero: the object has not been (in a sense) displaced at all, no net movement has occurred. Gravity is the most frequently mentioned example; in this lab the conservative force tested was that of a spring. A *non-conservative force* is a force that does not conserve energy, often letting energy dissipate (irreversibly) into heat energy. Such forces do not allow work to be “taken back” when returning an object to its original position. A prime example is friction.

In general, work is defined as the *scalar product* (also known as the *dot product*) of the force applied to an object and the distance the object moves, in the direction of the applied force, of course. This is accounted for by the dot product, with no manipulation of the vector components necessary. The scalar product is computed in the following manner, combining two vectors to produce a scalar:

[ units: dependent on situation ]

The scalar product can also be defined as the product of the magnitude of two vectors and the cosine of the angel between them:

[ units: dependent on situation ]

Using this definition, work is defined as the dot product for force and the displacement vectors, or, expressed as a more refined, calculus based definition, the integral of the dot product:

[ units: force: Newtons, distance: meters, work: joules ]

Work done over a closed integral results to zero and can be expressed as:

[ units: force: Newtons, distance: meters, work: joules ]

However, these theories were tested using the sums of the work required to cross components of the paths, as opposed to the theoretical integrals. These work components were estimated using the forces measured at certain points along the path (assumed to be the average force applied across the displacement segment), the length of the displacement segment, and the angles between these vector quantities:

[ units: force: Newtons, distance: meters, work: joules ]

The process of averaging was used in this lab, for finding the average force applied across secant lines in the first experiment and to find the average force applied in the direction of motion in the second experiment:

[ units: Newtons ]

A percent difference approach was neglected in this lab due to the theoretical value in experiment one being zero, as well as a lack of a truly theoretical value for experiment two that would have been found using the mass of the block, coefficient of friction between the block and the floor, etc. No other numerical approach to assess error was used either.

*Procedure*

This lab consisted of two experiments: one, to measure the work done by a conservative force on a closed path, and two, to measure the work done by a non-conservative force on a closed path.

The first experiment required a paper for marking the work path, a meter stick and protractor for measuring and straight edge purposes, and a spring scale (with an anchor point) to measure the force exerted along the path.

The paper was secured such that it would lie in front of the anchor point. A 180˚ arc using the protractor was drawn on the paper so that it was far enough from the anchor point that the spring scale was taught when placed at one of the endpoints of the arc. The arc was then marked off into 20˚ intervals, making nine segments, and the line making the diameter of the semi-circle was counted as the tenth distance interval. Secant lines were drawn from endpoint to endpoint of each minor arc, and this was used to approximate the length of the path, as opposed to measuring or calculating the actual arc length of each segment. These, along with the diameter-length segment, were measured and recorded. Then, lines from the anchor point were drawn through each of the endpoints of the minor arcs to represent the direction of the force that would be applied. The angle between the secant lines and the force lines was measured and recorded.

The experiment was then carried out by stretching the end of the spring scale to each endpoint of the minor arcs and recording the force at each point once. The measurements at each endpoint of a minor arc were averaged to represent the average force applied across the path segment. The work was then calculated using the scalar product formula that uses vector magnitudes and the angle between the vectors. The findings for work across each segment were summed and the result was compared with the theoretical outcome.

Figure 1: A schematic of the first experiment.

In the second experiment, a square path was indicated on the floor (with 1m sides), a large block, and a spring scale were used. The block was pulled along the path by the spring scale, and at intermittent points (12 points) the force reading on the spring scale was recorded. The segments along the path were not uniform. The force measurements were assumed to oppose to the force of the friction acting on the block as it was dragged along the square path, giving us reason to accept the measurement on the spring scale as the magnitude of the force applied in pulling the block. The block was pulled around the path once, and the path segments were measured and recorded. The work required to move the block was computed for each segment, and then summed. The same formula for work that was used in the previous segment, except that the cosine of the angle was always assumed to be 1, as the force always acted along the same direction as the block’s motion. It is important to note that movement and force vectors were recorded in accordance with a traditional coordinate system, with some values being negative and some being positive. The work accumulates because when movement is in a negative direction, force is also in a negative direction, resulting in a positive work value. Force was never applied in a direction differing from that of the block’s motion

Figure 2: A schematic of the second experiment

**Data**

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| *i* th Segment | Delta s Approximation to Arc Length (m) | Fi | Fi+1 | Average Force Across Segment (N) | Angle (degrees) | Work (J) |
| 1 | 0.028 | 0.0 | 1.2 | 0.6 | 23 | 0.016 |
| 2 | 0.028 | 1.2 | 2.4 | 1.8 | 43 | 0.037 |
| 3 | 0.028 | 2.4 | 3.2 | 2.8 | 60 | 0.040 |
| 4 | 0.028 | 3.2 | 3.4 | 3.3 | 80 | 0.016 |
| 5 | 0.028 | 3.4 | 3.4 | 3.4 | 90 | 0.000 |
| 6 | 0.028 | 3.4 | 3.0 | 3.2 | 110 | -0.031 |
| 7 | 0.028 | 3.0 | 2.2 | 2.6 | 125 | -0.042 |
| 8 | 0.028 | 2.2 | 1.0 | 1.6 | 142 | -0.035 |
| 9 | 0.028 | 1.0 | 0.0 | 0.5 | 162 | -0.013 |
| 10 | 0.155 | 0.0 | 0.0 | 0.0 | 270 | 0.000 |
|  |  |  |  | Total Work Done (J) | | -0.012 |

Table 1: Data collected from the conservative force experiment.

|  |  |  |  |
| --- | --- | --- | --- |
| Segment | Distance Moved (m) | Force (N) | Work (J) |
| 1 | 0.38 | 3.2 | 1.22 |
| 2 | 0.37 | 2.4 | 0.89 |
| 3 | 0.25 | 2.6 | 0.65 |
| 4 | -0.28 | -2.4 | 0.67 |
| 5 | -0.39 | -2.0 | 0.78 |
| 6 | -0.32 | -3.2 | 1.02 |
| 7 | -0.26 | -3.6 | 0.94 |
| 8 | -0.40 | -3.6 | 1.44 |
| 9 | -0.35 | -2.0 | 0.70 |
| 10 | 0.32 | 2.5 | 0.80 |
| 11 | 0.39 | 2.2 | 0.86 |
| 12 | 0.29 | 2.2 | 0.64 |
|  |  | Total Work (J) | 10.6 |

Table 2: The data collected from the non-conservative force experiment.

**Computations**

Computations were performed through the use of Microsoft Excel. There was no opportunity/need to perform computations using the integrals presented in the theory section, and doing so might not illustrate the object of the lab as well as the simple sums that were used. Sample calculations are as follows:

This is an example computation for the work done across a minor arc from the second minor arc in the first experiment:

= 0.037 J

This is an example of calculating force part of calculating the expected work in experiment two. The absolute value symbol brackets are used to indicate that all forces are added because they are in the same direction as the direction of motion, and therefore included in the sum of force applied:

*Uncertainty*

Three devices were used for collecting measurements in this experiment:

The meter stick. The meter stick was used to measure the length of the secant lines, and did so with a precision of 0.001 meters by being marked off at 0.001 m intervals, and is likely a trustworthy tool, as the meter stick appeared to be well manufactured and uniformly marked. When collecting measurements with the meter stick, I rounded measurements to the nearest millimeter, so I can say with relative confidence that the uncertainty of the meter stick was likely on the order of 0.0005 m, and contributed relatively little to the uncertainty of the results of calculations. The precision used in the calculations was 0.001 m, which was later trumped by less precise measurements, meaning result errors were likely not a product of inadequacy of the meter stick.

The protractor. The protractor was used to measure the angle between displacement vectors and force vectors. The protractor was marked off to indicate single degrees, and again, these appeared to be well done. The protractor measured to a precision of a single degree, and I rounded measurements to the nearest degree. I was led to assume the error of the protractor was 0.5 degrees. The protractor likely did not introduce very much uncertainty to the results, as the precision used by the protractor was again trumped by less precise measurements, again meaning overall uncertainty was not likely due to the protractor.

The spring scale. The spring scale was used to measure the force applied to the objects. The spring scale appeared warn and loose, and the built in adjustor to zero the scale was unreliable. The scale was marked off in 0.2 N increments, and when measuring I rounded to the nearest 0.2 N, amounting to an uncertainty of 0.1 N. This was a relatively imprecise implement, given that readings only spanned about 3 N, with many readings needing to be rounded more than was desirable. The spring scale likely introduce the most error into the calculations due to its lack of precision, overshadowing the other measurement devices in terms of uncertainty.

Overall, I would claim that my calculations have an uncertainty at the level of 0.01 Joules, in accordance with significant figure procedure.

**Results**

Results of the experiment are found in Tables 1 and 3. A summary of the results of interest are displayed in Table 3.

|  |  |  |  |
| --- | --- | --- | --- |
| Experiment | Total Work (J) | Expected Result (J) | Theory Supported? (Yes/No) |
| Work done by a Conservative Force | -0.012 | 0 | Yes |
| Work done by a Non-conservative Force | 10.6 | 10.63 | Yes |

Table 3: A reiteration of the results of the experiments. The expected result for the non-conservative force experiment was found using the average force applied in the direction of movement and the absolute distance the object traveled, and not a true expected value found through calculations involving the mass of the block, coefficients of friction, etc.

As seen in Table 3, both theories of this lab were supported, with a near zero work result of -0.012 J in experiment one, as might be expected for the work done on a closed path by a conservative force when allowing for measurement error, and a total work of 10.6 J in experiment 2 when expecting a result near 10.63 J. In fact, observing that work even accumulated over the closed path verified the theory tested in experiment two.

*Questions*

This question was extracted from the “Report” section at the end of the lab procedure document and not explicitly stated:

“…compare the values in the above two calculations ([experiments one and two])… Think about the two line integrals carefully, and explain why the values should be so very different.”

The values found were -0.012 J and 10.6 J respectively. They are so different because the first work result was done by a conservative force, and the second by a non-conservative force. In the case of the conservative force, while force may have been applied to extend the spring in the first half of the experiment, force was “given back,” in a sense, in the second half when the spring retracted. In the second experiment, no force was “given back,” force was only applied.

*Discussion*

This experiment tested the theories related to the total work done by conservative and non-conservative forces. This was done by using spring scales to measure applied force, and direction of motion was determined by drawn paths. In the first experiment a conservative force (provided by the spring) was measured at various points around a closed path, and in the second experiment, the non-conservative force necessary to counter friction was measured as a block was dragged around a square path. Again: as seen in Table 3, experiment one had a near zero work result of -0.012 J, and a total work of 10.6 J was seen in experiment two.

The results of this experiment supported the theories discussed in this lab. The result in the conservative force experiment was near zero; the result of the non-conservative force experiment indicated that work accumulated. Given that I claim the overall uncertainty of my measurements and calculations to be within 0.01 J, we can see that the result of experiment one appears to be even closer to zero, and the result of the second experiment becomes no less acceptable. However, the results of experiment one would have supported the associated theory even more definitively if the experimental result had not been outside the claimed uncertainty adjustment. However, the value of the result of experiment one is of such small magnitude, it is hard to believe that it would not be a deviation from a theoretical value of zero due to minor errors, measurement or otherwise. The result of experiment two supports its theory strongly, with or without uncertainty adjustments, simply because net work did accumulate through every portion of the path.

Sources of error in this lab would primarily have resulted from faulty measuring equipment and inadequacy of an experimenter’s ability to produce sound measurement readings. Most of the measuring devices were actually acceptable, except for the spring scale. It was loose and worn from years of use and abuse, was likely not manufactured to a high quality standard, and was rather imprecise as far as measuring devices go. As for inadequacy of the experimenter’s ability, the wide range of force readings in experiment two would indicate that the experimenter did not have a good feel for how to appropriately measure the force applied, which is not to disparage the experimenter who took these readings, as it can be difficult to assess the force needed to counter kinetic friction when intermittent stops that produce static friction (which is stronger than kinetic friction) are a requirement of dragging the block along the path.