LRC Circuit Observations

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**Objective**: To observe the behavior of an LRC (inductor-resistor-capacitor) circuit under varying frequencies of applied voltage. We will find the resonance frequency of the LRC circuit by finding the frequency that produces the largest current in the circuit.

**Theory**

First, as a reminder, current, resistance (and as will be discussed, impedance), and voltage are all related in the following manner, known as Ohm’s Law:

[1]

where I is the current is measured in amps (A), voltage V in volts (V), and resistance R in ohms (Ω).  
 A simple RC (Resistor Capacitor) circuit is a circuit composed of a resistor and capacitor circuit connected in series. A capacitor is used to store electrical energy so that it can be rapidly discharged later. This is accomplished by slowly and forcibly separating charge into the plates, positive on one plate, negative on the other. The resistor’s role is to have the capacitor’s energy discharged through it in the form of heat.

The energy stored in a capacitor is modeled by:

[2]

where electrical energy E is measured in Joules, capacitance C in Farads (F), and voltage V in volts (V).

Perhaps of interest, as a circuit is powered and the capacitor charges, the charge in the resistor can be found by:

[3]

where q is the current charge held by the capacitor and q0 is the maximum charge that can be held by the capacitor, both measured in Coulombs (C), *e* is the natural number, t is the time that the capacitor has been charging in seconds (s), and τ is the capacitive time constant, where , with R being resistance and C being capacitance).

An LR circuit is a circuit composed of an inductor (indicated by L) and a resistor. An inductor can be used to store electrical energy, similar to a capacitor, but via a magnetic field, rather than charge separation, and again, energy is dissipated as heat by the resistor. The energy stored in an inductor is modeled by:

[4]

where E is the electrical energy stored in Joules (J), L is the inductance in henries (H), and the current I is measured in amps (A).

An inductor is generally composed of a coil of wires, so that the magnetic field produced as the current flows through the inductor can be “stacked,” allowing for the coils to simultaneously produce several inductive magnetic fields as the current flows steadily through them. These magnetic fields can later produce a self-induced current in the coil via an *emf* that results from the fading magnetic field, which is how the energy is stored.

Combining these two types of circuits produces an LRC (inductor-resistor-capacitor) circuit, which is a circuit that can function as a harmonic oscillator, and produce electrical resonance. As a reminder, resonance is a state where a system oscillates at a frequency such that vibrational energy is stored, allowing a system to oscillate with greater amplitude than other, non-resonance frequencies. In terms of the LRC circuit, this means that different frequencies of current input (i.e. how rapidly an Alternating Current input oscillates) will produce different magnitudes (amplitudes) of measured current in the system, being at a maximum when a resonance state is achieved.

The LRC circuit has the ability to oscillate because energy is passed back and forth between the inductor and the capacitor. Initially, the capacitor is charged and the capacitor is allowed to discharge, current will flow through the inductor, charging the inductor. According to Lenz’ Law, as the current passes through a coil an *emf* in the opposite direction will be induced. This induced *emf* will drive current back towards the capacitor, transferring the energy back into the capacitor. Because an alternating current switches direction continuously, the current from the power source can add or detract from this energy exchanging rhythm between capacitor and inductor by flowing the same direction as the energy exchange to add to it, or by going the opposite way to detract from it. If the frequency of the power source is matched to the natural (or resonance) frequency of the energy exchange, then resonance will be achieved, and the maximum amount of energy can be stored in the circuit. An analogy that might be considered would be a playground swing: if the pushes are timed so that they only go with the direction of the swings current direction of travel, then the swing can eventually reach maximum height; if the pushes are not timed with the rhythm of the swing’s motion, then the rhythm will be disrupted and the swing will be prevented from going very high.

When discussing circuits that utilize AC currents, we must discuss impedance, the AC analog to DC resistance. Because AC currents introduce both self-inductance of voltage due to the magnetic fields of the current as well as the storage of charge due to the capacitance of the conductors of the circuit, resistance is insufficient to describe the opposition to electrical flow a circuit may present when AC currents involve. Thus impedance is what provides the dampening effect in an AC circuit, just as resistance does in the DC circuit. Impedance is measured both in magnitude and phase, the phase being the offset of the impedance from the applied oscillating voltage.

Impedance is, in fact, affected by the resistance of a circuit, but the frequency of the oscillation of applied voltage also matters:

[5]

where impedance Z is measured in ohms (Ω), XL and XC are the Inductive and Capacitive Reactance, respectively, also measured in ohms (Ω), and R is the resistance, again measured in ohms (Ω).

Reactance will only be discussed briefly here, and is simply a measure of a circuit’s opposition to change in current or voltage. In terms of inductive reactance, this is the opposition to a change in current in the circuit, where:

[6]

and capacitive reactance is opposition to changes in voltage in the circuit:

[7]

where L is the inductance measured in Henrys (H), capacitance C is measured in Farads (F), and ω is the angular frequency of the applied voltage/current to the circuit measured in radians per second (rad/s). ω can be found by:

[8]

where f is simple frequency measured in Hertz (Hz), and may be more convenient to work with manually, while ω is more convenient mathematically.

In an LRC circuit, the current and voltage are related similar to the way they are in a simple RC circuit (or similar to Ohm’s law, shown in [1] ), where:

[9]

where I is the current measured in amps (A), voltage V is measured in volts (V), and impedance Z is measured in ohms (Ω), the resistance analog.

Given that the resonance frequency is found by:

[10]

and combining this expression with [5], [6], and [7], it follows that = , thus . In this case, Z is at a minimum. This result, when inserted into [5], helps to demonstrate that current I is at maximum when the AC input is a resonant frequency for the circuit.

**Procedure**

This experiment requires Science Workshop Interface data collection software, a power amplifier, a voltage sensor, 4 patch cords, and a CI-6512 RLC circuit board.

The experiment is set up by connecting the power amplifier to the circuit board such that a resistor, inductor, and capacitor are included in series. The 10Ω resistor, and 100 µF resistor should be used. The voltage meter should be connected on either side of the resistor. The power source should be set to output an approximately 3V sinusoidal output at 10 Hz. The “Circuit Scope” window should be set to collect 1000 samples/second.

Data collection will proceed as follows: The power amplifier will be turned on, and the frequency should be set to 10 Hz in the signal generator window. Using the “Smart Cursor,” the peak voltage should be found from the red trace that will appear in the “Circuit Scope” window in the Science Workshop application. The frequency should then be increased by 10 Hz, and the peak voltage again recorded. This process should be recorded up to 50 Hz. At this point, the scope should be set to collect 5000 samples/second. Peak voltage readings will again be collected at 10 Hz increments up to 150 Hz. Data should be stored in an appropriate table In this table, values for the current should be calculated by dividing the measured voltage by the resistance.

Then using the data table, intuitively find a frequency likely to be a resonant one. This should be a frequency near the ones in the table where the output voltage and the voltage across the resistor are in phase. Make fine adjustments to the output voltage to find the resonance frequency. This can be done by switching the scope window to X-Y mode, which, using input data from the power amplifier and the voltmeter, will display the phases of the two voltages. When the scope displays an oblong shape, the two are out of phase, but when the two are in phase, a diagonal line should appear. A second data table should be used to store the values associated with this part of the experiment, including the inductance rating of the inductor, the resistance of the resistor, the capacitance of the capacitor, the linear frequency (the input frequency of the power amplifier at resonance), the converted resonant frequency to an angular frequency, and the theoretical angular frequency (found by [6]). Finally, the current should be graphed against the linear frequencies recorded.