Official Sovereign Debt*

Cristina Arellano

Federal Reserve Bank of Minneapolis
and University of Minnesota

Leonardo Barreto

University of Minnesota and
Federal Reserve Bank of Minneapolis

November 2025

Abstract

This paper studies sovereign debt from official lenders empirically and theoretically. Official sovereign debt is more than half of the total sovereign debt in emerging markets and tends to flow in during default episodes. We develop a model with a sovereign borrowing from official and private lenders that can partially default on each of its debts. A fraction of the defaulted debt accumulates during a default episode, which resolves when the sovereign pays back its accrued obligations. Official debt is longer-term and more concessional during defaults than private debt, and the prices of all debts compensate lenders for default losses. The contractual differences across debts allow our model to rationalize the stylized facts of emerging markets. Counterfactual analysis finds the feasibility of Pareto improving voluntary swaps that exchange one type of debt for another one. Our work rationalizes the involvement of official debt in the resolution of sovereign defaults.

*The views expressed herein are those of the authors and not necessarily those of the Federal Reserve Bank of Minneapolis or the Federal Reserve System. Contact information: arellano.cristina@gmail.com; barre843@umn.edu

1 Introduction

A large portion of the sovereign debt in emerging markets is held by official lenders, including bilateral loans with other sovereign governments and loans from multilateral organizations. The empirical work of Horn, Reinhart, and Trebesch (2020) emphasizes the importance of official lending for many countries historically and its role in coping with adverse shocks. Theoretical work on sovereign debt, however, has mainly focused on debt and default problems in contracts with private creditors. This paper provides an integrated framework with official and private debts and explores the role of official debt in helping during episodes of financial distress.

We start by analyzing a panel dataset covering 50 years and 30 emerging markets that contains debt series and their decomposition across official and private creditors. Building on the accounting framework of Arellano, Mateos-Planas, and Ríos-Rull (2023), we analyze official and private debts during partial defaults and default episodes. We find that official debt accounts for more than half of the total external sovereign debt for these countries. We document that official debt tends to grow during default episodes and accounts for much of the dynamics of total debt during these events. Private debt, in contrast, remains relatively stable during these episodes. We also present evidence that official lending has been an important component in the resolution of default episodes, such as the Brady Plan of the early 1990s.

We then develop a sovereign default model with official and private debts. Our framework consists of a sovereign in a small open economy that faces a stochastic endowment stream and can choose to partially default selectively on its coupon payments on official or private debt. Official debt differs from private debt in that it is of longer duration and more concessional, as it calls for lower recoveries during defaults. When the sovereign partially defaults on official or private debt, a fraction of that amount, which depends on a recovery factor parameter that is debt-specific, accumulates as arrears and is added to the total debt next period. Partial default is costly because it induces resource costs that depend on the intensity of the default, yet it is a flexible policy since the government decides the intensity of default, separately for each type of debt, as well as the length of the default spell. The sovereign raises funds by borrowing from official and private lenders at interest rates that compensate for potential default losses. Borrowing with these two assets is always possible, even during default episodes.

An important aspect of our framework is that partial default on each type of debt is a period-by-period decision by the sovereign and borrowing is permitted during default.[1] We show that partial

1. These features are different from traditional sovereign default theory with multiple debts, as in Arellano and Rama-

default incentives are higher when the payments due on official and private debt are high, as well as when bond prices on new borrowings of both types of debts are low. Bond prices matter for partial default because they encode the value of the accumulated debt in arrears; a low price lowers the value of those claims and increases the benefit to the sovereign for defaulting today because less will be repaid in expected value.

The intensity and duration of default episodes are endogenous and depend on the shape of the bond price functions and the dynamics of debts. A highly indebted sovereign may choose to default on most of its debt, but will tend to deleverage over time and reach a state with less debt and no default. The elasticities of bond prices with respect to debt tend to be unfavorable during defaults, which provides the sovereign with incentives to deleverage and exit the default. We find that during the deleveraging process, the sovereign tends to reduce private debt first by contracting consumption and temporarily issuing official debt, and only later reduce official debt. This pecking order reflects that the bond price function of the official debt tends to be more favorable. Importantly these dynamics resemble the data in emerging markets during default episodes.

A main theoretical finding is that in our model official debt gives greater debt capacity to the sovereign relative to private debt. We characterize this result theoretically in a simplified model economy with no shocks, assuming linear utility, and zero recovery factors upon a partial default. We show that borrowing with official debt expands the budget set of the sovereign more than borrowing with private debt. This is because official debt is a long-term duration asset, and default is a period-by-period decision. Official contracts can effectively constrain future governments from borrowing, as any pledgeable resources in the future, namely the resource costs from a potential default default, can be used to increase the commitment to a long-term official contract that includes coupon payments during these future periods. In contrast, only the one-period ahead default cost provides commitment for private short-term contracts. Future periods' default costs are irrelevant to the commitment of the short-term private contract because, after default on short-term debt, the sovereign can borrow fresh loans again with new private lenders. We also relate to traditional sovereign default theory and show that the debt capacity of both assets would be the same under the common assumption of full default, no borrowing during defaults, and permanent costs.

We perform a quantitative evaluation of the model and map it to emerging market data. We use unconditional moments in the parameterization of the model and show that the model can reproduce

narayanan (2012), Hatchondo, Martinez, and Sosa-Padilla (2016) and Aguiar et al. (2019), where default is assumed to be a long-lasting decision that eliminates current and future obligations of both types of debts and precludes any borrowing.

salient patterns during default episodes. We target first and second moments of official and private debt, debt service, and partial default. Our moment-matching process recovers parameters for the duration of debts, the recovery factors, and default costs. It finds that official debt is of longer duration and more concessional. Our baseline model reproduces an economy with debt ratios as in the data; official debt is about 2/3 of the total debt. Moreover, we show that in the model, as in the data, debt grows during defaults, and more so for official debt. The magnitudes of these increases are very similar to those in the data, which provides an important validation of the model. We also show that in the model, when the sovereign exits default episodes, it first reduces private debt consistent with the dynamics in the data.

We use the baseline model to perform counterfactuals. We first evaluate the feasibility of voluntary swaps of official and private debts that generate Pareto improvements. We find a sizable region of the state space where swaps are feasible, especially when private debt and default risk are high, highlighting that official debt can play an important role in resolving financial distress events. Second, we study the design of official contracts by comparing our baseline with economies that feature official debt that is shorter-term and less concessional. These experiments are motivated by various liquidity programs from multilateral organizations. We find limited welfare benefits from these programs and instead find that the best design consists of long-duration bonds.

Literature Review. Our work contributes to the literature on official lending. Horn, Reinhart, and Trebesch (2020) documents how extensive official lending has historically been for many countries around the world. They make a compelling case that an important role for official debt is in coping with the economic consequences of disasters, including natural and financial adverse shocks. Schlegl, Trebesch, and Wright (2019) studies the seniority of official and private debt. Using measures of debt in arrears and haircuts from default episodes, they argue that bilateral official lenders are more junior than private debt creditors. The increasing role of official Chinese lending to emerging markets since the 2000s, as documented by Horn, Reinhart, and Trebesch (2021), has also sparked some work focusing on official loans from China.[2] Our work complements these empirical findings by using a comprehensive data set on official and private debt and arrears. Our work documents a novel property, namely that official debt is used more heavily during default episodes.

Some theoretical work on official debt has studied multilateral lending and assumed that these loans are not defaultable. Boz (2011) and Kirsch and Rühmkorf (2017) enrich a sovereign default model with this type of multilateral lending and show that it can be useful for providing insurance. Our work on

2. See also Clayton et al. (2025) on the role of China's bond markets more broadly in international financial markets.

official lending also finds that this lending is useful in times of crises, but unlike previous work and consistent with evidence, it allows for this debt to also be defaultable.

A main property of official lending is its long maturity. As such, our work contributes to the large literature that has analyzed the interactions between default risk and the maturity of sovereign debt. Hatchondo and Martinez (2009) and Chatterjee and Eyigungor (2012) show that although the inclusion of long-term bonds improves the quantitative performance of sovereign debt models, it also tends to decrease welfare because of the so-called "debt dilution problem." Consistent with this work, Arellano and Ramanarayanan (2012) argue that short-term debt is better at providing incentives to repay, although long-term debt can provide hedging benefits. Aguiar et al. (2019) shows that these incentive benefits imply that the sovereign should actively manage only short-term debt while remaining passive in long-term bond markets, when income fluctuations are not a concern. Hatchondo, Martinez, and Sosa-Padilla (2016) studies the impact of debt dilution and shows that eliminating dilution can reduce default risk and increase welfare substantially.[3] The strength of the debt dilution force in canonical sovereign debt models also implies that upon debt restructurings, maturity should be shortened, while empirically maturity is generally extended in the data. Sánchez, Sapriza, and Yurdagul (2018) and Mihalache (2020) argue that additional elements, such as extended financial market exclusions or lenders' aversion to face value haircuts, are needed for the model to deliver maturity extensions during restructurings.

We re-consider these interactions in the framework of rich default episodes and partial default of Arellano, Mateos-Planas, and Ríos-Rull (2023). We find that studying maturity in this framework overturns the conventional result that short-term debt is preferred because it provides more repayment incentives and should be used upon exiting default episodes. In our framework, official long-term debt tends to welfare dominate short-term debt because it carries higher debt capacity, and maturity is extended upon exiting default episodes. These results are relevant because the partial default framework can better resemble emerging market data in that default episodes are long and do not result in a reduction in debt or a shortening of maturity. In our model, the sovereign exits defaults with more long-term official debt.

Our findings of the possibility of Pareto improvements through voluntary debt swaps relate to those in Hatchondo, Martinez, and Padilla (2014). They study voluntary restructurings in a framework of a single long-term defaultable bond and show that voluntary debt reductions can arise in equilibrium because the sovereign may borrow loans that reduce the value of the legacy debt. In our model with

3. Other important studies on maturity and sovereign default risk include, Dovis (2019), Bocola and Dovis (2019), and Bigio, Nuño, and Passadore (2023).

two types of debts, these forces are compounded as new private or official loans may dilute the value of both, private and official legacy debts.[4] Moreover, our emphasis on official lenders alleviates concerns in Bulow, Rogoff, and Dornbusch (1988) about the country being unable to obtain any benefits from buybacks, as official lenders can help coordinate the distribution of gains. We also show that the type of swaps we consider, which reduce private debt obligations with an increase in official debt, have empirical relevance in salient examples like the Brady Plan, which we discuss in the empirical section.

2 Empirical Properties of Official and Private Debt

In this section, we document some properties of sovereign debt from official and private creditors and of sovereign partial defaults using 50 years of data from emerging markets. We will extend the analysis in Arellano, Mateos-Planas, and Ríos-Rull (2023) (thereafter AMR) by focusing on differential patterns of debt across creditor type.

2.1 Accounting

To fix ideas for our work, we will revisit the accounting framework of AMR and apply it to our case of analyzing disaggregated debt from private and official lenders. We will apply this accounting framework to the data from emerging markets and to the simulated data from the model.

Flow Financial Variables. Each period, the sovereign owes official lenders an amount $\tilde{f}_t$ and private lenders an amount $\tilde{b}_t$, which are the sum of all the coupons from past issuances due at t. As we will see later, these official and private debts due include not only the promised coupons at t from newly issued bonds in previous periods but also the current obligations that result from past partial defaults. We consider a selective and flexible partial default policy that is applied to the payment dues on official and private debt, given by d_t^f and d_t^b respectively. Defaults $\{d_t^f, d_t^b\}$ imply that the sovereign pays in period t the amount $(1 - d_t^f)\tilde{f}_t$ to official lenders and $(1 - d_t^b)\tilde{b}_t$ to private lenders, and does not pay $d_t^f \tilde{f}_t$ and $d_t^b \tilde{b}_t$. Given the default policies, debt service for official and private debt are $(1 - d_t^f)\tilde{f}_t$ and $(1 - d_t^b)\tilde{b}_t$, debt service for total debt is the sum of these two debt services. Partial default for each type of debt is defined as the fraction of the debt due defaulted on and therefore equals d_t^f and d_t^b for official and private debt. Partial default for total debt is defined as $d_t = (d_t^b \tilde{b}_t + d_t^f \tilde{f}_t)/(\tilde{b}_t + \tilde{f}_t)$.

4. Also related is Aguiar and Amador (2024) which studies the feasibility of swaps across debts of different maturities in the presence of self-fulfilling runs and finds additional forces that can lead to feasible swaps.

Long-Term Official and Private Debt with Partial Default. We map the data into a tractable structure for two long-term debt contracts with official and private lenders that consist of perpetuity bonds with coupon payments that decay, as in Hatchondo and Martinez (2009), and with recovery rates as in AMR. We allow for different durations and recoveries for each type of debt by considering different decay rates for the bonds, ϑ^f and ϑ^b and differential recovery factors, κ^f and κ^b, for official and private debts respectively. We will show that in this structure, the sovereign's states every period are the official and private debts, $\{f, b\}$.

Consider a type of debt a. A borrowing contract specifies a price q_t^a and a value ℓ_t^a such that the sovereign receives $q_t^a \ell_t^a$ units in period t and promises to pay, conditional on not defaulting, $R^a(\vartheta^a)^{n-1}\ell_t^a$ units in every future period $t+n$ for $n = 1, 2, \ldots, \infty$. The coupon rate R^a, which we set to $R^a = R - \vartheta^a$, is a normalization that has no bearing on the analysis; it only changes the units of the debt due, such that $\tilde{a}_t = R^a a_t$. These settings mean that the default-free discount price for each contract is 1, where R is the risk-free rate. These contracts are tractable because they encode a rich structure of debt issuances into one state variable for each type of debt which evolves with a law of motion. Given a type of debt a, if sovereign that in period t pays in full its debts due, $R^a a_t$, and borrows ℓ_t^a, it will have in period $t+1$ a debt due equal to $a_{t+1} = \vartheta^a a_t + \ell_t^a$.

We assume that partial default of intensity d_t^a for each type of debt a, reduces the debt service and can trigger defaults on all future coupons, encoded in the legacy debt $\vartheta^a a_t$, of intensity $\mu^a d_t^a$. We interpret the parameter μ^a as arising from default acceleration clauses, which are common in bonds, and also to reflect that many restructurings include bonds with streams of payoffs due in the future.[5] As in AMR, we assume that the defaulted coupons and defaults on legacy debt, namely $R^a d_t^a a_t + \vartheta^a \mu^a d_t^a a_t$, result in new obligations, $\kappa^a d_t^a a_t$, that are due in the future. The factor κ^a is a parameter that captures the empirical observation that during default episodes, sovereigns accumulate their defaulted debt and, in some cases, restructure their obligations with their creditors. All else equal, contracts with lower κ^a are more concessional: defaults on those contracts result in higher discharge of debt.

The sovereign trades two types of debts, one with official lenders f and another one with private lenders b, which are the debt states of the sovereign. The duration and recovery of the contract is debt specific, and the laws of motion for the debts incorporate the legacy debts, the accumulation of defaulted

5. Acceleration clauses are a feature in sovereign debt contracts that entitle creditors to accelerate unmatured principal following a default event. They are common in bonds issued under New York law and typically require a minority vote of at least 25% of the value. Accelerations can also be revoked (de-accelerated) by majority bondholders. See Das, Papaioannou, and Trebesch (2012) and Stefanescu (2016).

coupons, and new borrowing:

$$f_{t+1} = \vartheta^f(1-\mu^f d_t^f)f_t + \kappa^f\ d_t^f\ f_t + \ell_t^f \tag{1}$$
$$b_{t+1} = \vartheta^b(1-\mu^b d_t^b)b_t + \kappa^b\ d_t^a\ b_t + \ell_t^b$$

Note that partial default does not necessarily reduce the debt. Debt can actually increase when recovery factors are sufficiently high because the defaulted coupons are accumulated with interest. Also, debts can increase if new borrowings ℓ_t^f and ℓ_t^b are positive. As we document below, we find differential patterns of official and private debt during partial defaults.

Debt, Spreads, and Default Episodes. We measure the level of official and private debts at t as the present value of the contractual payments due with flows discounted at the risk-free gross interest rate R. With our bond structure, the level of official and private debts for the sovereign are the end-of-the-period states $\{f_{t+1}, b_{t+1}\}$. We define the sovereign spreads on official and private debts, $\{s_t^f, s_t^b\}$, as the difference between the yield-to-maturity and the risk-free rate for each type of contract. As is standard, we use the market values of the debt and the streams of contractual payments to define the yield-to-maturities, which are the constant discount rates that equates these two. Applying these standard formulas, the sovereign spreads are inversely related to the bond prices and equal $s_t^a = (R-\vartheta^a)\left(\frac{1}{q_t^a}-1\right)$, for $a \in \{f, b\}$.

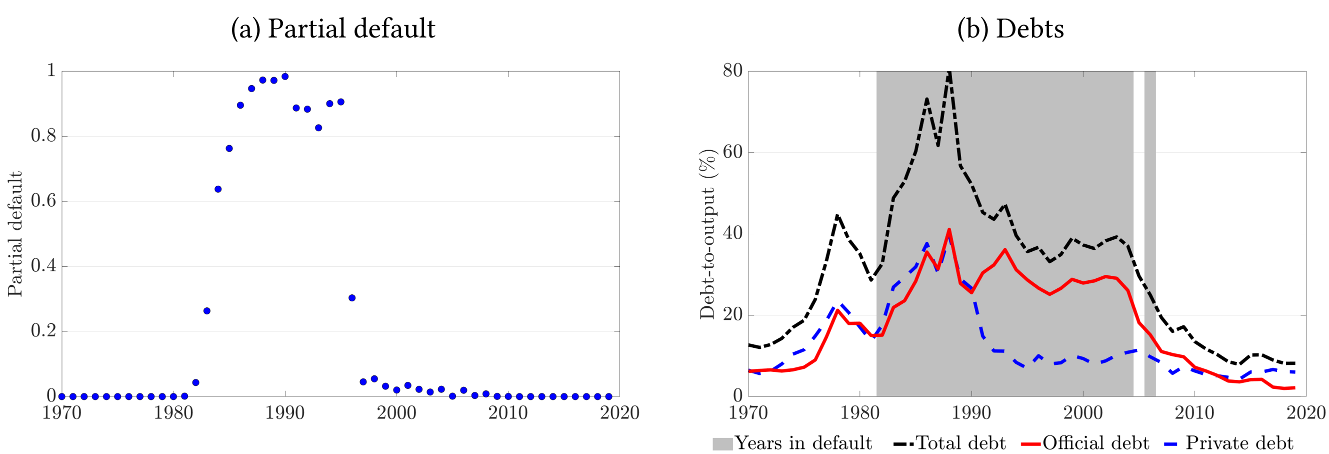
We flag a default episode as a sequence of periods with consecutive positive partial defaults and define its length by the number of such periods. An episode of length $N+1$, which starts in period t and ends in period $t+N$, has $d_{t+j} > 0$ for $j = 0, 1, \ldots, N$ and $d_{t-1} = d_{t+N+1} = 0$. The sequences of official and private debt level, debt service, as well as the sequence of partial default for official, private, and total, in the default episode, are given by $\{a_{t+j}, (1-d_{t+j})(R-\vartheta^a)a_{t+j}, d_{t+j}^a, d_{t+j}\}$ for $a \in \{f, b\}$ and $j = 0, 1, \ldots, N$.

2.2 Empirical Findings

We use the debt statistics from the World Development Indicators (WDI), International Debt Statistics (IDS), and the Debtor Reporting System, all from the World Bank, to empirically measure the variables of interest in our accounting framework at an annual frequency. From these data, we use the debt obligations for the government, defined as public and publicly guaranteed (PPG), for both flow and stock variables. We focus on the total debt obligations, as well as the decomposition across these obligations between official and private credit. Debt obligations with private creditors include debt in the form of bonds and loans, and trade credit, and debt with official creditors includes loans with bilateral governments and multinational organizations. We also collect data on Gross Domestic Product in constant dollars which we log and linearly detrend. We also use government EMBI+ spreads from the Global Financial Database. The dataset is annual and corresponds to a panel of 30 emerging countries from 1970 to 2019. Appendix A contains the list of countries and additional details on the data sources.

2.2.1 Case Studies

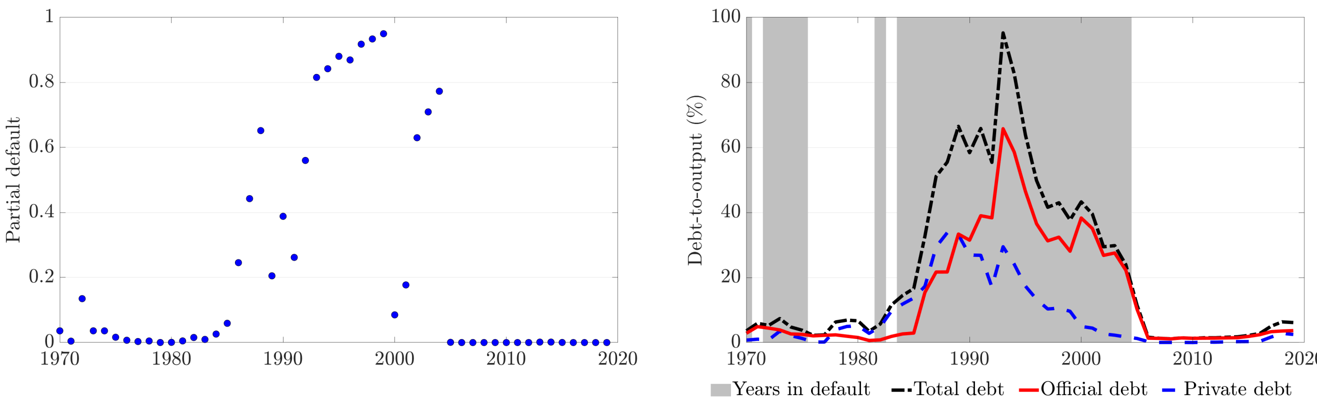
Figure 1: Peru: Partial Default, Official, Private, and Total Debt



Notes: Partial default is the ratio of total debt due in arrears over the sum to total debt service and debt in arrears due. Debt is external debt that is public and publicly guaranteed. Total debt corresponds to the total external, official is the sum of debt from bilateral and multilateral creditors, and private is the sum of debt for bonds and loans from private creditors, and trade credit. Data is from the World Bank.

We start by describing the times series properties of partial default and total, official, and private government debt for two countries, Peru and Nigeria. These countries feature time series patterns that are similar to the average patterns across countries. Figure 1 plots the times series for Peru; the left panel in the figure has the time series for partial default, and the right panel contains the time series for the debts. The left panel shows that Peru had a long default event, with partial default increasing from the early 1980s from 0 to about 1 in 1990. Partial default fell after that but remained positive well into the early 2000s. The right panel plots the three debt series, the black dotted line corresponds to the total debt to output, the red solid line is the official debt to output and the blue dashed line is the private debt to output. The sum of official debt and private debt is the total debt. The figure also contains shaded bars, that correspond to the periods of positive partial default. Before the default episode started, the level of

Figure 2: Nigeria: Partial Default, Official, Private, and Total Debt



Notes: See notes of Figure 1

official and private debt in Peru was similar and equal to about 15% of output. When the default episode starts, both debts grow and reach close to 40% in the late 1980s. The official debt remains elevated until the end of the episode at about 30% of output. The private debt in contrast falls during the episode and remains at about 10% of output for much of the latter part of the episode. The total debt at the end of the default episode is similar to before the beginning of the episode, but this end level is largely composed of official debt, in contrast to the beginning, where the shares of official and private were very similar. After the episode, official debt falls as well.

Figure 2 plots the time series for Nigeria. The structure of the figure is the same as for Peru. Nigeria experienced four default episodes according to our accounting. We will focus on the long episode that starts in the mid-1980s and runs through the mid-2000s. The left panel shows that partial default starts small and increases during the episode reaching its peak in the mid-1990s. Early 2000s partial default is minor but in the mid-2000s it increases again. The right panel shows that the patterns of debts in Nigeria were similar to those in Peru with some differences. Right before the default episode starts, Nigeria had mainly private government debt, about 15% of output. The beginning of the default episode features a rise in both official and private debt. By the late 1980s, private debt stops rising but official debt rises to about 60% of output. At the end of the default episode, Nigerian debt is mainly official, at about 25% of output. After the episode, official debt falls too.

These examples illustrate that the dynamics of official debt are crucial for understanding the evolution of debt during default episodes. Official debt is a major source of financing during default episodes, and in some cases substitutes the use of private debt. Moreover, the increase in total debt is driven mainly by official debt and private debt tends to fall earlier in the episode relative to official debt.

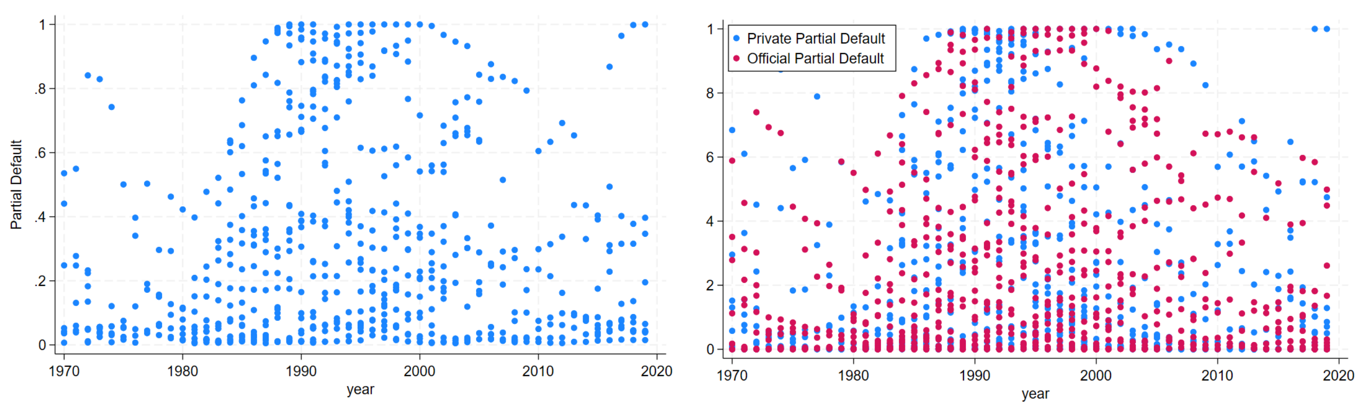
2.2.2 Descriptive Statistics

We now describe the panel data and start with the time series for partial default, for total, official, and private debt for the panel of countries. Panel (a) in Figure 3 plots the time series of partial default (conditional on positive) d_t and panel (b) plots partial default for official and private debts, d_t^b, d_t^f, for all the countries. The plots are conditional on positive partial default, which occurs with a frequency of 40%. There is a wide dispersion in partial defaults and partial defaults are very correlated. Moreover, during the 1980s and 1990s many countries partially defaulted on official and private debts.

We summarize these series in Table 1. Partial default, conditional on positive, is on average 32%, 31%, and 35%, for total, official, and private debt respectively. Partial default is highly volatile, with an average volatility across countries of 24%, 22%, and 27% for total, official, and private. All the partial default series are correlated, as reported in the bottom panel of the table. The mean correlation across countries between partial defaults on official and private debt is 62%, which leads to strong correlations with partial default on total debt, both at 85%.

Figure 3: Partial Default for Total, Official, and Private Debt

(a) Total Debt (b) Official and Private Debt



Notes: Partial default across countries and time. Panel (a) contains partial default based on total debt conditional on positive default. Panel (b) reports partial default separately for official and private debt. See the notes of Table 1 for more detailed descriptions of the series.

Table 1 also reports the first and second moments for debt levels and debt service for total, official, and private, all relative to output. The mean total debt to output in the panel data is 33%. The share of official debt is 61% and the share of private debt is 39%; official debt to output is 20% on average and private debt to output is 13%. Total debt service is 3.5% of output, about 50% of this debt service is paid to official lenders and half of it to private lenders. An interesting feature of this data is that although official debt is about 50% higher than private debt, the debt service of official debt is smaller than that for private debt. As we explore further in the calibration of our quantitative model, a higher level of debt relative to the coupon payments is consistent with longer duration debt.

The second column in Table 1 reports the standard deviations of the debts. The standard deviations reported are the average ones across the countries in the sample. The volatility of the debt is high, with comparable coefficients of variation across official and private debt. The volatility of debt service is 2% and about half of that for official and private. Official and private debt also tend to move together, with a correlation of 42%, which makes them very correlated with total debt (88% and 73% respectively). These last two number are not reported in the table

Table 1: Partial Defaults and Debts

		Mean	Std. dev.
Partial default | >0			
Total		32	24
Official		31	22
Private		35	27
Debt to output			
Total		33	18
Official		20	12
Private		13	8
Debt service to output			
Total		3.5	2.0
Official		1.6	1.0
Private		1.9	1.6
Corr. (official partial default, private partial default)	62		
Corr. (official debt, private debt)	42		

Notes: Data is from the World Bank Databases and measured as public and publicly guaranteed (PPG). Total corresponds to the total external PPG series. Official is the sum of debt from bilateral and multilateral. Private is the sum of debt for bonds, loans, and trade credit with private creditors. Partial default is the ratio of debt in arrears over the sum of debt service and debt in arrears for total, official, and private. The standard deviations are means across countries of the statistics using country time series data. All variables are expressed in %.

Official Debt and Default. We now assess how various variables of interest vary with partial default. Table 2 reports means across states with no partial default and positive partial default, based on total debt. Debt to output is about 20% higher when partial default is positive (44% vs 24%). The majority of this increase is due to an increase in official debt. Official debt to output increases by 16% while private debt to output increases only by 4%. Interestingly, we also find debt service to output increases on average with partial default—although the government is not paying all of its debt due, the higher debt implies that the government is paying more for servicing the non-defaulted portion of the coupons. The increase is more for official debt because of the additional inflows of this type of debt during these times. The table also illustrates that periods with positive partial default are associated with higher sovereign spreads and lower output; spreads are about 7% higher with partial default and output is 5% lower.[6]

We now further decompose the patterns of debt across finer partial default bins. In Figure 4, we report official and private debts across four bins for partial default. The bin *No default* contains the observations with no partial default. The bin *Small* contains observations with positive partial default but below the 25 percentile. The bin *Medium* contains observations with partial default between the 25 and 75 percentile. The bin *Large* contains the observations with partial default above the 75 percentile. The bars are the means of official and private debt to output across bins; the red bars correspond to official debt and the blue bars correspond to private debt. The figure illustrates that both debts increase with partial default and that the increase is sharper for official debt. Official debt is about 25 percentage points higher when partial default is in the top quartile (bin 4) relative to when partial default is zero (bin 0). Private debt in contrast is about 8% higher when partial default is in the top quartile relative to when it is zero.

We now study the properties of default episodes for the 30 emerging markets, by analyzing dynamics within default episodes for the variables of interest. Using our accounting framework we measure 62 default episodes in our dataset. The average length of the default episode is 10 years, but many defaults are shorter, about 35% of default episodes last less than or equal to 2 years.

Table 3 reports the patterns of partial default and debt to output for total debt, official debt, and private debt during default episodes. We also report the patterns of spreads and output. We report average statistics for these variables for the period before the start of the default episode, labeled *Before*, the first period of the default episode, which we label *Beginning*, the middle of the default episode, which

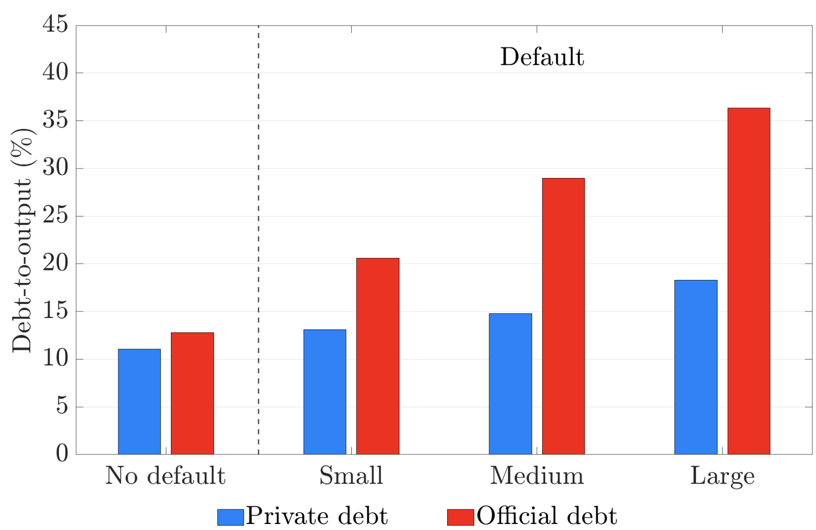
6. In the Appendix B, we show that these patterns are not driven by fluctuations in output. These conditional statistics for debts are very similar when constructed relative to trend output. We also show that the patterns of multilateral and bilateral debt are similar to that of aggregated official debt.

Table 2: Default Flag: Total, Official, and Private Debt

	No default	Partial default > 0
Debt to output		
Total	24	44
Official	13	29
Private	11	15
Debt service to output		
Total	3.0	4.1
Official	1.2	2.1
Private	1.8	2.0
Spreads	4	11
Output	2	-3

Notes: The statistics are means of the variables in the first column after partitioning the panel data set across two bins based on partial default based on total debt. All variables are expressed in %. The "No default" bin has all the observations with zero partial default; the "Partial default > 0" bin has the observations with positive partial default. Output is logged and detrended using the Hodrick-Prescott filter with a multiplier of 6.25. We measure private spreads with the spread series of the Emerging Market Bond Index (EMBI+) from the Global Financial Database for each of the countries in our sample. See also the notes of Table 1 for additional details.

Figure 4: Official and Private Debt across Partial Default Bins



Notes: Each bar shows the mean debt-to-output ratio within a given partial default bin. Red bars represent official debt and blue bars represent private debt. The "No default" bin corresponds to observations with no partial default. The "Small" bin includes observations with positive partial default below the 25th percentile, the "Medium" bin covers defaults between the 25th and 75th percentiles, and the "Large" bin contains defaults above the 75th percentile.

Table 3: Dynamics during Default Episodes

	Before	Beginning	Middle	After
Partial default	0	14	26	0
Official	0	8	22	0
Private	0	14	23	0
Debt to output (in %)				
Total	33	35	40	33
Official	17	18	24	19
Private	16	17	17	14
Spreads	11	21	16	5
Output	0	-2	-5	-3

Notes: The dynamics of the default episode are averages across the 62 episodes in our sample for the variables of interest. *Before* is the period before the start of the episode; *Beginning* is the first period of the episode; *Middle* is the midpoint of the episode; *After* is the period when partial default returns to zero. Debt is reported relative to output; output is logged and linearly detrended and reported relative to the level before the episode. See notes in Table 1 for the definitions of the variables.

we label *Middle*, and the period after the end of the episode, when partial default returns to zero, which we label *After*.[7] As in AMR, we find that partial default on total debt and total debt to output feature hump shape patterns within episodes. The patterns of partial default on official and private debt are very similar to those on total debt within the episode, illustrating the strong co-movement among these variables discussed above. In contrast, the dynamics across debt types feature some distinct patterns and implications. First, the decomposition shows that the dynamics of total debt are mainly driven by the dynamics of official debt. Official debt to output increases by about 7% during the episode while private debt to output increases only by 1%. This evidence reinforces the point that official debt flows in during sovereign defaults. A second point is that the deleveraging process prior to the end of the default episode occurs by reducing both types of debt. In fact, arguably private debt is reduced more aggressively as it reaches a level below that observed before the default episode, while official debt continues to be elevated. The table also shows that spreads feature a hump-shaped pattern while output features a U -pattern. In Appendix B, we also provide robustness results. We show that the patterns of official and private debt across defaults are similar when defining debt ratios with trend output. We also find that within official debt, both bilateral and multilateral debts increase with default.

7. We define the middle of the episode as the total length of the episode divided by 2, rounded to the nearest integer.

Restructurings under the Brady Plan. Many emerging markets in our sample experienced sizable debt crises during the 1980s, which were resolved under the Brady Plan. The Brady Plan provides an interesting example of official creditor involvement for the resolution of defaults. Here we review some of these details to shed light on how official debt increases during default episodes.

By late 1980s, many countries had substantial private debts with commercial banks and much of it was in default. The debt crises had been evolving for most of the decade and included multiple rounds of unsuccessful restructurings, with countries defaulting on the restructured debt numerous times. This multi-country decade-long crisis was resolved under the Brady Plan, named after U.S. Treasury Secretary Nicholas Brady. As described by Truman (2023), the program involved not only the emerging market sovereigns and their private lenders, but a collection of official lenders, notably the International Monetary Fund, the World Bank, lenders from The Paris Club, and the U.S. government.

The Brady plan consisted of a comprehensive restructuring program. The program was implemented first in 1989 by Mexico, followed by over 17 other sovereigns, and concluded in the mid-1990s. An important element of this process was the cash payments through market-based buybacks offered as part of debt restructuring to private lenders. As described by Zettelmeyer, Savastano, and Lui (2021), the IMF and the World Bank provided funds to facilitate these buybacks. In particular, the Debt and Debt-Service Operations (DDSRO) plan and the Debt Reduction facilities from these organizations provided important financing. This paper documents that 11 countries obtained IMF funding which was used to buy back debts and to purchase the collateral needed for the bonds. The IMF funds were effectively loans, which increased the indebtedness of the countries. These loans required changes in IMF's policies to allow lending to countries with debt in arrears.

A second important element of the Brady Plan, was that the new bonds used in the exchange, were partially collateralized. In most cases, the principal of the bonds was collateralized by 30-year U.S. treasury bonds and the escrow agent holding the collateral was the Federal Reserve. As argued by Truman (2023), this involvement of the U.S. government helped give the program increased credibility and was an important component in the resolution of the debt crises.

Summary. We conclude this section by summarizing our findings from our emerging market data. First, we document that official debt is large in emerging markets, and represents more than half of the external debt for governments. Second, we document that official and private debt grows with partial default, but the increase is sharper for official debt. Third, we find that private debt returns to lower levels after default episodes while official remains elevated. Finally, we find evidence of official credit involvement in the resolution of debt crises. In the next section, we develop a model with official and private sovereign debt and default to study theoretically the patterns of these debts during default episodes and rationalize these patterns.

3 The Model Economy

We consider an infinite horizon model of sovereign default with official and private debt. The sovereign has preferences over consumption

$$\mathbb{E}_t \sum \beta^t u(c_t)$$

where β is the discount factor and $u(c)$ is increasing and concave. The economy faces stochastic endowment z_t and can borrow from international official and private lenders. Official debt is denoted by f_t and private debt by b_t. Debt contracts are perpetuities with coupon payments equal to $R^f f_t$ and $R^b b_t$ and that decay at rate ϑ^f and ϑ^b, respectively for official and private debt.[8] The sovereign can selectively partially default on each type of debt; the partial default decision for official debt is given by $d_t^f \in [0,1]$ and that for private debt is $d_t^b \in [0,1]$. A partial default of intensity d_t^a for debt of type $a \in \{b, f\}$, means that the sovereign defaults on $d_t^a R^a a_t$ of the coupons due upon the partial default and also defaults on $\mu^a d_t^a$ of all future debt coupons for that type of debt. Partial defaults lower resources for absorption, such that output depends on the endowment as well as partial defaults, $y_t(z_t, d_t^f, d_t^b) \leq z_t$ when $d_t^f > 0$ or $d_t^b > 0$. Partial defaults reduce the payments on the official or private coupons, but a fraction of the defaulted coupons accumulate and are due in the future. The fraction of defaulted coupons that accumulate is κ^f for official debt and κ^b for private debt. We will consider the case of official debt being of longer duration and carrying lower recoveries with default.

The sovereign borrows new loans ℓ_t^f from official lenders at price q_t^f and loans ℓ_t^b from private lenders at price q_t^b to support consumption and pay off the existing debts due. The budget constraint for the sovereign is

$$c_t = y_t - (1 - d_t^f) R^f f_t - (1 - d_t^b) R^b b_t + q_t^f \ell_t^f + q_t^b \ell_t^b$$

Consumption equals output net of the non-defaulted coupon payments of official and private debt and

8. The coupon rates R^f and R^b are constant rates that scale up the units of the debt payments. They will be set without any loss so that the default-free discount prices of the perpetuity contracts are equal to one, $R^a = (R - \vartheta^a)$ for $a \in \{f, b\}$, where R is the gross risk-free interest rate.

new loans. The laws of motion for debts satisfy

$$a_{t+1} = \vartheta^a(1 - \mu^a d_t^a)a_t + \ell_t^a + \kappa^a d_t^a a_t \qquad \text{for } a \in \{f, b\} \tag{2}$$

These laws of motion incorporate the coupons from the legacy debt that are not defaulted on $\vartheta^a a_t(1-\mu^a d_t^a)$, the new issuances ℓ_t^a, and the accumulation of the defaulted coupons $\kappa^a d_t^a a_t$. Note that $\kappa^a/(R^a + \mu^a\vartheta^a)$ can be interpreted as the recovery rate for a one-period default. For this default, the present value of the recovered debt is $\kappa^a d_t^a a_t$ and the present value of the defaulted debt is $(R^a + \mu^a\vartheta^a)d_t^a a_t$; the ratio of these two is the recovery rate. Our specification incorporates as special cases a standard full default with zero recovery, when $\kappa^a = 0$, $\mu^a = 1$, and $d^a = 1$, and the case with full recovery for lenders, when $\kappa^a = R^a + \mu^a\vartheta^a$.

The prices for official and private loans are schedules that compensate lenders for the losses from default. As we will see below, this means that these schedules depend on the debt states in the following period, $\{f_{t+1}, b_{t+1}\}$, and on the endowment z_t, as it helps forecast the next period's endowment. These schedules are $q^f(f_{t+1}, b_{t+1}, z_t)$ and $q^b(f_{t+1}, b_{t+1}, z_t)$.

3.1 Recursive Problem for the Sovereign

We now set up the recursive problem for the sovereign using standard recursive notation. The state variables for the sovereign are the official and private debts due and the endowment, $\{f, b, z\}$, and future variables are represented with primes. The sovereign also takes as given the bond price functions for the two types of debt. Given these states and the bond price functions, the sovereign makes choices for partial defaults, official and private loans, and consumption to maximize its value

$$V(f, b, z) = \max_{c, d^f, d^b, \ell^f, \ell^b} u(c) + \beta\mathbb{E}\left[V(f', b', z')\right] \tag{3}$$

subject to the budget constraint

$$c = y(z, d^f, d^b) - (1 - d^f)R^f f - (1 - d^b)R^b b + q^f(f', b', z)\ell^f + q^b(f', b', z)\ell^b, \tag{4}$$

the accumulation equations of official and private debt in (2), and the restriction that partial default on official and private is bounded, $0 \leq d^f \leq 1$ and $0 \leq d^b \leq 1$. This problem results in decision rules for consumption, partial defaults, official and private borrowing, denoted by $\mathbf{c}(f, b, z)$, $\mathbf{d^f}(f, b, z)$, $\mathbf{d^b}(f, b, z)$,

$\ell^{\mathbf{f}}(f, b, z)$, and $\ell^{\mathbf{b}}(f, b, z)$. We can use the decision rules for partial default and borrowing, together with the laws of motion, to determine the decision rule for next period's debts, $\mathbf{f}'(f, b, z)$ and $\mathbf{b}'(f, b, z)$.

3.2 Loan Contracts

International lenders are competitive, discount the future at rate R, and do not have any recourse other than that dictated by the terms of the contracts. Lenders break even in expected value in each contract. Bond prices are functions that depend on (f', b', z) to compensate for the expected loss of default, which depends on these states. The bond price functions for private and official loans satisfy

$$q^a(f', b', z) = \frac{1}{R}\mathbb{E}\left[(1 - \mathbf{d}^{a\prime})R^a + (\vartheta^a(1 - \mu^a\mathbf{d}^{a\prime}) + \mathbf{d}^{a\prime}\kappa^a)q^a(\mathbf{f}'', \mathbf{b}'', z')\right] \qquad \text{for } a \in \{f, b\} \tag{5}$$

The expressions for bond prices encode the expected stream of payments per unit of the loan for the life of each perpetuity contract. The bold letters represent the equilibrium policy functions. The first term $(1 - \mathbf{d}^{a\prime})R^a$ is the expected payment of the first coupon of the bond in the period following the issuance and it takes into account a potential partial default. The second term encodes that the perpetuity contract calls for the long-term promise to pay ϑ^a fraction of the coupon the following period net of the reduction from a partial default $(1-\mu^a\mathbf{d}^{a\prime})$. The third term $\mathbf{d}^{a\prime}\kappa^a$ takes into account that κ^a fraction of the defaulted coupons remain as future obligations. These future obligations also contain default risk and a specific coupon structure, both of which are encoded in the continuation price $q^a(\mathbf{f}'', \mathbf{b}'', z')$. Importantly, the future bond prices are evaluated at the equilibrium policy functions given a particular choice $\{f', b'\}$.

It is also useful for some of the exercises we analyze below to define the values to official and private lenders given a state. Let $H^f(f, b, z)$ and $H^b(f, b, z)$ be the values for lenders per unit of private and official debt respectively, given a state $\{f, b, z\}$. These values equal the expected payments and satisfy

$$H^a(f, b, z) = \left[(1 - \mathbf{d}^a)R^a + (\vartheta^a(1 - \mu^a\mathbf{d}^a) + \mathbf{d}^a\kappa^a)q^a(\mathbf{f}', \mathbf{b}', z)\right] \qquad \text{for } a \in \{f, b\} \tag{6}$$

Lenders' values are low if default is high, recovery net of acceleration $\kappa^a - \mu^a\vartheta^a$ is low, and continuation prices $q^a(\mathbf{f}', \mathbf{b}', z)$ are low. These continuation prices reflect expected losses from future default and depend on the dynamics of debts.

3.3 Partial Defaults and Borrowings

We now characterize the partial default decisions. Given the structure of our model, we can recast the sovereign problem in two stages. In the first stage, the partial default policies for official and private debt are determined given a state and any potential choices of future states $\{f', b'\}$. In the second stage, the sovereign makes its portfolio choices for official and private $\{f', b'\}$. For the first stage, in an interior optimum, the partial default policies are chosen to expand the budget set of the sovereign and satisfy the following conditions:

$$\begin{aligned} -y_{d^f}(z, d^f, d^b) =& f[R - \vartheta^f + q^f(f', b', z)(\vartheta^f \mu^f - \kappa^f)] \\ -y_{d^b}(z, d^f, d^b) =& b[R - \vartheta^b + q^b(f', b', z)(\vartheta^b \mu^b - \kappa^b)] \end{aligned} \tag{7}$$

The left-hand sides are the marginal costs of partial default for each type of debt in terms of output losses. The right-hand sides are the marginal benefits, where we have plugged in the coupons $R^a = R - \vartheta^a$ to normalize the default-free prices for each debt to 1. To explain the partial default decisions, let us first consider the case when continuation prices do not carry any default and therefore are equal to 1. In this case, the marginal benefits from partial defaulting equal $R - \vartheta^a(1 - \mu^a) - \kappa^a$ for $a \in \{f, b\}$. These benefits are the expansion of resources from saving on the defaulting coupons net of any increase in future obligations, which depend on the acceleration and recovery terms of the contract. Note that all else equal, shorter-term debt increases the marginal benefits of partial default given a level of debt because it carries a larger coupon payment (lower ϑ^a). Also, contracts with higher acceleration terms μ^a and lower recoveries κ^a, then to increase partial default. As we will explore below, private debts tend to be shorter-term with larger coupon obligations, and are associated with higher default incentives. Partial default also depends on the continuation prices for the sovereign. Low bond prices increase the incentives to default because they reduce the value of the defaulted coupons.

At interior solutions, the partial default policies equate the marginal costs and benefits. These policies, however, are bounded, $0 \leq d^a \leq 1$ for $a \in \{f, b\}$. Therefore, when the marginal costs strictly exceed the marginal benefits for any positive partial default for debt type a then $d^a = 0$; conversely, if the marginal benefit exceeds the costs at $d^a = 1$, then default is full.

We now turn to the debt policies. Given the partial default policies, the sovereign chooses official and private debt to maximize its value taking as given the bond price functions. In our model, as in Arellano, Mateos-Planas, and Ríos-Rull (2023), the dynamics of debts are governed by intertemporal consumption smoothing incentives, as well as the shapes of the bond price functions. To illustrate these forces, we assume all functions are differentiable and optimal partial defaults are interior. The sovereign program results in the following Euler conditions:

$$u_c\left(1+\frac{\partial q^f}{\partial f'}\frac{(f'-f(\vartheta^f+d^f\hat{\kappa}^f))}{q^f}+\frac{\partial q^b}{\partial f'}\frac{(b'-b(\vartheta^b+d^b\hat{\kappa}^b))}{q^f}\right)=\beta R\,\mathbb{E}\left(u_c'\right)+\frac{\text{Cov}(u_c',H^f)}{q^f} \tag{8}$$

$$u_c\left(1+\frac{\partial q^b}{\partial b'}\frac{(b'-b(\vartheta^b+d^b\hat{\kappa}^b))}{q^b}+\frac{\partial q^b}{\partial f'}\frac{(f'-f(\vartheta^f+d^f\hat{\kappa}^f))}{q^b}\right)=\beta R\,\mathbb{E}\left(u_c'\right)+\frac{\text{Cov}(u_c',H^b)}{q^b} \tag{9}$$

where recall $q^a=\mathbb{E}\left(H^a\right)/R$, and $\hat{\kappa}^a=\kappa^a-\mu^a\vartheta^a$ for $a\in\{b,f\}$

These Euler conditions govern the dynamics of official and private debts. Importantly, these conditions determine the dynamics for the sovereign going into default and as well as for exiting the default episode. The left-hand-sides are the marginal benefits of debts, which take into account that bond prices react to bond issuances and that the changes in prices also affect the value of legacy debts. Increased official (private) borrowing is beneficial if the elasticities of both prices q^f, q^b with respect to official (private) debt are lenient, meaning prices do not fall as fast. In contrast, the sovereign has incentives to deleverage and reduce debts when price elasticities are unfavorable. Borrowing dilutes the values of legacy debts which is a net benefit to the sovereign. During defaults, the bond price functions tend to be unfavorable and they provide the incentives for the sovereign to deleverage and exit the default episode.

The marginal costs of debts in the right-hand sides incorporate the discounted expected marginal utility of consumption for repaying the debt. Note that in general, the net default costs would enter into the right-hand sides. However, when partial default is at an interior optimum, the marginal resource cost of default is equated to the marginal benefit of not repaying the debt, giving a zero net default cost. The right-hand sides also contain the hedging properties of the debts, which are also shaped by the bond price functions, and depend on the covariance of the marginal utility of consumption with the lender's values. In our model these covariances tend to negative, as the lenders tend to get paid less when the sovereign's marginal utility is high. The long-term debts therefore are a good hedge for the sovereign, and all else equal this force increases borrowing incentives.

These conditions also say that the portfolio of debts will depend on the relative elasticities of the bond price functions with respect to the quantities borrowed and the relative hedging benefits of the debts. The portfolio is also actively used to exit default episodes because in response to the dynamics of the forces. As we show below, bond prices of official debt tend to be less sensitive to quantities, which gives official debt an advantage for borrowing, especially during default. Before moving on to the quantitative analysis of this model, we provide some further characterizations of the relative debt capacity of the two types of debts in the next section.

4 Model Characterization

In this section, we simplify the model and characterize a few key properties of official and private debt contracts. To that end, we assume that official debt are perpetuities with $\vartheta^f = 1$ and private debt are short-term contracts $\vartheta^b = 0$, with $R^f = r$ and $R^b = 1+r$, where $r = R-1$. For simplicity, we also consider the case of linear utility, no accumulation of defaulted coupons, no acceleration and a fixed default cost with any positive default, such that $y_t = z_L$ if $d_t^f > 0$ or $d_t^b > 0$. We summarize these settings in the following assumption.

Assumption 1 (Simple Economy)**.** *In the simple economy,* $u(c) = c \geq 0$, $\vartheta^f = 1$, $\vartheta^b = 0$, $\kappa^f = \kappa^b = \mu^f = \mu^b = 0$, $R\beta < 1$, *and initial debt is zero,* $b_0 = f_0 = 0$. *Absent default, productivity is constant* $z_t = z$, *and it falls to* $z_t = z_L$ *if* $d_t^f > 0$ *or* $d_t^b > 0$. *Default does not prevent borrowing.*

Default and Budget Sets. As is standard in sovereign default models, default incentives shape the price schedules for debt, and these in turn determine the supply of loans. Here we use our simplified model to characterize default incentives and the price schedules. A main objective is to characterize how official and private debt differs in terms of default incentives. We relegate the detailed derivations of the proofs to Appendix C.

Under assumption 1, the recursive problem for the government is the following

$$V(f,b) = \max_{c, d^f \in [0,1], d^b \in [0,1], \ell^f, \ell^b} c + \beta V(f', b')$$

subject to $c \geq 0$ and the budget constraints. With no default, $d^f = d^b = 0$, the budget is

$$c = z - rf - (1+r)b + q^f(f', b')(f' - f) + q^b(f', b')b'$$

Given that the default cost is fixed and independent of the intensity of the default, if the sovereign chooses to default, it fully defaults on the coupons of both debts, namely $d^f = d^b = 1$. The budget constraint with default is then

$$c = z_L + q^f(f', b')(f' - f) + q^b(f', b')b'$$

Importantly, default does not preclude market access to borrowing or paying future debt. Default is a period-by-period decision that is costly only because it reduces resources if coupons are not paid. Given the setup, the sets of contracts available for official and private debt, namely $q^f(f', b')(f' - f)$ and $q^b(f', b')b'$, do not depend on whether the sovereign defaults or repays the coupons. This means that default will be chosen if it expands the budget set and that the default policies are:

$$d^f = d^b = \begin{cases} 0 & \text{if} \quad rf + (1+r)b \leq z - z_L \\ 1 & \text{otherwise} \end{cases} \tag{10}$$

The default policies map into price functions. To analyze the impact of default policies on bond prices and budget sets, it is useful to consider using one type of bonds at a time.

Suppose first that the sovereign uses only private debt. Given default policies, the private loan that maximizes the budget that guarantees repayment is $b_{\max} = \frac{z - z_L}{1+r}$ and the associated price is $q^b = 1$. Under Assumption 1, it is optimal for the sovereign to choose this loan in period 0. In period 1, the sovereign is committed to repay $(1+r)b_{\max}$, but it does not have any further commitments from the period 0 contract. Moreover, in all future periods, it is optimal for the sovereign to exhaust its borrowing capacity and not default. These policies imply that consumption paths with private debt contracts satisfy

$$\begin{aligned} c_0 &= z + \frac{z - z_L}{1+r} && \text{for } t = 0 \\ c_t &= z_L + \frac{z - z_L}{1+r} && \text{for } t \geq 1 \end{aligned}$$

Consumption in period 0 is expanded by $\frac{z - z_L}{1+r}$ and in future periods, consumption is independent of the period 0 private contract.

Suppose now that the sovereign only uses official debt. Unlike for private debt, the long-term nature of official debt implies its bond price function depends on all future default incentives and future borrowings. Consider a candidate official debt contract that gives the sovereign barely enough incentives to repay in the future. Given default decisions in (10), this contract has a coupon value that is equal to the cost of default, such that $rf = z - z_L$.

An official contract at $t = 0$, that incorporates a transfer to the sovereign of $f_1 = \frac{z-z_L}{r}$ and promises to pay $rf_t = z - z_L$ for $t \geq 1$, is the maximal contract that ensures repayment. It is optimal for the sovereign to choose this contract $f_{\max} = \frac{z-z_L}{r}$ with the associated price $q^f = 1$, as it maximizes its budget at $t = 0$. Consumption paths with official debt are therefore:

$$\begin{aligned} c_0 &= z + f_{\max} = z + \frac{z - z_L}{r} && \text{for } t = 0 \\ c_t &= z - rf_{\max} = z_L && \text{for } t \geq 1 \end{aligned}$$

With the maximal official contract, consumption in period 0 is expanded by $\frac{z-z_L}{r}$, and is reduced in all future periods as the sovereign pledges future resources to servicing the official debt coupons.

The analysis comparing private and official loans gives our first result.

Lemma 1 (Official expands budget more)**.** *Under Assumption 1, borrowing at $t = 0$ with official debt can increase the budget set more than borrowing with private debt,*

$$q^f(f'_{max}, b' = 0)f'_{max} = \frac{z - z_L}{r} > q^b(f' = 0, b'_{max})b'_{max} = \frac{z - z_L}{1 + r}.$$

This result arises because the official debt contract effectively constrains future governments from borrowing as the pledgeable resources, namely the default cost $z - z_L$, are already committed to the legacy official contract. The official contract can extract the present value of these resources, $(z - z_L)/r$. Private debt, in contrast, does not constrain future governments from borrowing, and therefore it can extract only the one period ahead pledgeable resources $(z - z_L)/(1 + r)$. This difference implies that private contracts cannot replicate the paths of consumption that are possible with official contracts.

In models with sovereign default, debt maturity generally matters for allocations.[9] But the standard result is the opposite: short-term debt can replicate long-term debt contracts, but not vice-versa. In our partial default model, however, the result is that only official debt, which is longer-term, can replicate the allocations with only private debt, which is short-term. Next, we explore how our model's main differences relative to the standard model, namely borrowing during default, default on legacy debt, and the timing of default costs, give rise to these differences.

Assumption 2 (Complete default, permanent exclusion, and output costs)**.** *In the simple economy, any positive default results in a permanent exclusion from financial markets and output costs.*

9. See for example, Aguiar et al. (2019) and Arellano and Ramanarayanan (2012).

This is a standard assumption in the sovereign default literature in the tradition of Eaton and Gersovitz (1981), namely that default triggers a permanent cost in the form of exclusion from borrowing and output costs. Under assumption 2, default decisions and the patterns of consumption are different. Default in this case is complete, which means that the default is on all the current and future coupons. Consumption with default is equal to output net of the cost of default, $c = z_L$, while consumption during repayment depends on the policy functions for future official and private loans. Given the preference assumption in (1), these policy functions are simple: the sovereign exhausts its borrowing capacity in every period. As above, let $b_{\max}$ and $f_{\max}$ be the maximum levels of private and official debt that prevent default. The following result shows that now both types feature the same debt limits.

Lemma 2. *Under Assumption 2, official and private loans expand equally the budget*

$$q^f(f'_{max}, b' = 0) f'_{max} = \frac{z - z_L}{r} = q^b(f' = 0, b'_{max}) b'_{max} = \frac{z - z_L}{r}.$$

The standard assumption in the sovereign default literature of complete default and exclusion from financial markets after default is at odds with the empirical evidence documented above. In practice, during periods of default, sovereigns continue to pay some of its debts, and new loans, particularly with official lenders, come in. We find interesting that under the more empirically relevant assumptions of our baseline model, that default is partial and does not preclude borrowing, private shorter-term debt features more limited debt capacity than official longer-term debt. As we show next in our quantitative evaluation, the higher debt capacity of official debt allows our model to replicate many of the patterns in the data.

5 Quantitative Evaluation

This section presents the quantitative evaluation of our model. We first describe the parameterization of the model with a moment-matching exercise that uses the panel data presented above. We describe the dynamics of the models and policy functions. We then compare the implications of the model for additional moments that characterize default episodes and find that it delivers patterns comparable to the data.

5.1 Specification and Parameterization

The utility function is $u(c) = \frac{c^{1-\sigma}-1}{1-\sigma}$. The potential endowment follows a log-normal AR(1) process $\log(z) = \rho \log(z_{t-1}) + \sigma_z \varepsilon_t$, with $\varepsilon_t \sim \mathcal{N}(0, 1)$. We discretize this process into 13 different states following Tauchen (1986). The output costs of default are increasing and convex in partial default and potential endowment, as in Arellano, Mateos-Planas, and Ríos-Rull (2023). These costs are realized only when z is higher than the mean $\bar{z}$, and are linearly increasing in z with slope ϕ. Similarly, output costs are increasing and convex in partial default d_f and d_b, with a slope parameter λ and a curvature parameter γ. The specific functional form for the output cost of default is given by: $y = z(1 - \lambda d_f^{\gamma})(1 - \lambda d_b^{\gamma})(1 - \mathcal{I}_{d,z}\phi(z - \bar{z}))$, where the indicator function equals one when partial default for either debt is positive, $d_f > 0$ or $d_b > 0$, and $z > \bar{z}$.

We calibrate the model at an annual frequency. We set some parameters to values from the literature and estimate others in a moment-matching exercise. We set the annual international risk-free rate to 2%, consistent with yields from U.S. Treasury bills, and set the coefficient of risk aversion to 2, a standard value in the literature. The autocorrelation of the endowment process is set to 0.87 consistent with our estimates for our panel of emerging markets. We also set the default cost exponent γ to 2, a value close to that estimated in Arellano, Mateos-Planas, and Ríos-Rull (2023), for computational simplicity as it delivers a closed-form expression for partial default.

In computing our model, we also incorporate discrete taste shocks following Dvorkin et al. (2018). These shocks slightly perturb the borrowing decision to achieve numerical stability and robust convergence in the computational algorithm.[10] The parameter ϱ governs the relative importance of the taste shocks for the choice of b' and is set to $5e^{-5}$, which is the smallest value that guarantees convergence in the model.

The parameters that differ between official and private debt are $\mu^a, \kappa^a, \vartheta^a$ for $a \in \{b, f\}$. It turns out that in our model, given a decay parameter ϑ^a, the parameters that control default on legacy and the accumulation of defaulted coupons, μ^a and κ^a, matter only as the linear combination $\kappa^a - \vartheta^a \mu^a$ in the sovereign program. Therefore, we only estimate μ^a and κ^a and set $\mu^a = 0.18$ for both types of bonds based on estimates by Stefanescu (2016) for the fraction of sovereign bonds in developing countries that contain acceleration clauses relative to the bonds that contain reverse acceleration clauses.[11] Note that this choice has no bearing on the results.

10. See the appendix F for details of the computational algorithm.

11. Stefanescu (2016) uses the Thomson One dataset covering 882 bonds in developing countries from 1990-2013 and tabulates the use of acceleration and reverse acceleration among these bonds, corresponding to 77% and 59% respectively.

Table 4: Parameter Values

Parameters Set Externally	
Risk-free interest rate	$R = 1.02$
Risk aversion coefficient	$\sigma = 2$
Endowment persistence	$\rho = 0.87$
Default cost exponent	$\gamma = 2$
Taste shock	$\varrho = 5e^{-5}$
Parameters Set Internally	
Endowment volatility	$\sigma_z = 0.052$
Discount factor	$\beta = 0.954$
Debt contracts	
Decay parameters	$\vartheta^f = 0.907$, $\vartheta^b = 0.794$
Recovery factors	$\kappa^f = 0.11$, $\kappa^b = 0.19$
Default Costs	
Cost based on partial default	$\lambda = 0.06$
Asymmetric endowment	$\phi = 0.8$

We perform a moment-matching exercise and estimate eight parameters. These parameters are the standard deviation of the potential endowment, the discount factor, the debt contract parameters, and the default cost parameters. We collect these parameters in $\Theta = \{\sigma_z, \beta, \vartheta^f, \vartheta^b, \kappa^f, \kappa^b, \lambda, \phi\}$. We target the volatility of output and 9 moments on the distribution of debts and partial defaults. These moments are the means and standard deviations of debt-to-output ratio for total debt, official debt, and private debt, the mean of partial default, and the mean of the ratios of debt service to output for total, official, and private debt. As in the data, partial default is measured as total debt in arrears due relative to total debt due, $\frac{d^f R^f f + d^b R^b b}{R^f f + R^b b}$. Table 4 shows all the baseline calibration values for the parameters of the model.

All parameters affect all moments, but some moments are more informative of certain parameters. The volatility of output maps into the volatility the shock. The level of total debt informs the default cost parameters, as higher default is associated with higher debt capacity. The net recovery parameters matter for the relative levels of private and official as well as their standard deviations. The discount factor and the default cost parameters also matter for the average partial default and the volatility of the debts. The ratios of debt service to output are informative on the decay parameters.

The resulting parameters controlling official and private contracts, namely $\{\vartheta^f, \vartheta^b, \kappa^f, \kappa^b\}$, imply two properties that are also consistent with external estimates. First, the exercise implies that the durations of official and private debt are 9 and 4.5 years, which are consistent with estimates in Arellano and Ramanarayanan (2012).[12] Second, official debt has lower recoveries; the estimated parameters imply that the recoveries, namely $\kappa^a/(R^a + \mu^a \vartheta^a)$, are on average 41% for official debt and 52% for private debt. The more concessional nature of official debt is consistent with the findings of Schlegl, Trebesch, and Wright (2019). They document an average of 40% recovery across 414 restructurings with official creditors from the Paris Club since 1978 and an average of 60% recovery across 187 restructurings with private creditors, in line with our results.

5.2 Moment-Matching Results

We start with the results from our moment matching exercise. Table 5 reports the model's implications for our target moments as well as for additional moments. The model statistics come from a long simulation of 200000 periods, after discarding the first 10000 observations. The model matches well the mean total debt and the breakdown of official and private. In the model debt to output is on average 34%, very similar to the data mean which is 33%. About two-thirds of that debt is official both in the model and data. Debt-to-output ratios are volatile with comparable magnitudes between data and model. The mean partial default is on average close to 30% in the model and data. Finally, in the model, the ratios of debt service to output are equal to 2% for each type of debt. These ratios are very similar in the data.

The table also reports some the correlations between official and private partial default and between official and private debt as additional moments. Both of these correlations are positive, as implied by the model, although in the model these correlations are stronger.

5.3 Policy Rules and Dynamics

Before confronting our model to additional patterns in emerging market data, we describe more details of the workings of the model by illustrating policy rules and dynamics.

Default and debt policies depend on the states, namely (f, b, z). Panel (a) in figure 5 illustrates the policy functions for partial default and debts. The figure is constructed for the mean z; the x-axis denotes the f state and the y-axis is the b state, reported relative to output. The shades are the color map for the partial default policies. The gray area corresponds to no default on either official or private debt, $d^f = d^b = 0$. The white area corresponds to $d^f = d^b = 1$. The blue area has positive partial default;

12. We use Macaulay duration which equal $\frac{R}{R-\vartheta^f}$ and $\frac{R}{R-\vartheta^b}$ for official and private debt, respectively.

Table 5: Model Fit (10 moments, 8 parameters)

	Data	Model
Targeted moments		
Total debt		
Mean	33	34
Std. dev.	18	18
Official debt		
Mean	20	21
Std. dev.	12	12
Private debt		
Mean	13	13
Std. dev.	8	6
Debt service to output		
Official	1.6	1.7
Private	1.9	2.3
Partial default	32	28
Output std. dev.	11	12
Other moments		
Corr (d^f, d^b)	62	98
Corr (f, b)	42	91

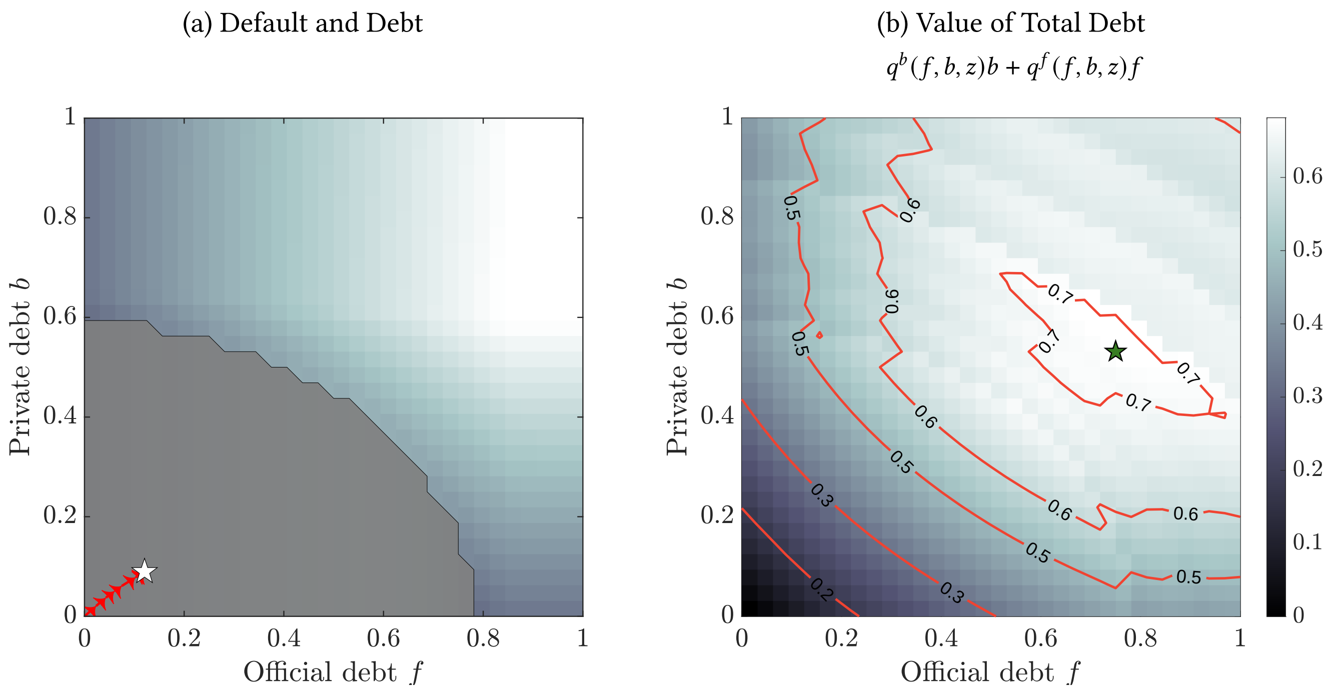
partial default on official (private) debt tends to be more elevated for higher values of official (private) debt.

As is standard, partial default tends to increase with debt. Interestingly, however, default incentives in our model vary with the type of debt. The sovereign can sustain more official debt without defaulting relative to private debt. The sovereign starts to partially default when official debt is above 60% of output for low levels of private debt, while that threshold is about 50% for private debt. The higher debt capacity of official debt is related to the theoretical results in Section 4. Moreover, partial default is higher with a portfolio that is tilted to one type of debt. As illustrated in the figure, the sovereign can sustain about 95% of total debt without defaulting for this shock when it holds a portfolio of about 55% official and 40% private, while it cannot sustain those levels of debt when holding only one type of debt.

We now turn to the debt policies. Recall that the debt policies, in equations (8) and (9), are shaped by consumption smoothing incentives and the shapes of the bond prices functions. We now illustrate some of these forces in our calibrated model with the red path with arrows in Panel (a) of Figure 5, which plots the dynamics of debts when the economy starts with zero debt and z is always at its mean. The white star in the figure is the point that the economy settles at, for a sequence of shocks z equal to the mean. Given that the sovereign is impatient relative to the risk-free rate, the sovereign frontloads consumption and borrows. It settles at a point with about 12% of official debt and 9% of private debt, a point of no partial default. This stationary portfolio is tilted towards official debt because of the higher debt capacity of this debt which is encoded in the elasticities of the bond price functions.

Partial default and borrowing decisions in turn affect bond price functions, which we turn to next. As in many sovereign default models, bond prices tend to fall with larger debts and low endowment shocks because these are the states where the sovereign defaults more. In Panel (b) Figure 5, we summarize these bond price functions with a heat map for the total resources borrowed, namely the total value of debt $q^b(f,b,z)b + q^f(f,b,z)f$, across the set (f,b) given mean z. Lighter colors in the figure correspond to higher values. The plot also illustrates with the red contour lines how the various combinations of (f,b) can give the same resources borrowed to the sovereign. Starting at the origin, increasing debts tend to increase the total resources borrowed but they are capped by a peak. The peak of the debt Laffer curve is illustrated by the green star: this is the maximum amount of resources that a sovereign can get when it starts with zero debt and is achieved with a portfolio that is tilted towards official debt. Note also that portfolios tilted towards official debt need less private debt to reach a total amount of resources borrowed than vice-versa: to get total resources borrowed of 60% of output requires 60% of official and

Figure 5: Default, Debt Dynamics, and Bond Prices

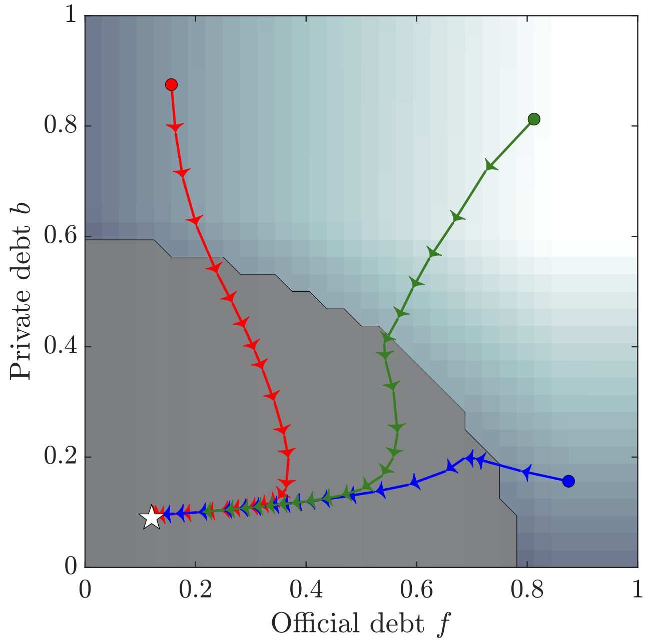


Notes: Panel (a) plots a heat map for partial default for the mean z across the debt states $\{f, b\}$. The gray area corresponds to full repayment; the white area corresponds to full default; the blue areas indicate partial defaults, with lighter colors indicating more intense defaults. The red arrows are the dynamic paths debts starting from zero. The white star is the stationary point for debts when the economy remains at mean z. Panel (b) plots a heat map for the total debt value $(q^b(f,b,z)b + q^f(f,b,z)f)$ for mean z across the debt states $\{f, b\}$. Lighter colors indicate higher debt values. The green star is the peak of the debt Laffer curve.

20% of private, or 60% of private and 30% of official. Of course, the shape of these functions crucially affects the choices of official and private debt, as seen in the first-order conditions presented above.

Exiting Defaults. In our partial default model, bond price functions and borrowing incentives are also at play when exiting default episodes. We now illustrate the forces behind these dynamics. Figure 10 plots the debt dynamics for the economy, when it starts at three different points with high levels of debt. When the sovereign is highly indebted, partial default is high; the sovereign deleverages to exit default and reach its stationary point. As the figure shows, the deleveraging process uses actively a portfolio of official and private debt and it does not feature a monotonic decrease of both debts. The paths in the figure also illustrate that the sovereign tends to issue official debt to reduce private debt at a faster rate. The three different starting points of the debt converge to a path that starts with relatively high official debt and low private (about .4 official and .12 private). Eventually, the sovereign also reduces the official debt to settle at the stationary point. These dynamics show that official debt is useful for the sovereign to exit faster the default episode. We will return to this point with counterfactual analysis

Figure 6: Exiting Default



Notes: This figure plots a heat map for partial default for the mean output across the debt states $\{b, f\}$ as well as dynamics of debts starting from 3 different points. See the notes to Figure 5.

below. These deleveraging dynamics are shaped by the bond price functions as well as default costs; the sovereign deleverages to reduce default costs and issues official debt because of more favorable bond price functions.

It is interesting to compare these dynamics to those in Aguiar et al. (2019). A main result of that paper is that the sovereign decreases only the short-term debt to exit the crisis zone and reach the no-default risk zone, without touching the long-term holdings. In that paper, this was because of the worse properties of long-term debt in terms of dilution. In our model, in contrast, the long-term bond which is the official debt, has higher debt capacity, and therefore the sovereign actively uses it in this process.

5.4 Official and Private Debt During Defaults

We now confront our model with additional empirical patterns in emerging markets. A main finding in Section 2.2 is that official debt tends to increase by more during defaults relative to private debt. We explore the model patterns for total, official, and private debt-to-output as well as partial default, and private spreads across bins with and without partial default in the model and data. In Table 6, we present the results; the *No default* bin corresponds to the observations with zero partial default, whereas the bin labeled *Partial default* corresponds to periods with positive values for partial default.

Table 6: Moments Conditional on Partial Default

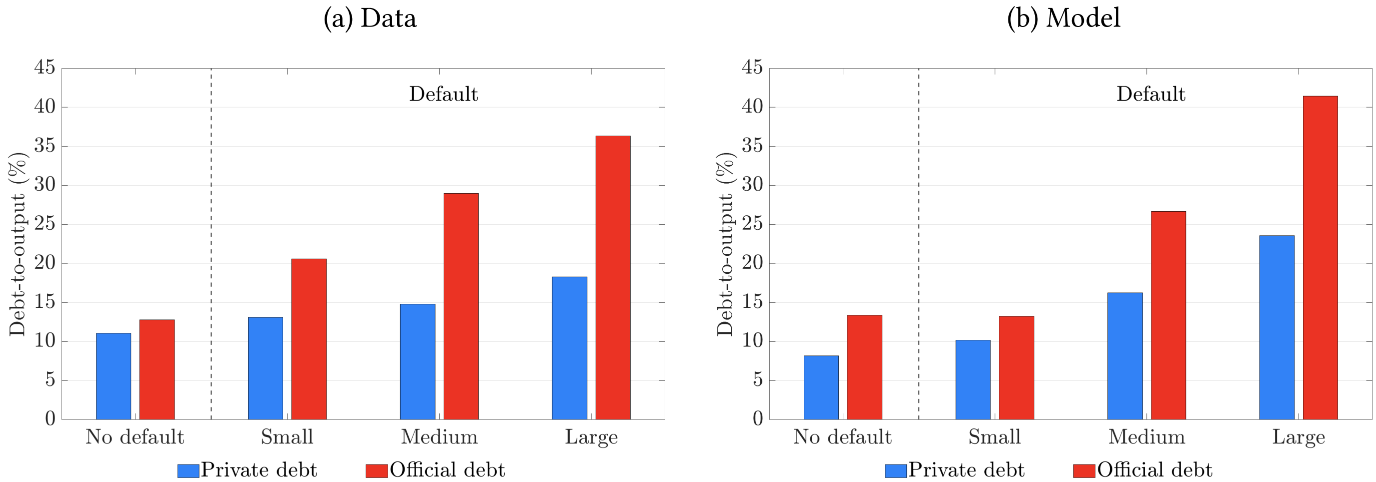
	Data		Model	
	No default	Partial default	No default	Partial default
Debt to output	24	44	21	44
Official	13	29	13	27
Private	11	15	8	17
Private spreads	4	8	1	5
Partial default	0	32	0	28

The model generates higher debt-to-output ratios with partial default as in the data. During periods of positive default, total debt-to-output is 44% while with no default it is 21%. These patterns are similar to those observed in the data, with levels of 44% vs. 24% in periods of positive and no partial default. Official debt increases more with default than private debt in both model and data. In the model, however, these differences across debts are a bit less accentuated than in the data. Spreads also increase with default. Spreads on private debt are about 4% higher when partial default is positive in the model and the data.

As in the empirical analysis, we further examine the patterns of official and private debts across finer partial default bins. In Figure 7, we report mean official and private debt across 4 bins, the same ones we used in Section 2.2. As before, the bars represent the averages of official and private debt-to-mean output across bins; the red bars correspond to official debt and the blue bars correspond to private debt; the left panel is the data and the right panel is the model. The figure illustrates that in the model, as in the data, debt increases monotonically with partial default. When partial defaults are small, under the 25 percentile of the distribution, debt to output is higher than when default is zero. Large defaults, over the 75 percentile of the distribution, feature the highest debt-to-output. The increase is more accentuated for official debt than for private debt in both the model and data.

This analysis illustrates that the model captures well the conditional patterns of debts across partial default. Periods of more intense defaults are associated with higher debt-to-output, especially so for official debt. In the model, periods of defaults are associated with a history of low endowment realizations and build-up of debt. The sovereign shifts its portfolio towards official debt as this debt offers greater debt capacity. We expand on these dynamics next, by analyzing patterns dynamics within default episodes.

Figure 7: Official and Private Debt across Default Bins



Notes: The figure plots mean debt (relative to mean output) across partial default bins for the data and the model. The "No default" bin corresponds to observations with no partial default. The "Small" bin includes observations with positive partial default below the 25th percentile, the "Medium" bin covers defaults between the 25th and 75th percentiles, and the "Large" bin contains defaults above the 75th percentile.

Default Episodes. We now study default episodes. As in the data, we use the simulated time series data to track default episodes: a period of time with continuous positive partial default. Default in the model is persistent, which gives rise to long default episodes. The average length of default episodes in the model is 10 years which is the same length found across episodes in the data.

We study how debts, partial defaults, and spreads evolve within default episodes. Table 7 reports the results of the model and compares them to data. During default episodes, total debt features a hump-shaped pattern in the model, as in the data. In the model, total debt to output increases about 8% from the year before the episode up to the middle of the episode (i.e. 37% minus 29%), which is similar to the 7% increase observed in the data. In the model, official debt increases more than private debt, as in the data. The contribution of private debt to this total increase is, however, somewhat more pronounced in the model relative to the data.

The model dynamics around episodes occur in response to a sequence of shocks and endogenous debt dynamics, where the sovereign increases and then decreases its debts. The sovereign enters the default episode when it receives an adverse endowment shock. As the shock recovers, the sovereign deleverages and exits. As explained above, the sovereign uses more heavily official debt during the episode because of a more lenient bond price function, which reflects the higher official debt capacity. As the sovereign exits the episode, it reduces private debt more aggressively than official debt. This effect is seen by comparing the levels after the episode; here the private debt level is very comparable to that before the episode, while the official debt level continues to be elevated.

Interestingly, the dynamics in the data are consistent with these forces. Official debt increases more in the ramp-up of debt of the episode, and private debt is reduced more rapidly, relative to official debt. In fact, in the data, this effect is accentuated as the level of private debt after the episode is below that seen before the episode, while official debt after the episode continues to be elevated. Through the lens of our model, the more aggressive deleverage of private debt results from its lower debt capacity and higher propensity to lengthier defaults.

Table 7: Default Episodes

	Dynamics of Debt			
	Before	Beginning	Middle	After
Data				
Total	33	35	40	33
Official	17	18	23	19
Private	16	17	17	14
Model				
Total	29	32	37	33
Official	18	20	23	21
Private	11	12	14	12

Notes: The mean length across default episodes is 10 years in the data and in the model. See the notes in Table 3 for further details.

6 Counterfactuals

We now use our baseline quantitative model to perform counterfactuals. In the first counterfactual, we evaluate the feasibility of voluntary swaps across official and private debt, that generate Pareto improvements for lenders and the sovereign. We find that swaps are feasible for a large region of the state space and can generate sizable welfare gains. Swaps tend to reduce default and spreads and tend to reduce private debt. Next, we study the design of official debt contracts by comparing our baseline with economies that feature official debt that is shorter-term and less concessional. This experiment is motivated by various liquidity programs from multilateral organizations. We find that short-term official debt offers limited welfare benefits and that the best design instead consists of long-duration bonds with few concessions.

6.1 Voluntary Swaps

In this section, we evaluate the feasibility of voluntary swaps, where the country and its official and private lenders agree to a debt exchange. A voluntary swap is feasible if such an exchange increases the value of the sovereign and the value of the debt to all lenders. Given a state (f, b), a feasible swap exists if there is a pair $(\hat{f}, \hat{b})$ such that the following two conditions hold:

$$V(\hat{f}, \hat{b}, z) \geq V(f, b, z) \tag{11}$$

$$H(\hat{f}, \hat{b}, z) \geq H(f, b, z), \tag{12}$$

with at least one with strict inequality and where $H(f, b, z) = H^f(f, b, z)f + H^b(f, b, z)b$, the sum of the values of debt to official and private lenders, defined in Section 3.2. Condition (11) indicates that the value of the sovereign must increase with the swap program and condition (12) says that the joint value to private and official lenders must also increase with the swap program. If a voluntary swap exists, then it constitutes a Pareto improvement because, under the new contract, the sovereign and the lenders are better off.

In models with constant debt prices, Pareto-improved swaps not feasible because the borrower's value decreases in debt while the lenders' value increases in debt. In our model, in contrast, bond price functions tend to decrease with debt due to default risk, which opens the possibility of feasible swaps because reducing debt could increase the value of lenders as debt price increases due to reduced default risk. This force makes swaps feasible in states of very high debt and very depressed prices–for example a price of zero that reflects 100% default risk. An important question, however, is whether the borrower would choose in equilibrium to borrow a level of debt that ex-post can be restructured to deliver welfare gains.

To understand why states with feasible swaps may arise in equilibrium, it is useful to consider small changes in debts. Swaps are feasible on the margin in a state (f, b) if the total differentials are positive $V_f df + V_b db > 0$ and $H_f df + H_b db > 0$ for small changes $\{df, db\}$. We can relate the marginal values of the sovereign and lenders using the sovereign optimality conditions and the bond price functions. By combining the lenders' values in (6) with the first order conditions with respect to b' and f', we can get that a small deviation $\{df, db\}$ around optimal choices satisfies the following condition

$$\frac{\beta}{u_c}\,\mathbb{E}\big[\underbrace{V_{b'}db' + V_{f'}df'}_{\text{gain sovereign}}\big] + \mathbb{E}\big[R\;\underbrace{\left(H_{b'}db' + H_{f'}df'\right)}_{\text{gain lenders}}\big] = \underbrace{\vartheta^b b\left(\frac{\partial q^b}{\partial b'}db' + \frac{\partial q^b}{\partial f'}df'\right) + \vartheta^f f\left(\frac{\partial q^f}{\partial b'}db' + \frac{\partial q^f}{\partial f'}df'\right)}_{\text{gain value of legacy debt}}. \tag{13}$$

This portfolio-deviation condition says that at an interior optimal, the expected marginal gain for the sovereign from a small deviation $\{db', df'\}$ is negatively related to the expected marginal gain for lenders and positively related to the gain in the value of the legacy debts. Consider an example with no uncertainty and a particular deviation $\{db', df'\}$ that gives the sovereign positive welfare gains, $V_{b'}db' + V_{f'}df' > 0$. Note that given that the value of the sovereign decreases with each type of debt, this deviation requires a reduction in at least one type of debt, $db' < 0$ and/or $df' < 0$. If the sovereign has a state of no legacy debt, $b = f = 0$, then the portfolio-deviation condition under no uncertainty immediately says that the sovereign would never choose a portfolio with feasible swaps ex-post. Any deviation ex-post that increases the value to the sovereign necessarily decreases the value to lenders: the RHS in (13) is zero and $u_c > 0$. This implies that legacy debt plays a key role in the feasibility of swaps along the equilibrium. With legacy debts, deviations that increase sufficiently their value could give gains to both the sovereign and the lenders. In addition, with uncertainty, particular ex-post realizations could give rise to feasible swaps ex-post for insurance reasons.

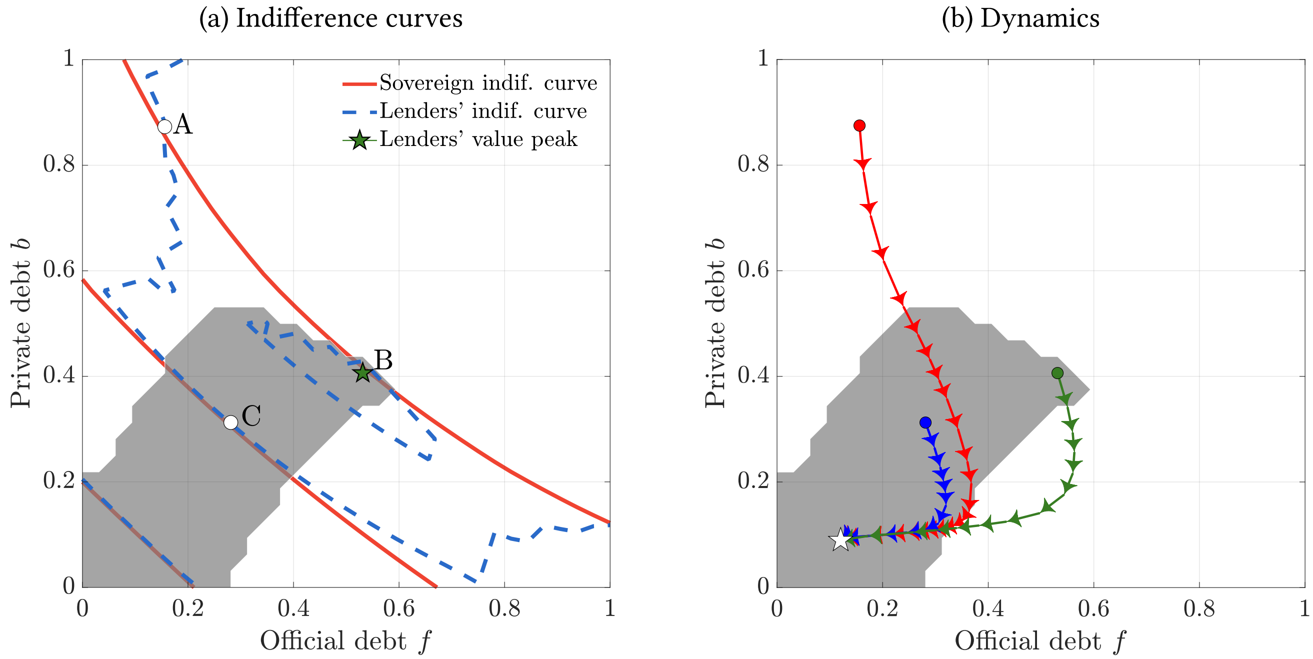
This analysis relates to Hatchondo, Martinez, and Padilla (2014), that illustrate that the sovereign may borrow beyond levels that maximize the value of debt to all lenders as the value of the new loans may increase with a reduction in the value of the legacy debt. In our model with two types of debts, private or official loans may dilute the value of both private and official legacy debts and debt accumulated as arrears. Therefore, Pareto improvements may arise not only because the economy has too much debt, but also because it has an inefficient portfolio of debts.

We now explore the feasibility of swaps in our baseline model. We find that across the state space, there are many states (f, b, z) where swaps are feasible. In the white area of panels (a) or (b) in Figure 8, swaps are feasible, while in the gray area, they are not. The state space features feasible swaps when the sovereign has too much debt, the north-east region of the state, or when it has a portfolio that is tilted too much to one type of debt, the north-west and south-east regions of the state.

Panel (a) of Figure 8 also plots the indifference curves for lenders and the sovereign across the portfolio of debts.[13] The lenders' indifference curves increase towards the star of the figure, which is the

13. The indifference curves of lenders are similar to those presented above in panel (b) Figure 5, only that here they are ex-post values.

Figure 8: Swaps



Notes: The white area in the figures are the states $\{f, b\}$ given mean z with feasible swaps, where conditions (11) and (12) are satisfied. In the grey area, swaps are not feasible. Panel (a) plots with solid lines indifference curves for the sovereign and in the dash lines indifference curves for lenders. The green star is the peak of the lenders' value. A swap from point A that maximizes lenders' value is point B, and one that maximizes the sovereign welfare is point C. Panel (b) plots the debt dynamics starting from points A, B, and C.

peak for lenders, while those for the sovereign increase towards the origin. Importantly, the lenders' indifference curves are more convex in the portfolio than the indifference curves of the sovereign and these relative convexities allow for voluntary swaps to arise. Consider, for example, the state corresponding to point A in the Figure. In this point, there is a set of feasible swaps that result in various combinations of welfare gains for the lenders and sovereign. For example, a swap from point A to point B, which corresponds to the peak, maximizes the gains for lenders, while keeping the sovereign at the same welfare level. Here lenders gain about 44% of their initial value (or 22% of mean output). A swap to point C, in contrast, maximizes the welfare of the sovereign while keeping the value to lenders unchanged. This swap results in a consumption equivalence gain of 1.3%.

In the limiting distribution, the economy visits states with feasible swaps about 5.3% of the time and welfare gains are modest.[14] The gains from these swaps if they maximize the sovereign' welfare are on average 0.04% of consumption equivalence. When they maximize the lenders' value, the swap gains are on average 1.1%. Across the state space, however, gains from swaps can be much larger. We

14. We find that swaps are feasible in the limiting distribution about 2.9% of the time when debts differ only in their recovery. Therefore both debt characteristics, duration and recovery, matter for the feasibility of swaps in equilibrium.

find that in general swaps that maximize the lenders' value tend to give higher gains in regions of the states tilted to one type of debt, whereas swaps that maximize the country tend to give higher gains with portfolios that contain high amounts of both debts.[15] These results are consistent with a limited role for swap contracts in our baseline economy's limiting distribution, which is subject to business cycle shocks. Our interpretation is that swaps may be more valuable when considering more extreme shocks, like a pandemic or natural and economic disasters, as these drive the economy to less common states.

To understand how a swap affects the economic outcomes, in panel (b) of the figure, we illustrate the debt dynamics after a swap. The red path is the original deleveraging path starting from point A, which we analyzed above in Figure (10). The green and blue paths are the ones that result after implementing swaps that maximize lenders and the sovereign, respectively; that is, swaps from A to point B and C. After the swaps, private debt decreases and settles at a lower level over time. Official debt, in contrast, tends to first increase and then decrease. As before, the increase is official debt helps the sovereign reduce private debt faster. Note that with these swaps, the new states on impact are in the gray region, but in the subsequent dynamics the economy travels through states where swaps are feasible. As explained above with the portfolio-deviation condition (13), the sovereign may choose in equilibrium debt levels with feasible swaps due to the dilution incentives for legacy debts; these dynamics illustrate these forces at play. Also, importantly, point A in the baseline model is associated with defaults in private debt and high private spreads. Swaps to points B or C eliminate the default and reduce spreads, which constitute a source of welfare gains.

6.2 Design of Official Debt

In the baseline model, official debt differs from private debt in that it is longer in duration and carries a lower recovery factor, making it more concessional. We now study how these properties affect the economy and the implications for welfare. To this end, we perform comparative statistics with the parameters ϑ^f and κ^f that change the duration and recovery of the official debt. We show that official debt of longer duration tends to be beneficial to the sovereign. Higher recovery factors tend to benefit the lenders and benefit the sovereign mainly in low-debt states. We also find that these welfare differences are linked to how different economies bear the costs associated with deleveraging episodes as they exit

15. In Figure 10 of the appendix, we present heat maps of the gains across the state space for mean z that illustrate these findings.

defaults.

In Table 8, we report the main summary statistics in various comparative statistics and compare them with the baseline model. Column (2) reduces the duration of the official debt from 9 to 7 years. Column (3) makes official debt more concessional by reducing the recovery factor of official debt to 30%. Column (4) makes official debt have the same duration and recovery as private debt, of 4.5 years and 54%, respectively. In column (5), official debt has a duration of 2 years and a higher recovery factor of 80%. We label this setting a "Multilateral" because it resembles certain official loans from multilateral organizations that are quite short-term and with very few concessional characteristics such as the Stand-By Agreement (SBA) or the Short-Term Liquidity Line (SLL) of the IMF and the Federal Reserve swap lines. In column (6), we analyze the case of longer and less concessional official debt, of 13 years and a recovery of 70%.

We first describe the top panel of Table 8. We find that official debt with shorter duration or lower recovery features a lower debt capacity, as seen by the comparative statics in columns (2) and (3), respectively. In these economies, the average official debt to output is reduced by about 5%. These characteristics also increase consumption volatility but lower default costs paid in equilibrium. Column (4) features official debt with equal characteristics as private debt, namely shorter duration and lower recovery. Here, we see that these forces are compounded, with further reductions in official debt capacity, higher consumption volatility, and reduced default costs. Column 5, our multilateral economy, displays outcomes of two offsetting forces; shorter duration reduces debt while higher recovery increases debt, which in net leads to a sizable level of official debt, but lower than in the baseline. However, consumption volatility is fairly elevated in the economy, but default costs are reduced. Finally, in column (6), the economy's official debt has the longest duration and high recoveries, and it features the highest debts, lowest consumption volatility, but highest default costs. Across these experiments, we also find that settings that increase official debt substantially also tend to increase modestly private debt.

The lower panel of the table contains welfare comparisons in these economies. Welfare for the sovereign is the consumption equivalence in each counterfactual economy relative to that of the baseline given a state, namely $((V^{\text{counter}}(f,b,z)/V(f,b,z))^{1/(1-\sigma)}-1)\times 100$. Welfare for lenders is the difference in values given a state in percent of the mean endowment, namely $(H^{\text{counter}}(f,b,z)-H(f,b,z))/\bar{z}\times 100$. We report results for the mean endowment in states with zero debt, the mean debts of the baseline economy, and high levels of debt in the baseline, which here are 50% and 25% for official and private debt, respectively.

We find that welfare for the sovereign tends to be lower in economies with shorter-term official debt. As seen in column (2), welfare is especially lower in states of high debt. In contrast, the welfare implications of varying recoveries for the sovereign are highly dependent on the state. As seen in column (3), the sovereign benefits from lower recoveries in high-debt states, but these are costly for low-debt states.[16] Lower recoveries help in high-debt states because default here reduces the burden of debt, and it is discharged at higher rates, while for low debts, the lower debt capacity of contracts with lower recoveries is detrimental. In the settings in column (4), the shorter duration effects dominate, and therefore welfare is qualitatively similar to column (2). In the multilateral economy of column (5), in contrast, the recovery effect dominates: higher recovery is beneficial with low debts, and costly in high debt states due to less debt discharge. Finally, the settings in column (6), with longer duration and higher recovery, have the highest welfare gains, especially for higher debts.

What are the sources of these welfare gains and losses for the sovereign? We find that in our economy the main sources of the gains for the sovereign arise from differences in consumption volatility and also the ability to exit defaults and debt crises without excessively costly deleveraging. In terms of consumption volatility, as shown in the table, economies with lower consumption volatility tend to feature higher welfare. For example, the much lower consumption volatility in column (6) is associated with the highest welfare gains. Moreover, we find that this volatility is correlated with the sensitivity of consumption to endowment shocks. For example, the elasticity of consumption to endowment shocks is 0.97 in the baseline while it is 0.95 in the economy of column (6).

Second, and importantly, the welfare rankings also reflect the consumption costs of deleveraging to exit debt crises, illustrated in Figure 10. Economies with longer loans tend to be able to exit these states with smoother consumption profiles. In Figure 9, we plot the consumption paths of the various economies starting in the state of high debts and assuming that the endowment remains in the mean z. In the baseline economy, consumption is about 5% lower than the endowment and increases about 3%, as the economy reduces its debt to stationary levels, which are associated with low risk of default. In the economy with shorter debt and lower recovery, the consumption decline is about 7% and features a more steep path. The economy with longer duration and higher recovery, in contrast, can roll over the debt completely and in fact debt increases further over time, given the benefits from tilting consumption. These consumption paths explain why welfare losses in an economy with shorter-term debt can be sizable in high debt states.

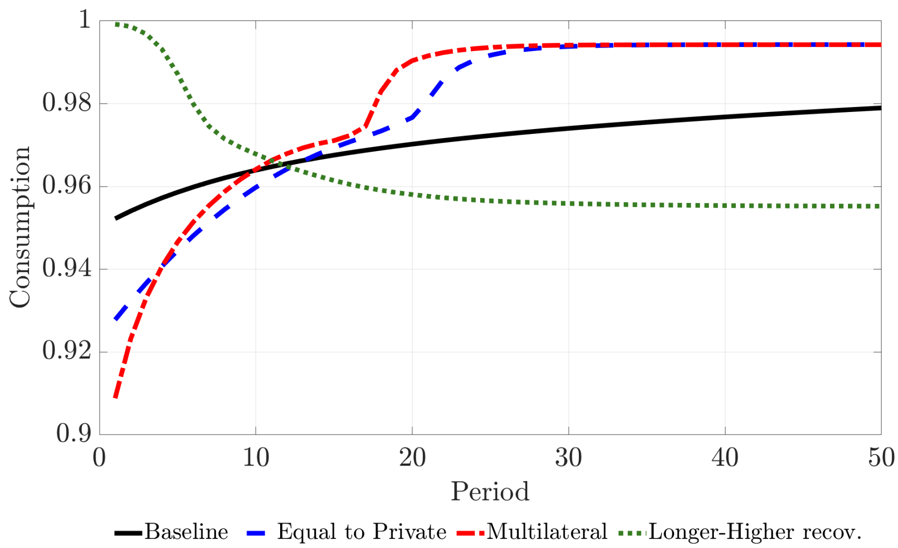
16. In Table 11 of the appendix, we present results when official debt differs from private debt in either duration or recovery, and contains a similar message.

Table 8: Counterfactual

	Baseline	Counterfactual Official Debt Contract				
		Shorter	Lower recov.	Shorter + Lower recov. (Equal to Private)	Shorter + Higher recov. (Multilateral)	Longer + Higher recov.
	(1)	(2)	(3)	(4)	(5)	(6)
Official debt	21	16	15	12	16	97
No default	13	10	10	8	11	78
Partial default	27	20	19	15	20	109
Private debt	13	13	13	12	11	16
No default	8	8	8	8	8	12
Partial default	17	16	16	15	14	19
Partial default	28	24	24	21	21	55
Consumption std. dev.	0.92	0.93	0.93	0.95	0.95	0.81
Default costs ???	0.62	0.51	0.50	0.39	0.35	1.88
Welfare (%)						
Sovereign: Consumption Equiv. Gain						
No debts	0.00	-0.002	-0.02	-0.0003	0.13	0.04
Mean debts	0.00	-0.03	0.004	-0.06	0.006	0.13
High debts	0.00	-0.07	0.06	-0.19	-0.41	0.17
Lenders: Change in Value						
No debts	0.00	0.00	0.00	0.00	0.00	0.00
Mean debts	0.00	0.65	-0.84	1.66	4.62	0.52
High Debts	0.00	0.78	-2.71	1.38	12.38	5.28

Notes: This table reports the main statistics and welfare comparison across comparative statistics that vary the duration and recovery of official debt, controlled by the parameters $\{\vartheta^f, \kappa^f\}$. Column (1) contains the results for the baseline calibration, with official duration of 9 years and recovery of 40%. Columns (2) and (3) report results comparative statics on ϑ^f and κ^f, which reduce official duration to 7 years and recovery to 30%, respectively. In column (4) official duration is 4.5 years and recovery is 52%, making official and private debt equal. In column (5) official has the properties of liquidity facilities from multilateral organizations, duration is 2 years and recovery is 80%. In column (6) official duration is 13 years and recovery is 70%. Welfare for the sovereign is consumption equivalence measures relative to baseline in percent. Welfare for the lenders is reported as the difference in values ΔH relative to the baseline, reported in units of mean output in percent.

Figure 9: Exiting Defaults: Consumption Paths during Deleveraging



Notes: This figure plots the consumption paths for the baseline model (column (1) in Table 8), and counterfactuals (4), (5) and (6) in Table 8 when the economy starts in the mean output and "High debts" state in period $t = 0$. Consumption is reported relative to mean output.

The sources of welfare gains in our model are related to the findings in Aguiar, Amador, and Fourakis (2020). They show that the sources of the welfare differences in the sovereign default models arise due to differences in consumption variability, the ability to tilt the consumption paths, and default costs. Our analysis contains these forces, but expands the interpretation of the ability to tilt consumption paths to environments where the sovereign needs to deleverage to exit defaults. We also find as they do that the benefits from consumption smoothing and tilting dominate the overall effect, while the costs of default play only a modest role and in our setting tend to offset the other sources of welfare differences.[17]

The properties of official debt also affect the welfare of lenders. Although lenders break even in expectation with new issuances, as seen from condition (6), the ex-post value depends on the level of debts, default decisions, recoveries, and continuation prices of the debts, which encode future losses from defaults. The duration of official debt has two offsetting effects on the value of lenders. One effect is that, given a state, shorter-duration debt is associated with more default, which tends to decrease the values of lenders. However, continuation prices tend to be higher with shorter duration because steep price schedules give the sovereign higher incentives to deleverage. The latter effect is the dominant one in columns (2). The recovery of official debt also directly affects the value of lenders; lower recoveries are associated with lower values for lenders, as these loans are more concessional, as seen in

17. Note, however, that unlike traditional default models, exiting default does not lead to a boom in borrowing because debt is essentially not reduced. This property limits the benefits from tilting consumption upon exiting default.

column (3). The effects on lenders values in the rest of the comparative statics are shaped by how these forces counteract each other. In column (4), the change in duration is more significant and, therefore, it dominates. However, we do find that for even higher levels of debt, this economy can lead to a loss for lenders because of large debt discharge with default. In columns (5) and (6), the high recovery effect dominates as lenders' benefits with shorter or longer duration debt.

These exercises show that overall, longer duration official debt with few concessions may be the best design for sovereigns and their lenders.

7 Conclusion

We proposed a framework to study the interplay of sovereign debt from private and official lenders and confronted it with evidence from 50 years of data in 30 countries. We documented that empirically debt from official lenders flows into countries during sovereign default episodes and that these flows keep the country's debt elevated upon exiting default. We showed that the key properties of the model necessary to capture this evidence are that default is partial and selective and sovereign debt from official lenders is of longer duration and more concessional than debt from private lenders. Our model overturned the traditional result in sovereign debt models that shorter term debt tends to give higher debt capacity and is welfare dominant. We also found the feasibility of Pareto improving debt swaps of one type of debt for another, especially during defaults, rationalizing the involvement of official lenders during emerging markets crisis. Finally, we provided guidelines for the design of official debt contracts and argued that even longer-term official debt would be beneficial for the sovereign and its lenders.

In developing this integrated framework that distinguishes debt from private and official lenders, we have abstracted from important differences between types of official debt, namely from bilateral governments and multilateral organizations. Chinese lending, for example, has been very prevalent in recent decades and linked to infrastructure projects. In contrast, central banks of advanced economies have designed liquidity facilities for certain emerging markets to access during periods of financial stress. We leave for future work the study of differences in objectives and goals among these official lenders and their implications for sovereign debt markets.

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A Data

We collect annual data from the International Debt Statistics (IDS), the Debtor Reporting System, the World Development Indicators (WDI), all from the World Bank, and the Global Financial Database (GFD) for the following countries: Argentina, Belize, Brazil, Bulgaria, China, Colombia, Dominican Republic, Ecuador, Egypt, El Salvador, Gabon, Ghana, Indonesia, Jamaica, Mexico, Morocco, Nigeria, Pakistan, Panama, Peru, Philippines, Russia, Serbia, South Africa, Sri Lanka, Tunisia, Turkey, Ukraine, Venezuela and Vietnam. The time span considered here is 1970-2019.

We consider the following variables:

1. **International Debt Statistics**
   (a) External debt stocks (current USD, PPG)
   (b) Debt service on external debt (current USD, PPG)
   (c) Principal repayments on external debt (current USD, PPG)
   (d) Interest payments on external debt (current USD, PPG)
   (e) Official creditors' debt (current USD, PPG)
   (f) Multilateral debt (current USD, PPG)
   (g) Bilateral debt (current USD, PPG)
   (h) Private creditors' debt (current USD, PPG)
2. **Debtor Reporting System**
   (a) Principal arrears official creditors (current USD, PPG)
   (b) Interest arrears official creditors (current USD, PPG)
   (c) Principal arrears private creditors (current USD, PPG)
   (d) Interest arrears private creditors (current USD, PPG)
3. **World Development Indicators**
   (a) Gross Domestic Product (constant 2010 USD)
   (b) Gross Domestic Product (current USD)

4. **Global Financial Database**

   (a) Emerging Markets Bond Index (EMBI+) spread

B Robustness on Empirical Analysis

This appendix provides robustness analysis for the patterns of total, official, and private debt during defaults. Table 9 reports debts related to trend output, constructed from nominal Gross Domestic Product in U.S. Dollars. This table shows that the patterns in Tables 2 and 3 remain unchanged with this different definition of debt. Official debt increases by more during partial defaults relative to private debt and also accounts for much of the hump-shaped patterns of debts during default episodes.

Table 9: Partial Default and Debt to Trend Output

Default Flag and Debts				
	Overall mean	No default	Partial default > 0	
Debt to output (in %)				
Total	32	23	43	
Official	19	13	28	
Private	13	11	15	
Dynamics during Default Episodes				
	Before	Beginning	Middle	After
Partial default	0	14	26	0
Debt to output (in %)				
Total	33	34	39	33
Official	17	18	23	19
Private	16	16	16	14

Notes: See notes to Tables 2 and 3. Debt ratios here are reported relative to trend output. Trend output is constructed with Hodrick-Prescott filter with smoothing parameter of 6.25.

Our baseline analysis decomposes debt between that with official and private creditors, and uncovers different patterns during defaults. Here we analyze patterns within official debt, namely with multilateral creditors and bilateral creditors. Multilateral creditors consist on multilateral organizations such as regional development banks, the World Bank, and the IMF. Bilateral creditors are governments from other countries and include the Paris Club lenders. Table 10 shows that bilateral is about 60% of official debt and that both multilateral and bilateral debts increase with partial default. The patterns across partial default bins and during episodes are similar across these two types of official debt, but the increases are more accentuated with bilateral debt.

Table 10: Partial Default and Official, Multilateral, and Bilateral Debts

Default Flag and Debts				
	Overall mean	No default	Partial default > 0	
Debt to output				
Official	20	13	29	
Multilateral	8	6	10	
Bilateral	12	7	19	
Dynamics during Default Episodes				
	Before	Beginning	Middle	After
Debt to output				
Official	17	18	24	19
Multilateral	7	7	9	8
Bilateral	10	11	15	11

Notes: All numbers are reported in percentage points. See notes to Tables 2 and 3. As described in Table 1, official debt is the sum of PPG debt from bilateral and multilateral.

C Theoretical Derivations

This appendix contains the proofs for Section 4.

Proof of Lemma 1

Consider an economy that satisfies Assumption 1. We start analyzing the case with only private debt. Default policies are:

$$d^b = \begin{cases} 0, & \text{if} \quad (1+r)b \le z - z_L \\ 1, & \text{otherwise.} \end{cases} \tag{14}$$

The bond price function for private debt depends on the one-period ahead default policy, such that

$$q^b(b) = \begin{cases} 1, & \text{if} \quad (1+r)b \le z - z_L \\ 0, & \text{otherwise.} \end{cases}$$

The private loan that maximizes the budget is $b_{\max} = \frac{z-z_L}{1+r}$ and the associated price is $q^b = 1$.

Next we consider using only official debt. Default policies in this case are

$$d^f = \begin{cases} 0, & \text{if} \quad rf \le z - z_L \\ 1, & \text{otherwise.} \end{cases} \tag{15}$$

For this debt, the bond price function depends on all future default incentives and future borrowings, and satisfies $q^f(f) = \frac{1}{R}[(1 - \mathbf{d^f}(f))r + q^f(\mathbf{f'}(f))]$.

Consider a candidate official debt level $\hat{f}$ that gives the sovereign barely enough incentives to repay in the future $\hat{f} = \frac{z-z_L}{r}$. If the sovereign remains with this level of official debt forever, namely $\mathbf{f'}(\hat{f}) = \hat{f}$, then the default policy and the bond price function imply that $q^f(\hat{f}) = 1$. At time zero, such contract is the maximum transfer that the sovereign can obtain, because it is the one that keeps the sovereign at its indifferent point of defaulting in every period in the future. To complete the proof we show next that in fact it is optimal for the sovereign to remain at $\hat{f}$.

When the sovereign starts at $\hat{f}$, any additional borrowing $f' > \hat{f}$ will necessarily generate default, at least temporarily as the default policy indicates that the following period the sovereign will default. This implies that $q(f) < 1$ for $f > \hat{f}$. Moreover, this price will be non-zero only if the sovereign chooses in the future to deleverage and lower its debt to the non-default region. However, it is never optimal for the sovereign to do so because this implies consumption falling below z_L during the deleveraging phase, with all the gains accruing to the legacy lenders. Given that it is never optimal for the sovereign to deleverage to levels below $\hat{f}$ when its debt is $f > \hat{f}$, then the price is zero in this range, such that

$$q^f(f) = \begin{cases} 1, & \text{if} \quad f \le \frac{z-z_L}{r} \\ 0, & \text{otherwise.} \end{cases}$$

Given these prices, it is never optimal for the sovereign to borrow beyond $\hat{f}$, as it gets zero from such loans without increases in consumption in the future. Moreover, given preferences, it is never optimal to lower its debt when it starts at $\hat{f}$. Therefore the official loan that maximizes the budget is $f_{\max} = \frac{z-z_L}{r}$ and the associated price is $q^f = 1$. The comparison between $f_{\max}$ and $b_{\max}$ give the result.

Proof of Lemma 2

The debt limits that prevent default equate the values of repayment and default. Given preferences and the fact that the bond price function jumps to zero beyond its limit, it is optimal for the sovereign to always borrowing to the limit. Moreover, the stationary and recursive structure imply that borrowing limits do not depend on time. Therefore, the private debt limit satisfies

$$\sum_{t=0}^{\infty} \beta^t [z - (1+r)b_{\max} + q^b b_{\max}] = \sum_{t=0}^{\infty} \beta^t z_L.$$

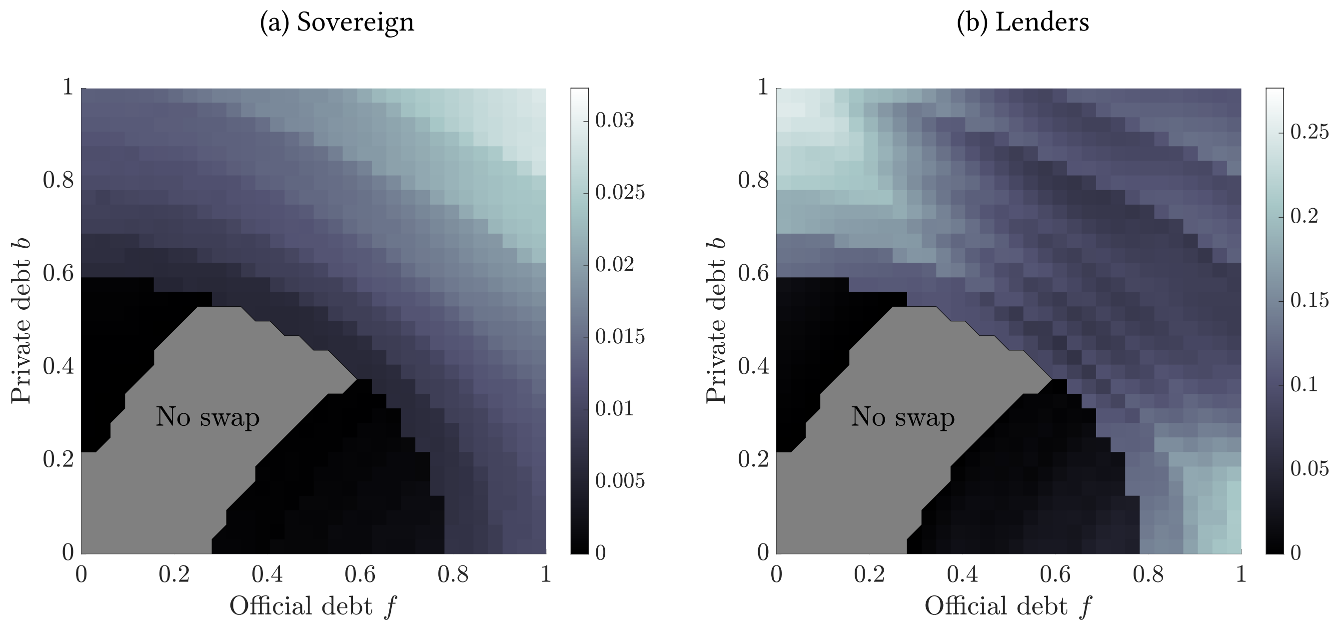
The official debt limits satisfies

$$\sum_{t=0}^{\infty} \beta^t [z - r f_{\max} + q^f (f_{\max} - f_{\max})] = \sum_{t=0}^{\infty} \beta^t z_L.$$

Prices without uncertainty are simple: $q^f = 1$ if $f \leq f_{\max}$ and $q^b = 1$ if $b \leq b_{\max}$. We solve for $f_{\max}$ and $b_{\max}$ using these prices and get the result in the Lemma.

D Swap Welfare Gains

Figure 10: Swap Welfare Gains



Notes: The gray area in the figures are the states $\{f, b\}$ given mean z with no feasible swaps. Panel (a) plots the welfare gains for the sovereign of implementing a swap that maximizes its value, measured in consumption equivalence units. Panel (b) plots the welfare gains for the lenders of implementing a swap that maximizes the total value of debt, reported relative to mean output.

E Additional Comparative Statics

Official debt in the baseline model is of longer duration and carries a lower recovery. We analyze here how each of these differences manifests in the main statistics of interest. Table 11 compares the main statistics for baseline model with an economies where official debt differs only in duration or in recovery. Column (2) is the economy where official debt has longer duration than private debt but equal recovery. In this economy, official debt is higher than the baseline and consumption volatility is lower. Welfare tend to be higher for the country and its lenders. In the economy of column (3), official debt has equal duration to private debt but lower recovery. This economy features lower debt levels, higher consumption volatility, and tends to deliver lower welfare.

Table 11: Comparative Statics

	Baseline	Same recovery	Same druation
	(1)	(2)	(3)
Official debt	21	30	9
No default	13	19	6
Partial default	27	38	11
Private debt	13	13	12
No default	8	8	8
Partial default	17	17	16
Partial default	28	32	20
Consumption std. dev.	0.92	0.90	0.95
Default costs ???	0.62	0.81	0.37
Welfare (%)			
Sovereign: Consumption Equiv. Gain			
No debts	0.00	0.04	-0.05
Mean debts	0.00	0.03	-0.09
High debts	0.00	-0.04	-0.10
Lenders: Change in Value			
No debts	0.00	0.00	0.00
Mean debts	0.00	1.18	0.26
High Debts	0.00	3.33	-3.35

Notes: This table reports the main statistics and welfare comparison for comparative statics that make official debt differ from private debt in either duration or recovery. Column (1) contains the results for the baseline calibration, with official duration of 9 years and recovery of 40% and private duration of 4.5 years and recovery of 52%. In column (2), the recovery of official debt is changed to 52% and in column (3) the duration of official debt is changed to 4.5 years. See notes in Table 8

F Computational Algorithn

The model is solved numerically by combining value function iteration with a root-finding routine for the partial default optimality conditions over a discretized state space. The algorithm proceeds as follows:

1. Discretize the exogenous income process $z \in \mathcal{Z}$ using Tauchen (1986), and define grids for the two debts: private debt $b \in [0, 1]$ and official debt $f \in [0, 1]$.

2. Initialize the value function $V_0(b, f, z)$ and bond prices $q_0^b(b', f', z)$ and $q_0^f(b', f', z)$. Set convergence tolerances ε_V, ε_{q^b}, and ε_{q^f} for the value function, private debt price, and official debt price, respectively.

3. For each tuple (b', f', z), compute the threshold levels $\bar{b}(b', f', z)$ and $\bar{f}(b', f', z)$ such that full default is optimal, and set $d_b(b, f, b', f', z) = 1$ and $d_f(b, f, b', f', z) = 1$ for all $b > \bar{b}(b', f', z)$ and $f > \bar{f}(b', f', z)$. For debt levels $b < \bar{b}(b', f', z)$ and $f < \bar{f}(b', f', z)$, solve for the partial default decision via root-finding by iterating on d_b and d_f using the two-first order conditions in equations (7):

$$
\begin{aligned}
-y_{d^f}(z, d^f, d^b) =& f[R - \vartheta^f + q^f(f', b', z)(\vartheta^f \mu^f - \kappa^f)] \\
-y_{d^b}(z, d^f, d^b) =& b[R - \vartheta^b + q^b(f', b', z)(\vartheta^b \mu^b - \kappa^b)],
\end{aligned}
$$

   The solution to this step yields candidate partial defaults $d_b(b, f, b', f', z)$ and $d_f(b, f, b', f', z)$.

4. Given candidates for partial defaults, compute new borrowing $\ell_b(b, f, b', f', z)$, $\ell_f(b, f, b', f', z)$ from the debt dynamics (2), and recover consumption $c^D(b, f, b', f', z)$ from the budget constraint (4).

5. Check if full repayment dominates partial default candidates. To do so, evaluate $d^b = 0$, $d^f = 0$ in equations (2) and (4) and compute the associated consumption under full repayment $c^R(b, f, b', f', z)$. Set $c(b, f, b', f', z) = \max\left(c^R(b, f, b', f', z), c^D(b, f, b', f', z)\right)$ and update optimal partial defaults and new borrowings accordingly.

6. Compute the flow value $W_0(b, f, b', f', z) = u(c(b, f, b', f', z)) + \beta\mathbb{E}\left[V_0(b', f', z')\right]$, and evaluate the value function using Gumbel taste shocks on each pair (b, f). This procedure gives the following

probability of choosing each (b', f'):

$$\text{Prob}(b', f' \mid b, f) = \frac{\exp\left[\left(W_0\left(b, f, b', f', z\right) - \bar{W_0}(b, f, z)\right)/\rho_s\right]}{\sum_{\tilde{b}', \tilde{f}'} \exp\left[\left(W_0\left(b, f, \tilde{b}', \tilde{f}', z\right) - \bar{W_0}(b, f, z)\right)/\rho_s\right]}$$

where $\bar{W_0}(b, f, z) = \max_{b', f'} W_0\left(b, f, b', f', z\right)$ and ρ_s is the variance of the taste shocks.

7. Update the value:

$$V_1(b, f, z) = \bar{W_0}(b, f, z) + \rho_s \log\left\{\sum_{b', f'} \exp\left[\left(W_0\left(b, f, b', f', z\right) - \bar{W_0}(b, f, z)\right)/\rho_s\right]\right\}$$

8. Compute decision rules $\mathbf{c}(f, b, z)$, $\mathbf{d}^{\mathbf{f}}(f, b, z)$, $\mathbf{d}^{\mathbf{b}}(f, b, z)$, $\ell^{\mathbf{f}}(f, b, z)$, and $\ell^{\mathbf{b}}(f, b, z)$ using the choice probabilities for (b', f').

9. Update prices using (5):

$$q_1^b(f', b', z) = \frac{1}{R}\mathbb{E}\left[(1 - \mathbf{d}^{\mathbf{b}'})R^b + (\vartheta^b(1 - \mu^b \mathbf{d}^{\mathbf{b}'}) + \mathbf{d}^{\mathbf{b}'}\kappa^b)q_0^b(\mathbf{f}'', \mathbf{b}'', z')\right]$$
$$q_1^f(f', b', z) = \frac{1}{R}\mathbb{E}\left[(1 - \mathbf{d}^{\mathbf{f}'})R^f + (\vartheta^f(1 - \mu^f \mathbf{d}^{\mathbf{f}'}) + \mathbf{d}^{\mathbf{f}'}\kappa^f)q_0^f(\mathbf{f}'', \mathbf{b}'', z')\right]$$

10. Iterate until the maximum absolute difference between successive iterations satisfies:

$$\|V_1 - V_0\| < \varepsilon_V, \quad \|q_1^b - q_0^b\| < \varepsilon_{q^b}, \quad \|q_1^f - q_0^f\| < \varepsilon_{q^f}$$

If all three converged, stop; otherwise, go back to step 3.