Sovereign Default Risk, Monetary Policy and Global Financial Conditions*

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Abstract

This paper explores the macroeconomic implications of tight global financial conditions in a small open economy New Keynesian model with sovereign default. My analysis shows that the interplay between the government incentives to repay the debt and inflation during periods of high world interest rates has distinct implications for monetary policy. I show that a monetary easing may emerge when a world interest rate hike induces a substantial increase in the probability of default, depressing domestic demand and leading firms to reduce inflation. This *default amplification* channel complements the expenditure-switching and expenditure-reducing channels found in standard open economy models, and rationalizes why emerging economies might reduce monetary policy rates when the Federal Reserve tightens. An increase in sovereign risk reduces aggregate domestic demand beyond these conventional channels, leading a real exchange rate depreciation to be contractionary for output.

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1 Introduction

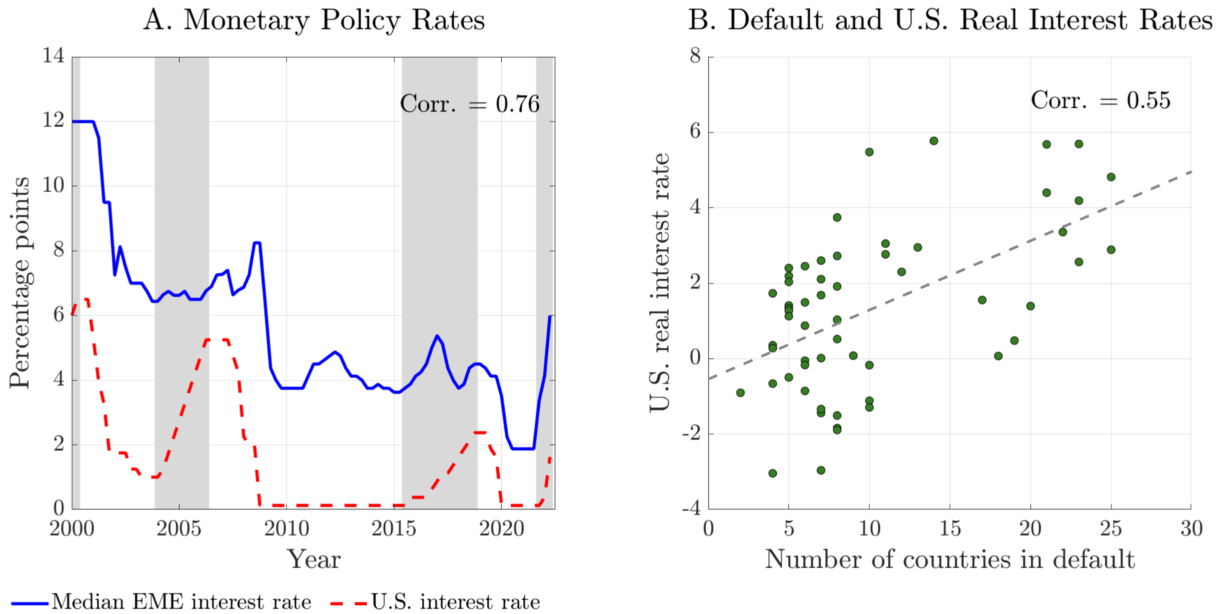
Emerging economies are highly exposed to global financial conditions. The monetary tightening in the United States that followed the COVID-19 pandemic has sparked a renewed interest in the international spillovers of such a policy. At the same time, governments' borrowing supported emerging economies through the pandemic, driving sovereign debt to unprecedented levels, and raising concerns about future debt crises. In this context, central banks face major challenges in the conduct of monetary policy, partly stemming from the uncertainty about how sovereign risk affects inflation in periods of financial distress (IMF (2023)).

There is a widespread consensus that fluctuations in the U.S. interest rate are an important driver of business cycles and inflation in these economies (Neumeyer and Perri (2005), Uribe and Yue (2006)). Beyond this general view lies substantial disagreement about how central banks typically react to such fluctuations. Panel A in Figure 1 shows that the median nominal interest rate in inflation-targeting emerging economies correlates strongly with that of the U.S. However, there is mixed evidence about the direction of the median rate during U.S. monetary tightenings. Some structural estimates of the effect of a 1% increase in the U.S. policy rate range from a large increase in domestic nominal interest rates of over 100 basis points (Vicondoa (2019)) to a moderate decline of less than 50 basis points (Kalemli-Özcan et al. (2023)). In parallel, panel B shows that periods with high U.S. real interest rates are typically associated with a higher number of countries in default. Understanding the mechanisms behind these relationships and the associated transmission channels is important for the conduct of monetary policy in small open economies whose financial conditions can be largely determined by global factors. Motivated by these empirical regularities, this paper provides a joint analysis of the interplay of monetary policy and sovereign risk during periods of tight global financial conditions.

To do so, I propose a model to study how the interaction between monetary policy and sovereign risk shapes the macroeconomic adjustment amid tighter global financial conditions, in the form of higher levels of the world interest rate[1]. I use this framework to explain the dynamic patterns documented in the empirical literature. In particular, I focus on the following questions: first, through which new channels are increases in the world interest rate transmitted to emerging economies? Second, does the model rationalize the mixed evidence on the mon-

[1]I use the U.S. interest rate as a proxy for the world interest rate, and use both terms interchangeably throughout the paper. As has been emphasized in the literature, the United States' position as the dominant country in the international monetary system has remained unchanged – persisting long after the conclusion of the Bretton Woods era (Miranda-Agrippino and Rey (2020), Farhi and Maggiori (2018)) – therefore providing a natural proxy for the world interest rate.

Figure 1: Monetary Policy, Default, and the U.S. Interest Rate



Notes: Panel A plots the median nominal interest rate for a sample of 8 emerging economies that adopted inflation targeting, and the U.S. nominal interest rate, from 2000Q1 to 2022Q2. The nominal interest rates correspond to the central bank monetary policy rates. The sample of countries includes Brazil, Chile, Colombia, Mexico, Peru, the Philippines, Poland, and South Africa. Shaded bars refer to periods of U.S. monetary tightening. Panel B displays the relationship between the U.S. real interest rate and the number of countries in default. The data of countries in default comes from Reinhart and Rogoff (2009), and the U.S. real interest rate is the U.S. Federal funds effective rate, deflated by the one-year ahead expected inflation. See Appendix B for details.

etary policy responses found in the empirical literature? Third, are real exchange rate depreciations expansionary or contractionary for output? The canonical answers to these questions are derived from benchmark open economy models, where world interest rates affect domestic macroeconomic aggregates through *expenditure-switching* and *expenditure-reducing* channels[2]. In the expenditure-switching channel, an increase in the world interest rate leads to a depreciation of the domestic currency, lowering the prices of domestic goods and services relative to those in the foreign economy, which expands output through higher exports and lower imports. This generates an increase in inflation and nominal interest rates. In the expenditure-reducing channel, the same increase in the world interest rate dampens aggregate consumption through lower transfers to households – as more resources are needed to service the debt – which reduces inflation and nominal interest rates. This analysis, however, abstracts from sovereign risk, which is a central ingredient in the fiscal and monetary policy discussions in emerging economies.

[2]See, for example, Blanchard et al. (2017), Akinci and Queraltó (2019) for a detailed description of these channels. Some recent papers propose additional channels based on households' heterogeneity (e.g. Auclert et al. (2021)) and domestic financial intermediaries (e.g. Ahmed et al. (2021)).

This paper aims to fill this gap by embedding a process for the world interest rate in Arellano et al. (2020) small open economy New Keynesian model with sovereign default. In this framework, households consume domestic and imported goods, and supply labor to firms that produce intermediate goods varieties. These firms face nominal rigidities and are subject to productivity shocks. Final goods producers are competitive and demand domestic varieties to produce domestic output, which is consumed by domestic and foreign households. The government borrows from international financial markets using long-term bonds denominated in foreign currency and transfers the proceeds of these operations to households. The government lacks commitment and can default on its outstanding obligations. Default is associated with a productivity loss, which dampens output and consumption. The central bank sets nominal interest rates using a Taylor rule that targets domestic inflation, which coincides with the optimal monetary policy rule in Galí and Monacelli (2005).

The analysis shows that fluctuations in world financial conditions are important driving forces of sovereign risk and business cycles, which in turn determine the monetary stance. The model displays significant state-contingency, whereby the response of the monetary authority is non-monotonic with respect to the level of indebtedness in the economy. At the heart of this result is an interaction between default risk and inflation. Consider a tightening in the world financial conditions that results from an increase in the U.S. interest rate. When the level of sovereign debt is either low or very high, the central bank follows a contractionary monetary policy. The intuition for this result is as follows: with such levels of debt, an increase in the U.S. interest rate does not unfold any substantial change in the probability of default. Here, the transmission to the domestic economy is governed by the standard expenditure-switching and expenditure-reducing channels. As such, higher U.S. rates translate into a tighter bond price schedule for the government and thus, a higher cost of rolling over the current debt. As a result, transfers to households fall, which in turn reduces aggregate consumption. Since external borrowing is used by the government to smooth imported goods' consumption, the overall reduction in consumption is mostly driven by a fall in consumption of imported goods, which results in a depreciation of the exchange rate. The depreciation makes imported goods relatively more expensive compared to domestic goods, leading domestic and foreign households to substitute towards domestic goods. In this case, domestic goods producers respond by increasing output and prices, leading the central bank to adopt a contractionary monetary policy. Here, U.S. and domestic monetary policy rates move in the same direction, and the real exchange rate depreciation is expansionary for output.

In sharp contrast, for intermediate debt levels, the monetary authority response does not coincide with the direction of the U.S. interest rate. In this case, the U.S. monetary tightening drastically increases sovereign risk – and therefore expectations about future default – and a novel *default amplification* channel comes into play. Since future default is associated with a fall in productivity, households reduce current consumption in anticipation of future lower consumption. Compared to the previous case, this further reduction in aggregate domestic demand leads firms to decrease output and prices, and the central bank follows an expansionary monetary policy to stimulate demand. This contraction in current consumption is the result of the conventional intertemporal consumption-smoothing behavior of households, and dominates the increase in foreign demand for domestic goods that results from a depreciation of the real exchange rate. The model's prediction of state-contingency helps rationalize the evidence provided by Kalemli-Özcan et al. (2023) whereby countries' dependence on global financial conditions generates a disconnection between the U.S. nominal interest rate and the policy rates in emerging economies. Similarly, it provides an explanation about why real exchange rate depreciations might result in a contraction of output.

In the quantitative section, I calibrate the model to the Mexican economy using data for the period 2001Q1-2019Q4. The model accounts for salient features of business cycles in Mexico. I then use the calibrated model to simulate the post-COVID-19 U.S. monetary tightening. To do so, I feed into the model the observed Federal Funds effective rate for the period 2022Q2-2023Q2 and simulate the model forward starting from two different initial debt levels, which I denote as "high" and "low". Along the path, I assume that the U.S. monetary tightening continues at a slower pace for the remaining periods of the simulation, and productivity remains constant at its mean level. This policy experiment confirms the main results of the paper; namely, the default amplification channel is able to break the monetary policy synchronization between the domestic economy and the U.S., and it limits or even reverses the positive comovement between the real exchange rate and output.

Related Literature. My paper is related to several strands of the literature. My work builds on the literature that has studied the international transmission of global shocks to macroeconomic fluctuations in emerging economies. Following the seminal contributions of Mundell (1960) and Fleming (1962), a large strand of macroeconomic research has explored both empirically and theoretically how global financial conditions affect business cycles and inflation in emerging countries. Some examples include Özge Akıncı (2013), Dedola et al. (2017), Iacoviello and Navarro (2019), Kalemli-Özcan (2019), Miranda-Agrippino and Rey (2020), Ilzetzki and Jin (2021), and Miranda-Agrippino and Nenova (2022). Several papers argue that central banks increase the policy rate when the Federal Reserve does (e.g. Uribe and Yue (2006), Vicondoa (2019)), while others find evidence of the opposite. Kalemli-Özcan et al. (2023), for example, document that a monetary policy tightening in the United States leads to an expansionary monetary policy in emerging economies. They argue that central banks do so to mitigate the negative impact of tighter global financial conditions on the domestic economy. I contribute to this literature by analyzing in a quantitative dynamic model how the interaction between monetary policy and sovereign risk leads to state-contingent effects of tighter global financial conditions on domestic macroeconomic variables.

This paper is also related to the open economy New Keynesian literature in the tradition of Galí and Monacelli (2005). In these models, output in the short-run is demand-determined, central banks stimulate aggregate demand by setting monetary policy rates (see, for example, Woodford (2003) and Galí (2015) for a textbook treatment), and shocks to the world interest rate affect output and inflation through expenditure-switching and expenditure-reducing channels. Recent examples studying the cross-border spillovers of foreign monetary policy in New Keynesian models include Aoki et al. (2018), Akinci and Queraltó (2019), Ahmed et al. (2021) and Auclert et al. (2021). I contribute to this literature by incorporating sovereign risk, which is an important driver of business cycles in emerging economies. I characterize how default risk shapes the macroeconomic adjustment of a small open economy after an increase in the world interest rate, and show that by accounting for default risk the model is able to rationalize the different monetary policy responses observed in the empirical literature of emerging economies.

Second, the paper also contributes to the literature studying the effects of the world interest rate in models with sovereign default. In recent work, Johri et al. (2022) incorporate an estimated process for the world interest rate with time-varying mean and volatility shocks in a standard default model, and they find that such time variation can generate sizable co-movement in sovereign spreads across countries. Centorrino et al. (2022) studies expected and realized interest rate movements in a sovereign default model with production and financial intermediaries, and they show that their model can generate reversals in the current account, recessions, and tighter domestic financial conditions during periods of U.S. interest rate hikes. Guimaraes (2011) build a default model of debt renegotiation with endowment and world interest rate shocks, and find that fluctuations in world interest rates can have a strong impact on the level of debt that economies can support in equilibrium. Almeida et al. (2023) consider a similar framework to study the Mexican default of 1982, and they find that renegotiation is key for reconciling sovereign default models with the narrative that the "Volcker shock" triggered the crisis. These studies abstract from the Keynesian channel, and hence they do not address the monetary policy implications that I examine in this paper.

Finally, my paper is related to the literature on exchange rate movements and their effect on output. Recent examples include Blanchard et al. (2017), who extend the set of assets included in the Mundell-Fleming model to explore whether capital flows are contractionary or expansionary for output. In related work, Auclert et al. (2021) study how open economies respond to exchange rate shocks in a HANK model. They show that in the Galí and Monacelli (2005) model a depreciation that results from an increase in the world interest rate is always expansionary, and only when import and export elasticities are low enough, the heterogeneous-agent model delivers a contractionary depreciation. Bianchi and Coulibaly (2023) propose a theory of "fear of floating" based on the vulnerability of the economy to self-fulfilling debt crises, and explore the conditions under which a depreciation increases or decreases output. I complement this literature by studying how default risk can break the co-movement between output and exchange rate.

Outline. The rest of the article proceeds as follows. Section 2 presents the model; Section 3 presents its recursive formulation and defines the equilibrium concept; Section 4 describes the calibration; Section 5 provides the quantitative results of the paper; and Section 6 concludes.

2 Model

The environment follows closely Arellano et al. (2020), which combines the workhorse small open New Keynesian model of Galí and Monacelli (2005) with the standard long-term debt sovereign default model, as in Hatchondo and Martinez (2009) and Chatterjee and Eyigungor (2012). Time is discrete and indexed by $t \in \{1, 2, \dots\}$. The economy is populated by households, domestic final goods firms, domestic intermediate goods firms, a government, and a central bank. In the following subsections, I describe each block of the model.

2.1 Domestic Households

Representative households have preferences given by:

$$U = \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left[u(c_t) - v(n_t) \right] \tag{1}$$

where $\mathbb{E}_t$ denotes the expectation operator at time t, $\beta \in (0, 1)$ is the discount factor, c_t denotes consumption in period t, and n_t denotes units of labor in period t. The utility function over final consumption $u : \mathbb{R}_+ \to \mathbb{R}$ satisfies the usual assumptions: $u' > 0$, $u'' < 0$ and $\lim_{c_t \to \infty} u'(c_t) = 0$. Similarly, $v : \mathbb{R}_+ \to \mathbb{R}$ denotes the disutility function of labor, which is increasing and convex:

$v' > 0$ and $v'' > 0$. The final consumption good is assumed to be a composite of domestic and imported consumption goods:

$$c_t = C\left(c_t^D, c_t^F\right) \tag{2}$$

where c_t^D is domestic consumption, c_t^F is imported consumption, and the function $C : \mathbb{R}_+^2 \to \mathbb{R}_+$ is differentiable, homogeneous of degree one, increasing in both arguments and concave.

Each period, households supply labor, and choose consumption and holdings of one-period non-state-contingent domestic bonds, B_t^D. These bonds can only be traded by domestic households, and are denominated in domestic currency. Households receive a wage rate, W_t, and government transfers, T_t. They also own firms and receive their profits Ψ_t. Their budget constraint in nominal terms is given by:

$$P_t^D c_t^D + (1 + \tau_F) P_t^F c_t^F + q_t^D B_{t+1}^D \leq W_t n_t + B_t^D + \Psi_t + T_t \tag{3}$$

where q_t^D is the nominal price of domestic bonds and τ_F is a constant consumption tax that households pay on imports. The left-hand side in (3) represents the total expenditure on domestic and foreign goods and purchases of domestic bonds, while the right-hand side represents the total income.

The problem of the household consists of choosing the sequences of $\{c_t^D, c_t^F, n_t, B_{t+1}^D\}$ to maximize the expected present discounted value of utility (1) subject to the budget constraint (3), taking as given the sequence of profits $\{\Psi_t\}$, transfers $\{T_t\}$, and prices $\{P_t^D, P_t^F, q_t^D, W_t\}$. I denote the real wage as $w_t := W_t / P_t^D$, the domestic goods gross inflation is $\pi_t := P_t^D / P_{t-1}^D$, and the nominal interest rate is the yield of the domestic bond price, $i_t := 1/q_t^D$.

The optimality conditions are:

$$-\frac{v_{n_t}}{u_{c_t^D}} = w_t \tag{4}$$

$$\frac{u_{c_t^F}}{u_{c_t^D}} = (1 + \tau_F) \frac{P_t^F}{P_t^D} \tag{5}$$

$$u_{c_t^D} = \beta i_t \mathbb{E}_t \left[\frac{u_{c_{t+1}^D}}{\pi_{t+1}} \right] \tag{6}$$

where $u_{c_t^D}$ and $u_{c_t^F}$ denote the marginal utility of domestic and foreign consumption in period t, respectively, and v_{n_t} the marginal disutility of labor. Condition (4) is the labor supply optimality condition, and equates the marginal rate of substitution between leisure and domestic consumption to the real wage. Condition (5) equates the marginal rate of substitution between domestic and foreign consumption to the after-tax relative price. Condition (6) is the Euler equation and equates the marginal benefit from saving in domestic bonds to the marginal cost of reducing current domestic consumption to buy the bonds.

2.2 Foreign Households

Foreign households consume domestic final goods. Their demand is given by:

$$X_t = \left(\frac{P_t^D}{\varepsilon_t P_t^\star}\right)^\gamma \xi$$

where $P_t^\star$ is the price of foreign goods in foreign currency, ε_t is the nominal exchange rate defined as the price of domestic currency in terms of foreign currency, γ is the trade elasticity, and ξ is the level of overall foreign demand. The law of one price holds for the foreign good, that is, $P_t^F = \varepsilon_t P_t^\star$. The real exchange rate e_t is given by:

$$e_t := \frac{\varepsilon_t P_t^\star}{P_t^D} = \frac{P_t^F}{P_t^D} \tag{7}$$

Then, foreign demand, which is equal to the level of exports in this economy, is a function of the real exchange rate and the level of overall foreign demand:

$$X_t = e_t^\gamma \xi \tag{8}$$

I assume that the foreign price $P_t^\star$ is equal to one in all periods.

2.3 Final Goods Firms

The final good is produced by a continuum of firms in a perfectly competitive market using a unit measure of differentiated intermediate goods y_{jt}, $j \in [0, 1]$. The production is defined by the Dixit-Stiglitz aggregator of all varieties:

$$Y_t = \left[\int_0^1 (y_{jt})^{\frac{\epsilon-1}{\epsilon}} dj\right]^{\frac{\epsilon}{\epsilon-1}} \tag{9}$$

Notice that $\epsilon > 1$ denotes the elasticity of substitution between varieties. The optimality condition of the final goods firms yields the standard demand function:

$$y_{jt} = \left(\frac{P_{jt}}{P_t^D}\right)^{-\epsilon} Y_t \tag{10}$$

where P_{jt} is the price of the intermediate good j at time t and P_t^D is the price index for domestic goods: $P_t^D = \left[\int_0^1 P_{jt}^{1-\epsilon} dj\right]^{\frac{1}{1-\epsilon}}$.

2.4 Intermediate Goods Firms

There is a continuum of intermediate goods firms, each one producing a variety $j \in [0, 1]$ with a constant return to scale technology with productivity z_t that uses labor as the sole input:

$$y_{jt} = z_t N_{jt} \tag{11}$$

Productivity depends on the aggregate shock $\tilde{z}$, and on the default decision of the government D_t. As we see below, when the government repays its debts ($D_t = 0$) productivity is equal to the shock, and when it defaults ($D_t = 1$) productivity is lower.

Firms are monopolistic competitors and are subject to Rotemberg (1982) price-adjustment costs when they change their prices away from a domestic inflation target $\bar{\pi}$. Government subsidies labor at a rate τ, and the intratemporal cost minimization problem implies that the marginal costs of production are $MC_t = (1 - \tau)W_t/z_t$. Taking as given the sequence for marginal costs $\{MC_t\}$ and final good prices $\{P_t^D\}$, a monopolist j chooses its price P_{jt} and labor N_{jt} to maximize the expected discounted stream of profits:

$$\mathbb{E}_0 \sum_{t=0}^{\infty} Q_{t,0} \left\{ \left(P_{jt} - MC_t\right) y_{jt} - \frac{\varphi}{2}\left(\frac{P_{jt}}{P_{jt-1}} - \bar{\pi}\right)^2 P_t^D Y_t \right\}$$

subject to the demand function (10) and the production function (11). Firms discount profits using the stochastic discount factor of households, $Q_{t,0} = \beta^t \frac{u_{c_t^D}/u_{c_0^D}}{P_t^D/P_0^D}$, and $\varphi \geq 0$ is the adjustment cost parameter. The first-order condition of the firm's problem yields the following optimal pricing rule:

$$(\pi_t - \bar{\pi})\pi_t = \frac{\epsilon - 1}{\varphi}\left(\frac{\epsilon}{\epsilon - 1} mc_t - 1\right) + \beta \mathbb{E}_t \left[\frac{u_{c_{t+1}^D}}{u_{c_t^D}} \frac{Y_{t+1}}{Y_t} (\pi_{t+1} - \bar{\pi})\pi_{t+1}\right] \tag{12}$$

where $mc_t := MC_t/P_t^D$ is the real marginal cost. The equation (12) is often referred to as the New Keynesian Phillips Curve (NKPC, for short) and relates current inflation to the real marginal cost and the expected inflation.

2.5 Central Bank

The central bank conducts monetary policy following a domestic inflation-based Taylor rule. The nominal interest rate i_t depends on a long-run value $\bar{i}$ and on the deviations of the inflation from the target $\bar{\pi}$:

$$i_t = \bar{i}\left(\frac{\pi_t}{\bar{\pi}}\right)^{\psi} \tag{13}$$

where $\psi > 1$ is the degree of responsiveness of the central bank to the inflation deviations and satisfies the Taylor principle. This formulation corresponds to the optimal monetary policy rule in Galí and Monacelli (2005).

2.6 Government

The government determines external borrowing and default decisions subject to a predetermined tax scheme. Its objective is to maximize the present discounted value of the flow utility derived from consumption and labor by households, $U_g = \mathbb{E}_0 \sum_{t=0}^{\infty} \beta_g^t \left[u(c_t) - v(n_t)\right]$. The discount factor of the government β_g is lower than that of the households, β. This assumption is often attributed to political economy frictions, as in Aguiar and Amador (2011) and Aguiar et al. (2020).

External borrowing. The government issues long-term bonds denominated in foreign currency. The bond specifies a price q_t and a quantity of new borrowing l_t such that the sovereign receives $q_t l_t$ units of foreign currency in period t. As in Hatchondo and Martinez (2009), I assume that a bond issued in period t promises in case of repayment $(r_t^\star + \delta)(1-\delta)^{j-1}$ units of foreign currency in period $t+j$ for all $j \geq 1$, where $r_t^\star$ is the world interest rate (to be described in more detail later). As such, the stream of coupons decay at an exogenous constant rate δ, and each unit of debt calls for a payment of $r_t^\star + \delta$ every period. Hence, debt dynamics are given by:

$$B_{t+1} = (1-\delta)B_t + l_t \tag{14}$$

where B_t is the stock of bonds at the beginning of period t, and $(1-\delta)B_t$ is the legacy debt that has not matured. This payment structure condenses all future payment obligations derived from past issuances into a one-dimensional state variable: the coupons that mature in the current period. As is standard in the sovereign default literature, I assume that B_t take values only in a finite grid Γ with J different points:

$$\Gamma = \left[B_1, B_2, \ldots, B_J\right] \tag{15}$$

Default. Debt contracts cannot be enforced, and each period the government can default on its debt. The default history determines whether it is in good or bad credit standing, which is encoded in the variable $\vartheta_t \in \{0, 1\}$. When the government repays the debt, $D_t = 0$, borrows in the international financial markets and decides the level of debt in the following period B_{t+1}. During these periods the credit standing is good: $\vartheta_t = 1$. When the government defaults, $D_t = 1$, it avoids paying the outstanding debt obligations but incurs three different costs:

i. The government is excluded from financial markets for a stochastic number of periods, in which it has a temporary bad credit standing $\vartheta_t = 0$. With probability ι it reenters the financial markets, regains good credit standing, and exits default with zero debt obligations.

ii. The government receives a direct utility *iid* cost ν_t, as in Aguiar et al. (2019). These disturbances can be interpreted as capturing time variation in the enforcement of sovereign debt, or financial shocks that affect the terms of government borrowing. I assume that $\nu \sim \text{Logistic}(\mu_\nu, \sigma_\nu)$.

iii. Productivity is depressed: $z(\tilde{z}, \vartheta_t = 0) < z(\tilde{z}, \vartheta_t = 1) = \tilde{z}$. This additional cost captures the disruption of trading arrangements, domestic financial markets, etc. that may accompany default.

Taxes. The tax scheme has three components: consumption tax, labor subsidy, and transfers. The tax that households pay on imported consumption is set to $\tau_F = \frac{1}{\rho - 1}$. As shown in Corsetti and Pesenti (2001) this is the optimal tariff on imports that offsets the distortions associated with the monopoly power of a country in trade. The labor subsidy is set to $\tau = 1/\epsilon$, as is standard in the New Keynesian literature. This subsidy corrects the markup of the monopolistic firms in the intermediate goods market.

The government provides lump-sum transfers T_t to households. Its budget constraint in domestic currency is:

$$T_t + \tau W_t n_t = (1 - \vartheta_t)\varepsilon_t \left[q_t(B_{t+1} - (1 - \delta)B_t) - (r_t^\star + \delta)B_t\right] + \tau_F P_t^F c_t^F \tag{16}$$

The left-hand side in (16) represents the total government expenditure, while the right-hand side represents its total revenue, which includes the net capital inflow from debt operations when the government has access to international financial markets.

2.7 Foreign Lenders

Sovereign bonds are traded with competitive risk-neutral foreign lenders with infinite collective wealth. In addition to these bonds, they have access to a one-period risk-free bond denominated in foreign currency that pays a net interest rate $r_t^\star$ in period t. By a no-arbitrage condition, lenders must break even in expectation, and this requires computing the probability of default. The bond price compensates them for any expected default losses, and is given by:

$$q_t = \frac{1}{1+r_t^\star}\mathbb{E}_t\left[(1-D_{t+1})((r_t^\star+\delta)+(1-\delta)q_{t+1})\right] \tag{17}$$

If the sovereign does not default next period, $D_{t+1} = 0$, and each unit of the bond pays the coupon $r_t^\star + \delta$, and the fraction that does not mature has market value $(1-\delta)q_{t+1}$. In states where the sovereign defaults, the associated payoff for the lenders is zero.

World interest rate. The international risk-free interest rate faced by foreign lenders follows an AR(1) time-varying process:

$$r_t^\star = \rho_r r_{t-1}^\star + (1-\rho_r)\bar{r^\star} + \epsilon_t^r \tag{18}$$

where $\rho_r \in (0,1)$ is the persistence of the process, $\bar{r^\star}$ in the long-run risk-free rate, and ϵ_t^r is an idiosyncratic shock. I assume that $\epsilon_t^r \sim \mathcal{N}(0,\sigma_r^2)$.

2.8 Resource Constraint and Market Clearing

Market clearing in the labor market requires that labor supplied by households equals the aggregate labor demand by firms N_t:

$$n_t = N_t \tag{19}$$

Domestic bonds are assumed to be in zero net supply, so the market clearing condition in this market implies:

$$B_t^D = 0 \tag{20}$$

The resource constraint for domestic goods requires that domestic production of final goods, net of the adjustment costs of inflation, equals domestic consumption and exports:

$$Y_t\left[1-\frac{\varphi}{2}(\pi_t-\bar{\pi})^2\right] = c_t^D + X_t \tag{21}$$

Notice that using the households' budget constraint (3), the government's budget constraint (16), the profits of the firms, the export's demand (8), and the market clearing conditions in (19), (20) and (21) we arrive at the balance of payments condition:

$$e_t c_t^F - e_t^\gamma \xi = (1-\vartheta_t)e_t\left[q_t(B_{t+1}-(1-\delta)B_t)-(r_t^\star+\delta)B_t\right] \tag{22}$$

which implies that the trade balance must be financed with the net capital inflows from debt operations.

3 Recursive Formulation and Equilibrium

In this section, I cast the optimality conditions in recursive form and I define the recursive equilibrium of the model. I focus on the notion of a Markov perfect equilibrium, in which policies depend on payoff-relevant states. As such, the government takes into account that its policies for default and borrowing affect the equilibrium allocations for households and firms, prices, and the monetary response of the central bank. In what follows, I denote the next-period value of the variables with a prime symbol.

3.1 Private and Monetary Equilibrium

Define $s := \{r^{\star}, \tilde{z}, \nu\}$ as the vector of exogenous states, composed by the world interest rate, the productivity, and the enforcement shock, respectively. The endogenous states for the Private and Monetary Equilibrium (a concept that I define formally below) are the current level of debt B, the credit standing ϑ, and the future level of debt B'.

In every period in which the government has access to international borrowing, it enters with good credit standing $\vartheta_{-1} = 1$ and a level of debt B, and chooses whether to repay or default. If the government repays it maintains access to financial markets, and the end-of-period credit standing is good, $\vartheta = 1$, whereas if it defaults it loses the good credit status, so at the end of the period $\vartheta = 0$. On the other hand, when the government enters the period with bad credit standing, ϑ_{-1}, it draws a random variable Λ, where $\Lambda \sim \text{Bernoulli}(\iota)$: with probability ι, $\Lambda = 1$ and the sovereign recovers the good credit standing, and with probability $(1-\iota)$, $\Lambda = 0$ and it maintains the bad credit standing $\vartheta = 0$. Then, the dynamics of the credit standing is given by:

$$\vartheta(s, \nu, B, \vartheta_{-1}) = \begin{cases} 1 & \text{if } (\vartheta_{-1} = 1 \text{ and } D = 0) \text{ or } (\vartheta_{-1} = 0 \text{ and } \Lambda = 1) \\ 0 & \text{otherwise} \end{cases} \tag{23}$$

Let $S := \{s, B, \vartheta, B'\}$ be the end-of-period vector of states. Now we can define the Private and Monetary Equilibrium (PME, for short) that characterizes the decisions of households, firms, and the monetary authority while taking as given the policy functions of the current and future governments, and the evolution of the credit standing.

Definition 1 (Private and Monetary Equilibrium). *Given the state $S = \{s, B, \vartheta, B'\}$, the government policy functions for future default $\mathcal{D}'(s', v', B')$, borrowing $\mathcal{B}'(s', B')$, the future credit standing $\vartheta'(s', v', B', \vartheta)$ and a transfer function $\mathcal{T}(S)$ that satisfies the government budget constraint, the Private and Monetary Equilibrium (PME) consists of household's policy functions $\{C(S), C^D(S), C^F(S), n(S), \mathcal{B}^D(S)\}$, firms' policy functions $\{N(S), \pi(S), \mathcal{Y}(S)\}$, exports $X(S)$, and prices $\{w(S), i(S), e(S)\}$ such that:*

1. *Policy functions for households solve their optimization problem, and conditions (4), (5) and (6) are satisfied*
2. *Policy functions for intermediate and final goods' firms solve their optimization problems, and conditions (9), (10), (11) and (12) are satisfied*
3. *Demand for exports (8) is satisfied*
4. *Domestic inflation-based Taylor rule (13) is satisfied*
5. *Markets clear, and conditions (19), (20), (21) and (22) are satisfied*

Notice that the PME depends on S because these state variables affect both the productivity and the transfers of the government. It also depends on the future policy functions of the government because the nature of the equilibrium is forward-looking. As shown in Appendix A, the PME allocations can be summarized by the policy functions for domestic and imported consumption $\{C^D(S), C^F(S)\}$, labor $n(S)$, inflation $\pi(S)$, the nominal interest rate $i(S)$, and the real exchange rate $e(S)$ that satisfy the following system of equations:

$$\frac{u_{c^F}(S)}{u_{c^D}(S)} = \frac{\rho}{\rho - 1} e(S) \tag{24}$$

$$u_{c^D}(S) = \beta i(S) M(s, B', \vartheta) \tag{25}$$

$$(\pi(S) - \bar{\pi})\pi(S) = \frac{\epsilon - 1}{\varphi}\left(-\frac{v_n(S)}{u_{c^D}(S) z(\tilde{z}, \vartheta)} - 1\right) + \frac{\beta}{u_{c^D}(S) z(\tilde{z}, \vartheta) n(S)} F(s, B', \vartheta) \tag{26}$$

$$i(S) = \bar{i}\left(\frac{\pi(S)}{\bar{\pi}}\right)^{\psi} \tag{27}$$

$$z(\tilde{z}, \vartheta) n(S)\left[1 - \frac{\varphi}{2}\left(\pi(S) - \bar{\pi}\right)^2\right] = C^D(S) + e(S)^{\gamma}\xi \tag{28}$$

$$e(S) C^F(S) - e(S)^{\gamma}\xi = (1 - \vartheta) e(S)\left[Q(s, B')(B' - (1 - \delta)B) - (r^{\star} + \delta)B\right] \tag{29}$$

where M and F are functions that describe the expectations in the domestic Euler equation and the NKPC, respectively. These functions are given by:

$$M(s, B', \vartheta) = \mathbb{E}\left[\frac{u_{c^D}(S')}{\pi(S')}\right] \tag{30}$$

$$F(s, B', \vartheta) = \mathbb{E}\left[z(\tilde{z}', \vartheta')n(S')u_{c^D}(S')(\pi(S') - \bar{\pi})\pi(S')\right] \tag{31}$$

where the future state $S' = (s', B', \vartheta'(s', v', B', \vartheta), \mathcal{B}'(s', B'))$ depend on the policy function for future borrowing and default, and the evolution of the credit standing in (23). Here $Q(s, B')$ is the bond price schedule, given by:

$$Q(s, B') = \frac{1}{1 + r^\star}\mathbb{E}\left\{(1 - \mathcal{D}'(s', v', B'))\left[(r^\star + \delta) + (1 - \delta)Q(s', B'')\right]\right\} \tag{32}$$

where $B'' = \mathcal{B}'(s', B')$. Notice that the bond price schedule depends on the world interest rate, productivity, enforcement shock, and the future level of debt because these state variables affect the probability of default. Similarly, future long-term obligations contain default risk, which is encoded in the continuation price $Q(s', \mathcal{B}'(s', B'))$. This future bond price is evaluated at the equilibrium policy function for debt, given a particular choice B'.

3.2 Recursive Government Problem

Every period, the government chooses the default decision, borrowing, and transfers subject to the PME, and taking as given its future policies. At any given state, the value of the option to default is given by:

$$W(s, v, B) = \max_{D \in \{0,1\}}\left\{(1 - D)W^R(s, B) + D\left[W^D(s) - v\right]\right\} \tag{33}$$

where $W^R(s, B)$ is the value associated with repaying and staying in the contract, and $W^D(s) - v$ is the value associated with default. Specifically, the value of repaying is given by:

$$W^R(s, B) = \max_{B'}\left\{u(C(S)) - v(n(S)) + \beta_g\mathbb{E}\left[W(s', v', B')\right]\right\} \tag{34}$$

subject to the PME, which is characterized by conditions (24)-(29) with $S = \{s, B, \vartheta = 1, B'\}$, and also subject to the bond price schedule in (32). Here $C(\mathcal{S})$ refers to the aggregator of $C^D(S)$ and $C^F(S)$, as in (2).

The value of default, net of the utility cost, is given by:

$$W^D(s) = u(C(S)) - v(n(S)) + \beta_g\mathbb{E}\left[\iota W(s', v', B' = 0) + (1 - \iota)W^D(s')\right] \tag{35}$$

subject to the PME with $S = \{s, B = 0, \vartheta = 0, B' = 0\}$ where s contains a reduced productivity. As is implied by the aggregate state, in this case the debt B is eliminated, and in the next period, with probability ι, the sovereign recovers access to financial markets, re-entering with zero debt.

Default decision. The value of the option to default in (35) implies that there exists a cutoff cost at which the value of repayment is equal to the value of default. At this particular value, the sovereign is indifferent between the two options. Let's denote such value as $v^\star(s, B)$. Then:

$$v^\star(s, B) = W^D(s) - W^R(s, B) \tag{36}$$

For realizations of the utility cost that do not exceed $v^\star(s, B)$ the government defaults on the current debt, and for those exceeding it, the government repays. Given this, the default policy can be characterized as follows:

$$\mathcal{D}(s, v, B) = \begin{cases} 1 & \text{if } v \leq v^\star(s, B) \\ 0 & \text{if } v > v^\star(s, B) \end{cases} \tag{37}$$

I assume that at the indifference point, the government defaults.

3.3 Recursive Equilibrium

I can now define the recursive equilibrium of this economy as follows:

Definition 2 (Recursive Equilibrium). *Given the state $\{s, v, B\}$, a Recursive Equilibrium consists of government policies for default $\mathcal{D}(s, v, B)$ and borrowing $\mathcal{B}(s, B)$, and government value functions $W(s, v, B)$, $W^R(s, B)$, and $W^D(s)$ such that:*

1. *Given future policies $\mathcal{D}'(s', v', B')$ and $\mathcal{B}'(s', B')$, and value functions $W(s, v, B)$, $W^R(s', v')$ and $W^D(s')$, government policies solve its optimization problem.*

2. *Government policies and value functions are consistent with future policies and value functions.*

4 Quantitative Analysis

4.1 Numerical Solution

The model is solved globally combining a root-finding algorithm with value function iteration on a discrete state space. For each state, I solve the PME numerically by searching over the allocations and prices that satisfy the private and monetary optimality conditions. For this step, I use a nonlinear optimization routine. Given the solution of the PME, I solve the recursive problem of the government using value function iteration. I incorporate discrete taste shocks following Dvorkin et al. (2021). These shocks slightly perturb the default and borrowing decisions of the government to achieve robust convergence and numerical stability in the computational algorithm. Appendix D.1 describes the perturbation approach with the taste shocks, and Appendix D.2 details the algorithm.

4.2 Calibration

The model is calibrated using Mexican data from 2001Q1 to 2019Q4. One period corresponds to one quarter. I fix some parameters to values from the literature and estimate the rest using Simulated Method of Moments (SMM). In this paper I focus on Mexico for several reasons. First, Mexico is a good example of an inflation-targeting small open economy in which sovereign risk has been an important source of business cycle fluctuations. Second, Mexico has both the median average inflation rate and the median average spread in the sample of inflation-targeting countries considered in Arellano et al. (2020). Third, the Central Bank of Mexico, *Banxico*, reacts to changes in the U.S. nominal interest rate, which is my proxy for the world interest rate. In 2015, for example, the monetary authorities in *Banxico* rescheduled all its policy meetings to take place exactly one day after those of the Federal Reserve.

Functional forms. I assume that the consumption utility function and the labor disutility function take the following forms:

$$u(c_t) = \frac{c_t^{1-\frac{1}{\sigma}} - 1}{1 - \frac{1}{\sigma}}, \qquad v(n_t) = \chi \frac{n_t^{1+\frac{1}{\eta}}}{1 + \frac{1}{\eta}}$$

where σ is the elasticity of intertemporal substitution, η is the Frisch elasticity of labor supply, and $\chi \geq 0$ is a labor disutility parameter. The final consumption good is a composite of domestic consumption c_t^D and imported consumption c_t^F according to a CES aggregator:

$$c_t = \left[\theta \left(c_t^D\right)^{\frac{\rho-1}{\rho}} + (1-\theta)\left(c_t^F\right)^{\frac{\rho-1}{\rho}}\right]^{\frac{\rho}{\rho-1}}$$

where $\theta \in [0,1]$ refers to the degree of home bias in preferences, and $\rho > 0$ measures the substitutability between domestic and foreign goods. As in Arellano et al. (2020), productivity with good credit standing follows an AR(1) process of the form:

$$\tilde{z}_t = \rho_z \tilde{z}_{t-1} + (1-\rho_z)\bar{\tilde{z}} + \epsilon_z$$

where $\epsilon_z \sim \mathcal{N}(0, \sigma_z^2)$. I assume that the productivity with bad credit standing is subject to a convex default cost as in Chatterjee and Eyigungor (2012):

$$z_t(\tilde{z}_t, \vartheta_t) = \begin{cases} \tilde{z} & \text{if } \vartheta = 1 \\ \tilde{z} - \max\left\{\lambda_0 \tilde{z} + \lambda_1 \tilde{z}^2, 0\right\} & \text{if } \vartheta = 0 \end{cases}$$

where $\lambda_0 \leq 0$ and $\lambda_1 \geq 0$, implying that default is most costly when the economy has higher levels of productivity. All selected parameter values used in the baseline calibration are shown in Table 1. The sources of the Mexican data used for moments targeted in the calibration are detailed in Appendix B.

Parameters set externally. I set the intertemporal elasticity of substitution to 1, which is a common value considered in the literature. I set the Frisch elasticity to 0.7, as in Pistaferri (2003), and I normalize the labor disutility parameter χ to 1. The domestic consumption weight in the consumption aggregator is set to $\theta = 0.66$, following Auclert et al. (2021) estimate, and the elasticity of substitution between varieties ϵ is 6, as in Arellano et al. (2020), inducing an average markup of 20%. For the parameters related to the debt market and the default decision, I calibrate the reentry probability ι to 4.1% to generate an average market exclusion spell of 6 years, in line with the estimates of Gelos et al. (2011) over the period 1980-2000 for 150 developing countries; and the value of δ, the decay rate of bonds, is chosen so that the average duration in the model corresponds to a Macaulay duration of 6 years, in line with the average bond duration reported in Cruces and Trebesch (2013).

The inflation coefficient in the Taylor rule ψ is 2.5, a standard value in the New Keynesian literature, and the parameter controlling the degree of price stickiness is set to $\varphi = 58$ following Arellano et al. (2020), which uses the first-order equivalence between the nominal rigidities

Table 1: Parameter Values

Parameters Set Externally	**Value**	**Target statistic/Source**
Preferences		
Intertemporal elasticity of substitution	$\sigma = 1$	Standard business cycles literature
Frisch elasticity	$\eta = 0.7$	Pistaferri (2003)
Labor disutility parameter	$\chi = 1$	Normalization
Domestic consumption weight	$\theta = 0.66$	Auclert et al. (2021)
Varieties elasticity of substitution	$\epsilon = 6$	Arellano et al. (2020)
Debt and Default		
Probability of reentry	$\iota = 0.041$	Average autarky spell (6 years)
Debt decay parameter	$\delta = 0.036$	Average Macaulay duration (6 years)
Monetary Policy		
Taylor rule coefficient	$\psi = 2.5$	Standard New Keynesian literature
Inflation adjustment cost	$\varphi = 58$	Frequency of price changes (1 year)
Foreign Demand		
Trade elasticity	$\gamma = 1.7$	Estimates for Mexico in literature
Exogenous Processes		
Persistence of AR(1) productivity	$\rho_z = 0.90$	Arellano et al. (2020)
Mean of AR(1) productivity	$\bar{\bar{z}} = 1$	Normalization
Persistence of AR(1) world interest rate	$\rho_r = 0.91$	OLS estimation
Mean of AR(1) world interest rate	$\bar{r^\star} = 0.003$	Average U.S. real interest rate
Standard dev. of world interest rate shock	$\sigma_{r^\star} = 0.002$	OLS estimation
Parameters Set Internally	**Value**	
Preferences		
Households discount factor	$\beta = 0.995$	Volatilities of cons. and inflation
Government discount factor	$\beta_g = 0.983$	Volatilities of cons. and inflation
Debt and Default		
Default taste shock	$\varrho_D = 1e^{-4}$	Numerical convergence
Borrowing taste shock	$\varrho_B = 5e^{-7}$	Numerical convergence
Parameter in default cost function	$\lambda_0 = -0.16$	Average spread
Parameter in default cost function	$\lambda_1 = 0.18$	Volatility of spread
Monetary Policy		
Inflation target	$\bar{\pi} = 1.010$	Average CPI inflation rate
Long-run nominal interest rate	$\bar{i} = 1.015$	Average Mexican nominal int. rate
Other Parameters		
Level of overall foreign demand	$\xi = 0.6$	Average cons.-to-output ratio
Standard dev. of productivity shock	$\sigma_z = 0.012$	Volatility of output

of Calvo and Rotemberg to calibrate an average frequency of price changes to once every year. Estimates for the trade elasticity in Mexico are uncertain, so I calibrate this parameter within the range of Caliendo and Parro (2014) and Auclert et al. (2021) estimates, implying a value of $\gamma = 1.7$. I consider alternative trade elasticities in Appendix C.

For the stochastic process of productivity, I normalize its mean to $\bar{\bar{z}} = 1$, and set the persistence parameter ρ_z to 0.9, as in Arellano et al. (2020). The world interest rate $r_t^\star$ is measured by the U.S. real interest rate, which is calculated by deflating the U.S. federal funds rate with the expected CPI inflation. I set its mean value to $\bar{r^\star} = 0.003$, consistent with an average annual real rate of 1.5%. I estimate the autoregressive process in (18) using the Hodrick-Prescott cyclical component of the U.S. real interest rate, which delivers estimated values for persistence and volatility of $\rho_r = 0.91$ and $\sigma_r = 0.002$, respectively.

Parameters set internally. The parameters ϱ_B and ϱ_D govern the relative importance for the choice of B' in the finite grid Γ, as well as the choice for D, and they are set to $5e^{-7}$ and $1e^{-4}$, respectively. These are the smallest values that guarantee convergence in the model. The remaining eight parameters $\Theta = \{\beta, \beta_g, \lambda_0, \lambda_1, \bar{\pi}, \bar{i}, \xi, \sigma_z\}$ include discount factors, parameters in the default cost function, the long-run nominal interest rate, the inflation target, the overall level of external demand and the standard deviation of the productivity shock. They are set to match the following moments from the data: (i) mean CPI inflation, (ii) mean nominal interest rate, (iii) mean spread, (iv) mean consumption-to-output ratio, (v) volatility of CPI inflation, (vi) volatility of output, (vii) volatility of final consumption, (viii) volatility of spread, and (ix) the correlation between spread and GDP.[3]

Although all parameters affect all moments in the calibration, some moments are governed mostly by certain parameters: the target rate of inflation $\bar{\pi}$ is calibrated to deliver an average CPI inflation rate in line with the observed average in the data of 4.11%; the long-run nominal interest rate $\bar{i}$ targets the average Mexican monetary policy rate (5.9% annually); the volatility of productivity shocks informs the volatility of output, and the discount factor of the household and the government, together with the default cost parameters, are crucial for the mean spread, which in the data is 2.2%, and the volatilities of consumption, inflation, and spread (1.8%, 1%, and 0.71%,

[3] Note that the consumer price index in the model P_t is a weighted average between the prices of the domestic and foreign consumption goods: $P_t = \left[\theta^\rho \left(P_t^D\right)^{1-\rho} + (1-\theta)^\rho \left(P_t^F\right)^{1-\rho}\right]^{\frac{1}{1-\rho}}$. Replacing the definition of the real exchange rate, the CPI inflation is:

$$\pi^{CPI} = \pi \left[\frac{\theta^\rho + (1-\theta)^\rho e_t^{1-\rho}}{\theta^\rho + (1-\theta)^\rho e_{t-1}^{1-\rho}}\right]^{\frac{1}{1-\rho}}$$

respectively). The level of overall foreign demand ξ targets a mean consumption-to-output ratio of about 66%. I also report some non-targeted moments: the correlation of output and spread, the correlation of inflation and spread, and the volatility of the nominal interest rate.

5 Results of the Quantitative Analysis

This section presents the main quantitative findings. The first subsection presents the fit of the model. The second subsection describes the policy functions coming from the model solution, and the third subsection simulates the model dynamics during the post-COVID-19 U.S. monetary tightening, starting from different initial levels of debt.

5.1 Model Fit

The model is able to account for salient features of business cycles in Mexico. Table 2 reports key moments from the baseline model and compares them with their data counterparts. To compute the moments in the model I simulate the exogenous stochastic processes and the enforcement shock for 10000 periods and trace the evolution of all the macroeconomic variables. The moments are computed by eliminating the first 1000 observations.

The model matches closely the targeted moments. In the model and data, the mean CPI inflation is about 4.1%, the mean nominal interest rate is 5.9% and the mean spread is 2.2%. The consumption-to-output ratio is about 6% lower in the model compared to its data counterpart. In terms of standard deviations, the model delivers similar volatilities of the CPI inflation and spread, but overestimates the volatility of output (2.47% vs. 1.68% in the data) and underestimates the volatility of consumption. Moreover, the model successfully replicates the countercyclical behavior of spreads observed in Mexico, with a correlation of -0.37, about the same as in the data, and the volatility of the nominal interest rate. It also matches the positive correlation of inflation and spread: 0.28 in the model, very close to the data counterpart of 0.23.

5.2 Policy functions

In this subsection, I show that default risk leads to an important state dependency in the monetary policy during periods of U.S. monetary tightening. In particular, the domestic inflation response depends crucially on the debt level position of the country, and so does the nominal interest rate. To illustrate this state-contingency Figure 2 presents the probability of default as a function of the gross world interest rate for three different levels of debt. Here, I keep productivity at the mean level.

Table 2: Moments in the Calibration

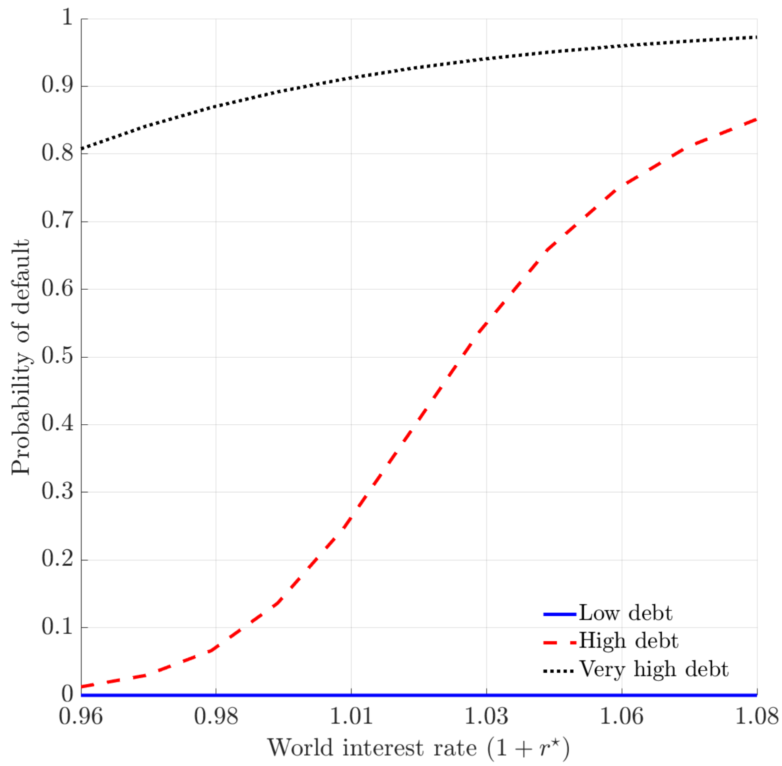
Moment	**Data**	**Model**
Means		
CPI inflation	4.11	4.17
Nominal interest rate	5.90	5.89
Spread	2.20	2.25
Consumption-to-output ratio	66	63
Standard Deviations		
CPI inflation	1.00	0.81
Nominal interest rate	1.94	1.92
Output	1.68	2.47
Consumption	1.85	1.28
Spread	0.71	0.81
Correlations		
Output, spread	-0.39	-0.37
Inflation, spread	0.23	0.28

Notes: Moments in the data are computed using data from 2001Q1 to 2019Q4. Moments in the model are computed using simulated time series. I simulate the model for 10000 periods, and eliminate the first 1000 observations. Means and standard deviations are reported in percentage points. For additional details, see Appendix B.

The solid blue line refers to the low level of debt while the dashed red line and the black dotted line refer to the high and very high debt levels, respectively. The figure shows that the default probability is less sensitive to hikes in the world interest rate when the economy has either too low or too high levels of debt. If the initial level of debt is low, the probability of default is zero for any level of $r^\star$, and the government always repays its outstanding obligations despite potentially large costs of servicing the debt. If current debt is high enough, default is already very likely, and as a result, increases in $r^\star$ are met with moderate increases in the probability of default. In sharp contrast, when the economy has current levels of debt between the previous two cases (the dashed red line) the sensitivity of the default risk to changes in the world interest rate is amplified. A contractionary monetary policy in the U.S., for example, triggers a potentially large increase in sovereign risk.

Such an increase in the world interest rate affects default incentives through different mechanisms: The first one is the "standard mechanism" highlighted in Almeida et al. (2023) and works through higher borrowing costs. International lenders discount debt payments at a higher rate, and this translates into a less favorable bond price schedule for the government. This implies a

Figure 2: Probability of Default



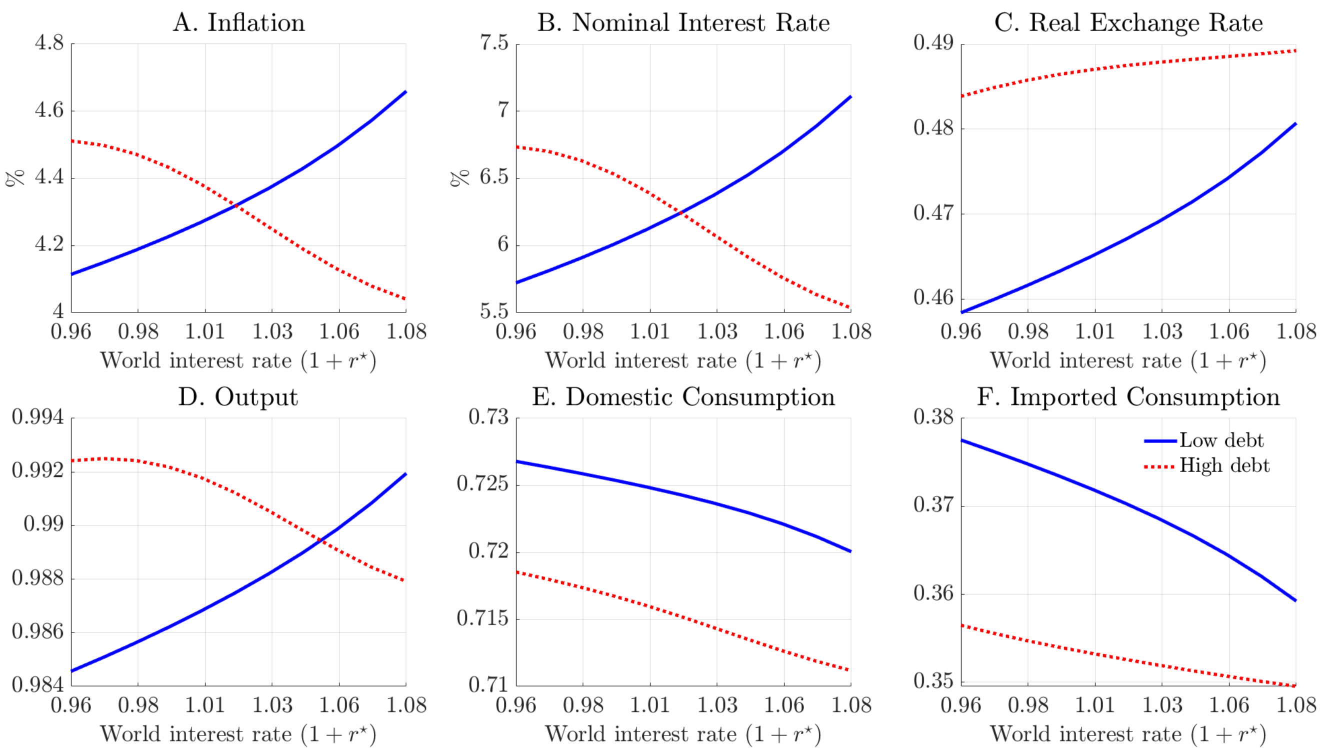
Notes: This figure plots the one-period ahead probability of default as a function of the gross world interest rate. The solid blue line is associated with a low level of debt, the dashed red line corresponds to a high level of debt, and the dotted black line to a very high level of debt, respectively. The productivity is assumed constant at the mean level.

higher cost of rolling over the current debt, as the sovereign would need to sacrifice more units of consumption to service the debt. The second mechanism is related to the exclusion from financial markets during default: due to the persistence of the process for $r^{\star}$, higher borrowing costs would remain in place for several periods in the future. This makes defaulting relatively more attractive as the periods that the sovereign is unable to borrow would coincide with those of high costs.

Default amplification. To shed light on the monetary policy implications, Figure 3 displays key variables of the private and monetary equilibrium. In these policy rules I focus only on the "low" and "high" debt levels.

Let's analyze first the decision rules associated with low debt. In this case, the default probability is zero for any level of the world interest rate. As such, a U.S. monetary tightening does not change expectations about a future debt crisis, and thus, it transmits to the domestic economy following the standard expenditure-switching channel that is present in the Galí and Monacelli

Figure 3: Policy Functions for Selected Variables



Notes: This figure plots the policy functions for selected variables of the PME as a function of the gross world interest rate. The solid blue lines correspond to those associated with a low level of debt, whereas the dashed red lines correspond to those with a high level of debt. The productivity is assumed constant at the mean level.

(2005) model, as well as an expenditure-reducing channel. Higher external interest rates reduce consumption by households, as more resources are needed to service the external debt. If the government were to repay the debt, the transfers to the households would be lower, and this reduces the amount of resources that are available for consumption. This is the expenditure-reducing effect seen in panels E and F: demand for both domestic and imported goods falls as the world interest rate increases. Such higher rates limit the availability of borrowing, which is used by the government to achieve the right tilting of consumption over time by smoothing the marginal utility of consumption of imported goods. This implies a sharper reduction in foreign goods' consumption, compared to domestic consumption, which results in a depreciation of the exchange rate, as displayed in panel C. The depreciation, in turn, makes imported goods relatively more expensive compared to domestic goods, leading domestic and foreign households to substitute towards domestic goods. This reallocation of spending corresponds to the traditional expenditure-switching effect. The stronger external demand for domestic goods boosts exports, as firms respond by increasing both output and prices (panels A and D). As shown in panel B, nominal interest rates rise to fight the higher inflation. In this case, the central bank adjusts its policy rate in line with the monetary policy stance abroad.

Now I focus on the decision rules for the high debt level, where the novel channel of *default amplification* is at play. As shown in Figure 2, a world interest rate hike is able to generate a large increase in sovereign risk. Default amplification works through the expectations of future consumption and inflation induced by the higher probability of default. Two key intertemporal conditions of the PME link current allocations with good credit standing to expected future allocations with a potential default: the Euler equation and the NKPC:

$$u_{c^D} = \beta i \mathbb{E}\left[\frac{u'_{c^D}}{\pi'}\right] \tag{38}$$

$$(\pi - \bar{\pi})\pi = \frac{\epsilon - 1}{\varphi}\left(-\frac{w}{z} - 1\right) + \beta\mathbb{E}\left[\frac{u'_{c^D}}{u_{c^D}}\frac{z'n'}{zn}(\pi' - \bar{\pi})\pi'\right] \tag{39}$$

When an increase in $r^\star$ triggers an increase in the probability of a debt crisis, expectations of low productivity next period are higher. This has two distinct effects: (i) lower future production, which renders lower future consumption, and a higher marginal utility of consumption next period, and (ii) higher real marginal costs in the future, as a firm with reduced productivity requires more units of labor to produce one unit of the domestic good, which yields a higher expected inflation rate. The first effect implies, through the Euler equation (38), that current domestic consumption decreases; and from the second effect, the NKPC in (39) calls for an increase in current inflation.

As in the previous "low debt" case, the expenditure-reducing channel induces lower consumption of domestic and imported goods. The fall tends to be stronger for imported goods due to the expenditure-switching channel, but now higher sovereign risk depresses more domestic consumption through default amplification. As a result, consumption of both goods decreases at comparable rates, and the real exchange rate depreciates less. Exports also increase less due to the lower depreciation, and this leads to a contraction of output given the reduction in domestic consumption. As seen in panel A, firms respond to this lower demand by decreasing prices. Note that the decline in inflation is larger despite the rise in the expected real marginal costs, as the effect of lower domestic demand dominates the overall effect on prices. In this case, the monetary authority lowers the nominal interest rate to stimulate demand and fight the low inflation. As a result, the domestic monetary policy does not coincide with that of the U.S., which contrasts with the synchronization observed when there is no sovereign risk. Remarkably, the effect on output is the opposite of the previous case: it falls, although the exchange rate still depreciates.

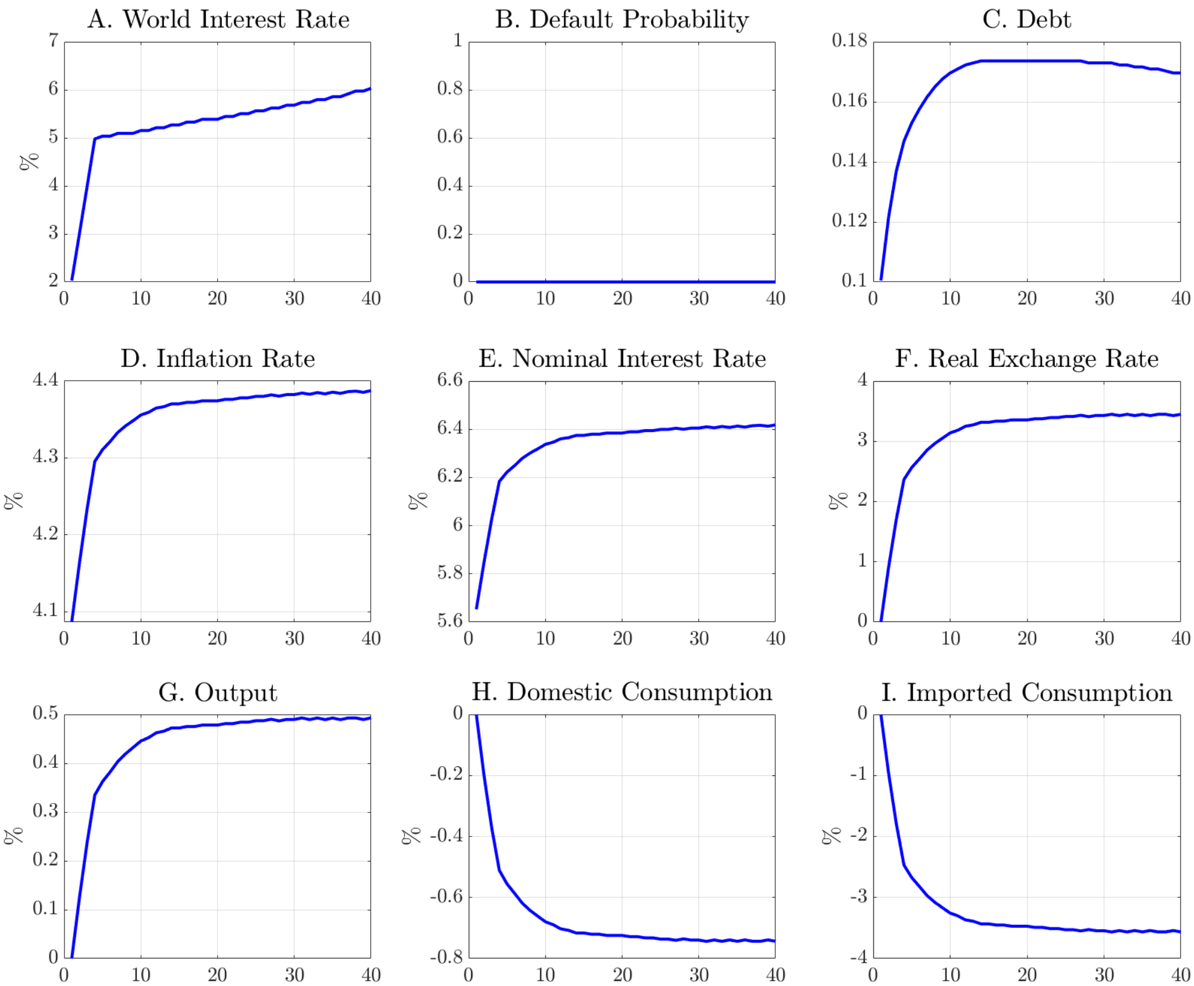
This happens because with default risk the overall demand for domestic goods decreases more. Hence, default amplification opens the door to contractionary depreciations.

5.3 Simulations

In this subsection, I present simulation results for a policy experiment. I consider a path for the world interest rate consistent with the post-COVID-19 path of the Federal Reserve funds rate and simulate the model forward under constant productivity, starting from two different initial levels of debt, which I denote as "high" and "low", as before. The idea is to feed into the model the observed Federal funds effective rate for the period 2022Q2-2023Q2, under the assumption that the monetary tightening continues at a slower pace for the remaining periods in the simulation. Appendix C.2 shows the results of the simulations considering an alternative constant path for the Federal Reserve funds rate. The exercise will highlight a main insight from the previous analysis of the decision rules; namely, that the domestic monetary response is highly state-contingent, and the direction of the nominal interest rate depends on the level of sovereign debt and the associated default probability. The quantitative analysis will also underscore how sovereign risk induces a negative co-movement between output and the real exchange rate.

Figure 4 shows the results of this exercise when the economy starts with an initial low debt level. Along the path, I report quantities and the real exchange rate as percentage deviations from their levels at $t = 0$. Panel A displays the path for the world interest rate: it increases from 2% to 5% during the first five periods, consistent with the Federal Reserve funds rate hike during the period 2022Q2-2023Q2. I assume that the tightening continues at a slower pace and reaches about 6% by the end of the simulation. In the first case, starting from a low level of debt, the government chooses an expansionary path for borrowing that increases debt during the beginning of the transition. Along the transition, the international lenders discount future payments at a higher rate, and eventually their discount factor reaches a level below that of the government, inducing a decrease in the debt level by the end of the simulation. The higher debt service reduces the amount of transfers that the government sends to households, and as shown in panels H and I, both the domestic and imported consumption decline. Note that imported consumption falls by almost 4% towards the end, while domestic consumption decreases only by around 1%. This calls for a depreciation of the real exchange rate of 3% which increases foreign demand for domestic goods. Output increases by 0.5%, as displayed in panel G, because the overall demand for domestic goods increases: the stronger demand from foreign households boots exports, and outweighs the reduced demand from domestic households. Panel D shows that firms also respond to higher demand by increasing prices, and the central bank adopts a contractionary monetary policy. In-

Figure 4: U.S. Monetary Tightening with Low Debt



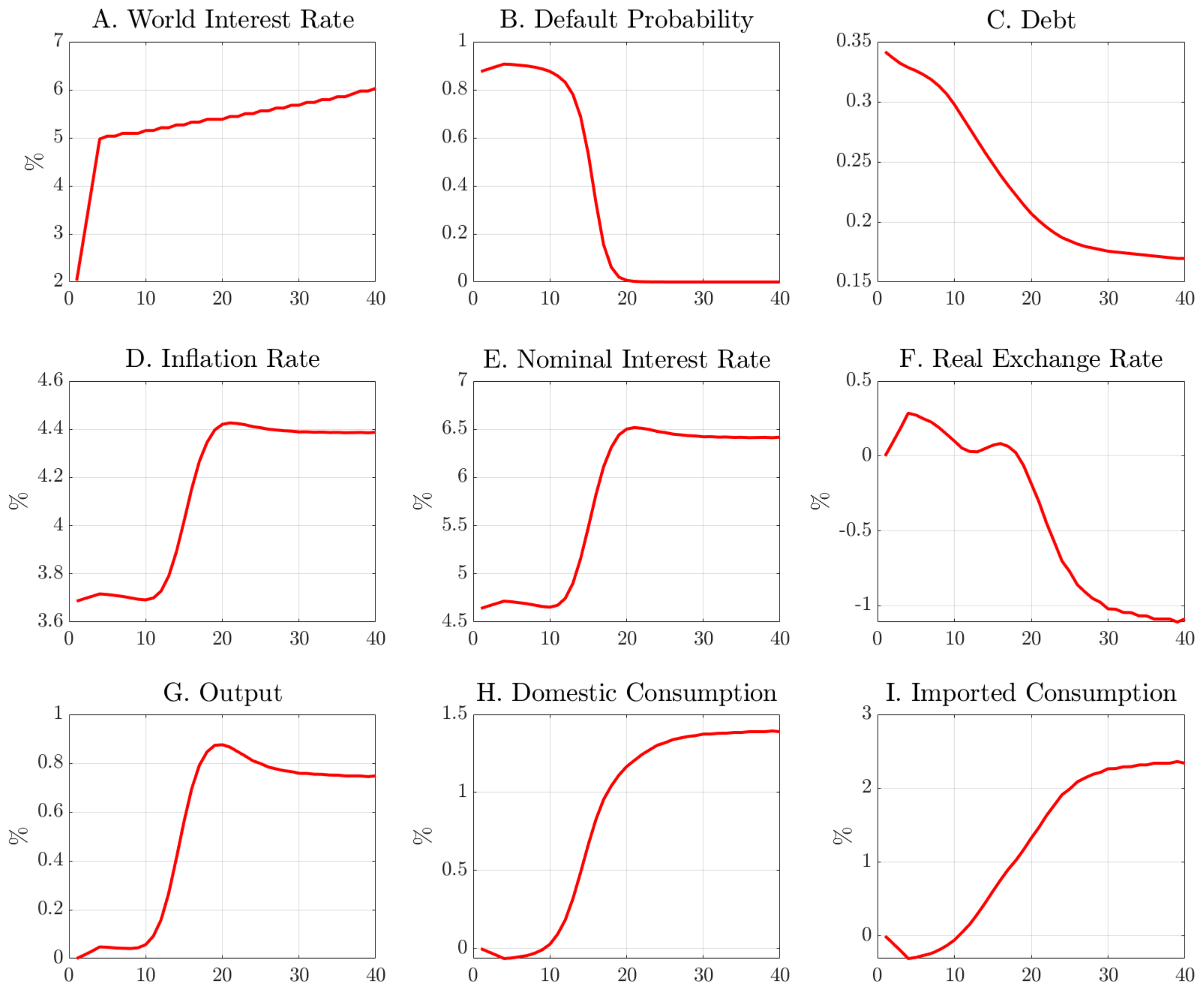
Notes: Simulations are obtained by feeding into the model a path for the world interest rate consistent with the post-COVID-19 U.S. monetary tightening for the period 2022Q2-2023Q2 (see panel A). The figure shows the response of macro variables when the domestic economy has an initial low debt level. Debt is reported in levels; real exchange rate, output, and domestic and imported consumption are reported in percentage deviations from the level in $t = 0$, while the world interest rate, inflation rate, and nominal interest rate are reported in percentage points.

flation rises by 0.2% over the first year, and by an additional 0.1% at the end of the simulation, inducing an interest rate hike of about 80 basis points.

Now I explore the aggregate dynamics starting from an initial high debt level. Figure 5 plots the variables of interest during this transition. The layout of the figure is the same as that of Figure 4.

Panel A presents the posited path of the world interest rate, which is the same as in Figure 4.

Figure 5: U.S. Monetary Tightening with High Debt



Notes: Simulations are obtained by feeding into the model a path for the world interest rate consistent with the post-COVID-19 U.S. monetary tightening for the period 2022Q2-2023Q2 (see panel A). The figure shows the response of macro variables when the domestic economy has an initial high debt level. Debt is reported in levels; real exchange rate, output, and domestic and imported consumption are reported in percentage deviations from the level in $t = 0$, while the world interest rate, inflation rate, and nominal interest rate are reported in percentage points.

Debt starts at a high level, decreases throughout the transition, and settles at a lower level. During the first periods of the transition, the default probability increases and remains high, owing to the higher world interest rate, and despite the reduction in government borrowing. Tighter bond price schedules and the repayment of the high debt due at the beginning of the simulation lead to low government transfers to households, which end up reducing consumption of both domestic and imported goods. Panels H and I show that the fall in imported consumption is more pronounced compared to domestic consumption, leading to a real exchange rate depreciation of about 30 basis points during the first year. Output remains fairly flat as a result of a lower increase in exports and a smaller decrease in demand for domestic goods. As demand for domestic goods is weak during these periods of high default risk, firms slightly decrease prices, and the central bank does not raise the monetary policy rate.

However, throughout the transition, the economy reaches a level of debt low enough that induces a large reduction in the probability of default. At this point, the default amplification channel shapes the dynamics of the macroeconomic variables. Facing lower default risk, the expected productivity is higher, implying higher expected consumption. As households smooth consumption over time, they increase the consumption of both domestic and imported goods, as shown in panels H and I. In this scenario, firms respond by both increasing production and prices. Note that this generates an increase in inflation of about 70 basis points, and this happens despite the lower expected inflation in the future due to the fall in the expected real marginal cost. The stronger demand dominates the effect of reduced inflation expectations in the response on inflation today. The real exchange rate appreciates by 1%, causing a downturn in exports. The stronger demand from domestic households offsets the weaker demand from foreign households, and the appreciation of the real exchange rate coincides with the expansion of output. The crucial result here is that the presence of sovereign risk breaks the positive comovement between the world interest rate and the domestic interest rate, and the positive comovement between the exchange rate and output.

6 Conclusion

This paper examines the role of fluctuations in the world interest rates in a small open economy New Keynesian model with sovereign default. My analysis shows that the interplay between the government incentives to repay the debt and domestic inflation has distinct implications for monetary policy. I show that a monetary easing may emerge when a world interest rate hike induces a substantial increase in the probability of default, depressing domestic demand and leading firms to reduce inflation. This default amplification channel complements the expenditure-switching and expenditure-reducing channels found in standard open economy models, and rationalizes why emerging economies might reduce monetary policy rates when the Federal Reserve tightens. An increase in sovereign risk reduces demand for domestic goods beyond these conventional channels, leading a real exchange rate depreciation to be contractionary for output.

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Appendices

A Recursive Private and Monetary Equilibrium

This section shows that the PME can be summarized by the system of nonlinear equations in (24), (25), (26), (27),(29) and (28), together with the definitions of expectations in the Euler equation and Phillips curve, given by (30) and (31). Let $\{C^D(S), C^F(S), n(S), \mathcal{Y}(S)\}$ be the policy functions for domestic, imported consumption, labor, and output; and $\{\pi(S), i(S), e(S)\}$ the inflation, nominal interest rate, and real exchange rate.

Relative demand. Let's start from the recursive form of the relative demand equation in (5):

$$\frac{u_{c^F}(S)}{u_{c^D}(S)} = (1+\tau_F)e(S)$$

where I replaced the real exchange rate definition in (7). Plugging the optimal tariff $\tau_F = \frac{1}{\rho-1}$, previous expression can be written as:

$$\frac{u_{c^F}(S)}{u_{c^D}(S)} = \frac{\rho}{\rho-1}e(S) \tag{40}$$

which coincides with equation (24).

Euler equation. From equation (6) we have:

$$u_{c^D}(S) = \beta i(S)\mathbb{E}\left[\frac{u_{c^D}(S')}{\pi(S')}\right]$$

Defining $M(s, B', \vartheta) := \mathbb{E}\left[\frac{u_{c^D}(S')}{\pi(S')}\right]$ as in (30), we obtain:

$$u_{c^D}(S) = \beta i(S) M(s, B', \vartheta) \tag{41}$$

which coincides with equation (25).

New Keynesian Phillips curve. From the NKPC in equation (12)

$$(\pi(S)-\bar{\pi})\pi(S) = \frac{\epsilon-1}{\varphi}\left(\frac{\epsilon}{\epsilon-1}mc(S)-1\right) + \beta\mathbb{E}\left[\frac{u_{c^D}(S')}{u_{c^D}(S)}\frac{\mathcal{Y}(S'))}{\mathcal{Y}(S)}(\pi(S'))-\bar{\pi})\pi(S'))\right]$$

Note that from the definition of the real marginal cost $mc(S) = \frac{(1-\tau)w(S)}{z(\tilde{z},\vartheta)}$, we can write the previous expression as:

$$(\pi(S) - \bar{\pi})\pi(S) = \frac{\epsilon - 1}{\varphi}\left(\frac{\epsilon}{\epsilon - 1}\frac{(1-\tau)w(S)}{z(\tilde{z},\vartheta)} - 1\right) + \beta\mathbb{E}\left[\frac{u_{c^D}(S')}{u_{c^D}(S)}\frac{\mathcal{Y}(S'))}{\mathcal{Y}(S)}(\pi(S')) - \bar{\pi})\pi(S'))\right]$$

Plugging the labor supply equation (4), given by $-\frac{v_n(S)}{u_{c^D}(S)} = w(S)$, and the optimal tax $\tau = 1/\epsilon$, we obtain:

$$(\pi(S) - \bar{\pi})\pi(S) = \frac{\epsilon - 1}{\varphi}\left(-\frac{v_n(S)}{u_{c^D}(S)z(\tilde{z},\vartheta)} - 1\right) + \beta\mathbb{E}\left[\frac{u_{c^D}(S')}{u_{c^D}(S)}\frac{\mathcal{Y}(S'))}{\mathcal{Y}(S)}(\pi(S')) - \bar{\pi})\pi(S'))\right] \tag{42}$$

Now, note that from (9) and (11) we have:

$$\begin{aligned}
\mathcal{Y}(S) &= \left[\int_0^1 \left(z(\tilde{z},\vartheta)N_j(S)\right)^{\frac{\epsilon-1}{\epsilon}} dj\right]^{\frac{\epsilon}{\epsilon-1}} \\
&= z(\tilde{z},\vartheta)\left[\int_0^1 \left(N_j(S)\right)^{\frac{\epsilon-1}{\epsilon}} dj\right]^{\frac{\epsilon}{\epsilon-1}} \\
&= z(\tilde{z},\vartheta)N(S)
\end{aligned}$$

where the last equality comes from defining $N(S) := \left[\int_0^1 \left(N_j(S)\right)^{\frac{\epsilon-1}{\epsilon}} dj\right]^{\frac{\epsilon}{\epsilon-1}}$. Imposing the market clearing condition in the labor market (19), given by $n(S) = N(S)$, final production in equilibrium can be written as:

$$\mathcal{Y}(S) = z(\tilde{z},\vartheta)n(S) \tag{43}$$

And replacing in (42) we obtain:

$$(\pi(S) - \bar{\pi})\pi(S) = \frac{\epsilon - 1}{\varphi}\left(-\frac{v_n(S)}{u_{c^D}(S)z(\tilde{z},\vartheta)} - 1\right) + \beta\mathbb{E}\left[\frac{u_{c^D}(S')}{u_{c^D}(S)}\frac{z(\tilde{z}',\vartheta')n(S'))}{z(\tilde{z},\vartheta)n(S)}(\pi(S')) - \bar{\pi})\pi(S'))\right]$$

Defining $F(s,B',\vartheta) := \mathbb{E}\left[z(\tilde{z}',\vartheta')n(S')u_{c^D}(S')(\pi(S') - \bar{\pi})\pi(S')\right]$ we get:

$$(\pi(S) - \bar{\pi})\pi(S) = \frac{\epsilon - 1}{\varphi}\left(-\frac{v_n(S)}{u_{c^D}(S)z(\tilde{z},\vartheta)} - 1\right) + \frac{\beta}{u_{c^D}(S)z(\tilde{z},\vartheta)n(S)}F(s,B',\vartheta) \tag{44}$$

which coincides with (26).

Taylor rule. From equation (13) written as a function of the aggregate state S we obtain:

$$i(S) = \bar{i}\left(\frac{\pi(S)}{\bar{\pi}}\right)^{\psi} \tag{45}$$

which coincides with (27).

Resource constraint. From (21) we have:

$$\mathcal{Y}(S)\left[1-\frac{\varphi}{2}\left(\pi(S)-\bar{\pi}\right)^2\right]=C^D+e(S)^\gamma\xi$$

where I replaced the export demand (8). Now, plugging (43) we obtain:

$$z(\tilde{z},\vartheta)n(S)\left[1-\frac{\varphi}{2}\left(\pi(S)-\bar{\pi}\right)^2\right]=C^D(S)+e(S)^\gamma\xi \tag{46}$$

which coincides with (28).

Balance of payments. From the budget constraint of the household (3) in terms of domestic goods:

$$C^D(S)+(1+\tau_F)e(S)C^F(S)=w(S)n(S)+\hat{\Psi}(S)+\mathcal{T}(S) \tag{47}$$

where I replaced the real exchange rate definition in (7), and I imposed the market clearing condition in the domestic bond market, $B^D(S)=0$, as in equation (20). Here $\hat{\Psi}(S)$ and $\mathcal{T}(S)$ refer to the profits of the firm, and the government transfers, both in terms of domestic consumption goods, respectively.

From the budget constraint of the government (16), we have:

$$\mathcal{T}(S)+\tau w(S)n(S)=(1-\vartheta)e(S)\left[Q(s,B')(B'-(1-\delta)B)-(r^\star+\delta)B\right]+\tau_F e(S)c^F(S) \tag{48}$$

Profits of the firm, in equilibrium, are given by:

$$\begin{aligned}\hat{\Psi}(S)&=\left(1-(1-\tau)\frac{w(S)}{z(\tilde{z},\vartheta)}\right)\mathcal{Y}(S)-\frac{\varphi}{2}\left(\pi(S)-\bar{\pi}\right)^2\mathcal{Y}(S)\\&=\mathcal{Y}(S)-(1-\tau)w(S)n(S)-\frac{\varphi}{2}\left(\pi(S)-\bar{\pi}\right)^2\mathcal{Y}(S)\end{aligned} \tag{49}$$

where I replaced $\mathcal{Y}(S)=z(\tilde{z},\vartheta)n(S)$. Now, replacing the resource constraint in (46) and the export demand in (8):

$$\begin{aligned}\hat{\Psi}(S)&=C^D(S)+e(S)^\gamma\xi+\frac{\varphi}{2}\left(\pi(S)-\bar{\pi}\right)^2\mathcal{Y}(S)-(1-\tau)w(S)n(S)-\frac{\varphi}{2}\left(\pi(S)-\bar{\pi}\right)^2\mathcal{Y}(S)\\&=C^D(S)+e(S)^\gamma\xi-(1-\tau)w(S)n(S)\end{aligned} \tag{50}$$

Plugging (50) into (47):

$$C^D(S) + (1 + \tau_F)e(S)C^F(S) = w(S)n(S) + C^D(S) + e(S)^\gamma\xi - (1-\tau)w(S)n(S) + \mathcal{T}(S)$$

Cancelling out similar terms:

$$(1 + \tau_F)e(S)C^F(S) = \tau w(S)n(S) + e(S)^\gamma\xi + \mathcal{T}(S)$$

Now, replacing the transfers from the government budget constraint, given by (48), we obtain:

$$\begin{aligned}(1 + \tau_F)e(S)C^F(S) &= \tau w(S)n(S) + e(S)^\gamma\xi \\ &\quad + (1-\vartheta)e(S)\left[Q(s, B')(B' - (1-\delta)B) - (r^\star + \delta)B\right] + \tau_F e(S)c^F(S) - \tau w(S)n(S)\end{aligned}$$

Cancelling out the tax and subsidy terms:

$$e(S)C^F(S) - e(S)^\gamma\xi = (1-\vartheta)e(S)\left[Q(s, B')(B' - (1-\delta)B) - (r^\star + \delta)B\right] \tag{51}$$

which coincides with (29). This completes the derivation of the equations (24) - (29) in the PME.

B Data sources

Nominal interest rate: Bank of Mexico monetary policy rate. From January 2008 onwards, it corresponds to the central bank target money market overnight rate, and from November 1998 to January 2008, it corresponds to the bank funding rate. Expressed in percentage points.
Source: Bank of International Settlements (BIS)

Consumer price index: Corresponds to a weighted geometric average of prices for a basket of goods and services representative of aggregate Mexican consumer spending. Normalized to 100 in base year 2018.
Source: International Financial Statistics (IFS). Ticker: PCPI_IX

Spread: Emerging Markets Bond Index (EMBI+) for Mexico. Corresponds to a weighted capitalization market benchmark that measures the financial returns obtained each day by a selected portfolio of Mexican government bonds, calculated by J.P. Morgan.
Source: Global Economic Monitor (GEM)

Output: Gross Domestic Product (GDP). Corresponds to the total final expenditures at pur-

chasers' prices (including the FOB value of exports of goods and services), less the FOB value of imports of goods and services. Expressed in national currency units. Seasonally adjusted.
Source: International Financial Statistics (IFS). Ticker: NGDP_SA_XDC

Consumption: Households' consumption expenditure plus expenditures of general government and non-profit institutions serving households (NPISHs) that directly benefit households, such as health care and education. Expressed in national currency units. Seasonally adjusted.
Source: International Financial Statistics (IFS). Ticker: NCP_SA_XDC

U.S. nominal interest rate: Federal funds effective rate. Corresponds to the interest rate at which depository institutions trade federal funds (balances held at Federal Reserve Banks) with each other overnight. Expressed in percentage points. Not seasonally adjusted.
Source: Federal Reserve Bank of St. Louis Economic Data (FRED). Ticker: FEDFUNDS

U.S. expected inflation: Estimated one-year ahead expectations on total inflation. Corresponds to the rate that inflation is expected to average over the next year, and is calculated with a model that uses Treasury yields, inflation data, inflation swaps, and survey-based measures of inflation expectations.
Source: Federal Reserve Bank of Cleveland

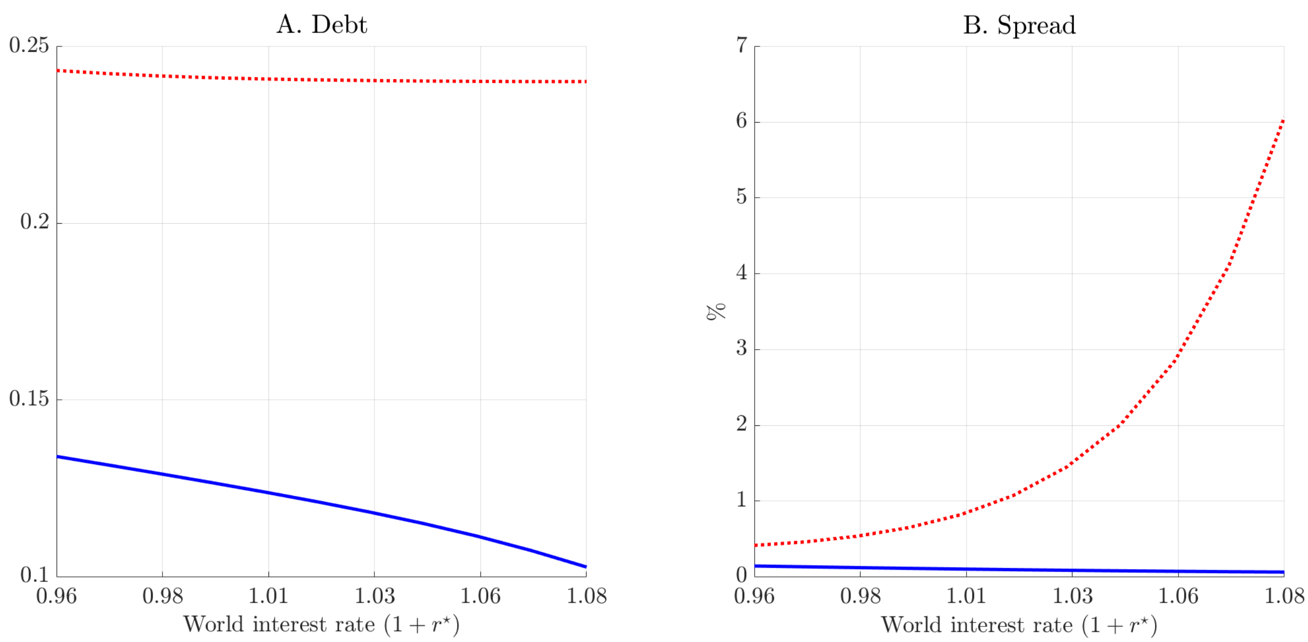
C Additional Results

C.1 Policy Functions

This subsection presents the policy functions for the sovereign debt and the spreads when the economy has the same "low" and "high" levels of debt considered in section 5.3. The spread is defined by the difference between the yield-to-maturity of the sovereign bond and the mean world interest rate that the risk-free bond pays:

$$\text{spread}_t = (\bar{r} + \delta)\left(\frac{1}{q_t} - 1\right) \tag{52}$$

Figure 6: Debt and Spread

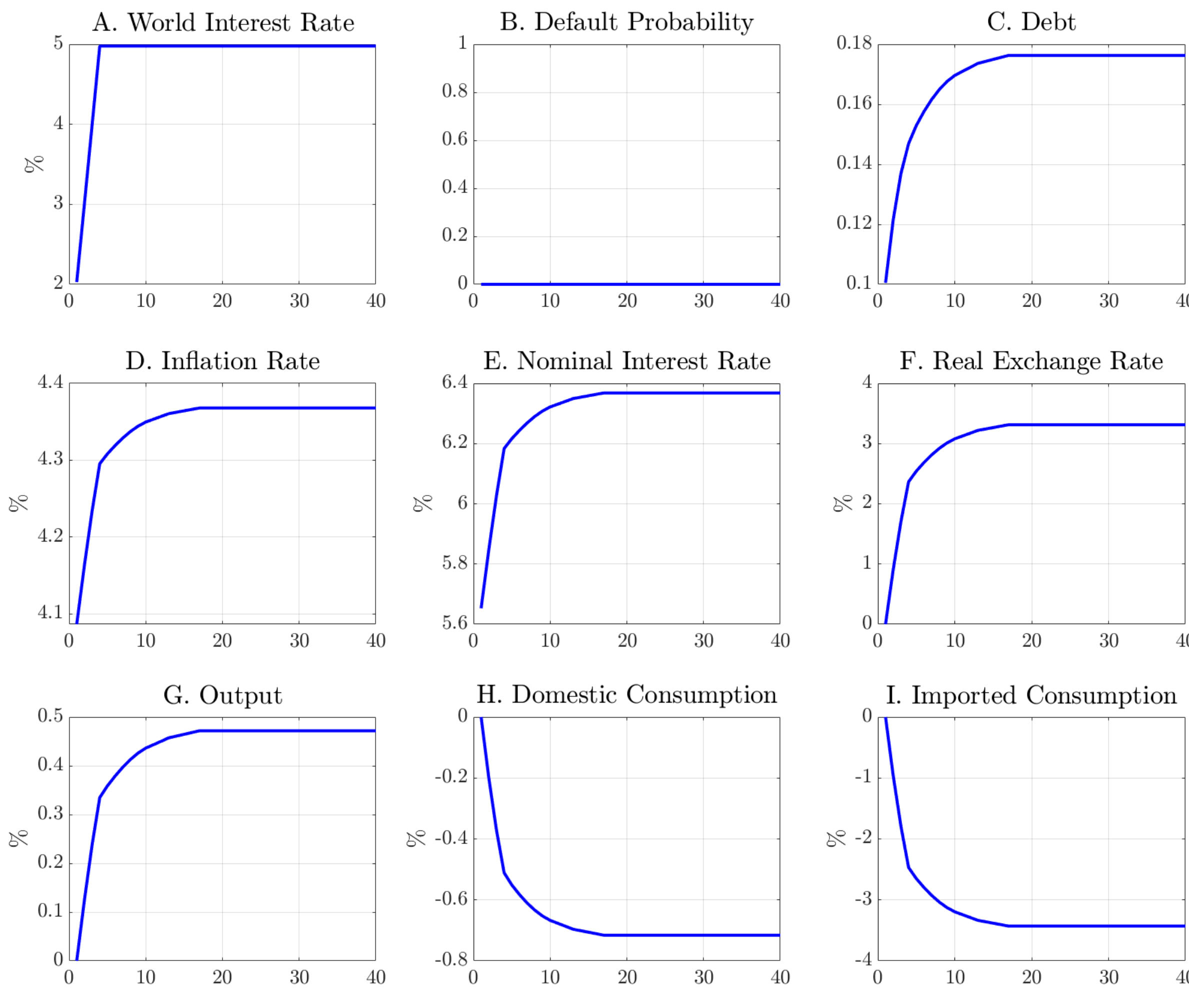


Notes: This figure plots the policy functions for debt and spread as a function of the gross world interest rate. The solid blue lines correspond to those associated with a low level of debt, whereas the dashed red lines correspond to those with a high level of debt. The productivity is assumed constant at the mean level.

C.2 Sensitivity Analysis

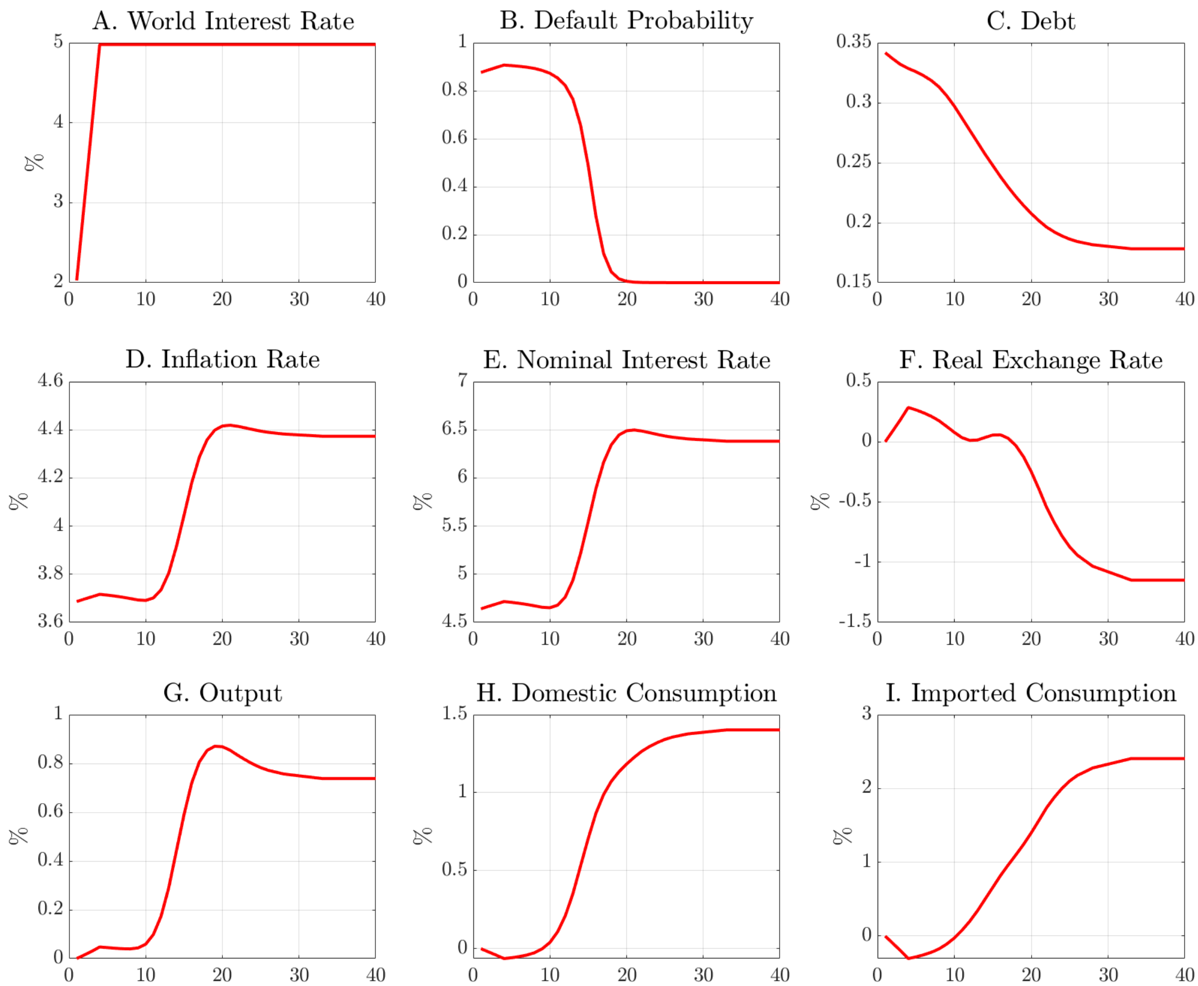
In this subsection, I consider an alternative path for the world interest rate in the simulations. In particular, I keep feeding into the model the observed post-COVID-19 U.S. monetary tightening for the period 2022Q2-2023Q2, but I assume that after 2023Q2 the interest rate remains constant at 5% for the remaining periods in the simulation. Figure 7 displays the results when the economy has an initial low level of sovereign debt, whereas Figure 8 shows the results starting from an initial high debt level.

Figure 7: U.S. Monetary Tightening with Low Debt



Notes: Simulations are obtained by feeding into the model a path for the world interest rate consistent with the post-COVID-19 U.S. monetary tightening for the period 2022Q2-2023Q2 (see panel A). The figure shows the response of macro variables when the domestic economy has an initial low debt level. Debt is reported in levels; real exchange rate, output, and domestic and imported consumption are reported in percentage deviations from the level in $t = 0$, while the world interest rate, inflation rate, and nominal interest rate are reported in percentage points.

Figure 8: U.S. Monetary Tightening with Low Debt



Notes: Simulations are obtained by feeding into the model a path for the world interest rate consistent with the post-COVID-19 U.S. monetary tightening for the period 2022Q2-2023Q2 (see panel A). The figure shows the response of macro variables when the domestic economy has an initial low debt level. Debt is reported in levels; real exchange rate, output, and domestic and imported consumption are reported in percentage deviations from the level in $t = 0$, while the world interest rate, inflation rate, and nominal interest rate are reported in percentage points.

D Global Solution Method

D.1 Description of Discrete Shocks Perturbations

I assume that sovereign debt only takes values within a finite and bounded support with J points. As shown in (15), the grid of debt positions is summarized by a vector Γ. I perturb the borrowing choice B' as follows: each period the sovereign draws a random vector $\boldsymbol{\epsilon}$ of additive taste shocks of size J, and each element of the vector is associated with a particular debt choice on Γ in case of repayment. From an ex-ante perspective, taste shocks make the debt choice decision stochastic. I denote $\epsilon(B_j) = \epsilon_j$.

The taste shock $\boldsymbol{\epsilon}$ is drawn from a multivariate distribution with joint cumulative density function $F(\boldsymbol{\epsilon}) = F(\epsilon_1, \epsilon_2, ..., \epsilon_J)$ and joint density function $f(\boldsymbol{\epsilon}) = f(\epsilon_1, \epsilon_2, ..., \epsilon_J)$. I use the following operator to denote the expectation of any function $Z(\boldsymbol{\epsilon})$ with respect to all elements of $\boldsymbol{\epsilon}$:

$$\mathbb{E}_{\boldsymbol{\epsilon}} Z(\boldsymbol{\epsilon}) = \int_{\epsilon_1} \int_{\epsilon_2} \cdots \int_{\epsilon_J} Z(\epsilon_1, \epsilon_2, ..., \epsilon_J) f(\epsilon_1, \epsilon_2, ..., \epsilon_J) d\epsilon_1 d\epsilon_2 ... d\epsilon_J$$

I assume that $\boldsymbol{\epsilon}$ is *iid* and follows a Type-I Generalized Extreme Value distribution:

$$F(\boldsymbol{\epsilon}) = \exp\left[-\left(\sum_{j=1}^{J} \exp\left(-\frac{\epsilon_j - \mu_B}{p\varrho_B}\right)\right)^p\right] \tag{53}$$

where μ_B is a parameter such that the shocks have mean zero, ϱ_B is a parameter that scales the variance of the shocks, and p is a constant, which I set to 1 hereafter. Let $F_j(\boldsymbol{\epsilon}) := \partial F(\boldsymbol{\epsilon})/\partial \epsilon_j$ be the partial derivative of the joint distribution with respect to ϵ_j. Then, for $j = 1, \ldots, J$:

$$F_j(\boldsymbol{\epsilon}) = \frac{1}{\varrho_B} F(\boldsymbol{\epsilon}) \exp\left(-\frac{\epsilon_j - \mu_B}{\varrho_B}\right)$$

I perturb the default decision in a similar fashion, so that at the start of each period the sovereign observes default decision taste shocks (ϵ_R, ϵ_D), drawn for a bivariate version of the probability distribution in (53). I denote such distribution as $F^d(\epsilon_R, \epsilon_D)$, and the corresponding partial derivatives with respect to the repayment and default shock as $F^d_R(\epsilon_R, \epsilon_D)$ and $F^d_D(\epsilon_R, \epsilon_D)$, respectively.

Before the realization of these shocks, the default decision is stochastic. A sovereign that has observed the current states $(\tilde{z}, r^\star, B)$ and the realization of (ϵ_R, ϵ_D) makes a unique deterministic decision on whether to default. By taking expectations over (ϵ_R, ϵ_D), we view the default decision as probabilistic. With these assumptions, the government problem becomes:

$$\begin{aligned}
\mathrm{W}(s,B) &= \mathbb{E}_{\epsilon_R,\epsilon_D}\left[\max\left\{\mathrm{W}^R(s,B)+\epsilon_R, \mathrm{W}^D(s)+\epsilon_D\right\}\right] \\
&= \mathbb{E}_{\epsilon_R,\epsilon_D}\left[\max_{D\in\{0,1\}}\left\{(1-D)\left[\mathrm{W}^R(s,B)+\epsilon_R\right]+D\left[\mathrm{W}^D(s)+\epsilon_D\right]\right\}\right]
\end{aligned}$$

Here $\mathrm{W}^R(s,B)$ is the expected optimal repayment value, and $\mathrm{W}^D(s)$ is the expected default value, given by:

$$\begin{aligned}
\mathrm{W}^R(s,B) &= \mathbb{E}_{\boldsymbol{\epsilon}}\left[\max_{B'_j\in\Gamma}\left\{W_j(s,B)+\epsilon_j\right\}\right] \\
W_j\,(s,B) &= u(C(S_j)) - v(n(S_j)) + \beta_g\mathbb{E}_{s'|s}\mathrm{W}\left(s',B'_j\right) \\
\mathrm{W}^D(s) &= u(C(S)) - v(n(S)) + \beta_g\mathbb{E}_{s'|s}\left[\iota\mathrm{W}\,(s',0)+(1-\iota)\mathrm{W}^D\,(s')\right]
\end{aligned}$$

where $W_j(s,B)$ is the repayment value if the sovereign chooses to borrow $B^{'}_j$, with the sub-index j referring to the position of B_j in Γ. As in the original unperturbed problem, the maximization in $\mathrm{W}^R(s,B)$ is subject to the PME with $S_j = \left\{s, B, \vartheta = 1, B^{'}_j\right\}$ and the bond price schedule $Q(s,B^{'}_j)$ (described in more detail later). Similarly, $\mathrm{W}^D(B)$ is subject to the PME with $S = \left\{s, B=0, \vartheta=0, B^{'}=0\right\}$.

In this case the utility shock ν maps directly into the discrete taste shocks: $\nu = \epsilon_R - \epsilon_D$. Since both ϵ_D and ϵ_R are distributed Type-I Generalized Extreme Value, the difference $(\epsilon_R - \epsilon_D)$ is distributed Logistic, consistent with the assumption in Section 2.6. I can compute the ex-ante policy functions of the sovereign in closed form. Let's denote the default probability as $\Pr(D = 1|s,B)$. Then:

$$
\begin{aligned}
\Pr(D=1|s,B) =& \Pr\left(\mathrm{W}^D(s)+\epsilon_D > \mathrm{W}^R(s,B)+\epsilon_R|s,B\right)\\
=& \int_{-\infty}^{\infty} F_D^d\left(\mathrm{W}^D(s)+\epsilon_D-\mathrm{W}^R(s,B)\right)d\epsilon_D\\
=& \int_{-\infty}^{\infty}\frac{1}{\varrho_D}F(\epsilon_R,\epsilon_D)\exp\left(-\frac{\epsilon_D-\mu_D}{\varrho_D}\right)d\epsilon_D\\
=& \int_{-\infty}^{\infty}\frac{1}{\varrho_D}\exp\Bigg[-\Bigg(\exp\left(-\frac{\mathrm{W}^D(s)+\epsilon_D-\mathrm{W}^R(s,B)-\mu_D}{\varrho_D}\right)\\
&+\exp\left(-\frac{\epsilon_D-\mu_D}{\varrho_D}\right)\Bigg)\Bigg]\exp\left(-\frac{\epsilon_D-\mu_D}{\varrho_D}\right)d\epsilon_D\\
=& \int_{-\infty}^{\infty}\frac{1}{\varrho_D}\exp\left[-\exp\left(-\frac{\epsilon_D-\mu_D}{\varrho_D}\right)\left(1+\exp\left(-\frac{\mathrm{W}^D(s)-\mathrm{W}^R(s,B)}{\varrho_D}\right)\right)\right]\exp\left(-\frac{\epsilon_D-\mu_D}{\varrho_D}\right)d\epsilon_D
\end{aligned}
$$

Define $\exp(\phi_D) := 1+\exp\left(-\frac{\mathrm{W}^D(s)-\mathrm{W}^R(s,B)}{\varrho_D}\right)$. Then I can write the previous expression as:

$$
\begin{aligned}
\Pr(D=1|s,B) &= \int_{-\infty}^{\infty}\frac{1}{\varrho_D}\exp\left[-\exp\left(-\frac{\epsilon_D-\mu_D}{\varrho_D}\right)\exp(\phi_D)\right]\exp\left(-\frac{\epsilon_D-\mu_D}{\varrho_D}\right)d\epsilon_D\\
&= \int_{-\infty}^{\infty}\frac{1}{\varrho_D}\exp\left[-\exp\left(-\frac{\epsilon_D-\mu_D-\varrho_D\phi_D}{\varrho_D}\right)\right]\exp\left(-\frac{\epsilon_D-\mu_D}{\varrho_D}\right)d\epsilon_D
\end{aligned}
$$

Let's multiply and divide the right-hand side of the last equality by $\exp(\phi_D)$. Then:

$$
\Pr(D=1|s,B) = \frac{1}{\exp(\phi_D)}\int_{-\infty}^{\infty}\frac{1}{\varrho_D}\exp\left[-\exp\left(-\frac{\epsilon_D-\mu_D-\varrho_D\phi_D}{\varrho_D}\right)\right]\exp\left(-\frac{\epsilon_D-\mu_D-\varrho_D\phi_D}{\varrho_D}\right)d\epsilon_D
$$

Since the probability distribution function integrates to 1:

$$
\int_{-\infty}^{\infty}\frac{1}{\varrho_D}\exp\left[-\exp\left(-\frac{\epsilon_D-\mu_D-\varrho_D\phi_D}{\varrho_D}\right)\right]\exp\left(-\frac{\epsilon_D-\mu_D-\varrho_D\phi_D}{\varrho_D}\right)d\epsilon_D = 1
$$

and the default probability simplifies to:

$$
\begin{aligned}
\Pr(D=1|s,B) =& \frac{1}{\exp(\phi_D)}\\
=& \frac{1}{1+\exp\left(-\frac{\mathrm{W}^D(s)-\mathrm{W}^R(s,B)}{\varrho_D}\right)}\\
=& \frac{\exp\left(\frac{\mathrm{W}^D(s)}{\varrho_D}\right)}{\exp\left(\frac{\mathrm{W}^D(s)}{\varrho_D}\right)+\exp\left(\frac{\mathrm{W}^R(s,B)}{\varrho_D}\right)}
\end{aligned}
\tag{54}
$$

The value of the government is:

$$
\begin{aligned}
\mathrm{W}(s,B) =& \int_{-\infty}^{\infty} \left(\mathrm{W}^D(s)+\epsilon_D\right) F_D^d\left(\mathrm{W}^D(s)+\epsilon_D-\mathrm{W}^R(s,B)\right) d\epsilon_D \\
&+ \int_{-\infty}^{\infty} \left(\mathrm{W}^R(s,B)+\epsilon_R\right) F_R^d\left(\mathrm{W}^R(s,B)+\epsilon_R-\mathrm{W}^D(s)\right) d\epsilon_R \\
=& \frac{1}{\varrho_D}\int_{-\infty}^{\infty} \left(\mathrm{W}^D(s)+\epsilon_D\right) \exp\left[-\left(\exp\left(-\frac{\mathrm{W}^D(s)+\epsilon_D-\mathrm{W}^R(s,B)-\mu_D}{\varrho_D}\right)\right.\right. \\
&\left.\left.+\exp\left(-\frac{\epsilon_D-\mu_D}{\varrho_D}\right)\right)\right] \exp\left(-\frac{\epsilon_D-\mu_D}{\varrho_D}\right) d\epsilon_D \\
&+ \frac{1}{\varrho_D}\int_{-\infty}^{\infty} \left(\mathrm{W}^R(s,B)+\epsilon_R\right) \exp\left[-\left(\exp\left(-\frac{\mathrm{W}^R(s,B)+\epsilon_R-\mathrm{W}^D(s)-\mu_D}{\varrho_D}\right)\right.\right. \\
&\left.\left.+\exp\left(-\frac{\epsilon_R-\mu_D}{\varrho_D}\right)\right)\right] \exp\left(-\frac{\epsilon_R-\mu_D}{\varrho_D}\right) d\epsilon_R
\end{aligned}
$$

I define $\exp(\eta_D) := 1+\exp\left(-\frac{\mathrm{W}^D(s)-\mathrm{W}^R(s,B)}{\varrho_D}\right)$ and $\exp(\eta_R) := 1+\exp\left(-\frac{\mathrm{W}^R(s,B)-\mathrm{W}^D(s)}{\varrho_D}\right)$. Then:

$$
\begin{aligned}
\mathrm{W}(s,B) =& \frac{1}{\varrho_D}\int_{-\infty}^{\infty} \left(\mathrm{W}^D(s)+\epsilon_D\right) \exp\left[-\exp\left(-\frac{\epsilon_D-\mu_D}{\varrho_D}\right)\exp(\eta_D)\right] \exp\left(-\frac{\epsilon_D-\mu_D}{\varrho_D}\right) d\epsilon_D \\
&+ \frac{1}{\varrho_D}\int_{-\infty}^{\infty} \left(\mathrm{W}^R(s,B)+\epsilon_R\right) \exp\left[-\exp\left(-\frac{\epsilon_R-\mu_D}{\varrho_D}\right)\exp(\eta_R)\right] \exp\left(-\frac{\epsilon_R-\mu_D}{\varrho_D}\right) d\epsilon_R \\
=& \frac{1}{\varrho_D}\int_{-\infty}^{\infty} \left(\mathrm{W}^D(s)+\epsilon_D\right) \exp\left[-\exp\left(-\frac{\epsilon_D-\mu_D-\varrho_D\eta_D}{\varrho_D}\right)\right] \exp\left(-\frac{\epsilon_D-\mu_D}{\varrho_D}\right) d\epsilon_D \\
&+ \frac{1}{\varrho_D}\int_{-\infty}^{\infty} \left(\mathrm{W}^R(s,B)+\epsilon_R\right) \exp\left[-\exp\left(-\frac{\epsilon_R-\mu_D-\varrho_D\eta_R}{\varrho_D}\right)\right] \exp\left(-\frac{\epsilon_R-\mu_D}{\varrho_D}\right) d\epsilon_R
\end{aligned}
$$

Let's multiply and divide the first integral on the right-hand side by $\exp(\eta_D)$, and the second integral by $\exp(\eta_R)$. Hence:

$$
\begin{aligned}
\mathrm{W}(s,B) =& \frac{\exp(-\eta_D)}{\varrho_D}\int_{-\infty}^{\infty} \left(\mathrm{W}^D(s)+\epsilon_D\right) \exp\left[-\exp\left(-\frac{\epsilon_D-\mu_D-\varrho_D\eta_D}{\varrho_D}\right)\right] \exp\left(-\frac{\epsilon_D-\mu_D-\varrho_D\eta_D}{\varrho_D}\right) d\epsilon_D \\
&+ \frac{\exp(-\eta_R)}{\varrho_D}\int_{-\infty}^{\infty} \left(\mathrm{W}^R(s,B)+\epsilon_R\right) \exp\left[-\exp\left(-\frac{\epsilon_R-\mu_D-\varrho_D\eta_R}{\varrho_D}\right)\right] \exp\left(-\frac{\epsilon_R-\mu_D-\varrho_D\eta_R}{\varrho_D}\right) d\epsilon_R
\end{aligned}
$$

Now I add and subtract $(\mu_D+\varrho_D\eta_D)$ to the first term of the first integral, and $(\mu_D+\varrho_D\eta_R)$ to the first term of the second integral:

$$
\begin{aligned}
\mathrm{W}(s,B) =& \exp(-\eta_D)\Bigg[\int_{-\infty}^{\infty}\left(\frac{\mathrm{W}^D(s)+\mu_D+\varrho_D\eta_D}{\varrho_D}\right)\exp\left[-\exp\left(-\frac{\epsilon_D-\mu_D-\varrho_D\eta_D}{\varrho_D}\right)\right]\exp\left(-\frac{\epsilon_D-\mu_D-\varrho_D\eta_D}{\varrho_D}\right)d\epsilon_D \\
&+\int_{-\infty}^{\infty}\left(\frac{\epsilon_D-\mu_D-\varrho_D\eta_D}{\varrho_D}\right)\exp\left[-\exp\left(-\frac{\epsilon_D-\mu_D-\varrho_D\eta_D}{\varrho_D}\right)\right]\exp\left(-\frac{\epsilon_D-\mu_D-\varrho_D\eta_D}{\varrho_D}\right)d\epsilon_D\Bigg] \\
&+\exp(-\eta_R)\Bigg[\int_{-\infty}^{\infty}\left(\frac{\mathrm{W}^R(s,B)+\mu_D+\varrho_D\eta_R}{\varrho_D}\right)\exp\left[-\exp\left(-\frac{\epsilon_R-\mu_D-\varrho_D\eta_R}{\varrho_D}\right)\right]\exp\left(-\frac{\epsilon_R-\mu_D-\varrho_D\eta_R}{\varrho_D}\right)d\epsilon_R \\
&+\int_{-\infty}^{\infty}\left(\frac{\epsilon_R-\mu_D-\varrho_D\eta_R}{\varrho_D}\right)\exp\left[-\exp\left(-\frac{\epsilon_R-\mu_D-\varrho_D\eta_R}{\varrho_D}\right)\right]\exp\left(-\frac{\epsilon_R-\mu_D-\varrho_D\eta_R}{\varrho_D}\right)d\epsilon_R\Bigg]
\end{aligned}
$$

Note that the first terms on the first and third integrals do not depend on the variables of integration. Therefore:

$$
\begin{aligned}
\mathrm{W}(s,B) =& \exp(-\eta_D)\Bigg[\left(\frac{\mathrm{W}^D(s)+\mu_D+\varrho_D\eta_D}{\varrho_D}\right)\int_{-\infty}^{\infty}\exp\left[-\exp\left(-\frac{\epsilon_D-\mu_D-\varrho_D\eta_D}{\varrho_D}\right)\right]\exp\left(-\frac{\epsilon_D-\mu_D-\varrho_D\eta_D}{\varrho_D}\right)d\epsilon_D \\
&+\int_{-\infty}^{\infty}\left(\frac{\epsilon_D-\mu_D-\varrho_D\eta_D}{\varrho_D}\right)\exp\left[-\exp\left(-\frac{\epsilon_D-\mu_D-\varrho_D\eta_D}{\varrho_D}\right)\right]\exp\left(-\frac{\epsilon_D-\mu_D-\varrho_D\eta_D}{\varrho_D}\right)d\epsilon_D\Bigg] \\
&+\exp(-\eta_R)\Bigg[\left(\frac{\mathrm{W}^R(s,B)+\mu_D+\varrho_D\eta_R}{\varrho_D}\right)\int_{-\infty}^{\infty}\exp\left[-\exp\left(-\frac{\epsilon_R-\mu_D-\varrho_D\eta_R}{\varrho_D}\right)\right]\exp\left(-\frac{\epsilon_R-\mu_D-\varrho_D\eta_R}{\varrho_D}\right)d\epsilon_R \\
&+\int_{-\infty}^{\infty}\left(\frac{\epsilon_R-\mu_D-\varrho_D\eta_R}{\varrho_D}\right)\exp\left[-\exp\left(-\frac{\epsilon_R-\mu_D-\varrho_D\eta_R}{\varrho_D}\right)\right]\exp\left(-\frac{\epsilon_R-\mu_D-\varrho_D\eta_R}{\varrho_D}\right)d\epsilon_R\Bigg]
\end{aligned}
$$

By properties of the type-I Generalized Extreme Value distribution:

$$
1 = \frac{1}{\varrho_D}\int_{-\infty}^{\infty}\exp\left(-\exp\left(-\frac{\epsilon_j-\mu_D-\varrho_D\eta_j}{\varrho_D}\right)\right)\exp\left(-\frac{\epsilon_j-\mu_D-\varrho_D\eta_j}{\varrho_D}\right)d\epsilon_j \tag{55}
$$

$$
\gamma = \frac{1}{\varrho_D}\int_{-\infty}^{\infty}\left(\frac{\epsilon_j-\mu_D-\varrho_D\eta_j}{\varrho_D}\right)\exp\left(-\exp\left(-\frac{\epsilon_j-\mu_D-\varrho_D\eta_j}{\varrho_D}\right)\right)\exp\left(-\frac{\epsilon_j-\mu_D-\varrho_D\eta_j}{\varrho_D}\right)d\epsilon_j \tag{56}
$$

for $j \in \{R, D\}$. Here $\gamma \approx 0.5772$ is the Euler-Mascheroni constant. Then, the value of the sovereign is:

$$
\begin{aligned}
\mathrm{W}(s,B) &= \exp(-\eta_D)\left[\left(\frac{\mathrm{W}^D(s)+\mu_D+\varrho_D\eta_D}{\varrho_D}\right)\varrho_D+\varrho_D\gamma\right]+\exp(-\eta_R)\left[\left(\frac{\mathrm{W}^R(s,B)+\mu_D+\varrho_D\eta_R}{\varrho_D}\right)\varrho_D+\varrho_D\gamma\right] \\
&= \exp(-\eta_D)\left(\mathrm{W}^D(s)+\varrho_D\eta_D\right)+\exp(-\eta_R)\left(\mathrm{W}^R(s,B)+\varrho_D\eta_R\right)
\end{aligned}
$$

where the last equivalence results from imposing $\mu_D = -\varrho_D\gamma$. Now, replacing the definitions of $\exp(\eta_D)$ and $\exp(\eta_R)$ we obtain:

$$
\begin{aligned}
\mathrm{W}(s,B) =& \frac{1}{1+\exp\left(-\frac{\mathrm{W}^D(s)-\mathrm{W}^R(s,B)}{\varrho_D}\right)}\left(\mathrm{W}^D(s)+\varrho_D\log\left(1+\exp\left(-\frac{\mathrm{W}^D(s)-\mathrm{W}^R(s,B)}{\varrho_D}\right)\right)\right)\\
&+\frac{1}{1+\exp\left(-\frac{\mathrm{W}^R(s,B)-\mathrm{W}^D(s)}{\varrho_D}\right)}\left(\mathrm{W}^R(s,B)+\varrho_D\log\left(1+\exp\left(-\frac{\mathrm{W}^R(s,B)-\mathrm{W}^D(s)}{\varrho_D}\right)\right)\right)\\
=&\frac{\exp\left(\frac{\mathrm{W}^D(s)}{\varrho_D}\right)}{\exp\left(\frac{\mathrm{W}^D(s)}{\varrho_D}\right)+\exp\left(\frac{\mathrm{W}^R(s,B)}{\varrho_D}\right)}\left[\varrho_D\log\left(\exp\left(\frac{\mathrm{W}^R(s,B)}{\varrho_D}\right)+\exp\left(\frac{\mathrm{W}^D(s)}{\varrho_D}\right)\right)\right]\\
&+\frac{\exp\left(\frac{\mathrm{W}^R(s,B)}{\varrho_D}\right)}{\exp\left(\frac{\mathrm{W}^D(s)}{\varrho_D}\right)+\exp\left(\frac{\mathrm{W}^R(s,B)}{\varrho_D}\right)}\left[\varrho_D\log\left(\exp\left(\frac{\mathrm{W}^R(s,B)}{\varrho_D}\right)+\exp\left(\frac{\mathrm{W}^D(s)}{\varrho_D}\right)\right)\right]\\
=&\varrho_D\log\left[\exp\left(\frac{\mathrm{W}^R(s,B)}{\varrho_D}\right)+\exp\left(\frac{\mathrm{W}^D(s)}{\varrho_D}\right)\right]
\end{aligned}
\tag{57}
$$

Now I derive the probability of choosing an amount of debt B'_j, given the current aggregate states (s,B). I denote this probability as $\Pr(B'=B'_j|s,B)$. For convenience, I drop the (s,B) states from the $W_j(s,B)$ expression in the following derivations.

$$
\begin{aligned}
\Pr(B'=B'_j|s,B) &= \int_{-\infty}^{\infty} F_j(W_j+\epsilon_j-W_1,\ldots,W_j+\epsilon_j-W_J)d\epsilon_j\\
&=\int_{-\infty}^{\infty}\frac{1}{\varrho_B}\exp\left[-\left(\sum_{k=1}^{J}\exp\left(-\frac{W_j+\epsilon_j-W_k-\mu_B}{\varrho_B}\right)\right)\right]\exp\left(-\frac{\epsilon_j-\mu_B}{\varrho_B}\right)d\epsilon_j
\end{aligned}
$$

Define $\exp(\eta_j) := \sum_{k=1}^{J}\exp\left(-\frac{W_j-W_k}{\varrho_B}\right)$. Then:

$$
\begin{aligned}
\Pr(B'=B'_j|s,B) =&\frac{1}{\varrho_B}\int_{-\infty}^{\infty}\exp\left(-\exp(\eta_j)\exp\left(-\frac{\epsilon_j-\mu_B}{\varrho_B}\right)\right)\exp\left(-\frac{\epsilon_j-\mu_B}{\varrho_B}\right)d\epsilon_j\\
=&\frac{1}{\varrho_B}\int_{-\infty}^{\infty}\exp\left(-\exp\left(-\frac{\epsilon_j-\mu_B-\varrho_B\eta_j}{\varrho_B}\right)\right)\exp\left(-\frac{\epsilon_j-\mu_B}{\varrho_B}\right)d\epsilon_j
\end{aligned}
$$

I multiply and divide the right-hand side by $\exp(\eta_j)$, so that:

$$
\Pr(B'=B'_j|s,B) =\frac{1}{\exp(\eta_j)\varrho_B}\int_{-\infty}^{\infty}\exp\left(-\exp\left(-\frac{\epsilon_j-\mu_B-\varrho_B\eta_j}{\varrho_B}\right)\right)\exp\left(-\frac{\epsilon_j-\mu_B-\varrho_B\eta_j}{\varrho_B}\right)d\epsilon_j
$$

Replacing the property (55) and the definition of $\exp(\eta_j)$ we obtain:

$$
\begin{aligned}
\Pr(B' = B'_j | s, B) =& \frac{1}{\exp(\eta_j)} \\
=& \frac{1}{\sum_{k=1}^{J} \exp\left(-\frac{W_j - W_k}{\varrho_B}\right)} \\
=& \frac{\exp\left(\frac{W_j}{\varrho_B}\right)}{\sum_{k=1}^{J} \exp\left(\frac{W_k}{\varrho_B}\right)}
\end{aligned} \tag{58}
$$

Finally, for the repayment value of the government:

$$
\begin{aligned}
\mathrm{W}^R(s,B) &= \sum_{j=1}^{J} \int_{-\infty}^{\infty} (W_j + \epsilon_j) F_j (W_j + \epsilon_j - W_1, \ldots, W_j + \epsilon_j - W_J) d\epsilon_j \\
&= \frac{1}{\varrho_B} \sum_{j=1}^{J} \int_{-\infty}^{\infty} (W_j + \epsilon_j) F(W_j + \epsilon_j - W_1, \ldots, W_j + \epsilon_j - W_J) \exp\left(-\frac{\epsilon_j - \mu_B}{\varrho_B}\right) d\epsilon_j \\
&= \frac{1}{\varrho_B} \sum_{j=1}^{J} \int_{-\infty}^{\infty} (W_j + \epsilon_j) \exp\left[-\left(\sum_{k=1}^{J} \exp\left(-\frac{W_j + \epsilon_j - W_k - \mu_B}{\varrho_B}\right)\right)\right] \exp\left(-\frac{\epsilon_j - \mu_B}{\varrho_B}\right) d\epsilon_j
\end{aligned}
$$

Replacing the definition of $\exp(\eta_j)$:

$$
\begin{aligned}
\mathrm{W}^R(s,B) &= \frac{1}{\varrho_B} \sum_{j=1}^{J} \int_{-\infty}^{\infty} (W_j + \epsilon_j) \exp\left(-\exp(\eta_j) \exp\left(-\frac{\epsilon_j - \mu_B}{\varrho_B}\right)\right) \exp\left(-\frac{\epsilon_j - \mu_B}{\varrho_B}\right) d\epsilon_j \\
&= \frac{1}{\varrho_B} \sum_{j=1}^{J} \int_{-\infty}^{\infty} (W_j + \epsilon_j) \exp\left(-\exp\left(-\frac{\epsilon_j - \mu_B - \varrho_B \eta_j}{\varrho_B}\right)\right) \exp\left(-\frac{\epsilon_j - \mu_B}{\varrho_B}\right) d\epsilon_j \\
&= \frac{1}{\varrho_B} \sum_{j=1}^{J} \exp(-\eta_j) \int_{-\infty}^{\infty} (W_j + \epsilon_j) \exp\left(-\exp\left(-\frac{\epsilon_j - \mu_B - \varrho_B \eta_j}{\varrho_B}\right)\right) \exp\left(-\frac{\epsilon_j - \mu_B - \varrho_B \eta_j}{\varrho_B}\right) d\epsilon_j
\end{aligned}
$$

By adding and subtracting $(\mu_B + \varrho_B \eta_j)$ to the first term in the integral I can write the previous expression as:

$$
\begin{aligned}
\mathrm{W}^R(s,B) = \sum_{j=1}^{J} \exp(-\eta_j) \Bigg[& \int_{-\infty}^{\infty} \left(\frac{W_j + \mu_B + \varrho_B \eta_j}{\varrho_B}\right) \exp\left(-\exp\left(-\frac{\epsilon_j - \mu_B - \varrho_B \eta_j}{\varrho_B}\right)\right) \exp\left(-\frac{\epsilon_j - \mu_B - \varrho_B \eta_j}{\varrho_B}\right) d\epsilon_j \\
& + \int_{-\infty}^{\infty} \left(\frac{\epsilon_j - \mu_B - \varrho_B \eta_j}{\varrho_B}\right) \exp\left(-\exp\left(-\frac{\epsilon_j - \mu_B - \varrho_B \eta_j}{\varrho_B}\right)\right) \exp\left(-\frac{\epsilon_j - \mu_B - \varrho_B \eta_j}{\varrho_B}\right) d\epsilon_j \Bigg]
\end{aligned}
$$

Since the first term in the first integral does not depend on ϵ_j:

$$\mathrm{W}^R(s,B) = \sum_{j=1}^{J} \exp(-\eta_j)\Bigg[\left(\frac{W_j+\mu_B+\varrho_B\eta_j}{\varrho_B}\right)\int_{-\infty}^{\infty}\exp\left(-\exp\left(-\frac{\epsilon_j-\mu_B-\varrho_B\eta_j}{\varrho_B}\right)\right)\exp\left(-\frac{\epsilon_j-\mu_B-\varrho_B\eta_j}{\varrho_B}\right)d\epsilon_j$$
$$+\int_{-\infty}^{\infty}\left(\frac{\epsilon_j-\mu_B-\varrho_B\eta_j}{\varrho_B}\right)\exp\left(-\exp\left(-\frac{\epsilon_j-\mu_B-\varrho_B\eta_j}{\varrho_B}\right)\right)\exp\left(-\frac{\epsilon_j-\mu_B-\varrho_B\eta_j}{\varrho_B}\right)d\epsilon_j\Bigg]$$

I simplify the integrals by using properties (55) and (56). The value of repayment collapses to:

$$\begin{aligned}
\mathrm{W}^R(s,B) &= \sum_{j=1}^{J}\exp(-\eta_j)\left[\left(\frac{W_j+\mu_B+\varrho_B\eta_j}{\varrho_B}\right)\varrho_B+\varrho_B\gamma\right] \\
&= \sum_{j=1}^{J}\exp(-\eta_j)\left(W_j+\mu_B+\varrho_B\eta_j+\varrho_B\gamma\right) \\
&= \sum_{j=1}^{J}\exp(-\eta_j)\left(W_j+\varrho_B\eta_j\right)
\end{aligned}$$

where the last equivalence uses the assumption that the taste shocks have mean zero, and then $\mu_B = -\varrho_B\gamma$. Replacing the definition of $\exp(\eta_j)$ the previous expression simplifies to:

$$\begin{aligned}
\mathrm{W}^R(s,B) &= \sum_{j=1}^{J}\frac{1}{\sum_{k=1}^{J}\exp\left(-\frac{W_j-W_k}{\varrho_B}\right)}\left(W_j+\varrho_B\log\left(\sum_{k=1}^{J}\exp\left(-\frac{W_j-W_k}{\varrho_B}\right)\right)\right) \\
&= \sum_{j=1}^{J}\frac{\exp\left(\frac{W_j}{\varrho_B}\right)}{\sum_{k=1}^{J}\exp\left(\frac{W_k}{\varrho_B}\right)}\left(W_j+\varrho_B\log\left(\exp\left(\frac{-W_j}{\varrho_B}\right)\right)+\varrho_B\log\left(\sum_{k=1}^{J}\exp\left(\frac{W_k}{\varrho_B}\right)\right)\right) \\
&=\varrho_B\log\left(\sum_{k=1}^{J}\exp\left(\frac{W_k}{\varrho_B}\right)\right)\frac{\sum_{j=1}^{J}\exp\left(\frac{W_j}{\varrho_B}\right)}{\sum_{k=1}^{J}\exp\left(\frac{W_k}{\varrho_B}\right)} \\
&=\varrho_B\log\left(\sum_{k=1}^{J}\exp\left(\frac{W_k}{\varrho_B}\right)\right) \qquad (59)
\end{aligned}$$

D.2 Computational Algorithm

I solve the PME using Powell (1970) dogleg method to find the allocations and prices that satisfy the PME nonlinear system of equations, and I solve the government problem using value function iteration. The AR(1) processes for the productivity and the world interest rate are discretized using Tauchen (1986) method. I set 20 and 15 equally spaced gridpoints for the productivity and the world interest rate, respectively. For the government debt, I use a grid of 200 equally spaced points on $\Gamma = [0, 0.4]$.

Here is a detailed description of the algorithm.

1. Create grids and discretize Markov processes for $\tilde{z}$ and $r^{\star}$. From the transition matrix of each Markov chain, generate the corresponding joint transition probability matrix across states in $(\tilde{z}, r^{\star})$.

2. Guess an initial value function for repayment and default, $W_0^R(s, B)$ and $W_0^D(s)$ and a bond price function $Q_0(s, B')$; guess the initial expectations functions $M_0(s, B', \vartheta)$ and $F_0(s, B', \vartheta)$. I assume that at these initial guesses, $\vartheta = 0$ and $B' = B$ with probability one for all points in the state-space (s, B, B').

3. For any $B' \in \Gamma$, solve for the Private and Monetary Equilibrium (PME) in each point $\hat{S} := (\tilde{z}, r^{\star}, B)$. To do so I start assuming that the government repays, and I collapse the PME (equations (40)-(51)) into a system of two equations and two unknowns in C^F and n. Here, I proceed as follows:

   - Guess a pair of values $\{C^F, n\}$ for each point in $\hat{S}$.
   - Solve for the real exchange rate e from (51).
   - Using the guess for C^F and the real exchange rate e, solve for the domestic consumption C^D from (40).
   - Solve for the level of foreign demand from (8).
   - Using C^D and the government borrowing B', solve for the nominal interest rate i from the Euler equation (41).
   - Using i, solve for the domestic inflation rate using the Taylor rule (45).
   - Compute residuals for the NKPC (44) and the resource constraint (46).

   Iterate using Powell (1970) method to solve for the values of $\{C^F, n\}$ that make the residuals equal to zero.

4. Repeat step 3 assuming that the government defaults on B. This implies that productivity is reduced, and the government is unable to borrow ($B' = 0$).
   The solution to steps 3 and 4 yields policy functions $\{C^D(S), C^F(S), n(S), \pi(S), i(S), e(S)\}$.

5. Given the PME, compute the optimal policy of the government. To do so I calculate the values under repayment and default using the Discrete Shocks Perturbations approach described in D.1. Call the updated values $W_1^R(s, B)$ and $W_1^D(s)$, respectively.

6. Compute probabilities of default and choice probabilities for B' using (54) and (58).

7. Update the bond price function using (32) and the expectations functions in the Euler equation and the NKPC using (30) and (31). Call these updated functions $\mathcal{Q}_1(s, B')$, $M_1(s, B', \vartheta)$ and $F_1(s, B', \vartheta)$, respectively.

8. Compute the distance between the updated value functions, expectations functions, and bond price schedule and the ones from the previous iteration. To so do, calculate the sup norm between $\left\{W_1^R(s, B), W_0^R(s, B)\right\}$, $\left\{W_1^D(s), W_0^D(s)\right\}$, $\{M_1(s, B, \vartheta), M_0(s, B, \vartheta)\}$, $\left\{F_1(s, B, \vartheta), F_0(s, B, \vartheta)\right\}$ and $\{\mathcal{Q}_1(s, B'), \mathcal{Q}_0(s, B')\}$.

9. If any of these distances are larger than the given tolerance levels, go back to step 3. Otherwise, stop.