

Linear Algebra 期中专场

时间：2022年4月30日

一、判断正误题(每小题2分，共10分)

1. Every matrix is row equivalent to a unique matrix in echelon form. (T/F?)
2. If A is a 3×3 matrix, then $\det(2A) = 2 \det(A)$. (T/F?)
3. If an augmented matrix $[A \quad \mathbf{b}]$ is transformed into $[C \quad \mathbf{d}]$ by elementary row operations, then the equations $A\mathbf{x} = \mathbf{b}$ and $C\mathbf{x} = \mathbf{d}$ have exactly the same solution sets. (T/F?)
4. $\text{Rank } A = \dim(\text{Nul } A)$. (T/F?)
5. If A is $m \times n$ and the linear transformation $\mathbf{x} \mapsto A\mathbf{x}$ is onto, then $\text{rank } A = m$. (T/F?)

二、填空题(每小题5分，共15分)

1. 若 $\begin{pmatrix} 2 & 5 \\ 1 & 3 \end{pmatrix} X = \begin{pmatrix} 4 & -6 \\ 2 & 1 \end{pmatrix}$, 则 $X =$ _____.

2. 已知向量组

$$\alpha_1 = [1, -1, 2]^T, \alpha_2 = [0, 3, 1]^T, \alpha_3 = [3, 0, 7]^T$$

与向量组

$$\beta_1 = [1, -2, 2]^T, \beta_2 = [2, 1, 5]^T, \beta_3 = [x, 3, 3]^T$$

等秩, 则 $x =$ _____.

3. 向量组 $\alpha_1 = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$, $\alpha_2 = \begin{pmatrix} 0 \\ 2 \\ 5 \end{pmatrix}$, $\alpha_3 = \begin{pmatrix} 2 \\ 4 \\ 7 \end{pmatrix}$, $\alpha_4 = \begin{pmatrix} 1 \\ 2 \\ 0 \end{pmatrix}$ 是线性_____ (填相关或无关)的, 它的一个极大线性无关组是_____.

| 判断正误题 | 1 | 2 | 3 | 4 | 5 |
|-------|---|---|---|---|---|
| 你的判断 | | | | | |

| 填空题 | 1 | 2 | 3(1) | 3(2) |
|------|---|---|------|------|
| 你的答案 | | | | |

三、计算与证明题(共75分)

| 计算与证明题 | 1 | 2 | 3 | 4 | 5 |
|--------|---|---|---|---|---|
| 得分 | | | | | |

1. (15分)求解下列齐次线性方程组:

$$\begin{cases} 3x_1 + 4x_2 - 5x_3 + 7x_4 = 0 \\ 2x_1 - 3x_2 + 3x_3 - 2x_4 = 0 \\ 4x_1 + 11x_2 - 13x_3 + 16x_4 = 0 \\ 7x_1 - 2x_2 + x_3 + 3x_4 = 0 \end{cases}$$

2. (15分)求可逆矩阵 P 和对角矩阵 D , 使 $A = PDP^{-1}$.

$$A = \begin{bmatrix} 5 & 0 & 0 & 0 \\ 0 & 5 & 0 & 0 \\ 1 & 4 & -3 & 0 \\ -1 & -2 & 0 & -3 \end{bmatrix}$$

3. (15分)设 $\mathcal{B} = \{\mathbf{b}_1, \mathbf{b}_2\}$ 和 $\mathcal{C} = \{\mathbf{c}_1, \mathbf{c}_2\}$ 是 \mathbb{R}^2 的两个基, 求由 \mathcal{B} 到 \mathcal{C} 的坐标变换矩阵和由 \mathcal{C} 到 \mathcal{B} 的坐标变换矩阵.

$$\mathbf{b}_1 = \begin{bmatrix} 7 \\ 5 \end{bmatrix}, \mathbf{b}_2 = \begin{bmatrix} -3 \\ -1 \end{bmatrix}, \mathbf{c}_1 = \begin{bmatrix} 1 \\ -5 \end{bmatrix}, \mathbf{c}_2 = \begin{bmatrix} -2 \\ 2 \end{bmatrix}$$

4. (15分)

设 A 是 $n \times m$ 矩阵, B 是 $m \times n$ 矩阵, 其中 $n < m$, 若 $AB = E$, 证明 B 的列向量线性无关.

5. (15分) 设

$$V_1 = \left\{ \mathbf{x} = (x_1, x_2, \dots, x_n)^T \mid x_1, \dots, x_n \in \mathbb{R} \text{ 满足 } x_1 + \dots + x_n = 0 \right\},$$

$$V_2 = \left\{ \mathbf{x} = (x_1, x_2, \dots, x_n)^T \mid x_1, \dots, x_n \in \mathbb{R} \text{ 满足 } x_1 + \dots + x_n = 1 \right\},$$

问 V_1, V_2 是不是向量空间? 证明之.