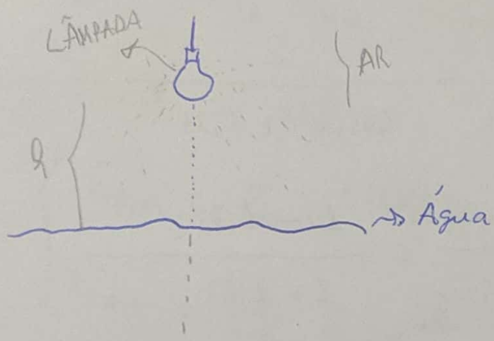
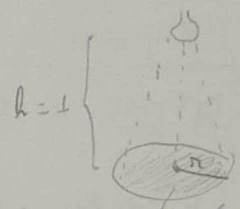
LISTA 2 | Eletromag 2 | LEONARDO C. ROSSATO

01) $n_{agua} = 1,33$; $n_{ar} = 1$; $h = 1$; $P = 60W$

Esquema:





$$I = \frac{P}{A}$$

↳ $Área = \pi r^2$, r → lâmpada

a)

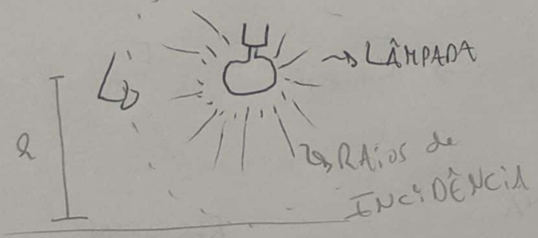
$$\rightarrow I = \frac{E_{rms}^2}{Z} \therefore E_{rms}^2 = IZ = \frac{PZ}{A} \Leftrightarrow \boxed{E_{rms} = \sqrt{\frac{PZ}{A}}}$$

→ Se $Z_{AR} \approx Z_0 = 377$ e $A \cong S_{\substack{Esfera \\ Pontual}} = 4\pi r^2$; com $r = 1$ (*)

$$\therefore \boxed{E_{rms} = \sqrt{\frac{60 \cdot 377}{4\pi}} \cong 42,427 \frac{V}{m}}$$

$$\rightarrow Z = \frac{E}{H} \Rightarrow H = \frac{E}{Z} \Rightarrow H_{rms} = \frac{E_{rms}}{Z} = \frac{42,427}{377} \cong 0,112 \ A/m$$

(*) Superfície Esfera Pontual } ÁREA de INCIDÊNCIA da LÂMPADA



$$\rightarrow A \cong S_{Esfera} = \int dA = \int R^2 \operatorname{sen}\theta \, d\theta \, d\varphi$$

$$= 4\pi R^2, \text{ para ESFERA PONTUAL}$$

$r = 1$

$h \approx$ raio de iluminação

p/ Área de Incidência

$$\therefore A \cong 4\pi$$

01) b) Equações Fresnel: $\theta_I = 0 \therefore \theta_T \cong 0$

$$\left(\frac{E_{T,m}}{E_{I,m}}\right)_\perp = \frac{2n_1 \cos\theta_I}{n_1\cos\theta_I + n_2\cos\theta_T} = \frac{2.1.\cos(0)}{1.\cos(0) + 1{,}33\cos(0)} = \frac{2}{2{,}33}$$

$$\left(\frac{E_{T,m}}{E_{I,m}}\right)_\parallel = \frac{2n_1 \cos\theta_I}{n_1\cos\theta_T + n_2\cos\theta_I} = \frac{2}{\cos(0) + 1{,}33} = \frac{2}{2{,}33}$$

$$\left(\frac{E_{R,m}}{E_{I,m}}\right)_\perp = \frac{n_1\cos\theta_I - n_2\cos\theta_T}{n_1\cos\theta_I + n_2\cos\theta_T} = \frac{1 - 1{,}33}{1 + 1{,}33} = \frac{-0{,}33}{2{,}33}$$

$$\left(\frac{E_{R,m}}{E_{I,m}}\right)_\parallel = \frac{n_1\cos\theta_T - n_2\cos\theta_I}{n_1\cos\theta_T + n_2\cos\theta_I} = \frac{1 - 1{,}33}{1 + 1{,}33} = -\frac{0{,}33}{2{,}33}$$

→ PARA ENCONTRAR OS VALORES RMS da Onda Refletida e transmitida:

$$\frac{E_{T,m}}{E_{I,m}} = \frac{E_T e^{i(\omega t - kz)}}{E_I e^{i(\omega t - kz)}} = \frac{E_T}{E_I} = \frac{\sqrt{2}\,E_{rms,T}}{\sqrt{2}\,E_{rms,I}} = \frac{E_{rms,T}}{E_{rms,I}}$$

$$\therefore \frac{E_{R,m}}{E_{I,m}} = \frac{E_R e^{i(\omega t - kz + \pi)}}{E_I e^{i(\omega t - kz)}} = \frac{E_R}{E_I} e^{i\pi} = \frac{E_R}{E_I}\left[\overset{-1}{\cos\pi} + i\,\overset{0}{\text{sen}\,\pi}\right] = -\frac{E_R}{E_I} = -\frac{E_{rms,R}}{E_{rms,I}}$$

$$\therefore \left(\frac{E_{T,m}}{E_{I,m}}\right) = \frac{2}{2{,}33} \iff \frac{E_{rms,T}}{E_{rms,I}} = \frac{2}{2{,}33} \iff E_{rms,T} = \frac{2}{2{,}33} E_{rms,I}$$

$$\left(\frac{E_{R,m}}{E_{I,m}}\right) = -\frac{0{,}33}{2{,}33} \iff \frac{E_{rms,R}}{E_{rms,I}} = -\frac{0{,}33}{2{,}33} \iff E_{rms,R} = \frac{0{,}33}{2{,}33} E_{rms,I}$$

PORTANTO

$$E_{rms,T} = \frac{2}{2{,}33} \cdot 42{,}427 = 36{,}418 \text{ V/m}$$

$$E_{rms,R} = \frac{0{,}33}{2{,}33} \cdot 42{,}427 = 6{,}009 \text{ V/m}$$

← RESPOSTAS LETRA A

$$H_{rms,T} = \frac{E_{rms,T}}{Z} \cong \frac{36{,}418}{377} = 0{,}0966 \text{ A/m}$$

$$H_{rms,R} = \frac{E_{rms,R}}{Z_0} = \frac{6{,}009}{377} = 0{,}0159 \text{ A/m}$$

02 $n_{air} = 1$; $n_{Diamante} = 2{,}42$

→ Construa Diagrama das Amplitudes dos $\{E_m, B_m\}$ em relação aos ângulos de incidência, tanto p/ Onda Refletida, quanto Transmitida.

→ PARA Onda TRANSMITIDA:

$$\left(\frac{E_{T,m}}{E_{I,m}}\right)_{\perp} = \frac{2n_1 \cos\theta_I}{n_1\cos\theta_I + n_2\cos\theta_T} = \frac{2\cos\theta_I}{\cos\theta_I + 2{,}42\cos\theta_T}$$

→ $\cos\theta_T = \sqrt{1-\text{Sen}^2\theta_T}$ e pela Lei de Snell : $\left\{ \text{Sen}\,\theta_T = \frac{n_1}{n_2}\text{Sen}\,\theta_I = \frac{1}{2{,}42}\text{Sen}\,\theta_I \right.$

$$= \sqrt{1-\left(\frac{1}{2{,}42}\right)^2 \text{Sen}^2\theta_I}$$

$$= \sqrt{1 - 0{,}17\,\text{Sen}^2\theta_I}$$

$$\therefore \left(\frac{E_{T,m}}{E_{I,m}}\right)_{\perp} = \frac{2\cos\theta_I}{\cos\theta_I + 2{,}42\sqrt{1-0{,}17\,\text{Sen}^2\theta_I}} = \boxed{-}_{I,T}$$

→ Como $$\frac{E_{T,m}}{E_{I,m}} = \frac{E_T\, e^{-i(\omega t - k_T z)}}{E_I\, e^{-i(\omega t - k_I z)}} = \frac{E_T e^{-i\omega t} e^{+ik_T z}}{E_I e^{-i\omega t} e^{+ik_I z}} = \frac{E_T[\cos k_T z + i\,\text{sen}\,k_T z]}{E_I[\cos k_I z + i\,\text{sen}\,k_I z]}$$

Como importa apenas: $\left\{ Re\left(\frac{E_{T,m}}{E_{I,m}}\right) = \frac{E_T \cos k_T z}{E_I \cos k_I z} \right.$, onde $\left\{ K_T = \frac{\omega_T}{c} = \frac{\omega_I}{c} = K_I \right.$

$\therefore \frac{E_T}{E_I} = \left(\frac{E_{T,m}}{E_{I,m}}\right)$ e como $E_{I,m} = E_I\left[\cos\frac{\omega z}{c} + i\,\text{sen}\frac{\omega z}{c}\right]$, p/ $t = 0$

→ $Re(E_{I,m}) = E_I \cos\left(\frac{\omega z}{c}\right)$

e $Re(E_{T,m}) = \boxed{-}_{I,T}\, Re(E_{I,m}) = \boxed{-}_{I,T}\, E_I \cos\left(\frac{\omega z}{c}\right)$

→ PARA ONDA REFLETIDA:

$$\therefore \left(\frac{E_{R,m}}{E_{I,m}}\right)_\perp = \frac{n_1 \cos\theta_I - n_2 \cos\theta_T}{n_1 \cos\theta_I + n_2 \cos\theta_T} = \frac{\cos\theta_I - 2{,}42\sqrt{1 - 0{,}17\,\text{Sen}^2\theta_I}}{\cos\theta_I + 2{,}42\sqrt{1 - 0{,}17\,\text{Sen}^2\theta_I}} = \boxminus_{I,R}$$

$$\therefore E_{R,m} = \boxminus_{I,R}\, E_{I,m} = \boxminus_{I,R}\, E_I\, e^{-i(\omega t - kz)}, \quad k = \frac{\omega}{c}$$

→ Se $t = 0$ e $\omega \approx c$:

*Obs: $E_{R,m}$ e $E_{I,m}$ ESTÃO DEFASADOS ↳ por "π"

$$\therefore \begin{cases} E_{I,m} = E_I\, e^{+i\left(\frac{\omega z}{c}\right)} = E_I\, e^{+iz} = E_I\left(\cos(z) + i\,\text{sen}(z)\right) \\ E_{R,m} = \boxminus_{I,R}\, E_I\, e^{i(z+\pi)} = \boxminus_{I,R}\, E_I\left(\cos(z+\pi) + i\,\text{sen}(z+\pi)\right) \end{cases}$$

→ Se usarmos só a parte real p/ fazer os Gráficos:

$$\therefore \begin{cases} Re\,(E_{I,m}) = E_i \cos z \\ Re\,(E_{R,m}) = \boxminus_{I,R}\, E_i \cos(z+\pi) \end{cases}$$

$$\therefore \left(\frac{E_{R,m}}{E_{I,m}}\right)_{\parallel} = \frac{n_1 \cos\theta_T - n_2 \cos\theta_I}{n_1 \cos\theta_T + n_2 \cos\theta_I} = \frac{\cos\theta_T - n_2 \cos\theta_I}{\cos\theta_T + n_2 \cos\theta_I} = \frac{\sqrt{1 - 0{,}17\,\text{sen}^2\theta_I} - n_2 \cos\theta_I}{\sqrt{1 - 0{,}17\,\text{sen}^2\theta_I} + n_2 \cos\theta_I} = \boxminus_{I,R_{\parallel}}$$

→ Se $t = 0$ e $\omega \approx c$:

$$\therefore \begin{cases} E_{I,m} = E_I\, e^{+i\left(\frac{\omega z}{c}\right)} = E_I\, e^{iz} = E_I\left(\cos(z) + i\,\text{sen}(z)\right) \\ E_{R,m} = E_R\, e^{i\left(\frac{\omega z}{c} + \pi\right)} = E_R\left[\cos(z+\pi) + i\,\text{sen}(z+\pi)\right] \cdot \boxminus_{I,R_{\parallel}} \end{cases}$$

→ Se usarmos só a parte real:

$$\therefore \begin{cases} Re\,(E_{I,m}) = E_I \cos(z) \\ Re\,(E_{R,m}) = E_R \cos(z+\pi) \cdot \boxminus_{I,R_{\parallel}} \end{cases}$$

QUESTÃO 04) Eq. Fresnel : $\left\{ \left(\frac{E_{R,m}}{E_{I,m}}\right)_{\parallel} = \frac{n_1 \cos\theta_T - n_2\cos\theta_I}{n_1\cos\theta_T + n_2\cos\theta_I} \right.$

→ Se $\cos\theta_T = -i\sqrt{\operatorname{sen}^2\theta_T - 1}$

$$\therefore \left(\frac{E_{R,m}}{E_{I,m}}\right)_{\parallel} = \frac{-(n_2/n_1)\cos\theta_I - i\sqrt{\operatorname{sen}^2\theta_T - 1}}{(n_2/n_1)\cos\theta_I + i\sqrt{\operatorname{sen}^2\theta_T - 1}} \quad (1)$$

→ Usando a lógica do Capítulo 8 onde: $\left\{ a = -\frac{n_2}{n_1}\cos\theta_I \; ; \; b = \sqrt{\operatorname{sen}^2\theta_T - 1} \right\}$

a Eq (1) muda p/ essa forma:

$$\therefore \left(\frac{E_{R,m}}{E_{I,m}}\right)_{\parallel} = \frac{|a+ib|\, e^{i\,atg(b/a)}}{|a-ib|\, e^{-i\,atg(b/a)}} = e^{2i\,atg(b/a)}$$

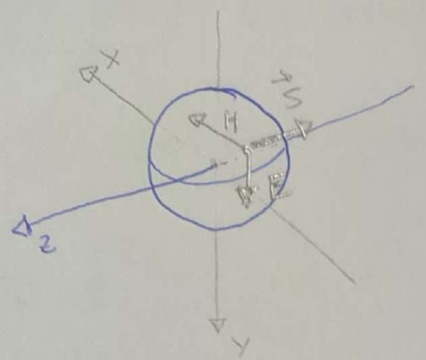
Com isso → $\left| \begin{aligned} |a+ib| &= \sqrt{\left(\frac{n_2}{n_1}\right)^2\cos^2\theta_I + \operatorname{sen}^2\theta_T - 1} = M \\ |a-ib| &= \sqrt{\left(\frac{n_2}{n_1}\right)^2\cos^2\theta_I + \operatorname{sen}^2\theta_T - 1} = M \end{aligned} \right.$

$$\therefore \left(\frac{E_{R,m}}{E_{I,m}}\right)_{\parallel} = \frac{M\, e^{i\,tg\left(\frac{-\sqrt{\operatorname{sen}^2\theta_T - 1}}{(n_2/n_1)\cos\theta_I}\right)}}{M\, e^{i\,tg\left(\frac{-\sqrt{\operatorname{sen}^2\theta_T - 1}}{-(n_2/n_1)\cos\theta_I}\right)}} = e^{-2i\,tg\left(\frac{\sqrt{\operatorname{sen}^2\theta_T - 1}}{(n_2/n_1)\cos\theta_I}\right)}$$

$$\rightarrow \phi_{\parallel} = -2\,tg\left(\frac{\sqrt{\operatorname{sen}^2\theta_T - 1}}{(n_2/n_1)\cos\theta_I}\right)$$

$$\therefore \left(\frac{E_{R,m}}{E_{I,m}}\right)_{\parallel} = e^{i\phi_{\parallel}}$$

05)



$$dQ = e\rho\, dV = e\rho\, r^2 \operatorname{sen}\varphi\, d\varphi\, d\theta\, dr$$

FORÇA de LORENTZ:

$$dF = dQ\,(E_T + v \times B)$$

$$= dQ\left(E_T + \frac{\sigma}{\rho e} E_T \times \mu_0 H_T\right)$$

$$= dQ\left(E_T + \frac{\mu_0\sigma}{\rho e} E_T \times H_T\right)$$

$$= dQ\left(\underbrace{E_T\hat{y}}_{0} + \frac{\mu_0\sigma}{\rho e} E_T H_T\,\hat{z}\right)$$

$$= dQ\,\frac{\mu_0\sigma}{\rho e} E_T H_T\,\hat{z}$$

$$= e\rho\, dV\,\frac{\mu_0\sigma}{\rho e} E_T H_T\,\hat{z}$$

$$= \mu_0\sigma\, E_T H_T\,\hat{z}\, dV \;\Big\}= \mu_0\sigma\,\vec{S}\, dV$$

Onde, $\vec{v} = \dfrac{\sigma E_T}{\rho e}\hat{y}$

$$\int dV = \int r^2 \operatorname{sen}\varphi\, d\varphi\, d\theta\, dr$$

$$= \iiint r^2 \operatorname{sen}\varphi\, d\varphi\, d\theta\, dr$$

$$= 4\pi \int_0^R r^2\, dr$$

$$= \frac{4\pi R^3}{3}$$

$$(E_T\hat{y}) \times (H_T\hat{x}) = E_T H_T\,\hat{z}$$

$\to$ MAS VETOR POYNTING $:\}\ \vec{S} = \vec{E} \times \vec{H}$

$$\therefore\ F = \int \mu_0\sigma E_T H_T\,\hat{z}\, dV = \mu_0\sigma E_T H_T\,\hat{z}\int dV$$

$$= \mu_0\sigma E_T H_T\,\hat{z}\left[\frac{4\pi R^3}{3}\right]$$

$$\boxed{F_{\text{Radiação}} = \frac{4\pi R^3}{3}\mu_0\sigma\, E_T H_T\,\hat{z}} \quad \text{ou} \quad \boxed{F = \frac{4\pi R^3}{3}\mu_0\sigma\,\vec{S}}$$

DENSIDADE DE FLUXO DE MOMENTO $\Big\}:\ \gamma = \dfrac{I}{c}$, Relacionando com a Força de Incidência $\Rightarrow \gamma = \dfrac{\bar{F}}{A}$, onde $\bar{F} \to$ FORÇA MÉDIA

$$\therefore\ \bar{F} = \frac{1}{2}\operatorname{Re}(F) = \frac{1}{2}\operatorname{Re}\left(\frac{4\pi R^3}{3}\mu_0\sigma\,\vec{S}\right) = \frac{2\pi R^3}{3}\mu_0\sigma\operatorname{Re}(S) \quad (1)$$

$$\to \operatorname{Re}(S) = \operatorname{Re}(E\times H) = \operatorname{Re}(EH^*\hat{z}) = \operatorname{Re}\left(E_m H_m e^{-2\alpha z}\cos\pi/4\right) = \frac{\sqrt{2}}{2} E_m H_m e^{-2\alpha z}$$

PARA BOM CONDUTOR

$$(1) \Rightarrow \bar{F} = \frac{2\pi R^3}{3}\mu_0\sigma \cdot \frac{\sqrt{2}}{2} E_m H_m e^{-2\alpha z} = \frac{2\sqrt{2}}{3}\pi R^3\mu_0\sigma\,|E_{RMS}|^2 e^{-2\alpha z}$$

$$\therefore\ \gamma = \frac{\bar{F}}{A} = \frac{\left(\frac{2\sqrt{2}}{3}\right)\pi R^3\mu_0\sigma\,|E_{RMS}|^2 e^{-2\alpha z}}{(4\pi R^2/2)} \Rightarrow \boxed{\frac{\sqrt{2}}{3} R\mu_0\sigma\,|E_{RMS}|^2 e^{-2\alpha z} = \gamma}$$

QUESTÃO 06 | Dado: $\overleftrightarrow{\sigma}_{ij} = \varepsilon_0\left(E_iE_j - \frac{1}{2}\delta_{ij}E^2\right) + \frac{1}{\mu_0}\left(B_iB_j - \frac{1}{2}\delta_{ij}B^2\right)$

$$\sigma_{xx} = \varepsilon_0\left(E_xE_x - \frac{1}{2}E^2\right) + \frac{1}{\mu_0}\left(B_xB_x - \frac{1}{2}B^2\right) = \varepsilon_0\left(E_x^2 - \frac{E^2}{2}\right) + \frac{1}{\mu_0}\left(B_x^2 - \frac{1}{2}B^2\right)$$

$\rightarrow$ Só que $\vec{E} = (E_x, 0, 0)$; $\vec{B} = (0, B_y, 0)$;

$$\therefore \sigma_{xx} = \varepsilon_0\left(E^2 - \frac{1}{2}E^2\right) + \frac{1}{\mu_0}\left(0 - \frac{1}{2}B^2\right) = \boxed{\frac{\varepsilon_0}{2}E^2 - \frac{B^2}{2\mu_0} = \sigma_{xx}}$$

$$\sigma_{xy} = \varepsilon_0\left(\cancel{E_x}E_y - \frac{1}{2}\cancel{\delta_{xy}}E^2\right) + \frac{1}{\mu_0}\left(\cancel{B_x}B_y - \frac{1}{2}\cancel{\delta_{xy}}B^2\right) = 0$$

↳ Obs: Como os Campos $\vec{E}$ e $\vec{B}$ dependem de apenas 1 coordenada, OS TERMOS CRUZADOS : $\{E_iE_j, B_iB_j$ e $\delta_{ij}\}$ são todos iguais a ZERO.

$$\therefore i \neq j \Rightarrow E_iE_j = 0 = B_iB_j = \delta_{ij} = 0$$

↳ Por isso: $\sigma_{xy} = \sigma_{xz} = \sigma_{yx} = \sigma_{yz} = \sigma_{zx} = \sigma_{zy} = 0$

$$\therefore \sigma_{yy} = \varepsilon_0\left(E_yE_y - \frac{1}{2}E^2\right) + \frac{1}{\mu_0}\left(B_yB_y - \frac{1}{2}B^2\right)$$

$$= \varepsilon_0\left(0 - \frac{1}{2}E^2\right) + \frac{1}{\mu_0}\left(B^2 - \frac{1}{2}B^2\right) \Rightarrow \boxed{\sigma_{yy} = -\frac{\varepsilon_0}{2}E^2 + \frac{B^2}{2\mu_0}}$$

$$\therefore \sigma_{zz} = \varepsilon_0\left(E_zE_z - \frac{1}{2}E^2\right) + \frac{1}{\mu_0}\left(B_zB_z - \frac{1}{2}B^2\right)$$

$$= \varepsilon_0\left(0 - \frac{1}{2}E^2\right) + \frac{1}{\mu_0}\left(0 - \frac{1}{2}B^2\right)$$

$$\Rightarrow \boxed{\sigma_{zz} = -\frac{\varepsilon_0}{2}E^2 - \frac{B^2}{2\mu_0}}$$