Aula 13: Runge-Kutta 4^a ordem

Primeiro repassemos ao RK2

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Primeiro repassemos ao RK2 Generico, EDO

$$\frac{df}{dt} = g(f(t), t)$$

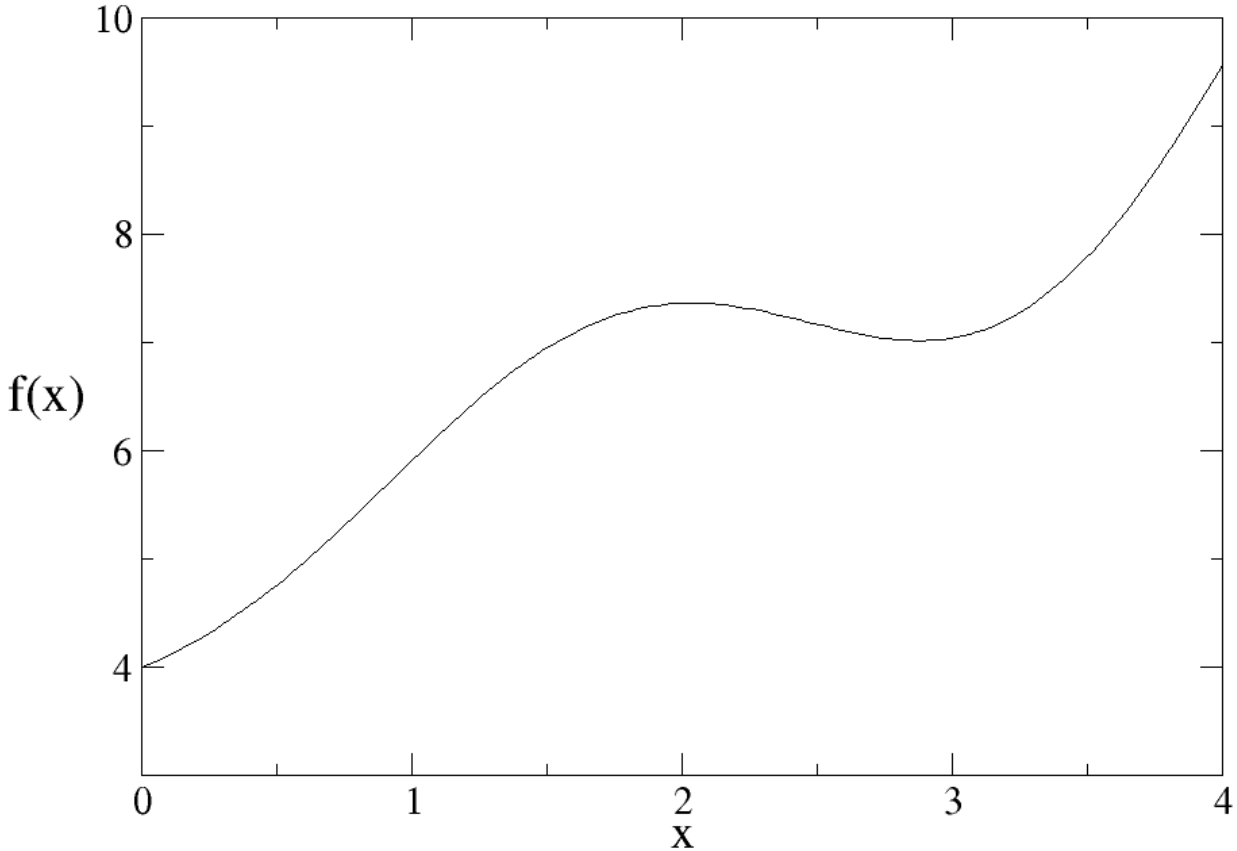
$$\boxed{\begin{aligned} f(t + \Delta t/2) &= f(t) + g(f(t), t)\Delta t/2 \\ f(t + \Delta t) &= f(t) + g(f(t + \Delta t/2), t + \Delta t/2)\Delta t \end{aligned}}$$

Eq de Newton

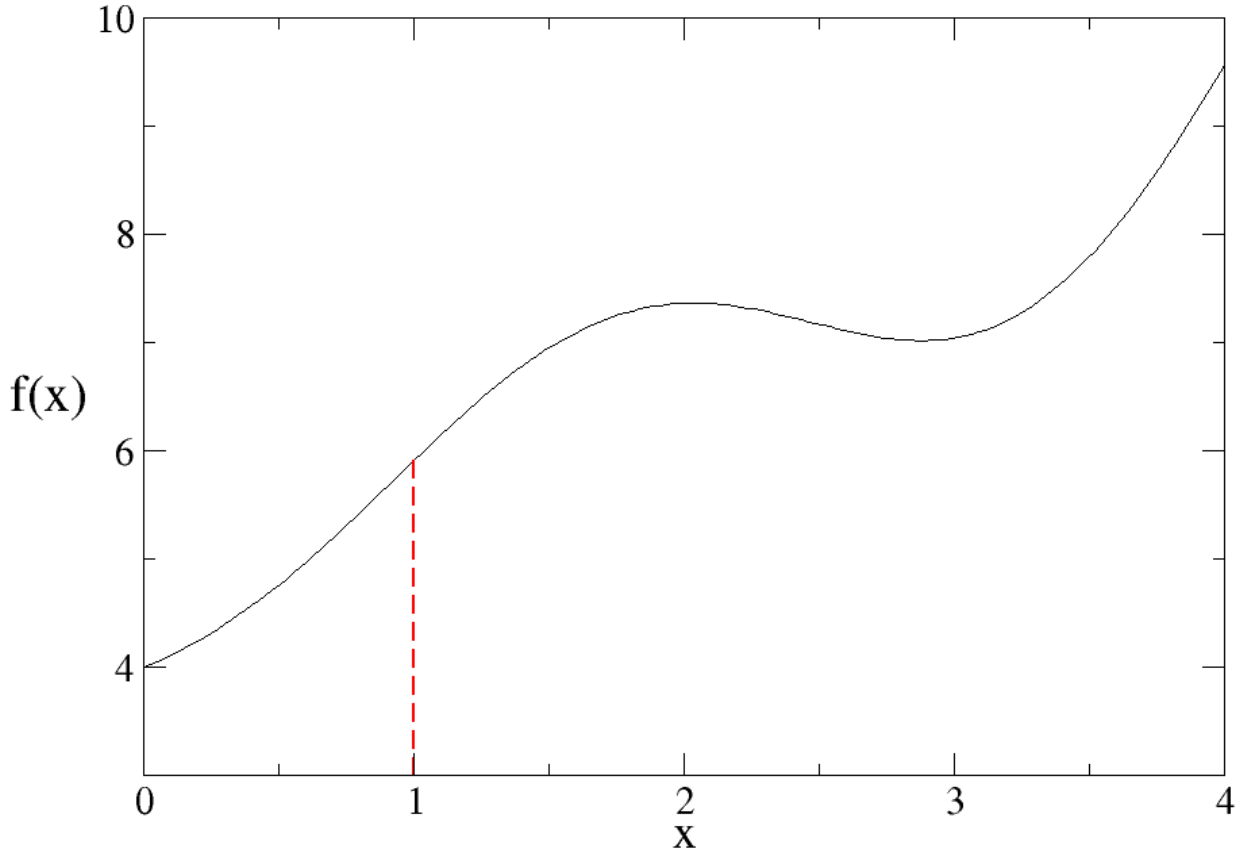
$$\boxed{\begin{aligned} x(t + \Delta t/2) &= x(t) + v(t)\Delta t/2 \\ v(t + \Delta t/2) &= v(t) + a(x(t))\Delta t/2 \\ x(t + \Delta t) &= x(t) + v(t + \Delta t/2)\Delta t \\ v(t + \Delta t) &= v(t) + a(x(t + \Delta t/2))\Delta t \end{aligned}}$$

RK4

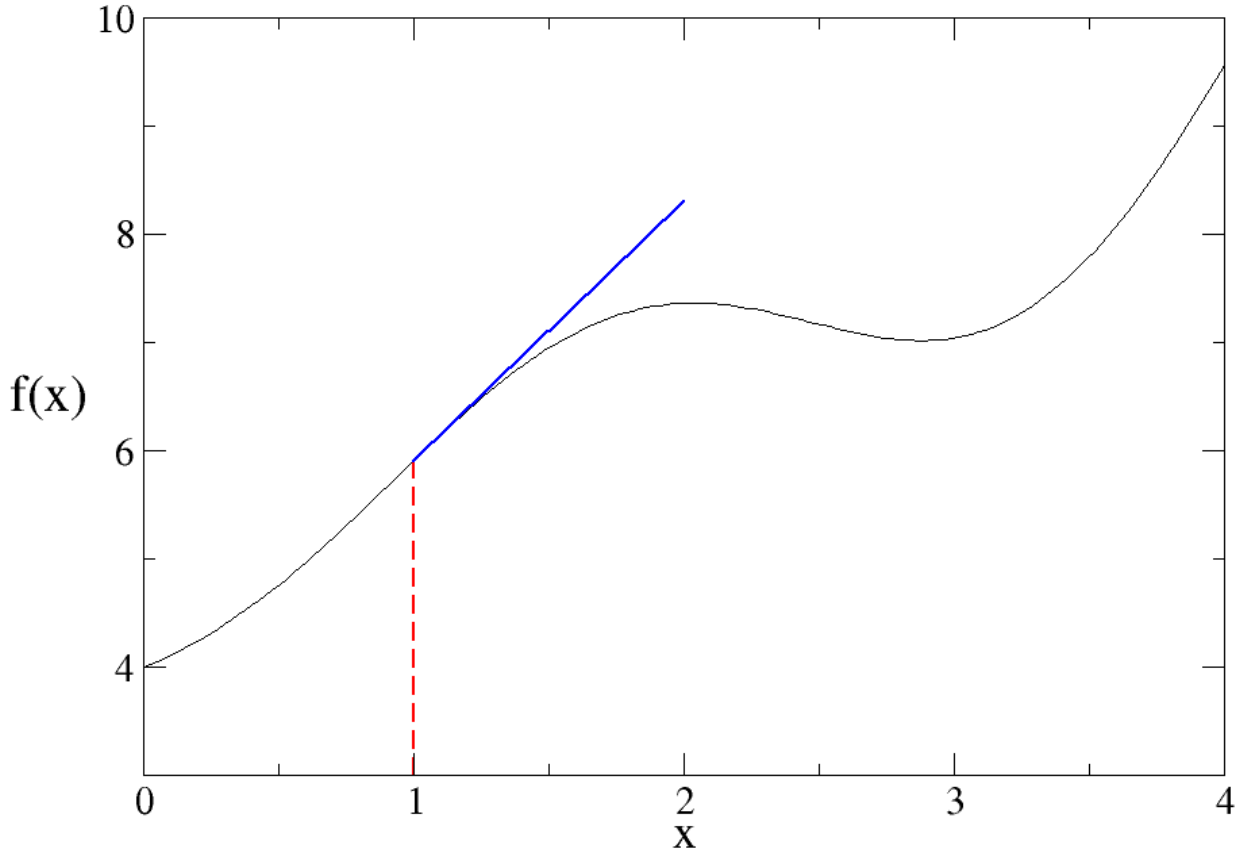
Runge-Kutta 4^a ordem



Runge-Kutta 4^a ordem



Runge-Kutta 4^a ordem



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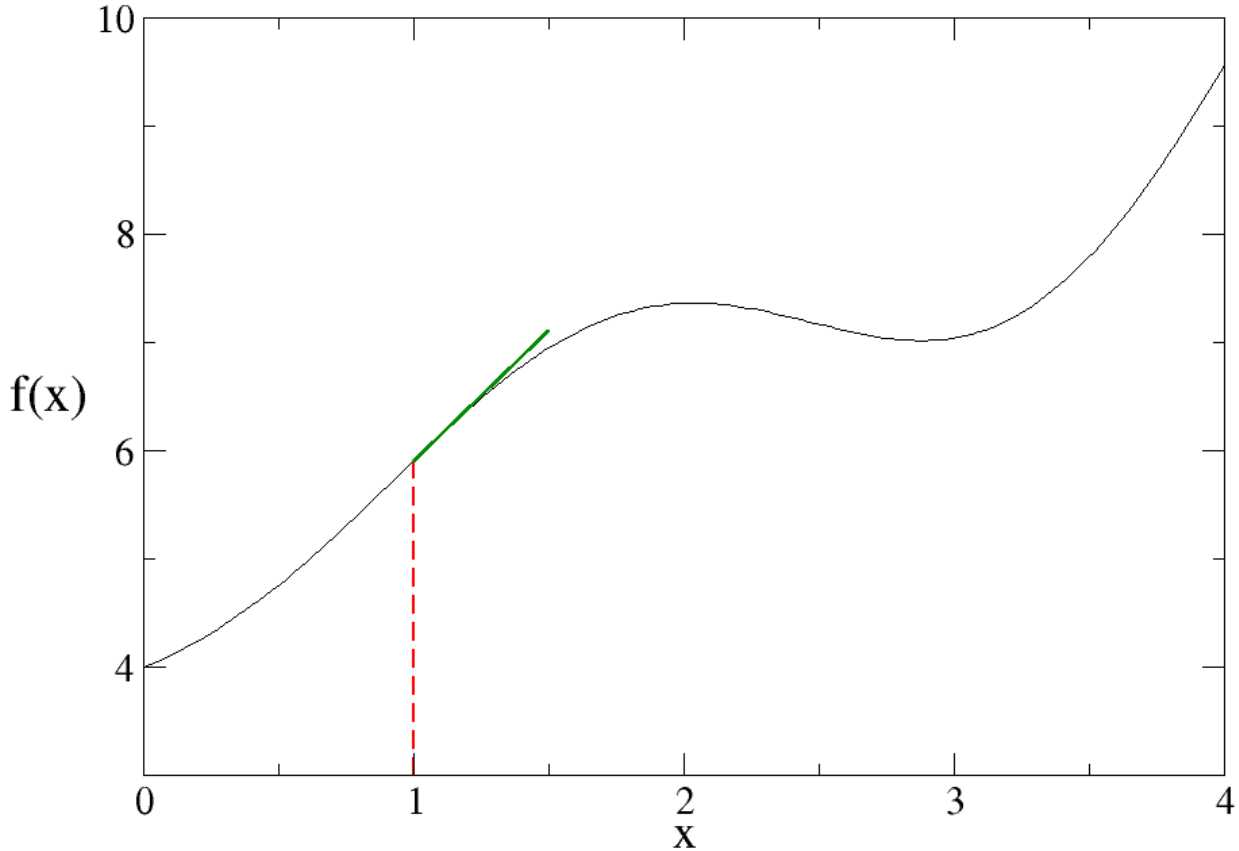
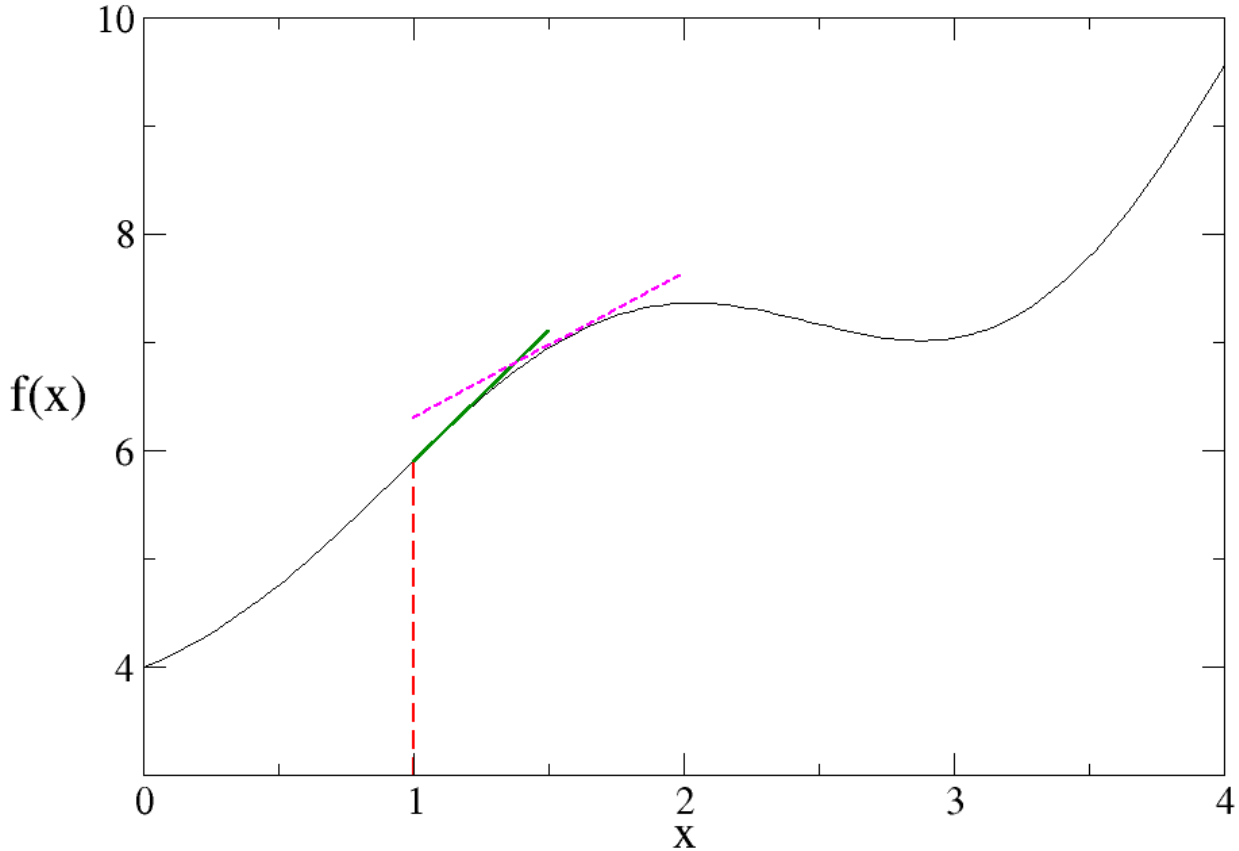


Runge-Kutta 4^a ordem



RK4

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RK4

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\end{aligned}
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RK4

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&\\
&\mathbf{g}(\mathbf{f}(\mathbf{t}+\mathbf{\Delta t}),\mathbf{t}+\mathbf{\Delta t})
\end{aligned}
$$

RK4

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f(t+\Delta t) &= f(t) + g(f(t+\Delta t/2),t+\Delta t/2)\Delta t \\
&\quad \mathbf{g}(\mathbf{f}(\mathbf{t}+\mathbf{\Delta t}),\mathbf{t}+\mathbf{\Delta t}) \\
\hat{g} &= 1/6\big[g_1 + 2g_2 + 2g_3 + g_4\big]
\end{aligned}
$$

RK4

$$
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f(t+\Delta t/2) &= f(t) + g(f(t+\Delta t/2),t+\Delta t/2)\Delta t/2 \\
f(t+\Delta t) &= f(t) + g(f(t+\Delta t/2),t+\Delta t/2)\Delta t \\
&\quad \mathbf{g}(\mathbf{f}(\mathbf{t}+\boldsymbol{\Delta}\mathbf{t}),\mathbf{t}+\boldsymbol{\Delta}\mathbf{t}) \\
\hat{g} &= 1/6\big[g_1 + 2g_2 + 2g_3 + g_4\big] \\
f(t+\Delta t) &= f(t) + \hat{g}\Delta t
\end{aligned}
$$

RK4 – Notação matemática formal

$$y'(t) = f(y, t)$$

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$$\begin{aligned}
k_1 &= f\left(t_n, y_n\right) \\
k_2 &= f\left(t_n + \frac{h}{2}, y_n + \frac{h}{2}k_1\right) \\
k_3 &= f\left(t_n + \frac{h}{2}, y_n + \frac{h}{2}k_2\right) \\
k_4 &= f\left(t_n + h, y_n + hk_3\right)
\end{aligned}$$

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k_4 &= f\left(t_n + h, y_n + hk_3\right)
\end{aligned}$$

$$y(t + \Delta t) = y(t) + \frac{1}{6}(k_1 + 2k_2 + 2k_3 + k4)\Delta t$$

Programas RK2 × RK4 (Newton)

```
# RK2: Newton
for i in range(1,np):
    t  = i*dt

    x2 = x + v*dt/2
    v2 = v + a(x)*dt/2

    x  = x + v2*dt
    v  = v + a(x2)*dt

    print(t,x,v)
```

```
# RK4: Newton
for i in range(1,np):
    t  = i*dt

    a1 = a(x); v1 = v

    x2 = x + v1*dt/2
    v2 = v + a1*dt/2;   a2=a(x2)

    x3 = x + v2*dt/2
    v3 = v + a2*dt/2;   a3=a(x3)

    x4 = x + v3*dt
    v4 = v + a3*dt;     a4=a(x4)

    x  = x + (v1+2*v2+2*v3+v4)*dt/6
    v  = v + (a1+2*a2+2*a3+a4)*dt/6

    print(t,x,v)
```