Aula 12: Revisão dos métodos (de Euler a Vel. Verlet)

Sob a ótica da expansão de Taylor

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Euler:

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$$\boxed{f(t + \Delta t) = f(t) + f'(t)\Delta t + \mathcal{O}(\Delta t^2)}$$

Verlet:

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$$\boxed{\begin{aligned} x(t+\Delta t) &= x(t) + v(t)\Delta t + (1/2)a(t)\Delta t^2 + (1/6)a'(t)\Delta t^3 \\ x(t-\Delta t) &= x(t) - v(t)\Delta t + (1/2)a(t)\Delta t^2 - (1/6)a'(t)\Delta t^3 \end{aligned}}$$

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Somando:

$$x(t+\Delta t) + x(t-\Delta t) = 2x(t) + a(t)\Delta t^2 + \mathcal{O}(\Delta t^4)$$

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Somando:

$$x(t+\Delta t) + x(t-\Delta t) = 2x(t) + a(t)\Delta t^2 + \mathcal{O}(\Delta t^4)$$

Restando:

$$x(t+\Delta t) - x(t-\Delta t) = 2v(t)\Delta t + \mathcal{O}(\Delta t^3)$$

Leap-frog

$$\boxed{\begin{aligned} v(t+\Delta t/2) &= v(t-\Delta t/2) + a(t)\Delta t \\ x(t+\Delta t) &= x(t) + v(t+\Delta t/2)\Delta t \end{aligned}}$$

Velocity Verlet

$$
\boxed{
\begin{aligned}
v(t + \Delta t/2) &= v(t) + a(t)\Delta t/2 \\
x(t + \Delta t) &= x(t) + v(t + \Delta t/2)\Delta t \\
v(t + \Delta t) &= v(t + \Delta t/2) + a(t + \Delta t)\Delta t/2
\end{aligned}
}
$$

Programas (trecho principal)

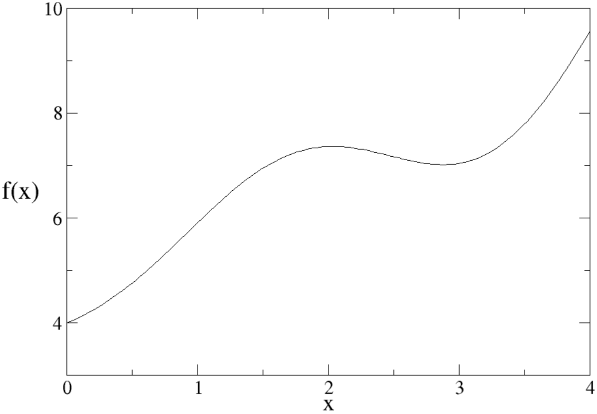
```python
# Euler
for i in range(1,np):
    t = i*dt
    vt = v + a(x)*dt
    x  = x + v*dt
    v  = vt
    print(t,x,v)
```

```python
# Leap-frog
v = v + a(x)*dt/2
for i in range(1,np):
    t = i*dt
    vt = v
    x  = x + v*dt
    v  = v + a(x)*dt
    vt = (vt + v)/2
    print(t,x,vt)
```

```python
# Verlet
xnew = x + v*dt
for i in range(1,np):
    t = i*dt
    xnew = 2*x-xold+a(x)*dt2
    v = (xnew - xold)/(2*dt)
    print(t,x,v)
    xold = x; x = xnew
```

```python
# Velocity Verlet
ax = a(x)
for i in range(1,np):
    t  = i*dt
    v  = v + ax*dt/2
    x  = x + v*dt;  ax = a(x)
    v  = v + ax*dt/2
    print(t,x,v)
```

Runge-Kutta 2^a ordem



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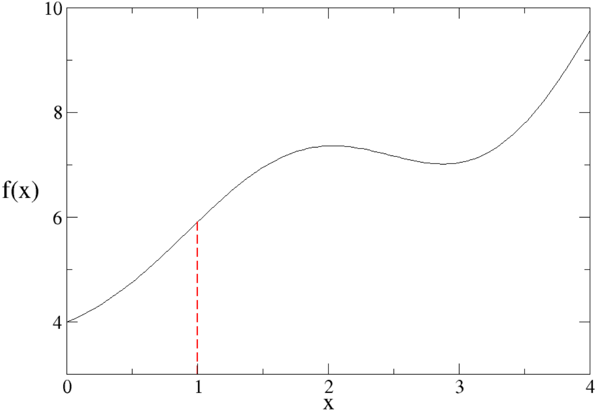

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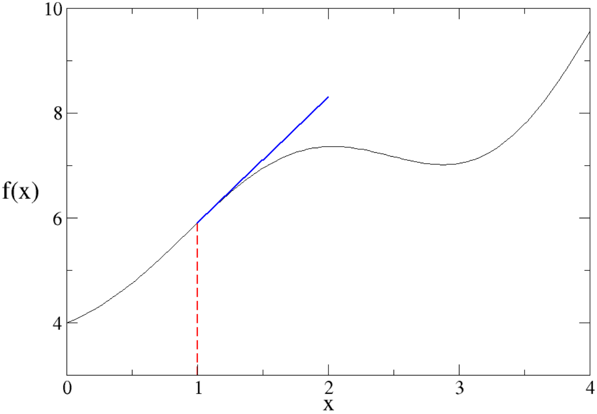

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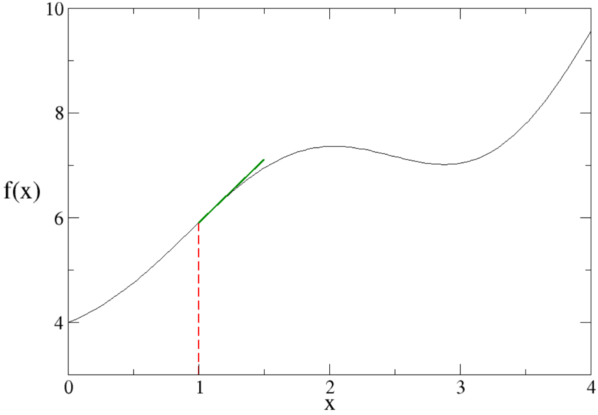

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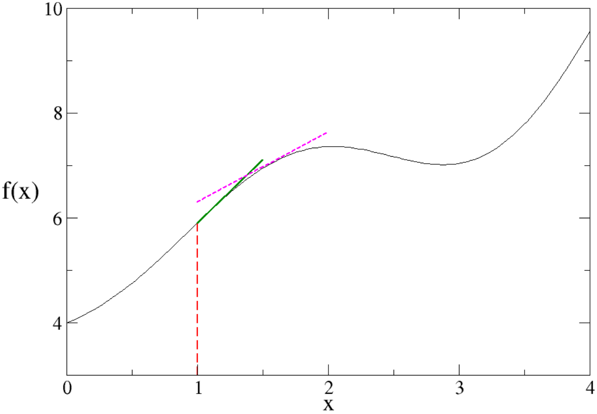

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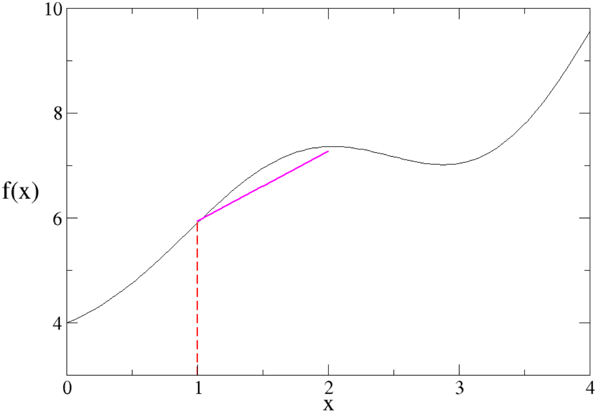

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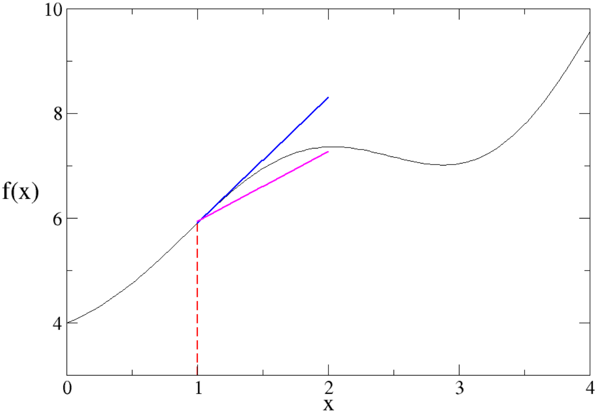

A linha azul representa a estimativa pelo Euler
A linha roxa, leo RK2.

Runge-Kutta 2^a ordem

Generico, EDO

$$\frac{df}{dt} = g(f(t), t)$$

$$\boxed{\begin{aligned} f(t+\Delta t/2) &= f(t) + g(f(t), t)\Delta t/2 \\ f(t+\Delta t) &= f(t) + g(f(t+\Delta t/2), t+\Delta t/2)\Delta t \end{aligned}}$$

Eq de Newton

$$\boxed{\begin{aligned} x(t+\Delta t/2) &= x(t) + v(t)\Delta t/2 \\ v(t+\Delta t/2) &= v(t) + a(x(t))\Delta t/2 \\ x(t+\Delta t) &= x(t) + v(t+\Delta t/2)\Delta t \\ v(t+\Delta t) &= v(t) + a(x(t+\Delta t/2)\Delta t \end{aligned}}$$

Programas (RK2)

```
# RK2: Newton
for i in range(1,np):
    t  = i*dt
    xi = x + v*dt/2
    vi = v + a(x)*dt/2
    x  = x + vi*dt
    v  = v + a(xi)*dt
    print(t,x,v)
```