NLO QCD with Massive Quarks

An extension of the NSC subtraction scheme

Bachelor Degree in Physics

Leonardo Cerasi (11410A)

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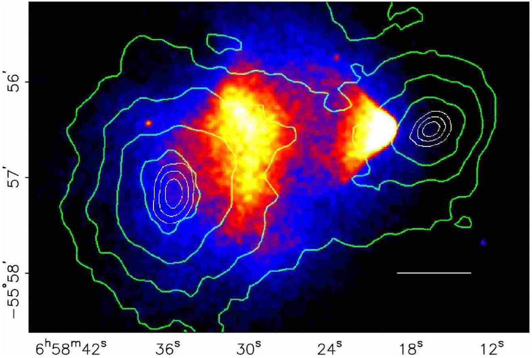
Precision estimates at the LHC

Evidence for Beyond-Standard-Model physics



Main BSM evidence

- dark matter and dark energy
- matter-antimatter asymmetry
- neutrino masses

Figure from Clowe et al. 2006.
Offset between the observed baryonic mass distribution and the gravitational potential in the Bullet Cluster (1E 0657-56).

Precision estimates at the LHC

BSM constraints and shift in research paradigm

Main BSM proposals

- supersymmetric models (MSSM, ...)
- dark matter models (WIMPs, axions, ...)
- extended gauge sectors ($SO(10)$, ...)
- SM Effective Field Theory (SMEFT)

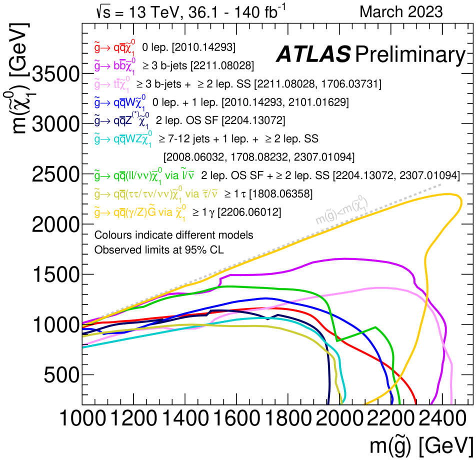
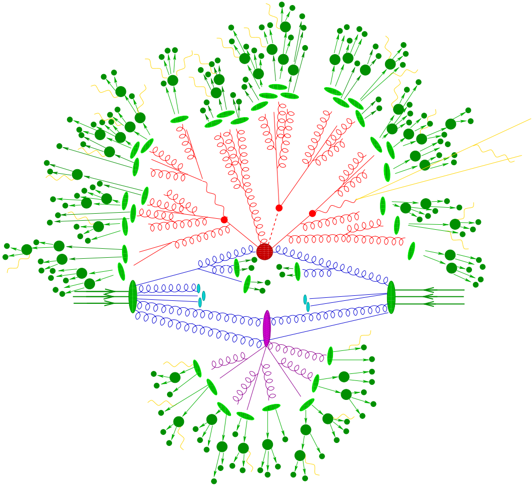
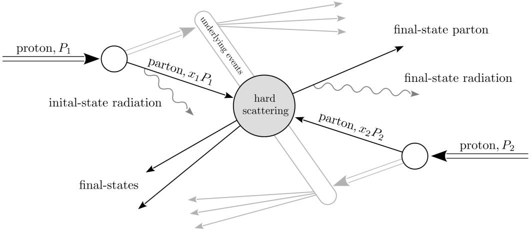


Figure from ATLA PUB Note 2023-025. Exclusion limits in the $\tilde{g} - \tilde{\chi}_1^0$ mass plane for various models for the decay of the gluino to the lightest supersymmetric particle.

Precision estimates at the LHC

Factorization theorem and perturbative QCD





Figures from Höche 2015 (left) and Asteriadis 2021 (right).
Hadronization of jets produced in hadronic scattering and detail of the underlying hard partonic scattering.

Hadronic and partonic physics decouple in hard scattering processes, and we can formulate a factorization theorem:

$$\mathrm{d}\sigma_{h_1,h_2}(P_1,P_2) = \sum_{a,b}\int_{[0,1]^2} \frac{\mathrm{d}\xi_1}{\xi_1}\frac{\mathrm{d}\xi_2}{\xi_2} f_a^{(h_1)}(\xi_1,\mu_\mathsf{F}^2) f_b^{(h_2)}(\xi_2,\mu_\mathsf{F}^2)\times$$
$$\times\, \mathrm{d}\hat{\sigma}_{a,b}(\xi_1 P_1, \xi_2 P_2, \alpha_\mathsf{s}, \mu_\mathsf{R}^2, \mu_\mathsf{F}^2)\left[1 + o\left(\frac{\Lambda^n_\mathsf{QCD}}{Q^n}\right)\right]$$

Asymptotic freedom allows for a perturbative analysis of the hard partonic scattering:

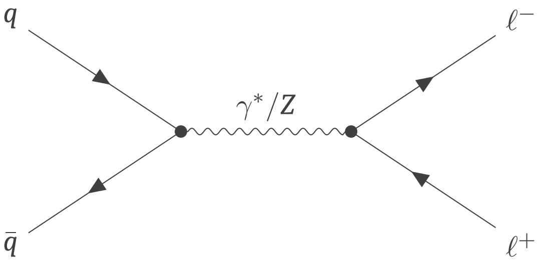
$$\mathrm{d}\hat{\sigma}_{a,b}(p_1,p_2) = \sum_{n\in\mathbb{N}_0} \mathrm{d}\hat{\sigma}^{(n)}_{a,b}(p_1,p_2)$$

$n \geq 1$ are denoted by N^{n}LO QCD corrections.

IR-pole structure of QCD

Radiative corrections to partonic processes

Our focus is on NLO QCD corrections. Consider e.g. the Drell-Yan process:



LO process

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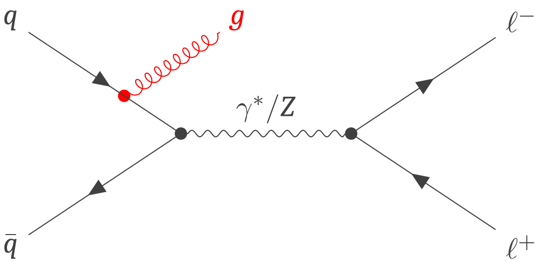

Real correction

IR-pole structure of QCD

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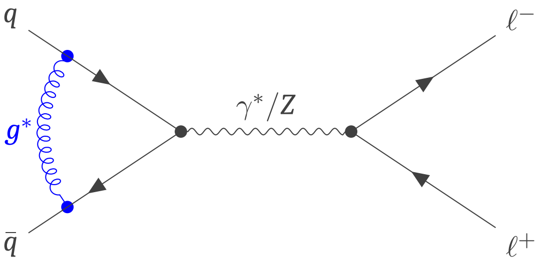
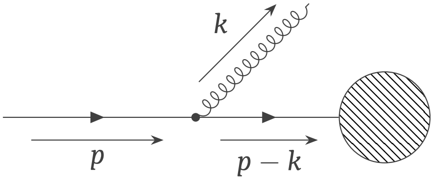


Virtual correction

IR-pole structure of QCD
Infrared singularities of scattering amplitudes

Main difficulty: infrared singularities in particular kinematic regimes.
Example in real corrections:



$$\sim \quad \frac{1}{(p-k)^2} = -\frac{1}{2E_p E_k(1-\cos\theta)}$$

IR-pole structure of QCD

Infrared singularities of scattering amplitudes

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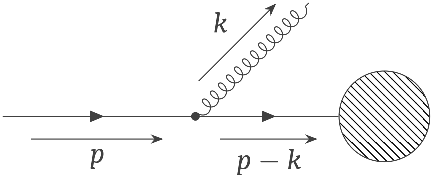

$$\sim \quad \frac{1}{(p-k)^2} = -\frac{1}{2E_p \color{red}{E_k}\color{black}(1-\cos\theta)}$$

Soft singularity: $E_k \to 0$

IR-pole structure of QCD

Infrared singularities of scattering amplitudes

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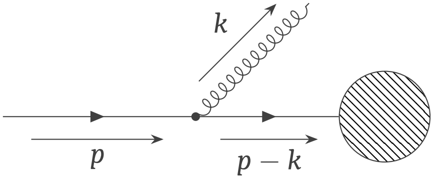
Example in real corrections:



$$\sim \quad \frac{1}{(p-k)^2} = -\frac{1}{2E_p E_k (1 - \cos\theta)}$$

Collinear singularity: $\theta \to 0$

IR-pole structure of QCD
Dimensional regularization

The key idea to regularize IR divergences is dimensional regularization:

$$d = 4 - 2\epsilon \quad , \quad \epsilon \in \mathbb{C} : \Re\epsilon < 0$$

Then, soft and collinear singularities are expressed as poles in ϵ:

$$\int \frac{\mathrm{d}^{d-1}k}{(2\pi)^{d-1}2E_k} \left|\mathcal{A}(p_k)\right|^2 \sim \int_0^{\mathcal{E}} \frac{\mathrm{d}E_k}{E_k^{5-d}} \int_0^{\pi} \mathrm{d}\theta \frac{\sin^{d-3}\theta}{1-\cos\theta} \left|\mathcal{A}_0\right|^2$$

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soft singularity $= -\dfrac{\mathcal{E}^{-2\epsilon}}{2\epsilon}$

IR-pole structure of QCD

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soft singularity $= -\dfrac{\mathcal{E}^{-2\epsilon}}{2\epsilon}$

collinear singularity $= -\dfrac{2^{-2+\epsilon}}{\epsilon}$

IR-pole structure of QCD

Subtraction schemes

ϵ-poles can be extracted from partonic cross-sections via subtraction methods.
General idea using a regular function $f(x)$:

$$I = \int_0^1 \frac{\mathrm{d}x}{x^{1+\epsilon}} f(x) = \int_0^1 \frac{\mathrm{d}x}{x^{1+\epsilon}} \left[f(x) - f(0)\right] + f(0) \int_0^1 \frac{\mathrm{d}x}{x^{1+\epsilon}}$$

- $\dfrac{f(x) - f(0)}{x^{1+\epsilon}}$ regular at $x = 0$, so it can be numerically integrated with $\epsilon \to 0$
- $f(0) \displaystyle\int_0^1 \frac{\mathrm{d}x}{x^{1+\epsilon}} = -\frac{f(0)}{\epsilon}$ contains the explicit ϵ-pole

Our aim is finding the most general subtraction terms $f(0)$ for partonic scattering.

NSC subtraction scheme

Extraction of poles via operators

introduce the NSC SS

NSC subtraction scheme

Pole cancellation

briefly show pole cancellation in the NSC SS

NSC SS with massive quarks

Mass-regualtion of soft and collinear limits

explain why massive quarks change $I_{\mathrm{S}}(\epsilon)$ and $I_{\mathrm{V}}(\epsilon)$, but not $I_{\mathrm{C}}(\epsilon)$

NSC SS with massive quarks

Generalized soft operator

show how $I_S(\epsilon)$ changes (in particular massive angular integrals)

NSC SS with massive quarks

Generalized virtual operator

show how $I_{\mathrm{V}}(\epsilon)$ changes (in particular, colour-correlated ϵ^{-2}-poles in $\mathcal{V}_{i,j}(\epsilon)$ coefficients)

NSC SS with massive quarks

Pole cancellation: generalized pole terms

highlights of pole cancellation in $I_{\mathrm{S+V}}(\epsilon)$, define $\chi_{i,j}(\epsilon)$ coefficients and explain their property

NSC SS with massive quarks

Pole cancellation: colour-correlated terms

show pole cancellation in the colour-correlated sum of $I_{\mathsf{S+V}}(\epsilon)$, leaving the same (and opposite) pole terms of $I_{\mathsf{C}}(\epsilon)$

NSC SS with massive quarks

Generalized integrated counterterms

show integrated counterterms and highlighting massive logs

Conclusions

Future developments

draw conclusions and point out possible further developments