

NLO QCD with Massive Quarks

An extension of the NSC subtraction scheme

Bachelor Degree in Physics

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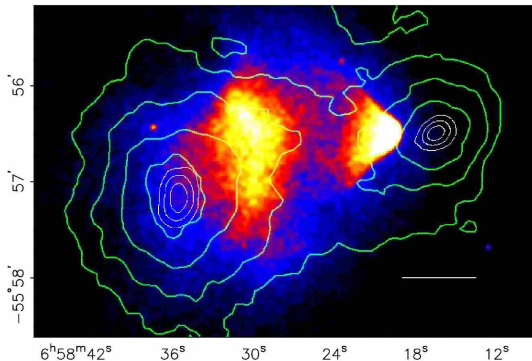


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Precision estimates at the LHC

Evidence for Beyond-Standard-Model physics



Main BSM evidence

- dark matter and dark energy
- matter-antimatter asymmetry
- neutrino masses

Figure from Clowe et al. 2006.

Offset between the observed baryonic mass distribution and the gravitational potential in the Bullet Cluster (1E 0657-56).



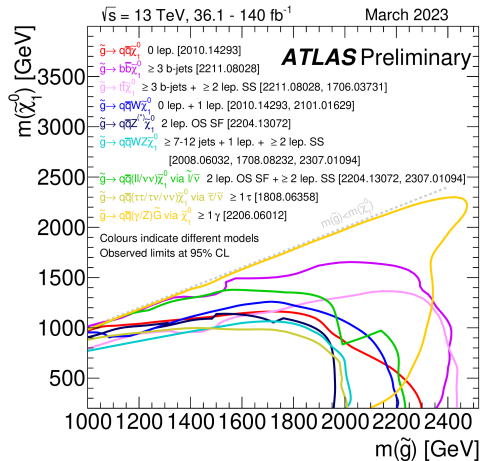
Precision estimates at the LHC

BSM constraints and shift in research paradigm

Main BSM proposals

- supersymmetric models (MSSM, ...)
- dark matter models (WIMPs, axions, ...)
- extended gauge sectors (SO(10), ...)
- SM Effective Field Theory (SMEFT)

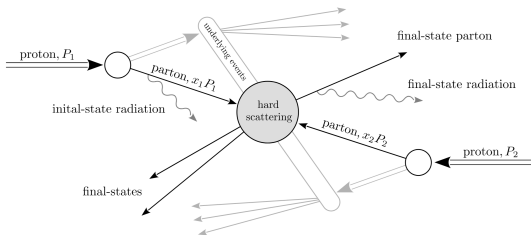
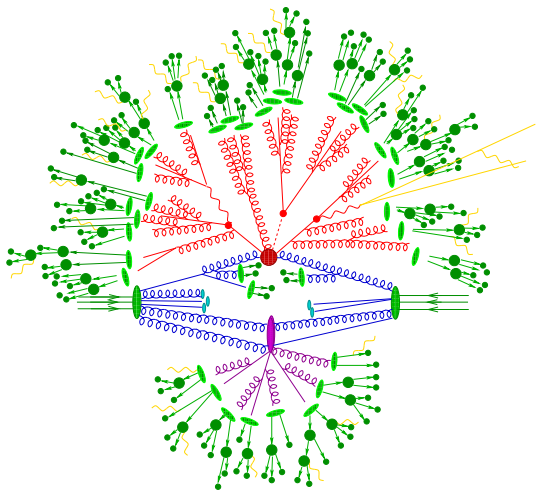
Figure from ATLA PUB Note 2023-025.
Exclusion limits in the $\tilde{g} - \tilde{\chi}_1^0$ mass plane for various models for the decay of the gluino to the lightest supersymmetric particle.





Precision estimates at the LHC

Factorization theorem and perturbative QCD



Figures from Höche 2015 (left) and Asteriadis 2021 (right).

Hadronization of jets produced in hadronic scattering and detail of the underlying hard partonic scattering.



Precision estimates at the LHC

Factorization theorem and perturbative QCD

Hadronic and partonic physics decouple in hard scattering processes, and we can formulate a factorization theorem:

$$\begin{aligned} d\sigma_{h_1, h_2}(P_1, P_2) = \sum_{a, b} \int_{[0, 1]^2} \frac{d\xi_1}{\xi_1} \frac{d\xi_2}{\xi_2} f_a^{(h_1)}(\xi_1, \mu_F^2) f_b^{(h_2)}(\xi_2, \mu_F^2) \times \\ \times d\hat{\sigma}_{a, b}(\xi_1 P_1, \xi_2 P_2, \alpha_s, \mu_R^2, \mu_F^2) \left[1 + o\left(\frac{\Lambda_{\text{QCD}}^n}{Q^n}\right) \right] \end{aligned}$$

Asymptotic freedom allows for a perturbative analysis of the hard partonic scattering:

$$d\hat{\sigma}_{a, b}(p_1, p_2) = \sum_{n \in \mathbb{N}_0} d\hat{\sigma}_{a, b}^{(n)}(p_1, p_2)$$

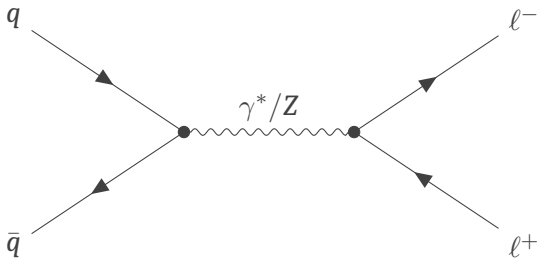
$n \geq 1$ are denoted by NⁿLO QCD corrections.



IR-pole structure of QCD

Radiative corrections to partonic processes

Our focus is on NLO QCD corrections. Consider e.g. the Drell-Yan process:



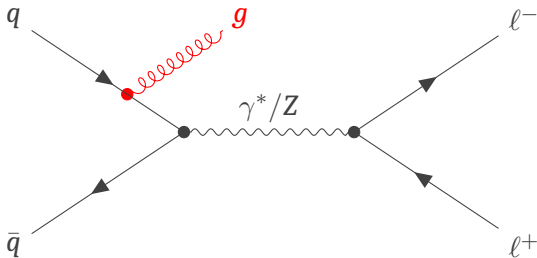
LO process



IR-pole structure of QCD

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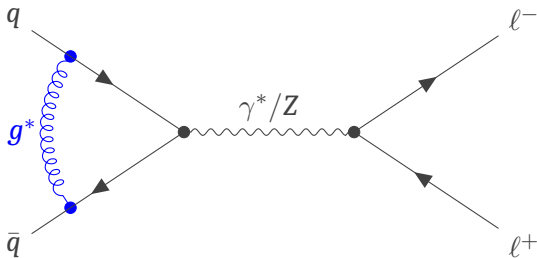
Real correction



IR-pole structure of QCD

Radiative corrections to partonic processes

Our focus is on NLO QCD corrections. Consider e.g. the Drell-Yan process:



Virtual correction



IR-pole structure of QCD

Infrared singularities of scattering amplitudes

Main difficulty: infrared singularities in particular kinematic regimes.

Example in real corrections:

$$\sim \frac{1}{(p-k)^2} = -\frac{1}{2E_p E_k (1 - \cos \theta)}$$

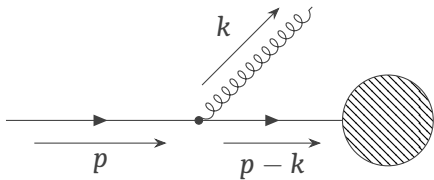


IR-pole structure of QCD

Infrared singularities of scattering amplitudes

Main difficulty: infrared singularities in particular kinematic regimes.

Example in real corrections:


$$\sim \frac{1}{(p-k)^2} = -\frac{1}{2E_p \mathbf{E}_k (1 - \cos \theta)}$$

Soft singularity: $E_k \rightarrow 0$



IR-pole structure of QCD

Infrared singularities of scattering amplitudes

Main difficulty: infrared singularities in particular kinematic regimes.

Example in real corrections:

$$\sim \frac{1}{(p-k)^2} = -\frac{1}{2E_p E_k (1 - \cos \theta)}$$

Collinear singularity: $\theta \rightarrow 0$



IR-pole structure of QCD

Dimensional regularization

The key idea to regularize IR divergences is dimensional regularization:

$$d = 4 - 2\epsilon \quad , \quad \epsilon \in \mathbb{C} : \Re \epsilon < 0$$

Then, soft and collinear singularities are expressed as poles in ϵ :

$$\int \frac{d^{d-1}k}{(2\pi)^{d-1}2E_k} |\mathcal{A}(p_k)|^2 \sim \int_0^\epsilon \frac{dE_k}{E_k^{5-d}} \int_0^\pi d\theta \frac{\sin^{d-3} \theta}{1 - \cos \theta} |\mathcal{A}_0|^2$$



IR-pole structure of QCD

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$$\int \frac{d^{d-1}k}{(2\pi)^{d-1} 2E_k} |\mathcal{A}(p_k)|^2 \sim \int_0^{\mathcal{E}} \frac{dE_k}{E_k^{5-d}} \int_0^\pi d\theta \frac{\sin^{d-3} \theta}{1 - \cos \theta} |\mathcal{A}_0|^2$$

soft singularity $= -\frac{\mathcal{E}^{-2\epsilon}}{2\epsilon}$



IR-pole structure of QCD

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$$\begin{array}{l} \text{soft singularity} \\ \text{collinear singularity} \end{array} = -\frac{\mathcal{E}^{-2\epsilon}}{2\epsilon} = -\frac{2^{-2+\epsilon}}{\epsilon}$$



IR-pole structure of QCD

Subtraction schemes

ϵ -poles can be extracted from partonic cross-sections via subtraction methods.

General idea using a regular function $f(x)$:

$$I = \int_0^1 \frac{dx}{x^{1+\epsilon}} f(x) = \int_0^1 \frac{dx}{x^{1+\epsilon}} [f(x) - f(0)] + f(0) \int_0^1 \frac{dx}{x^{1+\epsilon}}$$

- $\frac{f(x) - f(0)}{x^{1+\epsilon}}$ regular at $x = 0$, so it can be numerically integrated with $\epsilon \rightarrow 0$;
- $f(0) \int_0^1 \frac{dx}{x^{1+\epsilon}} = -\frac{f(0)}{\epsilon}$ contains the explicit ϵ -pole.

Our aim is finding the most general subtraction terms $f(0)$ for partonic scattering.



NSC subtraction scheme

Sequential extraction of singularities

The Nested Soft-Collinear (NSC) Subtraction Scheme (SS) at NLO is modular and local:

- singular limits extracted by operators: S_i soft limit $E_i \rightarrow 0$, C_{ij} collinear limit $\theta_{ij} \rightarrow 0$;



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- potentially-unresolved partons isolated by clever partitions of unity Δ^i and ω^{ij} ;
- first soft singularities removed applying $\bar{S}_m \equiv \text{id} - S_m$, then collinear ones removed using $\bar{C}_{im} \equiv \text{id} - C_{im}$, with m unresolved parton;



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- IR-safe part of cross-section extracted by:

$$O_{\text{NLO}}^m := \sum_{i \in \mathcal{H}_{m,0}^n} \bar{S}_m \bar{C}_{im} \omega^{mi}$$



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- counterterms contain the IR divergences.



NSC subtraction scheme

IR-safe cross-section

The total ϵ -finite NLO cross-section can be written as:

$$\begin{aligned} d\hat{\sigma}_{a,b}^{(1)} = d\hat{\sigma}_{a,b}^{\text{NLO,reg}} + \frac{1}{2\hat{s}} \sum_{n \in \mathcal{G}_m} \left[\frac{\alpha_s(\mu_R^2)}{2\pi} \left\langle I_T^{(0)} \mathcal{F}_{a,b}[\mathcal{X}_m^n] \right\rangle + \left\langle \mathcal{F}_{a,b}^{\text{fin}}[\mathcal{X}_m^n] \right\rangle \right] \\ + \frac{\alpha_s(\mu_R^2)}{2\pi} \sum_c \left[P_{f_c, f_a}^{\text{gen}} \otimes d\hat{\sigma}_{c,b}^{(0)} + P_{f_c, f_b}^{\text{gen}} \otimes d\hat{\sigma}_{a,c}^{(0)} \right] \end{aligned}$$



NSC subtraction scheme

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- NLO final reminder, defined using O_{NLO}^m ;



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- NLO final reminder, defined using O_{NLO}^m ;
- finite counterterm from virtual corrections;



NSC subtraction scheme

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- NLO final reminder, defined using O_{NLO}^m ;
- finite counterterm from virtual corrections;
- finite counterterm from PDF renormalization;



NSC subtraction scheme

IR-safe cross-section

The total ϵ -finite NLO cross-section can be written as:

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- NLO final reminder, defined using O_{NLO}^m ;
- finite counterterm from virtual corrections;
- finite counterterm from PDF renormalization;
- counterterm we are interested in.



NSC subtraction scheme

Pole extraction through operators

Soft singularities

$$\left\langle S_m \Delta^m \left| \begin{array}{c} a \\ \text{diagram} \\ b \end{array} \right| \right\rangle^2 = [\alpha_s] \left\langle I_S(\epsilon) \cdot \left| \begin{array}{c} a \\ \text{diagram} \\ b \end{array} \right| \right\rangle^2$$

The diagram shows a shaded circle with two external lines labeled 'a' and 'b', and several internal lines. A red wavy line labeled 'm' is shown connecting the circle to a red 'm'.

$$I_S(\epsilon) := -\frac{1}{\epsilon^2} \left(\frac{2\mathcal{E}}{\mu} \right)^{-2\epsilon} \frac{\Gamma^2(1-\epsilon)}{\Gamma(1-2\epsilon)} \sum_{i \neq j \in \mathcal{X}_{m,0}^n} \eta_{ij} (\mathbf{T}_i \cdot \mathbf{T}_j) {}_2F_1(1, 1, 1-\epsilon, 1-\eta_{ij})$$



NSC subtraction scheme

Pole extraction through operators

Hard-collinear singularities

$$\sum_{\rho=1}^{2n_f+1} \sum_{i \in \mathcal{H}_{m,0}^n(\mathfrak{m}_\rho)} \left\langle \bar{S}_m C_{i\mathfrak{m}_\rho} \Delta^{\mathfrak{m}_\rho} \right| \left| \begin{array}{c} a \\ \vdots \\ i \\ \vdots \\ b \end{array} \right\rangle^2 \sim [\alpha_s] \left\langle I_C(\epsilon) \cdot \left| \begin{array}{c} a \\ \vdots \\ [i\mathfrak{m}_\rho] \\ \vdots \\ b \end{array} \right| \right\rangle^2$$

$$I_C(\epsilon) := \sum_{i \in \mathcal{H}_{m,0}^n} \frac{\Gamma_{i f_i}}{\epsilon} = \frac{1}{\epsilon} \sum_{i \in \mathcal{H}_{m,0}^n} \left[\gamma_i + 2\mathbf{T}_i^2 \log \frac{\mathcal{E}}{E_i} \right] + o(\epsilon^0)$$



NSC subtraction scheme

Pole extraction through operators

Virtual singularities

$$\left\langle 2\Re \left\langle \begin{array}{c} a \\ \text{1-L} \\ b \end{array} \right\rangle \middle| \begin{array}{c} a \\ \text{shaded circle} \\ b \end{array} \right\rangle \right\rangle = [\alpha_s] \left\langle I_V(\epsilon) \cdot \left| \begin{array}{c} a \\ \text{shaded circle} \\ b \end{array} \right|^2 \right\rangle$$

$$I_V(\epsilon) \equiv \sum_{i \neq j \in \mathcal{X}_{m,0}^n} (\mathbf{T}_i \cdot \mathbf{T}_j) \left(\frac{\mu^2}{2p_i \cdot p_j} \right)^\epsilon \cos(\lambda_{ij}\pi\epsilon) \left[\frac{1}{\epsilon^2} + \frac{1}{\mathbf{T}_i^2} \frac{\gamma_i}{\epsilon} \right]$$



NSC subtraction scheme

Pole cancellation: $I_S(\epsilon) + I_V(\epsilon)$

$$\begin{aligned}
 I_{S+V} &= \sum_{i \neq j \in \mathcal{X}_{m,0}^n} (\mathbf{T}_i \cdot \mathbf{T}_j) \left[-\frac{1}{\epsilon^2} \left(\frac{2\mathcal{E}}{\mu} \right)^{-2\epsilon} K_{ij} + \left(\frac{\mu^2}{2p_i \cdot p_j} \right)^\epsilon \cos(\lambda_{ij}\pi\epsilon) \left(\frac{1}{\epsilon^2} + \frac{1}{\mathbf{T}_i^2} \frac{\gamma_i}{\epsilon} \right) \right] \\
 &\equiv \frac{\Gamma^2(1-\epsilon)}{\Gamma(1-2\epsilon)} \eta_{ij} {}_2F_1(1, 1, 1-\epsilon, 1-\eta_{ij}) = 1 - \epsilon \log \eta_{ij} + o(\epsilon^2) \\
 &= \sum_{i \neq j \in \mathcal{X}_{m,0}^n} (\mathbf{T}_i \cdot \mathbf{T}_j) \frac{1}{\epsilon^2} \left(\frac{2\mathcal{E}}{\mu} \right)^{-2\epsilon} \left[-1 + \epsilon \log \eta_{ij} + \left(\frac{\mathcal{E}^2}{E_i E_j} \right)^\epsilon \eta_{ij}^{-\epsilon} \cos(\lambda_{ij}\pi\epsilon) \left(1 + \epsilon \frac{\gamma_i}{\mathbf{T}_i^2} \right) \right] \\
 &= \frac{1}{\epsilon} \sum_{i \neq j \in \mathcal{X}_{m,0}^n} (\mathbf{T}_i \cdot \mathbf{T}_j) \left(L_i + L_j + \frac{\gamma_i}{\mathbf{T}_i^2} \right) + o(\epsilon^0) = -\frac{1}{\epsilon} \sum_{i \in \mathcal{H}_{m,0}^n} (\gamma_i + 2\mathbf{T}_i^2 L_i) + o(\epsilon^0)
 \end{aligned}$$



NSC subtraction scheme

Pole cancellation: $I_T(\epsilon)$

Compare:

$$I_{S+V}(\epsilon) = -\frac{1}{\epsilon} \sum_{i \in \mathcal{H}_{m,0}^n} (\gamma_i + 2\mathbf{T}_i^2 L_i) + I_{S+V}^{(0)} + o(\epsilon)$$

$$I_C(\epsilon) = -\frac{1}{\epsilon} \sum_{i \in \mathcal{H}_{m,0}^n} (\gamma_i + 2\mathbf{T}_i^2 L_i) + I_C^{(0)} + o(\epsilon)$$

Hence:

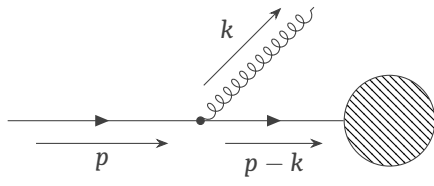
$$I_T(\epsilon) \equiv I_S(\epsilon) + I_C(\epsilon) + I_V(\epsilon) = I_T^{(0)} + o(\epsilon)$$



NSC SS with massive quarks

Mass-regulation of soft and collinear limits

Massive partons do not determine soft or collinear singularities:



The diagram shows a horizontal line representing a quark. An arrow labeled p points to the right. A vertex is marked with a black dot. From this vertex, a wavy line labeled k points upwards and to the right. The quark line continues to the right, labeled $p - k$ with an arrow. This line ends at a shaded circular blob. To the right of the diagram is an approximation symbol \sim followed by the expression $\frac{1}{(p - k)^2} = \frac{1}{\textcolor{red}{m}^2 - 2E_k \left(\sqrt{\textcolor{red}{m}^2 + |\mathbf{p}|^2} - |\mathbf{p}| \cos \theta \right)}$.

$$\sim \frac{1}{(p - k)^2} = \frac{1}{\textcolor{red}{m}^2 - 2E_k \left(\sqrt{\textcolor{red}{m}^2 + |\mathbf{p}|^2} - |\mathbf{p}| \cos \theta \right)}$$

Clearly, no more singularities due to $\textcolor{red}{m}^2$ in the denominator.



NSC SS with massive quarks

Generalized soft operator

- In presence of final-state massive partons, $I_S(\epsilon)$ becomes:

$$I_S(\epsilon) = \frac{1}{2\epsilon} \left(\frac{2\mathcal{E}}{\mu} \right)^{-2\epsilon} \frac{\Gamma^2(1-\epsilon)}{\Gamma(1-2\epsilon)} \sum_{i,j \in \mathcal{X}_{m,0}^n} (\mathbf{T}_i \cdot \mathbf{T}_j) I_{ij}(\epsilon)$$



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- $I_{i,j}(\epsilon)$ depends on the nature (massive or massless) of partons i and j ;



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- $I_{i,j}(\epsilon)$ depends on the nature (massive or massless) of partons i and j ;
- $I_{i,j}(\epsilon)$ is singular as $\sim \epsilon^{-1}$ if at least one parton is massless, otherwise it is ϵ -finite;



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- $I_{i,j}(\epsilon)$ is singular as $\sim \epsilon^{-1}$ if at least one parton is massless, otherwise it is ϵ -finite;
- ϵ^{-2} -poles of $I_S(\epsilon)$ are determined only by **massless partons**.



NSC SS with massive quarks

Generalized virtual operator

- In presence of final-state massive partons, $I_S(\epsilon)$ becomes:

$$I_V(\epsilon) := \Re I_1(\epsilon) = \sum_{i \neq j \in \mathcal{X}_{m,0}^n} (\mathbf{T}_i \cdot \mathbf{T}_j) \left(\frac{\mu^2}{|s_{ij}|} \right)^\epsilon \left[\mathcal{V}_{ij}(\epsilon) - \frac{1}{v_{ij}} \frac{\pi^2}{2} \theta(s_{ij}) \right] - \sum_{i \in \mathcal{X}_{m,0}^n} \Gamma_i(\epsilon)$$



NSC SS with massive quarks

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- $\mathcal{V}_{ij}(\epsilon)$ depends on the nature (massive or massless) of partons i and j ;
- $I_{ij}(\epsilon)$ is ϵ^{-2} -singular if at least one parton is massless, otherwise it is ϵ^{-1} -singular;



NSC SS with massive quarks

Generalized virtual operator

- In presence of final-state massive partons, $I_S(\epsilon)$ becomes:

$$I_V(\epsilon) := \Re I_1(\epsilon) = \sum_{i \neq j \in \mathcal{X}_{m,0}^n} (\mathbf{T}_i \cdot \mathbf{T}_j) \left(\frac{\mu^2}{|s_{ij}|} \right)^\epsilon \left[\mathcal{V}_{ij}(\epsilon) - \frac{1}{v_{ij}} \frac{\pi^2}{2} \theta(s_{ij}) \right] - \sum_{i \in \mathcal{X}_{m,0}^n} \Gamma_i(\epsilon)$$

- $\mathcal{V}_{ij}(\epsilon)$ depends on the nature (massive or massless) of partons i and j ;
- $I_{i,j}(\epsilon)$ is ϵ^{-2} -singular if at least one parton is massless, otherwise it is ϵ^{-1} -singular;
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- ϵ^{-2} -poles of $I_V(\epsilon)$ are determined only by **massless partons**.



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Pole cancellation: generalized pole terms

highlights of pole cancellation in $I_{S+V}(\epsilon)$, define $\chi_{i,j}(\epsilon)$ coefficients and explain their property



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Pole cancellation: colour-correlated terms

show pole cancellation in the colour-correlated sum of $I_{S+V}(\epsilon)$, leaving the same (and opposite) pole terms of $I_C(\epsilon)$



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Generalized integrated counterterms

show integrated counterterms and highlighting massive logs



Conclusions

Future developments

draw conclusions and point out possible further developments