Quantum Field Theory 1

Prof. S. Forte a.a. 2024-25

Leonardo Cerasi[1]

GitHub repository: LeonardoCerasi/notes

[1] leo.cerasi@pm.me

Contents

Part I

Classical Field Theory

Chapter 1

Discrete systems

1.1 One-dimensional harmonic crystal

Consider a simple one-dimensional model of a crystal where atoms of mass $m \equiv 1$ lie at rest on the x-axis, with equilibrium positions labelled by $n \in \mathbb{N}$ and equally spaced by a distance a.
Assuming these atoms are free to vibrate only in the x direction (longitudinal waves), and denoting the displacement of the atom at position n as η_n, one can write the Lagrangian for a *harmonic crystal* as:

$$L = \sum_n \left[\frac{1}{2}\dot{\eta}_n^2 - \frac{\lambda}{2}\left(\eta_n - \eta_{n-1}\right)^2 \right] \tag{1.1}$$

where λ is the spring constant. From the Lagrange equations, the classical equations of motions are:

$$\ddot{\eta}_n = \lambda\left(\eta_{n+1} - 2\eta_n + \eta_{n-1}\right) \tag{1.2}$$

The solutions can be written as complex travelling waves:

$$\eta_n(t) = e^{i(kn-\omega t)} \tag{1.3}$$

where the dispersion relation is:

$$\omega^2 = 2\lambda\left(1 - \cos k\right) \approx \lambda k^2 \tag{1.4}$$

Therefore, in the long-wavelength limit $k \ll 1$ waves propagate with velocity $c = \sqrt{\lambda}$.