Radial Distribution System Short Circuit Analysis with Lateral and Load Equivalencing: Solution Algorithms and Numerical Results

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Abstract: This paper presents two new approaches for short circuit analysis of radial distribution networks. The methods are based on the compensation method using two different lateral and loads equivalent methods. The proposed methods can be applied to balanced and unbalanced distribution networks and can handle different fault types. Comparisons results between the two new methods with the symmetrical components method and the compensation method from [1] are presented.

Keywords: unbalanced distribution system, short circuit analysis, compensation method, loads equivalent, lateral equivalent.

1. INTRODUCTION

In this paper, two new approaches based on the compensation method [1] for short circuit analysis in radial distribution systems are presented. As the distribution system becomes more heavily loaded and the need and ability to reconfigure the system for service restoration, load balancing and loss reduction grows; the network configuration will be changed more frequently. With each change protection device settings in the system may need updating. As more sophisticated protection devices are installed, the setting of these devices can be performed remotely as needed with each change in the network. Therefore, there is a need for fast and more accurate short circuit calculations.

The methods proposed in this paper calculate the short circuit current considering the loads in the system. Traditionally short circuit analysis for power systems omits the loads in the system [1-3]. Usually, only the path from the fault location to the root is considered. The reason why the loads have been omitted is because the loads are thought to have a very small effect on the short circuit current. But this assumption is not always precise enough in the distribution system. Thus in [4] detailed transformer and load models were used in the calculation of the post-fault voltages. This paper proposes to model beyond the fault path and to consider the loads in both the short circuit current calculations and the post-fault analysis.

The laterals and loads are handled in two ways. The first method uses initial condition and boundary matching to equivalence the laterals and loads in the system. The second method uses the Thevenin equivalent method to equivalence all laterals and loads in the system. The details of each method will be addressed in the following sections.

The methods developed in this paper model the system in three-phase coordinates. With the tremendous increase of computation power in recent years, solving power system in phase coordinates becomes practical. Therefore, both proposed methods in this paper solve the short circuit problem in phase coordinates. Floating point operation counts are recorded to analyze computational performance. Finally, comparison results between the two proposed methods with the symmetrical component method and the compensation method are presented.

2. LATERAL AND LOAD EQUIVALENCING

The proposed methods are based on the compensation method. Normally, when short circuit currents are calculated, the loads are omitted. If the loads are considered within the short circuit calculation, it is expected that fault currents and post-fault voltages will be more accurate.

Methods to determine circuit equivalents were discussed in [9]. The concept of equivalencing is utilized for more detailed network representation in short circuit analysis.

2.1 Using initial condition boundary matching (ICBM):

ICBM is applied to determine circuit equivalents for laterals and loads. The pre-fault system voltages and currents are treated as the initial condition for short circuit analysis. For the laterals branching from the buses in the fault path, the pre-fault information of the branch is used to get the lateral equivalent impedance matrix. An assumption that the mutual impedance between the phases on each lateral are zero is made. Thus, for 3-phase bus k, the lateral equivalent impedance matrix can be calculated by the following equation:

$$Z_k^{lat} = \begin{bmatrix} V_k^{a,pre} / I_k^{a,pre} & 0 & 0 \\ 0 & V_k^{b,pre} / I_k^{b,pre} & 0 \\ 0 & 0 & V_k^{c,pre} / I_k^{c,pre} \end{bmatrix} \quad (1)$$

where:

$V_k^{a,pre}, V_k^{b,pre}, V_k^{c,pre}$: pre-fault voltages of each phase at bus k.

$I_k^{a,pre}, I_k^{b,pre}, I_k^{c,pre}$: pre-fault currents of each phase flowing into the lateral at bus k.

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If there is more than one branch connected to a bus, the diagonal elements of Z_k^{lat} are placed in parallel. For example there are two laterals connected to bus k, then Z_k^{lat} of bus k can be calculated using (2):

$$Z_k^{lat} = Z_k^{lat1} .// Z_k^{lat2} \tag{2}$$

where: $.//$ represents placing two matrices in parallel element-wise. This matrix is used to represent all the laterals connected to the bus including the loads and branches in these laterals.

The loads of the distribution system are modeled as constant impedance loads. For a 3-phase load, the following represents the load equivalent Z_k^L:

$$Z_k^L = \begin{bmatrix} \frac{\left|V_k^{a,pre}\right|^2}{(S_{k.nom}^a)^*} & 0 & 0 \\ 0 & \frac{\left|V_k^{a,pre}\right|^2}{(S_{k.nom}^b)^*} & 0 \\ 0 & 0 & \frac{\left|V_k^{c,pre}\right|^2}{(S_{k.nom}^c)^*} \end{bmatrix} \tag{3}$$

where: $S_{k.nom}^a, S_{k.nom}^b, S_{k.nom}^c$: nominal complex load at bus k on each phase.

The mutual impedance between the loads on each phase are assumed to be zero. Using (3) each load connected to the buses in the fault path has a load impedance matrix.

For a given fault, first we should find all the buses in the fault path. Then for these buses, calculate the Z_k^{lat} and Z_k^L for them. If a load and laterals are attached to one bus, the diagonal elements of Z_k^L are in parallel with those of Z_k^{lat} to get Z_k^{eq} for this bus. For example, if there is load on bus k, and Z_k^{lat} is its lateral equivalent impedance matrix, then Z_k^{eq} can be calculated using (4):

$$Z_k^{eq} = Z_k^{lat} .// Z_k^L \tag{4}$$

So for each bus of the fault path, you can calculate one diagonal equivalent matrix Z_k^{eq} to equivalent all the loads and laterals connected to buses except those buses in the fault path.

Since Z_k^{eq} is a diagonal matrix, when the Thevenin compensation impedance matrix is formed, only the diagonal elements of the compensation impedance matrix will be affected. Therefore when line impedance matrix is in parallel with Z_k^{eq}, only the diagonal elements of the two matrices are placed in parallel and the off-diagonal elements will not change.

The following is a simple example to show the steps:

- The simple system structure likes the followings:

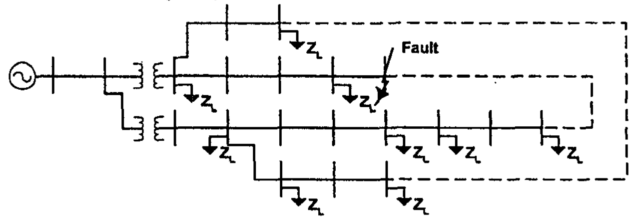

Fig. 1. System before Lateral Equivalencing

- After the lateral equivalent has been done the system will be like Figure 2:

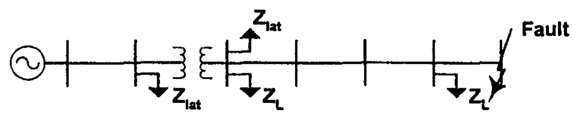

Fig. 2. System after Lateral Equivalencing

- Get the Z_{eq} by placing the Z_{lat} and Z_L in parallel. The figure 3 shows the result.

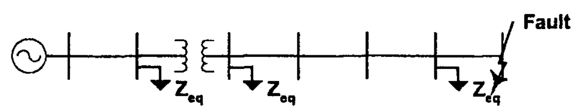

Fig. 3. Final System Diagram

The ICBM method for short circuit analysis determines circuit equivalents for all the laterals and loads, which lie in the fault path. ICBM does not search all the nodes in the system.

2.2 Using Thevenin Equivalent (TE):

In order to get even more accurate results, the Thevenin equivalent method accounts for all the nodes in the network. This method determines a Thevenin equivalent impedance matrix for every feeder lateral in the network using the pre-fault system information.

In the distribution system, there are many loads having one or two phases. For these loads the load impedance matrices are formed with very large impedance on the phases without loads to represent the open circuit.

The transformers of the distribution system are modeled as the three-phase admittance models from [2]. Transformer impedance models from [3] are used to express the transformer branch as a (3×3) impedance matrix, where a pseudo inverse is employed to handle the singular admittance matrix models for certain transformer connection models.

In order to get the Thevenin equivalent impedance matrix, branches and loads are placed in series and in parallel as dictated by the network. The equivalent impedance matrix of two (3×3) impedance matrices in parallel can be calculated by placing the corresponding elements of the two impedance matrices in parallel. For example, there are two impedance matrices Z_1 and Z_2, which are (3×3) impedance matrices. The result of the equivalent impedance matrix Z_3 can be calculated using the following equation:

$$Z_3 = Z_1.// Z_2 = \begin{bmatrix} Z_1^{aa} // Z_2^{aa} & Z_1^{ab} // Z_2^{ab} & Z_1^{ac} // Z_2^{ac} \\ Z_1^{ba} // Z_2^{ba} & Z_1^{bb} // Z_2^{bb} & Z_1^{bc} // Z_2^{bc} \\ Z_1^{ca} // Z_2^{ca} & Z_1^{cb} // Z_2^{cb} & Z_1^{cc} // Z_2^{cc} \end{bmatrix} \quad (6)$$

3. SOLUTION ALGORITHMS

Lateral and load equivalencing is proposed for short circuit analysis in distribution systems. These algorithms are based on the compensation method in [1]. The steps 5 through 10 follow the compensation method.

3.1 Using initial condition boundary matching

Step 1. Solve for the pre-fault three-phase power flow solution.
Step 2. Input the fault information.
Step 3. Find the path from the fault location to the root.
Step 4. Use the pre-fault condition as the initial condition and calculate Z_{lat} and Z_L for each bus in the fault path. And then get the Z_{eq} for these buses.
Step 5. Form the Thevenin compensation impedance matrix.
Step 6. According to each type of the fault, form the voltage mismatch vector V_f.
Step 7. Solve the I_f from the following equation:

$$Z_f I_f = V_f \quad (7)$$

Step 8. Update the node current injection with I_f.
Step 9. Solve three-phase power flow to obtain post-fault voltages
Step 10. Output the result of the short circuit calculation.

3.2 Using Thevenin equivalent:

Step 1. Solve for the pre-fault three-phase power flow solution.
Step 2. Input the fault information.
Step 3. Find the path from the fault location to the root.
Step 4. Find the Thevenin Equivalent impedance matrix representing all the laterals branching from the buses in the fault path.
Step 5. Form the Thevenin compensation impedance matrix.
Step 6. According to each type of the fault, form the voltage mismatch vector V_f.
Step 7. Solve the I_f from (7).
Step 8. Update the node current injection with I_f.
Step 9. Solve three-phase power flow to obtain post-fault bus voltages
Step 10. Output the result of the short circuit calculation.

4. NUMERICAL RESULTS

Based on the proposed algorithms, a distribution system short circuit analysis program has been developed in Matlab®. This program can be used to solve single-line to ground (SLG), three phases to ground (3LG), line-line fault (LL) and double-line to ground (DLG).

The methods were tested on two unbalanced distribution systems. Comparison results between these two methods, the symmetrical component method and the compensation method are presented in the following subsections. The number of floating point operations for each method is also listed.

4.1 20-bus System

The structure of the small system is shown in Fig. 4:

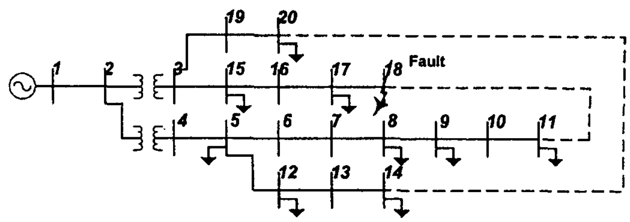

Fig. 4. A one-line diagram of the 20-bus system

Detailed information about the number of the components follows:

# of lines without breakers	=	11
# of lines with breakers	=	8
# of transformers	=	2
# of loads	=	9
# of unbalanced loads	=	2

Table 1-A: Comparison results of I_f for a single fault at bus 18 of the 20-bus system

Type of fault		I_f (A.)			
		SCM	CM	ICBM	TE
3	A	6531.056	6524.113	6541.24	6540.777
L	B	6531.056	6532.908	6542.629	6541.703
G	C	6531.056	6535.222	6540.777	6539.851
L	A	5655.712	5649.695	5664.97	5664.044
L	B	5655.712	5649.695	5657.564	5657.101
	C	0	0	0	0
D	A	6300.532	6291.737	6307.475	6307.013
L	B	6087.135	6089.45	6099.17	6098.708
G	C	0	0	0	0
S	A	5700.151	5692.744	5706.631	5706.631
L	B	0	0	0	0
G	C	0	0	0	0

Table 1-B: Comparison results of V_f for a single fault at bus 18 of the 20-bus system

Type of fault		V_f (p.u.) SCM	CM	ICBM	TE
3	A	1.9921	0.005	0.0003	0.0003
L	B	1.9960	0.0019	0.0003	0.0001
G	C	1.9957	0.0014	0.0002	0.0003
L	A	1.3164	0.4964	0.5003	0.5003
L	B	0.4976	0.5013	0.4998	0.4998
	C	0.9989	0.9988	0.9986	0.9986
D	A	0.9943	0.0048	0.0003	0.0003
L	B	0.9961	0.0017	0.0003	0.0003
G	C	1.0748	1.1122	1.1122	1.1122
S	A	1.9923	0.0046	0	0
L	B	0.9629	1.0491	1.0497	1.0497
G	C	0.9204	1.0867	1.0865	1.0865

Table 1-C: Comparison results of Flops for a single fault at bus 18 of the 20-bus system

Type of fault	flops SCM	CM	ICBM	TE
3LG	9544	9331	11774	20958
LL	9618	8200	10432	19256
DLG	9660	8186	10418	19242
SLG	9590	7741	9973	18797

Where:

- **SCM** : Symmetrical Component Method
- **CM** : Compensation Method
- **ICBM** : Compensation Method using Initial Condition Boundary Matching
- **TE** : Compensation Method using Thevenin Equivalent
- **LL** : A phase to B phase
- **DLG** : A phase, B phase to ground
- **SLG** : A phase to ground
- **3LG** : A,B,C phases to ground
- **flop** : Floating point operation count

From the results shown in the Table 1-A and B, the results of ICBM and TE are more accurate than the other two methods. For example, for the single-line to ground (SLG) fault, the voltage of the faulted phase is expected to be zero. Both proposed methods yield zero on the faulted buses while the results of the first two methods are not zero. From Table 1-C, we can see that ICBM and TE need more calculation time than SCM and CM. And ICBM is much faster than TE which required almost twice the number of flops of ICBM.

4.2 394-bus System

Detailed information about the number of the components follows:

# of lines without breakers	=	343
# of lines with breakers	=	69
# of transformers	=	8
# of loads	=	199
# of unbalanced loads	=	187

The results of the short circuit calculation of different types of fault on the bus 1175 using the four methods are stated in the Table 2. There are 17 buses between the fault bus and the source.

Table 2-A: Comparison results of I_f for a single fault at bus 1179 of the 394-bus system

Type of fault		I_f (A) SCM	CM	ICBM	TE
3	A	3578.217	3598.122	3768.469	3727.734
L	B	3578.217	3606.454	3790.225	3746.25
G	C	3578.217	3825.406	4001.771	3962.887
L	A	3098.653	3109.299	3257.89	3221.784
L	B	3098.653	3109.299	3267.611	3232.894
	C	0	0	0	0
D	A	3464.807	3460.178	3592.104	3561.09
L	B	3332.88	3312.05	3497.672	3500.913
G	C	0	0	0	0
S	A	3139.851	3084.303	3202.342	3246.781
L	B	0	0	0	0
G	C	0	0	0	0

Table 2-B: Comparison results of V_f for a single fault at bus 1179 of the 394-bus system

Type of fault		V_f (p.u.) SCM	CM	ICBM	TE
3	A	1.8625	0.0722	0.0153	0.0111
L	B	1.8552	0.0731	0.0174	0.0135
G	C	1.8515	0.0918	0.0794	0.0691
L	A	1.1918	0.4190	0.4804	0.4819
L	B	0.4175	0.5297	0.4647	0.4623
	C	0.9432	0.9436	0.9437	0.9438
D	A	0.8766	0.0578	0.0305	0.0155
L	B	0.8691	0.0935	0.0169	0.0083
G	C	1.0144	1.0375	1.0418	1.045
S	A	1.8610	0.0787	0.0186	0.0056
L	B	0.9233	0.9813	0.9899	0.9900
G	C	0.8688	1.0240	1.0219	1.0235

Table 2-C: Comparison results of Flops for a single fault at bus 1179 of the 394-bus system

Type of fault	Flops SCM	CM	ICBM	TE
3LG	20570	36772	50554	161765
LL	20644	36119	49001	160212
DLG	20686	36105	48987	160198
SLG	20616	35660	48542	159753

For the large-scale system, the results of ICBM and TE are slightly different. From the results of the fault voltages in Table 2-B, it can be seen that ICBM and TE are still more accurate than SCM and CM. Also, TE results are more accurate than ICBM. For the SLG fault, the results in the Table 2-B show that the phase *a* fault voltage of TE is smaller than that of ICBM. This is expected because the TE method models all laterals and loads in the network. However the improved results come with a tradeoff in speed, as the TE method required three times more flops than ICBM.

4.3 *Remarks*

- From the results of the two-test system it can be seen that both proposed methods are more accurate than SCM and CM. And TE is more accurate than ICBM.
- From Table 1-C and Table 2-C we can see that ICBM is significant faster than TE. Therefore if speed and improved accuracy over SCM and CM are more important, the ICBM is recommended. If accuracy is more important, the TE is recommended.
- The level of imbalance influences the accuracy of ICBM and TE. The 20-bus system is more balanced than 394-bus system. Thus ICBM and TE yielded better results in the small system than the large system.

5. CONCLUSION

In this paper, two new methods the Initial Condition Boundary Matching Method (ICBM) and the Thevenin Equivalent Method (TE) for short circuit analysis of radial distribution systems are presented. These two methods used two different ways to handle the laterals and loads in distribution systems. The approaches were tested on two unbalanced radial distribution systems. The comparison results between the two new methods and two traditional methods: the symmetrical component method and the compensation method were obtained. Both proposed methods yielded more accurate results than the other two. TE got more accurate results than ICBM while it needs more time to calculate. But ICBM can get more accurate results without increasing calculating time too much.

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