Fast Short Circuit Analysis Method for Unbalanced Distribution Systems

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Abstract: **A fast short circuit analysis method for unbalanced distribution systems is proposed in the paper. Two matrices developed based on the topological structures of distribution systems are used to analyze the variations of bus voltages, bus current injections and branch currents under fault conditions. The proposed short circuit analysis method with exact three-phase models can then be developed from these two matrices and be exploited to solve the various types of single or simultaneous unsymmetrical faults. Since the proposed method does not use the traditional admittance matrix, the time-consuming procedures including matrix tri-factorization and/or matrix inverse are not necessary; therefore, the proposed method can achieve the advantages of high speed, robust convergence and accuracy. Test results show that the proposed method has great potential to be integrated into the existent distribution automation.**

Keywords: **Short Circuit Analysis, Network Topology, Unbalanced Distribution Systems, Unsymmetrical Fault, Symmetrical Component.**

1. INTRODUCTION

In order to efficiently construct Distribution Automation (DA), many real-time applications, such as network optimization, VAR. planning, feeder reconfiguration, state estimation, short circuit analysis and so forth, are necessary. Among those applications, a robust, efficient and accurate short circuit analysis program is very important for real-time study of the protective needs of DA. The results of such short circuit studies can be used for both off-line planning and real-time applications. For example, the results can be used for real-time distribution relay coordination and settings when feeder reconfigurations are performed automatically after fault identification and isolation [1-10].

The symmetrical component based methods for short circuit analysis have been used for years in power industries [1-6]. These methods extend the per-phase algorithm to analyze the power systems with unbalanced faults. The main computational advantage of the conventional symmetrical component based methods is that a three-phase network can be treated as three separate sequence networks; therefore, the difficulties in the integration of exact three-phase models into short circuit analysis can be avoided. For that reason, fault analysis based on those methods has been used for several decades. Although these methods work well for balanced electric power systems, their value is limited when the system becomes unbalanced.

In practical applications, the distribution system configurations are basically unbalanced; therefore, short circuit analysis methods based on the actual three-phase models are desirable. Several short circuit analysis methods based on the phase coordinates have been proposed. Kersting and Phillips [7] proposed the method based on a three-phase impedance matrix to incorporate different types of faults into a personal computer program. The solution method of [8] was an iterative compensation method, which used the tri-factorization of bus admittance matrix to simulate the fault conditions. The backward /forward sweep based short circuit analysis algorithm for distribution systems was proposed in [9-10]. This algorithm used a multi-port hybrid compensation method. It can also be used for distribution relay coordination and setting.

This paper proposes a short-circuit analysis method based on the actual three-phase models for unbalanced distribution systems. Two matrices, the bus-injection to branch-current matrix (***BIBC***) and the branch-current to bus-voltage matrix (***BCBV***), and their building algorithms are developed. These two matrices can be used to analyze the relationship between bus voltages, bus current injections and branch currents. These two matrices can also be used to solve the pre-fault system status including bus voltages, line flows and bus injections and to analyze the variations of bus voltages and branch currents under fault conditions. The proposed short circuit analysis method is then developed from these two matrices and used to solve the various types of single or simultaneous unsymmetrical faults. Test results demonstrate the performance of the proposed method.

2. BASIC FORMUALTIONS

The proposed method is developed based on the ***BIBC*** and ***BCBV*** matrices. The detailed derivation of these two matrices can be found in [11]. In this paper, only the building algorithms of these two matrices are shown. The relationship of bus current injections and branch currents can be expressed as

$$[\mathbf{B}] = [\mathbf{BIBC}]\,[\mathbf{I}] \qquad (1)$$

where $\boldsymbol{B}$ and $\boldsymbol{I}$ are the vectors of branch currents and bus current injections, respectively.

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The constant ***BIBC*** matrix is an upper triangular matrix and has non-zero entries of +1 only. The building algorithm for ***BIBC*** matrix can be developed as follows:

Procedure 1) - For a distribution system with m branch sections and n buses, the dimension of ***BIBC*** matrix is mx(n-1).
Procedure 2) - If a line section (B_k) is located between bus i and bus j, copy the column of the i-th bus of ***BIBC*** matrix to the column of the j-th bus and fill a +1 to the position of the k-th row and the j-th bus column.
Procedure 3) - Repeat Procedure 2) until all line sections are included in the ***BIBC*** matrix.

The relationship between branch currents and bus voltages can be written as

$$[\mathbf{V}_0]-[\mathbf{V}]=[\mathbf{BCBV}][\mathbf{B}] \tag{2}$$

where $\mathbf{V}$ and $\mathbf{V}_0$ are the vectors of bus voltages and no-load bus voltages, respectively.

Equation (2) can also be rewritten as

$$[\Delta\mathbf{V}]=[\mathbf{BCBV}][\mathbf{B}] \tag{3}$$

where $[\Delta\mathbf{V}]=[\mathbf{V}_0]-[\mathbf{V}]$.

The constant ***BCBV*** matrix has non-zero entries consisted by line impedance values. The building algorithm for ***BCBV*** matrix can be developed as follows:

Procedure 4) - For a distribution system with m branch sections and n buses, the dimension of ***BCBV*** matrix is (n-1)xm.
Procedure 5) - If a line section (B_k) is located between bus i and bus j, copy the row of the i-th bus of ***BCBV*** matrix to the row of the j-th bus and fill the line impedance (Z_{ij}) to the position of the j-th bus row and the k-th column.
Procedure 6) - Repeat Procedure 5) until all line sections are included in the ***BCBV*** matrix.

Note that for radial distribution networks we have m=n-1. In additional, for multiple line sections and buses, the algorithm can easily be expanded. Equation (1) shows that the ***BIBC*** matrix represents the relationship between bus current injections and branch currents. The corresponding variations of branch currents, generated by the current injections of fault conditions, can be calculated directly by using the ***BIBC*** matrix. Equation (3) shows that the ***BCBV*** matrix represents the relationship between branch currents and bus voltages. The corresponding variations of bus voltages, generated by the branch currents of fault conditions, can be calculated directly by using the ***BCBV*** matrix. These two matrices are very useful for the derivation of the proposed short circuit analysis method.

3. SOLUTION TECHNIQUES

Most faults that occur on distribution systems are unsymmetrical faults. Unsymmetrical faults include single line-to-ground fault, double line-to-ground fault, line-to-line fault and three line-to-ground fault etc. For unsymmetrical faults, appropriate fault boundary conditions can be obtained and then (1) and (3) can be used to calculate their effects on branch currents and bus voltages.

3.1 Single Line-to-Ground Fault

When a single line-to-ground fault occurs on phase a of bus i, the boundary conditions can be written as

$$I_i^a=I_{i,f}^a,\ \ I_i^b=0,\ \ I_i^c=0 \tag{4a}$$

$$\Delta V_{i,f}^a=V_{i,0}^a \tag{4b}$$

where $I_{i,f}^a$, $I_{i,f}^b$ and $I_{i,f}^c$ are the currents to ground of phase a, b and c at the fault positions, repsectively. I_i^a, I_i^b and I_i^c are the bus injection currents of phase a, b and c, repsectively. $V_{i,0}^a$ and $\Delta V_{i,f}^a$ is the prefault voltage and voltage variation caused by the fault, respectively.

Equation (4) means that if a single line-to-ground fault occurred on phase a of bus i, the current of $I_{i,f}^a$ will flow into ground and make the voltage of bus i have a variation of $V_{i,0}^a$.

Substituting (4a) into (1), the variations of the branch currents generated by the fault current can be expressed as

$$[\mathbf{B}_f]=[\mathbf{BIBC}]\begin{bmatrix}0 & \cdots & I_{i,f}^a & 0 & \cdots & \cdots\end{bmatrix}^T \tag{5}$$

Equation (5) can be rewritten as

$$[\mathbf{B}_f]=[\mathbf{BIBC}_i^a]I_{i,f}^a \tag{6}$$

where $[\mathbf{BIBC}_i^a]$ is the column vector of $[\mathbf{BIBC}]$ matrix corresponding to phase a of bus i.

The variations of bus voltages caused by the fault branch currents can be obtained by substituting (6) into (3) and expressed as

$$[\Delta\mathbf{V}_f]=[\mathbf{BCBV}][\mathbf{BIBC}_i^a]I_{i,f}^a \tag{7}$$

The bus voltage at fault position can be formulated as

$$\Delta V_{i,f}^{a} = \lfloor \mathbf{BCBV}_i^a \rfloor \lfloor \mathbf{BIBC}_i^a \rfloor I_{i,f}^{a} \tag{8}$$

where $\lfloor \mathbf{BCBV}_i^a \rfloor$ is the row vector of $[\mathbf{BCBV}]$ matrix corresponding to phase a of bus i.

Note that $\Delta V_{i,f}^{a} = V_{i,0}^{a}$; therefore, the fault current can be calculated by

$$\begin{aligned} I_{i,f}^{a} &= (\lfloor \mathbf{BCBV}_i^a \rfloor \lfloor \mathbf{BIBC}_i^a \rfloor)^{-1} (V_{i,0}^{a}) \\ &= ([\mathbf{SC}^a])(V_{i,0}^{a}) \end{aligned} \tag{9}$$

where $[\mathbf{SC}^a]$ is the matrix related to the voltage and current at the fault bus. It is a 1x1 matrix for a single line-to-ground fault.

After the fault current was calculated, the variations of branch currents and bus voltages caused by the single line-to-ground fault can be obtained immediately by (6) and (7), respectively.

3.2 Double Line-to-Ground Fault

When a double line-to-ground fault occurs on phases a, b of bus i, the boundary conditions can be written as

$$I_i^a = I_{i,f}^a, \quad I_i^b = I_{i,f}^b, \quad I_i^c = 0 \tag{10a}$$

$$\Delta V_{i,f}^a = V_{i,0}^a, \quad \Delta V_{i,f}^b = V_{i,0}^b \tag{10b}$$

Equation (10) means that if a double line-to-ground fault occurred at bus i, the currents of $I_{i,f}^a$ and $I_{i,f}^b$ will flow into ground and make the voltages of bus i have the variations of $V_{i,0}^a$ and $V_{i,0}^b$ at phases a, b of bus i, respectively.

Substituting (10a) into (1), the variations of the branch currents generated by the fault currents can be expressed as

$$[\mathbf{B}_f] = [\mathbf{BIBC}] \begin{bmatrix} 0 & \cdots & I_{i,f}^a & I_{i,f}^b & 0 & \cdots \end{bmatrix}^T \tag{11}$$

Equation (11) can be rewritten as

$$[\mathbf{B}_f] = \begin{bmatrix} \mathbf{BIBC}_i^a & \mathbf{BIBC}_i^b \end{bmatrix} \begin{bmatrix} I_{i,f}^a \\ I_{i,f}^b \end{bmatrix} \tag{12}$$

where $\lfloor \mathbf{BIBC}_i^a \rfloor$ and $\lfloor \mathbf{BIBC}_i^b \rfloor$ are the column vectors of $[\mathbf{BIBC}]$ matrix corresponding to phase a and b of bus i, respectively.

The variations of the bus voltages caused by the fault branch currents can be obtained by substituting (12) into (3) and expressed as

$$[\Delta \mathbf{V}_f] = [\mathbf{BCBV}] \begin{bmatrix} \mathbf{BIBC}_i^a & \mathbf{BIBC}_i^b \end{bmatrix} \begin{bmatrix} I_{i,f}^a \\ I_{i,f}^b \end{bmatrix} \tag{13}$$

The bus voltages at fault positions can be formulated as

$$\begin{bmatrix} \Delta V_{i,f}^a \\ \Delta V_{i,f}^b \end{bmatrix} = \begin{bmatrix} \mathbf{BCBV}_i^a \\ \mathbf{BCBV}_i^b \end{bmatrix} \begin{bmatrix} \mathbf{BIBC}_i^a & \mathbf{BIBC}_i^b \end{bmatrix} \begin{bmatrix} I_{i,f}^a \\ I_{i,f}^b \end{bmatrix} \tag{14}$$

where $\lfloor \mathbf{BCBV}_i^a \rfloor$ and $\lfloor \mathbf{BCBV}_i^b \rfloor$ are the row vectors of $[\mathbf{BCBV}]$ matrix corresponding to phase a and b of bus i, respectively.

Note that $\Delta V_{f,0}^a = V_{i,0}^a$ and $\Delta V_{f,0}^b = V_{i,0}^b$; therefore, the fault currents can be calculated by

$$\begin{aligned} \begin{bmatrix} I_{i,f}^a \\ I_{i,f}^b \end{bmatrix} &= \left(\begin{bmatrix} \mathbf{BCBV}_i^a \\ \mathbf{BCBV}_i^b \end{bmatrix} \begin{bmatrix} \mathbf{BIBC}_i^a & \mathbf{BIBC}_i^b \end{bmatrix} \right)^{-1} \begin{bmatrix} V_{i,0}^a \\ V_{i,0}^b \end{bmatrix} \\ &= ([\mathbf{SC}^{ab}])^{-1} \begin{bmatrix} V_{i,0}^a \\ V_{i,0}^b \end{bmatrix} \end{aligned} \tag{15}$$

where $[\mathbf{SC}^{ab}]$ is the matrix related to the voltages and currents at the fault bus. It is a 2x2 matrix for a double line-to-ground fault.

After the currents to ground at the fault positions are calculated, the variations of branch currents and bus voltages caused by the double line-to-ground fault can be obtained immediately by (12) and (13), respectively.

3.3 Three Line-to-Ground Fault

When a three line-to-ground fault occurs on phases a, b and c of bus i, the boundary conditions can be written as

$$I_i^a = I_{i,f}^a, \quad I_i^b = I_{i,f}^b, \quad I_i^c = I_{i,f}^c \tag{16a}$$

$$\Delta V_{i,f}^a = V_{i,0}^a, \quad \Delta V_{i,f}^b = V_{i,0}^b, \quad \Delta V_{i,f}^c = V_{i,0}^c \tag{16b}$$

The fault currents can be calculated by

$$\begin{aligned} \begin{bmatrix} I_{i,f}^a \\ I_{i,f}^b \\ I_{i,f}^c \end{bmatrix} &= \left(\begin{bmatrix} \mathbf{BCBV}_i^a \\ \mathbf{BCBV}_i^b \\ \mathbf{BCBV}_i^c \end{bmatrix} \begin{bmatrix} \mathbf{BIBC}_i^a \\ \mathbf{BIBC}_i^b \\ \mathbf{BIBC}_i^c \end{bmatrix}^{\mathrm{T}} \right)^{-1} \begin{bmatrix} V_{i,0}^a \\ V_{i,0}^b \\ V_{i,0}^c \end{bmatrix} \\ &= ([\mathbf{SC}^{abc}])^{-1} \begin{bmatrix} V_{i,0}^a \\ V_{i,0}^b \\ V_{i,0}^c \end{bmatrix} \end{aligned} \tag{17}$$

where $[\mathbf{SC}^{abc}]$ is the matrix related to the fault bus voltages and currents at the fault positions. It is a 3x3 matrix for a three line-to-ground fault. $\left[\mathbf{BIBC}_i^a\right]$, $\left[\mathbf{BIBC}_i^b\right]$ and $\left[\mathbf{BIBC}_i^c\right]$ are the column vectors of $[\mathbf{BIBC}]$ matrix corresponding to phase *a*, *b* and *c* of bus *i*, respectively. $\left[\mathbf{BCBV}_i^a\right]$, $\left[\mathbf{BCBV}_i^b\right]$ and $\left[\mathbf{BCBV}_i^c\right]$ are the row vectors of $[\mathbf{BCBV}]$ matrix corresponding to phase *a*, *b* and *c* of bus *i*, respectively.

The variations of branch currents and bus voltages caused by the three line-to-ground fault can be obtained directly by (18) and (19), respectively.

$$\left[\mathbf{B}_f\right] = \left[\mathbf{BIBC}_i^a \quad \mathbf{BIBC}_i^b \quad \mathbf{BIBC}_i^c\right]\begin{bmatrix} I_{i,f}^a \\ I_{i,f}^b \\ I_{i,f}^c \end{bmatrix} \tag{18}$$

$$\left[\Delta\mathbf{V}_f\right] = [\mathbf{BCBV}]\left[\mathbf{B}_f\right] \tag{19}$$

The detailed derivation is similar to the process shown in Section 3.2; therefore, it is not shown here.

The proposed method can be used to deal with the simultaneous faults as necessary. The key-point is to build the $[\mathbf{SC}]$ matrix. For example, if a single line-to-ground fault on phase *a* of bus *i*, a double line-to-ground fault on phase *b* and *c* of bus *j* and a three line-to-ground fault on bus *k* occurred simultaneously; the following matrices can be obtained

$$[\mathbf{SC}] = \begin{bmatrix} \mathbf{BCBV}_i^a \\ \mathbf{BCBV}_j^b \\ \mathbf{BCBV}_j^c \\ \mathbf{BCBV}_k^a \\ \mathbf{BCBV}_k^b \\ \mathbf{BCBV}_k^c \end{bmatrix}\begin{bmatrix} \mathbf{BIBC}_i^a \\ \mathbf{BIBC}_j^b \\ \mathbf{BIBC}_j^c \\ \mathbf{BIBC}_k^a \\ \mathbf{BIBC}_k^b \\ \mathbf{BIBC}_k^c \end{bmatrix}^{\mathrm{T}} \tag{20a}$$

$$[\mathbf{V}_{SC}] = \left[V_{i,0}^a \quad V_{j,0}^b \quad V_{j,0}^c \quad V_{k,0}^a \quad V_{k,0}^b \quad V_{k,0}^c\right]^T \tag{20b}$$

$$[\mathbf{I}_{SC}] = \left[I_{i,f}^a \quad I_{j,f}^b \quad I_{j,f}^c \quad I_{k,f}^a \quad I_{k,f}^b \quad I_{k,f}^c\right]^T \tag{20c}$$

where the $[\mathbf{SC}]$, $[\mathbf{V}_{SC}]$ and $[\mathbf{I}_{SC}]$ matrices are 6x6 matrix, 6x1 vector and 6x1 vector, respectively.

The fault currents can be calculated by

$$[\mathbf{I}_{SC}] = ([\mathbf{SC}])^{-1}\,[\mathbf{V}_{SC}] \tag{21}$$

After the fault currents obtained, (1) and (3) can be used to calculate the branch currents and bus voltage caused by the simultaneous faults, respectively.

4. TEST RESULTS AND DISCUSSIONS

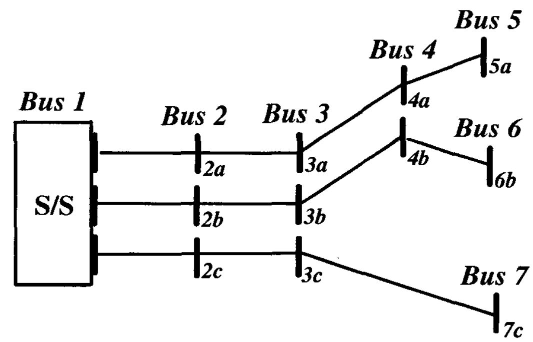

Fig. 1: A Simple 7-bus Distribution System

The proposed short-circuit analysis method was implemented by Borland C++ language and tested on a Windows-98 based Pentium PC. Many distribution systems have been tested by the proposed method; however, only the results of a simple 7-bus system (equivalent 14-bus system) are shown here. Fig. 1 shows the topology of the test system, it consists of three-phase, double-phase and single-phase line sections and buses. The following cases are used to test the proposed short-circuit analysis program:

Case 1: A single line-to-ground fault occurred on phase *b* of bus 6;
Case 2: A double line-to-ground fault occurred on phase *b* and *c* of bus 4.

The unbalanced three-phase line models and parameters can be calculated by the formulas presented in [12]. The nonzero terms of the $[\mathbf{BIBC}]$ and $[\mathbf{BCBV}]$ matrices are expressed in (22) and (23), respectively (shown in the next page). The substation bus (swing bus) will not be in those matrices since bus voltages of substation are scheduled as constant. For Case 1, the $[\mathbf{SC}]$, $[\mathbf{V}_{SC}]$ and $[\mathbf{I}_{SC}]$ matrices can be expressed as

$$[\mathbf{V}_{SC}] = \left[V_{6,0}^b\right], \quad [\mathbf{I}_{SC}] = \left[I_{6,f}^b\right]$$
$$\left[\mathbf{SC}^c\right] = [\mathbf{BCBV}_6^b][\mathbf{BIBC}_6^b] = \left[Z_{12}^{bb} + Z_{23}^{bb} + Z_{34}^{bb} + Z_{46}^{bb}\right]$$

After the fault current was calculated, the variations of the branch currents and bus voltages caused by the fault can be obtained. The voltage profiles before and after the fault of Case 1 are shown in Fig. 2 and 3, respectively.

For Case 2, the $[\mathbf{SC}]$, $[\mathbf{V}_{SC}]$ and $[\mathbf{I}_{SC}]$ matrices can be expressed as

$$[\mathbf{V}_{SC}]=\begin{bmatrix} V_{4,0}^{a} \\ V_{4,0}^{b} \end{bmatrix}, \quad [\mathbf{I}_{SC}]=\begin{bmatrix} I_{4,f}^{a} \\ I_{4,f}^{b} \end{bmatrix}$$

$$[\mathbf{SC}]=\begin{bmatrix} \mathbf{BCBV}_4^a \\ \mathbf{BCBV}_4^b \end{bmatrix}\begin{bmatrix} \mathbf{BIBC}_4^a \\ \mathbf{BIBC}_4^b \end{bmatrix}^T$$

$$=\begin{bmatrix} Z_{12}^{aa}+Z_{23}^{aa}+Z_{34}^{aa} & Z_{12}^{ab}+Z_{23}^{ab}+Z_{34}^{ab} \\ Z_{12}^{ba}+Z_{23}^{ba}+Z_{34}^{ba} & Z_{12}^{bb}+Z_{23}^{bb}+Z_{34}^{bb} \end{bmatrix}$$

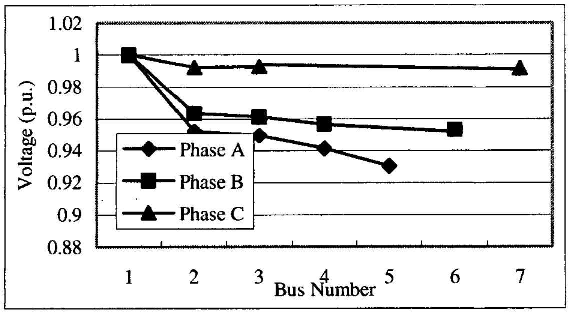

Fig. 2: The Voltage Profiles of Case 1 Before Fault

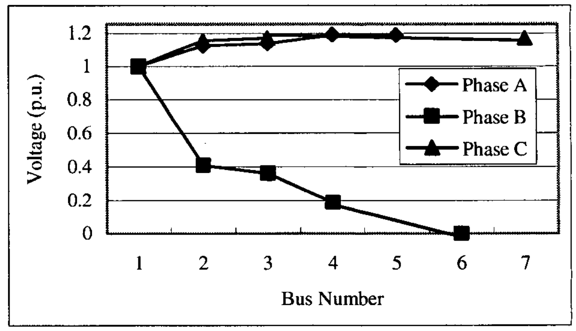

Fig. 3: The Voltage Profiles of Case 1 After Fault

The voltage profiles of Case 2 are not shown here due to limited space. The proposed method can be used to solve short circuit problems of large-scale distribution systems efficiently. For example, a distribution system with 294 buses, the computional time needed to calculate the pre-fault load flow solution, simulate three simultaneous faults and obtain the variations of branch currents and bus voltages caused by the faults is only 3.015 s.

$$\begin{array}{c|ccccccccccc|c}
\text{Bus} & 2a & 2b & 2c & 3a & 3b & 3c & 4a & 4b & 5a & 6b & 7c & \text{Branch} \\
\hline
 & 1 & & & 1 & & & 1 & & 1 & & & 1\text{-}2a \\
 & & 1 & & & 1 & & & 1 & & 1 & & 1\text{-}2b \\
 & & & 1 & & & 1 & & & & & 1 & 1\text{-}2c \\
 & & & & 1 & & & 1 & & 1 & & & 2\text{-}3a \\
[\mathbf{BIBC}]= & & & & & 1 & & & 1 & & 1 & & 2\text{-}3b \\
 & & & & & & 1 & & & & & 1 & 2\text{-}3c \\
 & & & & & & & 1 & & 1 & & & 3\text{-}4a \\
 & & & & & & & & 1 & & 1 & & 3\text{-}4b \\
 & & & & & & & & & 1 & & & 4\text{-}5a \\
 & & & & & & & & & & 1 & & 4\text{-}6b \\
 & & & & & & & & & & & 1 & 3\text{-}7c \\
\hline
\end{array} \quad (22)$$

$$\begin{array}{c|ccccccccccc|c}
\text{Branch} & 1\text{-}2a & 1\text{-}2b & 1\text{-}2c & 2\text{-}3a & 2\text{-}3b & 2\text{-}3c & 3\text{-}4a & 3\text{-}4b & 4\text{-}5a & 4\text{-}6b & 3\text{-}7c & \text{Bus} \\
\hline
 & Z_{12}^{aa} & Z_{12}^{ab} & Z_{12}^{ac} & & & & & & & & & 2a \\
 & Z_{12}^{ba} & Z_{12}^{bb} & Z_{12}^{bc} & & & & & & & & & 2b \\
 & Z_{12}^{ca} & Z_{12}^{cb} & Z_{12}^{cc} & & & & & & & & & 2c \\
 & Z_{12}^{aa} & Z_{12}^{ab} & Z_{12}^{ac} & Z_{23}^{aa} & Z_{23}^{ab} & Z_{23}^{ac} & & & & & & 3a \\
[\mathbf{BCBV}]= & Z_{12}^{ba} & Z_{12}^{bb} & Z_{12}^{bc} & Z_{23}^{ba} & Z_{23}^{bb} & Z_{23}^{bc} & & & & & & 3b \\
 & Z_{12}^{ca} & Z_{12}^{cb} & Z_{12}^{cc} & Z_{23}^{ca} & Z_{23}^{cb} & Z_{23}^{cc} & & & & & & 3c \\
 & Z_{12}^{aa} & Z_{12}^{ab} & Z_{12}^{ac} & Z_{23}^{aa} & Z_{23}^{ab} & Z_{23}^{ac} & Z_{34}^{aa} & Z_{34}^{ab} & & & & 4a \\
 & Z_{12}^{ba} & Z_{12}^{bb} & Z_{12}^{bc} & Z_{23}^{ba} & Z_{23}^{bb} & Z_{23}^{bc} & Z_{34}^{ba} & Z_{34}^{bb} & & & & 4b \\
 & Z_{12}^{aa} & Z_{12}^{ab} & Z_{12}^{ac} & Z_{23}^{aa} & Z_{23}^{ab} & Z_{23}^{ac} & Z_{34}^{aa} & Z_{34}^{ab} & Z_{45}^{aa} & & & 5a \\
 & Z_{12}^{ba} & Z_{12}^{bb} & Z_{12}^{bc} & Z_{23}^{ba} & Z_{23}^{bb} & Z_{23}^{bc} & Z_{34}^{ba} & Z_{34}^{bb} & & Z_{46}^{bb} & & 6b \\
 & Z_{12}^{ca} & Z_{12}^{cb} & Z_{12}^{cc} & Z_{23}^{ca} & Z_{23}^{cb} & Z_{23}^{cc} & & & & & Z_{37}^{cc} & 7c \\
\hline
\end{array} \quad (23)$$

From the above test results, it can be seen that the proposed algorithm can be used to handle unsymmetrical faults efficiently. For a double line-to-ground fault, the traditional method needs to build the sequence networks and then construct the impedance matrix of each sequence network by impedance building algorithm. Those are all time-consuming procedures. However, the proposed algorithm only needs to build the ***BIBC*** and ***BCBV*** matrices and do the inverse of a 2x2 [**SC**] matrix, then the fault currents can be obtained easily. It can save considerable computational time. In additonal, the variations of branch currents and bus voltages caused by faults can also be calculated efficiently. Those information obtained are very useful for real-time distribution adaptive relay coordination and setting when feeder reconfiguration is performed automatically after fault identification and isolation.

5. CONCLUSIONS

A short-circuit analysis method for unbalanced distribution systems was proposed in the paper. Two matrices are developed to analyze the variations of bus voltages, bus current injections and branch currents under fault conditions. The proposed short circuit method can then be developed from these two matrices and be used to solve the various types of single or simultaneous unsymmetrical faults. Since the proposed method employs the actual three-phase models for short circuit analysis and does not require building the traditional admittance matrix, the proposed method can achieve the advantages of high speed, robust convergence and accuracy. The integration of other fault types such as line-to-line fault and unsymmetrical faults through impedance, and distributed generators into the proposed method will be investigated in the future works.

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