Calculation of Electric Field and Potential Distributions Into Soil and Air Media for a Ground Electrode of a HVDC System

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***Abstract*—The present article describes a methodology that allows suitable calculations of the electric field as well as the potentials and current densities due to current in the ground electrode of a HVDC system in any point of nonhomogeneous soils and air media, taking into account the complexities of the problems involved. Results of these variables are shown for the case of a toroidal ground electrode of a HVDC system installed in homogenous and nonhomogenous soils.**

***Index Terms*—Electric field, ground electrode, heterogeneous soil.**

I. Introduction

A higher or lower complexity in the calculation of electric fields or potentials, as a result of current injection into the soil through ground electrodes of a HVDC system (in a monopolar or homopolar modes), can be related to the following aspects:

a) soil heterogeneous concerning the spatial distribution of the electrical resistivity;
b) occasional modifications of the electrical resistivity as a consequence of natural causes or by the effect of soil heating due to the injected current and the associated modification of humidity distribution involving the electroosmosis phenomenon.

The first aspect involves higher depths and distances from the ground electrodes (approximately 10 km). The latter, in general, affects only the ground electrode vicinities (until 10° m) [1]. Additionally, the safety criteria concerning human beings must be satisfied for all unfavorable operating conditions. These particularly include conditions leading to more pessimistic situations related to the electrical resistivity, in the ground electrode vicinities.

II. Soil Medium

The soil represents a fundamental role concerning the ground electrode behavior, since a current circulates in it continuously. As the soil does not behave as a well-defined material, it is necessary to consider a high number of aspects and parameters, with more or less relative importance, according to the case, and to analyze their characteristics, not only for higher distances from the ground electrodes, but also for significant depths. Fig. 1 displays a composition of gross layering of the Earth and the order of magnitude of the related electrical resistivities [2]. The current injected into the soil circulates between the ground electrodes through the earth crust and the mantle, being predominant the earth crust concerning the characteristics and effects that rule the behavior and the design of ground electrodes.

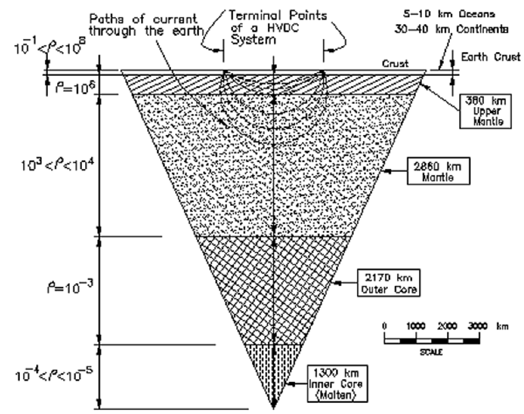

Fig. 1. Generic composition of earth layers and related typical values of electrical resistivity ρ (in $\Omega \cdot$ m).

III. Methodology Proposed to Electric Field and Soil Potential Calculations

A. Homogeneous Soils

For the hypothesis of homogeneous soil configuration (uniform electrical resistivity) and simple electrode geometries, the electric field and potential calculations can be carried using analytical methods, as indicated in [1]. Fig. 2 shows a ground electrode with toroidal geometry, represented by a section conductor of radius r, in a toroid of radius R, in a depth h of a homogenous soil having electrical resistivity ρ, being X the horizontal distance to the center of the electrode and x, the distance referred to the center of a circular section of the conductor, in the ground electrode boundary.

Under these conditions, the "exact" solution of the electric field and potential can be expressed in terms of elliptic integrals. If $r \ll h$ and $h \ll R$, two regions of interest can be assumed, where simple analytical solutions of potential distribution and electric field can be obtained, as indicated in "(1)"–"(4)".

Manuscript received June 18, 2002.

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Digital Object Identifier 10.1109/TPWRD.2003.809741

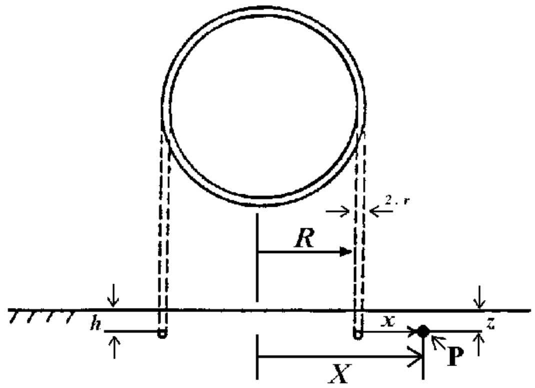

Fig. 2. Schematic representation of a toroidal ground electrode.

1) *Region 1—$|x| \ll R$ and $z \ll R$*

a) Potential in a generic point

$$U \cong \frac{\rho \cdot I}{4 \cdot \pi^2 \cdot R} \cdot \ln \frac{128 \cdot R^2 \cdot h \cdot r}{(h^2 + z^2 + x^2) - (2 \cdot h \cdot z)^2}. \tag{1}$$

b) Electric field $(z = 0)$

$$E \cong \frac{\rho \cdot I}{\pi^2 \cdot R} \cdot \frac{x}{h^2 + x^2}. \tag{2}$$

c) Maximum electric field $(x = \pm h, z = 0)$

$$E \max \cong \frac{\rho \cdot I}{2 \cdot \pi^2 \cdot R \cdot h}. \tag{3}$$

2) *Region 2—$X \gg R$*

a) Potential in a generic point

$$U \cong \frac{\rho \cdot I}{2 \cdot \pi \cdot X}. \tag{4}$$

b) Electric field

$$E \cong \frac{\rho \cdot I}{2 \cdot \pi \cdot X^2}. \tag{5}$$

Figs. 3 and 4 show, respectively, the potential U and electric field E results on the soil surface $(z = 0)$, for a toroidal ground electrode with the following data $\rho = 100\ \Omega \cdot$ m, $I = 1000$ A, $R = 200$ m, $r = 0.1$ m, and $h = 2$ m.

B. Heterogeneous Soils

In case of nonhomogeneous soils, if the soil can be assumed homogeneous within regions separated by a few simple shape surfaces (e.g., horizontal, vertical, or inclined planes, spheres, cuboids), it is also possible to obtain analytic solutions, considering "images" or "transmission-reflection" routine procedures, in eventual series form. It is also possible to consider heterogeneity of soil in vicinity of electrode, by example due to local heating and humidity changes, by means of an additive correction procedure. For instance, let us consider a soil that can be assumed homogeneous within each of two "layers" separated by a horizontal plane. Analytic solutions can be expressed in simple series form, corresponding to successive "transmission-reflection" factors, in soil and in the horizontal plane that separates the two layers. The resulting formula for different positions of

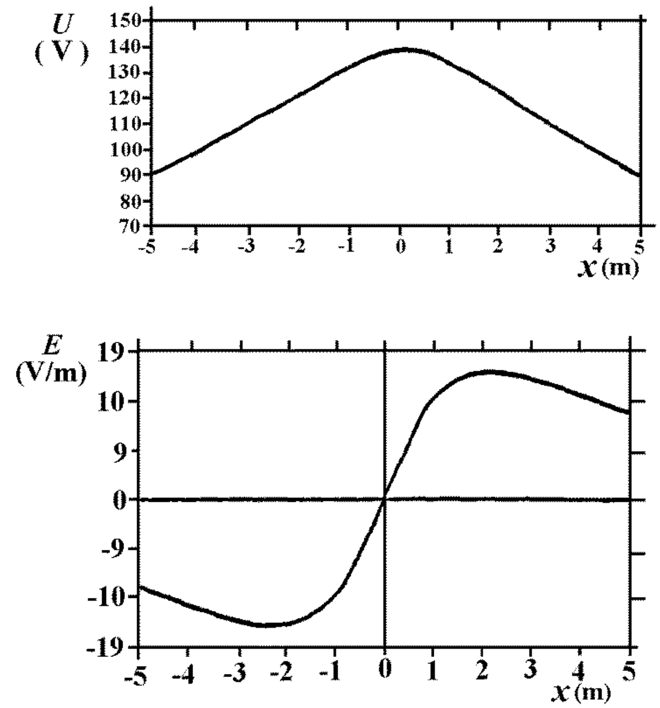

Fig. 3. Potential U related to a remote earth and electric field E in direction x on soil $(z = 0)$, in the earth electrode vicinity.

a point current source for a ground electrode located in a upper or the lower layer of a two layer soil is presented herein. By integration procedures, line or surface current sources formulation are easily obtained from point current source formula. If the soil parameters have no simple distribution form, different methods can be justified [e.g., solutions expressed in series of particular solutions of electric field in cylindrical, spherical, or ellipsoidal coordinates, and/or routine procedures of numerical solution, within the "global" or "integral" concept (such as in charge simulation method and similar procedures)]; "differential"-type methods, such finite differences and finite-elements methods.

1) Ground Electrode (Current Source) Located in the Upper Layer of a Two-Layer Ground: Assume a ground electrode (source current i) located in point Q, with coordinates (x_0, y_0, h), situated in an upper layer of a two layer soil (electrical resistivities $\rho_1 e \rho_2$ and upper layer depth H—see Fig. 5). The resulting electric field, as an immediate consequence of a perfect reflection condition reflection coefficient $(k_r = 1)$ in the soil, is equivalent, in media 1 and 2, to two punctual current sources Q and Q', under the conditions described.

Taking into account successive reflections in the P_2 and P_2' planes of "waves" from Q and Q', the potential U and the electric field E and current density J in a generic point located in medium (1), are identical to those corresponding to a succession of punctual current sources, symmetrical in relation to the soil, in the vertical of Q, for the following distances from the soil (in algebraic means, countered positively from the soil, in downward direction): $\pm h$; $\pm(2 \cdot H - h)$; $\pm(2 \cdot H + h)$; $\ldots$; $\pm(2 \cdot n \cdot H - h)$; $\pm(2 \cdot n \cdot H + h)$, in a uniform medium of resistivity ρ_1 (see Fig. 6), and the currents, in those sources, of

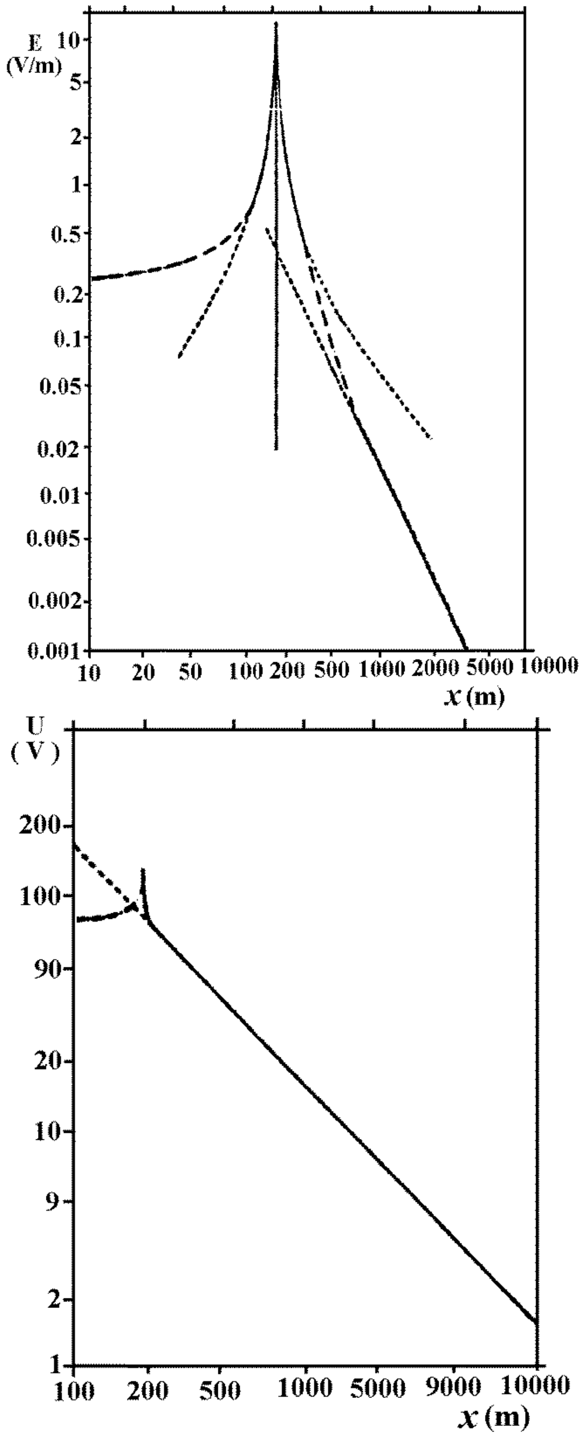

Fig. 4. Electric field E in direction x on the soil surface ($z = 0$) and potential U related to a remote earth. —— approximated solution for $x \ll re$, $X \gg R$, —— approximated solution for $x \ll re$, $X \gg R$ out of validity domain. —.— correct values.

the form $i \cdot k_r^n$. The electric field in medium (2) is equivalent to one created in an uniform soil of electrical resistivity ρ_2, by a succession of punctual current sources, in a vertical of Q, for the following soil distances: $\pm h$; $-(2 \cdot H - h)$; $-(2 \cdot H + h)$; $\ldots$; $-(2 \cdot n \cdot H - h)$; $-(2 \cdot n \cdot H + h)$; and the currents, in these sources, of the form $i \cdot (1 - k_r) \cdot k_r^n$ (see Fig. 7).

The electric field above the soil surface is identical to one in a homogeneous medium of electrical resistivity ρ_1 associated

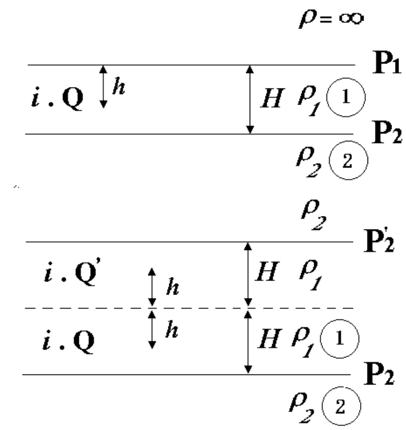

Fig. 5. Current source located in the upper layer of a two layer soil.

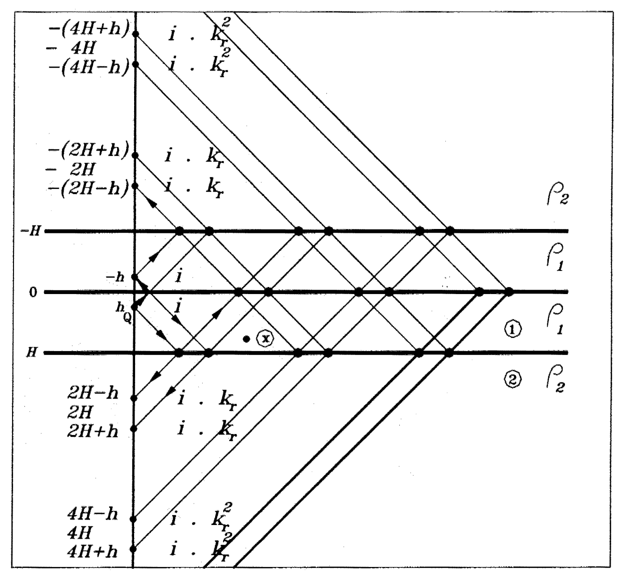

Fig. 6. Point ⊗ in an upper layer—ground electrode in the upper layer.

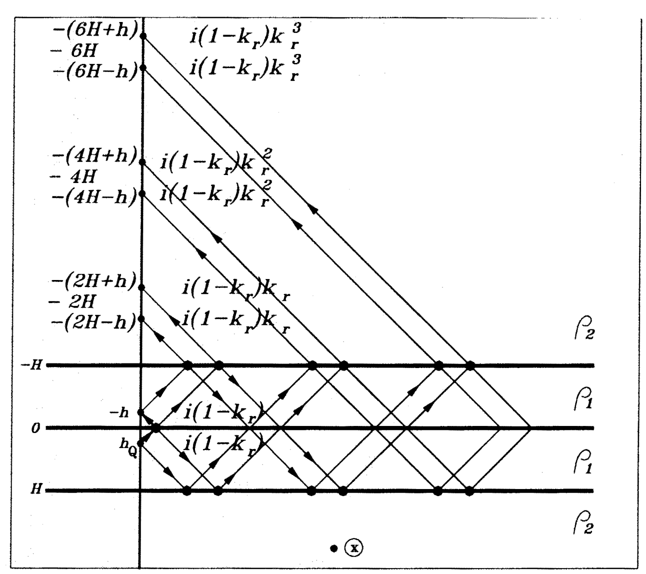

Fig. 7. Point ⊗ in a lower layer—ground electrode in the upper layer.

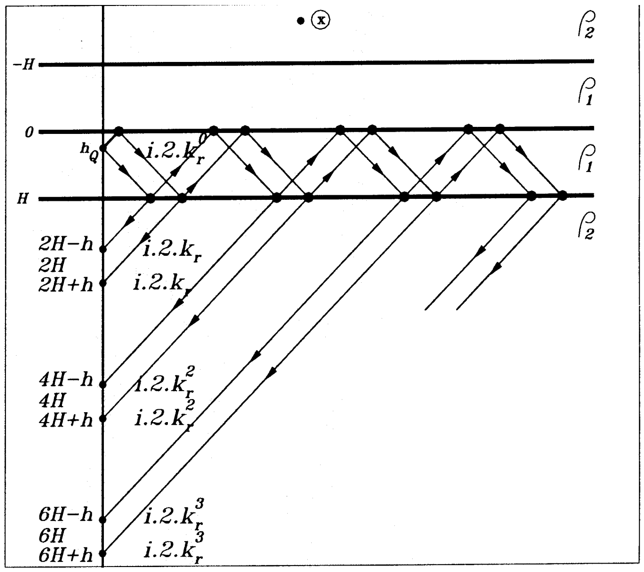

Fig. 8. Point $\otimes$ above soil surface—ground electrode in the upper layer.

with a succession of punctual current sources, in the vertical of Q, for the soil distances $+h$; $+(2\cdot H-h)$; $+(2\cdot H+h)$; $\ldots$; $+(2\cdot n\cdot H-h)$; $+(2\cdot n\cdot H+h)$; and with currents in these sources of the type $i\cdot 2\cdot k_r^n$ (see Fig. 8).

The expressions for calculating the potential U for generic points located in medium (1), medium (2), and above the soil surface are indicated in "(6)"–"(8)", as well as the related electric and density fields E and J.

1) *Electric Potential U in a (x,y,z) Point Located*

a) In medium 1

$$U=\frac{\rho_1\cdot i}{4\cdot\pi}\cdot\Bigg\{\frac{1}{\sqrt{(x-x_0)^2+(y-y_0)^2+(z-h)^2}}+\frac{1}{\sqrt{(x-x_0)^2+(y-y_0)^2+(z+h)^2}}+\sum_{n=1}^{\infty}k_r^n\cdot\Bigg[\frac{1}{\sqrt{(x-x_0)^2+(y-y_0)^2+(z-2\cdot n\cdot H+h)^2}}+\frac{1}{\sqrt{(x-x_0)^2+(y-y_0)^2+(z-2\cdot n\cdot H-h)^2}}+\frac{1}{\sqrt{(x-x_0)^2+(y-y_0)^2+(z+2\cdot n\cdot H-h)^2}}+\frac{1}{\sqrt{(x-x_0)^2+(y-y_0)^2+(z+2\cdot n\cdot H+h)^2}}\Bigg]\Bigg\}. \tag{6}$$

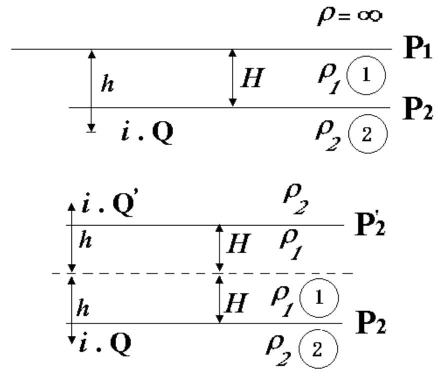

Fig. 9. Current source located in a lower layer of a two layer soil.

b) In medium 2

$$U=\frac{\rho_2\cdot i}{4\cdot\pi}\cdot(1-k_r)\cdot\Bigg\{\frac{1}{\sqrt{(x-x_0)^2+(y-y_0)^2+(z-h)^2}}+\frac{1}{\sqrt{(x-x_0)^2+(y-y_0)^2+(z+h)^2}}+\sum_{n=1}^{\infty}k_r^n\cdot\Bigg[\frac{1}{\sqrt{(x-x_0)^2+(y-y_0)^2+(z+2\cdot n\cdot H-h)^2}}+\frac{1}{\sqrt{(x-x_0)^2+(y-y_0)^2+(z+2\cdot n\cdot H+h)^2}}\Bigg]\Bigg\}. \tag{7}$$

c) Above soil surface

$$U=\frac{\rho_1\cdot i}{2\cdot\pi}\cdot\Bigg\{\frac{1}{\sqrt{(x-x_0)^2+(y-y_0)^2+(z-h)^2}}+\sum_{n=1}^{\infty}k_r^n\cdot\Bigg[\frac{1}{\sqrt{(x-x_0)^2+(y-y_0)^2+(z-2\cdot n\cdot H+h)^2}}+\frac{1}{\sqrt{(x-x_0)^2+(y-y_0)^2+(z-2\cdot n\cdot H-h)^2}}\Bigg]\Bigg\}. \tag{8}$$

2) Earth Electrode (Current Source) Located in the Lower Layer of a Two Layer Ground: In a similar way, from the conditions of Fig. 9, the series of punctual current sources as a result of successive reflections in P_2 and P_2' planes is placed, resulting in the succession of punctual current sources for different locations of the point of interest for voltage and electric field calculations. The calculation of the electric E and density current J fields, and the potential U, for generic points located in

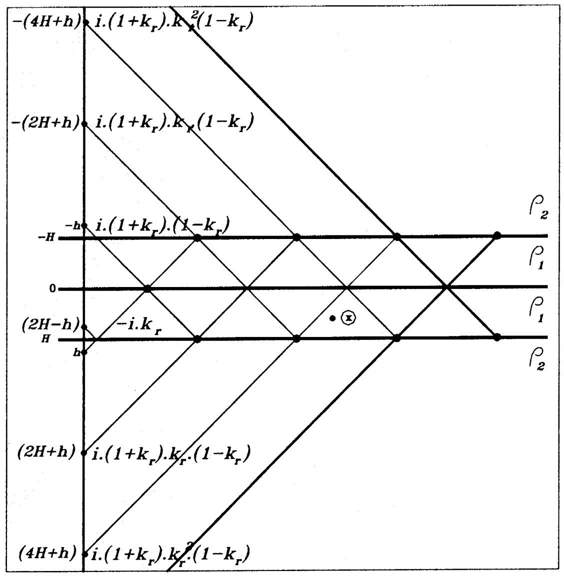

Fig. 10. Point ⊗ in an upper layer—ground electrode in the lower layer.

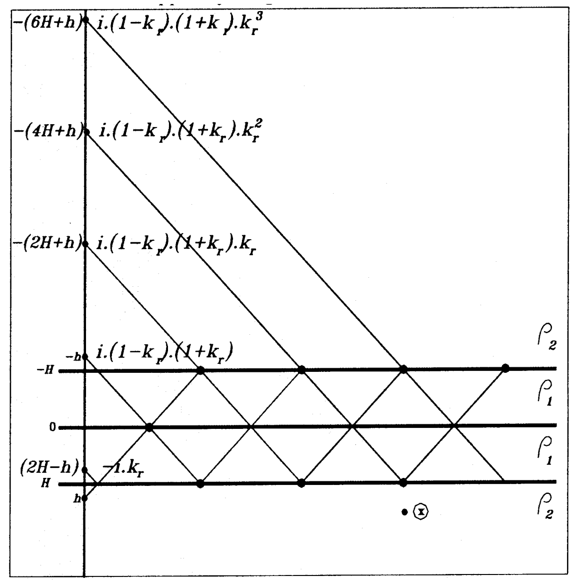

Fig. 11. Point ⊗ in a lower layer—ground electrode in the lower layer.

media (1), (2) and above soil surface (see Figs. 10–12), is done by means of "(10)"–"(12)" (in stationary conditions).

1) Electric Potential U in a (x, y, z) point located

a) In medium 1

$$U = \frac{\rho_1 \cdot i}{4 \cdot \pi} \cdot \left\{ \frac{1}{\sqrt{(x-x_0)^2 + (y-y_0)^2 + (z-h)^2}} - k_r \cdot \frac{1}{\sqrt{(x-x_0)^2 + (y-y_0)^2 + (z-2\cdot H+h)^2}} + \left(1-k_r^2\right) \cdot \sum_{n=0}^{\infty} k_r^n \cdot \left[\frac{1}{\sqrt{(x-x_0)^2 + (y-y_0)^2 + (z+2\cdot n\cdot H+h)^2}} + \frac{1}{\sqrt{(x-x_0)^2 + (y-y_0)^2 + (z-2\cdot n\cdot H-h)^2}} \right] \right\}. \tag{9}$$

b) In medium 2

$$U = \frac{\rho_2 \cdot i}{4 \cdot \pi} \cdot \left\{ \frac{1}{\sqrt{(x-x_0)^2 + (y-y_0)^2 + (z-h)^2}} - k_r \cdot \frac{1}{\sqrt{(x-x_0)^2 + (y-y_0)^2 + (z-2\cdot H+h)^2}} + \left(1-k_r^2\right) \rho \cdot \sum_{n=0}^{\infty} k_r^n \cdot \left[\frac{1}{\sqrt{(x-x_0)^2 + (y-y_0)^2 + (z+2\cdot n\cdot H+h)^2}} \right] \right\}. \tag{10}$$

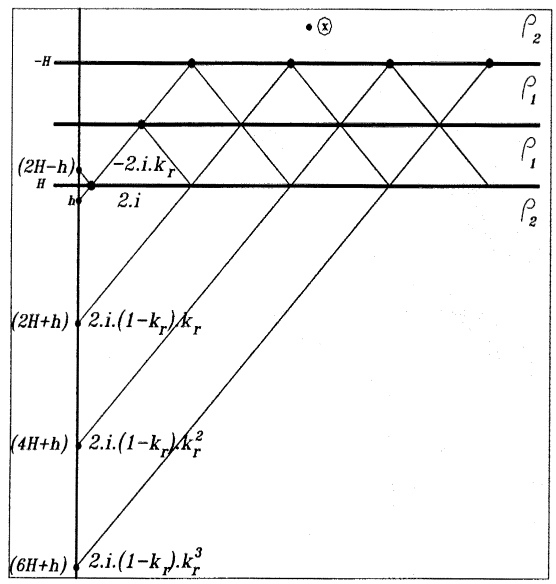

Fig. 12. Point ⊗ above soil surface—ground electrode in the lower layer.

c) Above soil surface

$$U = \frac{\rho_1 \cdot i}{2 \cdot \pi} \cdot \left\{ \frac{1}{\sqrt{(x-x_0)^2 + (y-y_0)^2 + (z-h)^2}} - k_r \cdot \frac{1}{\sqrt{(x-x_0)^2 + (y-y_0)^2 + (z-2\cdot H+h)^2}} + \left(1-k_r^2\right) \cdot \sum_{n=0}^{\infty} k_r^n \rho \cdot \left[\frac{1}{\sqrt{(x-x_0)^2 + (y-y_0)^2 + (z-2\cdot n\cdot H-h)^2}} \right] \right\} \tag{11}$$

$$E = -\operatorname{grad} U. \tag{12}$$

C. Current Density

The current density J components in axes x, y, and z in any point of interest can be calculated through the expressions below, using the electric field E obtained for that point as a function of the electrical conductivity (σ) and the ground electrode geometry under analysis

$$J = E \cdot \sigma. \tag{13}$$

D. Resistance, Potential Rise, and Dissipated Power

The resistance of a ground electrode depends on soil characteristics (spatial distribution of electrical resistivity) and its geometry and may vary in time as a result of the following phenomena affecting the electrical resistivity

- meteorological conditions (rain, solar radiation, temperature and air humidity) that originate variations of the electrical resistivity, especially for lower depths, resulting in soil humidity modification, including temperature heterogeneity;
- climatic and hydrologic effects, causing occasional variation of the freathic level;
- effects from the current injected into the soil, through the ground electrode, resulting in soil heating, electroosmosis and change of humidity distribution and electrochemical effects;
- eventual humidification or drainage in the soil, caused by external factors, in the ground electrode vicinities.

The ground electrode resistance R_e means, for a given current injected I through it, the potential rise U_e of a ground electrode in relation to a remote earth. The dissipated power P_e in the soil by the ground electrode is given by

$$U_e = R_e \cdot I \tag{14}$$

$$P_e = R_e \cdot I^2. \tag{15}$$

The potential in the ground electrode represents an upper limit of the touch, step, and transferred potentials. The dissipated power in the soil, even for lower values of the ground electrode resistance, may be not negligible in terms of losses of the converters' system.

E. Safety Restraints for Human Beings

Concerning the direct current, human beings have a higher threshold current when compared with the alternating current in industrial frequency (50 or 60 Hz) with tolerable voltage limits indicated in [5]. A safe electrode design must take into account these aspects due to the risk of electrical shocks involved.

IV. Example Cases

The main characteristics of the ground electrode and the soil considered in the simulations under analysis will be given. The resulting profiles of electric fields calculated for homogeneous and nonhomogeneous soils are indicated in Fig. 13, in the boundaries of the ground electrode in its plane of installation $(z = 2.7\text{ m})$. The profile of current densities around the ground electrode in its vicinities as well as the potentials are indicated,

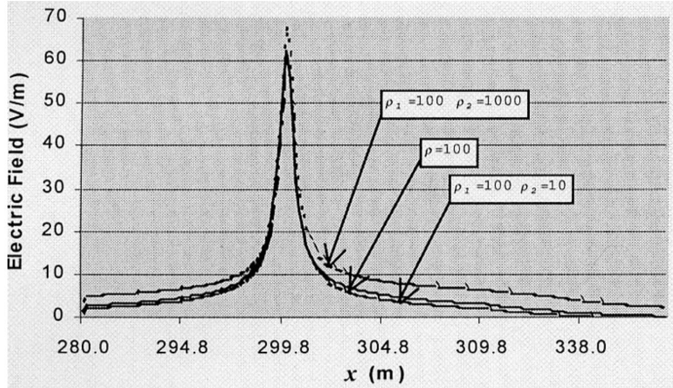

Fig. 13. Electric field profiles (Case 1, ρ_i in $\Omega \cdot$ m).

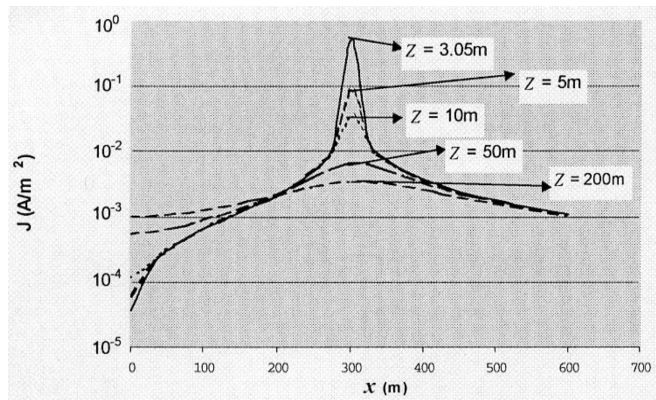

Fig. 14. Current densities.

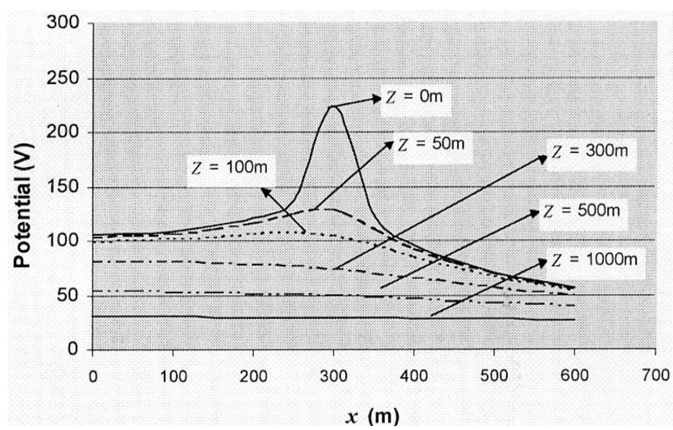

Fig. 15. Potential profile (Case 2).

respectively, in Figs. 14 and 15 for different depths of interest for a homogeneous soil (Case 2). Fig. 16 show the spatial distribution of the electric field related to the results shown in Fig. 13.

These results fit well with those reported in [6], allowing a good conclusion of the effectiveness of the proposed technique.

a) Ground electrode
- geometry: ring (toroidal section);
- electrode diameter: 300 m;
- conductor diameter: 0.6 m;
- depth: 2.7 m;

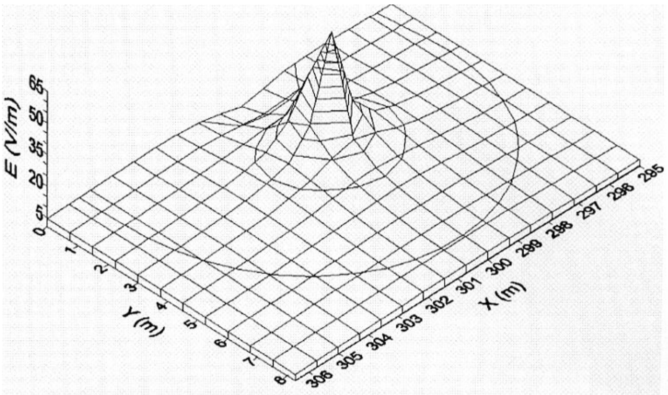

Fig. 16. Electric field spatial distribution.

- number of segments used for ground electrode modeling: 144;
- injected current: 2000 A.

b) Soil

Case 1) nonhomogeneous soil $\rho_1 = 100\ \Omega \cdot \mathrm{m}$; $\rho_2 = 1000\ \Omega \cdot \mathrm{m}$; $H = 14.0$ m;

Case 2) homogeneous soil $\rho = 100 \Omega \cdot \mathrm{m}$.

V. CONCLUSION

- The equations presented in this article and the methodology described allow sufficiently accurate calculations of electric field, density current, and potential in any point of the soil medium for a HVDC system ground electrode configurations immersed in homogeneous and nonhomogeneous soils;
- When symmetries in ground electrodes are present, like spherical and cylindrical geometries, the use of two-dimensional (2-D) approximations is valid and advisable, aiming memory storage and time computer less processing;
- For a proper analysis of the electroosmosis and soil heating phenomena, the correct calculation of the electric field in generic points of the soil and above it (air) is essential. The electric field variation as a function of the temporal changes of soil electrical resistivity by migration of the humidity present in the medium is also a mandatory aspect to be observed.

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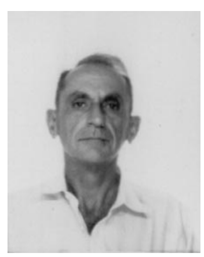

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