Theoretical Analysis of Grounding Resistance for the Rod Buried into the Multi-Layered Earth

Masashi Mitani
Graduate School of Engineering
Kanto-gakuin University
Yokohama, Japan

Takehiko Takahashi, Member, IEEE
Dept. of Architectural Environmental Engineering
Kanto-gakuin University
Yokohama, Japan

Abstract: The theoretical analysis has already been performed for the cylindrical rod electrode that is driven vertically into the earth with its upper end at earth surface. However, there is almost no reports to describe the grounding resistance when the rod is buried into the multi-layered earth. It is important to understand how changing various buried depth for electrodes affects the potential distribution and the grounding resistance.

In this paper, the electric potential distribution in the underground and the voltage along the earth surface for the buried rod with buried depth t into the multi-layered earth are analyzed. Based on the results, the authors proposed a calculating equation of the grounding resistance for the rod buried into the multi-layered earth.

The validity of those estimated values for the buried rod is verified with the numerical calculation.

Keywords: buried rod, multi-layer earth structure, grounding resistance, potential distribution, numerical calculation

I. INTRODUCTION

For protective grounding mainly to prevent electrical shock and protect equipment in facilities, it is essential to estimate the grounding resistance of the electrode itself caused by the potential build-up and the potential distribution around the electrode at the flowing-in of fault current and the like.

Meantime the electrode is available in various shapes and compositions and that suitable for a specific application has been selected in actual use. Among them a cylindrical rod electrode is widely used today. The rod electrode can be a fundamental element in analyzing the electrodes in other shapes. The installation method of the rod can be classified into two types. The method called the driving method is to drive the rod into the earth with its upper end at the earth surface, while another called the burying method is to bury it fully into the earth without exposing its upper end at the earth surface. The former method is appreciated in practical application as a simple installation method. However in Japan, Technological Standards define that the electrodes shall be buried with the depth of 75cm or more when the electrode is installed in a place where the grounding system is likely to be touched by human body. While estimating the potential distribution and grounding resistance of buried rods beforehand will be useful in securing safety during the actual operation of grounding systems, and in reducing labor at installation work as the burying depth can previously be determined.

The study to introduce the calculating equations for the potential distribution and grounding resistance when the rod electrode is driven into the multi-layer structure of the earth has been performed by Tagg for 2 layers, and Takahashi et al for multi-layer. However, the study to examine the potential distribution and grounding resistance of the buried rod for multi-layered earth can not be found by our investigation. This paper describes the process to introduce the calculating equation and the result of the numerical simulation for the potential distribution and grounding resistance of the rod buried into the multi-layered earth, applying to the analyzing method of driven rod electrodes.

II. THEORETICAL MODEL AND POTENTIAL CALCULATING EQUATION

<2.1> Fundamental Equation

At First, the potential in the earth due to a point current source is clarified.

When a point current source i_o exists at the position of buried depth of z' under the homogeneous earth with the resistivity of ρ , the potential $V_o(x,z)$ caused by the point current source at any points (x,z) in the earth can be expressed as follows;

$$V_o(x,z) = \frac{\rho i_o}{4\pi}\int_0^\infty \left\{e^{-\lambda|z'-z|} + e^{-\lambda(z'+z)}\right\} J_0(\lambda x)\,d\lambda \qquad (1)$$

where $J_0(x)$ is the Bessel function of the first kind of order 0. And it should be noted that the origin of xz orthogonal coordinate is set to the earth surface immediately above the point current source, the x-axis is set along the earth surface and the z-axis is set by making the depth direction positive.

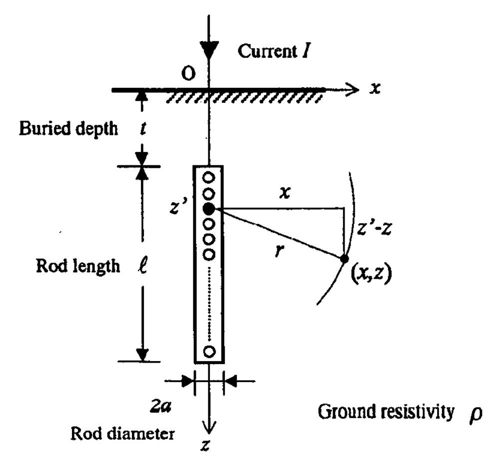


Fig.1 Theoretical model for buried rod electrode

0-7803-6420-1/00/$10.00 (c) 2000 IEEE

<2.2> Setting of Theoretical Model for Buried Rod Electrode

As shown in Fig.1, the rod electrode with the length ℓ and radius a is being buried at the depth t, and the case when current I is flowing to this electrode is supposed. The rod electrode is buried vertically against the earth surface, and the z-axis is set so that the center line of the rod electrode will be aligned. The theoretical model of the rod is supposed as the assemblage of the point current source with the uniform current density ($= I/\ell$) in the direction of depth.

The potential $V(x,z)$ generated by the buried rod electrode can be obtained by integrating the potential due to the point current source (equation (1)) with the electrode length ℓ in the direction of the depth, that can be expressed as follows;

$$V(x,z) = \frac{I\rho}{4\pi\ell}\cdot\left\{\ln\frac{\ell+t-z+\sqrt{x^2+(\ell+t-z)^2}}{t-z+\sqrt{x^2+(t-z)^2}} + \ln\frac{\ell+t+z+\sqrt{x^2+(\ell+t+z)^2}}{t+z+\sqrt{x^2+(t+z)^2}}\right\} \quad (2)$$

From this equation, the status of the potential distribution by the buried rod electrode can analytically be grasped. The equipotential curve is shown by numerical calculation in the next clause.

<2.3> Numerical calculation of potential distribution

Taking a rod electrode with length ℓ = 1.5[m], radius a = 0.007[m] as an example, the equipotential curve on xz plan and the potential distribution on the earth surface both at different depth are shown in Fig.2 by using equation (2).

Since the potential is proportional to both the current I and the resistivity ρ as can be found from the calculating equation, the current and the resistivity are normalized to I = 1 [A] and ρ = 100 [Ωm] for expression in these figures.

The figures are shown with the buried depth t = 0, 0.5[m].

Fig.2(a); t = 0 represents the state when the rod is driven but exposing its upper end at earth surface using the driving method so called. However, as can be found from the introducing procedure, it can be handled as same as the bury method in this model, and equation (2) can be applied to both driving and burying methods.

From the underground equipotential curve shown in Fig.2, it can be confirmed that the earth surface and equipotential line cross vertically due to the total reflection on the earth surface. In addition, it can be found that the equipotential surface becomes a hemisphere at more distant point from the electrode. These findings agree with the general views.

When the potential distribution on the earth surface of Fig.2 (a) and (b) are compared, it can be found that a high potential gradient of the earth surface around the electrode is shown especially at t = 0. It is also found that the potential gradient around the buried point can greatly be reduced by burying the electrode instead of driving it. In this example, the maximum potential on the ground surface counted for 64.3V at t = 0, 14.7V at t = 50cm, 11.7V at t = 75cm and 4.3V at t = 3m (although the last two examples are not shown in the figure). In the case when the electrode is buried deeply to some extent, the installation at further deeper position will give a small effect.

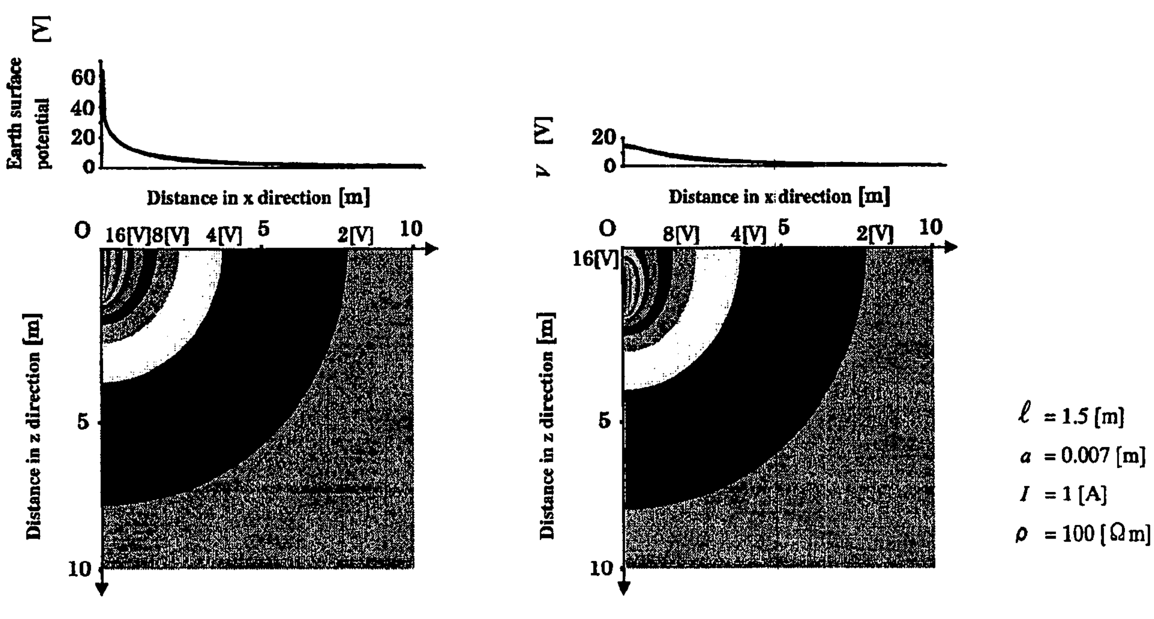
The equipotential curve at buried depth t = 0 forms a semi-oval group having a same focus, while when the buried depth is sufficiently large, it is observed that the equipotential surface around the rod electrode becomes a spheroid.

III. GROUNDING RESISTANCE OF BURIED ROD

<3.1> Calculating Equation for Grounding Resistance

Next the grounding resistance due to the buried rod electrode is considered.

As clearly understood from Equation (2) showing potential



(a) At buried depth t = 0 [m] (b) At buried depth t = 0.5 [m]

Fig.2 Underground potential distribution and earth surface potential at different buried depths : (a) t = 0 [m] ; (b) t = 0.5 [m] distribution, the potential differs depending on the measuring points (x,z). This also can be adapted to the potential at any point a distant from the z-axis on the electrode surface, and so the potential on the rod surface differs depending on the position in the direction of the depth. According to the definition of the grounding resistance that it is to be obtained by dividing the potential build-up of the electrode itself by the current, it will be a problem how to select the electrode position of which potential is taken for the reference point in calculating the grounding resistance.

Under the circumstance, the calculating equations of grounding resistance set by the plural reference points of the possible electrode position are obtained to compare them individually by numerical calculus. (See Fig.3)

(1) When the top end point of rod surface is selected for reference point

Supposing from the analytical method to obtain the grounding resistance of the rod electrode at driving, the calculating equation of the grounding resistance at the burying of the rod is attempted to be introduced. In analyzing for the driving, the value obtained by dividing the potential at the electrode top end on the earth surface by the current is used as the grounding resistance in general. Taking this as a reference, the potential V_{top} at the top end of electrode surface (a,t) is selected as one of the calculation reference points also for the burying. When the grounding resistance in this case is represented by R_{top}, the calculating equation of R_{top} can be obtained by dividing the potential at the point (a,t) obtainable from equation (2) by the current I as follows;

$$R_{top}=\frac{V_{top}}{I}=\frac{V(a,t)}{I}$$
$$=\frac{\rho}{4\pi\ell}\left\{\ln\frac{\ell+\sqrt{a^2+\ell^2}}{a}+\ln\frac{\ell+2t+\sqrt{a^2+(\ell+2t)^2}}{2t+\sqrt{a^2+(2t)^2}}\right\} \tag{3}$$

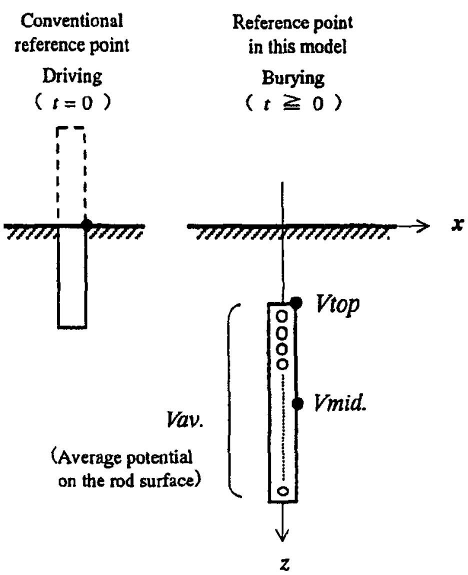


Fig.3 Potential reference points of buried rod electrode

(2) When the middle point of rod surface is selected for reference point

When the electrode top end at driving is understood as the position taking its image into account, it can be regarded to be the middle position of the electrode that includes the image portion. This thinking is adapted to the case of burying. The potential at the middle point $(a\ ,\ t+\ell/2)$ in the direction of length on the rod surface is expressed with $V_{mid.}$, and the value obtained by dividing this value by the current is defined to be the grounding resistance $R_{mid.}$. $R_{mid.}$ is expressed as the equation below;

$$R_{mid.}=\frac{V_{mid.}}{I}=\frac{V(a,t+\ell/2)}{I} \tag{4}$$
$$=\frac{\rho}{2\pi\ell}\left\{\ln\frac{\frac{\ell}{2}+\sqrt{a^2+\left(\frac{\ell}{2}\right)^2}}{a}+\frac{1}{2}\ln\frac{\frac{3}{2}\ell+2t+\sqrt{a^2+\left(\frac{3}{2}\ell+2t\right)^2}}{\frac{1}{2}\ell+2t+\sqrt{a^2+\left(\frac{1}{2}\ell+2t\right)^2}}\right\}$$

(3) When the average potential on rod surface is referenced

By introducing equation (2), the potential at any points on the electrode surface can be obtained. Therefore, as one of the calculating equations for grounding resistance, and the grounding resistance $R_{av.}$ is set by dividing the average potential $V_{av.}$ on the rod surface by the current I flowing into the rod. We can find the average potential $V_{av.}$ on the rod surface by setting the distance x in the equation (2) to the radius a of the rod and by integrating the equation for the rod length ℓ in the direction of z. Therefore the grounding resistance $R_{av.}$ can be rearranged as follows;

$$R_{av.}=\frac{V_{av.}}{I}=\frac{1}{I}\cdot\frac{1}{\ell}\cdot\int_t^{\ell+t}V(a,z)dz$$
$$=\frac{\rho}{2\pi\ell}\Bigg[\ln\frac{\ell}{a}\left(1+\sqrt{1+\left(\frac{a}{\ell}\right)^2}\right)+\frac{a}{\ell}-\sqrt{1+\left(\frac{a}{\ell}\right)^2}$$
$$+\ln\left(\frac{2(\ell+t)\left\{1+\sqrt{1+\left\{\frac{a}{2(\ell+t)}\right\}^2}\right\}}{(\ell+2t)\left\{1+\sqrt{1+\left(\frac{a}{\ell+2t}\right)^2}\right\}}\right)$$
$$+\frac{t}{\ell}\ln\left(\frac{4t(\ell+t)\cdot\left\{1+\sqrt{1+\left\{\frac{a}{2(\ell+t)}\right\}^2}\right\}\left\{1+\sqrt{1+\left(\frac{a}{2t}\right)^2}\right\}}{(\ell+2t)^2\cdot\left\{1+\sqrt{1+\left(\frac{a}{\ell+2t}\right)^2}\right\}^2}\right)$$
$$-\left(1+\frac{t}{\ell}\right)\cdot\sqrt{1+\left\{\frac{a}{2(\ell+t)}\right\}^2}+\left(1+\frac{2t}{\ell}\right)\cdot\sqrt{1+\left(\frac{a}{\ell+2t}\right)^2}$$
$$-\frac{t}{\ell}\cdot\sqrt{1+\left(\frac{a}{2t}\right)^2}\ \Bigg] \tag{5}$$

<3.2> Calculation of Grounding Resistance

For the rod electrode having the same length and radius as that in obtaining the potential distribution, the difference of the calculated values of the grounding resistance R_{top}, R_{mid} and $V_{av.}$ due to changes in the buried depth is shown in Fig.4. This figure also includes the calculated values obtained by

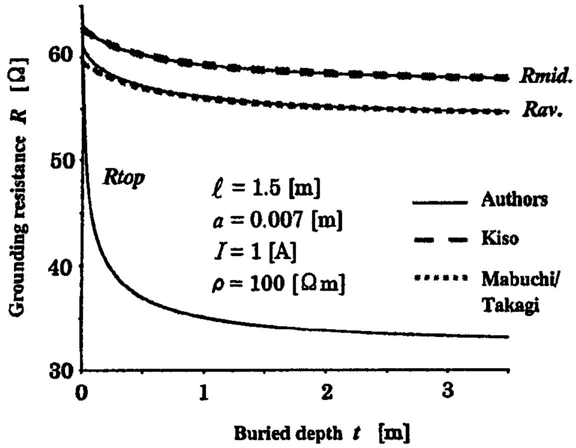


Fig.4 Relationship between buried depth and grounding resistance

the formulas for grounding resistance.

R_{top} referring the top end point does not differ significantly from other values at $t = 0$, and is acceptable as one of the calculated values for grounding resistance at driving. However, the potential at the top end falls rapidly as the buried depth increases due to the decrease of the reflection component from the earth surface thus not indicating the trend of the entire electrode. Although it depends on the buried depth, a low value equivalent to about 60% of other grounding resistance calculated values is presented. From these results, R_{top} is considered to be not suitable for the calculating point of the grounding resistance at burying.

On the other hand, $R_{mid.}$ referring the middle point and $V_{av.}$ obtained from the average potential are mostly agreed with the conventional formulas. This is because of the fact that the positions of the potential were grasped under same thinking. It can be found that the difference of the values is caused by approximation with $\ell, t >> a$ through the comparison of the calculating equations.

To calculate the grounding resistance of the buried rod, it will be appropriate to use $R_{mid.}$ referring the middle point or $V_{av.}$ obtained from the average potential from the above result. Further in considering the grounding resistance of the rod buried into multi-layer earth, it will be better to use $V_{av.}$ covering the entire electrode than the middle point $R_{mid.}$ referring a specific point as the status of the potential distribution differs in each layer.

IV. ROD ELECTRODE BURIED INTO MULTI-LAYERED EARTH

While the calculation was conducted on the assumption that the earth structure is homogeneous in the former chapter, this chapter analyzes the status of the rod electrode buried into multi-layered earth.

For the earth models in Fig.5, we designate the thickness of each layer as h_n, the depth of each boundary as H_n, the ground resistivity of each layer as ρ_n, and the current source for each layer as i_m. The general solutions for the potential due to the current source in each layer are designated as follows;

Potential due to i_L : $V_{L1}, V_{L2}, \dots V_{Ln}, \dots, V_{LN}$

:

Potential due to i_m : $V_{m1}, V_{m2}, \dots V_{mn}, \dots, V_{mN}$

:

Potential due to i_M : $V_{M1}, V_{M2}, \dots V_{Mn}, \dots, V_{MN}$

The potential V_{mn} due to the current source i_m in the n-th layer can be expressed as the equations below;

$$V_{mn} = \frac{\rho_m i_m}{4\pi}\int_0^\infty \left\{\phi_{mn}(\lambda)e^{+\lambda(z-z')} + \varphi_{mn}(\lambda)e^{-\lambda(z-z')}\right\}J_0(\lambda x)d\lambda \qquad (6)$$

in case m = n ;

$$V_{mm} = \frac{\rho_m i_m}{4\pi}\int_0^\infty \left\{e^{-\lambda|z-z'|} + \phi_{mm}(\lambda)e^{+\lambda(z-z')} + \varphi_{mm}(\lambda)e^{-\lambda(z-z')}\right\}J_0(\lambda x)d\lambda$$

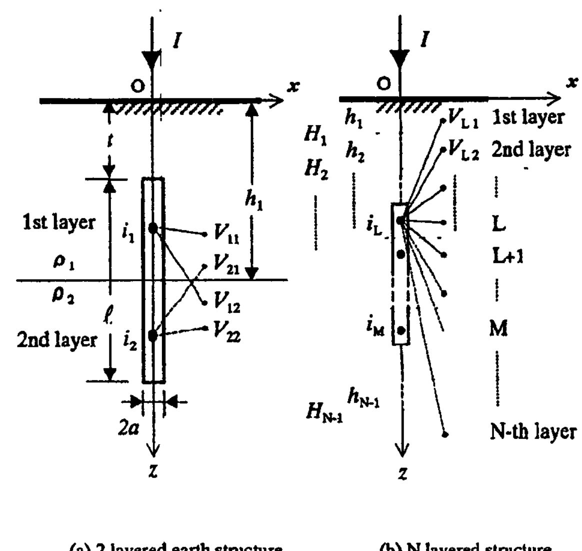
The unknown functions $\phi_{mn}(\lambda)$, $\varphi_{mn}(\lambda)$ in the above equations can be determined by assuming the following boundary conditions;

1) the potential in the lowest layer as z goes to infinity is 0;
2) at the earth surface z = 0, the current flow will be along the surface;
3) the potential at a boundary is the same for the layers on either side of the boundary;
4) the current is continuous at the boundary between two layers.

It is obvious that 2N unknown functions can be determined by 2N boundary conditions.

Thus, if we solve the unknown functions, the potential V_n at any points in the n-th layer due to a buried rod in a horizontally stratified N-layer earth is expressed as;

$$V_n(x,z) = \int_t^{H_L} V_{Ln}dz' + \int_{H_L}^{H_{L+1}} V_{L+1,n}dz' + \cdots + \int_{H_{M-2}}^{H_{M-1}} V_{M-1,n}dz' + \int_{H_{M-1}}^{\ell+t} V_{Mn}dz' \qquad (7)$$



(a) 2 layered earth structure (b) N layered structure

Fig.5 Multi-layered earth model and buried rod

Next considered was the current density distribution for the rod buried into the multi-layered earth.

For a homogeneous earth, it has been assumed that the current flowing out from the rod to the earth is uniform. Though, for a multi-layered structure, the above assumption can not be made as the resistivity ρ is different for each layer. However, if we assume that the current density is inversely proportional to the resistivity ρ, it can be considered that the current in each layer is uniform. Thus, for the horizontally stratified N-layer earth model, we obtain the following expressions;

$$\begin{aligned} &\rho_L i_L = \rho_{L+1} i_{L+1} = \cdots = \rho_m i_m = \cdots = \rho_M i_M \\ &\rho_L h_L = \rho_{L+1} h_{L+1} = \cdots = \rho_m h_m = \cdots = \rho_M h_M = I \end{aligned} \tag{8}$$

And if we solve the above, we get the relation between each i_m and the total current I as follows;

$$\rho_m i_m = \frac{I}{\sum_{i=L}^{M} \frac{h_i}{\rho_i}} \tag{9}$$

We can thus derive the equation for calculation of the potential at any points using equation (7) and (9) in an N-layer model. And we can also get the equation for grounding resistance buried into the N layered earth as follows;

$$R = \frac{1}{I}\cdot\frac{1}{\ell}\Big\{ \int_t^{H_L} V_L(a,z)dz + \int_{H_L}^{H_{L+1}} V_{L+1}(a,z)dz + \cdots + \int_{H_{M-2}}^{H_{M-1}} V_{M-1}(a,z)dz + \int_{H_{M-2}}^{\ell+t} V_M(a,z)dz \Big\} \tag{10}$$

In order to conduct numerical simulations, the problem is simplified by considering the earth to be horizontal two layered structure, and expressing the resistivity of the upper layer with ρ_1, the lower layer with ρ_2, and the depth of the layer boundary with h_1.

In two layered earth, the potential equations can be classified into 3 patterns.

1) In case when electrode exists only at lower layer

Making substitutions and rearranging of equation (7) and (9), we obtain the following expression for the potential of a buried rod into 2 layer earth;

$$\begin{aligned} V_1(x,z) = \frac{\rho_2 I}{4\pi\ell}\cdot(1-k_1)\times &\sum_{n=0}^{\infty} k_1^n \Bigg\{ \ln\frac{2nh+z+\ell+t+\sqrt{x^2+(2nh+z+\ell+t)^2}}{2nh+z+t+\sqrt{x^2+(2nh+z+t)^2}} \\ &+ \ln\frac{2nh-z+\ell+t+\sqrt{x^2+(2nh-z+\ell+t)^2}}{2nh-z+t+\sqrt{x^2+(2nh-z+t)^2}} \Bigg\} \end{aligned} \tag{12}$$

$$\begin{aligned} V_2(x,z) = \frac{\rho_2 I}{4\pi\ell}\times \Bigg\{ &\ln\frac{z-t+\sqrt{x^2+(z-t)^2}}{z-\ell-t+\sqrt{x^2+(z-\ell-t)^2}} \\ &+ k_1\cdot\ln\frac{-2h+z+t+\sqrt{x^2+(-2h+z+t)^2}}{-2h+z+\ell+t+\sqrt{x^2+(-2h+z+\ell+t)^2}} \\ &+ (1-k_1^2)\cdot\sum_{n=0}^{\infty} k_1^n\cdot\ln\frac{2nh+z+\ell+t+\sqrt{x^2+(2nh+z+\ell+t)^2}}{2nh+z+t+\sqrt{x^2+(2nh+z+t)^2}} \Bigg\} \end{aligned} \tag{13}$$

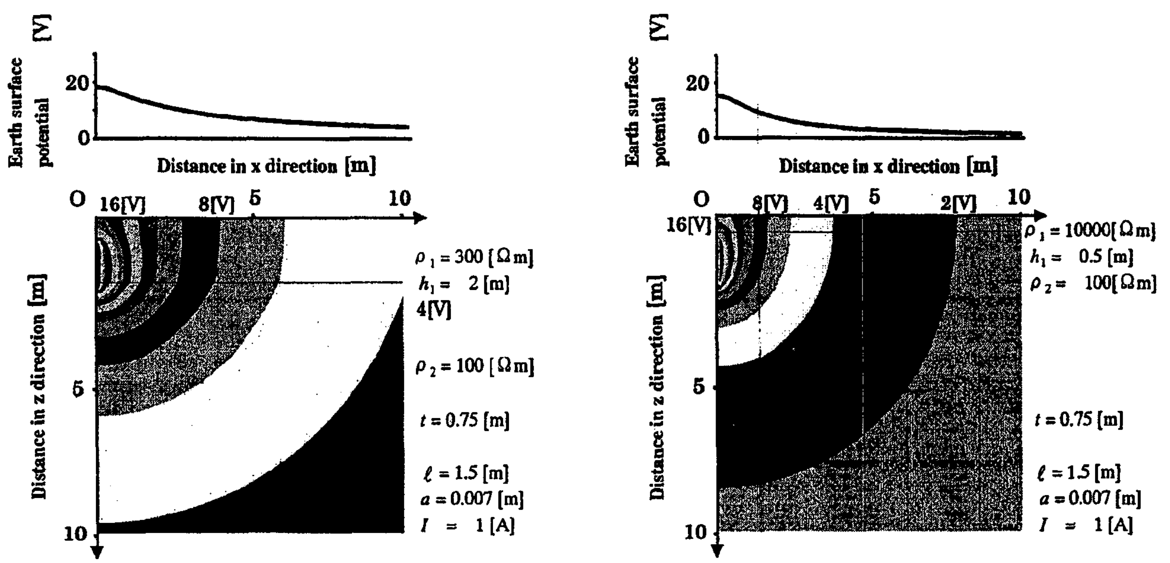
where the value k_1 is the reflectivity at the boundary between the first and second layers. In general for an upper and lower layer resistivity of ρ_i and ρ_{i+1}, the resistivity can be expressed as below;

$$k_i = \frac{\rho_{i+1} - \rho_i}{\rho_{i+1} + \rho_i} \tag{14}$$

2) In case when electrode exists covering 2 layers

3) In case when electrode exists only at upper layer

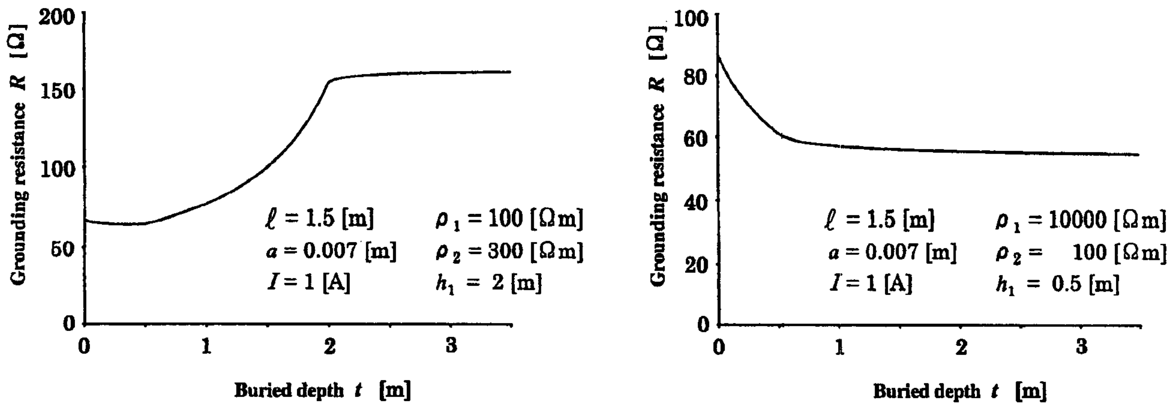
We can get the potential equations in each case through the same process above.



(a) In case of high resistivity at lower layer

(b) In case of high resistivity at upper layer

Fig.6 Underground potential distribution and earth surface potential of the rod electrode buried into 2-layered earth ; the buried depth t = 0.75[m]



(a) In case of high resistivity at lower layer

(b) In case of high resistivity at upper layer

Fig.7 Grounding resistance of the rod electrode buried into 2-layered earth

Using these equations, Fig.6 shows the status of the potential distribution due to the buried rod into 2-layered earth, taking two numerical examples. In addition, using equation (10), the change of the calculated values of the grounding resistance presented when varying the buried depth in 2-layered earth under each earth conditions is shown in Fig.7. From these results, it was found that the grounding resistance largely depends on the ground resistivity of the layer where the electrode is existing. Also it can be found by the numerical simulation that the effect of the layer with high ground resistivity if existed will mostly be avoided by burying the electrode isolating from the layer in the length of the electrode.

V. CONCLUSION

The calculating equations for the potential distribution and grounding resistance applicable to the buried rod electrode at the buried depth $t \geqq 0$ was introduced. By the numerical simulation using these equations, the status of the potential distribution due to the buried rod was visualized contributing to help understand the characteristics. It was confirmed that the potential gradient on earth surface can be reduced by burying the electrode instead of the driving.

For the grounding resistance in addition, it was clarified that the setting of the reference points of potential causes a difference in the calculating equations of grounding resistance. For the buried rod electrode, *Rav.* obtained from the average potential was found to be appropriate.

Further the introduction process to obtain calculating equations for the potential distribution and grounding resistance of the rod electrode buried in multi-layered earth were mainly described and examined with plural numerical examples. From these studies, the status of the rod electrode at multi-layered earth was grasped.

Backed by the fruits thus obtained, the authors wish to adopt them in analyzing the electrodes in other shapes buried into the multi-layered earth structure in future.

References

[1] G.F.Tagg: " Earth Resistance ", George Newnes Ltd., (1964)

[2] E.D.Sunde: " Earth Condition Effects in Transmission Systems ", Dover Pub., Inc. (1967)

[3] T.Takahashi, T.Kawase: "Calculation of Earth Resistance for a Deep-driven rod in a Multi-layer Earth Structure," IEEE Trans. Power Delivery, Vol.6, No.2, April 1991, pp.608-614

Masashi Mitani was born in Hiroshima Japan on January 25, 1964. He received the B.S. and the M.S. degrees in electro-communications engineering from University of Electro-Communications, Tokyo Japan in 1986 and 1988, respectively. In 1988, he joined Taisei Corporation, since then he has been engaged mainly in design of electrical installation in buildings. He is currently a Ph.D student majoring in electrical engineering at Kanto-gakuin University, Japan. Mr. Mitani is a sub-member of IEIEJ.

Takehiko Takahashi (M' 83) was born in Fukushima Japan on July 10, 1947. He received the B.S. degree in Electrical Engineering from Tokyo Denki University in 1970 and Dr. Eng. degree from University of Tokyo. After working at Chiba University, Dr. Takahashi is now a Professor of the department of Architectural Environmental Engineering Kanto-Gakuin University. His research has included the study of Grounding and Lighting Protection and Electrical Safety. Dr. Takahashi is a director of the Institute of Electrical Installation Engineers of Japan and a member of TC64, TC81 of IEC.