Analysis of Transmission Tower Potentials During Ground Faults

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Abstract—**Since ground fault currents in high-voltage systems are on the increase, the hazards associated with transmission tower voltages during ground faults have become of concern. An analytical method is presented to determine these voltages for long and short lines, arbitrary ground network terminations, and any combination of skywires and counterpoise conductors. The proportion in which the fault current is supplied by the line on both sides of the fault location is taken into account, and the effect of fault currents flowing in parallel external circuits is also considered. The touch and step potentials at the faulted tower are determined and an assessment is given of the hazard they may represent. In a numerical example, several measures aimed at reducing this hazard are compared.**

Introduction

During ground faults on transmission lines, a number of towers near the fault are likely to acquire high potentials to ground. These tower voltages, if excessive, may present a hazard to humans or animals. In the past the number of accidents due to such potentials has been very low indeed and, therefore, the hazard associated with high tower voltages during ground faults generally has been considered negligible. As circumstances change, however, this conclusion may need to be reexamined.

The low accident rate undoubtedly can be attributed to two specific circumstances. First, the fault currents at most places are of magnitudes which make fatal accidents from excessive tower voltages unlikely. Second, and more important, the probability that concurrently with a fault a person would be in contact with a tower nearby is usually very low. However, complete reliance on these factors may not be as well justified in the future as it has been in the past. It is generally believed, for example, that the fault current capacities of high-voltage systems will continue to increase; in Ontario Hydro's 230-kV system the magnitude of ground fault currents is expected to approach 60 000 amperes within the next few years. Also, transmission line rights-of-way nowadays are located more often in densely populated areas so that the likelihood of persons touching the towers at any time is increased. As a consequence, concern has been expressed in several publications abroad about possible hazards at towers, associated with these factors. A selection of such papers is given in the References.[1]–[4]

For these reasons, a closer examination of tower potential conditions and of the protective effects of various transmission line grounding schemes seems to be timely. Such an analysis is undertaken in this study. In the first five sections an analytical solution for transmission line ground networks is described; in the subsequent sections reference to field tests is made and a method of evaluating shock hazard at towers is proposed.

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General Considerations

For similar situations, the greatest danger exists where the tower voltage reaches the highest value. Since during a ground fault the maximum voltage will appear at the tower nearest to the fault, attention in this study will be focused on that tower.

The voltage rise of the faulted tower will depend on a number of factors, the most important of which are:

1) magnitudes of fault currents on both sides of the fault location and in parallel circuits
2) fault location with respect to the line terminals
3) station ground resistances at the line terminals
4) conductor arrangement on the tower and the location of the faulted phase
5) average tower ground resistance of the line and the ground resistance of the faulted tower
6) number, material, and size of skywires
7) number, material, and size of counterpoise conductors
8) soil resistivity.

In exploring the effects of these factors an important assumption will be that the magnitudes of the fault currents, as supplied by the line on both sides of the fault location, are known from system studies; no attempt will be made, therefore, to determine these quantities. Also, in comparing various line grounding schemes, it will be assumed that all the fault currents are unaffected by changes in the line grounding arrangement.

Impedance of Half-Lines

A phase-to-ground fault occurring on a transmission line divides the line into two sections, each extending from the fault towards one end of the line. These sections will be called half-lines. For the whole line, consisting of two half-lines, the term full-line will be used. In the following discussions the half-lines will be studied first, and then the analysis of full-lines can be accomplished by regarding them as a composite of two half-lines. Half-lines may be considered infinite if certain conditions are met; otherwise, they must be regarded as finite.

Infinite Half-Line

As far as its grounding is concerned, an infinite half-line can be represented by the ladder network in Fig. 1. Here Z_p in the parallel branches represents the tower ground resistances, all assumed equal; and Z_s in the series branches the self-impedance per span of the ground conductor–ground loop. For the moment, only one ground conductor, a skywire, will be considered. It should be noted that the tower at the fault is not included; this arrangement will make it easier to apply the findings of this section when discussing full-lines.

The impedance of the ladder network, as seen from the fault location can be determined using either the lumped parameters describing the actual line or the distributed parameters providing an approximate representation of the line. In the following both methods will be reviewed.

Using lumped parameters first, it is apparent from Fig. 1 that, once the line is infinite, the addition of one more series–parallel unit cannot change its impedance, Z_∞. Thus, the equa-

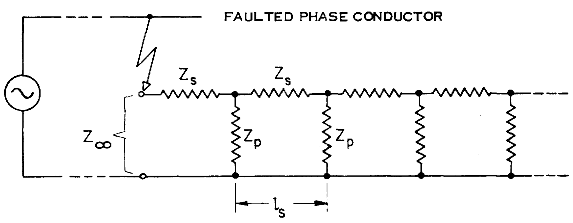


Fig. 1. Equivalent ladder network for an infinite half-line.

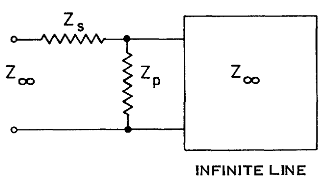


Fig. 2. Equivalent scheme for an infinite ladder network.

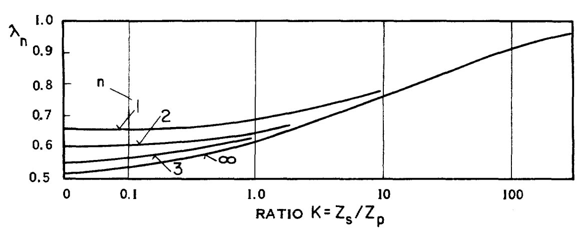


Fig. 3. Curves for factor λ in (5) and (8).

tion obtained from Fig. 2 is

$$\frac{Z_p Z_\infty}{Z_p + Z_\infty} + Z_s = Z_\infty \tag{1}$$

and solving this for Z_∞ one obtains

$$Z_\infty = \frac{Z_s}{2} + \sqrt{\frac{Z_s^2}{4} + Z_s Z_p}. \tag{2}$$

It is noteworthy that, if $Z_s = Z_p$, then $Z_\infty/Z_p = 1.618\ldots$, a number known since ancient times and also related to the so-called Fibonacci numbers which were first published in 1202. Today, the Fibonacci numbers are frequently used in ladder network computations.[5]–[7]

Returning to (2), in most practical cases $Z_s \ll Z_p$, and then

$$Z_\infty \approx \frac{Z_s}{2} + \sqrt{Z_s Z_p}. \tag{3}$$

In the distributed parameter representation the line parameters describing the network are $z_s = Z_s/l_s$ and $z_p = Z_p l_s$, where l_s is the span length assumed to be uniform for the entire length of the line. By solving differential equations (19) in Appendix I it can be shown that the impedance Z_0 of an infinite line with distributed parameters is

$$Z_0 = \sqrt{z_s z_p}. \tag{4}$$

It is easy to show that, as long as the fault is fed from the side opposite to the network [as in Fig. 16(a)], a line of length l can be considered infinite if $l\sqrt{z_s/z_p} > 2$.

Considering that $\sqrt{Z_s Z_p} = \sqrt{z_s z_p}$, (4) resembles (3) except for the $Z_s/2$ term in (3). It follows that to obtain an infinite line with lumped parameters having the same impedance as a line with equivalent distributed parameters, the former must be modified by removal of about half its first series impedance element. Therefore, if distributed parameters are to be used to describe the actual line, the missing half of the first series impedance must be accounted for by adding a correction term to (4). Consequently, (4) is modified to read

$$Z_\infty = \lambda_\infty Z_s + Z_0 \tag{5}$$

where the exact values of λ_∞ can be determined by comparing (2) and (5), and are plotted in Fig. 3. As can be anticipated from (3), λ_∞ is close to 0.5 for the usual small values of Z_s/Z_p, and, therefore, in most practical cases this value can be used without undue error.

There is little to choose between the two methods as long as only infinite lines are involved. In a more general case, however, the accurate lumped parameter method may not be available, while the distributed parameter approach with the correction term can still be readily applied.

Finite Half-Line

In the previous section it was noted that, if $l\sqrt{z_s/z_p} < 2$, a half-line cannot be considered infinite. In the case of a finite line, the impedance of its ground network may depend greatly on the termination of the network at the end opposite to the fault.

Network open-ended: Using distributed parameters, the impedance Z_l of an open-ended finite network of length l is

$$Z_l = Z_0 \coth l\sqrt{\frac{z_s}{z_p}}. \tag{6}$$

This equation can be derived from (19) in Appendix I. If $l \to \infty$, (6) simplifies to (4). If per-span parameters are preferred, (6) can be rewritten as

$$Z_{n0} = Z_0 \coth n\sqrt{K} \tag{7}$$

where $K = Z_s/Z_p$, and considering that

$$l\sqrt{\frac{z_s}{z_p}} = \frac{l}{l_s}\sqrt{\frac{Z_s}{Z_p}} = n\sqrt{\frac{Z_s}{Z_p}} = n\sqrt{K}.$$

Here $n = l/l_s$ is the number of parallel branches (towers) in the line.

Since the true line representation is by lumped parameters, (7) must be corrected in a manner similar to the one described for infinite lines. If the corrected impedance of the finite half-line is Z_n, then in analogy with (5)

$$Z_n = \lambda_n Z_s + Z_{n0}. \tag{8}$$

Values of λ_n can be determined by comparing Z_{n0}, as calculated from (7), with the corresponding Z_n values obtained by elementary methods. Curves of λ_n are shown in Fig. 3; it should be noted that even λ_1 does not differ very much from λ_∞. With the appropriate value of λ_n from Fig. 3, it is easier to calculate Z_n from (8) than by the rather cumbersome elementary methods.

The ratio Z_n/Z_p is shown in terms of K and n in Fig. 4 by the curves labeled ∞.

Network closed by impedance: Real line ground networks are seldom open-ended. They terminate in stations, with the ground conductors tied to the station grounding system. Consequently, the ladder network representing such a line must be closed by an impedance representing the station ground resistance.

If the distributed parameter method is to be used, this terminating impedance must include a term of the form $\lambda_n' Z_s$ in addition to R_{st}, the station ground resistance. The reason for this is the same as for adding $\lambda_n Z_s$ to the ladder network near the fault: the station ground is connected to the last tower through an extra span and only part of this is included in the network represented by distributed parameters. The complete equivalent scheme is illustrated in Fig. 5. This scheme is an approximation since the station ground electrode and the groundings of the first few towers may not be completely in-

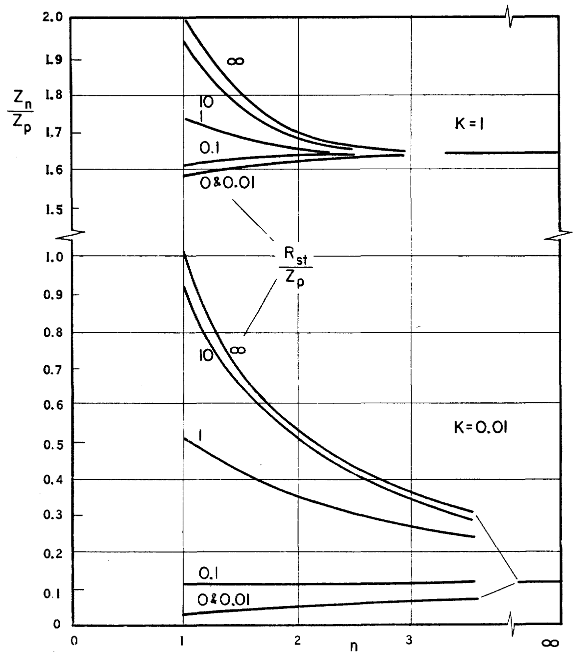


Fig. 4. Curves for impedance of finite half-line opposite infeed.

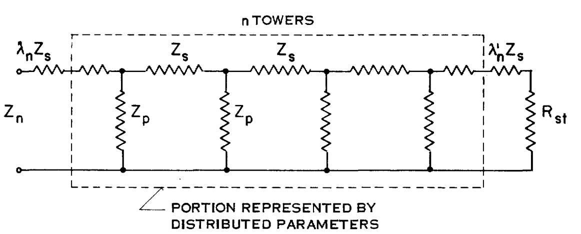


Fig. 5. Ground network of finite half-line terminated by R_{st}.

dependent; however, the error introduced thereby is usually small. In the computations that follow $\lambda_n' Z_s + R_{st}$ will be denoted by Z_g.

As for the factor λ_n', its exact determination is quite complicated since in addition to being a function of n and K, it depends on λ_n and R_{st}/Z_p as well. Only a slight error is introduced, however, if it is assumed that $\lambda_n' = 1 - \lambda_n$, and in most cases it is quite satisfactory to use the value $\lambda_n' = 0.5$.

The impedance of the half-line can be determined by solving the set of differential equations (19) in Appendix I with appropriate boundary conditions. As a result, the following equation is obtained

$$Z_n = \lambda_n Z_s + Z_0 A_n \tag{9}$$

where

$$A_n = \frac{Z_g \cosh n\sqrt{K} + Z_0 \sinh n\sqrt{K}}{Z_g \sinh n\sqrt{K} + Z_0 \cosh n\sqrt{K}}.$$

If $R_{st} \to \infty$, that is, if the station ground resistance is so high that the network practically becomes open-ended,

$$A_n \to \coth n\sqrt{K}$$

and thus (9) simplifies to (8).

The values of Z_n/Z_p are plotted in Fig. 4. It should be noted that the curves in Figs. 3 and 4 are exact if both Z_s and Z_p are real. This is usually true for Z_p, but not always for Z_s. In most cases, however, correct values of λ_n or Z_n/Z_p will deviate only slightly from those indicated in the diagrams.

Full-Lines—Fault Fed from Only One Direction

In determining the fault current distribution in full-lines, the case is first investigated where the fault is supplied from one direction only. Then the more general case, with the fault fed from both ends, is analyzed as a combination of two such *one-sided* current schemes, with the fault currents supplied from opposite ends.

Line Infinite on Infeed Side

To investigate this case, the scheme of Fig. 6 is constructed. Here R_t represents the ground resistance of the tower at the fault location, and the half-line opposite the infeed may be finite or infinite. The coupling between the faulted phase conductor and the ground conductor is taken into account by z_m, the mutual impedance per unit length of line. Using distributed parameters, the circuit is described by the set of differential equations (20) in Appendix I, and with the appropriate boundary conditions, the following solution can be obtained for V, the voltage rise of the faulted tower:

$$V = (1 - \mu) I_f Z_{\infty n} \tag{10}$$

where $\mu = z_m/z_s$, often called coupling factor, I_f is the total fault current, and

$$Z_{\infty n} = \frac{1}{\dfrac{1}{Z_\infty} + \dfrac{1}{R_t} + \dfrac{1}{Z_n}}. \tag{11}$$

It can be shown that between the fault and source, at a great distance from both, the portion of the fault current flowing in the ground conductor is μI_f while $(1 - \mu) I_f$ is returning in the ground, and there is no interchange of currents through the towers. It is apparent that in the absence of coupling between the phase and ground conductors, the total of I_f will gradually flow into the ground through the towers, and, if the line is long enough, no current remains in the skywire; whereas in the presence of such coupling a portion of the current will stay in the skywire. The voltage rise at the fault location is, according to (10), the product of this latter current portion and $Z_{\infty n}$ which, in turn, is simply the resultant impedance of the two half-lines and the ground resistance of the tower at the fault in parallel.

For a line infinite in both directions, $Z_{\infty\infty}$ is the reciprocal of $2/Z_\infty + 1/R_t$. Values of $Z_{\infty\infty}/Z_p$ are plotted for various R_t/Z_p ratios in Fig. 7. Z_p is the average ground resistance of the towers in the line and if there is no evidence to the contrary, it is reasonable to assume that for the tower at the fault location $R_t = Z_p$. However, it can be observed that even a significant deviation from the average will have only a very limited influence on $Z_{\infty\infty}$ and thus on V, especially for the usual low K ratios. The effect is particularly small if R_t exceeds the average tower ground resistance, a case which might be of undue concern otherwise.

Line Finite on Infeed Side

To investigate this case, the scheme of Fig. 8 was constructed. Except for indicating more details, this is analogous to the scheme in Fig. 6. The line has n towers on the infeed side and N towers (where N may be infinite) on the opposite side. Solving the differential equations (20) in Appendix I will now yield the following expression for V:

$$V = (1 - \mu) I_f Z_{nN}$$

where

$$Z_{nN} = \frac{1}{\dfrac{1}{Z_n'} + \dfrac{1}{D_n}\left(\dfrac{1}{R_t} + \dfrac{1}{Z_N}\right)}. \tag{12}$$

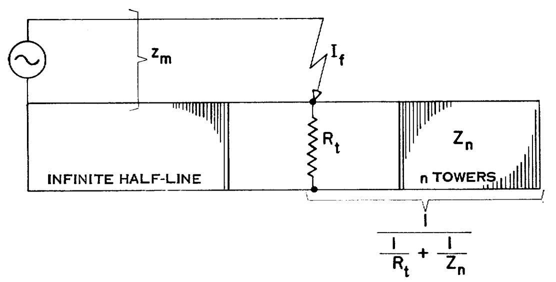


Fig. 6. Full-line, infinite on infeed side.

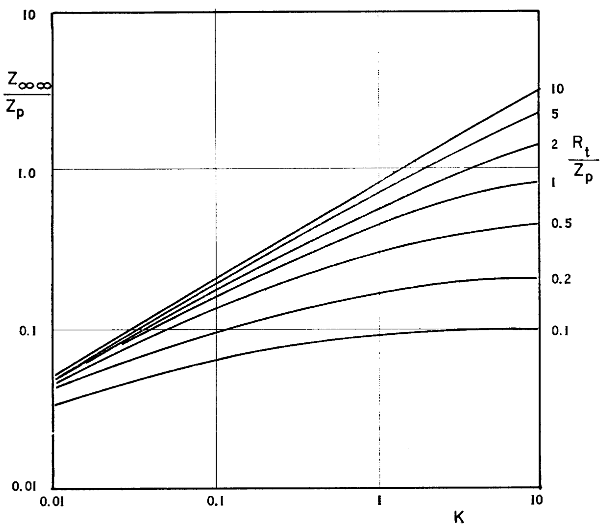


Fig. 7. Curves for impedance of infinite full-lines.

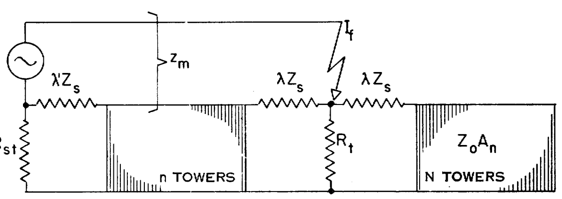


Fig. 8. Full-line, finite on infeed side.

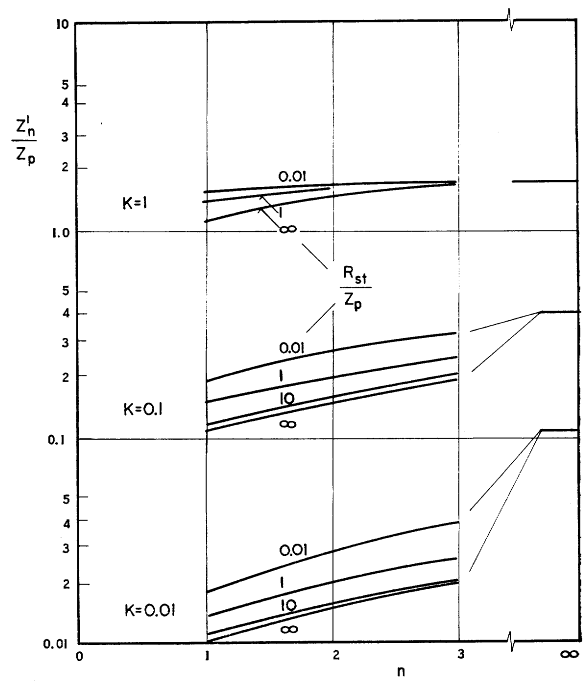


Fig. 9. Curves for impedance of finite half-line between source and fault.

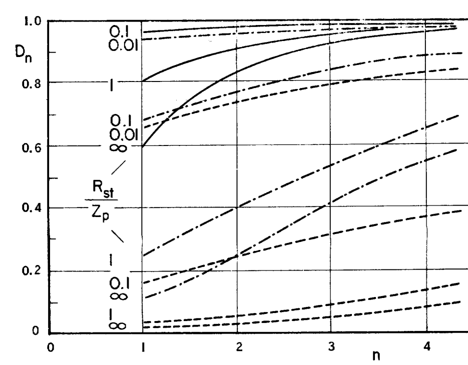


Fig. 10. Values of factor D_n: —— $K = 1$, –·–·– $K = 0.1$, ----- $K = 0.01$.

Here

$$Z_n' = \lambda_\infty Z_s + Z_0 B_n \tag{13}$$

$$B_n = \frac{Z_g \cosh n\sqrt{K} - R_{st} + Z_0 \sinh n\sqrt{K}}{Z_g \sinh n\sqrt{K} + Z_0 \cosh n\sqrt{K}}$$

and

$$D_n = 1 - \frac{R_{st}}{(Z_g + \lambda_\infty Z_s)\cosh n\sqrt{K} + (Z_0 + \lambda_\infty Z_g\sqrt{K})\sinh n\sqrt{K}}. \tag{14}$$

It should be noted that in (13) and (14) λ_∞ is substituted as it closely approximates the exact values required. In (14) R_{st} and Z_g refer to the line termination at the infeed end.

Values of Z_n'/Z_p are plotted in Fig. 9. An important difference should be observed between Z_n in (9) and Z_n' in (13). It can be best illustrated if the half-lines are assumed to be open-ended in both cases. Now $Z_n \to \infty$ if $n \to 0$, whereas $Z_n' \to Z_s$ which is normally quite small in comparison with Z_o. In other words, the shorter an open-ended line opposite the infeed, the higher its V/I_f ratio, and the shorter it is on the infeed side, the lower the ratio. This latter finding is quite obvious considering that as $n \to 0$, the voltage drop on the ground conductor becomes less and less and that V is only part (one half if the network is open-ended) of this drop.

If $n \to \infty$, $B_n \to 1$ and (13) reduces to (6). However, the condition for considering the line infinite is more restrictive than for lines opposite the infeed side. Now, in an infinite line, enough length must be available to allow the total of $(1 - \mu)I_f$ to enter the ground near the fault and then emerge again near the source; therefore, the minimum total length must be twice the length required in the former case. Hence, (6) may be used in lieu of (13) only if $n\sqrt{K} > 4$.

An interesting feature in (12) is the presence of factor D_n. Without it, (12) would follow the pattern of (11). In the present case, however, the resultant of R_t and Z_N in parallel must be derated by the factor D_n (this is always less than 1), and this effect may be quite pronounced if R_{st} is not much smaller than Z_0. Some typical values of D_n are shown in Fig. 10. It should be noted that, again, the curves in Figs. 7, 9, and 10 are exact only if both Z_s and Z_p are real.

Full-Lines—Fault Fed From Both Directions

General Case

A fault supplied from both ends in a line infinite in both directions is illustrated in Fig. 11. If the contribution to the total fault current I_f is I_a from one side and I_b from the other, so that $I_a + I_b = I_f$, then, if only I_a were present, the voltage rise V' of the faulted tower could be determined by (10), namely

$$V' = (1 - \mu)I_a Z_{\infty\infty}.$$

Similarly, for I_b acting alone,

$$V'' = (1 - \mu)I_b Z_{\infty\infty}.$$

If both I_a and I_f are present, the two current schemes can be superimposed (distribution of return currents in Fig. 11 should be noted) and the voltage rise of the faulted tower becomes

$$V = V' + V'' = (1 - \mu)I_f Z_{\infty\infty} \tag{15}$$

recalling that $I_a + I_b = I_f$. Equation (15) is the same as (10) with $n = \infty$, indicating that if the line may be considered infinite with identical parameters in both directions, then, for the same total fault current, the tower voltage rise will be the same regardless of the proportion in which the fault current is divided between the two sides.

Now, if one of the half-lines on the two sides of the fault is finite, or both are, with n towers on the side from where I_a is supplied and N towers on I_b's side, the voltage rise of the faulted tower becomes

$$V = V' + V'' = (1 - \mu)(I_a Z_{nN} + I_b Z_{Nn}). \tag{16}$$

Impedances Z_{nN} and Z_{Nn} are both patterned after (12), with the subscripts switched in the second case. If, say, $N = \infty$, the formula for Z_{Nn} simplifies to that in (11).

Induction from Parallel Circuits

If several circuits run parallel and are bused together at both ends of the line, all the circuits will carry fault currents when a ground fault occurs on one of them. Since there is coupling between the ground conductors and the phase conductors in the parallel circuits, it is of interest to investigate the effect of the fault currents flowing in these circuits on the voltage rise of the faulted tower.

A line is considered, identical and infinite in both directions, as shown in Fig. 12. There are k external parallel conductors carrying the fault currents I_i to I_k. In principle, any one of these conductors can be replaced by two circuits, both grounded at the fault location; for the ith conductor this is indicated by dotted lines in the drawing. For the left-hand section, (10) with $n = \infty$ directly applies, and therefore the additional tower voltage rise $\Delta V_i'$ caused by I_i in this section is

$$\Delta V_i' = (1 - \mu_i)I_i Z_{\infty\infty}.$$

It can be readily seen that $\Delta V_i''$ due to I_i in the right-hand section is equal to $\Delta V_i'$ in magnitude, but opposite in sign; thus

$$\Delta V_i = \Delta V_i' + \Delta V_i'' = 0;$$

that is, in the given case currents in external parallel circuits do not have any influence on the voltage rise of the faulted tower.

If, however, the half-lines on the two sides are different, resulting in different μ_i values, the effect of I_i may have to be taken into account. This may also be the case if one or both of the half-lines cannot be considered infinite. For example, if the line consists of n towers on the left-hand side and N towers on the right-hand side in Fig. 12, then

$$V_i = (1 - \mu_i)I_i(Z_{Nn} - Z_{nN}) \tag{17}$$

where the minus sign of the Z_{nN} term indicates that the I_i current in the left-hand loop, unlike the one in the right-hand loop, flows out of the skywire and into the phase conductor at the fault location. In (17), I_i is positive if flowing in the same direction as I_b, otherwise its sign must be changed.

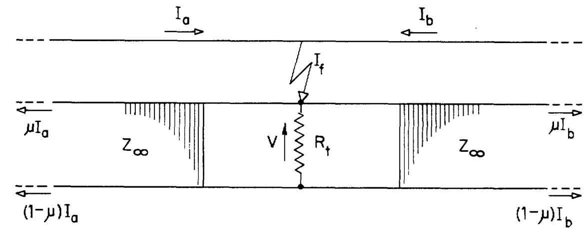


Fig. 11. Infinite full-line—fault fed from both directions.

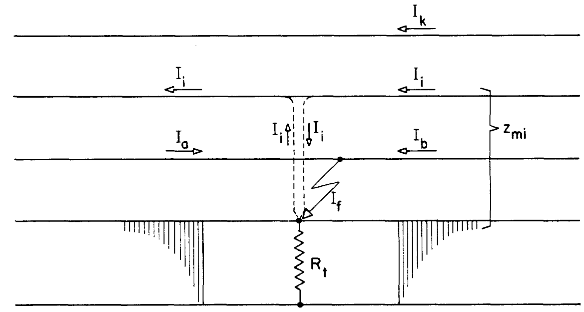


Fig. 12. Scheme for determining the effect of induction from parallel circuits.

With the values of ΔV_i computed through (17), the total extra voltage impressed on the faulted tower will be

$$\Delta V = \sum_{i=1}^{k} \Delta V_i.$$

From (17) it is apparent that if the half-lines on the two sides of the fault are about equal, that is, if $Z_{Nn} \approx Z_{nN}$, the effect of external conductors can be disregarded.

Multiple Ground Conductors

It has been assumed so far that only one overhead ground conductor (skywire) interconnected the towers in a line. Quite often, however, there are more than one installed. Additional skywires will reduce the overall series impedance and thus, everything else being equal, the tower voltages during a ground fault will be lower than in the case of a single skywire.

In some cases continuous buried counterpoise conductors are installed between towers to assist the skywires in carrying heavy fault currents. Since the counterpoise conductors parallel the skywires and are in contact with the earth, they will reduce both the overall series and parallel impedances of the ground network and so they, too, will decrease the tower voltages during ground faults.

In considering multiple ground conductors, basically the methods and equations presented for single skywires can still be applied. However, the z_p, z_s, and z_m values in any of the previous equations must represent the whole group of conductors. In developing equations for the self and mutual impedances of the group, the mutual coupling between the individual ground conductors must also be taken into consideration.

Series Impedances

Formulas for the self-impedance of a ground-return circuit and for the mutual impedance between two such circuits are available from the literature.[8] Involved in the equations are the radii of the ground conductors and the distance between the circuits. In the case of a single skywire, these equations are directly applicable to obtain z_s and the mutual impedance z_m between the skywire and the faulted phase conductor.

If certain conditions are met, a group of n ground conductors may be represented by an imaginary single conductor whose self and mutual impedances are given by the same equations as before; only now the geometric mean radius of the group and the geometric mean distance relative to the faulted phase[8] must be substituted. The conditions to be met are: the current distribution between the n conductors should be uniform; or the n conductors should be in a mutually symmetrical position. If the self impedances of the individual conductors in the group are equal and the mutual impedances within the group do not differ very markedly, a uniform current distribution may be assumed without undue error.

In most practical cases, the group of skywires or the group of counterpoise conductors can be dealt with by the method just described. On the other hand, the skywires *and* counterpoise conductors usually cannot be combined in such a manner because of the different conductor materials and sizes used in the two groups and the resulting uneven current distribution. In this case, it will be necessary to determine the overall self and mutual impedances by equations whose validity is rather general. For $n = 2$, such equations are derived in Appendix II.

Parallel Impedance

With only skywires present, the parallel impedances of a ground network consist of the tower ground resistances. The ground resistance R_t of a tower can be expressed as

$$R_t = \frac{\rho}{2\pi r_o}$$

where ρ is the resistivity of soil and r_o is the radius of an equivalent hemispherical electrode having the same ground resistance as the tower. This radius is in the order of two to three meters for an average single- or double-circuit tower and may be six to ten meters for bridge-type towers with multiple tower legs.

If Z_p is the average tower ground resistance in a line, the parallel impedance per unit length of the ground network is defined as $z_p = l_s Z_p$. With this, the parallel admittance is $y_p = 1/z_p$.

The presence of counterpoise conductors completely changes the situation because these conductors will discharge current into the ground through a considerable distance on both sides of a tower under potential. To obtain an estimate of this distance, the buried conductor can be represented by a distributed parameter network with series impedance z_c and parallel admittance y_c, both per unit length. It will be recalled that such a network may be considered infinite if its length is greater than $l_0 = 2/\sqrt{z_c y_c}$, and thus the portion beyond the distance l_o will have only a negligible rate of current discharge into the ground.

In practical cases, l_0 proves to be at least three to four times as long as the span length. Under these circumstances, an equivalent scheme can be constructed where the overall admittance per unit line length y_p is defined as the weighed sum of the admittances per unit line length of the towers and of the counterpoise conductors. In other words, if $y_t = 1/l_s R_t$

$$y_p = \alpha y_t + \beta y_b \tag{18}$$

where α and β are weighing factors.

It remains to determine y_c, α, and β. A method for determining y_c is given in Appendix III, and the following equation is obtained for the parallel admittance:

$$y_c = \frac{1000}{\rho(2.65 + 0.34 \log \rho)} \quad \frac{\text{mho}}{\text{km}}$$

where ρ is the soil resistivity in ohm·meters.

The factor α accounts for the fact that the tower groundings and the counterpoise conductor(s) are not independent but interfere with each other as ground electrodes. Hence, the tower groundings are only partly effective and in (18) y_t must be derated. Thus, factor α lies between 0 and 1; as to the exact value, there is insufficient information available on which to base its determination. From the evaluation of a few actual cases the use of $\alpha = 0.35$ is recommended.

The factor β must be applied if the number of counterpoise conductors is more than one; for one counterpoise conductor $\beta = 1$. If there are n_c such conductors with the distances between them much shorter than l_o, their total parallel admittance will be less than $n_c y_c$ because the conductors interfere with each other in discharging current into the ground. Obviously, β must lie between 1 and n_c, and, for $n_c = 2$, $\beta = 1.5$ is recommended as long as there are no more accurate values available.

Simplifications for Lines with Continuous Counterpoise

The first of these concerns the factors λ and λ'. For only skywires present, the branches λZ_s and $\lambda' Z_s$ were introduced in the equivalent schemes to compensate for the inaccuracies inherent in the distributed parameter representation. If the groundings in the line consisted of the counterpoise conductors only with no tower grounds present, the distributed parameter representation would be exact.

In the presence of both counterpoise conductors and tower groundings the largest part of the current entering the ground will be discharged by the former, since in most cases $\beta y_c \gg \alpha y_t$. The error will be insignificant, therefore, if $\lambda = \lambda' = 0$ is assumed for counterpoised lines. In the case of finite half-lines, however, n must be defined now as the number of spans between the fault and the end of the line, i.e., one more than the number of towers along the half-line.

Another simplification permissible for counterpoised lines is the omission of the term $1/R_t$ in (11) and (12). Since the tower ground resistances are represented by y_t, a distributed parameter, this will take care of the tower at the fault as well.

With the foregoing changes, most of the equations in the previous sections can be reduced to much simpler forms. For example, (16), which provides the tower voltage in the most general case, becomes

$$V = (1 - \mu) Z_0 \frac{I_a B_n A_N + I_b B_N A_n}{A_n + A_N}.$$

It is easy to show that the following simple equation now interrelates A_n, B_n, and D_n:

$$B_n = A_n D_n.$$

Field Tests

To check the accuracy with which predictions can be made using the equations and curves developed in the previous sections, it was deemed desirable to perform measurements in the field. An opportunity for such tests arose when Ontario Hydro became concerned about the possible tower voltages during ground faults, which may occur on the 230-kV line between Lakeview Generating Station and Manby Transformer Station.

A detailed description of the tests is given in a previous paper by the author.[9] Here it will be only noted that, basically, the tests consisted of simulating ground faults on a deenergized circuit at three towers; one in a counterpoised line section, the other in a section without counterpoise, and the third at the meeting of the two sections. Skywires were installed throughout. The faults were supplied from a portable generator located at the end of the line; thus, the case was represented where faults were fed from one direction only. Involved were both infinite and finite half-lines.

Comparing test results with corresponding calculated values, they were found to be, in all three cases, within 15 percent. Even allowing for inaccuracies in the measurement, it was estimated that the deviation between the true and calculated values would not exceed 30 percent. In assessing the degree of hazard, such an accuracy can still be considered satisfactory.

Evaluation of Shock Hazard

Determination of Maximum Tower Voltage

Maximum voltage for a given tower: If the voltage conditions of a given tower are to be investigated, it can be accomplished through the findings in the previous sections, provided that the ground fault currents are known. To obtain the highest voltage rise, the fault should be assumed to occur at the particular tower in question and, since the lowest coupling between the phase and ground conductors will produce the highest tower voltage, on the phase which is the furthest from the ground conductors. For example, in a vertical arrangement of phases and with only skywires present, the lowest phase should be assumed faulty.

Often, however, the object of investigation is not a given tower but a whole line. In such a case, the *worst* fault location, producing there the highest tower voltage, has to be found. This is fairly easy to do for long lines and somewhat more involved for short ones.

Worst fault location in long lines: A long line is one which has a middle section where the line is divided by each tower into infinite halves. This middle section will be called the core of the line. Within the core, the ground impedance does not vary and it is the available fault current alone on which the voltage rise of the faulted tower depends. Therefore, the maximum tower voltage will appear during a fault that occurs at the point where the fault current is at its maximum.

In an end section, the available fault current may be considered approximately constant and the worst fault location will depend on the network termination Z_g. If $Z_g < Z_0$, the impedance seen by the fault current component arriving from the far end will decrease as the fault is moved from the core towards the end of the line, and if $Z_g > Z_o$, it will increase. The impedance seen by the current component arriving from the near end will always be smaller closer to the line end, because both Z_n' and D_n are the smallest there (see Figs. 9 and 10). The worst fault location is, therefore, at the core end of the section (at about $n = 4/\sqrt{K}$) if $Z_g < Z_o$; and may be either there or at the station end, depending on the ratio of the two fault current components, if $Z_g > Z_o$.

For the whole line, then, the following can be said. If $Z_g < Z_o$ at both ends, which is the most likely case, the worst fault location will be in the core and where the fault current is at maximum. If for any one of the terminating stations $Z_g > Z_o$, the foregoing may still hold, but the first tower near that station should also be checked.

Worst fault location in short lines: There is no simple way to predict the worst location of a ground fault in a short line. This location will depend on the total length of the line and on the ratios I_a/I_b, Z_{ga}/Z_0, and Z_{gb}/Z_0. In a given case, the simplest procedure is to compare the tower voltages near both ends and in the middle of the line. In Fig. 13, a few characteristic voltage curves are shown as an example, assuming that I_f is constant throughout the entire line, $I_a = 0.25\, I_f$, and $R_{sta} = R_{stb}$.

Touch and Step Voltages Near Towers

The measure of hazard associated with tower voltages is the maximum touch or step voltage that can be contacted by a person near a tower. With the assumption that the greatest distance from which a tower can be touched is one meter, the maximum touch voltage may be defined as the largest potential difference measured between the tower structure and any point on the earth surface at a distance of one meter from the structure. Further, considering that a human step of one meter would be relatively long, and that the step potentials due to the current in a ground electrode are at their maximum nearest to the elec, trode (if no grading conductors are installed around the tower)- the foregoing definition also describes the maximum step voltage. From these definitions it follows that the touch and step voltages to which a person can be exposed near a tower are only a fraction of the total tower voltage rise.

Since analytical determination of the exact ratio of maximum touch voltage to total tower voltage is quite complicated, most investigators have preferred measurements for the purpose, either on models in electrolytic tanks or on real towers.[2], [4], [10], [11] The equivalent radii of the towers dealt with in these works were in the order of $r_o = 1.5$ to 2.5 meters. During Ontario Hydro's simulated fault tests,[9] touch and step potentials were measured near towers with equivalent radii in the order of 6 to 8 meters.

By comparing the test results it is apparent that, in general, the higher r_o is, the smaller is the ratio of the maximum touch voltage to the total tower voltage rise. For large towers where $r_o = 6$ to 8 meters, this ratio will rarely exceed 15 percent, and for towers with $r_o = 1$ to 3 meters, it may be as high as 25 to 30 percent.

Another conclusion drawn from Ontario Hydro's tests is that the ratio of maximum touch voltage to the total tower voltage cannot be expected to be lower because of the presence of counterpoise conductors. It may be reduced by installing grading conductors all around the footing of a tower.

Shock Hazard in a Typical Case

At this point, a numerical example may serve well to illustrate the tower voltage conditions in a typical case. A double circuit suspension tower is considered in a long 230-kV transmission line, located in the core, i.e., where the half-lines on both sides of the tower can be considered infinite. The line will be assumed to have two 3/8-inch steel skywires, but no counterpoise. The ground fault current at the location in question is 5000 amperes and the soil resistivity in the area is 300 ohm·meters.

For the conditions described, a tower voltage rise of 5410 volts is calculated. Considering that $r_o = 5$ meters for the tower in question, it can be assumed that the maximum touch voltage is about 15 percent of the total voltage rise. Hence, a maximum touch voltage of 810 volts must be anticipated.

The effect of an 810-volt shock can be estimated from the curve of Fig. 14, illustrating the manner in which the probability of death increases with the voltage contacted hand-to-hand or hand-to-foot. Based on the distribution of fault durations for Ontario Hydro's 115- and 230-kV systems, shown in Fig. 15, the curve in Fig. 14 was obtained by combining the test results and conclusions of several investigations.[12]–[14] It is assumed that firm contacts are made, with bare hands and feet, to the objects completing the circuit. Since comparatively little is known of the magnitudes of human skin resistance during extremely short exposures, the curve in Fig. 14 is primarily an indication of the relative values of the foregoing probability. In any case, under the given conditions the probability of death for an 810-volt shock could be quite high (almost 40 percent if the assumptions used in computing the probabilities in Fig. 14 are considered).

Although such high probabilities may be associated with many locations, tower lines have seldom been considered dangerous for that reason. The reasons for this are the following: 1) under usual circumstances, the probability of an *accident*, that is, the simultaneous occurrence of a ground fault and of a person being in contact with a tower affected by the fault, is extremely low; 2) in not every accident will the person involved be subjected to the maximum theoretical touch voltage since this is available only at the faulted tower; 3) in many cases the contact resistance of footwear (neglected in this investigation) effectively reduces the danger. It would appear that the level of safety is often determined by factor 1) rather than by the probability of fatality associated with the contacted voltage.

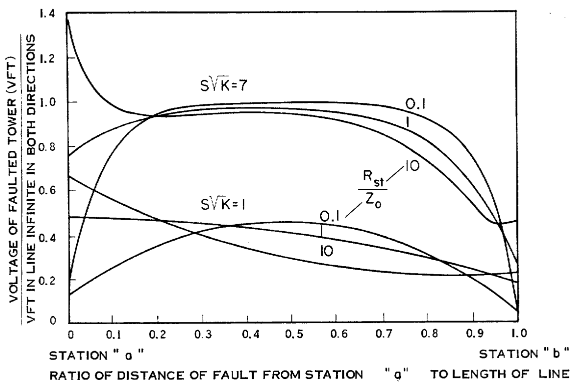


Fig. 13. Voltage rise of faulted tower in a short line. S = total number of spans in line.

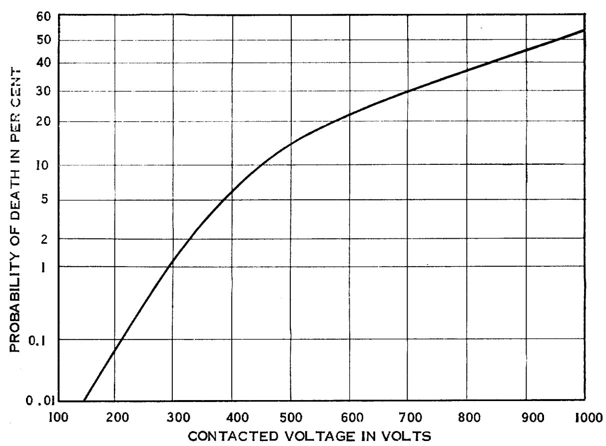


Fig. 14. Probability of fatality vs. voltage contacted hand-to-hand or hand-to-foot. (Assumed skin resistances as in Elek[12] and Dalziel *et al.*;[13] assumed fault duration as shown in Fig. 15.)

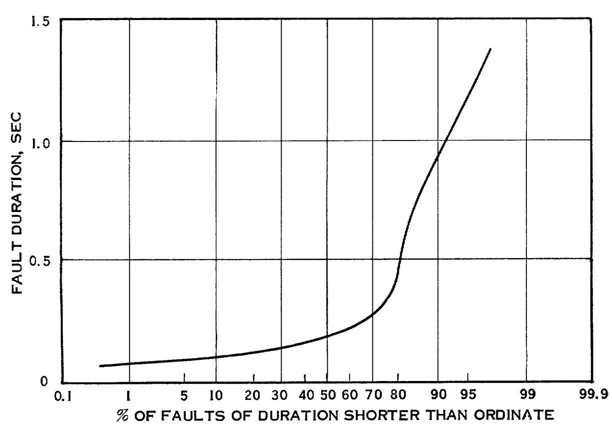


Fig. 15. Probability distribution of fault durations for Ontario Hydro's 115/230-kV system.

There are many cases, however, where factor 1) cannot be assumed to prevail. In such an event, extra measures must be taken. These measures are aimed at reducing the tower voltages. For example, the German VDE-Regulations (0141/1958, paragraph 29) require that at towers located in towns, villages, or other inhabited areas the maximum touch voltage shall not exceed 65 to 165 volts, depending on the fault duration (1 to 0.1 second).

There are several methods by which tower voltages can be reduced. Some of these are described in the next few paragraphs.

TABLE I

Comparison of Tower Voltages For Various Ground Conductor Arrangements

Case	Skywires	Counterpoise Conductors	Total Tower Voltage, V	Maximum Touch Voltage, V
A	2–steel, 3/8 inch		5410	810
B	2–al-weld, 7 no. 5		2245	336
C	2–steel, 3/8 inch	1–copper, no. 2	1447	217
D	2–al-weld, 7 no. 5	1–copper, no. 2	880	132
E	2–steel, 3/8 inch	2–copper, no. 2	837	126

Reduction of Tower Voltages

One group of possible measures is aimed at reducing the total tower voltages during ground faults. This can be achieved by installing higher-conductivity skywires, more skywires, counterpoise conductors, or a combination of these.

To illustrate the effect of some of these steps on the example described in the previous section, a few alternatives are compared in Table I.

In Case A, the probability of fatality was found to approach possibly 40 percent. Solution B would reduce this probability to about 1/20th of the former value; solution C would effect a further reduction by about one order of magnitude. This last arrangement (and, of course, D and E as well) would practically exclude fatal shock, even if an accident should occur.

Another group of methods for improving dangerous tower voltage conditions seeks to reduce the ratio of maximum touch voltage to the total tower voltage. Included among these methods are the installation of special grading conductors as parts of the tower grounding scheme, the application of crushed stone layers, or the provision of nonconducting fences around tower structures. A more detailed discussion of these measures and their relative merits is outside the scope of this study.

Conclusion

An analytical method has been developed, as presented in this study, to determine the tower voltages during ground faults and to determine the degree of shock hazard caused by such potential rises. The theory is sufficiently general to cover long and short lines, consider terminations, and account for any ground conductor arrangements and combinations in the line.

A limited number of field tests were carried out on transmission towers during simulated ground faults, to check the theory. The agreement between the theoretical and test results was found satisfactory.

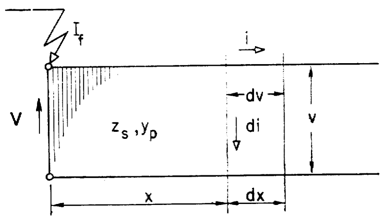
It was found that contacting a tower coincident with a ground fault nearby may present substantial hazard even in an average case and the present very low accident rate at towers is primarily due to the very low probability of this coincidence that normally prevails. Where this factor cannot be relied upon, however, special measures should be considered. Some of these measures, such as selecting better conductivity skywires and/or installing counterpoise conductors, are described and their protective effects compared.

Appendix I

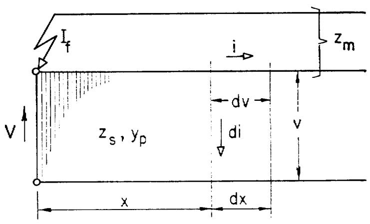
Distributed Parameter Representation of Transmission Line Ground Circuits

If the distributed line parameters are z_s, the series impedance per unit length, and $y_p = 1/z_p$, the parallel admittance per unit length, then the following differential equations describe the circuit illustrated in Fig. 16(a):

$$-di = vy_p dx$$
$$-dv = iz_s dx. \qquad (19)$$



(a)



(b)

Fig. 16. Half-line with distributed parameters. (a) Half-line and infeed on opposite sides of fault. (b) Half-line and infeed on the same side of fault.

If the fault current is supplied from the line side, as in Fig. 16(b), the coupling between the faulted phase conductor and ground conductor must be observed. This case is described by the following modified version of (19):

$$-di = vy_p dx$$
$$-dv = iz_s dx - I_f z_m dx. \tag{20}$$

Here z_m is the mutual impedance per unit length between the faulted phase and ground conductors, and I_f is the total ground fault current.

With the appropriate boundary conditions, the solutions of these equations will provide the basic formulas for the tower voltage rises in the various cases. For example, (12)–(14) were obtained by solving (20) with the boundary conditions

$$v(0) = V + \lambda l_s[I_f z_m - i(0)z_s]$$

$$i(l) = I_f + \frac{1}{Z_g}[v(l) - I_f \lambda' l_s(z_s - z_m)].$$

These can be written up with the help of Figs. 8 and 16(b). The tower voltage rise V is then given by

$$V = \frac{R_t Z_N}{R_t + Z_N}[I_f - i(0)].$$

Appendix II

Self and Mutual Impedances of Multiple Ground Conductors

The number of ground conductors is n. According to Fig. 17, and with the notation used in the drawing, the following equations apply:

$$\begin{aligned} E_a &= z_{aa}I_a + z_{ab}I_b + \dots + z_{an}I_n \\ E_b &= z_{ab}I_a + z_{bb}I_b + \dots + z_{bn}I_n \\ &\vdots \\ E_n &= z_{an}I_a + z_{bn}I_b + \dots + z_{nn}I_n. \end{aligned} \tag{21}$$

For a group of ground conductors interconnected at both ends, $E_a = E_b = \dots = E_n = E$, and in such a case the self impedance of the group can be defined as

$$z_s = \frac{E}{I_a + I_b + \dots + I_n}. \tag{22}$$

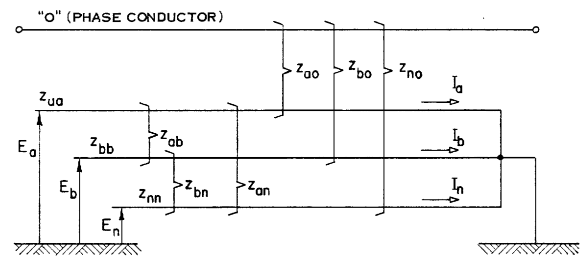


Fig. 17. Multiple ground conductors—general arrangement.

The mutual impedance between the group and an external conductor ("o" in Fig. 17) is

$$z_m = \frac{I_a z_{ao} + I_b z_{bo} + \dots + I_n z_{no}}{I_a + I_b + \dots + I_n}. \tag{23}$$

For substitution into (22) and (23), the currents $I_a, I_b, \dots, I_n$ must be determined by inverting (21). For example, if $n = 2$, one obtains

$$z_s = \frac{z_{aa}z_{bb} - z^2_{ab}}{z_{aa} + z_{bb} - 2z_{ab}}$$

and

$$z_m = \frac{z_{ao}(z_{bb} - z_{ab}) + z_{bo}(z_{aa} - z_{ab})}{z_{aa} + z_{bb} - 2z_{ab}}.$$

Appendix III

Parallel Admittance of Counterpoise Conductors

The parallel admittance of a buried conductor is a fictitious quantity. The concept originates from the assumption that the conductor and the surrounding soil can be considered as a ladder network with distributed parameters.[15] If z_c is the series impedance, y_c the parallel admittance (both p.u. length), and l the length of conductor, its resistance to ground will be

$$R_1 = \sqrt{\frac{z_c}{y_c}} \coth l\sqrt{z_c y_c}$$

in analogy with (6). If R_1 is known, from the preceding equation y_c can be determined.

The value of R_1 is, however, not known in advance and for the analytical determination of y_c another approach must be used. In this, the ground resistance of the conductor will first be obtained by a different method. By assuming equipotential conductor surface it can be shown[16] that the resistance of a not-too-long conductor of radius a and buried near the surface is

$$R_2 = \frac{\rho}{\pi l} \ln \frac{l}{a}.$$

By using

$$R_1 = R_2 \tag{24}$$

an equation is obtained containing two unknowns, y_2 and l.

To determine y_c, a second equation is required. To obtain this, an idea described by Sailer[17] will be utilized. If, for a given conductor, curves of R_1 for various y_c values and a curve for R_2 are plotted as functions of l, there will be one R_1 curve which almost coincides with R_2 for a wide range of l. This will be considered the *true* R_1 curve for the case and the y_c parameter of this curve will be the solution of the problem.

This idea suggests that the second equation needed to determine y_c will be

$$\frac{dR_1}{dl} = \frac{dR_2}{dl}. \tag{25}$$

Solving (24) and (25) for y_c, the following equation is obtained:

$$y_c = \frac{1000}{\rho(2.65 + 0.34 \log\rho)} \approx \frac{300}{\rho} \tag{26}$$

where ρ must be substituted in ohm·meters and y_c is obtained in mho/km. In obtaining (26), the counterpoise conductor was assumed to be no. 2 copper, but the equation will provide good approximation for most other conductors as well.

Acknowledgment

The author gratefully acknowledges the assistance of both Ontario Hydro and the University of Waterloo in making this project successful. Special thanks are due to Dr. H. C. Ratz of the University of Waterloo for numerous helpful suggestions.

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Discussion

S. G. Pann (Los Angeles Department of Water and Power, Los Angeles, Calif.): The author has presented a very useful and timely paper which gives transmission line designers an adequate analytical method for predicting the magnitude of tower voltages during ground faults. As the introduction points out, this has not been too much of a problem in the past, simply because of the remote possibility of having personnel adjacent to or touching transmission towers when a ground fault occurs.

Because of the air pollutants and infrequent precipitation in the Los Angeles area, the Department has found it necessary to wash insulators on energized lines for many years. During this process there is always the possibility of flashing an insulator string. We have actually experienced flashovers of this type on a few occasions. Although we have had our wash crews on and near the towers at the time of these faults, the voltage levels have never been of an unusually hazardous nature.

We believe these low voltages are a result of our transmission line design, which includes a low tower footing resistance in addition to overhead ground wires. The method we use to insure a low footing resistance is to tie the tower stubs to the reinforcing steel and then use either buried grounding rings, ground rods placed beneath the tower footings, or counterpoise wires as a grounding scheme.

As the practice of washing insulators on hot lines becomes more widespread, utilities must become aware of this problem and take adequate steps as proposed in this paper to reduce step and touch voltages.

Manuscript received July 19, 1966.

J. Endrenyi: Tower voltages during ground faults certainly depend to a large extent on the tower grounding arrangement. With the type of grounding described by Mr. Pann, it is quite possible that dangerous situations are prevented even if the fault currents are high, as can be expected in urban areas. Furthermore, by installing grounding rings around the tower structures, the ratio of the maximum touch voltage to the total tower voltage can be effectively reduced and, thereby, the hazard further decreased. In many cases, however, grounding rings or other improvements in tower grounding may not be as economical as installing better conductivity skywires or counterpoise conductors. Economic considerations will, obviously, play a most important role in selecting the best method when designing or improving the grounding arrangement of a transmission line.

Manuscript received September 8, 1966.