On the calculation of the potential profile and step potential of an arbitrary multiple driven rod earth electrode system

A. C. Liew
Department of Electrical Engineering, National University of Singapore, 10 Kent Ridge Crescent, Singapore 0511 (Singapore)

(Received July 28, 1992)

Abstract

This paper presents a simplified approximate method for the calculation of the potential profile and step potential of an arbitrary multiple driven rod earth electrode system. It is based on the equal potential assumption and driven rods are represented by their equivalent hemispheres. The expression is therefore simplified and computations significantly reduced. The routine is hence efficient, which renders it suitable for implementation on personal computers.

1. Introduction

The calculation of the potential profile and step potential of an earth electrode system when subject to an injection of current is of importance in the design of earthing systems. For simple and symmetrical electrode systems, it may be possible to obtain analytical solutions to the problem [1–3]. For more complicated systems, numerical solutions are invariably required. Very complicated computer programs are needed and these have been developed to operate on mainframe computers for such analysis.

This paper presents a simplified and approximate method for the calculation of the potential profile and step potential of an arbitrarily located multiple driven rod earth electrode system. This approximate method has the advantage of significantly reducing the tedium of vast amounts of computation, rendering it suitable and attractive for implementation on a personal computer (PC). Such a program using this approximate analysis has been developed. It is very user friendly with attractive graphic outputs. The program has proved to be useful in the design of medium-sized earthing systems.

2. Calculation of the potential profile in the vicinity of an earth electrode system

It has been shown in the book by Tagg [1] that the general expression for the resistance of any form of earth electrode is given by

$$R = \rho/2\pi C \tag{1}$$

where ρ is the soil resistivity (Ω m), and C is the electrostatic capacity of the electrode combined with its image above the surface of the earth, the combined electrode being considered as being in air.

Another very useful concept introduced in the analysis of the earth electrode resistance is the radius of the equivalent hemisphere of the electrode. This is the radius of an equivalent hemisphere giving the same earth resistance as the electrode under consideration. The resistance of a hemispherical electrode of radius r is easily derived to be

$$R = \rho/2\pi r \tag{2}$$

To obtain the equivalent hemisphere of a particular electrode, the resistance of the electrode is equated to that of the hemispherical electrode. In the case of a single driven rod of depth l and diameter d,

$$\frac{\rho}{2\pi r} = \frac{\rho}{2\pi l}\left(\ln\frac{8l}{d} - 1\right) \tag{3}$$

giving the radius of its equivalent hemisphere as

$$r = l\bigg/\left(\ln\frac{8l}{d} - 1\right) \tag{4}$$

0378-7796/93/$6.00

In the use of the above concepts, it is necessary to ensure that the spacing between the driven rods is greater than twice the radius of the equivalent hemisphere.

The derivation of the expression for the potential profile in the vicinity of a driven rod earth electrode system injected with a current I begins with three basic assumptions:

(a) all driven rods are approximated by their equivalent hemispherical electrodes;

(b) the flow of current from each equivalent hemisphere out into the soil is uniform in all directions;

(c) the potentials of all driven rod or equivalent hemispherical electrodes in the electrode system are equal, that is, an equal voltage or potential assumption is made [4].

Consider a driven rod bed electrode system of n rods arranged in a general arbitrary non-symmetrical configuration as shown in Fig. 1. The coordinates of the n rods are (x_i, y_i) for $i = 1, 2, \ldots, n$; each carries a charge Q_i and the current entering it is I_i.

For a current I_i entering the ith electrode of equivalent hemispherical radius r_i at (x_i, y_i) (Fig. 2), the voltage difference or drop between electrode i and a point P at a distance r_{ip} is given by

$$V_{\text{drop}ip} = \frac{\rho I_i}{2\pi}\left(\frac{1}{r_i} - \frac{1}{r_{ip}}\right) \tag{5}$$

where r_{ip} is the distance from (x_i, y_i) to (x_p, y_p).

If V_i is the voltage of electrode i, then the voltage contribution at P in position (x_p, y_p) due to current injection I_i at electrode i is

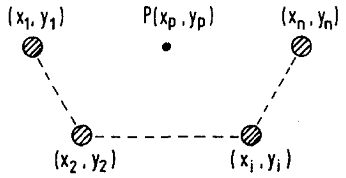

Fig. 1. System of n rods in any configuration.

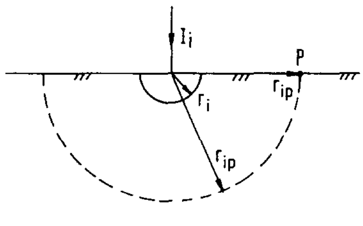

Fig. 2. Effect of current injection at ith electrode on a point P at a distance r_{ip}.

$$V(x_p, y_p) = V_i - V_{\text{drop}ip} \tag{6}$$

The total voltage at P due to current injection in all the electrodes can be obtained by summing all such contributions from the effects of the system on n electrodes as

$$V(x_p, y_p) = \sum_{i=1}^{n} (V_i - V_{\text{drop}ip}) \tag{7}$$

Based on the equal potential assumption, since all the electrodes are electrically connected, we have

$$\begin{aligned} V(x_1, y_1) &= V(x_2, y_2) = \cdots = V(x_i, y_i) \\ &= \cdots = V(x_n, y_n) \end{aligned} \tag{8}$$

Therefore,

$$\begin{aligned} \sum_{i=1}^{n} (V_i - V_{\text{drop}i1}) &= \sum_{i=1}^{n} (V_i - V_{\text{drop}i2}) \\ &= \cdots = \sum_{i=1}^{n} (V_i - V_{\text{drop}ik}) \\ &= \cdots = \sum_{i=1}^{n} (V_i - V_{\text{drop}in}) = \text{GPR} \end{aligned} \tag{9}$$

where $V_{\text{drop}ik}$ is the potential drop across the soil between electrodes i and k, and GPR is the ground potential rise of the electrode system.

From the series of equations in (9), we can rewrite

$$\begin{aligned} &\sum_{i=1}^{n} (V_{\text{drop}i1} - V_{\text{drop}i2}) = 0 \\ &\sum_{i=1}^{n} (V_{\text{drop}i1} - V_{\text{drop}i3}) = 0 \\ &\vdots \\ &\sum_{i=1}^{n} (V_{\text{drop}i1} - V_{\text{drop}in}) = 0 \end{aligned} \tag{10}$$

Equation (10) can in turn be simplified as

$$\begin{aligned} &\frac{\rho}{2\pi} \sum_{i=1}^{n} I_i\left(\frac{1}{r_{i2}} - \frac{1}{r_{i1}}\right) = 0 \\ &\frac{\rho}{2\pi} \sum_{i=1}^{n} I_i\left(\frac{1}{r_{i3}} - \frac{1}{r_{i1}}\right) = 0 \\ &\vdots \\ &\frac{\rho}{2\pi} \sum_{i=1}^{n} I_i\left(\frac{1}{r_{in}} - \frac{1}{r_{i1}} = 0\right) \end{aligned} \tag{11}$$

Rearranging in matrix form and in terms of current I_1, we have

$$\begin{bmatrix} \frac{1}{r_2}-\frac{1}{r_{21}} & \frac{1}{r_{32}}-\frac{1}{r_{31}} & \cdots & \frac{1}{r_{n2}}-\frac{1}{r_{n1}} \\ \frac{1}{r_{23}}-\frac{1}{r_{21}} & \frac{1}{r_3}-\frac{1}{r_{31}} & \cdots & \frac{1}{r_{n3}}-\frac{1}{r_{n1}} \\ \frac{1}{r_{24}}-\frac{1}{r_{21}} & \frac{1}{r_{34}}-\frac{1}{r_{31}} & \cdots & \frac{1}{r_{n4}}-\frac{1}{r_{n1}} \\ \vdots & & & \\ \frac{1}{r_{2n}}-\frac{1}{r_{21}} & \frac{1}{r_{3n}}-\frac{1}{r_{31}} & \cdots & \frac{1}{r_n}-\frac{1}{r_{n1}} \end{bmatrix} \begin{bmatrix} I_2 \\ I_3 \\ I_4 \\ \vdots \\ I_n \end{bmatrix} = - \begin{bmatrix} \frac{1}{r_{12}}-\frac{1}{r_1} \\ \frac{1}{r_{13}}-\frac{1}{r_1} \\ \frac{1}{r_{14}}-\frac{1}{r_1} \\ \vdots \\ \frac{1}{r_{in}}-\frac{1}{r_1} \end{bmatrix} [I_1] \tag{12}$$

The component currents $I_2, I_3, \ldots, I_n$ can be solved by Gaussian elimination in terms of I_1. As the total injection fault current I is known, then

$$I_1 + I_2 + I_3 + \cdots + I_n = I \tag{13}$$

and the individual current components I_i can each be determined.

The potential at any point $P(x_p, y_p)$ can then be calculated using eqn. (7). With the potential at any point known, the potential profile and step potentials can be determined.

3. Example using a nine-rod solid square symmetrical configuration

To verify the routine and to demonstrate the applicability of the expressions derived, an example using a nine-rod solid square symmetrical configuration as shown in Fig. 3 is presented. Computations were done using the following parameters:

depth of driven rods, l	1.2 m
diameter of driven rods, d	15 mm
resistivity of soil, ρ	100 Ω m
spacing between rods, s	4 m
total current injected into earth system	100 A

Equation (12) becomes

$$\begin{bmatrix} -4.301 & -0.125 & 0.073 & -0.073 & -0.065 & 0.013 & -0.013 & -0.023 \\ 0.000 & -4.426 & 0.138 & 0.000 & -0.138 & 0.037 & 0.000 & -0.037 \\ 0.073 & 0.013 & -4.301 & -0.073 & -0.013 & -0.125 & -0.065 & -0.023 \\ 0.000 & -0.052 & 0.000 & -4.374 & -0.138 & -0.052 & -0.138 & -0.088 \\ 0.073 & -0.125 & 0.125 & -0.073 & -4.439 & 0.013 & -0.065 & -0.162 \\ 0.138 & 0.037 & 0.000 & 0.000 & 0.000 & -4.426 & -0.138 & -0.037 \\ 0.125 & 0.013 & 0.073 & -0.073 & -0.065 & -0.125 & -4.439 & -0.162 \\ 0.138 & 0.000 & 0.138 & 0.000 & -0.138 & 0.000 & -0.138 & -4.463 \end{bmatrix} \begin{bmatrix} I_2 \\ I_3 \\ I_4 \\ I_5 \\ I_6 \\ I_7 \\ I_8 \\ I_9 \end{bmatrix} = \begin{bmatrix} -4.301 \\ -4.426 \\ -4.301 \\ -4.374 \\ -4.439 \\ -4.426 \\ -4.439 \\ -4.463 \end{bmatrix} [I_1] \tag{14}$$

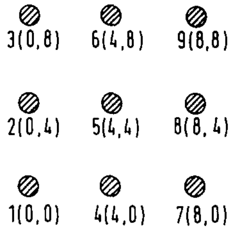

Fig. 3. Coordinates in metres of a nine-rod solid square configuration.

TABLE 1. Current distribution among individual rods in a nine-rod solid square electrode configuration

Rod no.	Location (x, y) (m)	Current (A)
1	(0, 0)	11.488
2	(0, 4)	10.939
3	(0, 8)	11.488
4	(4, 0)	10.939
5	(4, 4)	10.293
6	(4, 8)	10.939
7	(8, 0)	11.488
8	(8, 4)	10.939
9	(8, 8)	11.488
Total current		100.000

and eqn. (13) becomes

$$I_1 + I_2 + \cdots + I_9 = 100\ \mathrm{A} \tag{15}$$

Solving eqns. (14) and (15) gives the currents $I_1, I_2, \ldots, I_9$ entering each driven rod as shown in Table 1. As expected, the distribution of injected

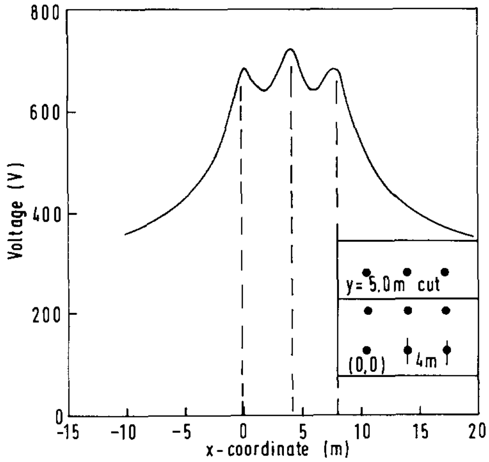

Fig. 4. Potential profile across a $y = 5.0$ m cut in the nine-rod solid square electrode configuration (inset) analysed.

current into the rods is symmetrical, and the currents through the rods located at the corners are larger than those at the centre of the square configuration.

Using an earlier derived expression for earth resistance for an arbitrarily arranged system of driven rod electrodes [4] based on the equal potential assumption, the earth resistance of the above electrode system equals 10.574 Ω. The corresponding ground potential rise (GPR) of the electrode system is thus 1057.4 V.

Application of eqn. (7) gives the potential at any point $P(x_p, y_p)$ on the rod bed system. With this, potential profiles and step potentials between any two points can be determined. The above expressions are implemented on a personal computer and plots of potential profiles along any selected cut can be displayed graphically. Figure 4 shows the potential profile across the cut $y = 5.0$ m.

The above example illustrates the applicability of the expressions derived for an electrode system which is symmetrical. Clearly, the expressions are also applicable for a system of arbitrarily located multiple driven rods. The latter is important for the optimal location of a system of driven rods to achieve a given value of resistance in a given area and shape of land available. Such optimal design capabilities have also been included in the software developed, but this is not the emphasis and scope of the present paper.

4. Conclusions

A simplified and approximate method for the calculation of the potential profile and step potential of an arbitrarily located multiple driven rod earth electrode system has been presented. This simplified method is very efficient and renders it suitable for implementation on a personal computer. Such a program has been developed and includes many features for the design and optimization of earth electrode systems.

References

1 E. F. Tagg, *Earth Resistances*, Pitman, London, 1964.
2 A. P. Sakis Meliopoulos, *Power System Grounding and Transients*, Marcel Dekker, New York, 1988.
3 *ANSI/IEEE Std. 80-1986, IEEE Guide for Safety in AC Substation Grounding*, IEEE, New York, 1986.
4 A. C. Liew and C. F. Lim, On the calculation of resistance of the multiple driven rod electrode, *Electr. Power Syst. Res.*, 7 (1984) 34–44.