Spring 2020

Homework: Models from Data
DUE: July 16, 2022

Consider the following historical (and classic) data set concerning Canadian lynx and snowshoe hare populations from 1845 to 1903.

Year	Snowshoe Hare Pelts (thousands)	Canada Lynx Pelts (thousands)
1845	20	32
1847	20	50
1849	52	12
1851	83	10
1853	64	13
1855	68	36
1857	83	15
1859	12	12
1861	36	6
1863	150	6
1865	110	65
1867	60	70
1869	7	40
1871	10	9
1873	70	20

Year	Snowshoe Hare Pelts (thousands)	Canada Lynx Pelts (thousands)
1875	100	34
1877	92	45
1879	70	40
1881	10	15
1883	11	15
1885	137	60
1887	137	80
1889	18	26
1891	22	18
1893	52	37
1895	83	50
1897	18	35
1899	10	12
1901	9	12
1903	65	25

Figure 1: Population data.

1. Develop a DMD model to forecast the future population states (note, DMD with two time series can only give you two eigenvalues). Try also bagging the DMD to improve the results. See: https://github.com/duqbo/optdmd (MATLAB) and https://github.com/kunert/py-optDMD (Python)
2. Do a time-delay DMD model to produce a forecast and compare with regular DMD. Determine if it is likely that there are latent variables. Try also bagging the DMD to improve the results.
3. Empirical Predator-Prey models such as Lotka-Volterra are commonly used to models such phenomenon. Consider the model $\dot{x} = (b - py)x$ and $\dot{y} = (rx - d)y$. Use the data to approximately fit values of b, p, r and d.
4. Find the best fit nonlinear, dynamical systems model to the data using bagging sparse regression SINDy (See: https://github.com/dynamicslab/pysindy/blob/master/examples/13_ensembling.ipynb)

I've included in your emails MATLAB/Python code for solving (i) A reaction-diffusion system of equations, and (ii) The Kuramoto-Sivashinsky (KS) equation.

1. Train a NN that can advance the solution from t to $t + \Delta t$ for the KS equation
2. Compare your evolution trajectories for your NN against using the ODE time-stepper provided with different initial conditions
3. For the reaction-diffusion system, first project to a low-dimensional subspace via the SVD and see how forecasting works in the low-rank variables using a NN.

For the Lorenz equations, consider the following.

1. Train a NN to advance the solution from t to $t + \Delta t$ for $\rho = 10, 28$ and 35. Now see how well your NN works for future state prediction for $\rho = 17$ and $\rho = 40$.

This is an exploratory homework. So play around with the data and make sure to make lots of plots. Good luck, and have fun.

Grading and Homework Write Ups and GitHub

This homework should be written as if it were an article/tutorial being prepared for submission, parts of which can be part of your GitHub page. The following is the expected format for homework submission in addition to porting the write-up to your GitHub:

MAXIMUM NUMBER OF PAGES: 10 (All your code will be on GitHub)

Title/author/abstract Title, author/address lines, and short (100 words or less) abstract.
Sec. I. Introduction and Overview
Sec. II. Theoretical Background
Sec. III. Algorithm Implementation and Development
Sec. IV. Computational Results
Sec. V. Summary and Conclusions

I will grade based upon how completely you solved the homework as well as neatness and little things like: did you label your graphs and include figure captions.

NOTE 1: The report does not have to be long. But it does have to be complete.

NOTE 2: This report is not for me, it is for you! Specifically, for the future you. So write a nice report so that you could reproduce the results if you need the methods addressed here in another year or more.

A few things should be kept in mind when generating your reports:

1. Use a professional grade word processor (Latex or MSword, for example)
2. For equations: Latex already does a nice job, but in Word, use Microsoft Equation Editor
3. Label your graphs. Include brief figure captions. Reference the figure in the text.
4. Figures should be set flush with the top or bottom of a page.
5. Label all equations.
6. Provide references where appropriate.
7. All coding should be shuffled to Appendix A and B. Reference it when necessary.
8. Always remember: this report is being written for YOU! So be clear and concise.
9. Spellcheck.