Chapter 6
$\mathcal{LODE}$ - The Logic of Knowledge Bases

6.1 Basic Concepts

Exercise 6.1 (Well-formed formulas of $\mathcal{LOE}$ or $\mathcal{LOD}$) Indicate which of the following statements are correct:

1. $A \sqcap B$ is a nonatomic well-formed formula of $\mathcal{LOD}$
2. $A \sqsubseteq \exists R.C \sqcap B$ is a well-formed formula of $\mathcal{LOD}$
3. $A \sqsubseteq \exists R.\top \sqcap B$ is a well-formed atomic formula of $\mathcal{LOD}$
4. $A \equiv R(a,b)$ is a well-formed formula of $\mathcal{LOD}$
5. $A \sqsubseteq \exists R.(\neg\forall S.C)$ is a well-formed formula of $\mathcal{LOD}$

Exercise 6.2 Given a Tbox $\mathcal{T}$ and two w.f.f. P,Q, which of the following statements are true?

1. P and Q are satisfiable with respect to $\mathcal{T}$ if and only if $\mathcal{T} \models P \wedge Q$
2. $\mathcal{T} \models \neg$ $(P \subseteq Q)$ is a logical consequence of $\mathcal{T} \models P \cap Q \subseteq \bot$
3. $\mathcal{T} \models P \cap Q \subseteq \bot$ is a logical consequence of $\mathcal{T} \models P \subseteq Q$
4. $\mathcal{T} \models P \subseteq Q$ is a logical consequence of $\mathcal{T} \models P \cap Q \subseteq \bot$
5. $\mathcal{T} \models \neg((Q \subseteq P) \cup (P \subseteq Q)$ is a logical consequence of $\mathcal{T} \models P \cap Q \subseteq \bot$
6. $\mathcal{T} \models \neg((Q \subseteq P) \cup (P \subseteq Q))$ if and only if $\mathcal{T} \models P \cap Q \subseteq \bot$

Exercise 6.3 Given $\mathcal{T}$ a terminology in Description Logics written in a language L, and I the interpretation function that maps $\mathcal{T}$ to the domain Δ. Having C, C_1, C_2 in $\mathcal{T}$, say which of the following statements are true:

1. if $\mathcal{T} \models C_1$ then $\mathcal{T} \models C_1 \sqsubseteq C_2$ for every formula C2
2. $I(\exists R.\top)=\{a\in \Delta \mid \text{there exists b so that } (a,b)\in I(R)\}$
3. $I(C_1 \sqsubseteq C_2)=\top$ iff $I(C_1)=\top$ and $I(C_2)=\top$
4. $\mathcal{T} \models C$ if there exists an interpretation I so that $I \models C_i$ for all $C_i \in \mathcal{T}$ and $I(C)=\top$

Exercise 6.4 [Definition of expansion and unfolding in $\mathcal{LODE}$] Say which of the following statements are true (one or more):

1. The conceptual expansion (”expansion”) of an ABox with respect to a reference $\mathcal{LODE}$ definitional TBox applies only after the TBox has been developed (”unfolded”).
2. The result of the exhaustive expansion (”expansion”) of an ABox with respect to all concepts defined in a reference TBox is always and only an Entity Graph ($\mathcal{EG}$).
3. The expansion (”expansion”) of an ABox with respect to a reference TBox cannot extend the original Entity Graph, as formalized by the ABox, with new arcs (”links”).
4. Expansion (”expansion”) of an ABox with respect to a reference TBox may extend the original Entity Graph, as formalized by the ABox, with a new node whose entity is not anonymous.

6.2 Translations

Exercise 6.5 Given the knowledge base $\mathcal{K}$

$$\mathcal{T} = \begin{cases} Lion \sqsubseteq Mammal \\ Whale \sqsubseteq Mammal \\ Mammal \sqsubseteq Animal \\ Mammal \iff livesIn.Habitat \\ Animal \iff feedsOf.Food \end{cases}$$

$$\mathcal{A} = \begin{cases} Willy : Whale \\ PacificOcean : Habitat \\ Krill : Food \\ (Willy, Krill) : feedsOf \\ (Willy, PacificOcean) : livesIn \\ Simba : Lion \\ Africa : Habitat \\ Springbok : Food \\ (Simba, Springbok) : feedsOf \\ (Simba, Africa) : livesIn \end{cases}$$

Draw a Knowledge Graph representation of $\mathcal{K}$.

Exercise 6.6 Formulate $\mathcal{LODE}$ concepts: for each of the following concepts, build a suitable $\mathcal{LODE}$ concept description, using only the concept names

Person, Happy, Animal, Cat,Old, Fish

and the role name

owns

1. Happy person
2. Happy pet owner
3. Person who owns only cats
4. Unhappy pet owners who own an old cat
5. Pet owners who own only cat and fish

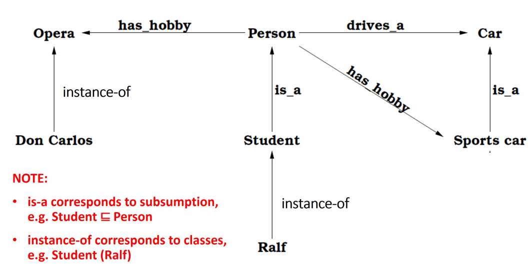
Then, draw a set representation that depicts the described situation

Exercise 6.7 Given the knowledge base

- Car ≡ Vehicle ⊓ ∃hasPart.Wheel ⊓ ∃poweredBy.Engine
- Bicycle ≡ Vehicle ⊓ ∃hasPart.Wheel ⊓ ∃poweredBy.Human
- Boat ≡ Vehicle ⊓ ∃travelsOn.Water
- Boat ⊆ ∀hasPart.¬Wheel
- Car ⊔ Bicycle ⊆ ∀travelsOn.¬Water
- Wheel ≡ Device ⊓ ∃hasPart.Axle ⊓ ∃capableOf.Rotation
- Driver ≡ Human ⊓ ∃controls.Vehicle
- Driver ⊓ ∃controls.Car ⊆ Adult
- Human ⊆ ¬Vehicle
- Wheel ⊔ Engine ⊆ ¬Human
- Human ⊆ Adult ⊔ Child
- Adult ⊆ ¬Child
- Bob :(∃controls.Car)
- Bob : Human
- (Bob, QE2) : controls
- QE2 :(Vehicle ⊓ ∃travelsOn.Water)

draw a possible interpretation of the given knowledge base as a Schema knowledge graph

Exercise 6.8 (Define a $\mathcal{LODE}$ theory) Define a $\mathcal{LODE}$ theory for the following knowledge graph:



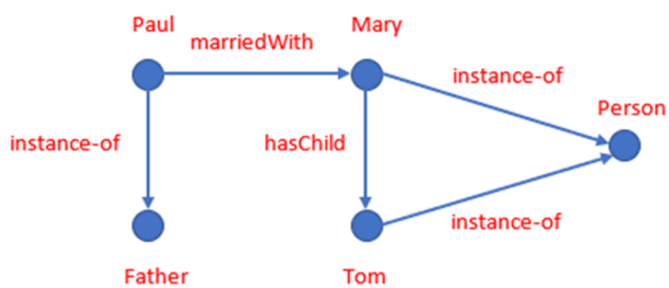
Exercise 6.9 (Define a $\mathcal{LODE}$ theory) Define a $\mathcal{LODE}$ theory for the following problem: In a hospital patients, doctors and computers are equipped with proximity

sensors able to detect whether doctors curated a patient or worked at their computer. The system detected that doctor Peter curated the patient Smith.

Exercise 6.10 Given the LOD etype graph ($\mathcal{ETG}$) corresponding to the following TBOX:

$$\mathcal{T} = \begin{cases} \text{Mother} \equiv \text{Woman} \sqcap \exists \text{hasChild.Person} \\ \text{Father} \equiv \text{Man} \sqcap \exists \text{hasChild.Person} \\ \text{Wife} \equiv \text{Woman} \sqcap \forall \text{marriedWith.Father} \\ \text{Husband} \equiv \text{Man} \sqcap \exists \text{marriedWith.Mother} \end{cases}$$

and given the LOE entity graph ($\mathcal{EG}$) depicted in the figure: Construct the LODE



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Entity Graph ($\mathcal{EG}$) that results from the composition, through development ("unfolding") of the TBOX and expansion ("expansion") of the ABOX and indicate which of the following statements are true (one or more):

1. The EG consists of 6 arcs and 8 nodes
2. EG consists of 8 arcs and 7 nodes
3. EG consists of 8 arcs and 6 nodes
4. The EG contains two nodes representing anonymous entities
5. The EG contains one node representing an anonymous entity
6. The EG contains 4 entities of type Person

6.3 Reasoning

6.3.1 Entailment

Exercise 6.11 Given the following TBOX in $\mathcal{LODE}$ language:

$$\mathcal{T} = \begin{cases} \text{Mother} \equiv \text{Woman} \sqcap \exists \text{hasChild.Person} \\ \text{Father} \equiv \text{Man} \sqcap \exists \text{hasChild.Person} \\ \text{Wife} \equiv \text{Woman} \sqcap \forall \text{marriedWith.Father} \\ \text{Husband} \equiv \text{Man} \sqcap \exists \text{marriedWith.Mother} \end{cases}$$

And the following ABOX in $\mathcal{LODE}$ language:

$$\mathcal{A} = \Big\{ \text{Father(Paul)Person(Mary)Person(Tom)hasChild(Mary, Tom)marriedWith(Paul, Mary)}$$

Indicate which of the following statements are true (one or more):

1. $T \models$ Man(Tom)
2. $T \models$ Man(Paul)
3. $T \models$ Husband(Paul)
4. $T \models$ hasChild(Paul, Tom)
5. $T \models$ Mother(Mary)

Exercise 6.12 Given the following Knowledge Base:

$$\mathcal{T} = \begin{cases} A \iff B \sqcap C \\ C \iff D \sqcap E \\ E \subseteq F \sqcap G \end{cases}$$

$$\mathcal{A} = \Big\{ A(1)$$

Provide $\mathcal{A}'$ obtained by expanding $\mathcal{A}$ with respect to $\mathcal{T}$.

Exercise 6.13 Extend the Knowledge Base $\mathcal{K}$ in exercise 6.7 to a Knowledge Base $\mathcal{K}'$ with a translation of the following sentences:

- A human who legally controls a car holds a driving license and is an adult
- A car with a broken part is broken
- Bob controls a car with a wheel that has a broken axle

Then, say whether the following statements are true or false:

- $\mathcal{K}'$ is consistent
- $\exists$legallyControls.$\top$ is subsumed by $\exists$controls.$\top$ w.r.t. $\mathcal{K}'$
- Bob is an instance of $\exists$ controls.(Car $\sqcap$ Broken) w.r.t. $\mathcal{K}'$

Exercise 6.14 Given the following Knowledge Base:

$$\mathcal{T} = \begin{cases} A \equiv B \sqcap C \\ C \equiv D \sqcap E \\ E \subseteq F \sqcap G \end{cases}$$

$$\mathcal{A} = \Big\{ A(1)$$

Provide $\mathcal{A}'$ obtained by extending $\mathcal{A}$ with respect to $\mathcal{T}$.

Exercise 6.15 (Expansion of a $\mathcal{LODE}$ concept) Given the following TBOX, compute the expansion of the ABox A = StepMother(Mary):

- Mother ≡ Woman ⊓ ∃hasChild.Person
- Father ≡ Man ⊓ ∃hasChild.Person
- StepMother ≡ Woman ⊓ ∃marriedWith.Father
- StepFather ≡ Man ⊓ ∃marriedWith.Mother
- Parent ≡ Father ⊔ Mother ⊔ StepFather ⊔ StepMother

Exercise 6.16 (Expansion of a $\mathcal{LODE}$ concept) Given the following TBOX, compute the expansion of the ABox A = StepMother(Mary), marriedWith(Paul):

- Mother ≡ Woman ⊓ ∃hasChild.Person
- Father ≡ Man ⊓ ∃hasChild.Person
- StepMother ≡ Woman ⊓ ∃marriedWith.Father
- StepFather ≡ Man ⊓ ∃marriedWith.Mother
- Parent ≡ Father ⊔ Mother ⊔ StepFather ⊔ StepMother

Exercise 6.17 (Instance checking in $\mathcal{LODE}$) Given the following $\mathcal{LODE}$ theory T, does T |= Professor(John)?

- Lecturer ≡ ∀Teaches.Course ⊓ ¬Undergrad ⊓ Professor
- Lecturer (John)
- Teaches(John, Logics)
- Course(Logics)

Exercise 6.18 (Instance retrieval in $\mathcal{LODE}$) Given the following $\mathcal{LODE}$ theory T, find all the instances of Lecturer.

- Lecturer ≡ ∀Teaches.Course ⊓ ¬Undergrad ⊓ Professor
- Lecturer (John)
- Teaches(John, Logics)
- Course(Logics)
- Teaches(Paul, Logics)
- ¬Undergrad(Paul)
- Professor(Paul)

Exercise 6.19 (Concept realization in $\mathcal{LODE}$) Given the following $\mathcal{LODE}$ theory T, find the most specific concept for Paul.

- Lecturer ≡ ∀Teaches.Course ⊓ ¬Undergrad ⊓ Professor
- Lecturer (John)
- Teaches(John, Logics)
- Course(Logics)
- Teaches(Paul, Logics)
- ¬Undergrad(Paul)
- Professor(Paul)