World models Logics and Representations

(HP2T)

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World models - intuition

Intuition (World models): **Models** depict a specific instance of the world, e.g., the world under observation. **Theories** describe the contents of models. To be unambiguous, a representation of the world should have three components:

- The intended model
- A theory describing the intended model
- A mapping from the theory to the intended model which describes which element of the theory describes which element of the model

A **World Model** is any representation which encodes the three types of information described above.

Formal world Models – intuition

Intuition (Formal world models). **Formal world models** are world models, also called **logical world models**, where:

- The intended **model** is formalized in **model theory**;
- A **theory** describing the intended model, written in a formally defined **language**;
- A functional mapping, called the **Interpretation function**, from the language to the domain of interpretation.

Types of world models - observation

Observation (Language, informal, semi-formal, Logical) There are three types of **languages** and, correspondingly, three types of **world models:**

- **Informal world models**, namely world models where the grammar of the language is defined informally, for instance, in natural language and without using production rules.
- **Semi-formal models,** namely world models where the grammar of the language is formally defined.
- **Formal (Logical) world models**, namely world models where where the grammar as well as the interpretation of the language are formally defined.

Terminology (Formality of the world model). From now on, when no confusion arises, we will leave implicit the type of world model (e.g., we will drop the word "formal" in "formal world model")

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Interpretation function

Definition (Interpretation function) Let L_a be a language of assertions and D a domain. Then an **Interpretation Function I_a** is defined as

$$I_a : L_a \rightarrow D$$

We say that a fact f ∈ M is **the interpretation of** $a \in I_a$, and write

$$f = I_a(a) = a^I$$

to mean that the assertion a is a linguistic description of f.

We say that the fact f is **the interpretation of** the assertion a, or, equivalently, that a **denotes** f.

Observation (Interpretation function). Interpretation functions apply to assertions and generate facts. Different world models refine the definition of interpretation function based on the percepts they consider. See later, the definition of theinterpretation functions of the different world logics. This is the key element distinguishing one logic from another.

Interpretation function (example)

- I_a(Sofia è una persona) = *Sofia ϵ person*
- I_a(Paolo è un uomo) = *Paolo ϵ man*
- I_a(Rocky is a dog) = *Rocky ϵ dog*
- I_a(Sofia is near Paolo) = *<Sofia, Paolo > ϵ near*
- I_a(Rocky è il cane di Sofia) = {*Rocky ϵ dog, <Sofia,Rocky > ϵ Owns*}
- I_a(Sofia è un’amica di Paolo) = *<Sofia, Paolo > ϵ friend*
- I_a(Sofia è bionda) = *Sofia ϵ blond*
- ….

Interpretation function – non-ambiguity

Observation (Non-ambiguity) Interpretation functions, being functions, are not ambiguous. If two facts f_1 and f_2 are different it cannot be that $\mathrm{I}_a(a) = f_1$ and $\mathrm{I}_a(a) = f_2$.

Observation 1 (Polysemy) Natural language words have multiple meanings, e.g., Java, Car, bank. This phenomenon, called *polysemy,* is pervasive. The average polysemy in the lexical resource *WordNet, the world de-facto* standard for lexical resources (i.e., digitized natural language vocabularies) is around 2.

Observation 2 (Polysemy). Polisemy is one of the key problems in natural language processing (NLP), with a lot of research on Word Sense Disambiguation (WSD) algorithms.

Interpretation function - synonymity

Observation (Interpretation function, synonymity). Two assertions are synonyms when they have the same meaning, that is, the interpretation of two different assertions a_1 and a_2, may denote the same fact, i.e., $I_a(a_1) = I_a(a_2)$. In logic synonymity is not a problem as the denotation of a word is a single percept.

Observation (Natural language, synonymity). Synonymous words are pervasive in natural languages (e.g., *car* and *automobile*).

Observation (Relational DB, synonymity). In relational DBs synonymity is not allowed, essentially for efficiency reasons (so-called *unique name assumption*). This means different strings always mean different things. That is, words behave like concepts or unique identifiers.

Interpretation function – totality and surjectivity

Observation (Totality). Interpretation functions are total. This guarantees that any element of the language has an interpretation.

Observation (Non-surjectivity). Interpretation functions are not necessarily surjective. In other words, if $I_a : L_a \rightarrow D$, L_a may not be able to name all the facts in D. This property formalizes the fact that linguistic representations do not necessarily describe all facts. It is also useful with infinite domains .

Proposition (Maximal theory). Given a world model, modulo synonyms, there is always one and only one maximal theory, that is a theory which contains one or more assertions for each fact in the intended model.

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World Model

Definition (World model). Given a Domain of interpretation D, a world model W is defined as

$$W = \langle L_a, D, I_a \rangle$$

where L_a is an assertional language and $I_a : L_a \rightarrow D$ is an interpretation function.

World model*

$$
\begin{tikzcd}
a & T_A \arrow[l, "e"'] \arrow[r, "\subseteq"] \arrow[d, dashed, "\mathcal{I}_A"] & \mathcal{L}_A \arrow[d, "\mathcal{I}_A"] \\
\mathbf{f} & \mathbf{M} \arrow[l, "e"'] \arrow[r, "\subseteq"] & \mathbf{D}
\arrow[from=1-1, to=2-1, "\mathcal{I}_A"]
\end{tikzcd}
$$

***Errata corridge:** «L_A» should be «L_a», «T_A» should be «T_a», «I_A» should be «I_a»*

World model – observations on diversity

Observation (Analogical and linguistic representations). Earlier on we informally introduced the fact that representations of the same reality are subjective and **highly diversified**. The notion of world model makes this intuition precise. Two world models describing the same reality may differ in:

- The domain D: the percepts (types and specific instances) and facts they consider;
- The language L: the alphabet, how words in the alphabet are mapped to percepts and how assertions are built via formation rules. We have multiple languages describing the same domain;
- The specific model, that is, the specific set of facts considered;
- The specific theory, that is the specific set of assertions used to describe facts. We have multiple theories for the same model.

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Truth and Falsity

Terminology (True and False assertion).

- Given a domain D,
- given an assertional language L_A that we use to describe D,
- given and an interpretation function $I_a : L_a \rightarrow D$,
- given an intended model $M \subseteq D$ which depicts what are interested in

we say that an assertion

$a \in L_a$ is **True** in M if the fact $f = I_a(a) \in M$, **False** otherwise.

Observation (Truth / Falsity). The notions of Truth and Falsity are meaningful only if made with respect to a reference model, i.e., the intended model.

Terminology – Syntax and Semantics

Terminology (Syntax and semantics). When talking about world models, people informally talk of **syntax** meaning the language of the world model, and of **semantics** meaning the domain of interpretation, associated to the syntax, via the **interpretation function,** informally or formally defined.

Observation (Syntax and semantics). Without a formal understanding of the intended semantics of a given syntax, that is, without the interpretation function, it is impossible to univocally assert whether a certain assertion (sentence) is true or false.

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Language correctness and completeness

Definition (Language correctness and completeness). Let W = ⟨L_a, D, I_a ⟩ be a World model. Let L_A and D be an assertional language and a domain of interpretation, respectively. Then we have two possible situations, as follows

Correctness. Let $a \in L_a$ be an assertion. If for all a, if $a \in L_a$ then $I_a(a)$ =f ∈ D, then we say that L_a is **correct** with respect to D, or that D **is a domain** for L_a;

Completeness. Let f ∈ D be a fact. If, for all f, if f ∈ D then there is an assertion $a \in L_a$ such that $I_a(a)$ = f, then we say that L_a is **complete** with respect to D.

The notions of **incorrectness** and **incompleteness** are defined in the obvious way.

Language correctness and completeness (continued)

Observation (Correctness of an assertional language L$_a$ with respect to a domain D). An assertional language, to be used for a given domain, **must be correct**, that is to contain *only* assertions which denote facts in the reference domain. If this not the case then we say that D is NOT a domain of L$_a$ or, vice versa, that L$_a$ is not a language for D. This in order to avoid nonsensical assertions (e.g., the assertion "gdhaosdf").

Observation (Completeness of an assertional language L$_a$ with respect to a domain D). An assertional language is not necessarily complete, that is, it does not necessarily contain assertions for all the facts in a domain (which, among other things, are in principle infinite). The key feature is that it should contain all the assertions deemed relevant.

Theory correctness and completeness

Definition (Theory correctness and completeness). Let W = ⟨L_a, D, I_a⟩ be a World model. Let $T_a \subseteq L_a$ and M ⊆ D be an assertional theory and a model, respectively. Then we have two possible situations, as follows

Correctness. Let $a \in L_a$ be an assertion. If for all a, if $a \in T_a$ then a is **True** in M, then we say that T_a is **correct** with respect to M;

Completeness. Let f ∈ M be a fact. If, for all f, if f ∈ M then there is an assertion $a \in T_a$ such that $I_a(a)$ = f, then we say that T_a is **complete** with respect to M.

The notions of **incorrectness** and **incompleteness** are defined in the obvious way.

Theory correctness and completeness (continued)

Observation (Correctness of an assertional theory T_a with respect to a model M). An assertional theory T_a,to be used to describe an intended model, **must be correct** contain *only* assertions about facts in M for M of T_a. If this not the case then we say that M is NOT a model of T_a or, vice versa, that T_a is not a theory for M. This in order to avoid false assertions.

Observation (Completeness of an assertional theory T_a with respect to a model M). An assertional theory **may be incomplete**, namely there can be facts of the model for which T_a does contain assertions. Incomplete assertional theories are the default.

Observation (Correctness and completeness). The requirement on theories is the same as that on languages and domains.

World models, theories and models

Definition (World theory). Given a world model

$$W = \langle L_a, D, I_a \rangle$$

then, given M and T_a defined as follows,

$$M = \{f\} \subseteq D$$
$$T_a = \{a\} \subseteq L_a$$

M and T_a are, respectively, a **model** of T_a and a **theory** of M, if $\mathbf{T_a}$ **is correct for M.**

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World model Entailment

Definition (Entailment relation). Let W =< L_a, D, I_a > be a world model . Let $T_a = \{a\} \subseteq L_a$ be a theory and M = {f} ⊆ D a model of W. Then, we write

$$M \models T_a$$

and say that M **entails** T_a if M is **a model of** T_a.

Observation (Entailment relation). Given a world model, an entailment relation defines which sets of facts are models of which theories, namely how truth propagates from analogical representations to linguistic representations.

World model Entailment

Proposition (World model entailment). Let W =< L$_a$, D, I$_a$ > be a world model. Let T$_a$ ⊆ L$_a$ and M ⊆ D. Then

$$M \mid= T_a \text{ if and only if } I_a(a) \in M, \text{ for all } a \in T_a$$

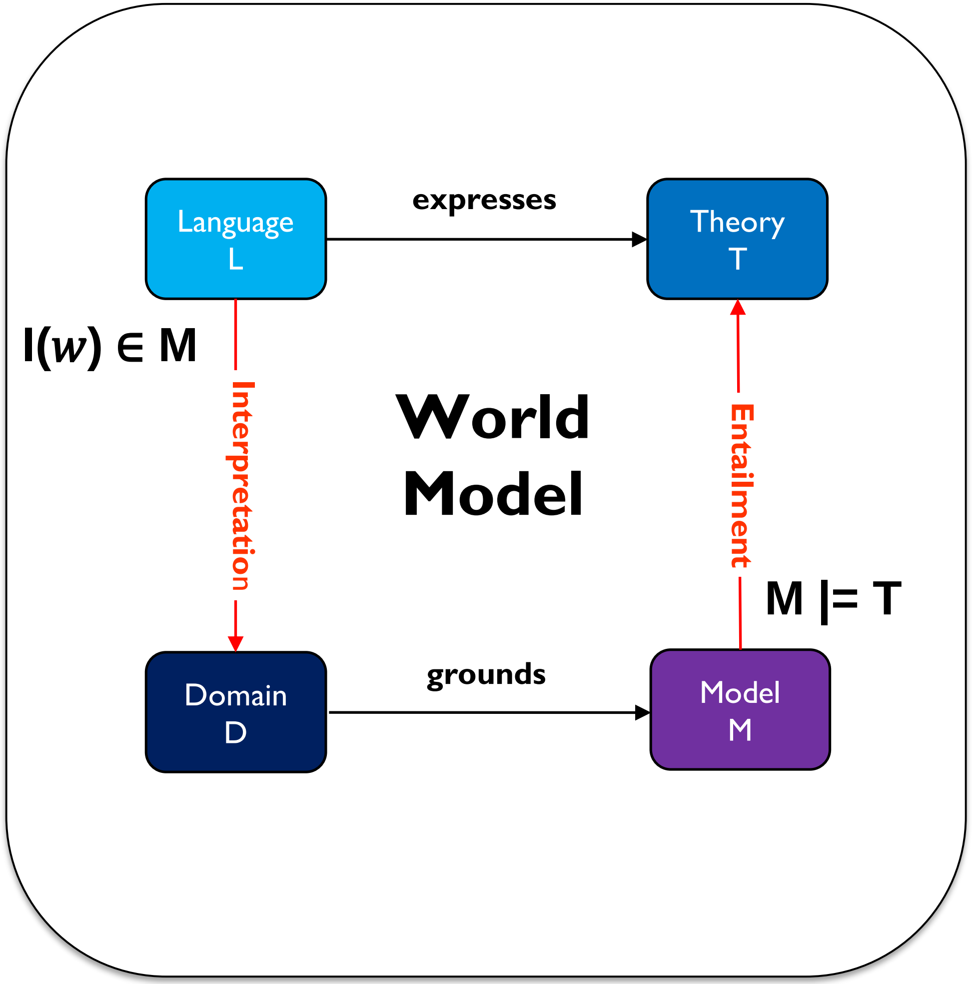
$$M \mid= \{a\} \text{ if and only if } I_a(a) \in M$$

Evidence (World model entailment). Trivial consequence of previous definitions.

Observation 1 (World model entailment). World model entailment reduces entailment to checking, via the interpretation function, for truth / falsity in the model. That is, a model M entails a theory if all its assertions are true in M.

Observation 2 (World model entailment). But how to select M and T$_a$? This is a **search** problem!

Entailment and interpretation



The process is as follows:

1. Via the interpretation function one defines the meaning of words and assertions thus providing the ability to name percepts and facts;
2. Using the interpretation function one can build models as sets of true facts;
3. Once the model is constructed, entailment allows to identify which sets of assertions are theories describing the intended model
4. A theory is by construction a subset of the language

Entailment and interpretation (observations)

Observation 1 (Entailment and interpretation).

- Interpretation is a function between a language and a domain;
- Entailment is a relation between a model and a theory;
- Interpretation takes a formula in input and establishes which fact is described by that formula, thus allowing to determine what is true/ false in a model;
- Entailment takes a model in input and establishes which theories describe that model (which theories are correct for that model).

Observation 2 (Entailment relation). Entailment is a many-to-many relation. There may be multiple theories that entail a model and, symmetrically, for the same theory there may be multiple models entailed by it. Finding the "right" pair of theory and model (a search problem) can become arbitrarily complex, in space and time.

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World logics

Definition (World logic). Given a **world model**

$$W = \langle L_a, D, I_a \rangle$$

a **world logic** L_W for W is defined as

$$L_W = \langle W, \models \rangle$$

where $\models$ is an **entailment relation**.

World models and world logics

$$\begin{tikzcd}
a & \mathcal{T}_A \arrow[l, "\in"'] \arrow[r, "\subseteq"] \arrow[d, dashed, "\mathcal{I}_A"] & \mathcal{L}_A \arrow[d, "\mathcal{I}_A"] \\
\mathfrak{f} & \mathbb{M} \arrow[l, "\in"'] \arrow[r, "\subseteq"] & \mathbb{D}
\arrow[from=1-1, to=2-1, "\mathcal{I}_A"]
\end{tikzcd}$$

World models*

$$\begin{tikzcd}
a & \mathcal{T} \arrow[l, "\in"'] \arrow[r, "\subseteq"] \arrow[d, dashed, "\models"] & \mathcal{L}_a \arrow[d, "\mathcal{I}"] \\
\mathfrak{f} & \mathbb{M} \arrow[l, "\in"'] \arrow[r, "\subseteq"] & \mathbb{D}
\arrow[from=1-1, to=2-1, "\mathcal{I}"]
\end{tikzcd}$$

World logics*

Observation: In the extension from world models to world logics: (1) The domain does not change, (2) the entailment relation is added, (3) the language is (optionally) extended; (4) the interpretation is extended (optionally, if the language is extended).

***Errata corridge:** *«A» should be «a», «a» in «L_a» in Logics should be dropped*

World logics – observations

Observation (World logics - intuition). There are different families of world logics. They key idea behind all world logics is as follows:

- Each family of world logic has the same domain of interpretation and the same notion of interpretation function and world model entailment;
- The simplest world logic ha a language which allows to express just the (selected) percepts of the reference world model.
- A world logic is obtained from a simpler one by enriching the expressiveness of the language of the world model, without changing the domain of interpretation, and by extending accordingly the interpretation function and the entailment relation.

World logics and reasoning logics – observations

Observation (World logics). World logics formalize how truth in a model can be reasoned in a (logical) theory (that is, a linguistic representation of the world). They are the key element, via the entailment relation, for the formalization of (**logical) reasoning.**

Observation (Reasoning logics). Logical reasoning is linguistic reasoning, that is, reasoning in a predefined language. Logical reasoning is implemented using **reasoning logics** which allow to draw conclusions from the true facts computed by world logics. They use world logics as oracles which provide information about what is true/ false in the intended model.

Note: The first part of this course focuses on the world logics of KGs. The second part will focus of reasoning logics exploiting the world logics of KGs.

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World representation

Definition (World theory). Given a world model

$$W = \langle L_a, D, I_a \rangle$$

then

$$R = \langle T_a, M \rangle$$

is a (**static**) **world representation**, with

$$M = \{f\} \subseteq D$$
$$T_a = \{a\} \subseteq L_a$$

where M and T_a are, respectively, a **model** of T_a and a **theory** of M in W.

World models and representations

Observations (World models). World models provide all the formal notions which are necessary to unambiguously describe what is perceived (analogic and linguistic representations). They specify the extent to which what will be perceived will be described.

Observation (World representations). World representations describe the contents of the analogic representations which have been perceived.

Observation (World model and representation, extension). World models, once selected, do not change. World representations get extended in time with the evolution of what is being perceived and/ or described.

Observation (Ask / Tell operations). World representations are extended via the Ask / Tell operations

Tell a world representation

Definition (TellL, Language extension). Let L = ⟨W, |=⟩ be a world logic with W = ⟨L_a,D, I_a ⟩, L_a=<A_a, FR>, A_a ={w} . Let R = ⟨T_a, M⟩ be a world representation. Let A = {w} be an alphabet. Then, the operation **TellL (R,A)** is such that

TellL (R, A) = R'

with

A_a' = A $_a$ U A

D' = D U {I'$_a$(w)}, for all w ∈ A

We also write **D!= A** to mean **TellL** (R,A).

Observation (TellL). Language (alphabet) extensions increase the number of percepts of the domain and, therefore, rewuires also the extension of the interpretation function.

Tell a world representation

Definition (TellT, Theory extension). Let L = ⟨W, |=⟩ be a world logic. Let R = ⟨T_a, M⟩ be a world representation. Let T = $\{a\}$ be a theory. Then, the operation **TellT (R,T)** is such that

$$\textbf{TellT}\ (R, T) = R'$$

with

$$T_a' = T_a \cup T,$$
$$M' = M \cup \{I_a(a)\}, \text{ for all } a \in T$$

We also write **M != T** to mean **TellT** (R,T).

Observation (TellT). Theory (Assertion) extensions increase the number of facts true in the model.

Ask a world representation

Definition (AskC, Model checking). Let L = ⟨W, |=⟩ be a world logic. Let R = ⟨T_a, M⟩ be a world representation. Let T = {a} be a theory. Then, the operation **AskC** (R,T) is such that

AskC (R,T) returns **yes** if M |= T, **no** otherwise

We also write **M ?= T** to mean **Ask** (R,T).

Observation 1 (AskC). Model checking is the key reasoning step. It checks the (in)correctness of T with respect to R, namely whether M is a model of T

Observation 2 (AskC). Most logics implement more complex ASK questions, all based on the basic model checking question.

Ask a world representation

Definition (AskS, Model finding, satisfiability). Let L = ⟨W, |=⟩ be a world logic. Let T = {a} be a theory. Then the operation **AskS** (W,T) is such that

AskS (W,T) returns

yes if there exists an M = {f} ⊆ D such that M |= T, **no** otherwise

We also write **M ??= T** to mean **AskS** (W,T). We say that T is **satisfiable** in D if AskS succeeds

Observation 1 (AskS). AskS requires searching for all possible models and then model checking them.

Observation 2 (AskS). AskS uses AskC for model checking

World representations (final definition)

Definition (World representation). Given a world Logic $L_W = \langle W, |= \rangle$, with $W = \langle L_a, D, I_a \rangle$, a world representation R for L_W is defined as

$$R = \langle T_a, M, Ask/Tell \rangle$$

where Ask/Tell is any combination of TellL, TellT, AskC, AskS.

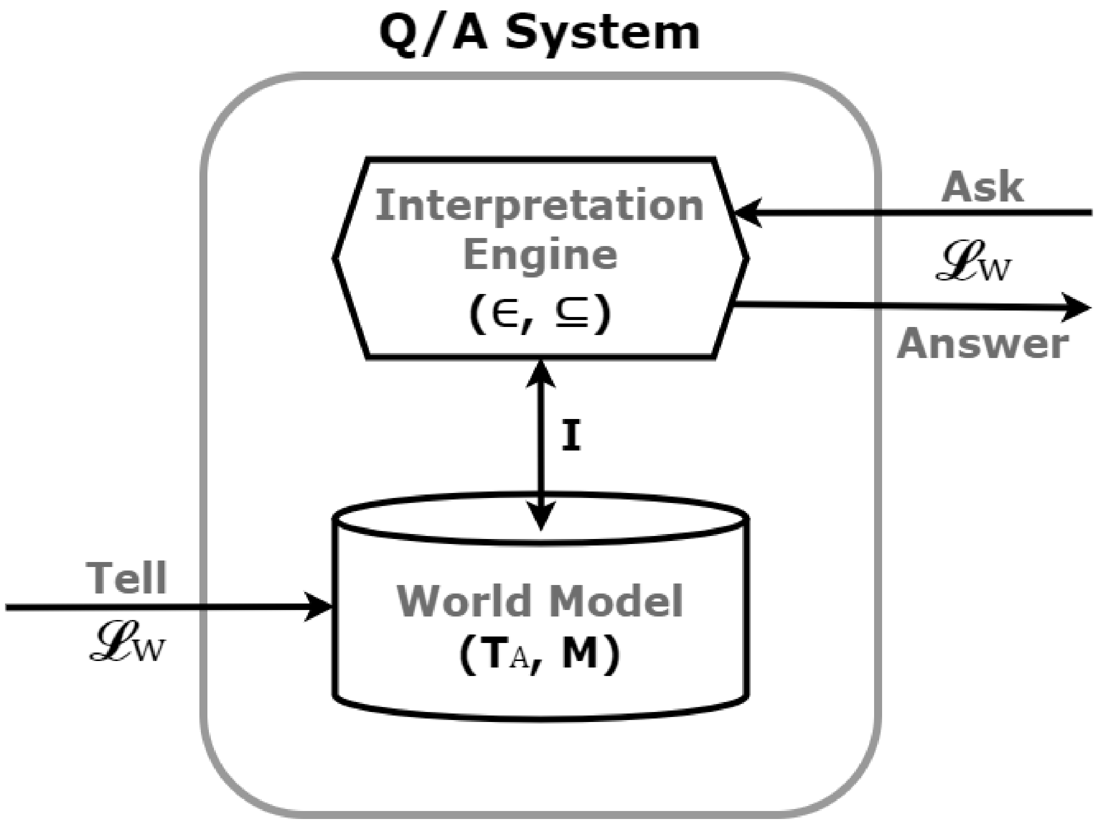
Observation (World representation). Any implemented world model is a world representation defined in terms of its theory T, model M, and the Ask/Tell primitives for interacting with them.

Observation (Ask/Tell language). In many world representations (e.g., DB, OWL/SparQL in the Web) the language used for Ask/Tell is different from L_a. There are three main reasons why this may happen: (1) it is more convenient, (2) it is more expressive, (3) it is a reference standard language.

Ask / Tell a world representation

$M \; ?\!= T_{L'}$

$M \; ??\!= T_{L'}$



$M \; !\!= T_{L'}$

$D \, !\!= A_{L'}$

The subscript **L'** means that the Ask/Tell language is not necessarily the same as the language L_a used inside the world model

Expressivity vs. Efficiency

Observation (Logic, selection trade-offs) Any logic, in particular beyond world logics, can be characterized by two main parameters:

- **Expressivity of the language** (beyond assertions), that is, the level of richness at which the problem is expressed, depending on the syntax of the language (for instance relating to partial or negative knowledge, see later);
- **Computational efficiency**, that is how much it costs, in terms of space and time, to reason and answer satisfiability queries in that language.

Expressivity vs. Efficiency (cont.)

- More expressivity allows for a more refined and precise modeling of the world but it also generates more complex formulas.
- The modeler must find the right trade-off between **expressiveness** and **computational complexity**.
- Here the choice of the representation language is crucial. The computational complexity of reasoning ranges in fact from polynomial to exponential and beyond.
- There is also an issue of **(un)decidability**, namely the possibility for the reasoner, on certain queries, to get into an infinite loop, never terminate and, therefore, never return an answer.

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How to use a world representation

1. **Select** the domain of interpretation (set of analogical representations) D

2. **Agree** on L_a, I_a - thus defining $W = \langle L_a, D, I_a \rangle$

3. **Tell** D != A, M != T

4. **Ask** M ?= T, M ??= T

World models and representations – observations (continued)

Observation (Select / Agree about the world model) These two steps are the two main modeling decisions. First choose the reference model to use, then select the language and, via the interpretation function, the intended model. These choices are done by the SW Engineers at design time.

Observation (Tell / Ask the world representation). These operations are performed when the system is in production, as part of its “normal” functioning.

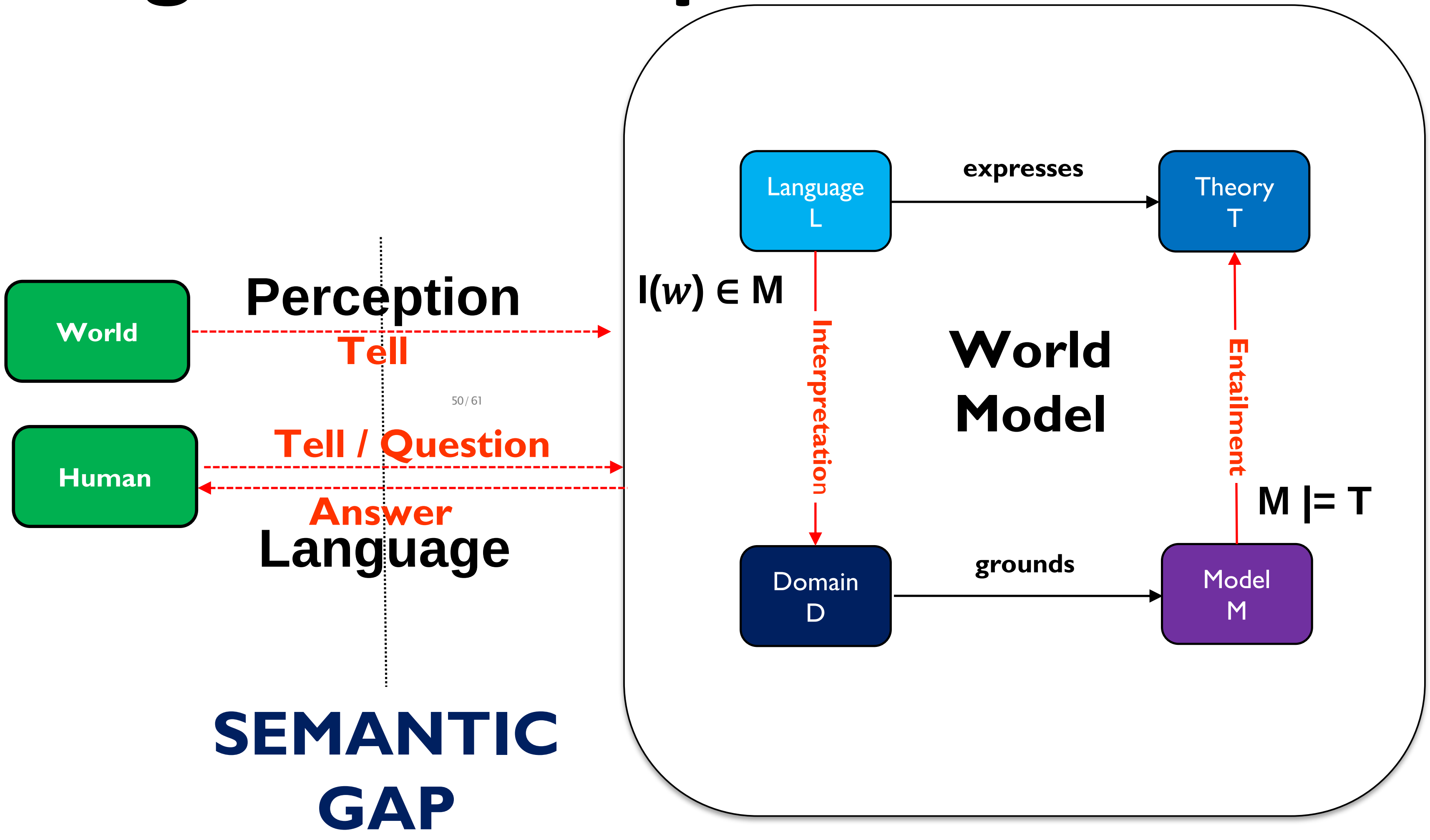
Using a world representation (example)

Example (Query Answering Q/A in DBs): a DB is the implementation of a (semi-formal world model and) world representation

- The DB language is the world model,
- The contents of the DB are the world representation,
- The DB is continuously told about new facts, possibly non monotonically (what was true before become false and vice versa),
- The query is the theory to be model checked,
- The answer is the set of instantiations which make the input theory correct (or the single tuple if there is no variable in query).

Observation (Generality of Q/A mechanism) Q/A can be applied to all world models (e.g., the logical formalizations of DBs, ER models, …).

Using a world representation



Using perception requires being able to compute the assertion denoting the input fact (as in, e.g., Computer Vision)

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World models Logics and Representations

(HP2T)