

## LOD

### Basic Concepts

Solution **5.1**

$\sqcap, \neg, \top, \equiv, \sqcup, \sqsubseteq, \perp, \models$

Solution **5.2** 2, 3 and 4 are not wffs in LOD.

Solution **5.3** Answer: a, c, d, e, f.

Solution **5.4** Answer: Yes, because it is acyclic and there are only equivalences.

Solution **5.5** Answer: No, because it contains a subsumption.

Solution **5.6** Answer: Yes, for instance as follows:

- $\text{Mother} \equiv \text{Woman} \sqcap \exists \text{hasChild.Person}$
- $\text{Father} \equiv \text{Man} \sqcap \exists \text{hasChild.Person}$
- $\text{StepMother} \equiv \text{Woman} \sqcap \exists \text{marriedWith.Father}$
- $\text{StepFather} \equiv \text{Man} \sqcap \exists \text{marriedWith.Mother}$
- $\text{Parent} \equiv \text{Father} \sqcup \text{Mother} \sqcup \text{StepFather} \sqcup \text{StepMother}$

Solution **5.7** By now you should be able to do it without help.

Solution **5.8** ANSWERS:

- is true because the interpretation of  $\top$  (top) is the entire domain and their difference is given precisely by the empty set that coincides with the interpretation of  $\perp$  (bottom).
- is false because by definition it must be  $e \in \mathcal{I}(C)$ .
- is true because they correspond to their respective definitions
- same as 3
- same as 3

### Translation

Solution **5.9**

- $\exists \text{studiesIn.Library}$
- $\text{Book}$
- $\exists \text{reads.Book}$
- $\forall \text{reads.ComicBook}$
- $\forall \text{friendsWith.}(\exists \text{ studiesIn.Library})$

Solution **5.10**

- $\text{Employee} \sqcap \exists \text{worksAt.Library}$
- $\text{BlackTea} \sqcup \text{GreenTea}$

- $\text{Person} \sqcap \neg \exists \text{drinks.GreenTea}$   
(or  $\text{Person} \sqcap \forall \text{drinks.}\neg \text{GreenTea}$ )
- $\forall \text{drinks.}\neg \text{BlackTea}$   
(or  $\neg \exists \text{drinks.BlackTea}$ )
- $\text{Person} \sqcap \exists \text{drinks.GreenTea} \sqcap \neg \exists \text{drinks.BlackTea}$   
(or  $\text{Person} \sqcap \exists \text{drinks.GreenTea} \sqcap \forall \text{drinks.}\neg \text{BlackTea}$ )

Solution **5.11**

- $\{Mario, Anna\}$
- $\{Toby, Gigi, Sara\}$
- $\{Anna\}$
- $\{Anna, Mario\}$
- $\emptyset$
- $U$
- $\{Mario, Anna\}$

Solution **5.12** ANSWER:

- $D = \langle E, C, P \rangle$
- $E = 0, \dots, 9$
- $C = A, B, C$
- $P = \emptyset$
- $T = C \equiv A \sqcap B$

Note that: (a) we need to define the language and the interpretation functions; (b) we can define different languages and interpretations

Solution **5.13** ANSWER:

- $D = \langle E, C, P \rangle$
- $E = \text{Fausto, Rui, Bisu, Italian, Chinese, Indian}$
- $C = \text{Employee, Professor, Student, Nationality}$
- $P = \text{hasNationality, hasSupervisor}$
- $T = \text{Professor} \sqsubseteq \text{Employee}; \text{Student} \sqsubseteq \text{Employee}; \text{Employee} \sqsubseteq \exists \text{hasNationality.Nationality}$   
 $\sqcap \exists \text{hasSupervisor.Employee}$

Solution **5.14** ANSWER:

- $\text{Producer} \sqsubseteq \text{Entity} \sqcap (\exists \text{ProductType.T} \sqcup \exists \text{Location.T} \sqcup \exists \text{ProductValue.T} \sqcup \exists \text{Transaction.Consumer})$
- $\text{Consumer} \sqsubseteq \text{Entity} \sqcap (\exists \text{Gender.T} \sqcup \exists \text{Age.T} \sqcup \exists \text{AverageSpend.T})$

NOTE: in this ER diagram there is no cardinality, but in general we may have it. Consider also the implicit direction of arcs (from left to right).

Solution **5.15** ANSWER:

- $D = \langle E, C, P \rangle$
- $E = \text{Alice, Bob, The Mona Lisa, Leonardo Da Vinci, La Joconde à Washington, 14 July 1990}$
- $C = \text{Entity, Person, Picture, File, Date}$

- $P = \text{isFriendOf}, \text{interestedIn}, \text{isAbout}, \text{wasCreatedBy}, \text{isBornOn}$

Solution **5.16** The theory  $T$  can be as follows:

- $\text{Person} \sqsubseteq \text{Entity} \sqcap (\exists \text{isFriendOf}.\text{Person} \sqcup \exists \text{interestedIn}.\text{Picture} \sqcup \exists \text{isBornOn}.\text{Date})$
- $\text{Picture} \sqsubseteq \text{Entity} \sqcap \exists \text{wasCreatedBy}.\text{Person}$
- $\text{File} \sqsubseteq \text{Entity} \sqcap \exists \text{isAbout}.\text{Picture}$

Solution **5.17** (Translate in  $\mathcal{LOD}$ ). The translation is pretty straightforward:

- Niente:  $\perp$
- Tutto:  $\top$
- Humans and vehicles:  $\text{Human} \sqcap \text{Vehicle}$
- Vehicles and not boats:  $\text{Vehicle} \sqcap \neg \text{Boat}$
- Wheels or engines and humans:  $(\text{Wheel} \sqcup \text{Engine}) \sqcap \text{Human}$
- Adults or children:  $\text{Adult} \sqcup \text{Child}$

Solution **5.18** (Translate in  $\mathcal{LOD}$ ). The translation of the concepts and role names is:

1.  $\text{Vehicle} \sqcap \exists \text{hasPart}.\text{Wheel} \sqcap \exists \text{poweredBy}.\text{Engine}$
2.  $\text{Vehicle} \sqcap \exists \text{hasPart}.\text{Wheel} \sqcap \exists \text{poweredBy}.\text{Human}$
3.  $\text{Vehicle} \sqcap \exists \text{travelsOn}.\text{Water}$
4.  $\forall \text{hasPart}.\neg \text{Wheel}$
5.  $\forall \text{travelsOn}.\neg \text{Water}$
6.  $\text{Device} \sqcap \exists \text{hasPart}.\text{Axle} \sqcap \exists \text{capableOf}.\text{Rotation}$
7.  $\text{Human} \sqcap \exists \text{controls}.\text{Vehicle}$
8.  $\text{Driver} \sqcap \exists \text{controls}.\text{Car}$

Solution **5.19** (Translate in  $\mathcal{LOD}$ ). The translation is:

1.  $\text{Boat} \sqsubseteq \forall \text{hasPart}.\neg \text{Wheel}$
2.  $\text{Car} \sqcup \text{Bicycle} \sqsubseteq \forall \text{travelsOn}.\neg \text{Water}$
3.  $\text{Driver} \sqcap \exists \text{controls}.\text{Car} \sqsubseteq \text{Adult}$
4.  $\text{Human} \sqsubseteq \neg \text{Vehicle}$
5.  $\text{Wheel} \sqcup \text{Engine} \sqsubseteq \neg \text{Human}$
6.  $\text{Human} \sqsubseteq \text{Adult} \sqcup \text{Child}$
7.  $\text{Adult} \sqsubseteq \neg \text{Child}$

Solution **5.20** (Translate in  $\mathcal{LOD}$ ). The translation of the phrases is:

1.  $\text{Car} \equiv \text{Vehicle} \sqcap \exists \text{hasPart}.\text{Wheel} \sqcap \exists \text{poweredBy}.\text{Engine}$
2.  $\text{Bicycle} \equiv \text{Vehicle} \sqcap \exists \text{hasPart}.\text{Wheel} \sqcap \exists \text{poweredBy}.\text{Human}$
3.  $\text{Boat} \equiv \text{Vehicle} \sqcap \exists \text{travelsOn}.\text{Water}$
4.  $\text{Wheel} \equiv \text{Device} \sqcap \exists \text{hasPart}.\text{Axle} \sqcap \exists \text{capableOf}.\text{Rotation}$
5.  $\text{Driver} \equiv \text{Human} \sqcap \exists \text{controls}.\text{Vehicle}$

Solution **5.21**

- $\text{Game} \sqcap \neg \text{Legal}$
- $\text{Lake} \sqsubseteq \text{Location}$

- $\text{Lake} \sqsubseteq \text{Location} \sqcap \exists \text{Madeof.Water}$
- $\text{Person} \sqsubseteq \text{Male} \sqcup \text{Female}$
- $\text{Male} \sqsubseteq \neg \text{Female}$
- $\text{Person} \sqsubseteq \exists \text{hasBirthPlace.T}$
- $\text{JavaProgramming} \sqsubseteq \text{ProgrammingLanguage} \sqcap \text{ComputerScience}$

Solution **5.22**

- $\text{MasterStudent} \sqsubseteq \text{Student}$ 
  - $\text{Unicorn} \sqsubseteq \text{mythical} \sqcap \text{horse} \sqcap \exists \text{has.Horn}$
  - $\text{PhDStudent} \sqsubseteq \text{Student} \sqcap \exists \text{hasTask.Research}$

Solution **5.23** Answer:

- " $\text{Lion} \equiv \text{Feline} \sqcap \text{Large} \sqcap \text{Gregarious} \sqcap \text{Predatory} \sqcap \forall \text{livesIn.}(\text{Africa} \sqcup \text{India}) \sqcap \exists \text{livesIn.}(\text{Africa} \sqcup \text{India})$ " and " $\text{MaleLion} \equiv \text{Lion} \sqcap \text{Male} \sqcap \forall \text{has.ShaggyMane} \sqcap \exists \text{has.ShaggyMane}$ "
- " $\text{Penguin} \equiv \text{Bird} \sqcap \neg \text{Fly} \sqcap \forall \text{livesIn.Antarctica} \sqcap \exists \text{livesIn.Antarctica} \sqcap \forall \text{has.WebbedFeet} \sqcap \exists \text{has.WebbedFeet}$ "

Solution **5.24** Answer:

- $\text{Vehicle} \equiv \text{Conveyance} \sqcap \exists \text{transports.}(\text{Person} \sqcup \text{Object}) \sqcap \forall \text{transports.}(\text{Person} \sqcup \text{Object})$
- $\text{Car} \equiv \text{Vehicle} \sqcap \neg \text{Bicycle} \sqcap \exists \text{hasPart.Wheel} \sqcap \exists \text{hasPart.Engine}$
- $\text{Bicycle} \equiv \text{Vehicle} \sqcap \exists \text{hasPart.Wheel} \sqcap \exists \text{hasPart.FootPedal} \sqcap \exists \text{movedBy.FootPedal} \sqcap \forall \text{movedBy.FootPedal}$
- $\text{Wheel} \equiv \text{Object} \sqcap \text{CircularShape} \sqcap \exists \text{moves.Vehicle} \sqcap \forall \text{moves.Vehicle}$

Solution **5.25** ANSWERS:

- is true because all drivers drive a vehicle, and consequently there is an AND between the two conditions.
- is false because it is not the driver who is electric.
- is right because the formula in parentheses indicates an electric vehicle.
- is true for the same reason as (1) plus it is specified that they do NOT drink alcohol.
- is false because the correct translation requires the existential quantifier  $\exists$  as in question (3).

Solution **5.26** ANSWERS:

- is true (although not necessarily complete) where because the values of the attributes have not been specified we assign  $\top$  as the space of possible values.
- is true because the universal quantifier tells us that the student is only enrolled in courses.
- is false because ER diagrams always read from left to right, and thus it is the student who is enrolled in the course, not the other way around. This is also reiterated in the text of the exercise when indicating what 1-n means. Possibly the inverse function should have been used, but this was not done in class.

- is false because it is not apparent from the diagram.

Solution **5.27** By assuming the schema as complete (otherwise it is not a terminology) we have:

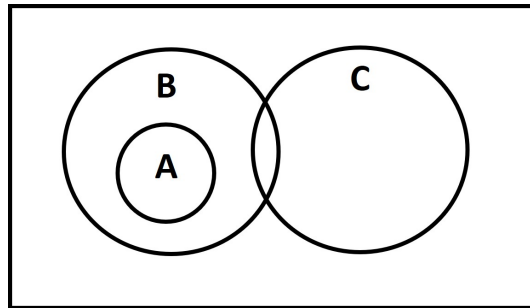
Event  $\equiv$  Thing  $\sqcap$

$\forall \text{ about.Thing } \sqcap \exists \text{ about.Thing } \sqcap$

$\forall \text{ actor.Person } \sqcap \exists \text{ actor.Person } \sqcap$

$\forall \text{ attendee.(Person } \sqcup \text{ Organization) } \sqcap \exists \text{ attendee.(Person } \sqcup \text{ Organization)}$

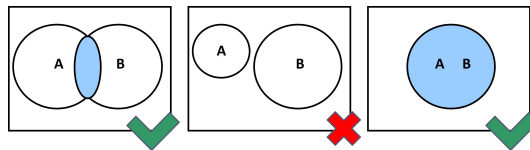
Solution **5.28**



## Reasoning

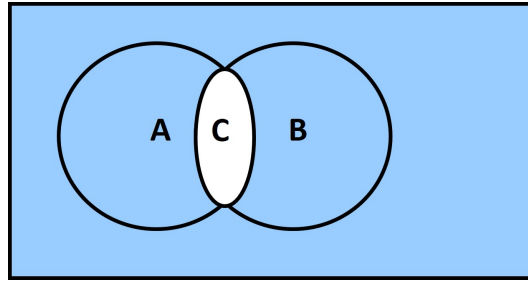
### Entailment

Solution **5.29** ANSWER:

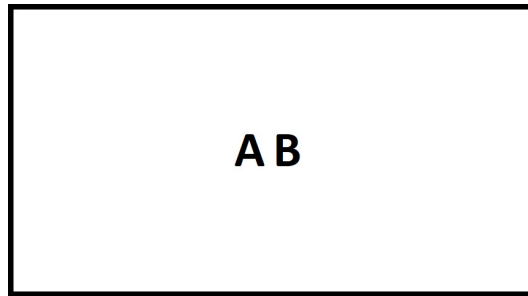


By using Venn Diagrams, we can easily observe that the fact that A and B are not empty does not imply that  $A \sqcap B$  is also not empty. Think to the case in which their extensions are disjoint.

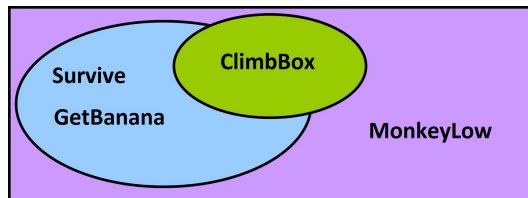
Solution **5.30** ANSWER: Yes. A case is described below with a Venn Diagram.



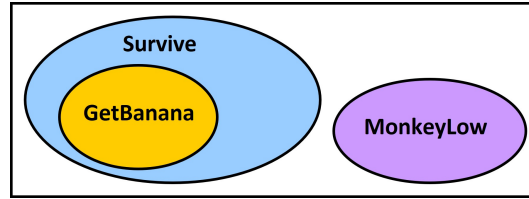
Solution **5.31** ANSWER: No. In fact, we can find a counterexample in which  $I(A) = I(B)$  but the  $I(\neg(A \sqcap B))$  is empty.



Solution **5.32** Answer: Yes. It is enough to find one model for it, represented graphically with the Venn Diagram below.

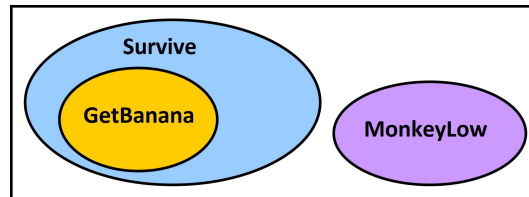


Solution **5.33** Answer: We can restate the problem as follow: does  $T \models \Box \text{GetBanana} \Box \text{Survive}$  at least in one model? Yes. We can find a model in which both all the assertions in  $T$  and  $\Box \text{GetBanana} \Box \text{Survive}$  are not empty.



Solution **5.34** Answer: We can restate the problem as follow: does  $T \models \text{Assistant} \sqsubseteq \text{Undergraduate}$ ? We need to prove that this is true in all models (via the method of unfolding):  $\text{Assistant} \equiv \text{PhD} \sqcap \text{Teach} \equiv \text{Master} \sqcap \text{Research} \sqcap \text{Teach} \equiv \text{Student} \sqcap \sqcap \text{Undergraduate} \sqcap \text{Research} \sqcap \text{Teach}$ . Answer is No. Assistants are actually students who are not undergraduate.

Solution **5.35** Answer: We can restate the problem as follow: does  $T \models \text{ClimbBox} \sqcap \sqcap \text{Survive}$  at least in one model? Yes. We can find a model in which both all the assertions in  $T$  and  $\text{ClimbBox} \sqcap \sqcap \text{Survive}$  are not empty.



Solution **5.36** Answer: We can restate the problem as follow: does  $T \models \text{Bachelor} \sqcap \text{Master} \sqsubseteq \perp$ ? We need to prove that this is true in all models (via the method of unfolding) Answer is obviously Yes because they contain two opposite constraints.

## Unfolding

Solution **5.37** Answer:  $\text{ColouredGuitar} \equiv \text{Guitar} \sqcap \forall \text{hasSoundAmplification.withInputJack} \sqcap \exists \text{hasColour.String}$

Solution **5.38** Answer: No, because by unfolding all concepts I never obtain the same concept on the left and on the right of the equivalences.

Solution **5.39** Answer: Yes, because by unfolding it I get  $\text{Female} \equiv \neg(\neg \text{Female})$  that is  $\text{Female} \equiv \text{Female}$ .

Solution **5.40** Answer: a, b, c, e, f.