Models
(model theory)
(HP2T)

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- **Intuition**
- Things, entities
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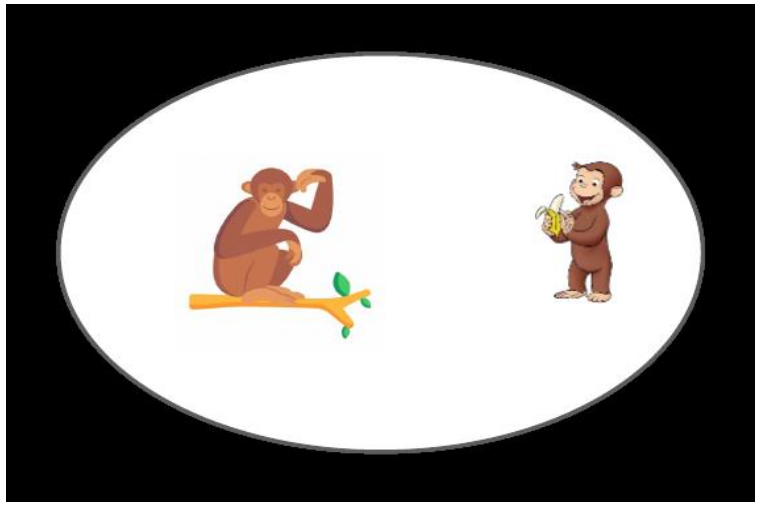
Linguistic vs. analogic representations

- There is a tree
- There is a banana
- The monkey is eating a banana
- The monkey is sitting on a tree
- The monkey is scratching its head



Observation 1. Which monkey? There are two monkeys! The language used in the linguistic representation is ambiguous! At the same time there is general way to "link" the language descriptions of the linguistic representation to the element of the analogic representation.

Observation 2. Preliminary to any attempt of disambiguation of representations we need to make precise what analogic representations depict.

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Analogic representations – Things

Observation (What can be seen in a picture: things). The core element of perception is **things**, where:

- Any thing can be clearly **distinguished** from any other thing, e.g., people, objects, dogs, hair;
- There cannot be two things in the **same** spatio-temporal coordinates;
- A thing moves **smoothly** in space-time (facilitates recognition in time);
- We see them multiple times, from **occurrences** (of things) to **things**, modulo ability to recognize two occurrences to be the same thing;
- Things are all **different** from one another (there are no two things which are identical);
- Things are all in **different** mutual relations.

We always perceive **multiple things**, that we can distinguish from one another, grounded in their space-time coordinates

Analogic representations – from Things to Entities

Observation (From things to entities). We call **entity** a single **thing** that we recognize as such,e.g., Sofia, a dog;

- The **specific entity** being recognized from the thing we perceive is largely **arbitrary.** For instance, what do you see in the picture on the right: an organism, an organism which is a alive, a person, a woman, a black person, a black woman, Sofia, a friend, an enemy a victim to kill, …?
- The first motivation underlying the selection of a specific entity is the **partiality** of the input. For instance: did you see the thing from the back, from far away, for a second, …?;
- The second motivation is your **background** (previous knowledge, memory): you are an alien, a social scientist analyzing a specific social phenomenon, a suprematist, a friend, a killer …?
- The third main motivation is the current **purpose**. For instance: you are looking for a form of life, for a friend, for Sofia?
- Partiality, background and purpose are element of the current **context (external, internal, motivational).**

Our perception of an entity out of a thing does not change, it is **stable**, and **as specific as possible**, given the current context. **But changes across contexts**. This is the main source **of representation diversity.**

Analogic representations – Entities

Observation (Entities). We have the following:

- **Entities**, that is, what we perceive from **things**,e.g., Sofia, a dog;
- **Entity types**, also called **etypes**, that is, **multiple entities which** somehow **look the same**, e.g., the etype woman, dog, tree;
- **Entity properties**, that is, **entities with certain characteristics**, e.g., a woman with blond hair, a dogs barking;
- **Property types**, that is, **multiple entities with certain characteristics**, similar among them, e.g., two women with blond hair, two dogs barking;
- **Entity Relations**, that, is **multiple entities in some relation among them**, e.g., a woman talking to a man with a phone, a dog between a woman and a man, a woman friend of a man;
- **Relation types**, that is, **groups of multiple entities in some relation among them**, similar among them, e.g., two groups of a man and woman talking to one another with a phone, two couples woman and man, both with a dog in between.

Relations and properties are **identified after** the **identification** of the entity: Do you see a woman which is black (together with a white one) or a black woman?

Analogical representations – Sameness vs. difference

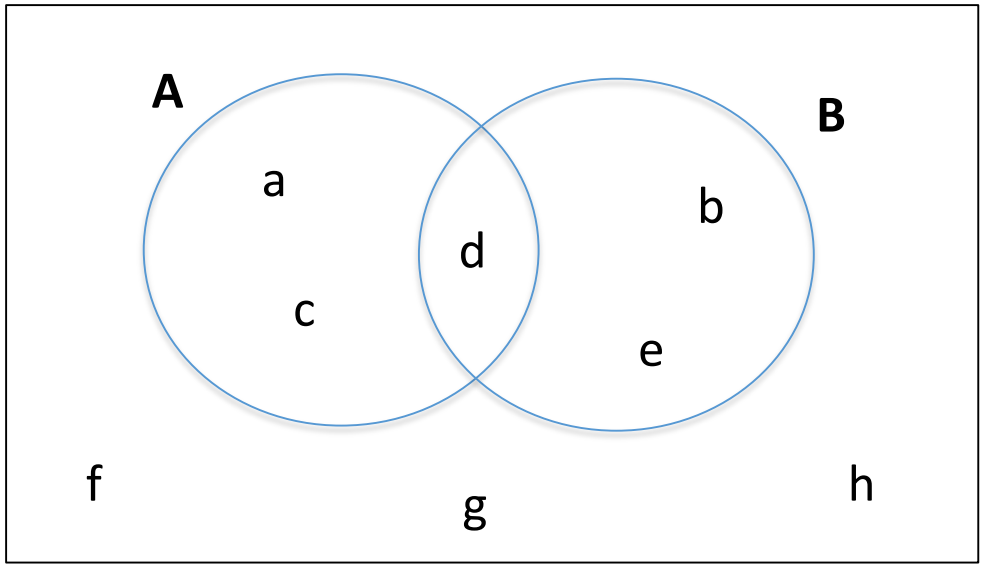
Observation (Entity sameness). Entities are identified by a set of properties which always hold about them. For instance, a black person is no longer a black person if it changes the color of his/he skin.

Observation (Entity difference). What makes an entity identical to itself (more or less, from your selected point of view) makes it different from every other entity:

- We must **distinguish** any **entity occurrence** from any other entity occurrence;
- Given **two distinct occurrences of two entities**, we must decide whether they are occurrences of the **same** entity or of two **different** entities (the two entities must be different if perceived at the same time);
- Given **two distinct entities**, we must decide whether they are members of the **same** etype or of two **different** etypes;
- Given **two distinct entity tuples**, we must decide whether they are the **same** tuple or of two **different** tuples;
- Given **two distinct entity tuples**, we must decide whether they are members of the **same** property type or relation type or of two **different** property types or relation types.

equality / inequality
vs.
similarity / dissimilarity
vs.
sameness / difference
of entities

An analogical representation -What do you see?



Intuition (Analogic representations in set theory). We model analogic representations using **sets,** where things are modeled as **single elements** and groups of things with properties or relations in common are modeled as **sets**.

Why set theory?

- It is *self-evident* – it formalizes the **similarity** and **dissimilarity** (as **set equality/ inequality**), i.e., what is different from what and what is similar to what,
  - of **entities** and **etypes**
  - of **entity properties** and **property types**
  - of **entity relations** and **relation types**
- It is *universal* – it can be used to represent ANY real world situation.
- *It both represents and depicts the world*. It has a linguistic representation (set-theory) and an analogic representation (Venn Diagrams)
- There is a *one-to-one mapping* between any set-theoretic representation of the world and Venn Diagrams

Set theory – Terminology

Terms naming components of linguistic representations

- *Entity*
- *Etype*
- *Entity property*
- *Property type*
- *Entity relation*
- *Relation type*
- *???*
- *???*
- *???*
- *???*

Corresponds to

Set-theoretic terms naming components of analogic representations

- *Element, Unit*
- *Class / set*
- *Tuple*
- *Relation*
- *Tuple*
- *Relation*
- *???*
- *???*
- *???*
- *???*

Each terms on the left (e.g., entity) is the name of the element which plays the same role as the element whose name is in the same line, on the right (e.g., unit).

Notation (terms used). In informal writing, whenever no confusion arises we will use terms on the right in place of terms on the left, to facilitate the interpretation.

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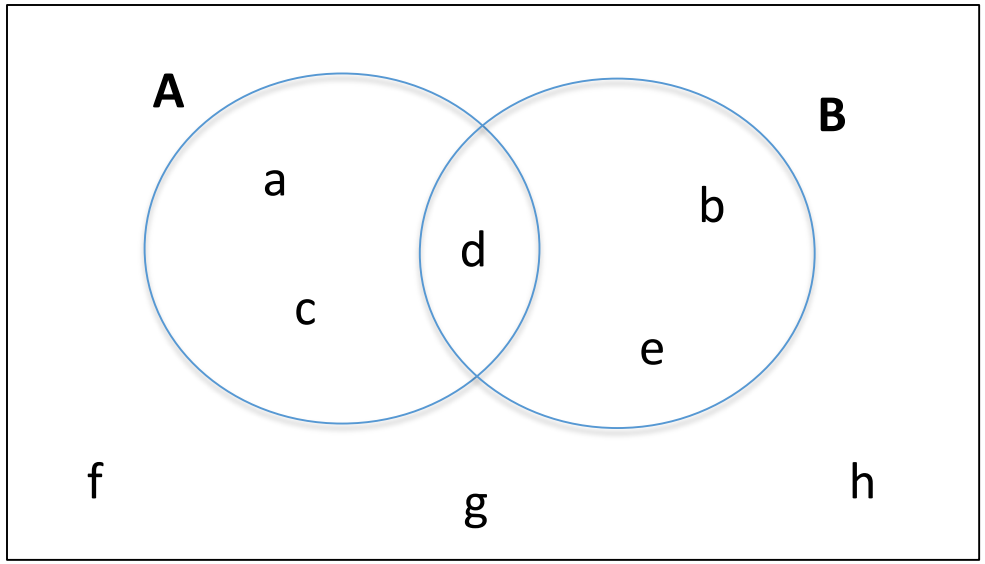
An analogical representation -What do you see?

Observation 1. Analogic representations are formalized using set theory



Observation 2 (Percepts, domain). Percepts are all the things or combinations of things which can be distinctively perceived, inside the selected **domain** (the box).

Example (Domain). The set of percepts of the picture on the left.

Percepts – Intuition

Intuition (percept) A **percept** is an entity or a combination of entities which is perceived as distinct from others.

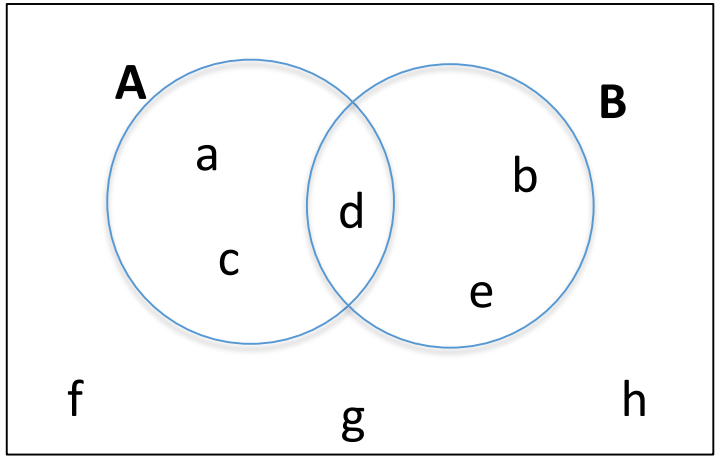
Examples: The percepts described by the sentences below

1. An *entity* you know *(e.g., Rocky)* or don't know (e.g., *woman#1*)
2. The *etype People* populated by entities who are persons (an etype)
3. The *property blond holding* of a *blond* entity (e.g., *person#1)*
4. The *property type blond* holding of the people who are blond (a property of a subset of the etype *people*)
5. The *relation near* holding of Sofia when *near* Paolo (a relation between entities)
6. The *relation type "talking to"* holding of *women talking to men* (a relation between two subsets of the two etypes *woman* and *man*)

Percepts - definition

Definition (percept) A **percept** is any of the six cases below.

- **Entities,** $e_1, \ldots, e_n$, perceived as distinct *units* (in space and time, e.g., Sofia);
- **Entity properties,** $P_1, \ldots, P_n$, perceived as *relations* between entities (e.g., Sofia is blond)
- **Entity relations,** $R_1, \ldots, R_n$, perceived as *n-ary relations* among entities (e.g., Rocky is between Sofia and Mark)
- **Etypes,** $E_1, \ldots, E_n$, as *classes* of entities, (e.g., person)
- **Property types,** $P_1, \ldots, P_n$, as *classes* of tuples of entities (e.g., Swedish people are blond)
- **Relation types,** $R_1, \ldots, R_n$, as *classes* of tuples of entities (e.g., breaks happen between lectures)



Example –Which percepts do you see?

Observation. The strings used in the text of the example are **pointers** to a region in the analogic representation. They are **NOT** words in some language

Observation (Which percepts can be seen in a picture). We have the following percepts (those which are underlined)

Entities

Rocky, tree#1, person#12, Sofia, animal#1, hair#1, hair#2, entity#1,...

Entity types (etypes)

Person = {Sofia, Paolo, woman#1, ...}

Entity properties

< hair#2, black>, < hair#1, blond>, ...

Property types

haircolor = {<hair#2, black>,<hair#1,blond>, ...}

from which we can write:

< hair#2, haircolor black>,< hair#1, haircolor, blond>

Entity relations

<Rocky, Sofia, Paolo>, <Sofia, tree#1, Paolo>, ...

Relations types

between = {<Rocky, Sofia, Paolo>, < Sofia, tree#1, Paolo >, ...}

from which we can write:

<between, Rocky, Sofia, Paolo>, ...

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Domain of interpretation - intuition

Observation (Percepts within a boundary). A **percept** is anything which is perceived as distinct from others. But perceived when and where? We need to define a boundary within which we select what we perceive.

Intuition (Domain of interpretation). We call **Domain of interpretation** the set of percepts that is within the scope of what we consider. That is, if **D** is a Universe of interpretation and **p** is a percept, then

$$D = \{p\}$$

where **{p}** is the set of selected percepts.

Observation (Different types of percepts). We can distinguish percepts based on our analysis of what we perceive.

Domain of interpretation - definition

Definition (Domain of interpretation) A **Domain (of interpretation)** D is defined as

$$D = < U, \{C\}, \{R\} >$$

where:

- U = {u} is called the **Universe (of interpretation)** of D.
- {u} is a set of **units** $u_1, \ldots, u_n$, for some n
- {C} is a set of **classes** $C_1, \ldots, C_m$ of elements, for some m, with $C_i \subseteq U$
- {R} is a set of n-ary **relations** $R_1^n, \ldots, R_p^n$ among elements, for some p, with $R_i^n \subseteq U \times \ldots \times U$
- $u_1, \ldots, u_n, C_1, \ldots, C_m, R_1^n, \ldots R_p^n$ are **percepts.**

Domain of interpretation - intuition

Observation (Diversity of percepts). From a set theoretic point of view we have three different types of percepts:

- (i) units: u
- (ii) classes: C
- (iii) relations: R^n

where:

- Units depict entities
- Classes depict sets on entities
- Relations depict properties of sets of entities
- Relations depict relations of sets of entities

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Facts - Intuition

Intuition (Fact) A **fact f** is something, involving percepts, happening at certain spacetime coordinates.

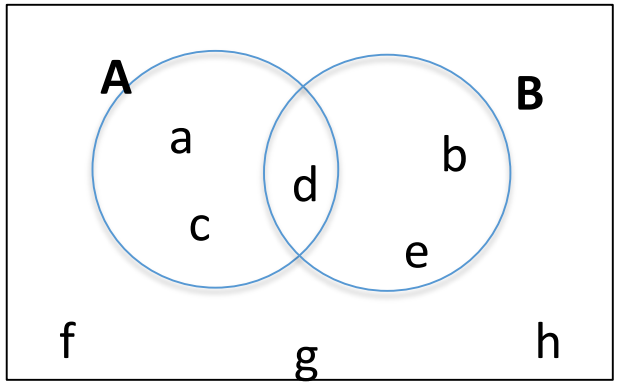
Example (fact). Facts involving percepts and facts denoted by the sentences below

- *Spacetime invariant facts,* e.g., dogs are animals, a woman is a person
- *Time invariant facts,* e.g., there is a church in Trento near the square
- *Space invariant facts,* e.g., moving across continents requires flying
- *Spacetime variant facts,* e.g., Sofia has blond hair, Sofia is a friend of Paolo, Sofia is walking, Paolo is talking to Sofia

Facts – intuition (continued)

Observation 1 (Fact). A **fact** is a **relation** between percepts. All and only the facts are as follows

- A **MemberOf** relation
  - between a unit and a class
  - between a tuple of units and a relation
- A **SubsetOf / SupersetOf** relation
  - between two classes
  - between two relations
  - between a n-ary relation and a tuple of n classes



Example –Which facts do you see?

Observation (Which facts can be seen in a picture). We have the following facts:

Element memberOf class

woman#1 ∈ Person

Tuple of elements memberOf relation

< Paolo, Rocky> ∈ HasDog

Class subsetOf class

Woman ⊆ Person

Relation SubsetOf relation

near ⊆ friendOf

Tuple of classes subsetOf relation

Person x Dog ⊆ HasDog

Relation subsetOf tuple of classes

HasDog ⊆ Person x Animal

Facts - definition

Definition (Fact). A **fact f** has one of the following five forms

- *Unit memberOf Class:* $u_i \in C_j$,
- *Tuple of Units memberOf relation:* $< u_1, \ldots, u_n > \in R^n$,
- *Class subsetOf Class:* $C_i \subseteq C_j$,
- *Relation subsetOf relation:* $R_i^n \subseteq R_j^n$
- *Relation subsetOf tuple of classes and viceversa:*
  - $R^n \subseteq C_1 \times \ldots \times C_n$
  - $C_1 \times \ldots \times C_n \subseteq R^n$

with: $u_i \in U$, $C_i \subseteq U$, $R^n \subseteq U \times \ldots \times U$.

Example –Which facts do you see?

Observation. Percepts define the space of possible facts. **Facts** are **relations among percepts** which describe what is the case. Facts are relations/ properties relating entities or sets of entities to one another. Elements alone cannot be facts!

Observation (Which facts involving percepts can be seen in a picture). We have the following (yellow)

Entities (Elements)

Rocky, tree#1, person#12, Sofia, animal#1, Thing#1,...

Entity properties (Element binary relations)

<haircolor, hair#2, black>, < haircolor,hair#5,brown>, ...

Entity relations (Element n-ary relations)

<between, Rocky, Sofia, Paolo>,
<between, Sofia, tree#1, Paolo>, ...

Etypes (Classes)

Person = {Sofia, Alfredo, Martin, ...}

Etype properties (Class binary relations)

color = {<hair#2, black>,<hair#1,blond>}

Etype relations (Class n-ary relations)

between = {<Rocky, Sofia, Paolo>, < Sofia, tree#1, Paolo >, ...}

Facts and domains

Observation 1 (percept, domain, fact). As defined before we have that a domain **D** is defined as a set of percepts, that is **D = {p}**. From the definition of fact we can also see **D** as the set of facts, that is

$$\mathbf{D = \{f\}}$$

where a fact **f** ∈ **{f}** is built from percepts by applying any of the equations used in the construction of fact.

Observation 2 (percept, domain, possible fact). The construction of a **domain** depends on two modeling choices:

- The selection of the set of **percepts** {p}
- The selection of the set of **possible facts** {f}. Domains are usually assumed to contain all **possible facts** which can be composed from a selected set of percepts {p}.

where by "possible fact" we mean a fact (relation among percepts) which may eventually be what is the case

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Models - Intuition

Intuition (Model) A **model** is what you perceive, that is the set of facts you perceive, not necessarily all of them

Example: A possible model depicting the picture above is the following set of facts:

$$\{Sofia \in person, Paolo \in man, Rocky \in dog,$$
$$<Sofia, Paolo> \subseteq near, <Sofia, hair\#32> \in blond,$$
$$<Sofia, Paolo> \in friendOf, man \subseteq person$$
$$Rocky \in animal, <Rocky, Sofia> \in dogOf, \ldots\}$$

Models - definition

Definition 3.1 (Model) A **model** M is a set of facts M = {f}

$$\mathbf{M = \{f\}}$$

Observation (Facts and models) Facts are the atomic, not further decomposable, elements of a model.

Observation (Mutually (in)consistent facts in a model) The example model above could be extended by asserting the fact that "Sofia is a woman". But NOT by adding the fact that "Paolo is a woman", as we would have two mutually inconsistent facts. **A model cannot contain facts which are mutually inconsistent.**

Facts, models and domains

Proposition (Domain and facts). A **Domain** D is a set of facts {f}.

$$D = \{f\}$$

Proposition (Model). Given a domain D, a model M in D is a subset of D.

$$M = \{f\} \subseteq D$$

Observation (Domain, model). A domain is the set of all facts that we are willing to consider. A model is just the subset of facts that we define when depicting what is the case in the current situation.

Facts, models and domains

Observation (Domain) A Domain defines all and only what can be potentially perceived. It can be thought of as the set of all the possible things and also as the set of all the possible models.

Observation (Mutually inconsistent facts in a domain). A domain, differently from a model, **can contain facts which are mutually inconsistent**. Given a domain, there are many potential models, some of which are potentially mutually inconsistent.

Observation. Domains must allow for the possible instantiation of distinct mutually inconsistent models, as it is normally the case in the world.

Model theory (terminology)

Observation (Model theory) . Model theory, that is, the theory of models, is the discipline that studies the structure of models as a function of specific types of percepts and facts.

In model theory, models are formalized as algebraic structures

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Set theory – Terminology

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- *Etype*
- *Entity property*
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- *Entity relation*
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- *???*

Corresponds to

Set-theoretic terms naming components of analogic representations

- *Element*
- *Class / set*
- *Tuple*
- *Relation*
- *Tuple*
- *Relation*
- *Percept*
- *Domain of interpretation*
- *Fact*
- *Model / analogic representation*

Each terms on the left (e.g., entity) is the name of the element which plays the same role as the element whose name is in the same line, on the right (e.g., element).

Notation (terms used). In informal writing, whenever no confusion arises we will use terms on the right in place of terms on the left, to facilitate the interpretation.

Key notions

- Formalization of analogical representations as model theory
- Model theory as set theory
- Thing
- Representation diversity (leading to subjectivity, ...)
- Entity
- Percept
- Domain
- Fact
- Model
- Correlations between percepts, facts, models and domains
- Model theory

Models
(model theory)
(HP2T)