École Polytechnique Fédérale de Lausanne (EPFL)

Analysis and Modelling of Locomotion

BIOENG-404

Simulation of a SLIP model

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1 Deliverable 1

1.1 Q.1

The Spring Loaded Inverse Pendulum (SLIP) model is a widely used tool for the prediction of ground reaction forces and the trajectory of the center of mass (CoM). Moreover, it is employed in the modelling of running. Thus, if we consider a bouncing gait during running, we can model it by a simple spring, having stiffness constant k (representing the stiffness of the entire musculoskeletal system), and a point mass into which the whole body mass is concentrated. In the ground-contact phase, the spring compresses during the first half of the movement and relaxes during the second half [1].
The SLIP model is defined as being a hybrid system because of its mixed nature, being at the same time a spring-mass model and an inverted pendulum model, and its mixed dynamics. It is based on the description of two main phases: a ground-contact phase and a flight phase. Both phases are described through continuous dynamics defined by the common motion equations obtained by applying classical Newtonian mechanics. However, the switch between one state and the other, i.e. the switch between the flight phase (indicating the swing phase during a gait) and the ground-contact phase (indicating the stance phase during gait) occurs in a discrete fashion. In the stance phase, the gravitational force $M\vec{g}$, and the force caused by the spring compression $\vec{F_s}$ are applied to the center of mass (CoM); in the swing phase, only the gravitational force is present. These forces are shown in Figure 1. The motion equations defining the dynamics for both phases are presented below:

Equations of motion for the stance phase:

$$\begin{cases} m\ddot{x} = k(l_0 \cos\varphi - x) \\ m\ddot{y} = -mg + k(l_0 \sin\varphi - x\tan\varphi) \end{cases}$$

Equations of motion for the swing phase:

$$\begin{cases} \ddot{x} = 0 \\ \ddot{y} = -g \end{cases}$$

1.2 Q.2

The three main parameters we can play with in the simulation of the SLIP model are the *angle of attack*, which is defined as the angle at which the SLIP touches the ground, the *forward speed*, which is the horizontal speed of the model and the *spring stiffness* k, which represents the level of stiffness of the spring. In order to find a good combination of these three parameters to obtain a stable gait, we should fix one of them and change the others. For example we can set the forward speed $V_{0,x}$ at a value of $7m/s$. In order to find the right combination of the other two parameters, we can think about the possible bio-mechanical relation between the angle of attack and the spring stiffness. In particular, we know that the higher the angle of attack, the higher the spring stiffness should be. Finally, a good combination of the three parameters was found to be ($V_{0,x}$ = 7m/s, $\alpha = 62°$, k = 7,35KN/m).

1.3 Q.3

The SLIP model is an energy-conservative model, which means that there is no dissipation of energy during the gait (in the case of horizontal ground). In particular, during the stance phase the potential and kinetic energies are at their highest level at the moment of ground-contact. These two energies decrease progressively by making the spring (elastic) energy increase (this is represented by spring compression). The elastic energy reaches its maximum at the mid-stance, then the spring starts to relax again in order to arrive at the take-off moment(end of stance phase) with the same distribution of energies as the touch-down moment (beginning of stance phase). The process of spring compression is shown in Figure 2. In the case of up/down slopes the conservation of energy is no longer fulfilled and thus slopes would not allow non-stationary stable gait. Indeed, if we consider the constant energy C = mgh (potential) + $\frac{1}{2}mv^2$ (kinetic), where m is the mass, g is the gravitational acceleration, h the vertical height and v is the speed), we can see that a positive slope will lead to a progressive increase in the potential energy, which in turn will make the kinetic energy decrease. The speed will eventually become zero and the SLIP model will stop. In the case of a negative slope, the potential energy will decrease which in turn will increase the kinetic energy. This will result in the SLIP falling over due to too high speed. In order to reach a stable gait, there would be the need for a corrective active component in the system, which should add or absorb the energy required. Thus, in conclusion we can affirm that the SLIP model can not achieve a stable (non-stationary) gait in the case of up/down slopes.

2 Deliverable 2

2.1 Q.1-2

A graph showing the stability of the model with different combinations of the spring stiffness k and angle of contact α is presented in Figure 3(a). The map is obtained by the use of a systematic grid search. The stability is assessed by counting the number of steps performed. The initial horizontal velocity $V_{0,x}$ was fixed to a value of $7m/s$. As expected, we can observe the typical J-shape of stability [3]. We can see that the higher the angle of attack, the higher the spring stiffness constant. When the angle of attack is high, the corresponding potential energy will be higher too. By taking into account the fact that the forward velocity is fixed, this means that the total energy will be higher. This energy will be transferred to the spring and, since the elastic energy is given by $\frac{1}{2}kx^2$ (where x here represents the compression of the spring), the spring constant k will need to be higher in order to ensure a stable gait. The reverse is valid for small angles of attack. In addition, it is important to notice that it would be meaningless to take into account an $\alpha < 45°$, as this would not be physiologically possible. The same is true for $\alpha > 90°$, hence we considered the interval $\alpha \in [45°,90°]$.

2.2 Q.3

The same procedure as the one followed in point 2.1 was applied in order to analyse different combinations of horizontal velocities $V_{0,x}$ and spring stiffness constants K by keeping the value of α constant (we fixed it at **62°**). The result is shown in Figure 3(b). We can see that in this case the stability curve is given by an almost "linear" relationship between the two variables. In particular, it is interesting to notice that the higher the forward velocity, the larger the range of spring stiffness constants k allowing a stable gait.

2.3 Q.4

In order to prove that stability (here represented by a high number of steps performed during the simulation) was not possible in the case of up/down slopes, the same grid search used in points 2.1 and 2.2 was employed. The slope was fixed at a value of $\pm$ 0.2 in order to try both positive and negative slopes. The results are shown in Figure 4 for a 20% slope, and in Figure 5 for a -20% slope. As we can easily see, the stability was not reached for any combination. In particular, it is interesting to notice that the general J-shape in the case of the k-α map and the "linear" relationship in the case of the dx-k map were somehow maintained. Moreover, we can observe that with a positive slope it was still possible to perform more steps than in the case of a negative slope. In particular, we can argue that in the case of a positive slope, for a given angle, the spring constant will need to be higher (with respect to the flat-ground case) in order to ensure a more stable gait. In the case of a negative slope, the opposite is valid (i.e. the stiffness constant should be lower).

2.4 Bonus: Alternative score function

We can affirm that the number of steps is surely a good approximation of the stability of the system. It is certainly an indirect measure of the system's stability, as it is based on the assumption that every step made is also stable. In order to implement a different score function, we exploit one of the conditions required for stability, i.e. $\frac{y_{i+1}}{y_i} \approx 1$, where y_i represents the apex height of step i. We know that in the case of stability, after a certain number of steps, the system will converge to the stable fixed point, corresponding to a stable height. This is represented in the condition: $y_i = y_{i+1} = y*$. This convergence is also illustrated in Figure 8. In our implementation this ratio was computed for every step performed. Then the mean was calculated and this values was considered as being the score that quantifies stability. When only one step was performed or when the last y value was negative (due to the falling of the system "crossing" the ground), the score was automatically set to 0.01. This was done according to the fact that in unstable systems the score should be $\ll 1$. The result obtained with our score function is shown in Figure 6. We can see that the result is well comparable to the one presented in Figure 3(a). Indeed, the J-curve representing the stability of the system is still present. However, we can notice that in the Figure 6, the transition between stable and unstable looks more sharp, which means that our score function does not easily capture the middle states between unstable and stable conditions.

3 Deliverable 3

3.1 Q.1-2

The graphs presented in Figure 7 show both the return maps and the eigenvalues for the apex height y and the horizontal velocity dx of the system. In order to assess stability, we know that two conditions have to be fulfilled. First, a periodic solution should be found, i.e. $y_i = y_{i+1} = y*$ ($y_i = y_{APEX}$ of step i); second, the perturbations should decrease step by step, i.e. $\left|\frac{y_{i+1}}{y_i}\right| < 1$. The existence of such a periodic solution is given by the presence of a fixed point in the return map. In order to assess the stability of this point, one can look at the slope of the "real input-output function" at the intersection with the periodicity line (i.e. the diagonal). Its absolute value should be < 1. In Figure 7, we can see that this is the case. If we now want to compare the return maps with the y_{APEX} and the dx, we can see that there is a relationship between the two of them. While the system evolves towards the fixed point, the two values (the height and the forward velocity) are inversely related: while y_{APEX} decreases, the forward speed increases. This is related to the conservation of energy, since if the potential energy (proportional to y_{APEX}) decreases, the kinetic energy (proportional to the forward velocity) has to increase.
A sequence of stable hopping is shown in Figure 8.

3.2 Q.3-4

The term **stability** refers to the condition that a specific dynamic system can reach over time. In particular, it refers to the ability of the system remaining constant (i.e. stable). Another important aspect is the fact that if a perturbation occurs the system is able to converge again towards the stability condition. In the case of the SLIP model, reaching the stable condition leads to a ever-lasting hopping.
The term **periodicity** refers to the fact that a certain state of the system repeats itself over time in a precisely defined fashion. In mathematical terms, $State(t) = State(t+T)$, with T the period. In the specific case of the SLIP model, we can encounter two different frames: either the system reaches the stable fixed point and therefore every step will be identical to the previous one ($T = 1$ step), or the system will approach the fixed point by oscillating around it, which leads to a period > 1 Step. The **basin of attraction** represents a range of values for which a dynamical system will surely converge towards a fixed point. In the case of the SLIP model this corresponds to a range of either the y_{APEX} and/or forward speed dx.
An example of convergence of the system to a fixed point on a return map is shown in Figure 7(a).

4 Deliverable 4

4.1 Q.1

As it is visible in Figure 9, the vertical trajectory of the center of mass (CoM) converges to a stable height more rapidly in the case where leg retraction is added to the system (the red trajectory). As the leg retraction component increases (from retraction speed $\omega_r = 0.05$ rad/s to $\omega_r = 0.25$), the convergence is reached more quickly. Notice that when $\omega_r = 0.25$, the convergence to the stable height is reached even only after one single step. If we think about this in terms of return map, we can argue that the "real input-output function" will be flatter in the case of leg retraction, which will lead to a faster convergence towards the fixed point.

References

[1] Farley CT and Ferris DP. Biomechanics of walking and running: center of mass movements to muscle action. *Exerc Sport Sci Rev. 1998;26:253-85*, 1998.

[2] Hartmut Geyer, Andre Seyfarth, and Reinhard Blickhan. Spring-mass running: simple approximate solution and application to gait stability. *Journal of Theoretical Biology*, 232(3):315–328, February 2005.

[3] Andre Seyfarth, Hartmut Geyer, Michael Günther, and Reinhard Blickhan. A movement criterion for running. *Journal of Biomechanics*, 35(5):649–655, May 2002.

5 Appendix

5.1 Figures

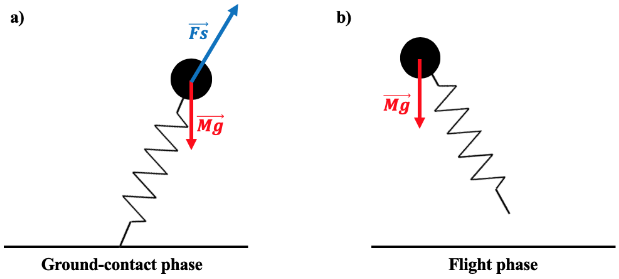


Figure 1: Forces acting on the center of mass (CoM) during the a) ground-contact (= stance) phase, and b) the flight phase (= swing)of the SLIP dynamics.

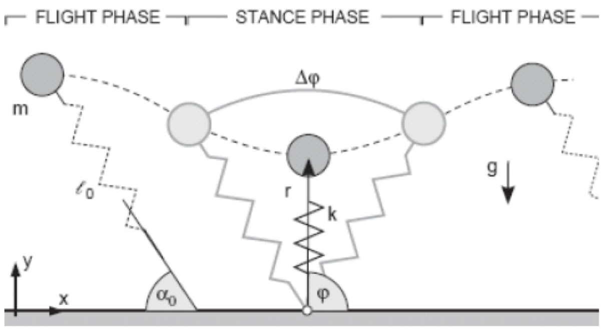
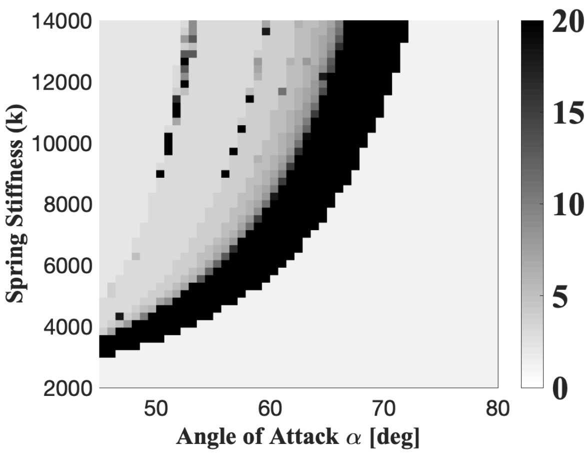
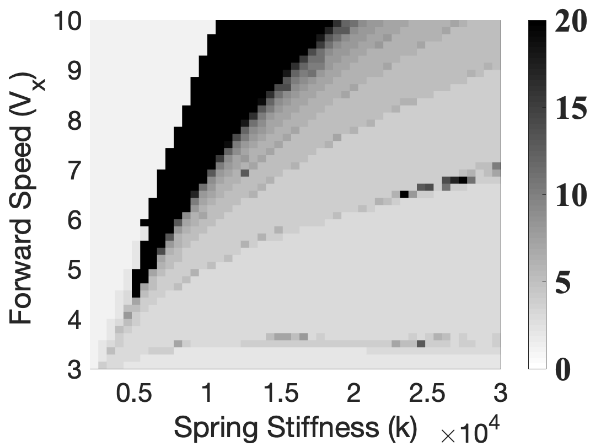


Figure 2: Scheme illustrating the process of spring compression from the flight phase to the stance phase [2].

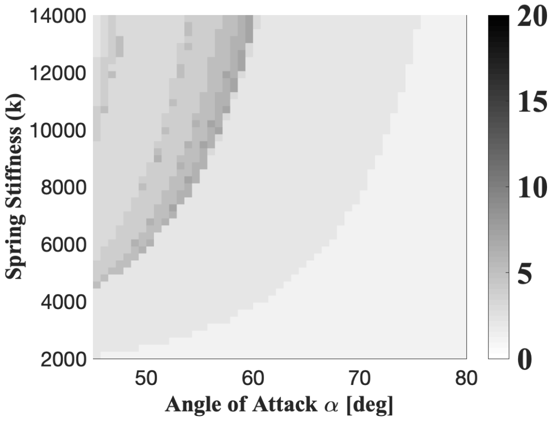


((a)) Map showing the relationship between the stiffness constant k and the ankle of attack *alpha* for a constant velocity V_0 =7m/s.

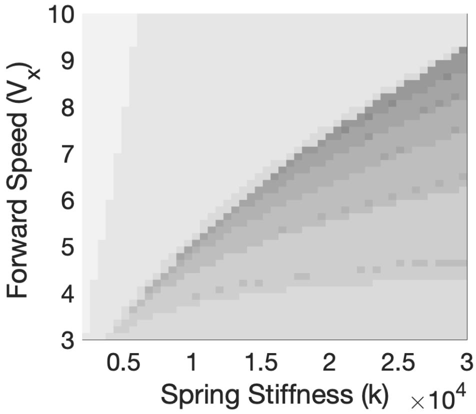


((b)) Map showing the relationship between the stiffness constant k and the forward speed V_0 for a constant angle of attack α =62°

Figure 3: Distribution of the stability of the system (black areas) when studying the change of angle of attack, spring stiffness and forward speed.

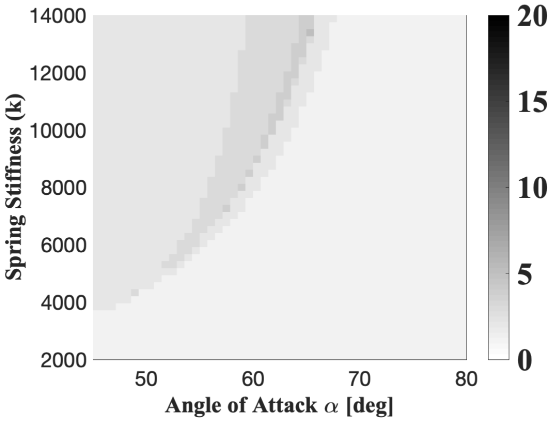


((a)) Map showing the relationship between the stiffness constant k and the ankle of attack *alpha* for a constant velocity V_0 =7m/s, when having a simulation ground with a slope of 0.2.

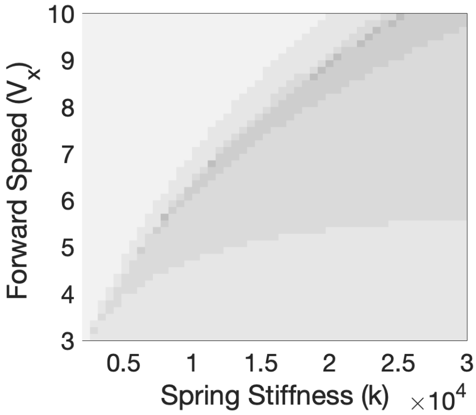


((b)) Map showing the relationship between the stiffness constant k and the forward speed V_0 for a constant angle of attack α =62°, when having a simulation ground with a slope of 0.2.

Figure 4: Distribution of the stability of the system when studying the change of angle of attack, spring stiffness and forward speed when considering a slope of 0.2.



((a)) Map showing the relationship between the stiffness constant k and the ankle of attack *alpha* for a constant velocity V_0 =7m/s, when having a simulation ground with a slope of -0.2.



((b)) Map showing the relationship between the stiffness constant k and the forward speed V_0 for a constant angle of attack α =62°, when having a simulation ground with a slope of -0.2.

Figure 5: Distribution of the stability of the system when studying the change of angle of attack, spring stiffness and forward speed when considering a slope of -0.2.

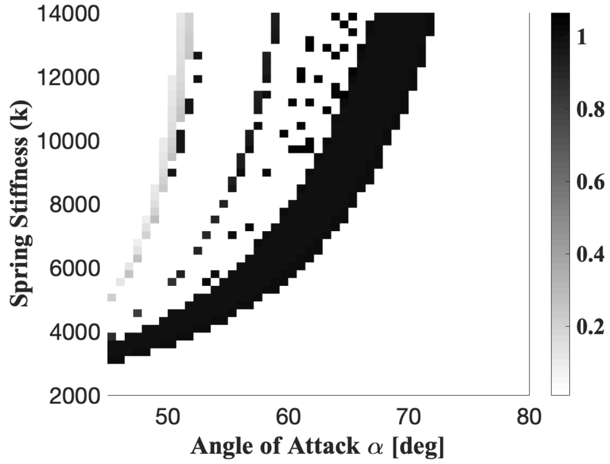
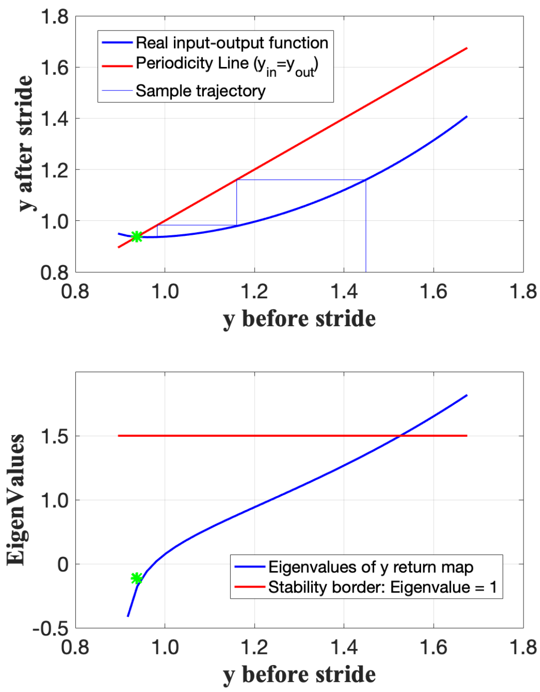
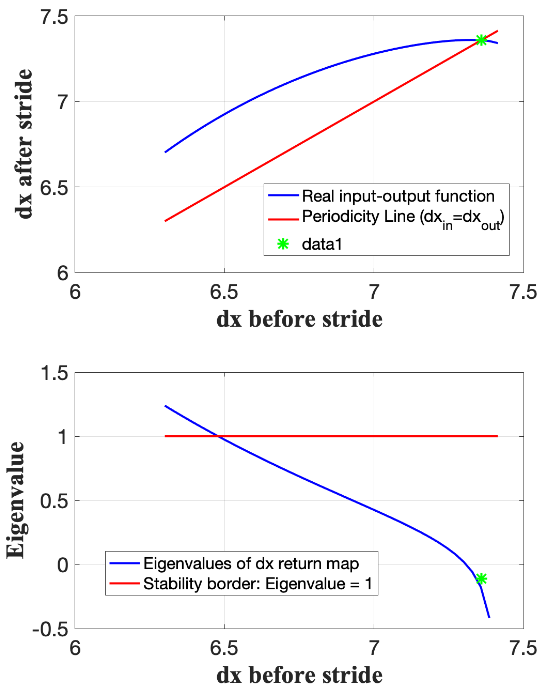


Figure 6: Map of the system stability when using our new implemented score function. For the simulation we kept the forward speed V_0 constant, while changing the stiffness constant k and the angle of attack α.



((a)) Return map of the body height (y) 7m/s, 62°, 7350N/m



((b)) Return map of the forward velocity (dx), initial parameters of 7 m/s, 62°, 7350 N/m

Figure 7: Return maps showing the location of the stable fixed point for the body height (y) and the forward velocity (dx).

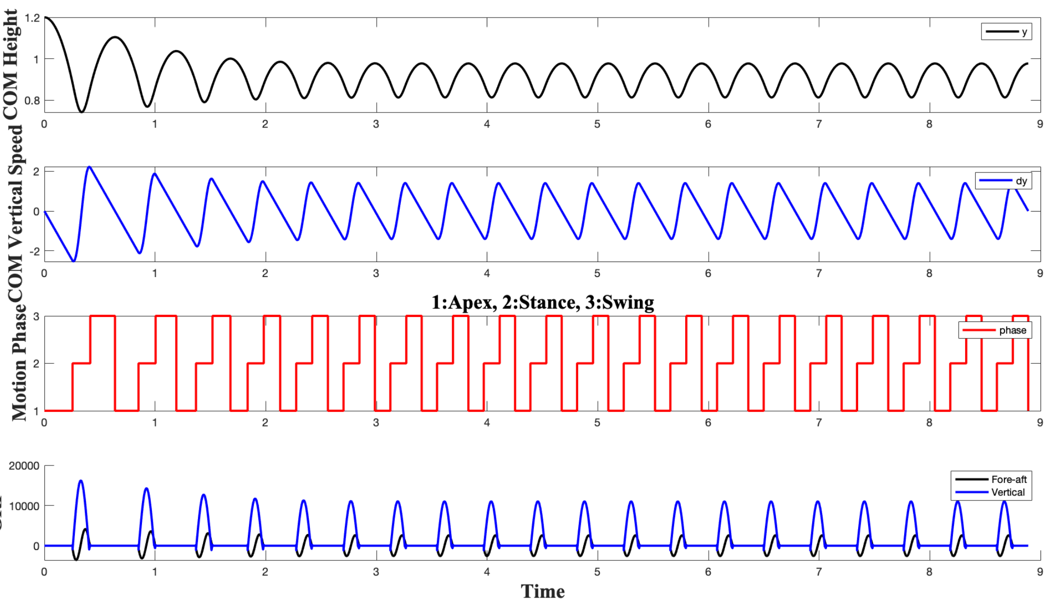
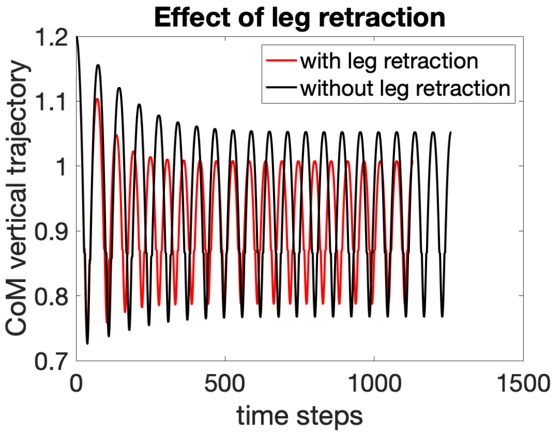
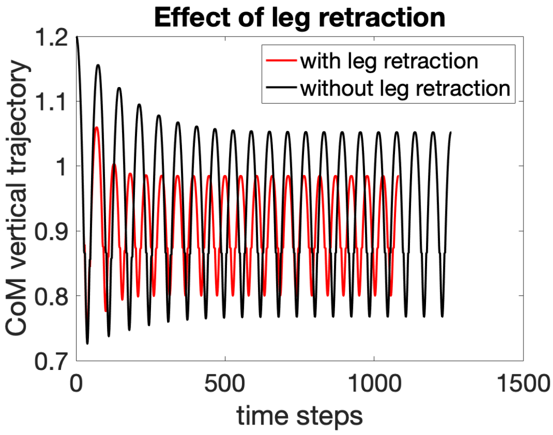


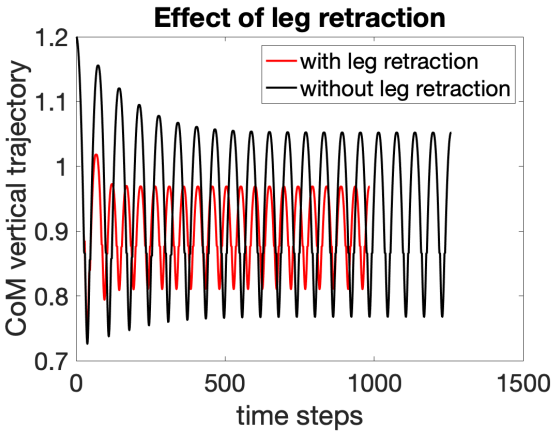
Figure 8: Sequence of stable hopping showing the motion phase, the vertical speed of the center of mass and the height of the center of mass.



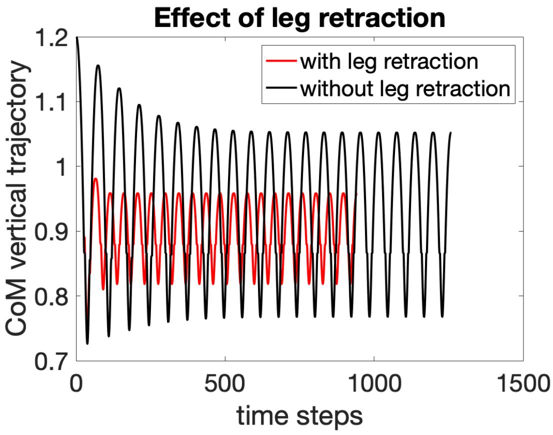
((a)) Leg retraction at $\omega_r = 0.05 rad/s$



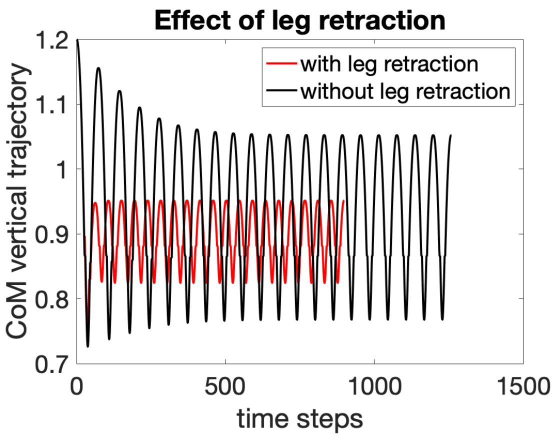
((b)) Leg retraction at $\omega_r = 0.10 rad/s$



((c)) Leg retraction at $\omega_r = 0.15 rad/s$



((d)) Leg retraction at $\omega_r = 0.20 rad/s$



((e)) Leg retraction at $\omega_r = 0.25 rad/s$

Figure 9: Evolution of the vertical trajectory of the center of mass (CoM) without or with leg retraction (ω_r from 0.05 to 0.25 [rad/s]). The retraction angle was fixed at 60° and the forward speed at $6.8 m/s$

5.2 Score function code

```
function [new_score] = implemented_alternative_score_function(apexes_vector)
% This function implementes our suggestion for an alternative score
% function in order to assess the stability of the SLIP model.
% The function has to be called in the grid-search algor
%
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%          - Leonardo Pollina
%          - Ludovica Romanin

% Check if the system performed only one step. If that is the case,
% arbitrarly set the score to 0.01 (according to the idea that un unstable
% system should give a value << 1).

if (length(apexes_vector) == 1)
    ratios_ = 0.01;
else

    % If the system performed multiple steps, we want to be sure that the
    % last one did not give a negative Y_apex. Again, it that is the case,
    % we set the score to 0.01.
    if (apexes_vector(end) < 0)
        ratios_ = 0.01;
    else
        % In all other cases we compute the ration between Y_{i+1} and
        % Y_{i}.
        ratios_ = zeros(length(apexes_vector) -1 ,1);
        for step = 1:length(apexes_vector)-1
            ratios_(step) = apexes_vector(step+1)/apexes_vector(step);
        end
    end
end

% The new score is given by the mean of the previously computed ratios.
new_score = mean(ratios_);
end
```