Report on numerical simulation of femtosecond pulse propagation in nonlinear centrosymmetric lossless media

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The following numerical simulation is based on the theory of pulse propagation in nonlinear centrosymmetric lossless media presented in Chapter 15 of the Nonlinear Optics Script-v8 of the course *Experimental Nonlinear Optics* held at Friedrich Schiller Universität of Jena.

1 Implementation

The numerical simulation has been implemented in the Matlab programming language and saved as .m file.

Material information necessary for the simulation:
- $n_0(\lambda)\mid_{\lambda_0}$ linear refractive index as a function of the wavelength evaluated at the central wavelength of the pulse, calculated via the Sellmeier equation with the coefficients taken from [1],
- $n_2(\lambda)\mid_{\lambda_0}$ nonlinear refractive index as a function of the wavelength evaluated at the central wavelength of the pulse. The experimental values of the nonlinear refractive index are sporadic and not available as a function of the wavelength. Therefore, the constant values of n_2 are taken from [2] and [3] for the available wavelengths,
- $k_0'' = \frac{\partial^2 k}{\partial w^2}\mid_{w_0} = \frac{\lambda^3}{2\pi c^2}\frac{\partial^2 n_0(\lambda)}{\partial \lambda^2}\mid_{\lambda_0}$ group velocity dispersion as a function of the central wavelength of the pulse, calculated via successive derivatives of the Sellmeier equation.

Initial pulse information necessary for the simulation:
- Gaussian Fourier Transform Limited (FTL) initial condition,
- λ_0 central wavelength in vacuum,
- $\Delta t_{FWHM} = \Delta t_0$ initial FWHM temporal pulse duration,
- I_{peak} instantaneous peak intensity,
- CEP_{in} input Carrier Envelope Phase to explore sensitivity of the CEP_{out} to changes of the input one.

The pulse has been eventually described as $E(z=0,t) = E_0 \cdot e^{-2ln2\frac{t^2}{\Delta t_{FWHM}^2}} \cdot e^{i(w_0 t + CEP_{in})}$, with the convention $\mathcal{E}(z,t) = ElectricField = E(z,t) + E^*(z,t)$, therefore $E_0 = \sqrt{\frac{Z_0 I_{peak}}{2n_0(\lambda_0)}}$ (Z_0 vacuum impedance).

The Nonlinear Schrödinger Equation, that describes the propagation of the pulse in the nonlinear media in the pulse reference frame (see Script-v8), has been solved using the Split-Step method [4] .

The code is structured as follows:
1- definition of the parameters for the pulse and medium description,
2- construction of time t and frequency f axis, and their normalized version τ and f_{norm},
3- construction of initial electric field $E(t)$ and normalized electric field $U(\tau)$,
4- Split-Step method for the propagation of the normalized electric field $U(z,\tau)$ saving it for each step,
5- back conversion to $E(z,t)$ and calculation of its spectrum $S(z,w)$,
6- calculation of the spectral phase $\phi(w)$, and fitting with polynomial in a region of w where the pulse spectral intensity is $\geq 10^{-3} S_{peak}(w)$ to find GDD, TOD, etc.... These are calculated to be such that:

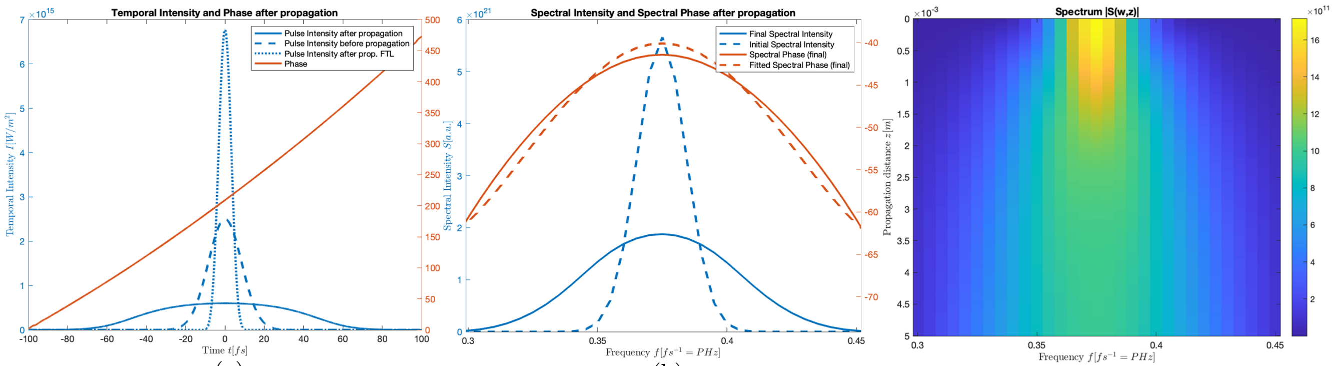
$$\phi(w) = \phi_0 + GD \cdot (w-w_0) + \frac{GDD}{2!}\cdot(w-w_0)^2 + \frac{TOD}{3!}\cdot(w-w_0)^3 + \frac{4^{th}}{4!}\cdot(w-w_0)^4 + \dots$$

7- generation of plots and save them,
8- functions to calculate n_0, k_0'', dispersion length L_D, nonlinear length L_N, and fit $\phi(w)$ with a polynomial.

2 Test cases

2.1 White light generation in a thin piece of glass: fused silica

Input pulse parameters: $I_{peak} = 1TW/cm^2$, $\Delta t_{FWHM} = 20fs$, and $\lambda_0 = 800nm$, resulting in $L_D \approx 2mm$ and $L_N \approx 0.6mm$ ($L_D \sim L_N$). At this wavelength fused silica has a normal dispersion ($GDD > 0$). Qualitatively, if we consider the dispersion and the nonlinear effect separately, we see that both effects induce an up-chirp on the pulse (formulas for the instantaneous frequency: dispersion of Gaussian pulse $\rightarrow$ $w(t) = w_0 + sign(k_0'')\frac{z' \cdot t'}{(1+z'^2)\Delta t_0}$ with $z' = z/L_D$, and SPM of a generic pulse $\rightarrow$ $w(t) = w_0 - \frac{w_0 n_2 z}{c}\frac{dI}{dt}$).
The results of the simulation are shown in Figure 1. Figure 1a illustrates that the pulse has increased its initial temporal duration and that the phase is quadratic in time with positive concavity, indicator of an up-chirp. Figure 1b shows that the pulse has broadened its spectrum by a factor $\gtrsim 2$, while acquiring a quadratic spectral phase. To compensate the dispersion introduced by the nonlinear propagation in 5mm of glass it would be necessary to apply a $GDD = -286fs^2/rad$, and a $4^{th}OrderDispersion = 5120fs^4/rad^3$ (the fitting is not perfect, these values have to be handled gingerly). Figure 1c shows the broadening of the spectrum during the propagation.

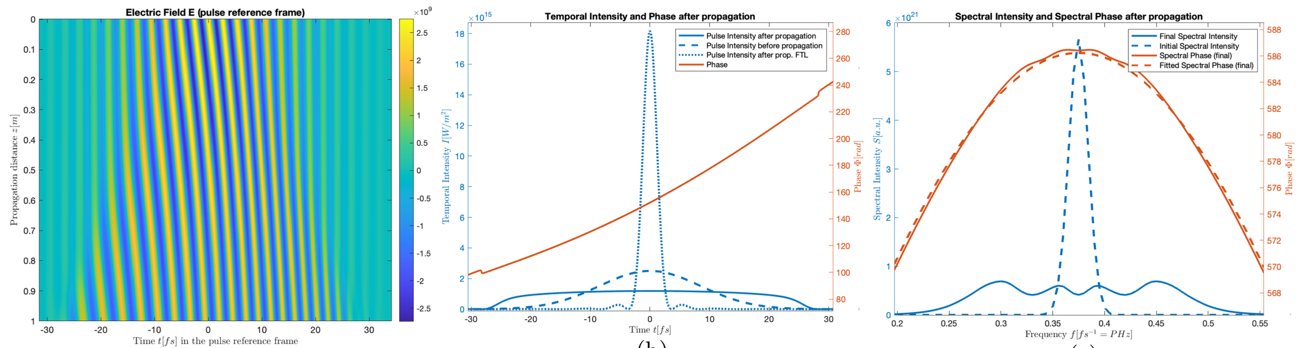


(a) (b) (c)

Figure 1: Results of the numerical simulation of a high intensity pulse propagating in fused silica

2.2 Pulse spectral broadening in Argon (e.g. hollow core fiber filled with Ar @1bar)

Input pulse parameters: $I_{peak} = 1TW/cm^2$, $\Delta t_{FWHM} = 20fs$, and $\lambda_0 = 800nm$, resulting in $L_D \approx 3.7m$ and $L_N \approx 6.6cm$. At this wavelength Ar has a normal dispersion ($GDD > 0$). With respect to the previous example, now the nonlinear effects are much stronger than the dispersion ones ($L_D \gg L_N$).
The results of the simulation are shown in Figure 2. Figure 2a illustrates the electric field in the pulse reference frame as a function of the propagation distance (y-axis). It is clearly visible that the pulse is broadening in time and is acquiring an up-chirp. Figure 2c evidences that the pulse has broadened its spectrum even more than in the previous case, due to the decrease of the strength of dispersion effects. It is possible to see from Figure 2b that the FTL pulse of such spectra has a FWHM temporal duration of $\Delta t_{FWHM} = 2.7fs$, with pre- and post-pulse whose relative peak intensity to the main peak of the pulse is $\approx$ 4 orders of magnitude smaller. To partially compensate the dispersion introduced by the nonlinear propagation, it is necessary to apply a $GDD = -28.05fs^2/rad$, and a $4^{th}OrderDispersion = 22.1fs^4/rad^3$. Running the simulation for different values of CEP_{in}, a linear relationship between it and CEP_{out} is verifiable.



(a) (b) (c)

Figure 2: Results of the numerical simulation of a high intensity pulse propagating in Ar at p=1bar

2.3 Self-compression and soliton formation

Material considered: fused silica.
Input pulse parameters: $I_{peak} = 3GW/cm^2$, $\Delta t_{FWHM} = 235fs$, and $\lambda_0 = 1.55\mu m$, resulting in $L_D \approx L_N = 0.375m$. At this wavelength fused silica has an anomalous dispersion ($GDD < 0$). Therefore, in general, we expect the dispersion and the nonlinear effects to slightly counterbalance each other. Due to the particular choice of the pulse characteristics (i.e. such that $L_D \approx L_N$), we expect the two phenomena to be almost perfectly counteracting each other, giving rise to a stationary solution, a wave packets that can propagate undistorted over long distances (infinite in our case since we are neglecting absorption/scatterings/etc...), called soliton.
The results of the simulation are shown in Figure 3. Figure 3a illustrates the formation of the soliton in a distance that is $\approx 3L_N$. Figure 3b evidences the propagation of the soliton, which maintain its shape also after long distances. Figure 3c shows the intensity of the Gaussian pulse after propagating $1m$ in fused silica. It is visible that the pulse has decreased its temporal duration by a factor ≈ 2 and that it is now $sech^2$-shaped and no more a Gaussian-shaped. Figure 3d shows that the spectra of the pulse after a propagation distance of $1m$ is slightly broadened to sustain the compression in time domain.

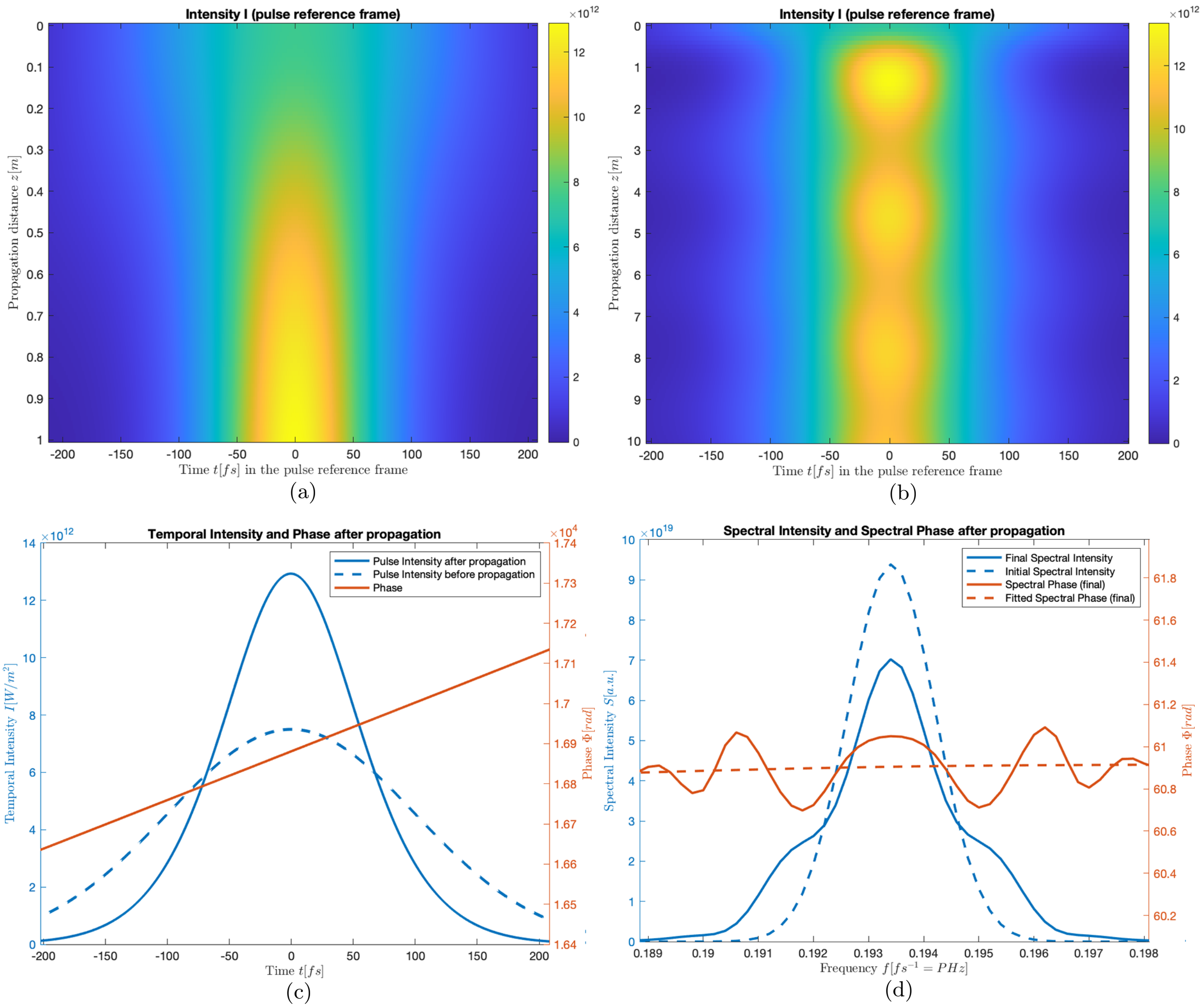


Figure 3: Results of the numerical simulation of a self-compression phenomena (and soliton formation) in fused silica

2.4 Parameters for telecommunication in optical fibers

Material: fused silica.
In the following discussion about the parameters of the pulse that are suitable for telecommunication we will assume no absorption/scattering/losses/gain during the propagation in the optical fiber, and a simple amplitude modulation to encode the digitized signal and transmit it (i.e. pulse = bit 1, no pulse = bit 0).

Under these assumptions, the important quantities are the bit-rate (BR) and the distinguishability of transmitted bits. The best condition corresponds to a high BR and low cross-talk between different bits. The upper limit for the BR is set by the electronics capability of integrating the signal over $t_{integration} \gtrsim 1ns$ temporal scale. The cross-talk between different bits sets a further limit to the temporal duration of pulses. Depending on the Bit-Error-Rate (BER) (i.e. $\frac{\#mistakes}{\#transmitted\_bits}$) that one want to achieve, the following condition could be less or more restrictive: $\Delta t_{FWHM} \lesssim 0.5 \cdot t_{integration}$.
Therefore, the maximum temporal duration of the pulses we are interested in is $\Delta t_{FWHM} \lesssim 500ps$. We want the "bit 1" pulses to travel kilometers without interfering between each others and with the bits 0. Consequently, we ask for L_D and L_N to be $\gg 1km$, in such a way that the dispersive and nonlinear effects will play a role only on long spatial scales.
The following simulation is done setting $L_D \approx L_N = 500km$ and $\lambda_0 = 1.55\mu m$. This request freezes our input pulse parameters to $I_{peak} = 2.25KW/cm^2$ and $\Delta t_{FWHM} = 268ps$. Note that the condition on Δt_{FWHM} is satisfied. Moreover, at $\lambda_0 = 1.55\mu m$ fused silica exhibits a minimum in attenuation ($\sim 0.2dB/km$), making our assumption of no losses valid for reasonable propagation distances. Qualitatively, a decrease in the intensity of the pulse during the propagation will also make the nonlinear effects decrease, resulting in an increase in the temporal duration of the pulse until the new balance between dispersive and nonlinear effect is reached.
Figure 4a shows that a propagation of $1km$ doesn't affect the pulse at all. Figure 4b shows the same simulation repeated with a step length (for the split-step method) of $100km$ instead of $200m$. In conclusion, these pulse parameters are suitable (neglecting losses) for telecommunication because the pulse temporal duration is such that the maximum information that can be transferred is limited by the electronics.

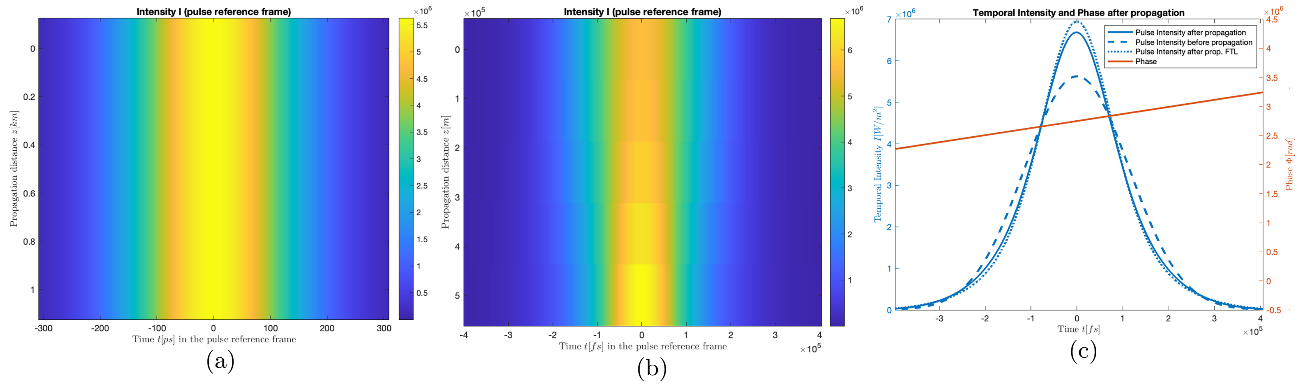


Figure 4: Results of the numerical simulation of a possible situation of interest for telecommunication in optical fibers

3 How to run the program

The program can be executed in Matlab.
The input laser pulse parameters can be set in the first 10 lines of the code, and the material can be selected during the execution of the code among the ones listed by the program itself list.
New materials can be added in the code as follows:
i) create a vector with the following elements: $[A, B_1, C_1, \ldots, B_n, C_n, n_2]$ where A, B_i, C_i are the ones appearing in the Sellmeier equation $n(\lambda) = A + \sum_{i=1}^{n} \frac{B_i \lambda^2}{\lambda^2 - C_i}$ and n_2 is the nonlinear refractive index,
ii) add in the print for the input of the material selection the correspondent material name,
iii) add in the switch-case command the correspondent case.

References

[1] Mikhail Polyanskiy. *refractiveindex.info*. 2023. URL: https://refractiveindex.info (visited on 01/14/2023).

[2] Dr. Rüdiger Paschotta. *Nonlinear Index*. 2023. URL: https://www.rp-photonics.com/nonlinear_index.html (visited on 01/14/2023).

[3] Á. Börzsönyi et al. "Measurement of pressure dependent nonlinear refractive index of inert gases". In: *Opt. Express* 25 (2010), pp. 25847–25854. DOI: 10.1364/OE.18.025847.

[4] Wikipedia. *Split-step method*. 2022. URL: https://en.wikipedia.org/wiki/Split-step_method (visited on 01/14/2023).