Cálculo 2 - Semana 2

Exercícios dos Testes de Convergência

① Aplique o teste da divergência

a) $\sum_{K=1}^{\infty} \frac{K^2+K+3}{2K^2+1}$ c) $\sum_{K=1}^{\infty} \cos K\pi$

b) $\sum_{K=1}^{\infty} \left(1+\frac{1}{K}\right)^K$ d) $\sum_{K=1}^{\infty} \frac{1}{K!}$

a) $\sum_{K=1}^{\infty} \frac{K^2+K+3}{2K^2+1} \rightarrow \lim_{K\to\infty} \frac{K^2+K+3}{2K^2+1} \begin{matrix}\div K^2 \\ = \\ \div K^2\end{matrix} \lim_{K\to\infty} \frac{1+\cancel{1/K}^{0}+\cancel{3/K^2}^{0}}{2+\cancel{1/K^2}^{0}} = \lim_{K\to\infty} \frac{1}{2} = \frac{1}{2}$ ∴ $\lim_{K\to\infty} \neq 0$, logo a série diverge.

b) $\sum_{K=1}^{\infty} \left(1+\frac{1}{K}\right)^K \rightarrow \lim_{K\to\infty} \left(1+\frac{1}{K}\right)^K = e$ (limite fund.) ∴ $\lim_{K\to\infty} \neq 0$, logo a série diverge

c) $\sum_{K=1}^{\infty} \cos K\pi \rightarrow \lim_{K\to\infty} \cos(K\pi) = \pm \rightarrow$ diverge

d) $\sum_{K=1}^{\infty} \frac{1}{K!} \rightarrow \lim_{K\to\infty} \frac{1}{K!} = 0 \rightarrow$ inconclusivo

② Aplique o teste da integral

$$\sum_{K=1}^{\infty} \frac{1}{5K+2} = \int_1^{\infty} f(x)\,dx = \int_1^{\infty} \frac{1}{5x+2}\,dx = \lim_{t\to\infty}\left[\int_1^{\infty} \frac{1}{5x+2}dx\right] = \lim_{t\to\infty} \frac{\ln|5x+2|}{5}\Big|_1^t =$$

$u = 5x+2$

$\frac{du}{dx} = 5$

$$\lim_{t\to\infty} \frac{\ln|5t+2|}{5}^{\nearrow\infty} - \frac{\ln(7)}{5} \Rightarrow \text{diverge}$$

③ Use o teste da comparação

a) $\sum_{n=1}^{\infty} \frac{1}{5n^2-n}$

a) $b_n = \frac{1}{5n^2} \rightarrow p>1$ ↳ converge

$\lim_{n\to\infty} \frac{a_n}{b_n} = \frac{1/(5n^2-n)}{1/5n^2} =$

$= \lim_{n\to\infty} \frac{1}{5n^2-n} \cdot 5n^2 =$

$= \lim_{n\to\infty} \frac{5n^2}{5n^2-n} \begin{matrix}\div n^2 \\ = \\ \div n^2\end{matrix}$

$= \lim_{n\to\infty} \frac{5}{5-\cancel{1/n}^{0}} = \frac{5}{5} = 1$

∴ Como $\lim_{n\to\infty} \frac{a_n}{b_n} = c > 0$

a série a_n converge

b) $\sum_{K=1}^{\infty} \frac{K+1}{K^2-K}$

$b_n = \frac{K}{K^2} = \frac{1}{K}$ diver.

$a_n \geq b_n$

$\frac{K+1}{K^2-K} \geq \frac{1}{K}$

$K^2+1 \geq K^2-K$

Como $a_n \geq b_n$ + b_n é divergente a_n diverge.

c) $\sum_{K=1}^{\infty} \frac{3}{K-1/4}$

$b_n = \frac{3}{K} \rightarrow p=1$ ↳ diverge

$a_n \geq b_n$

$\frac{3}{K-1/4} \geq \frac{3}{K}$

$3K \geq 3K - \frac{3}{4}$

Como $a_n \geq b_n$ + b_n diverge a série a_n é divergente

d) $\sum_{K=1}^{\infty} \frac{2}{K^4+K}$

$b_n = \frac{2}{K^4} \rightarrow p = 4 > 1$ ↳ converge

$a_n \leq b_n$

$\frac{2}{K^4+K} \leq \frac{2}{K^4}$

$2K^4 \leq 2K^4 + 2K$

Como $a_n \leq b_n$ + b_n é convergente, a série a_n é convergente

4) Use o teste da raiz ou razão

a) $\sum_{K=1}^{\infty} \frac{3^K}{K!}$ b) $\sum_{K=1}^{\infty} \frac{4^K}{K^2}$ c) $\sum_{K=1}^{\infty} \frac{1}{5K}$ d) $\sum_{K=1}^{\infty} K\left(\frac{1}{2}\right)^K$ e) $\sum_{K=1}^{\infty} \frac{K!}{K^3}$

f) $\sum_{K=1}^{\infty} \frac{K}{K^2+1}$ g) $\sum_{K=1}^{\infty} \left(\frac{3K+2}{2K-1}\right)^K$ h) $\sum_{K=1}^{\infty} \left(\frac{K}{100}\right)^K$ i) $\sum_{K=1}^{\infty} \frac{K}{5^K}$

a) $\rho = \lim_{K\to\infty} \frac{a_{n+1}}{a_n} = \lim_{K\to\infty} \frac{3^{K+1}/(K+1)!}{3^K/K!} = \lim_{K\to\infty} \frac{\cancel{3^K}\cdot 3}{(K+1)\cdot \cancel{K!}} \cdot \frac{\cancel{K!}}{\cancel{3^K}} = \lim_{K\to\infty} \frac{3}{K+1} = 3 > 1$

$\lim_{K\to\infty} \frac{a_{n+1}}{a_n} > 1 \to$ diverge

b) $\rho = \lim_{K\to\infty} \frac{a_{n+1}}{a_n} = \lim_{K\to\infty} \frac{4^{K+1}/(K+1)^2}{4^K/K^2} = \lim_{K\to\infty} \frac{\cancel{4^K}\cdot 4}{(K+1)^2} \cdot \frac{K^2}{\cancel{4^K}} \to \lim_{K\to\infty} \frac{4K^2}{K^2+2K+1} \begin{matrix}\div K^2 \\ = \\ \div K^2\end{matrix}$

$\lim_{K\to\infty} \frac{4}{1+\cancel{2/K}^{0} + \cancel{1/K^2}^{0}} = 4 \quad \rho > 1 \to$ diverge

c) $\rho = \lim_{K\to\infty} \frac{a_{n+1}}{a_n} = \lim_{K\to\infty} \frac{\frac{1}{5(K+1)}}{\frac{1}{5K}} = \lim_{K\to\infty} \frac{1}{5(K+1)} \cdot 5K = \lim_{K\to\infty} \frac{5K}{5K+5} \begin{matrix}\div K \\ = \\ \div K\end{matrix}$

$\lim_{K\to\infty} \frac{5}{5+\cancel{5/K}^{0}} = \frac{5}{5} = 1 \quad \rho = 1 \to$ nada se conclui

d) $\rho = \lim_{K\to\infty} \sqrt[n]{a_n} = \lim_{K\to\infty} \sqrt[K]{K\left(\frac{1}{2}\right)^{K}} = \lim_{K\to\infty} \sqrt[K]{K}^{\,1}\, \frac{1}{2} = \frac{1}{2} \quad \rho < 1 \to$ converge

e) $\rho = \lim_{K\to\infty} \frac{a_{n+1}}{a_n} = \lim_{K\to\infty} \frac{\frac{(K+1)!}{(K+1)^3}}{K!/K^3} = \lim_{K\to\infty} \frac{(K+1)\cancel{K!}}{\cancel{(K+1)^3}} \cdot \frac{K^3}{\cancel{K!}} = \lim_{K\to\infty} \frac{K^3}{(K+1)^2} =$

$\lim_{K\to\infty} \frac{K^3}{K^2+2K+1} \overset{LH}{=} \lim_{K\to\infty} \frac{3K^2}{2K+2} \overset{LH}{=} \lim_{K\to\infty} \frac{6K}{2} = \infty \to$ diverge

f) $\rho = \lim_{K\to\infty} \frac{a_{n+1}}{a_n} = \lim_{K\to\infty} \frac{\frac{K+1}{(K+1)^2+1}}{\frac{K}{K^2+1}} = \lim_{K\to\infty} \frac{\cancel{K+1}}{(K+1)^2+1} \cdot \frac{K^2+1}{K} = \lim_{K\to\infty} \frac{1}{K+2} \cdot \frac{K^2+1}{K} =$

$= \lim_{K\to\infty} \frac{K^2+1}{K^2+2K} \overset{\div K^2}{=} \lim_{K\to\infty} \frac{1+\cancel{1/K^2}^{0}}{1+\cancel{2/K}^{0}} = 1 \to \rho = 1 \to$ inconclusivo.

g) $\rho = \lim_{K\to\infty} \sqrt[K]{a_K} = \lim_{K\to\infty} \sqrt[K]{\left(\frac{3K+2}{2K-1}\right)^K} = \lim_{K\to\infty} \frac{3K+2}{2K+1} = \lim_{K\to\infty} \frac{3}{2} = \frac{3}{2} > 1 \to$ diverge

h) $\rho = \lim_{K\to\infty} \sqrt[K]{a_K} = \lim_{K\to\infty} \frac{K}{100} = \infty \to$ diverge

i) $\rho = \lim_{K\to\infty} \sqrt[K]{a_K} = \lim_{K\to\infty} \sqrt[K]{\frac{K}{5^K}} = \lim_{K\to\infty} \frac{\sqrt[K]{K}^{\,1}}{5} = \frac{1}{5} < 1 \to$ converge

⑤ Determine se as séries alternadas divergem ou convergem.

① $\sum_{k=1}^{\infty} (-1)^{k+1} \frac{k+1}{3k+1}$ — an

$a_n \geq 0, \forall n$ (ok)

$a_n \geq a_{n+1}$ (X)

$\frac{k+1}{3k+1} \geq \frac{k+2}{3k+2}$ ×

Como uma das condições não é satisfeita a série diverge.

② $\sum_{k=1}^{\infty} (-1)^{k+1} \frac{k+1}{\sqrt{k}+1}$ — an

$a_n \geq 0, \forall n$ (ok)

$a_n \geq a_{n+1}$ (ok)

$\frac{k+1}{\sqrt{k}+1} \geq \frac{(k+1)+1}{\sqrt{k+1}+1}$ ✓

$\lim_{k\to\infty} a_n = 0$ (X) diverge

$\lim_{k\to\infty} \frac{k+1}{\sqrt{k}+1} \overset{LH}{=} \lim_{k\to\infty} \frac{1}{\frac{1}{2\sqrt{k}}} =$

$= \lim_{k\to\infty} \frac{\sqrt{k}}{2} = \infty$

③ $\sum_{k=1}^{\infty} (-1)^{k+1} e^{-k}$ — an

$a_n > 0$ (ok)

$a_n \geq a_{n+1}$ (ok)

$e^{-k} \geq e^{-k+1}$

$\frac{1}{e^k} \geq \frac{1}{e^{k+1}}$

$e^k e \geq e^k$

$\lim_{k\to\infty} a_n = 0$

$\lim_{k\to\infty} \frac{1}{e^k} = 0$

∴ a série alternada converge.

⑥ Classifique as séries em absolutamente convergente ou condicionalmente convergente.