Gaussian Elimination Exercises

Due Feb. 24

1. Solve the following systems of equations. For each,

   - Write it in matrix form as an augmented system.
   - Perform Gaussian Elimination with the augmented system.
   - Solve the system.
   - Give L and A so that $A = LU$.
   - Solve $Ax = b$ via two triangular solves (with L and U).

   (a)

$$\begin{array}{rrrl} 2\chi_0 & +(-1)\chi_1 & +\quad 0\chi_2 & = -3 \\ -2\chi_0 & +(-1)\chi_1 & +\quad 1\chi_2 & = \quad 3 \\ -8\chi_0 & +\quad 10\chi_1 & +(-4)\chi_2 & = 10 \end{array}$$

   (b)

$$\begin{array}{rrrl} 2\chi_0 & +(-1)\chi_1 & -2\chi_2 & = -6 \\ 6\chi_0 & +(-5)\chi_1 & +(-5)\chi_2 & = -19 \\ -2\chi_0 & +(-7)\chi_1 & +\quad 8\chi_2 & = \quad 12 \end{array}$$

2. With octave, generate a nontrivial (e.g., not diagonal) system of four equations in four unknowns with the following properties:

   - All coefficients are integers.
   - The right-hand side consist of integers.
   - The solution consists of integers.

3. Challenge question: In class I said that one can generate a matrix A so that if one does Gaussian Elimination one never runs into a fraction by making $A = LU$ and picking L and U to only have integers (and have their special structure). Explain why this works. (You may illustrate it with a 3×3 system, but then you would want to explain it for general size).