Active Anti Roll Bar for a Formula Student Racing Car - Design and Control

SAPIENZA
UNIVERSITÀ DI ROMA

Vehicle System Dynamics

Project presentation

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Team Presentation

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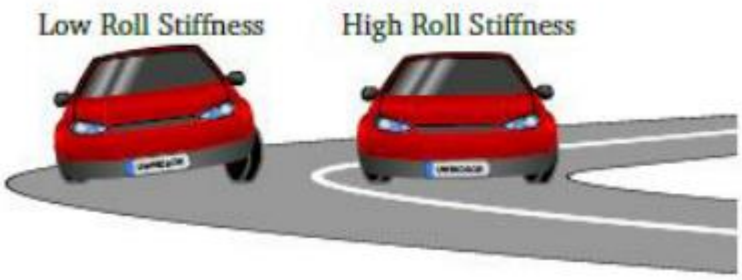
Problem Description

- During cornering, the sprung mass of the body will naturally be shifted towards the outer side of the car otherwise known as body roll.

- As the car leans the wheels start to tilt, reducing the contact patch of the tires.

- Excessive roll also results in a car that isn’t as responsive, as it’ll take longer to react to your commands while trying to settle.



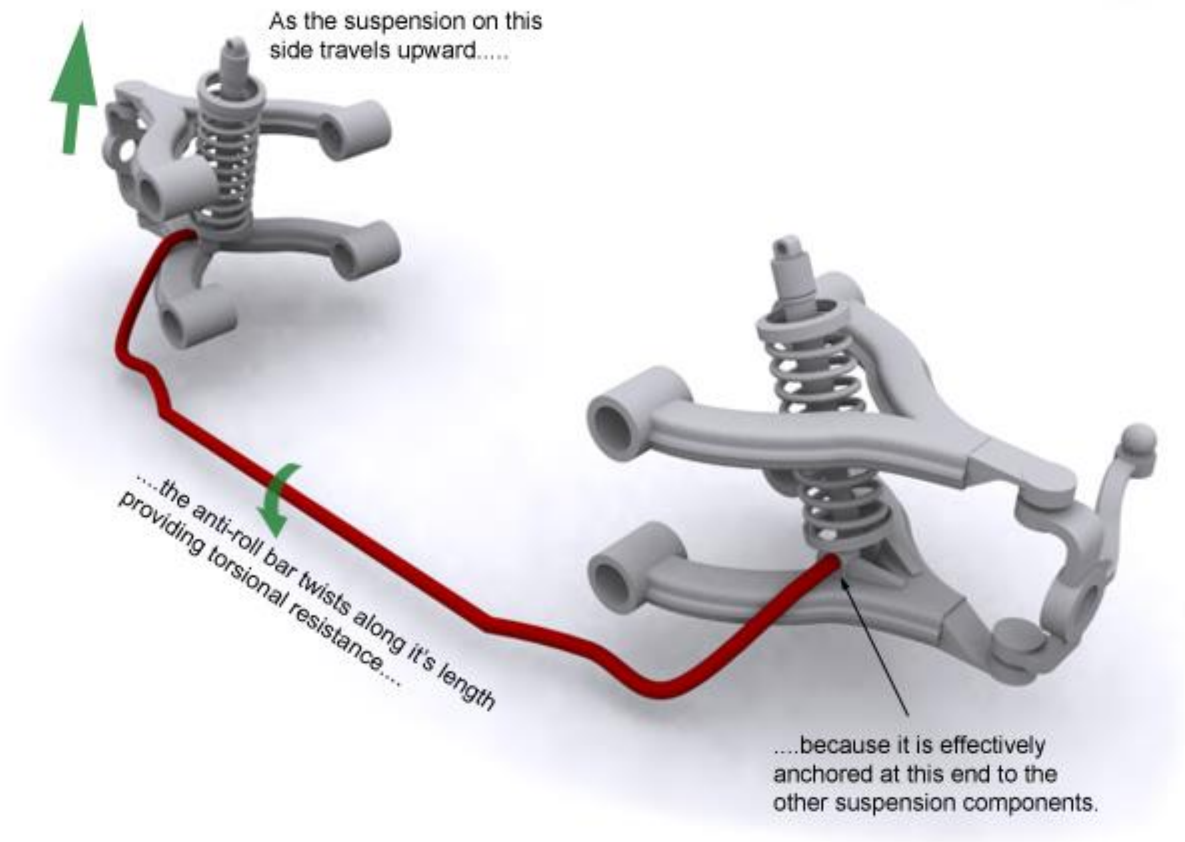
Objectives

- Design 2 active anti roll bars for a formula student racing car.
- Optimize the car behaviour during hard cornering in terms of reducing body roll and roll velocity.

Control Problem

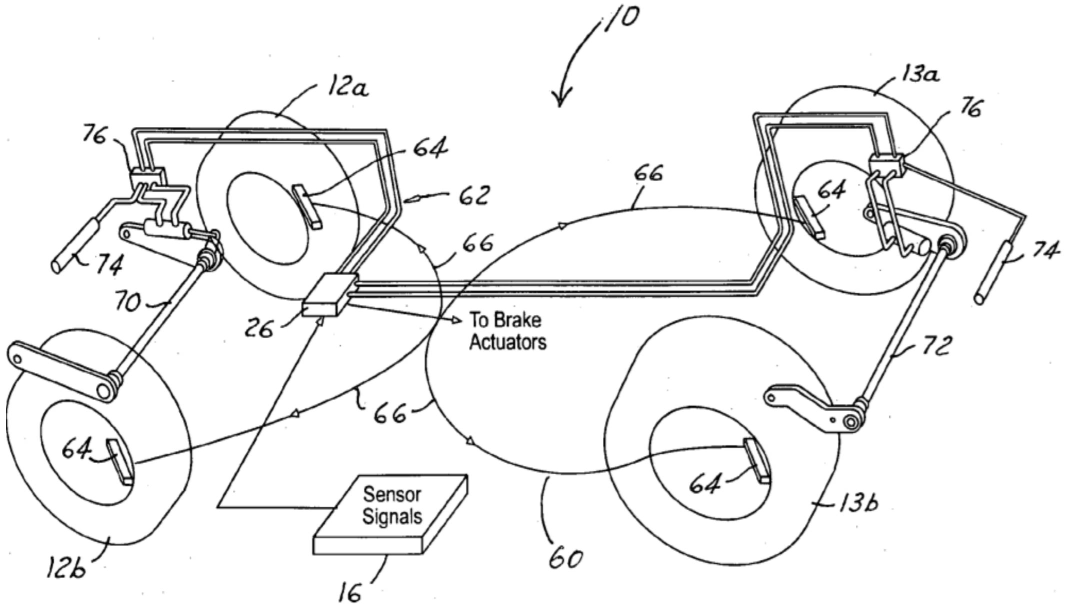
- Modelling and Linearization
- Optimal control over the active anti roll bar
- State Estimation

Anti Roll Bar



Patents – US 2005/0131604 A1

- **Ford Global Technology**
- An actuatable device (70) to vary the torsional stiffness.
- Apparatus for controlling the roll characteristics of the vehicle.
- Combination of active anti roll bars and active breaking system.
- Roll over prevention.



Patents – EP 0 974 447 A1

- **Thompson Ramo Wooldridge (TRW)**
- An actuatable device (70) for varying the torsional stiffness of an anti roll bar.
- Electric control system that vary the energy field in order to change fluid viscosity in the clutch mechanism.

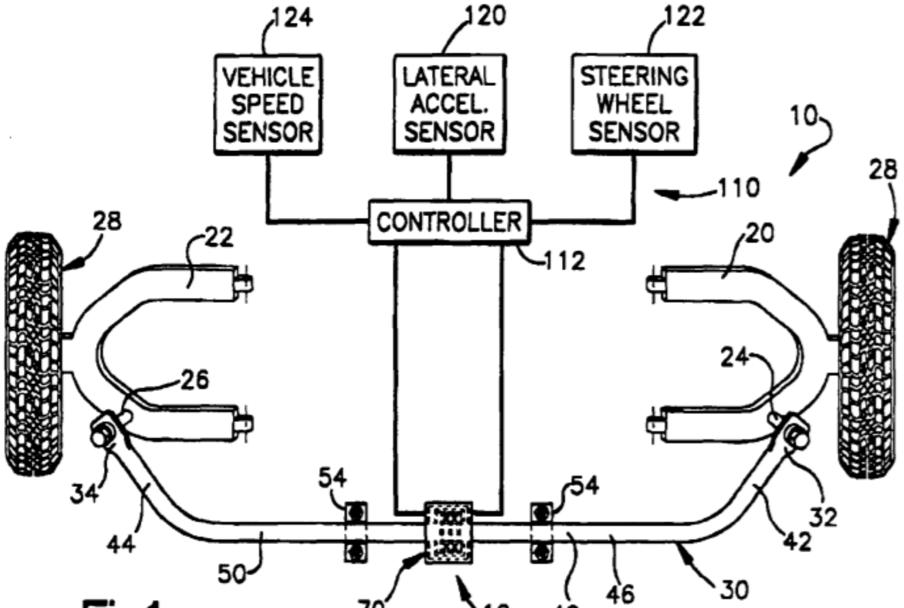


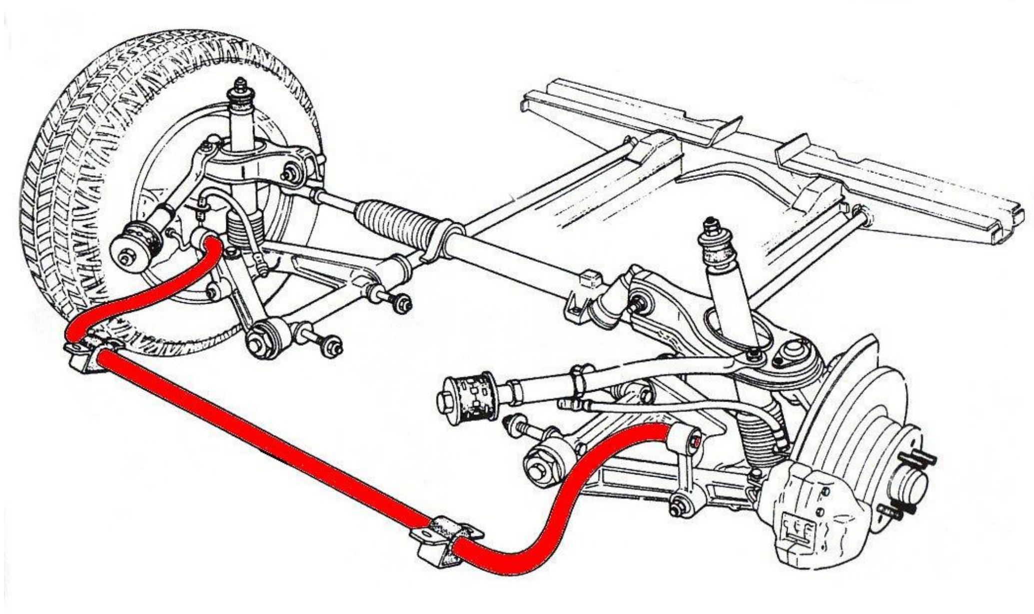
Fig.1

Patents – Porsche Active Balance

Active Anti Roll Bar

- **Passive**

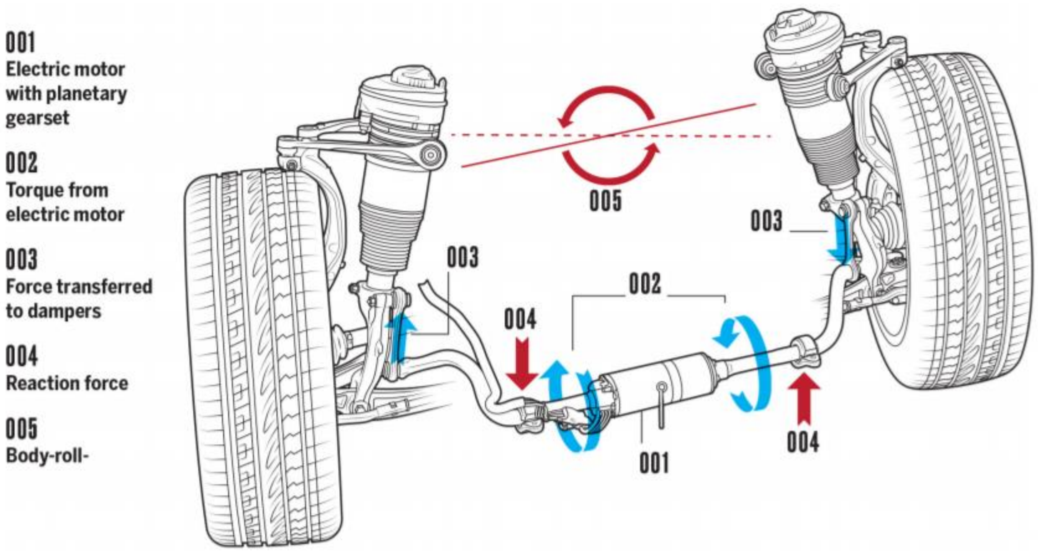
- Balance load transfer
- Increasing roll stiffness
- Purely mechanical element



- **Active**

- Twisting torque we talked about earlier is applied electronically

- Easy to be tweaked giving a quick way to adjust the balance of the car to the preferences of the driver.



Assumption

- Tires are considered undeformable
- Pitch angle is considered fixed
- Flat road surface
- Wheel’s suspension not modeled
- Heave motion of the vehicle excluded
- Camber effect not considered
- Small-angle approximation: $\begin{cases} sin\theta \approx \theta \\ cos\theta \approx 1 \end{cases}$

Vehicle Model

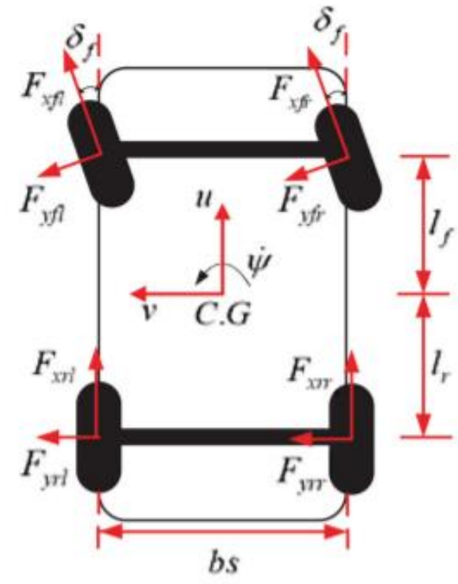
- 8 DoF model :
  - planar motion with yaw and roll
  - four-wheel rolling

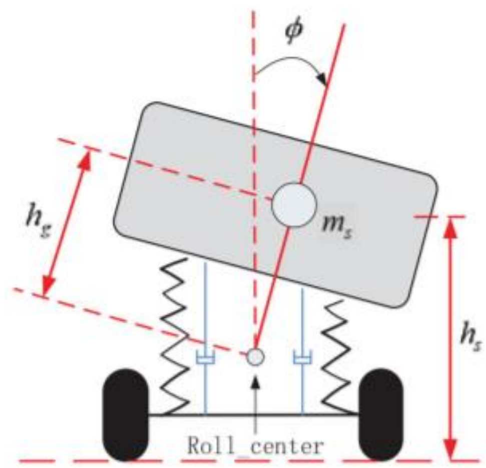
- Vehicle's kinematics:

$$\begin{cases} \dot{X} = u\cos\psi - v\sin\psi \\ \dot{Y} = u\sin\psi + v\cos\psi \end{cases}$$

With:

- X, Y : vehicle's position in the global coordinate system
- u, v : longitudinal and lateral vehicle's velocity
- ψ, ϕ : yaw and roll angle of the vehicle





Vehicle Model

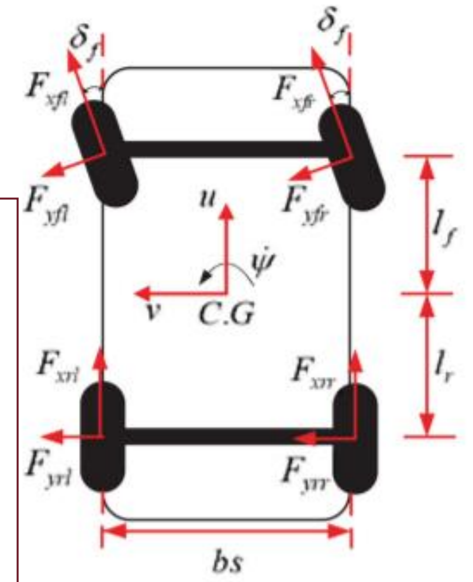
- Vehicle's dynamics:

$$\hat{x}:\quad m(\dot{u}-\dot{\psi}v)+m_s h_s\ddot{\psi}\phi=(F_{xfl}+F_{xfr})cos\delta+F_{xrl}+F_{xrr}-(F_{yfl}+F_{yfr})sin\delta-F_x^{air}$$

$$\hat{y}:\quad m(\dot{v}-\dot{\psi}u)-m_s h_s\ddot{\phi}=(F_{yfl}+F_{yfr})cos\delta+F_{yrl}+F_{yrr}+(F_{xfl}+F_{xfr})sin\delta$$

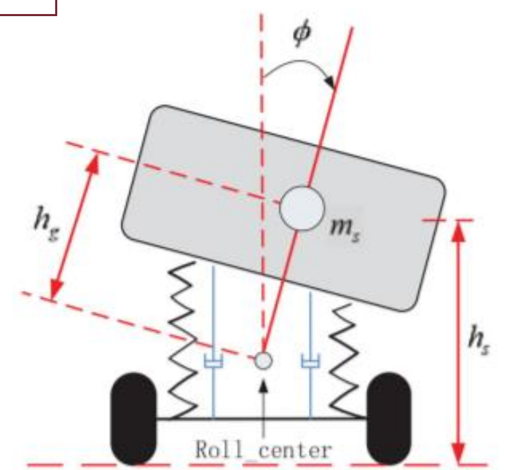
$$\hat{\psi}:\quad \mathrm{I}_z\ddot{\psi}=\frac{1}{2}b_{sf}\left[(F_{xfr}cos\delta-F_{yfr}sin\delta)-(F_{xfl}cos\delta-F_{yfl}sin\delta)\right]+\frac{1}{2}b_{sr}(F_{xrr}-F_{xrl})$$
$$+l_f\left[(F_{yfr}+F_{yfl})cos\delta+(F_{xfr}+F_{xfl})sin\delta\right]-l_r(F_{yrr}-F_{yrl})$$

$$\hat{\phi}:\quad \mathrm{I}_x\ddot{\phi}-m_s h_s(\dot{v}+\dot{\psi}u)=m_s h_s g\phi-(C_{\phi f}+C_{\phi r})\dot{\phi}-(K_{\phi f}+K_{\phi r})\phi$$



With:

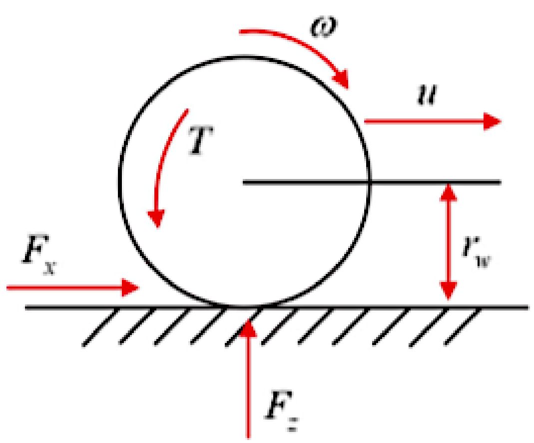
- m, m_s : vehicle mass and sprung mass
- C_ϕ, K_ϕ : damping coefficent and roll stiffness
- h_s, h_g: c.g. center and roll center height
- $\mathrm{I}_x, \mathrm{I}_z$: moment of inertia about longitudinal and vertical axes through the vehicle's c.g.
- δ : steering angle
- F_x^{air} : wind force on the longitudinal axle such that $F_x^{air}=\frac{1}{2}C_x\rho_{air}Au^2$



Vehicle Model

- Four-wheels rotational dynamics:

$$\begin{cases} J_{wfl}\dot{\omega}_{fl} = T_{dfl} - T_{bfl} - F_{xfl}r_w - T_{volv} \\ J_{wfr}\dot{\omega}_{fr} = T_{dfr} - T_{bfr} - F_{xfr}r_w - T_{volv} \\ J_{wrl}\dot{\omega}_{rl} = T_{drl} - T_{brl} - F_{xrl}r_w - T_{volv} \\ J_{wrr}\dot{\omega}_{rr} = T_{drr} - T_{brr} - F_{xrr}r_w - T_{volv} \end{cases}$$



With:

- J_{wij} : moment of inertia about the four wheels
- $\dot{\omega}_{ij}$: rate of the wheel angular speed
- T_{dij} : driving torques on the four wheels
- T_{bij} : braking torque on the four wheels
- F_{xij}, F_{yij} : longitudinal and lateral forces of the four tires
- r_w : radius of the wheels
- T_{volv}: rolling resistance torque on the four wheels, such that $T_{volv} = (\mu_0 + \mu_1 \omega_{ij}^2 r_w^2) F_{zij}$

Vehicle Model

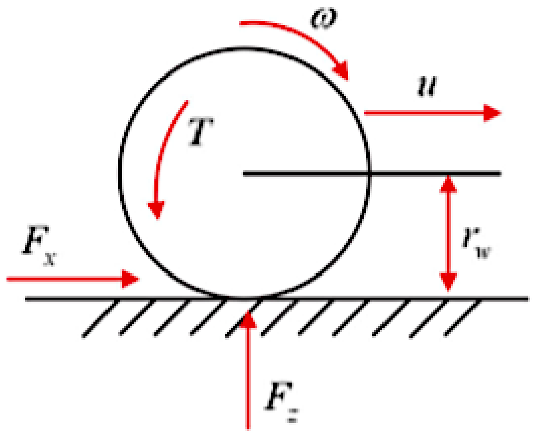
- In order to consider the non-linear region in which the tires perform, the Pacejka Magic Formula is used:

$$\sigma_{fl} = \frac{r_w\omega_{fl} - (u_{gfl}cos\delta + v_{gfl}sin\delta)}{|u_{gfl}cos\delta + v_{gfl}sin\delta|} \qquad \sigma_{rl} = \frac{r_w\omega_{rl} - u_{grl}}{|u_{grl}|}$$

$$\sigma_{fr} = \frac{r_w\omega_{fr} - (u_{gfr}cos\delta + v_{gfr}sin\delta)}{|u_{gfr}cos\delta + v_{gfr}sin\delta|} \qquad \sigma_{rr} = \frac{r_w\omega_{rr} - u_{grr}}{|u_{grr}|}$$

$$\alpha_{fl} = \tan^{-1}\left(\frac{v_{gfl}}{u_{gfl}}\right) - \delta \qquad \alpha_{rl} = \tan^{-1}\left(\frac{v_{grl}}{u_{grl}}\right)$$

$$\alpha_{fr} = \tan^{-1}\left(\frac{v_{gfr}}{u_{gfr}}\right) - \delta \qquad \alpha_{rr} = \tan^{-1}\left(\frac{v_{grr}}{u_{grr}}\right)$$

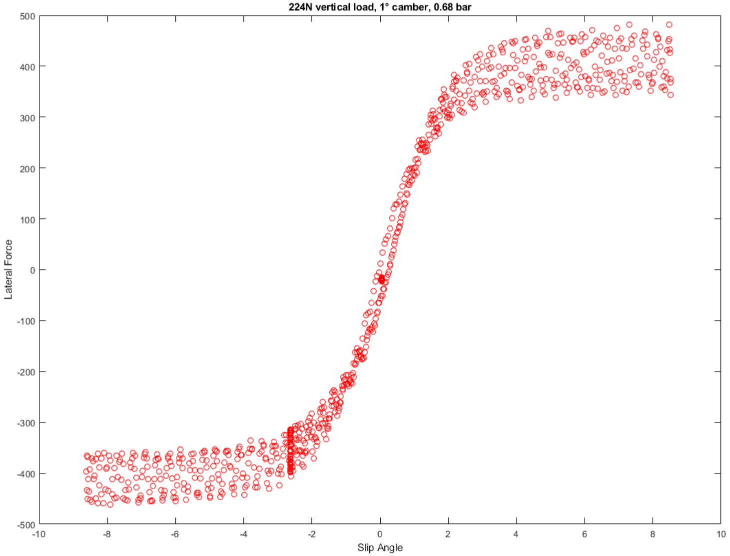


With:

- σ_{ij} : longitudinal slip ratio
- α_{ij} : lateral slip angle
- u_{gij}, v_{gij}: tire contact patch velocities

Pacejka Tire Model

- Model developed from Calspan tire data.
- Tire test center, working on both road and racing tire models, collaborates with SAE.
- FSAE Calspan data regards different tire models, commonly used in single-seaters.
- Each tire test contains data about the type of rolling test, slip angle, slip ratio, forces and moments values, inflating pressure, temperatures, camber…
- We chose the tire model of our car, the Hoosier LCO, with same rim size and working conditions.

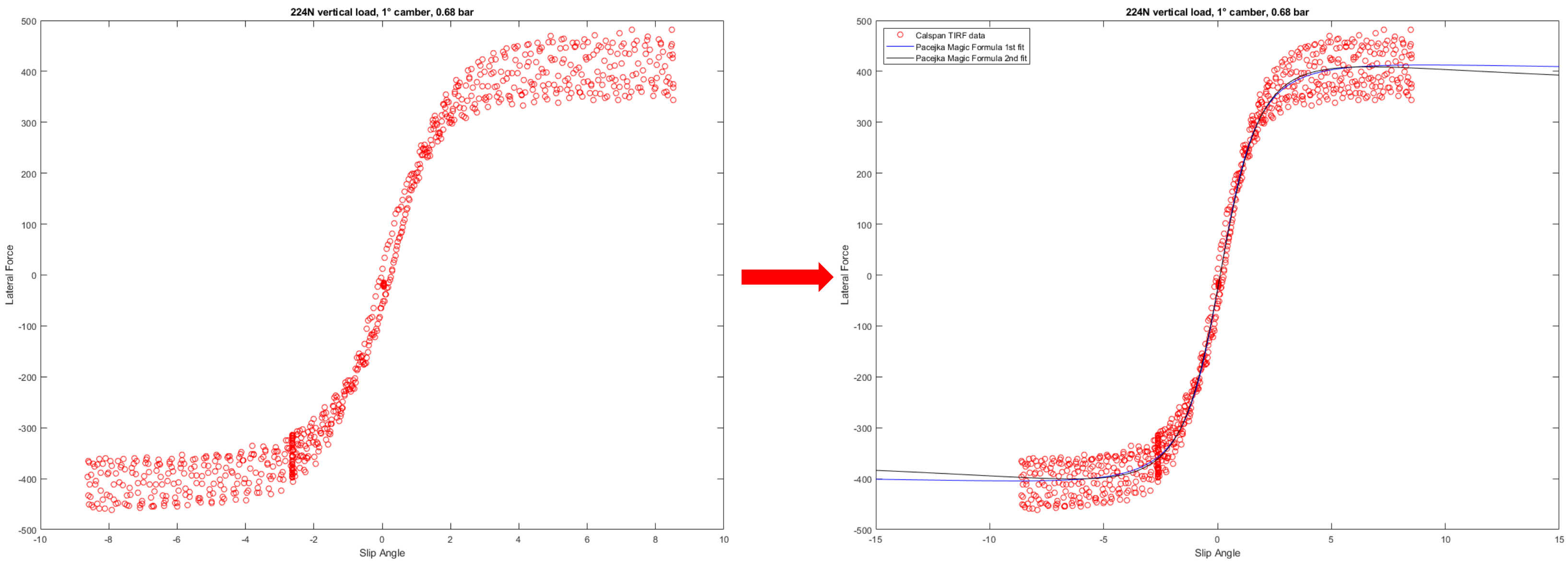


Pacejka Tire Model

- Starting from raw data, we made a Least Square Curve Fitting for the Magic Formula:

$$y = D\sin\{C\arctan[Bx - E(Bx - \arctan(Bx))]\}$$

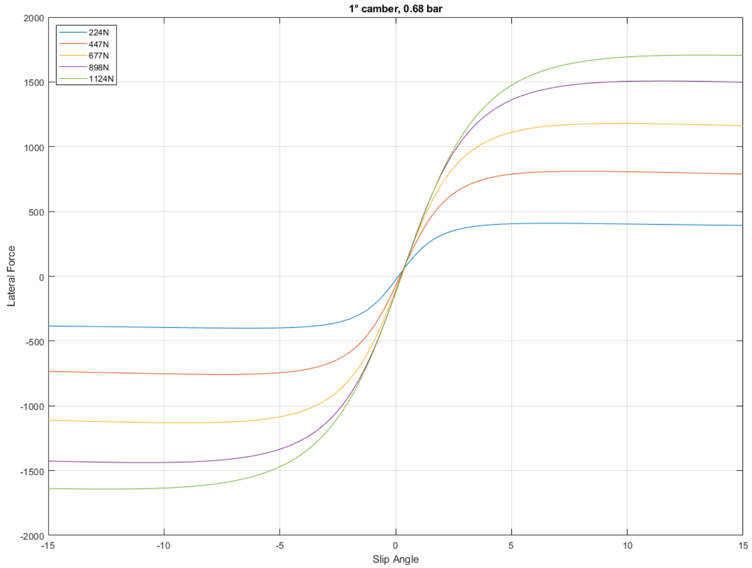
B: stiffness factor, C: shape factor, D: peak factor, E: curvature factor

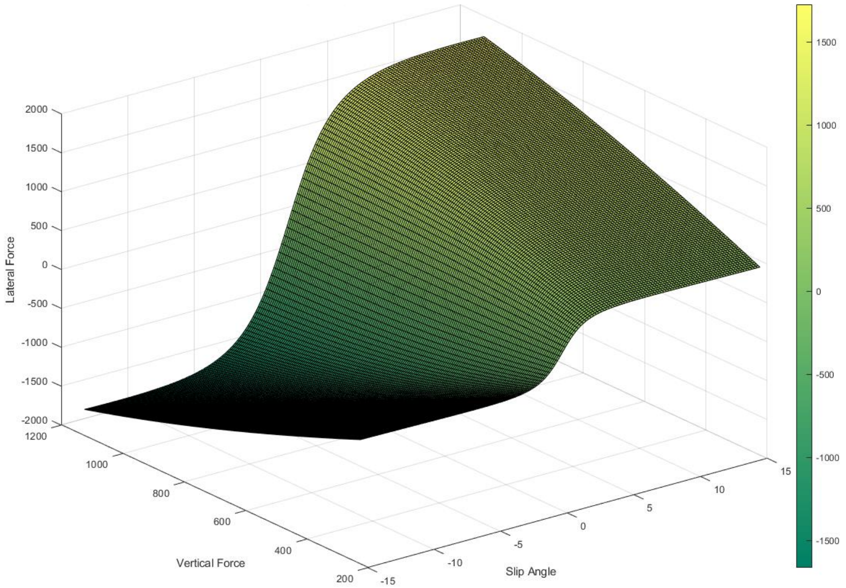


- With fixed camber and inflating pressure, we analyzed five vertical load (Fz) cases, for both pure lateral slip and pure longitudinal slip conditions. For each test condition, we obtained a B, C, D and E coefficient set that best fits the function over the data

Pacejka Tire Model

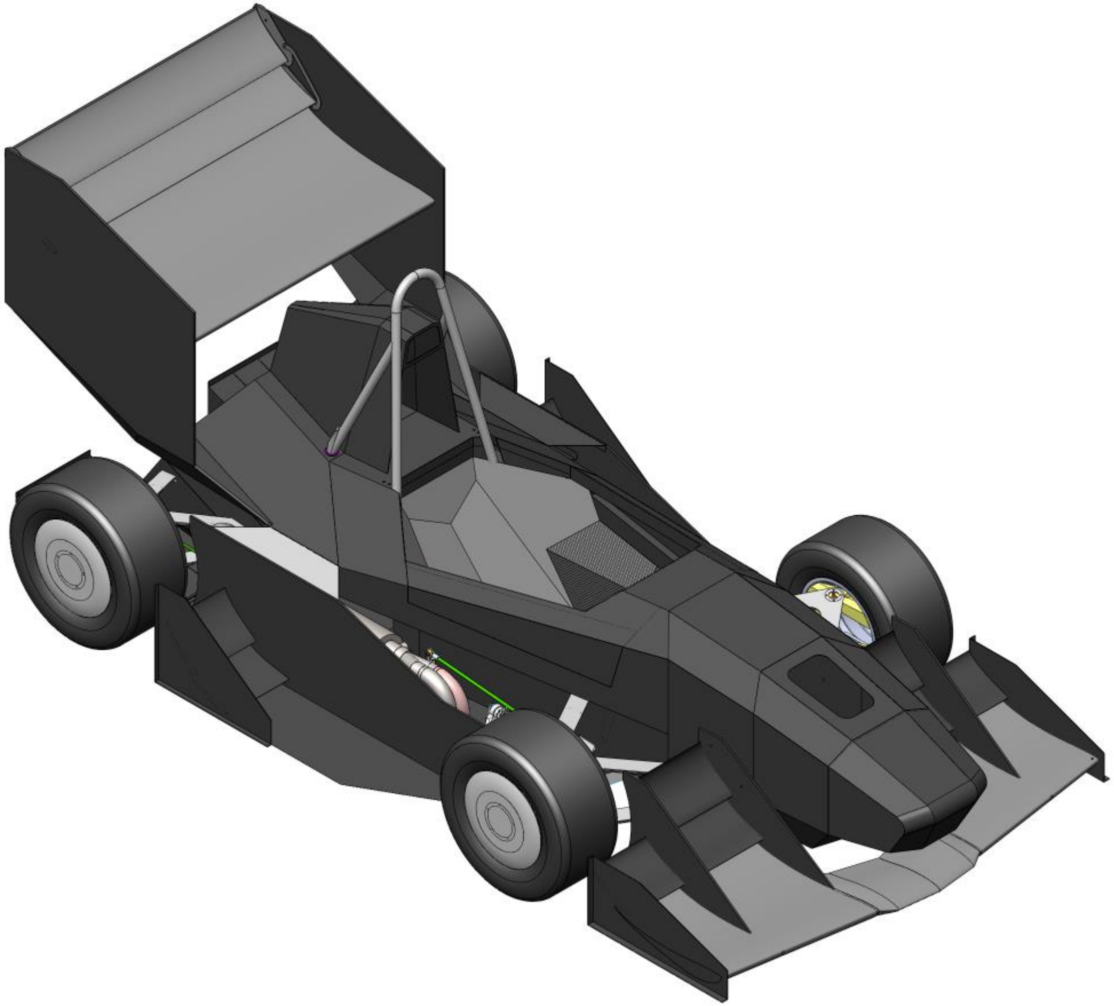
- After that, we interpolated the B, C, D and E discrete value sets over polynomial functions of Fz, with the aim of using those results for any vertical load
- At the end, we obtained two surfaces, one for lateral behavior and one for longitudinal behavior
- We saved the results into custom MATLAB Functions, to be used in our simulations





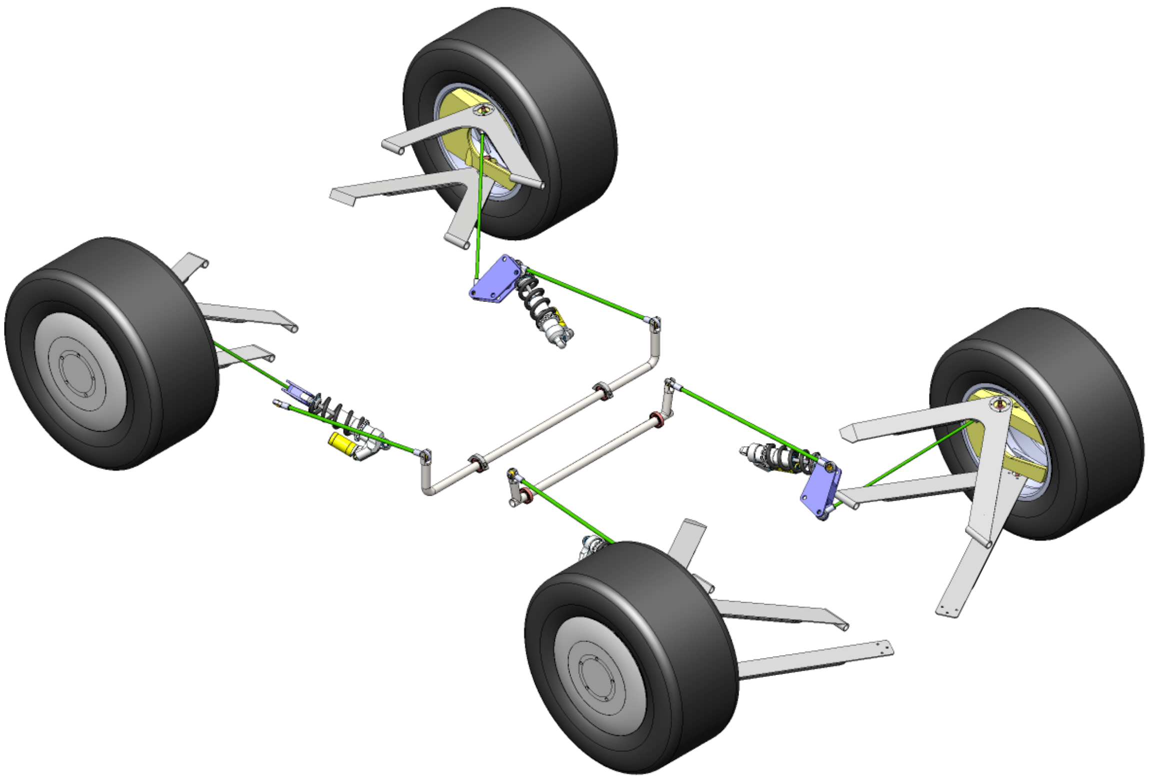
Gajarda AWD Driverless FSAE race car

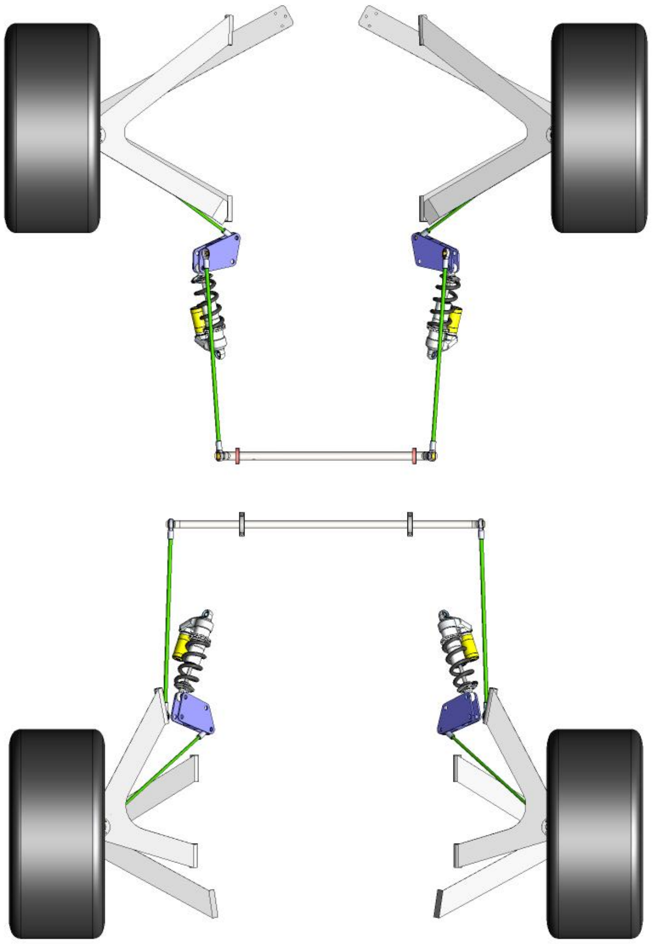
- The Formula SAE race car of our university is the starting point of the project
- Honda CBR600F engine
- Full CFRP body
- Full CFRP aero kit
- Full CFRP rims
- All-Wheel Drive system, unique in the competition
- Formula Driverless Italy 2019 champion



Suspension system

- The suspension system is a pull-rod actuated double wishbone type; the wishbones behave like flexural springs, so they participate to the total ride stiffness

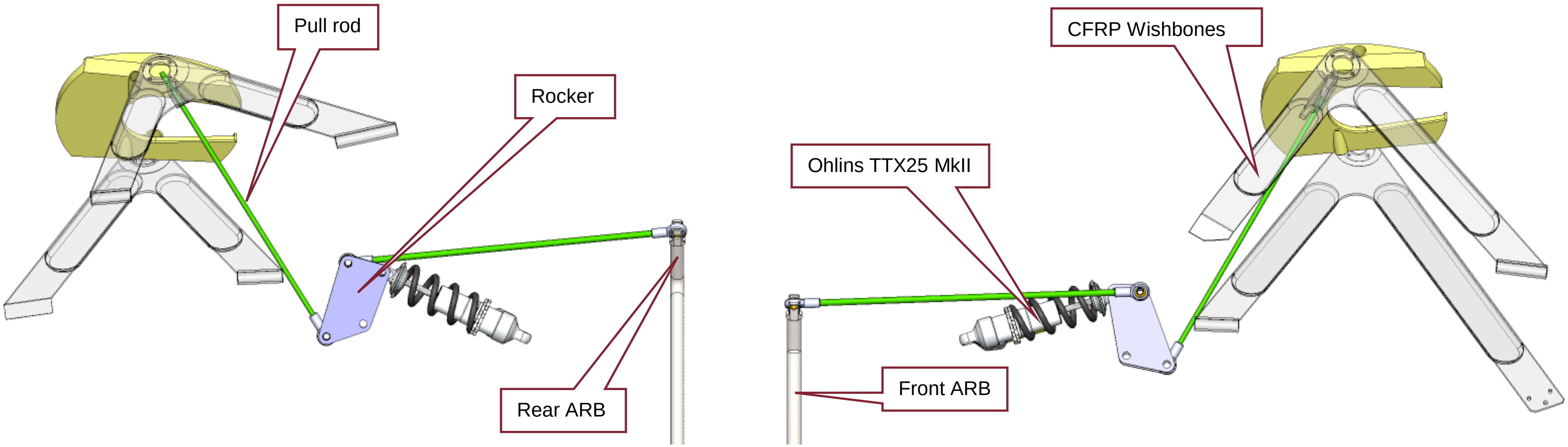




Suspension system

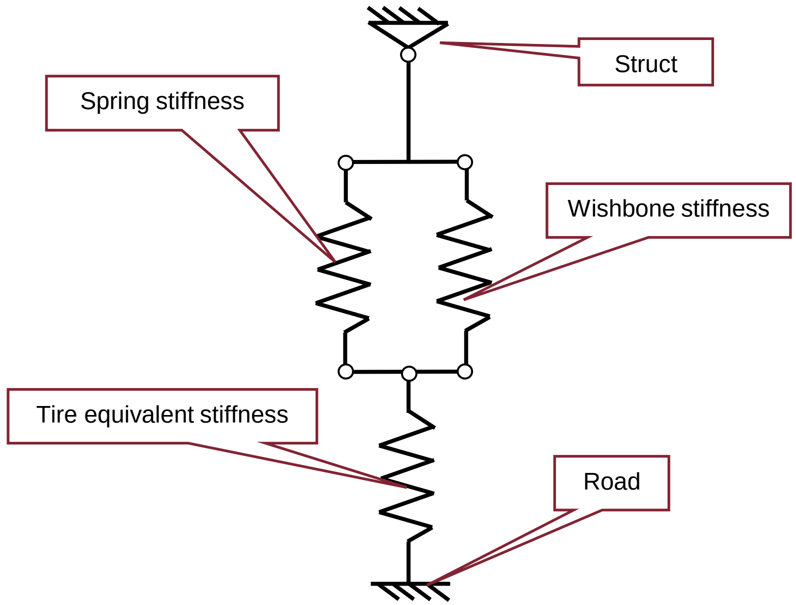
- In our 8DOF model, the vehicle stiffness is considered as torsional, since we are focusing on roll behavior. To do so we considered, for each axis, the roll stiffness as the sum of two contributions, one from the ARB and one from the ride stiffness:

$$K_{roll} = K_{roll,\,ride} + K_{roll,\,ARB}$$

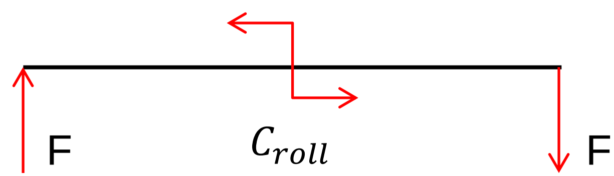


Suspension system

- For the ride stiffness contribution we considered, for each quarter car, the parallel sum of wishbones and springs stiffness with their installation ratios, in series with tire equivalent stiffness.
- To convert the linear ride rate into torsional contribution to roll stiffness, we considered the relationship between roll and wheel ride, considering small-angle approximation:



$$\phi * \frac{wheelbase}{2} = x$$



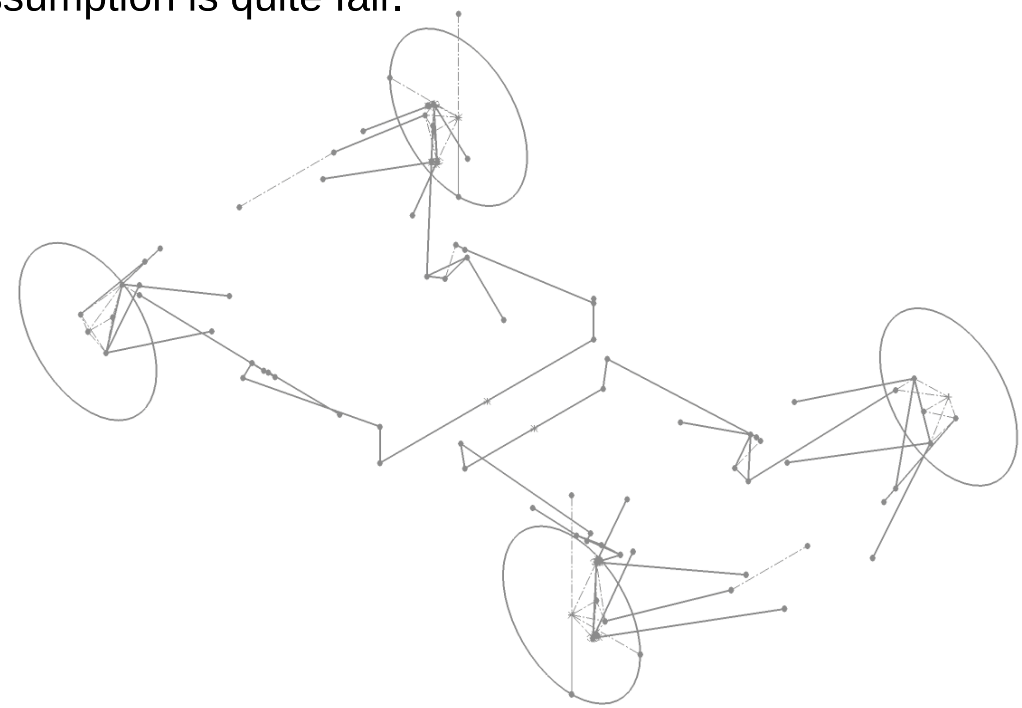
Then:

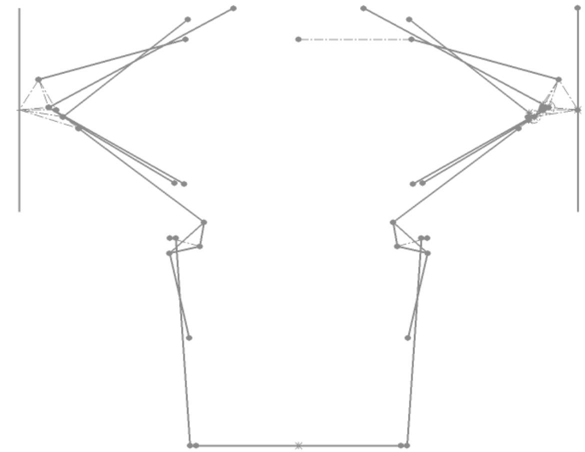
$$C_{roll} - \frac{Fw}{2} - \frac{Fw}{2} = 0 \rightarrow \frac{C_{roll}}{w} = F = K_{ride}\, x \rightarrow C_{roll} = w\, K_{ride} x = K_{roll}\, \phi$$

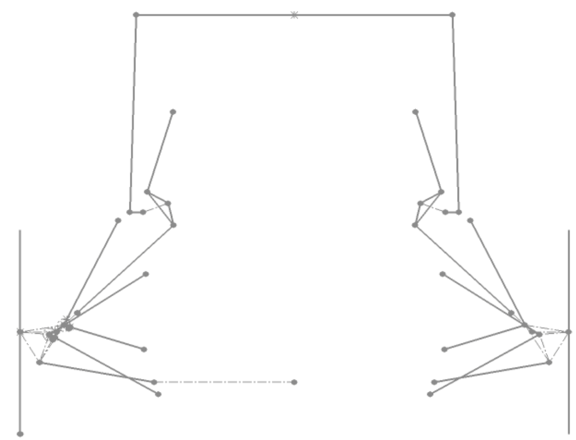
$$K_{roll,ride} = \frac{w}{\phi} x\, K_{ride} = \frac{w^2}{2} K_{ride}$$

Suspension system

- From the 3D sketch of the suspension system, the installation ratio of the anti-roll bar with respect to the roll angle is $i_{ARB} = 6$.
- Considering that the camber gain is around 0.5 deg per roll deg, due to the wheel rotation center positioning, fixed camber assumption is quite fair.







Simulation’s scenarios

1. The first simulation is with:

   - ✓ Constant speed for the vehicle.
   - ✓ δ grows as a ramp of slope $\frac{\pi}{6}$ up to 1 second.
   - ✓ u_0 = 40 Km/h = 11.1111 m/s.

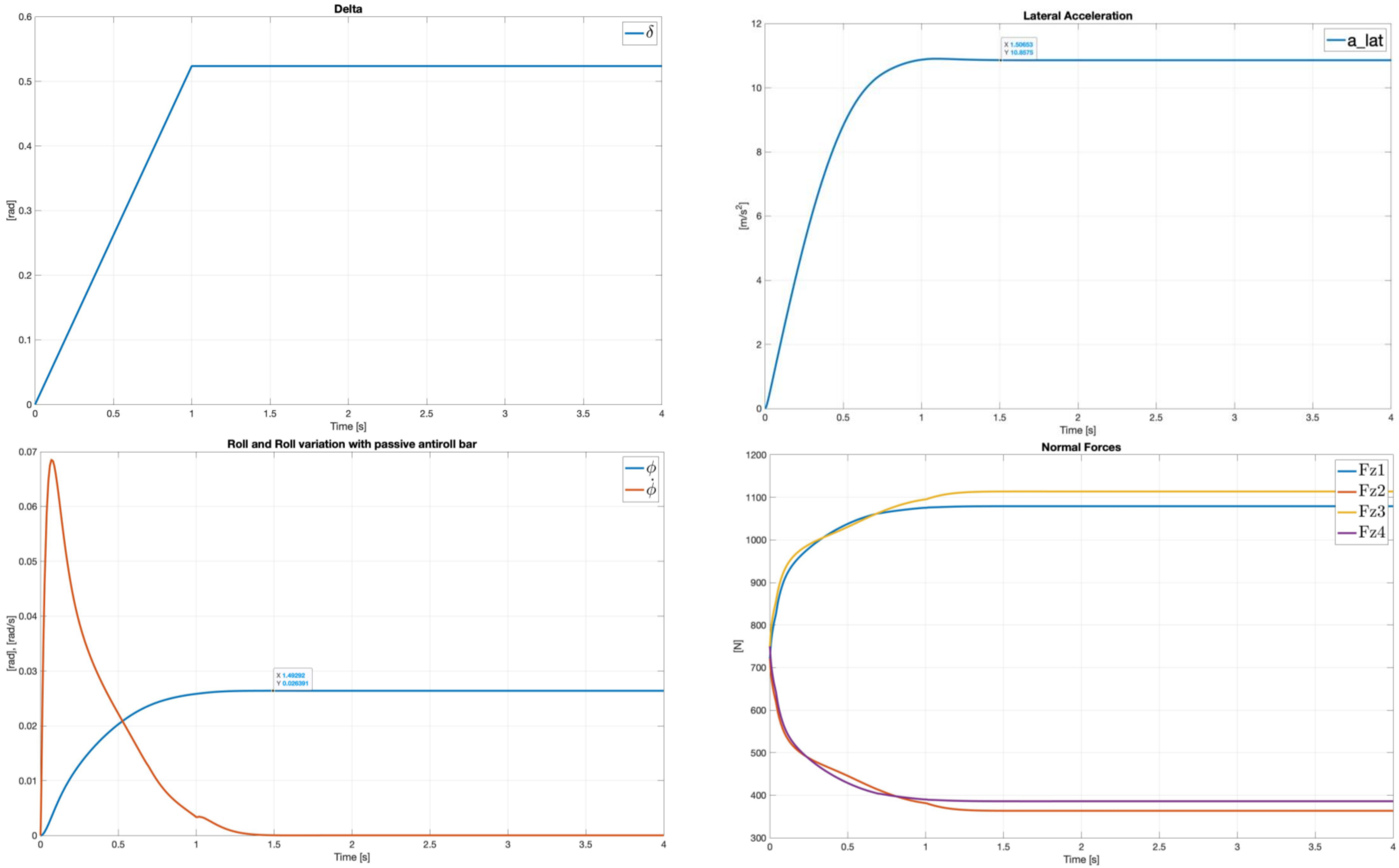
2. The second simulation is with:

   - ✓ Step torque at 1 second
   - ✓ δ goes as a single sinusoid branch of amplitude $\frac{\pi}{12}$
   - ✓ u_0 = 36 Km/h = 10 m/s.

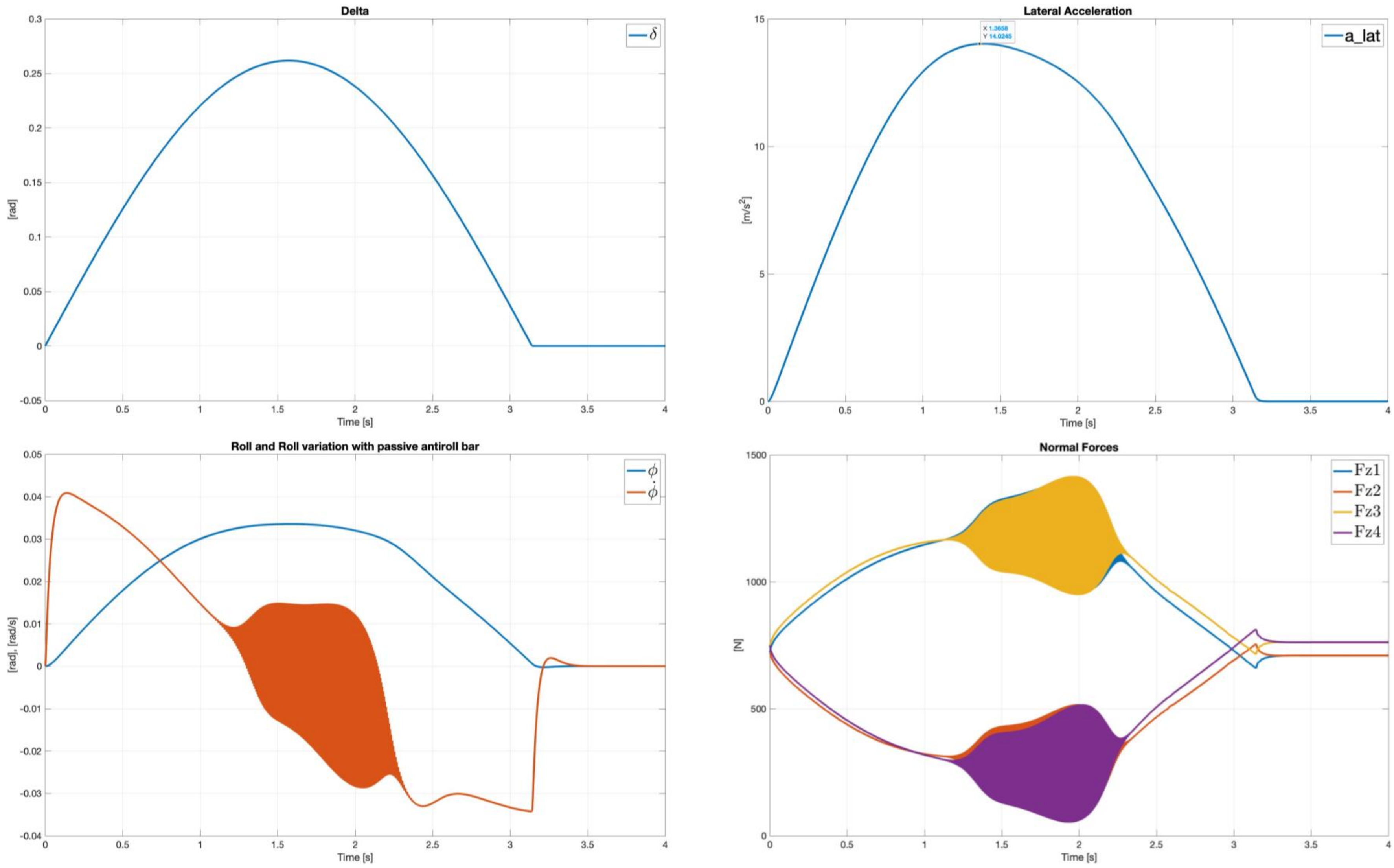
3. The third simulation is with:

   - ✓ Constant speed for the vehicle.
   - ✓ δ goes as a single sinusoid branch of amplitude $\frac{\pi}{12}$
   - ✓ u_0 = 40 Km/h = 11.1111 m/s.

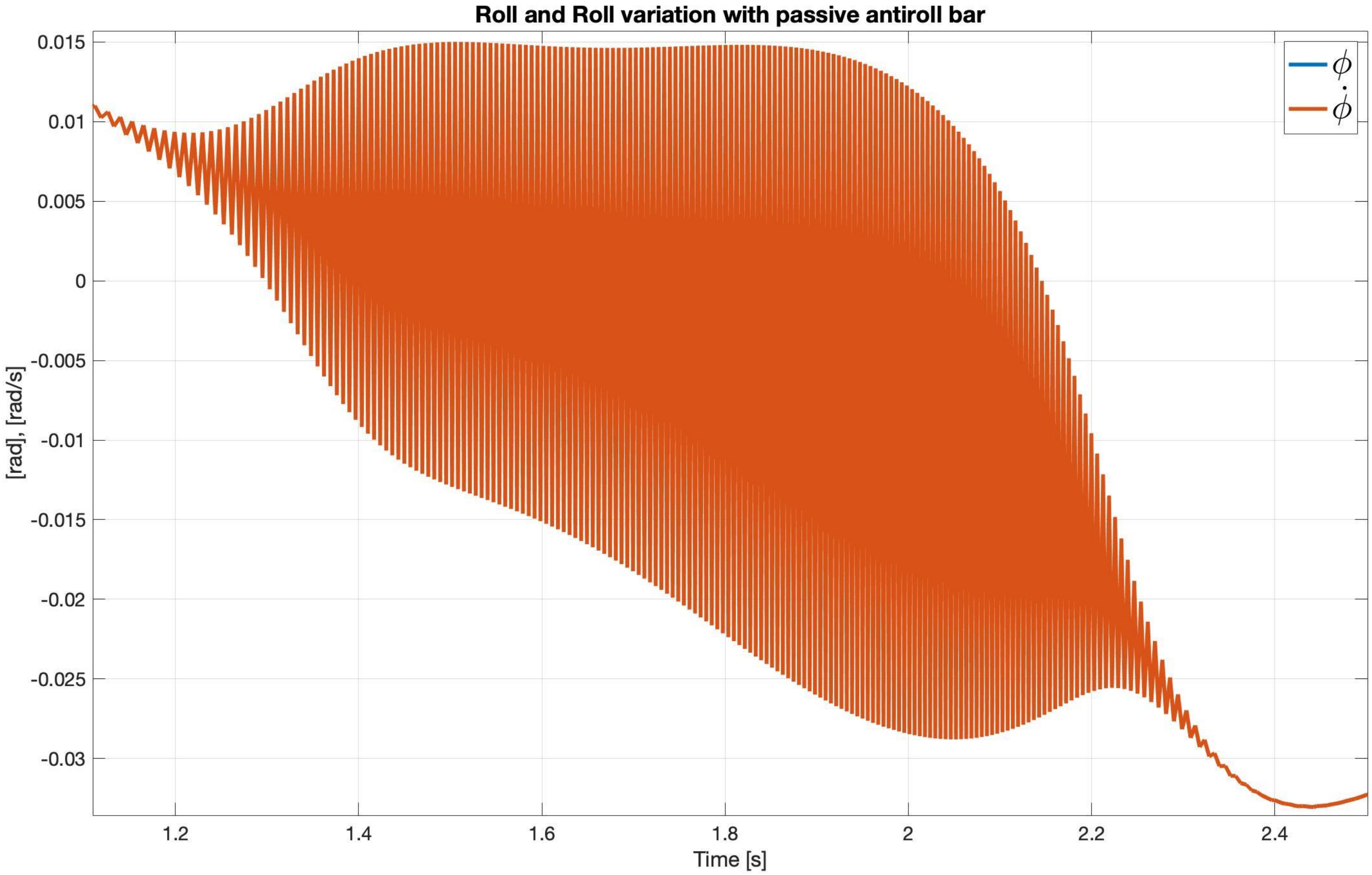
First Simulation, with passive anti-roll bar



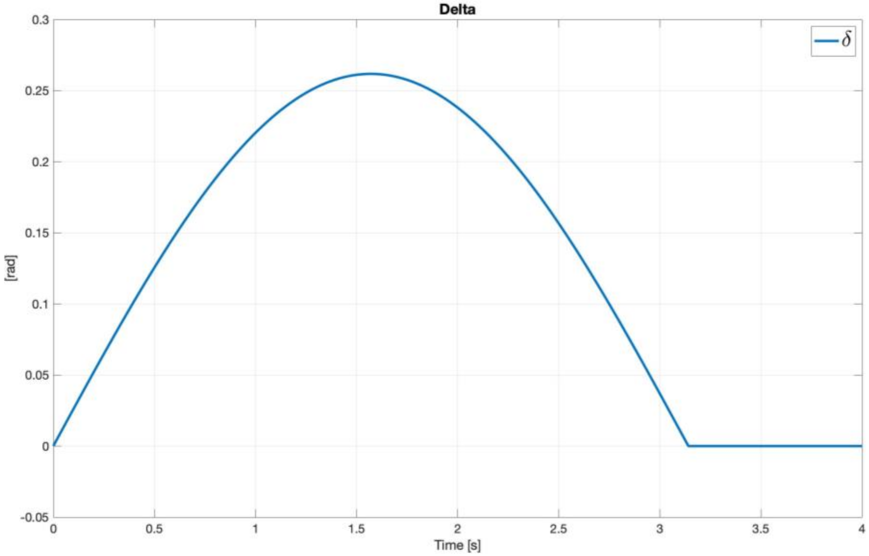
Second Simulation, with passive anti-roll bar

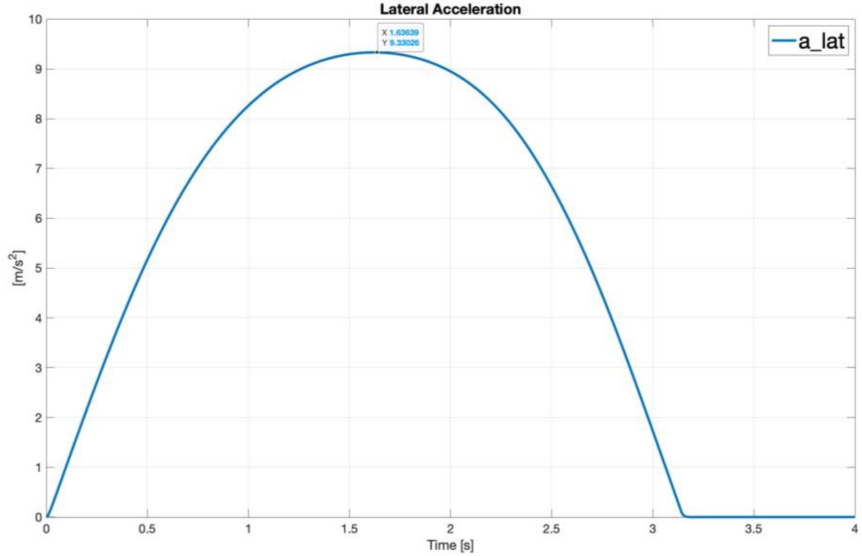


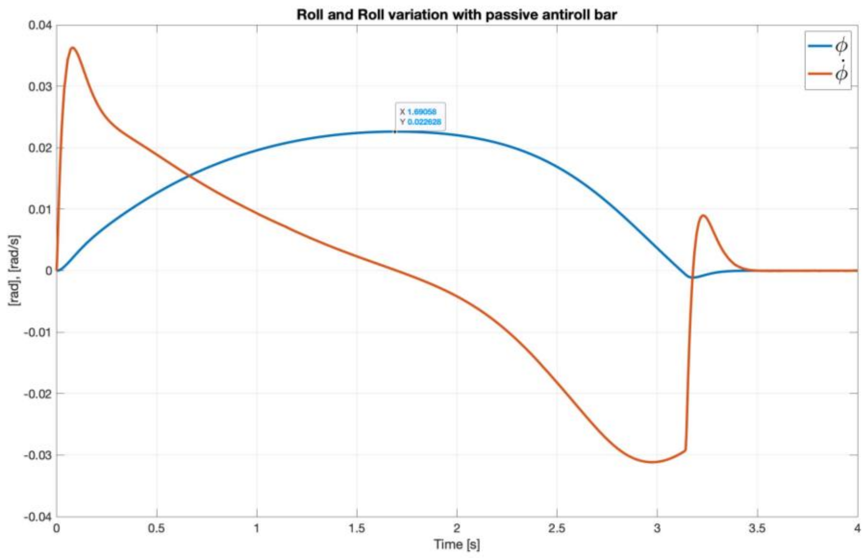
Second Simulation, with passive anti-roll bar

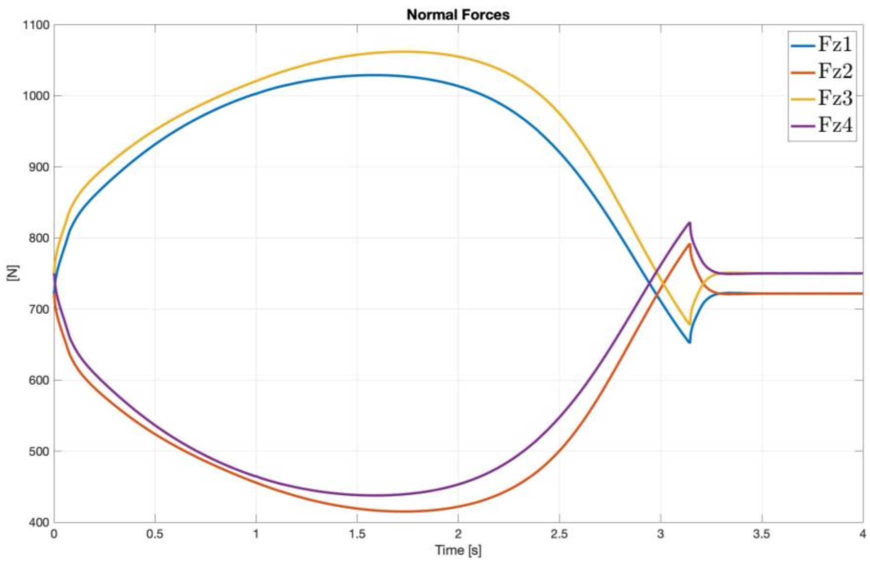


Third Simulation, with passive anti-roll bar



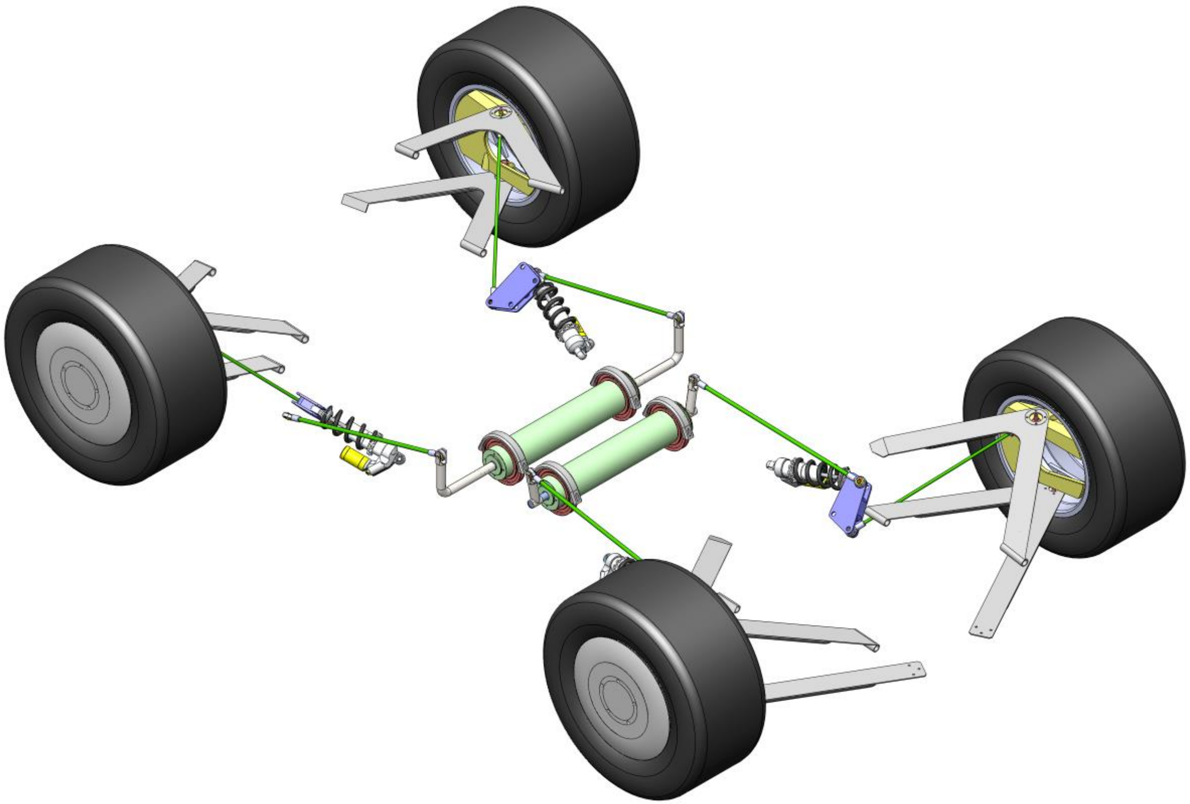


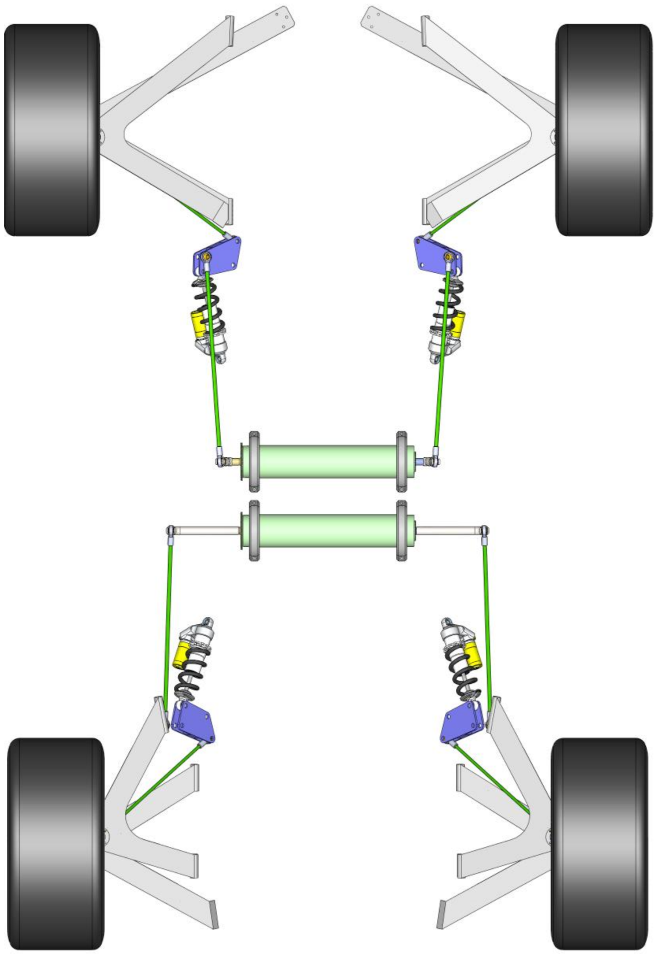




Suspension system modifications

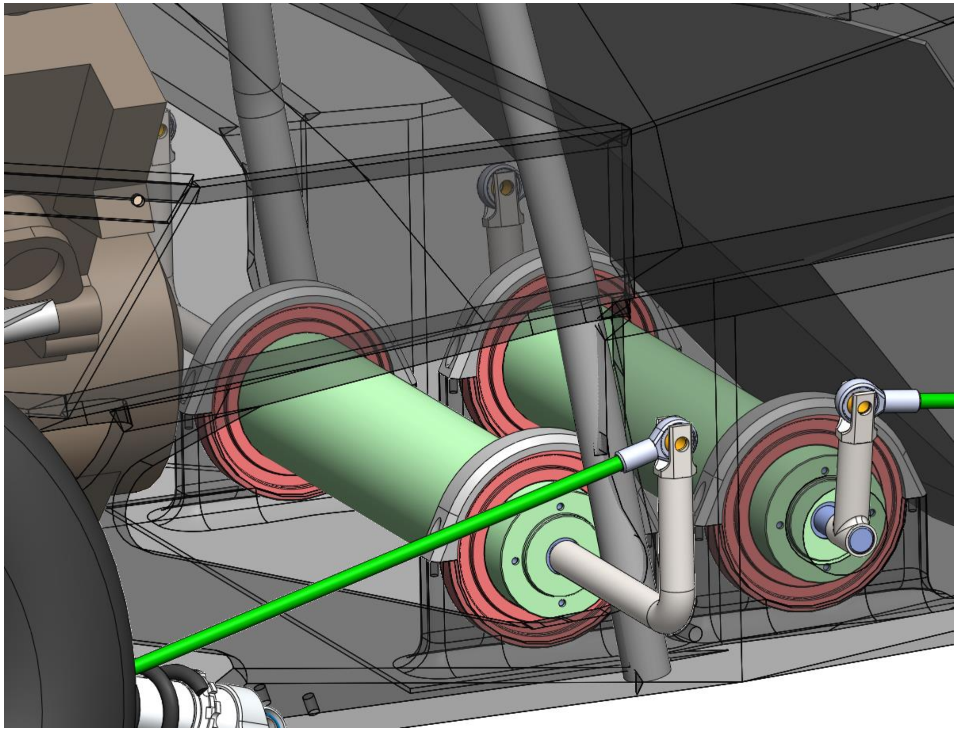
- Minimum modifications of the existing system
- Unchanged kinematics of the suspensions

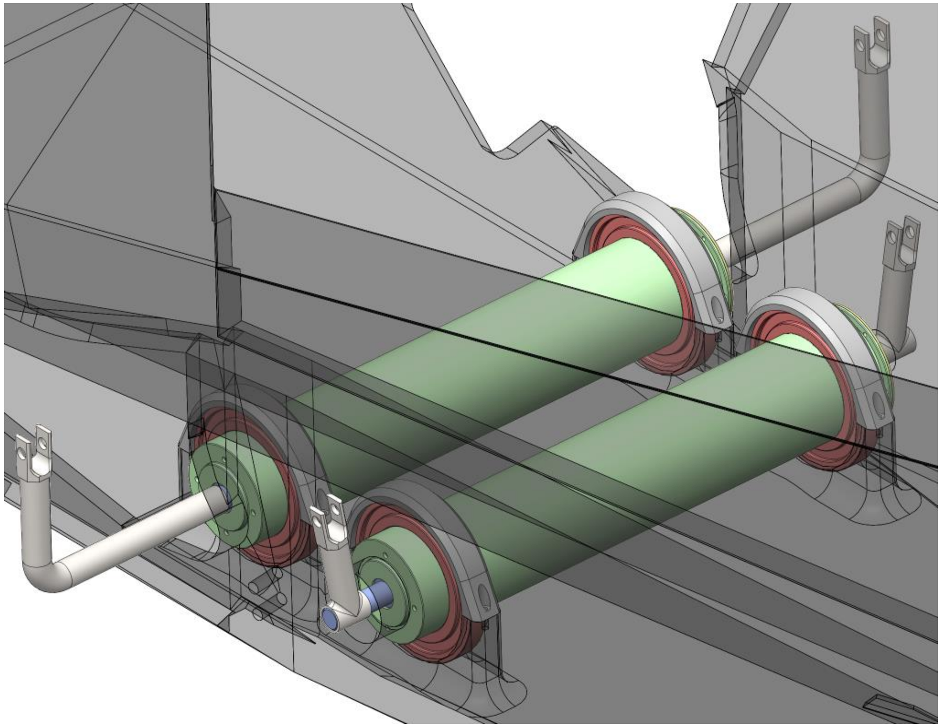




Anti-roll bar installation in the chassis

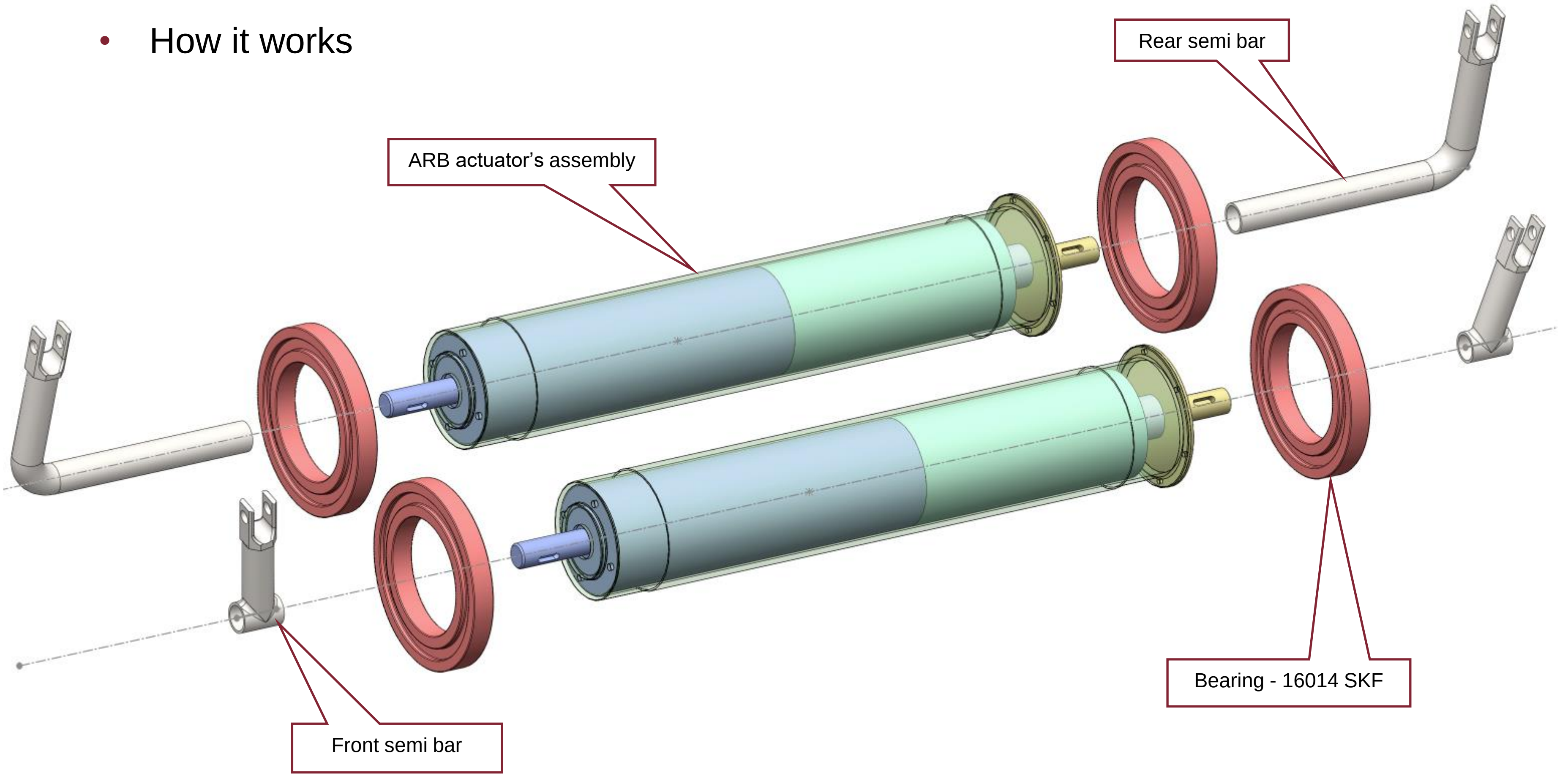
- Size issues
- Carbon fibre bearing supports
- Plug & play approach





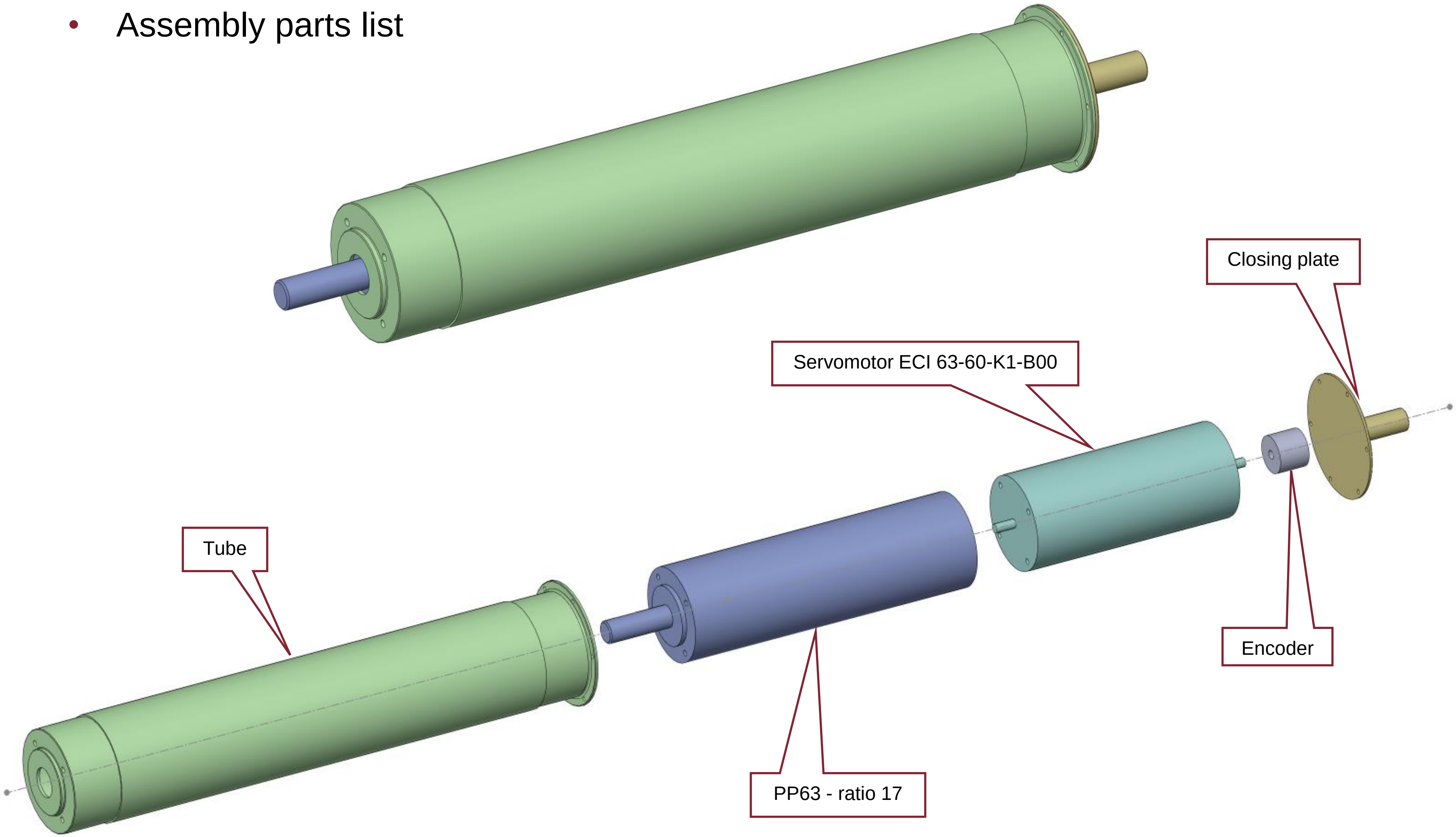
Exploded assembly view

- Assembly parts list
- How it works



Anti-roll bar actuator assembly & explosion

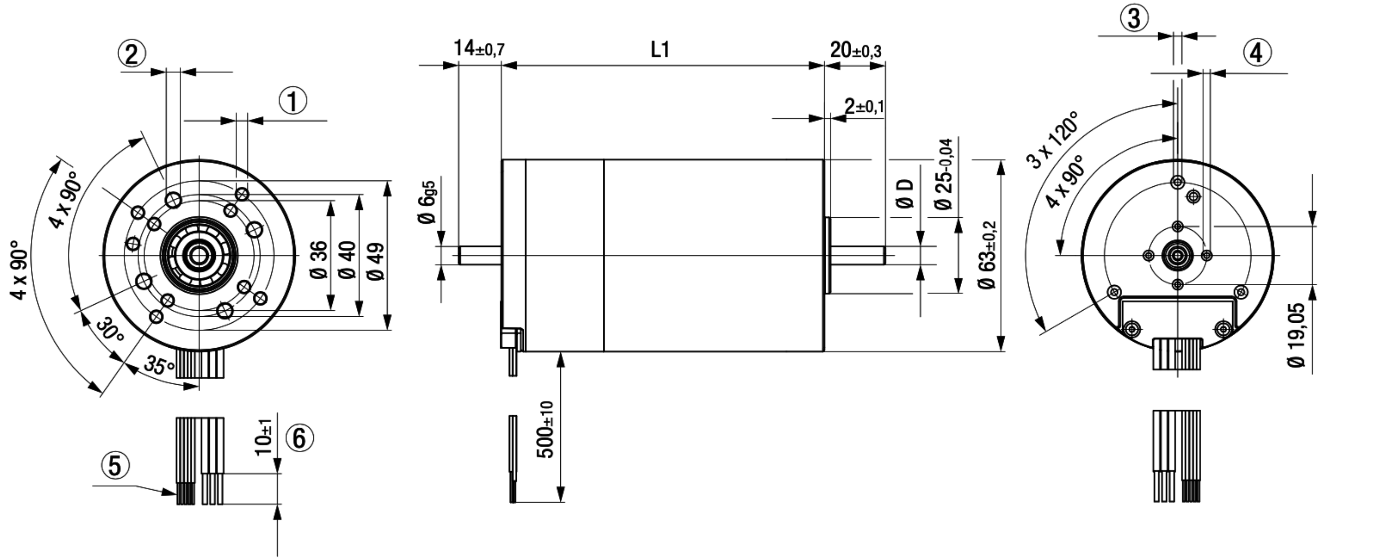
- Assembly parts list



Servomotor ECI 63-60-K1-B00

Technical drawing (Standard, wire interface) All dimensions in mm

Type	L	ØD
ECI-63.20	106.1±0.4	6_{g5}
ECI-63.40	126.1±0.4	6_{g5}
ECI-63.60	146.1±0.4	10_{g5}



① 8 x for thread-forming screws M4 according to DIN7500, screw-in depth max. 10 mm

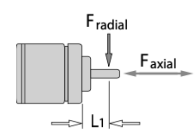
② 4 x for thread-forming screws M5 according to DIN7500, screw-in depth max. 10 mm

③ 3 x For thread-forming screws M3 according to DIN7500, screw-in depth max. 6 mm

④ 4 x for thread-forming screws M2,5 according to DIN7500, screw-in depth max. 6 mm

⑤ 5x ferrule

⑥ Tin-plated

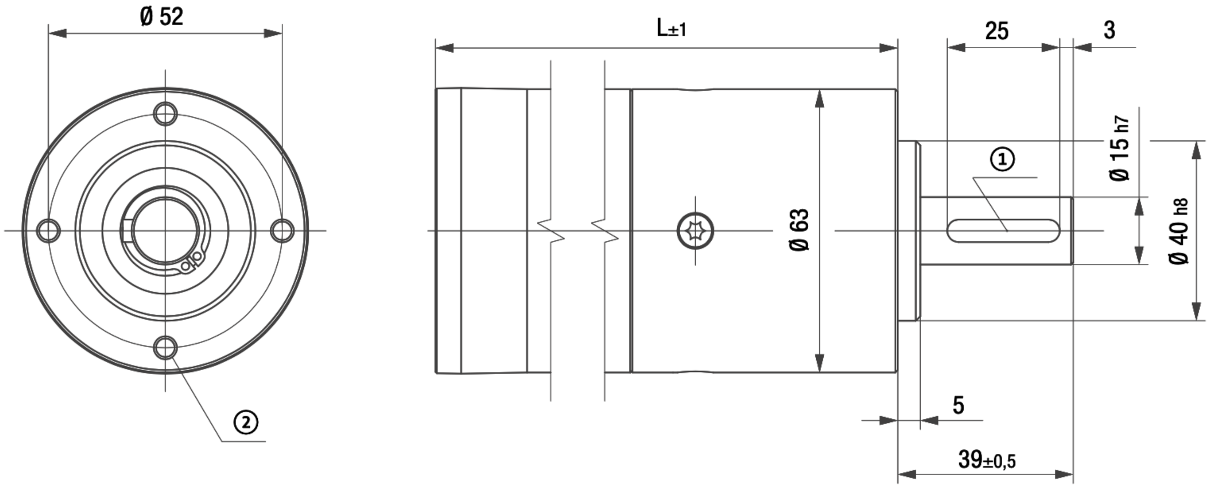


Permissible shaft load

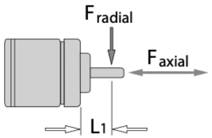
F_{axial}:	150 N	Permissible simultaneous shaft loads at rated speed and service life expectancy L_{10} (in rated operation) from 20 000 h (at T_U max. 40 °C)
F_{radial}:	150 N	
L_1:	20 mm	

PP63 - ratio 17

Technical drawing Image of 1-stage gearhead All dimensions in mm



① Fitted key DIN 6885 A-5x5x25
② 4 x M5, 10 deep

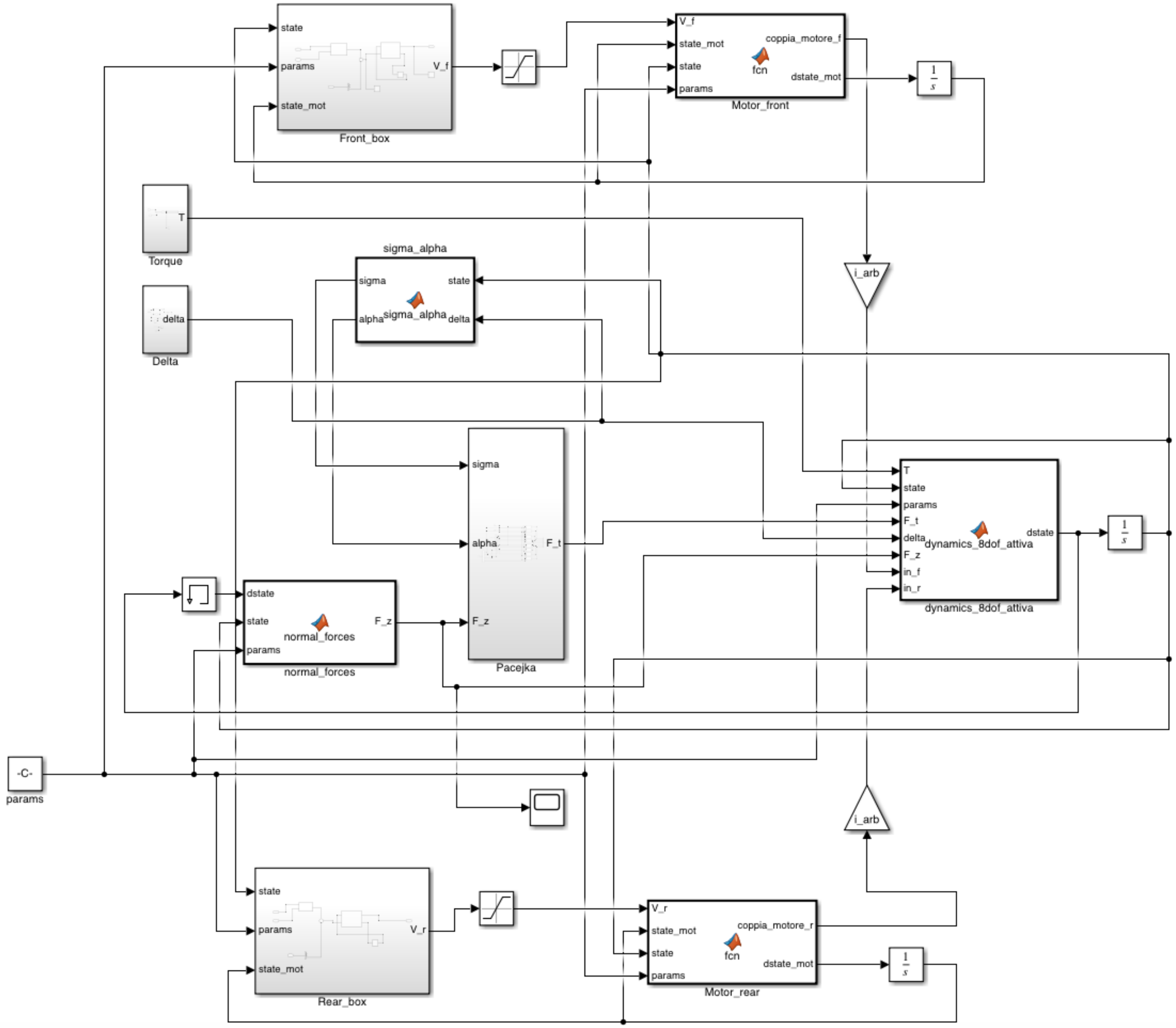


Permissible shaft load

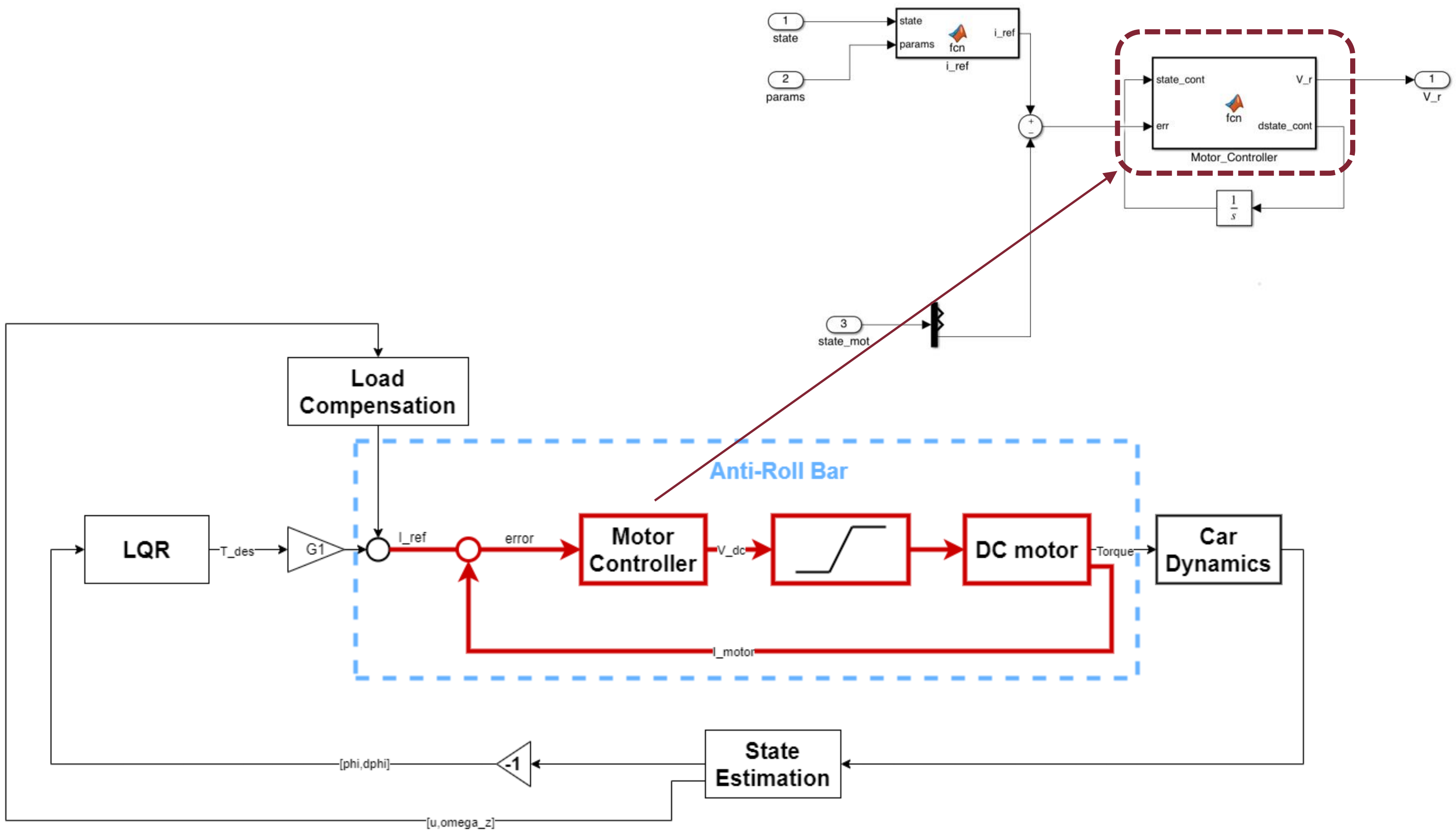
F_{axial}: 1000 N
F_{radial}: see table
L_1: 19 mm

At rated speed, operating factor C_B=1 and a service life expectancy L_{10} from 10 000 h (at T_U max. 40°C in rated operation)

General Active Simulink scheme



Control Approach



DC Motor

DC Motor State Space

$$\boxed{\begin{aligned} \dot{x} &= Ax + Bu + WT_{load} \\ y &= Cx \end{aligned}}$$

$$A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & -\frac{b}{J} & \frac{K_t}{J} \\ 0 & -\frac{K_e}{L} & -\frac{R}{L} \end{bmatrix}$$

$$B = \begin{bmatrix} 0 \\ 0 \\ \frac{1}{L} \end{bmatrix} \qquad C = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \qquad W = \begin{bmatrix} 0 \\ -\frac{1}{J} \\ 0 \end{bmatrix}$$

Parameter	**Value**	**Unit measure**
Nom. Voltage	24	V
Nom. Torque	0.8	Nm
Peak Current	150	A
Speed (no load)	6100	RPM
J	0.03	Kg m^2
K_e	0.404	V s/rad
R	0.04	Ω
L	0.09	H
m	1.5	Kg
b	0.02	N m s/rad
Reduction	1:17	

DC Motor Control, Loop-Shaping

- The main idea is to first consider the sensitivity transfer function “S“ related to the ***voltage-current*** IO channel.

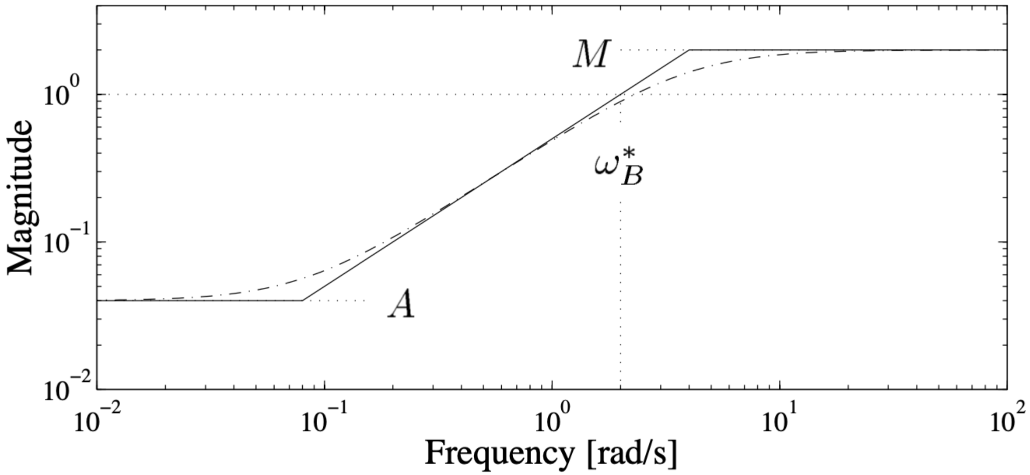
$$S = \frac{1}{1 + PC}$$

- Then, since “S“ is a really good indicator of closed-loop performances, we aim to find “$\omega_P(s)$“ such as:

$$||\omega_P S||_\infty < 1$$

- In this way we prevent amplification of noise at high frequencies, and we also introduce a margin of robustness.

DC Motor Control, Loop-Shaping



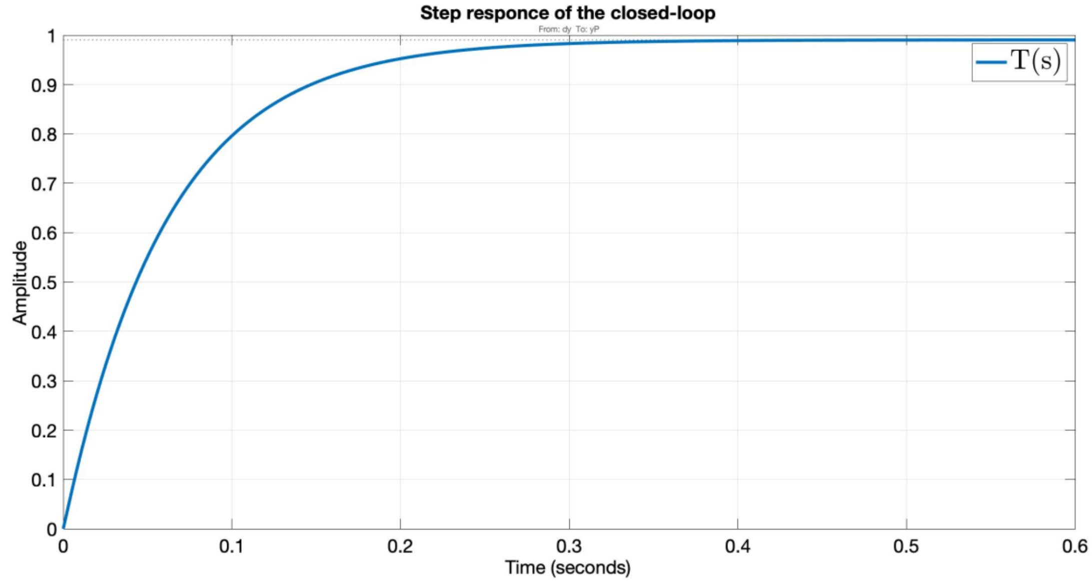
$$\omega_P(s) = \frac{s/M + \omega_B^*}{s + \omega_B^* A}$$

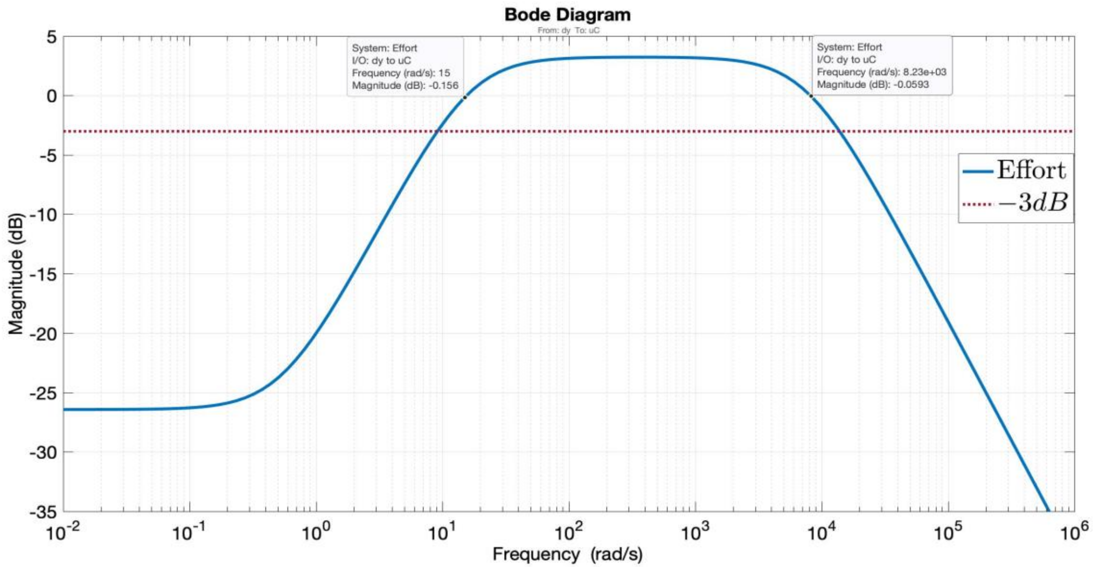
$$\omega_P(s) = \frac{0.95238(s + 16.43)}{s + 0.1565}$$

$$\frac{V(s)}{I(s)} = \frac{11.111(s + 6.667)}{(s + 6.568)(s + 0.5432)}$$

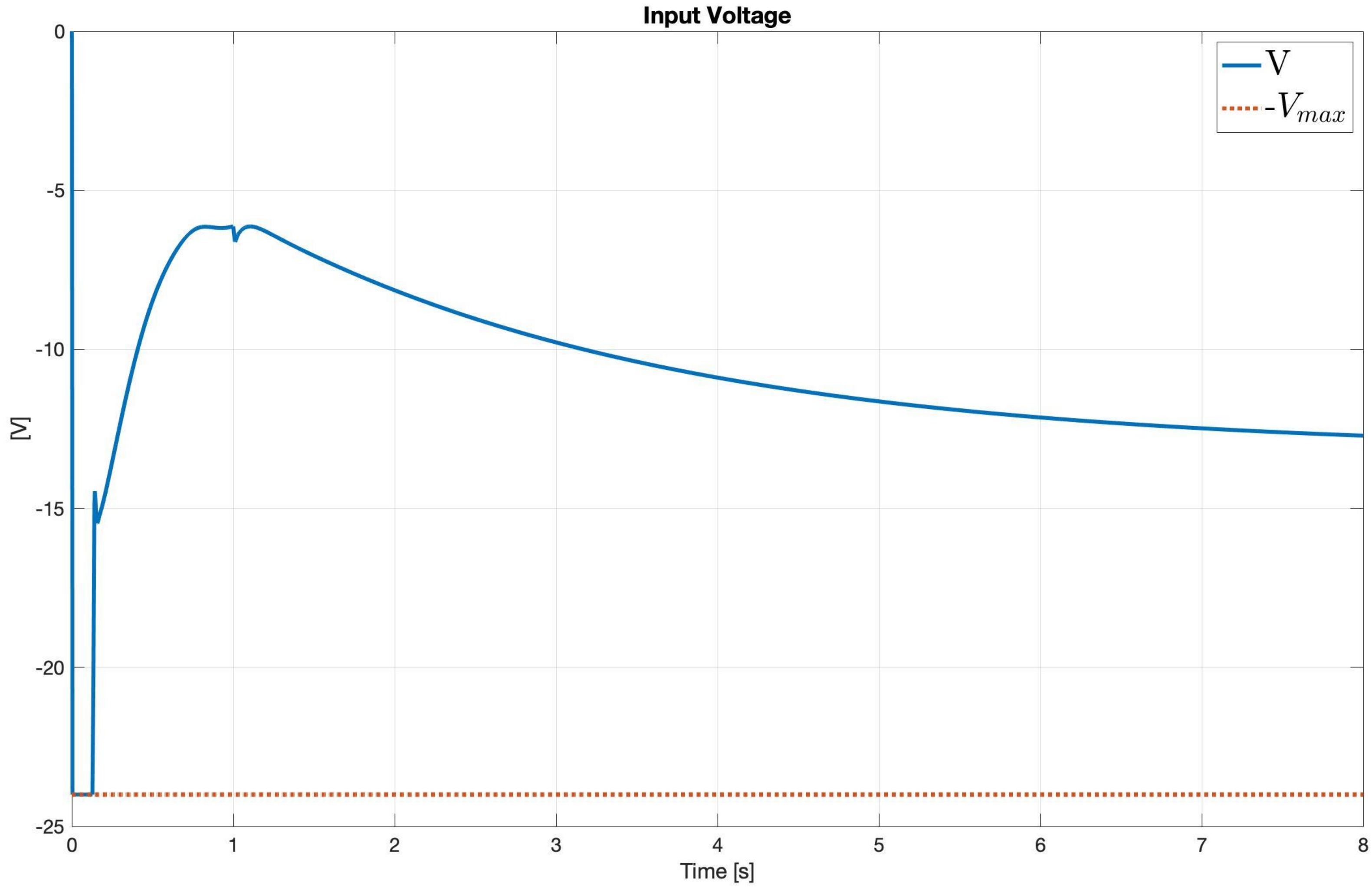
- Considering this weight transfer function we have obtained:

$$\|\, \omega_P(s)\ S(s) \,\|_\infty = 0.9618 < 1$$

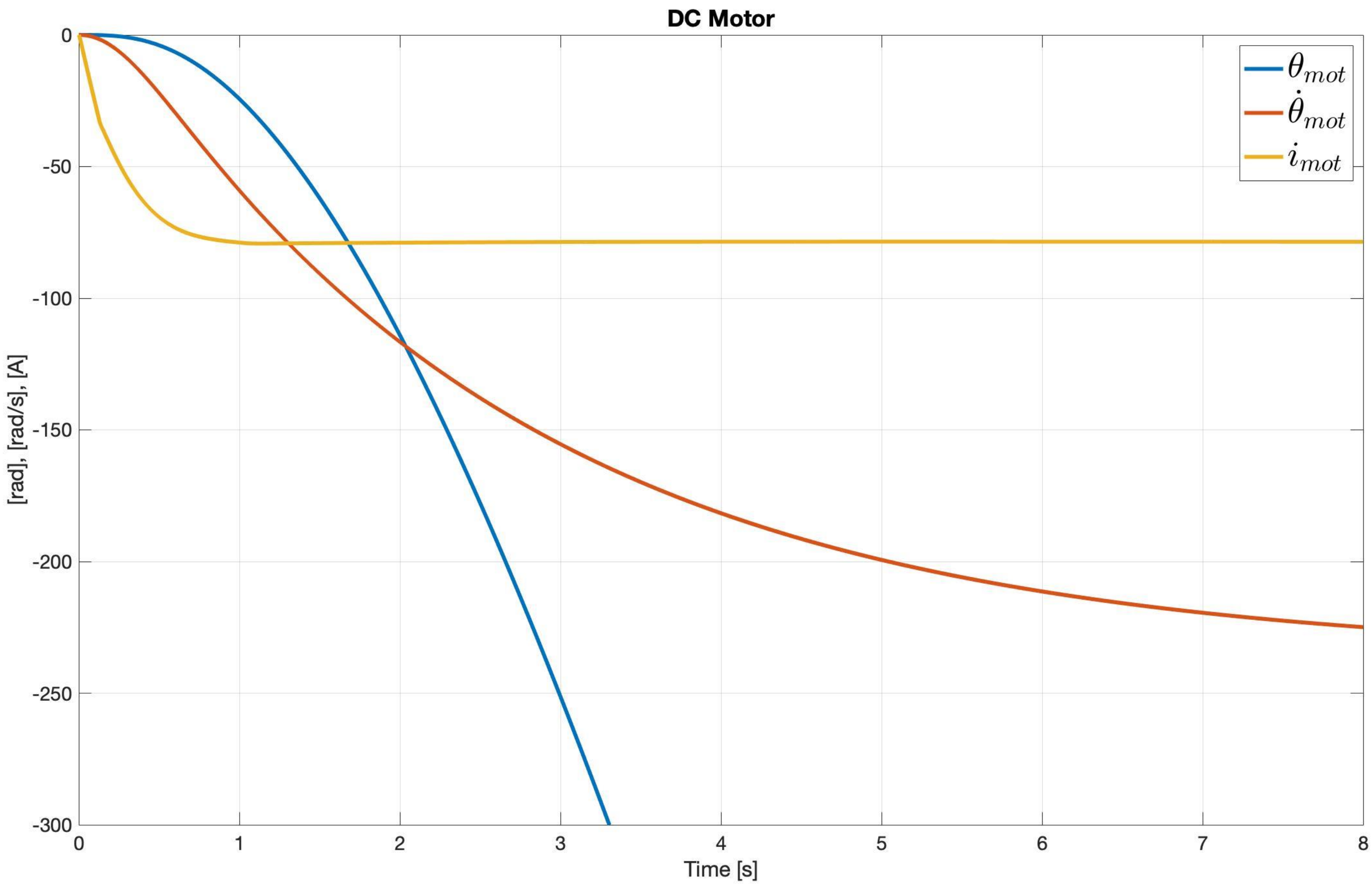

Step responce of the closed-loop
From: dy To: yP
T(s)
Amplitude
Time (seconds)


Bode Diagram
From: dy To: uC
System: Effort
I/O: dy to uC
Frequency (rad/s): 15
Magnitude (dB): -0.156
System: Effort
I/O: dy to uC
Frequency (rad/s): 8.23e+03
Magnitude (dB): -0.0593
Effort
$-3dB$
Magnitude (dB)
Frequency (rad/s)

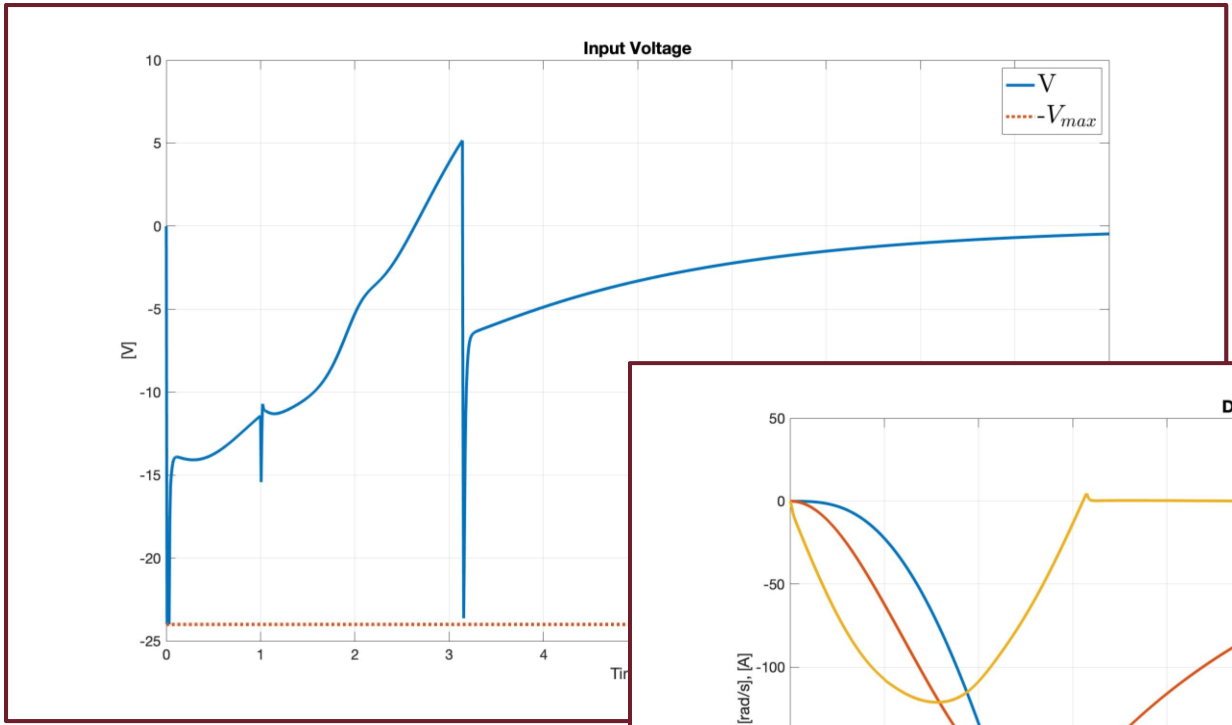
First Simulation, with active anti-roll bar

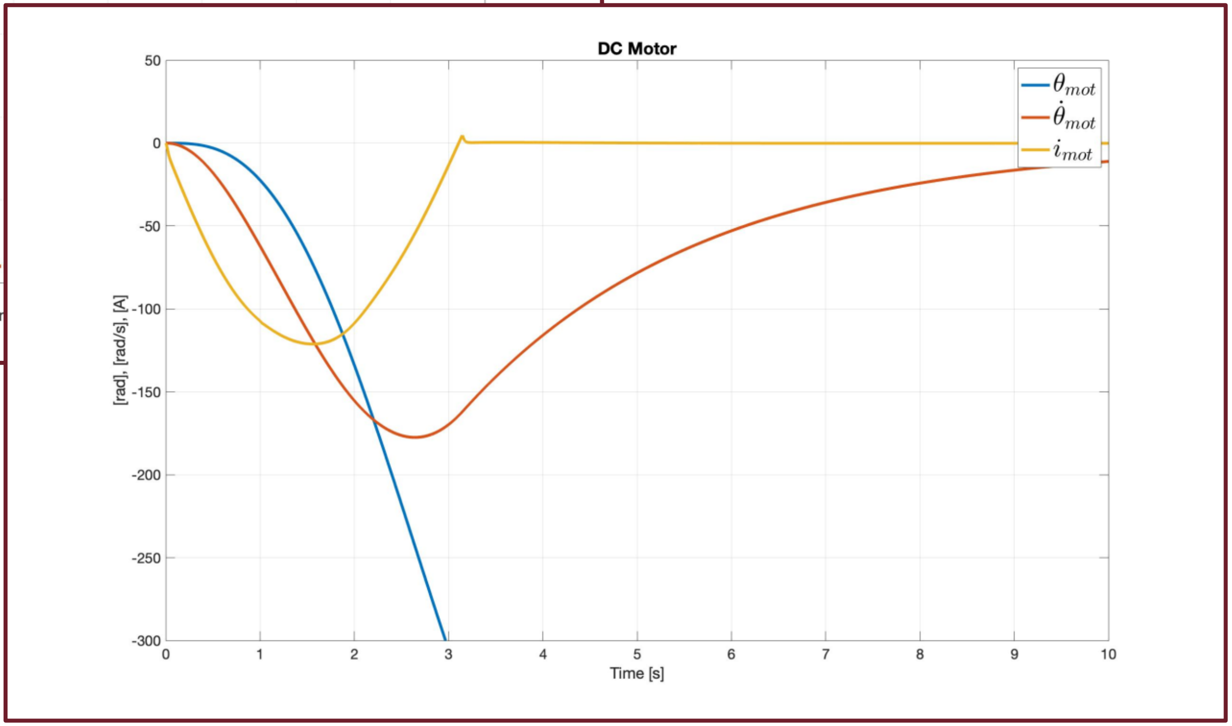


First Simulation, with active anti-roll bar



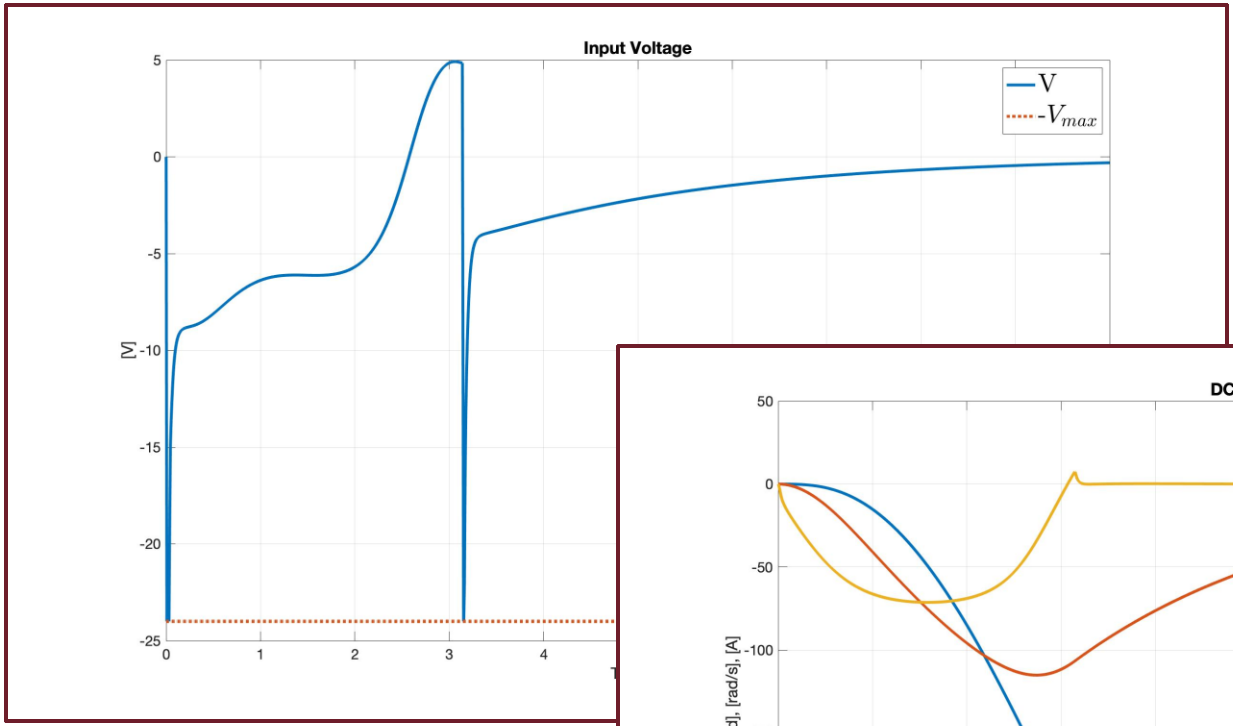
Second Simulation, with active anti-roll bar



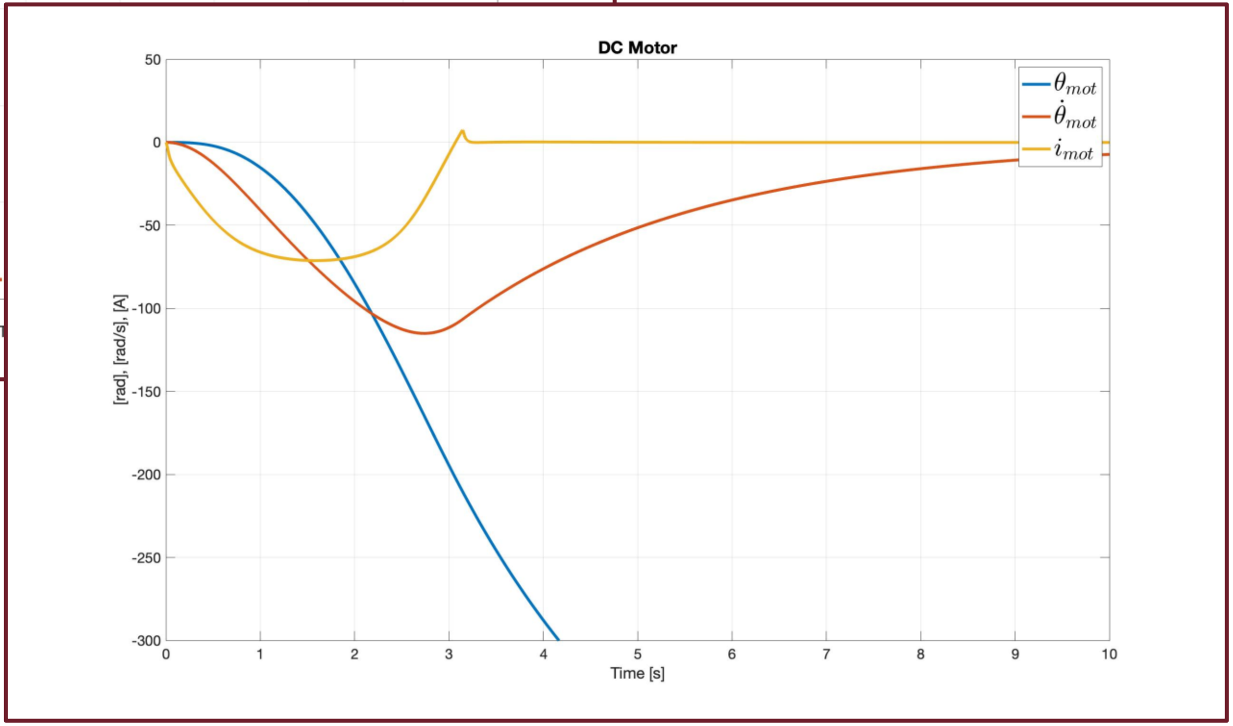


- Step torque at 1 second
- δ goes as a single sinusoid branch of amplitude $\frac{\pi}{12}$
- u_0 = 36 Km/h = 10 m/s.

Third Simulation, with active anti-roll bar



- Constant speed for the vehicle.
- δ goes as a single sinusoid branch of amplitude $\frac{\pi}{12}$
- u_0 = 40 Km/h = 11.1111 m/s.



Model with active anti-roll bars

- The dynamic model of the car is

$$
\begin{aligned}
\dot{x} &= u\cos(\psi) - v\sin(\psi) \\
\dot{y} &= u\sin(\psi) + v\cos(\psi) \\
\dot{u} &= \dot{\psi}v + \tfrac{1}{m}\Big(m_s h\ddot{\psi}\phi + (F_{\mathrm{xfl}} + F_{\mathrm{xfr}})\cos(\delta) + F_{\mathrm{xrl}} + F_{\mathrm{xrr}} - (F_{\mathrm{yfl}} + F_{\mathrm{yfr}})\sin(\delta)\Big) \\
\begin{bmatrix} \dot{v} \\ \ddot{\phi} \end{bmatrix} &= \begin{bmatrix} m & -m_s h \\ -m_s h & J_x \end{bmatrix}^{-1} \begin{bmatrix} -\dot{\psi}u + (F_{\mathrm{yfl}} + F_{\mathrm{yfr}})\cos(\delta) + F_{\mathrm{yrl}} + F_{\mathrm{yrr}} + (F_{\mathrm{xfl}} + F_{\mathrm{xfr}})\sin(\delta) \\ m_s h\dot{\psi}u + m_s hg\phi - (b_{sf} + b_{sr})\dot{\phi} - (k_{sf} + k_{sr})\phi + \boxed{\mathrm{in}_f + \mathrm{in}_r} \end{bmatrix} \\
\ddot{\psi} &= \tfrac{1}{J_z}\Big(\tfrac{c_f}{2}(F_{\mathrm{xfr}}\cos(\delta) - F_{\mathrm{yfr}}\sin(\delta)) - (F_{\mathrm{xfl}}\cos(\delta) - F_{\mathrm{yfl}}\sin(\delta)) + \tfrac{c_r}{2}(F_{\mathrm{xrr}} - F_{\mathrm{xrl}}) + \\
&\quad + d_f((F_{\mathrm{yfl}} + F_{\mathrm{yfr}})\cos(\delta) + (F_{\mathrm{xfr}} + F_{\mathrm{xfl}})\sin(\delta)) - d_r(F_{\mathrm{yrl}} + F_{\mathrm{yrr}}\Big) \\
\dot{\omega}_i &= \tfrac{1}{J_\omega}\left(T_i - r_0 F_{\mathrm{tix}} - T_{\mathrm{volv}}\right)
\end{aligned}
$$

- The torque inputs given by the two anti-roll bars are in_f and in_r

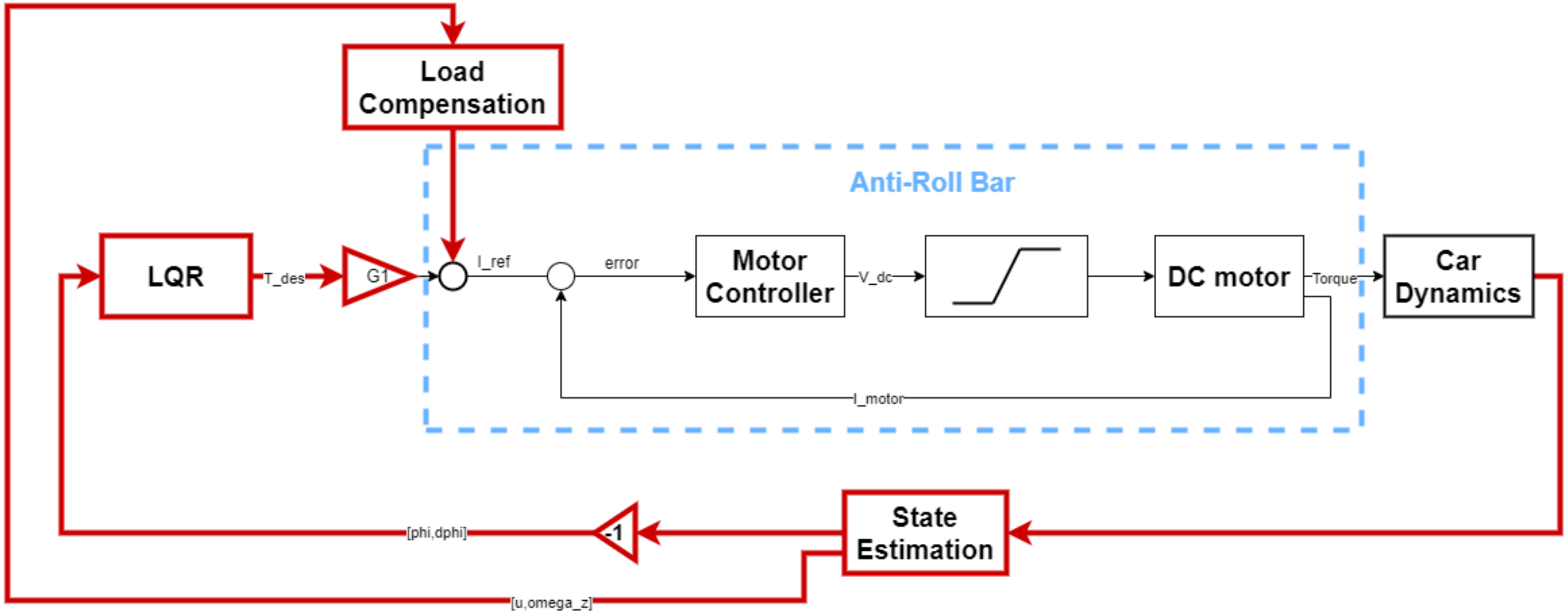
Outer loop

- The goal is to generate a reference for the DC motor's armature current.

- This is achieved through two actions:

  - Feedback generated by the LQR that aims to drive the roll and roll-rate to zero.

  - Compensation of the load torque with $I_{\text{comp}} = \dfrac{m_s h_{rc} u \omega_z t_{\text{ratio}_{f/r}}}{i_{\text{arb}} \cdot r \cdot k_t}$



LQR - Linearization

- The matrices **A** and **B** used in the LQR algorithm are the state space matrices of the linearized system. In particular:

  $$A = \left.\frac{\partial f}{\partial x}\right|_{x_{\text{work}},u_{\text{work}}}$$, and $$B = \left.\frac{\partial f}{\partial u}\right|_{x_{\text{work}},u_{\text{work}}}$$

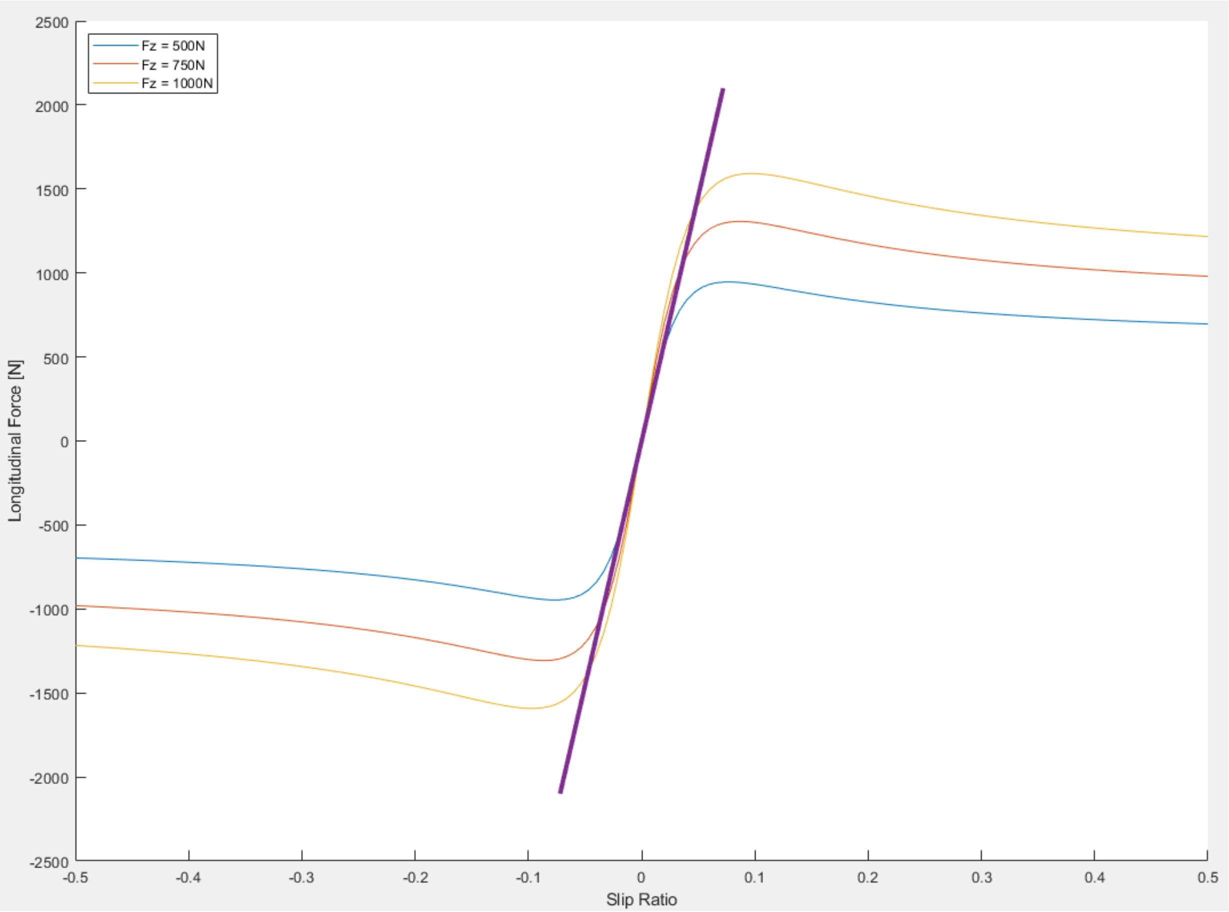
- The feedback action is given by $-F\begin{bmatrix}\phi & \dot{\phi}\end{bmatrix}^T$ where: $\mathrm{F{=}lqr(A,B,Q,R)}$, with:

  $$Q = \begin{bmatrix}10^6 & 0 \\ 0 & 10^5\end{bmatrix}$$, and $$R = 10^{-4}$$

- In the end the reference for the current is $I_{\text{ref}} = -I_{\text{comp}} - F\begin{bmatrix}\phi \\ \dot{\phi}\end{bmatrix}$

Linearization - Pacejka

- The forces at the contact patch are nonlinear, described according to Pacejka's magic formula. In order to perform the linearization we used a linear model for the aforementioned forces



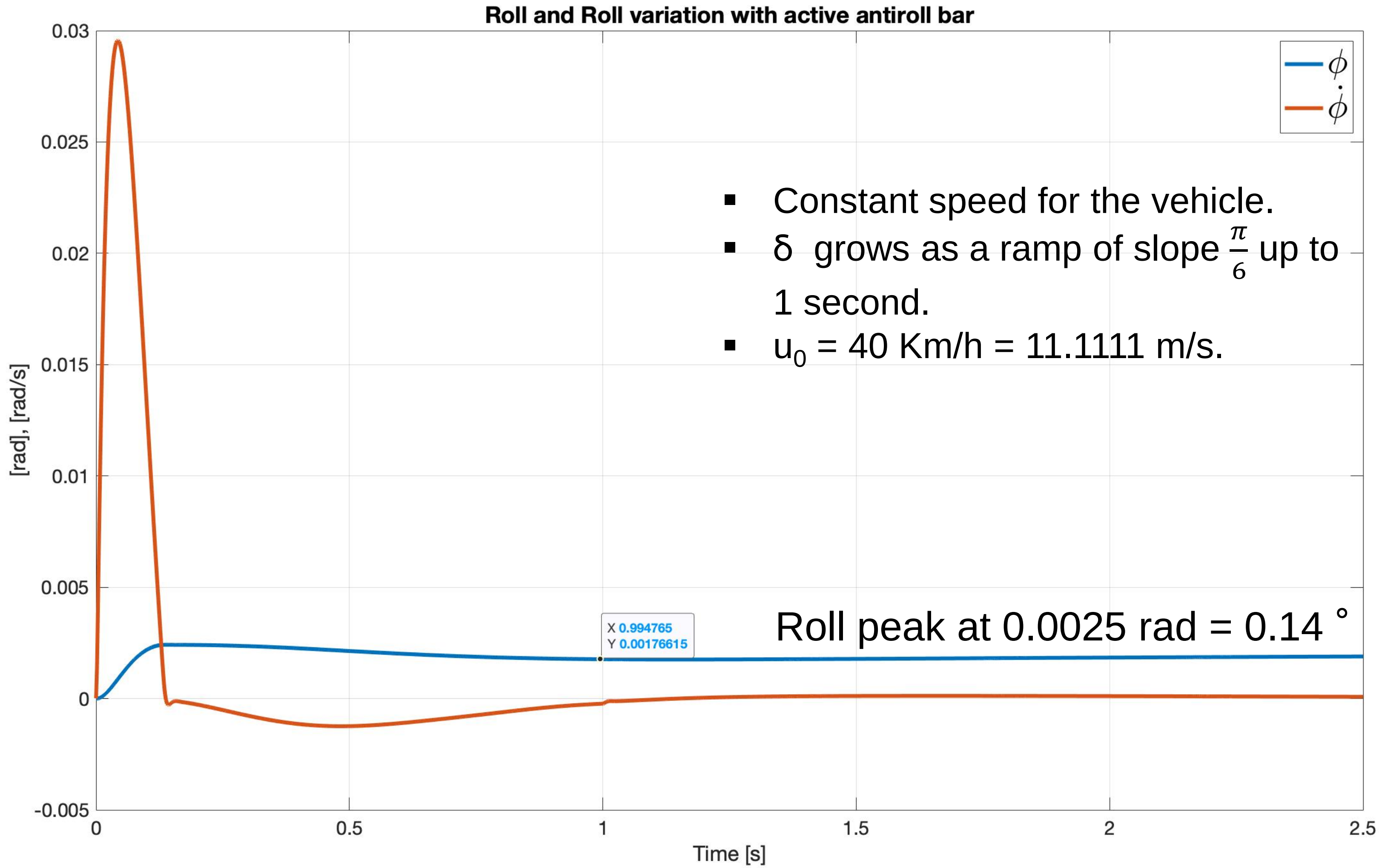
$$F_{tix} = k_{\text{pac,lon}} \sigma_i$$

$$F_{tiy} = k_{\text{pac,lat}} \alpha_i$$

σ_i longitudinal slip

α_i lateral slip

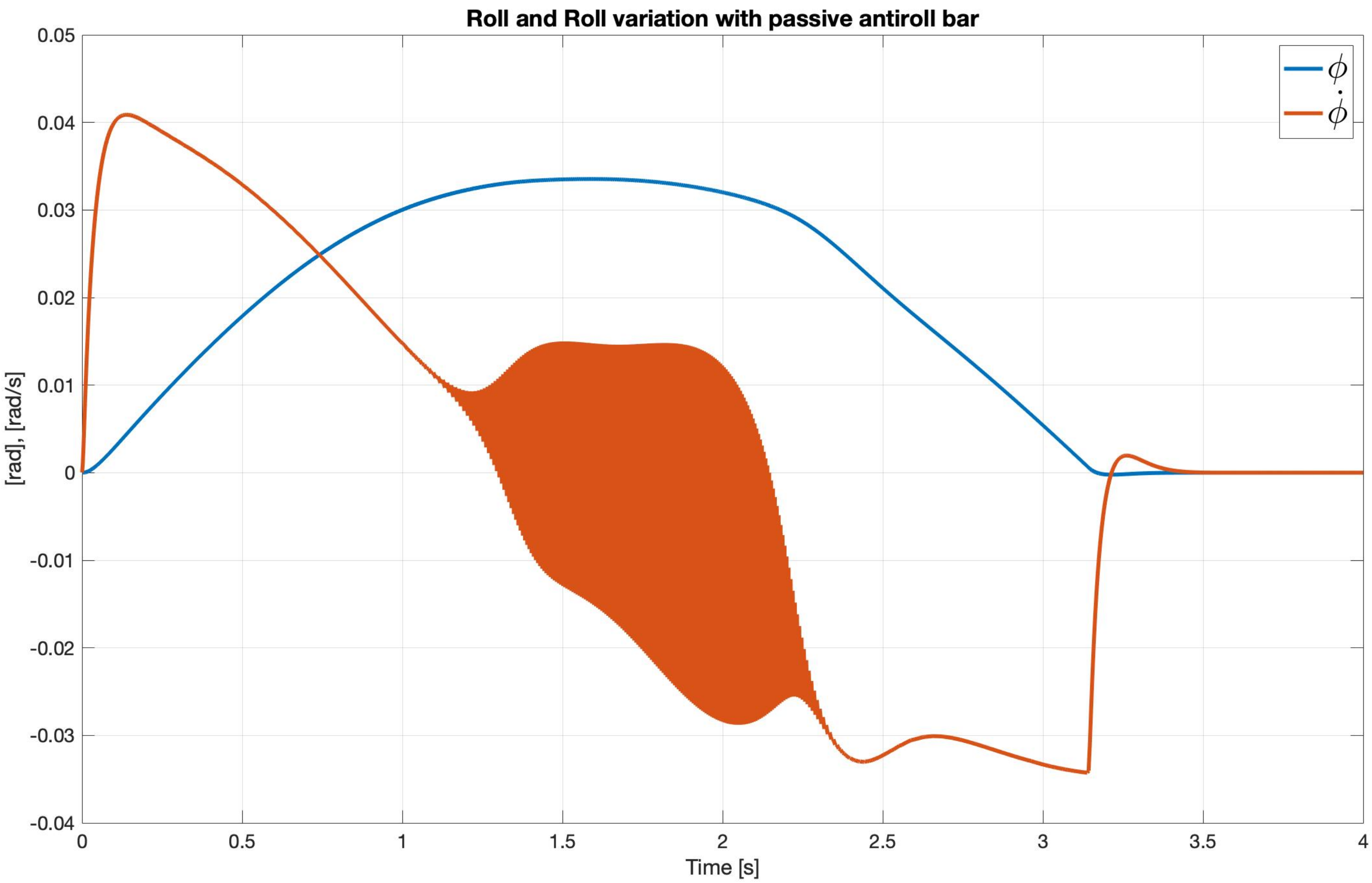
First Simulation, with active anti-roll bar



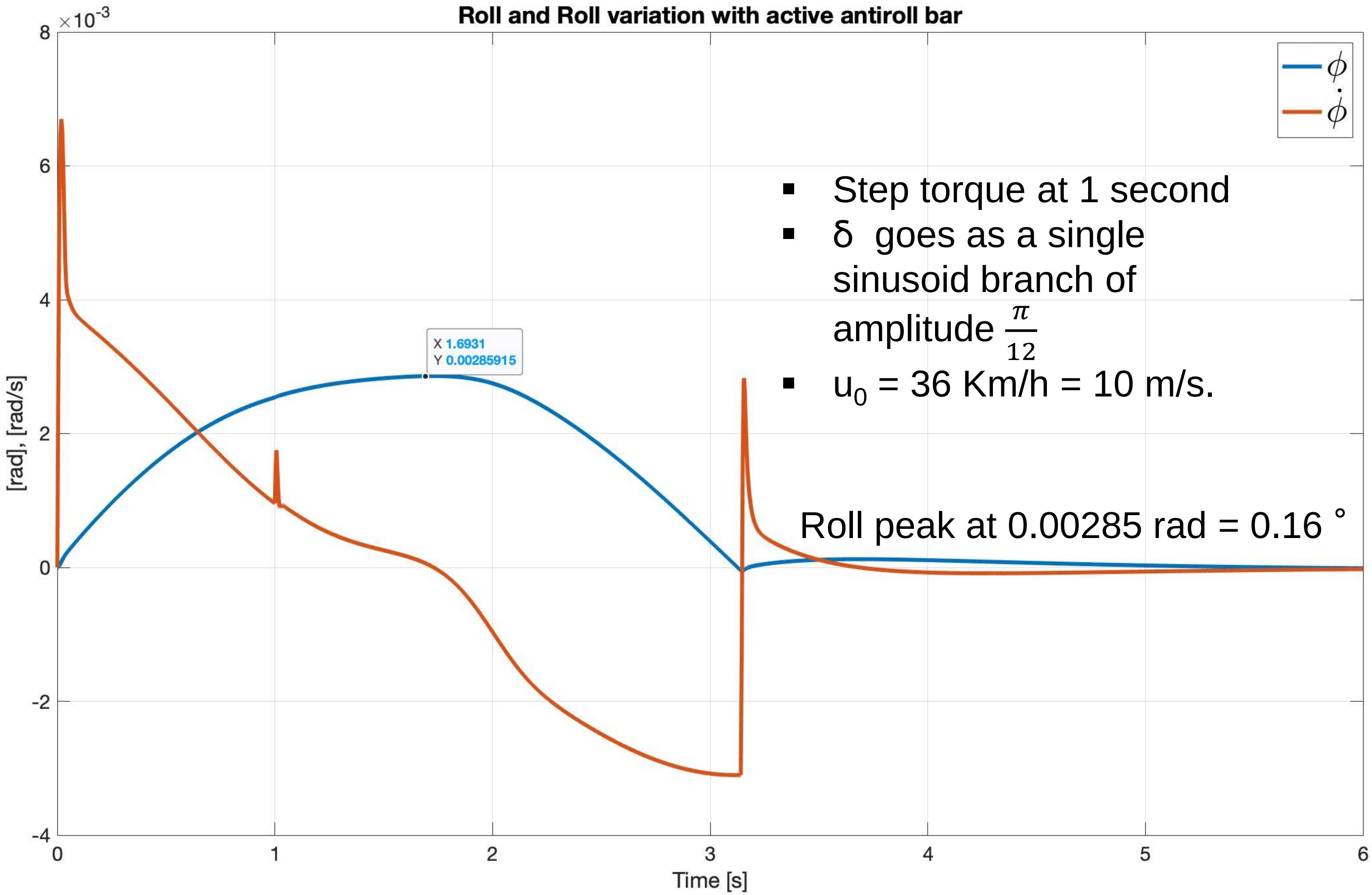
- Constant speed for the vehicle.
- δ grows as a ramp of slope $\frac{\pi}{6}$ up to 1 second.
- u_0 = 40 Km/h = 11.1111 m/s.

Roll peak at 0.0025 rad = 0.14 °

Second Simulation, with PASSIVE anti-roll bar

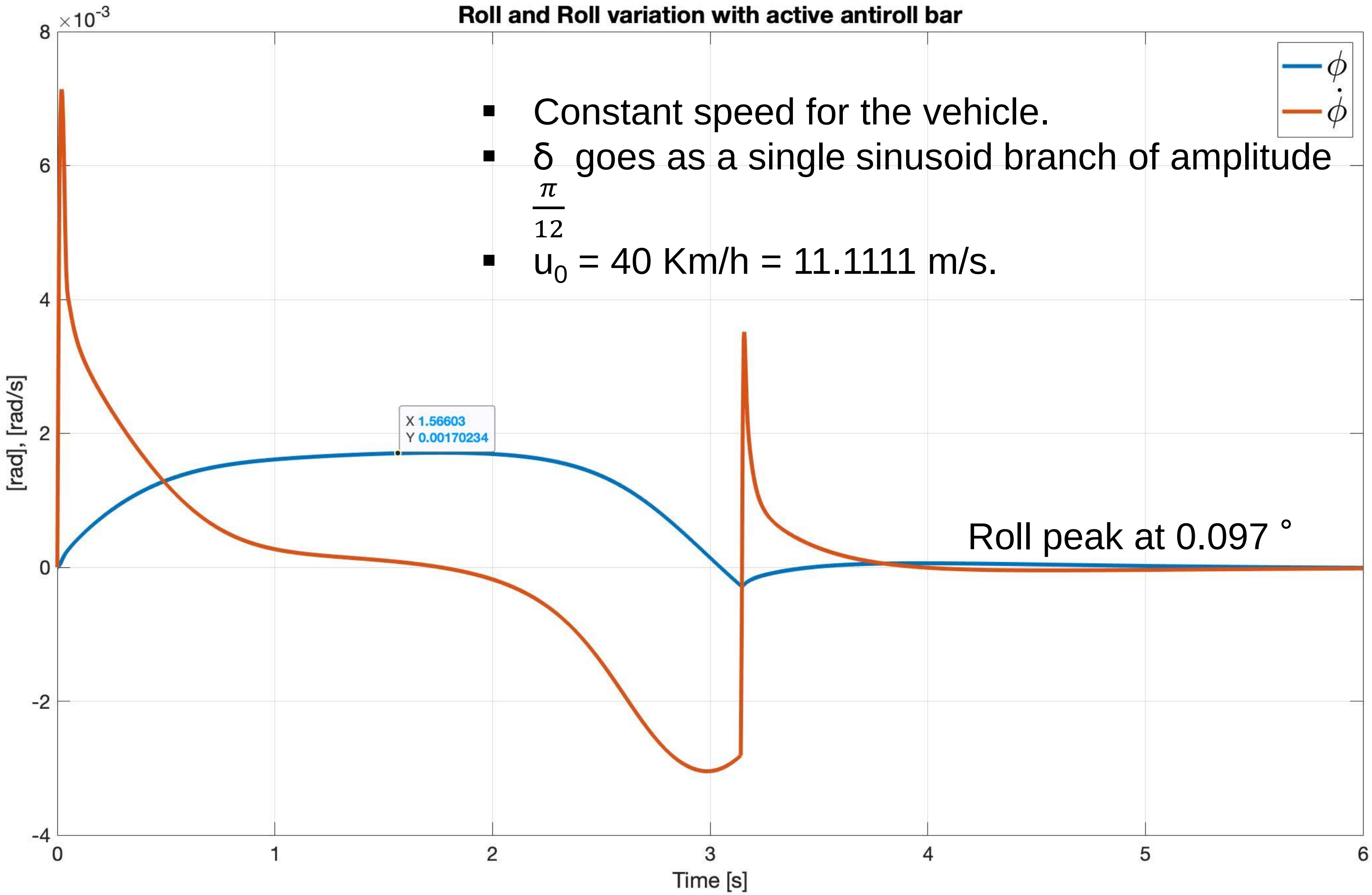


Second Simulation, with active anti-roll bar



Third Simulation, with active anti-roll bar

- Constant speed for the vehicle.
- δ goes as a single sinusoid branch of amplitude $\frac{\pi}{12}$
- u_0 = 40 Km/h = 11.1111 m/s.



Roll peak at 0.097 °

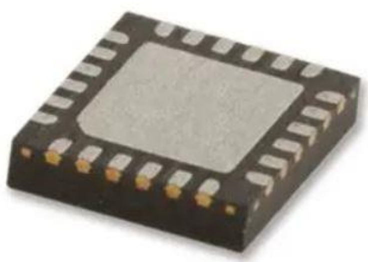
Sensors

- The car is equipped with several sensors:

    1. IMU sensor to measure the roll-rate and the yaw-rate through the gyroscope.

    2. Encoders mounted on the four wheels in order to measure the angular velocity.

    3. GPS sensor to measure the longitudinal and lateral velocity.

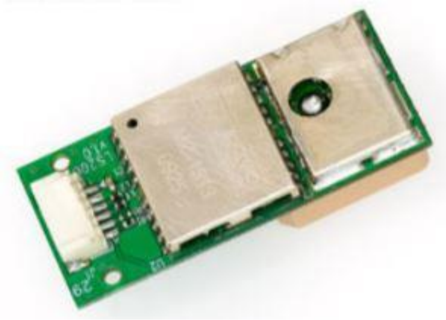


IMU-3000

Gyro accuracy < 0.01 °/s
Noise @100Hz 0.1 °/s-rms

Encoder RI-42

Noise @1000Hz 0.0001



GPS-LS20030

Position accuracy 3 m (2D-rms)
Velocity accuracy < 0.2 km/h

Estimation

- In order to perform the control action we need the estimate of $\begin{bmatrix} u & \phi & \dot{\phi} & \dot{\psi} \end{bmatrix}$

- The longitudinal velocity can be measured using the GPS sensor.

- The gyroscope will be used for measuring the roll-rate and the yaw-rate.

- As a first approach we won't use a measure for the roll angle, we will use the discrete time model to integrate the roll-rate, recovering ϕ.

Estimation

- We have simplified the dynamics used for the estimation, in fact have used an odometric model for the translational dynamics.

- With the hypothesis of free-rolling condition we will use the mean of the four wheels' velocities to estimate the longitudinal velocity.

$$
\begin{aligned}
u(k+1) &= \frac{\sum_{i=1}^{4} \omega_i r_0}{4} \cos(\alpha(k)) \\
v(k+1) &= \frac{\sum_{i=1}^{4} \omega_i r_0}{4} \sin(\alpha(k)) \\
\phi(k+1) &= \phi(k) + \omega_x T_s \\
\psi(k+1) &= \psi(k) + \omega_z T_s \\
\omega_x(k+1) &= \omega_x(k) + \frac{1}{J_x} \Big(m_s h \frac{\sum_{i=1}^{4} \omega_i r_0}{4} \sin(\alpha(k)) + \\
&\quad + m_s h \omega_z(k) u(k) + m_s h g \phi(k) - (b_s f + b_s r)\omega_x(k) - (k_s f + k_s r)\phi(k) T_s \\
\omega_z(k+1) &= \omega_z(k) + \frac{\sum_{i=1}^{4} \omega_i r_0}{4} \cos(\alpha(k)) \frac{(d_f + d_r}{(\cos(\delta(k))^2 (d_r^2 \tan(\delta(k))^2 + (d_f + d_r)^2)} \dot{\delta}(k) T_s \\
\omega_1(k+1) &= \omega_1(k) \\
\omega_2(k+1) &= \omega_2(k) \\
\omega_3(k+1) &= \omega_3(k) \\
\omega_4(k+1) &= \omega_4(k)
\end{aligned}
$$

- The outputs of the systems are the measures (+ noise) of

$$
h(k) = \begin{bmatrix} u(k) & v(k) & \omega_x(k) & \omega_z(k) & \omega_1(k) & \omega_2(k) & \omega_3(k) & \omega_4(k) \end{bmatrix}^T
$$

.

Estimation - EKF

- We have used an Extended Kalman Filter in order to estimate the reduced state.

$$
\begin{aligned}
\hat{x}_{k+1|k} &= f_k(\hat{x}_k, u_k) \\
P_{k+1|k} &= F_k P_k F_k^T + V_k \\
\hat{x}_{k+1} &= \hat{x}_{k+1|k} + R_{k+1}(y_{k+1} - C_{k+1}\hat{x}_{k+1|k}) \\
P_{k+1} &= P_{k+1|k} - R_{k+1} C_{k+1} P_{k+1|k}
\end{aligned}
$$

where:

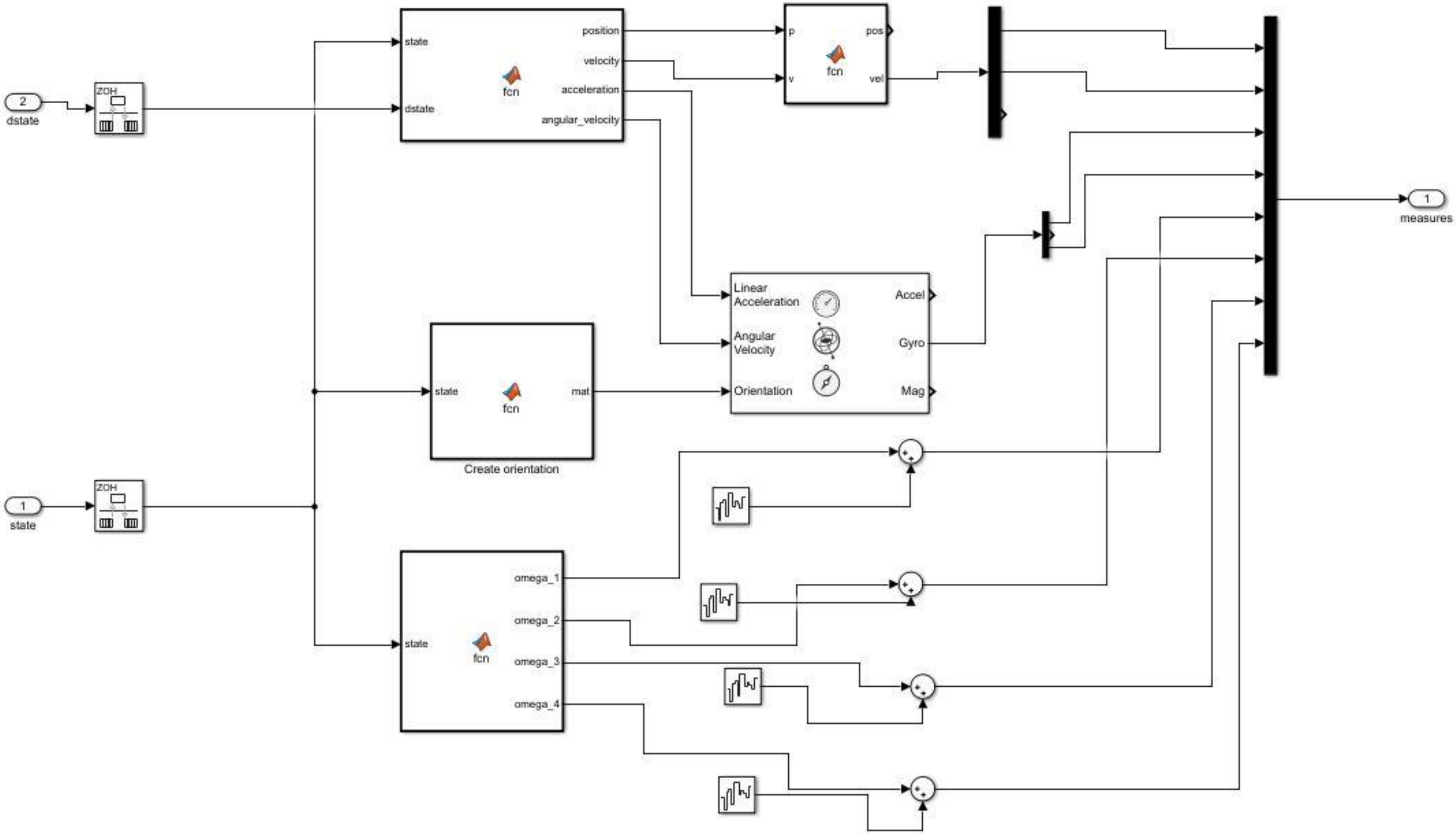
$$F_k = \left.\frac{\partial f_k}{\partial x}\right|_{x_k, u_k} \qquad C_{k+1} = \left.\frac{\partial h}{\partial x}\right|_{x_{k+1|k}}$$

$$R_{k+1} = P_{k+1|k} C_{k+1}^T (C_{k+1} P_{k+1|k} C_{k+1}^T + W_{k+1})^{-1}$$

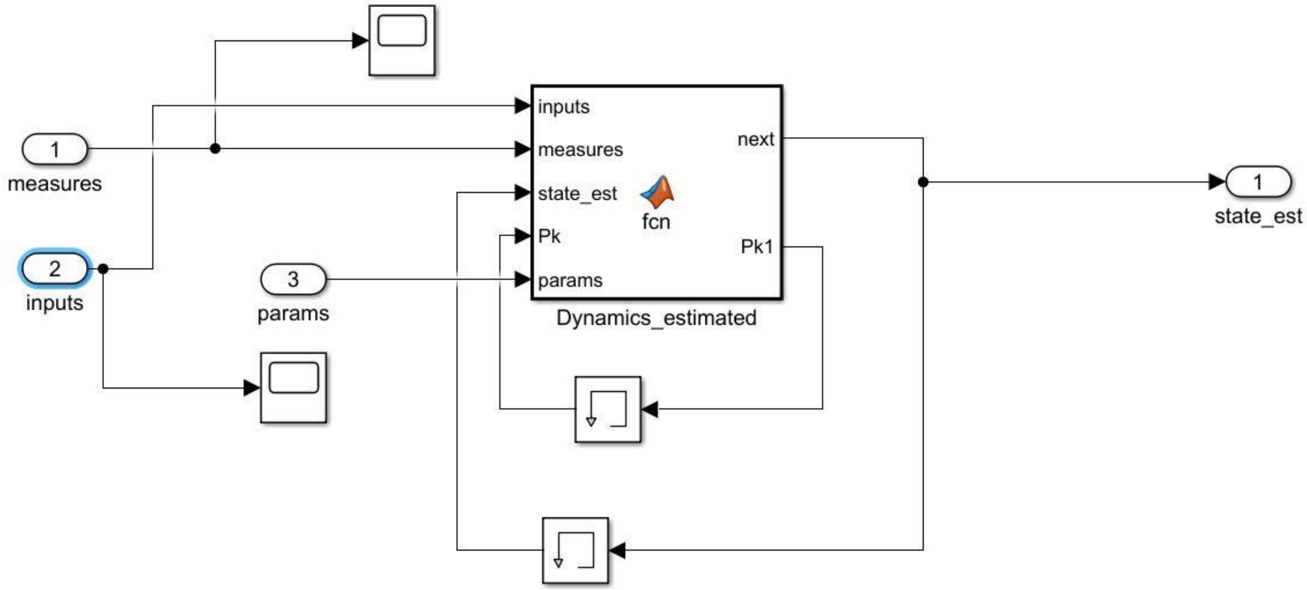
$$
V = \begin{bmatrix}
10^5 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 10^4 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 10^6 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 10^4 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 10^2 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 10^6 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1
\end{bmatrix}
$$

$$
W = \begin{bmatrix}
10^6 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 10^5 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 10^{15} & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 10^{10} & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 10^3 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 10^3 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 10^3 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 10^3
\end{bmatrix}
$$

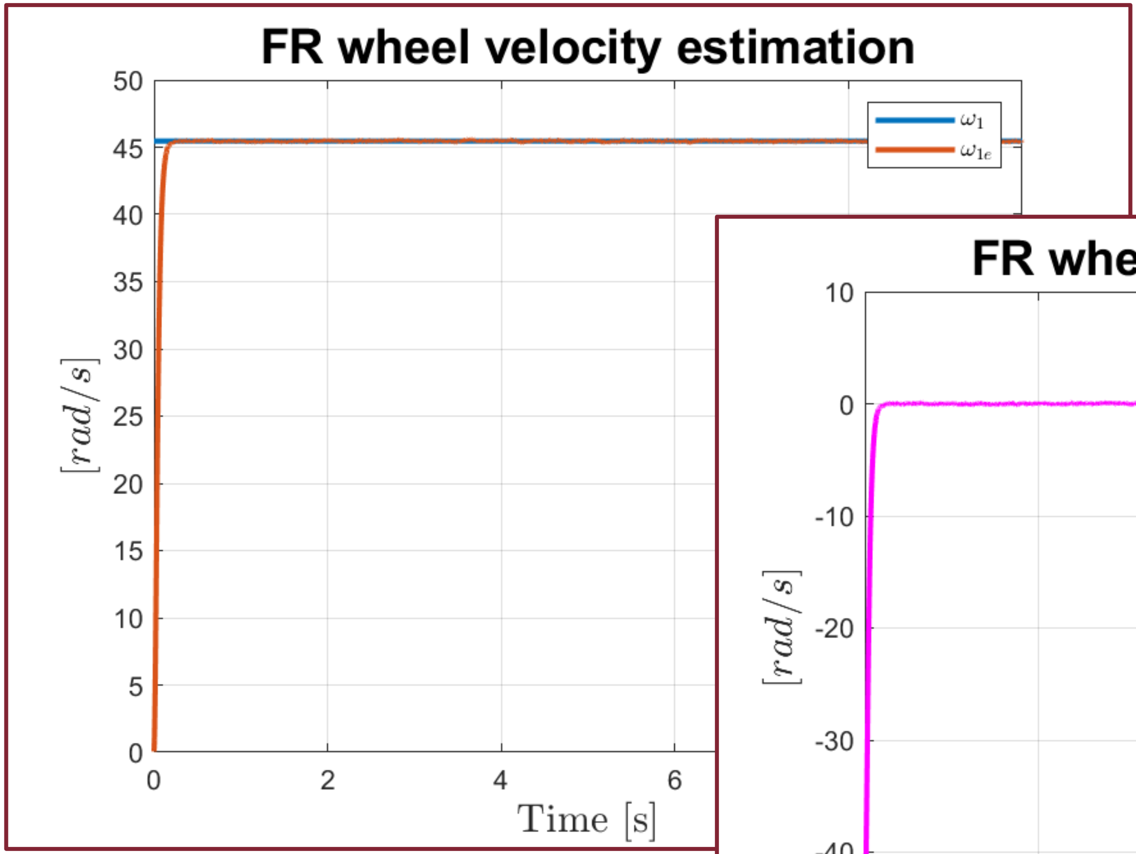
Estimation - Data acquisition

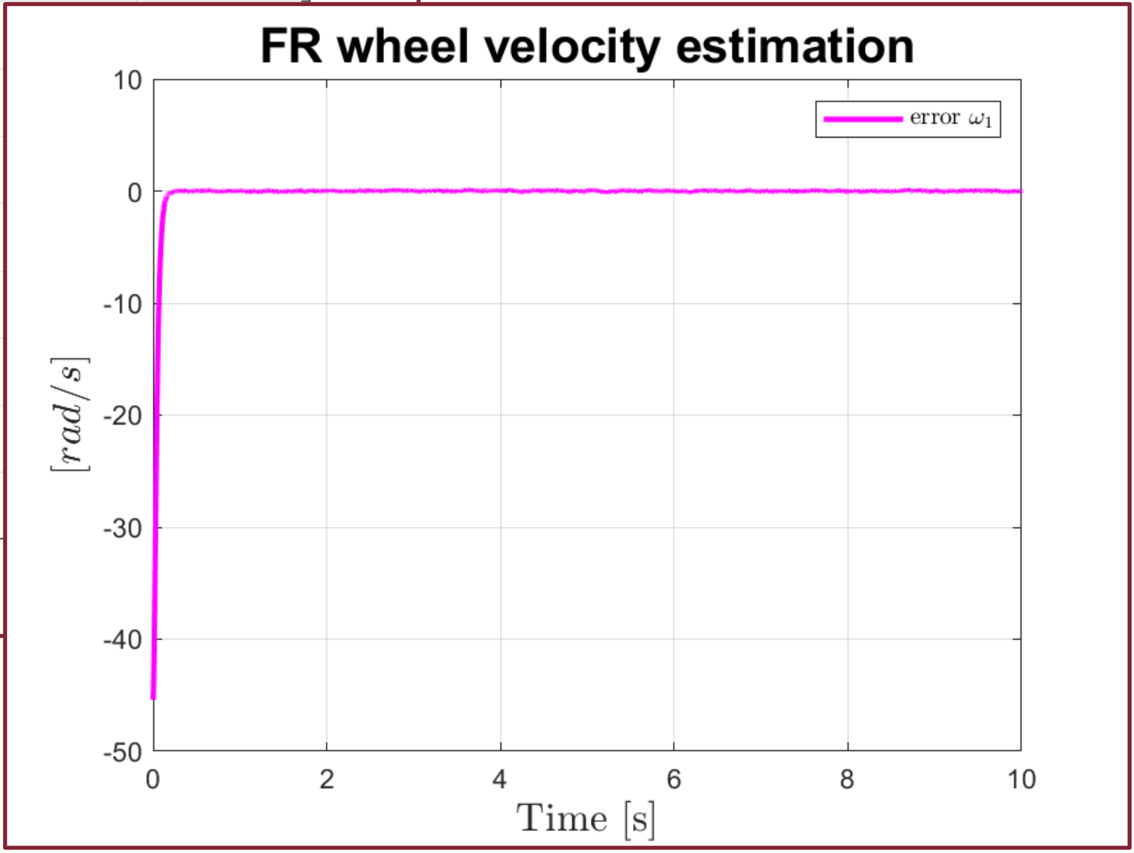


Estimation – EKF implementation

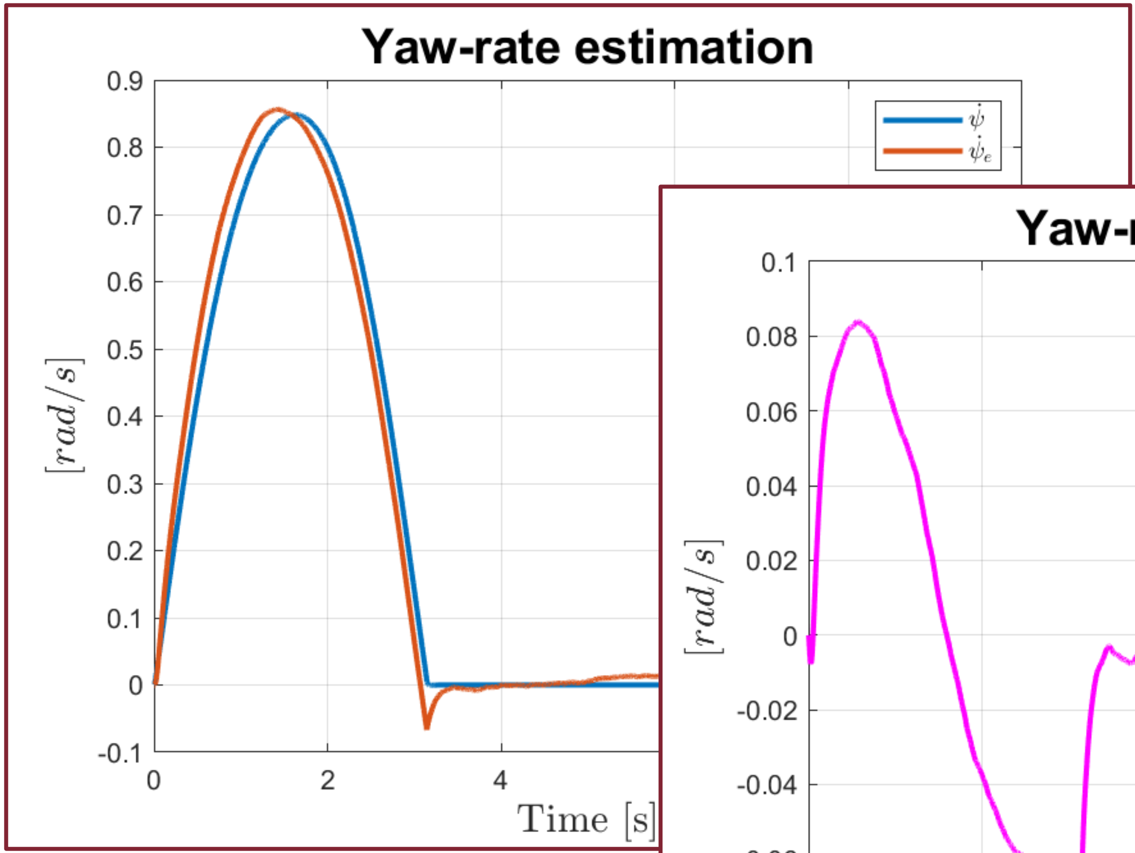


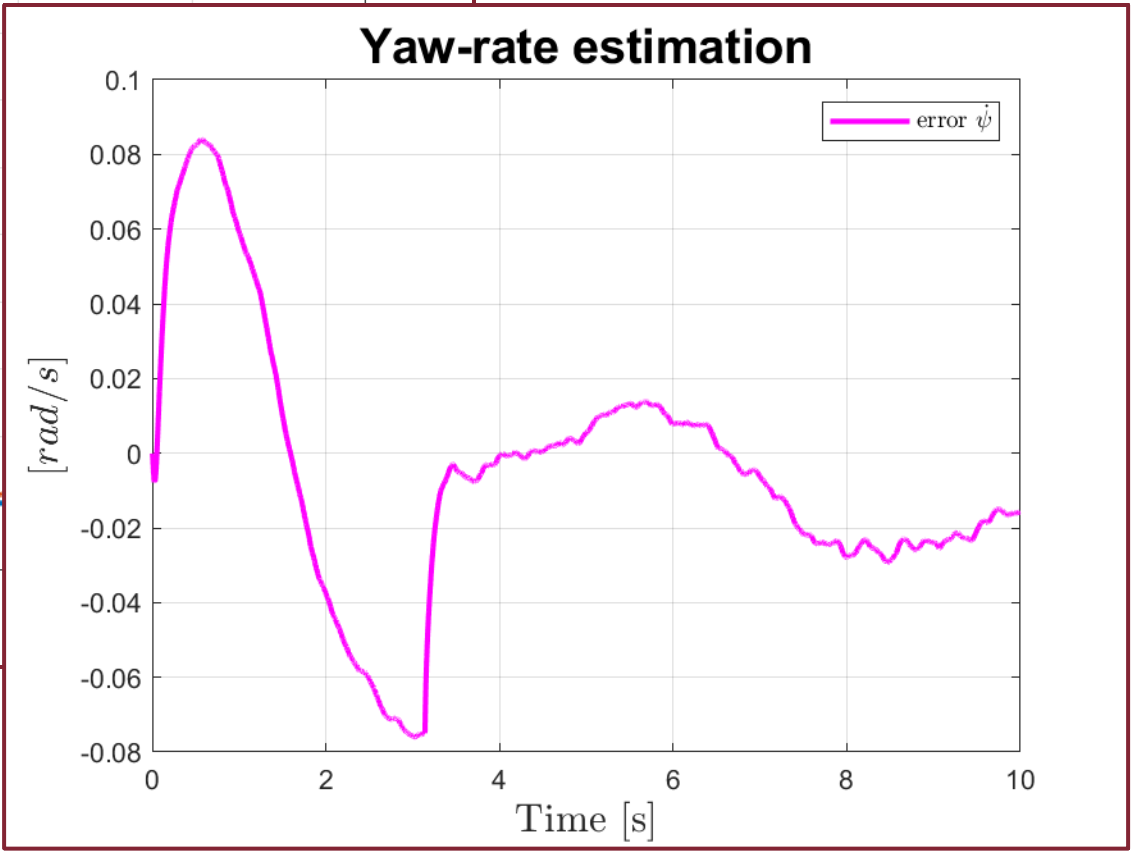
Estimation - Simulations



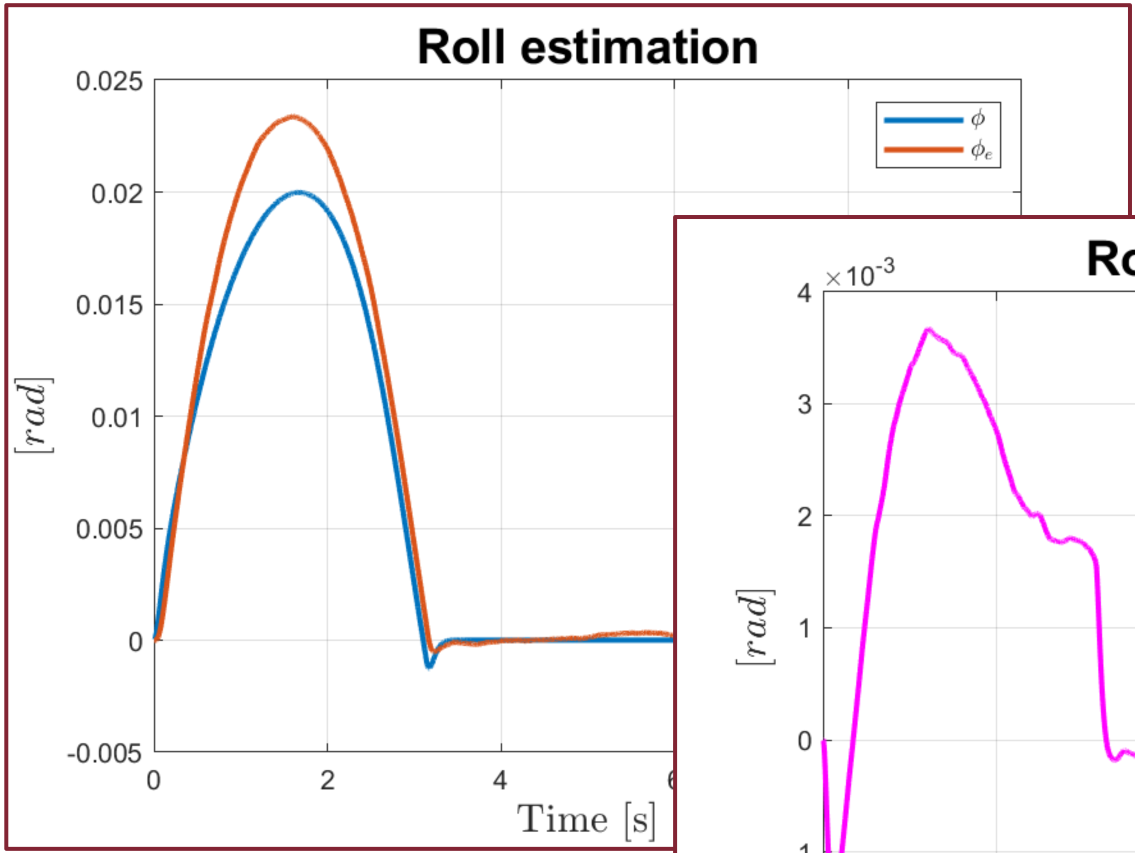


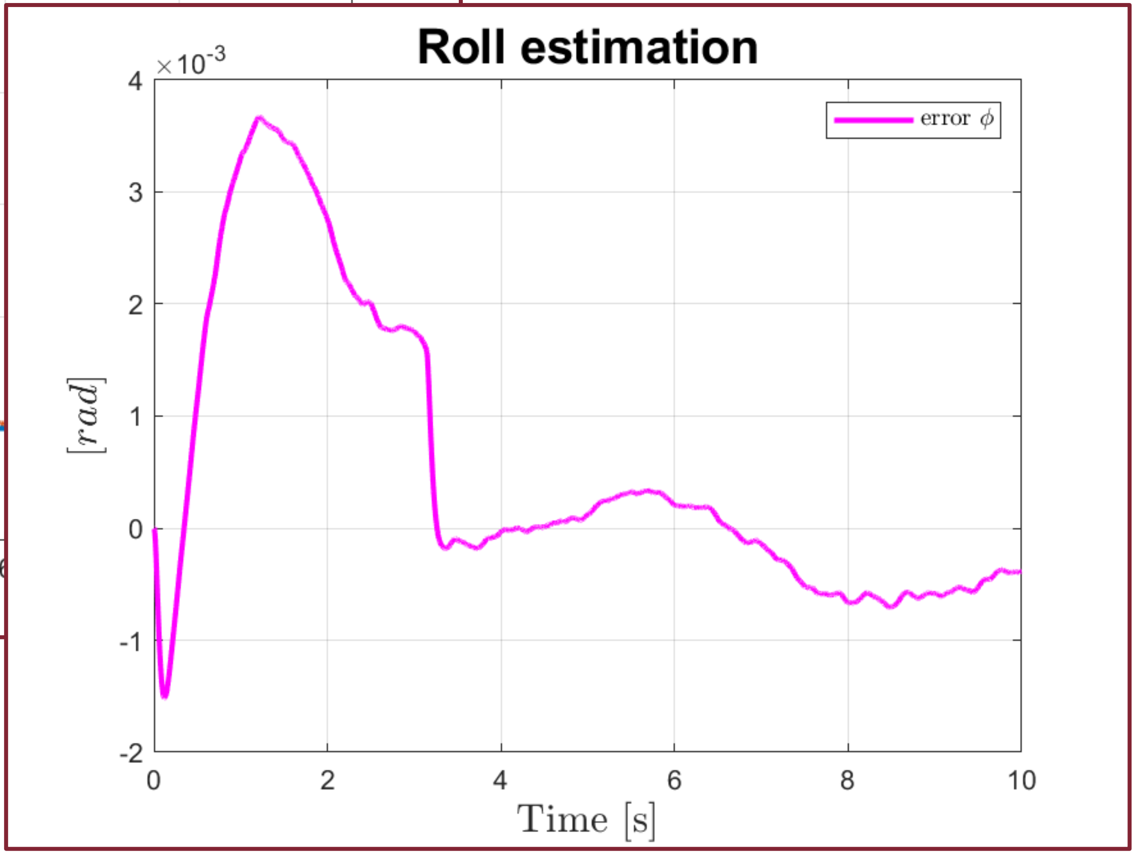
Estimation - Simulations



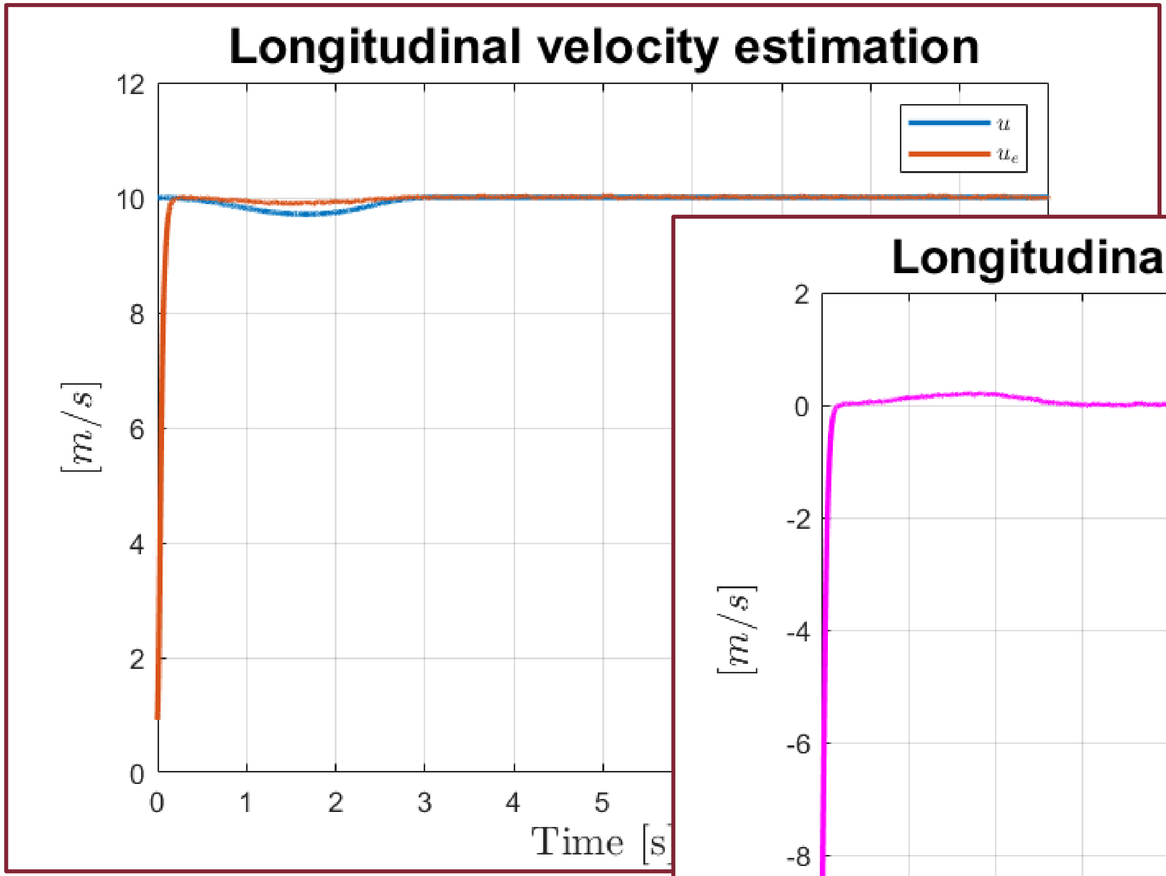


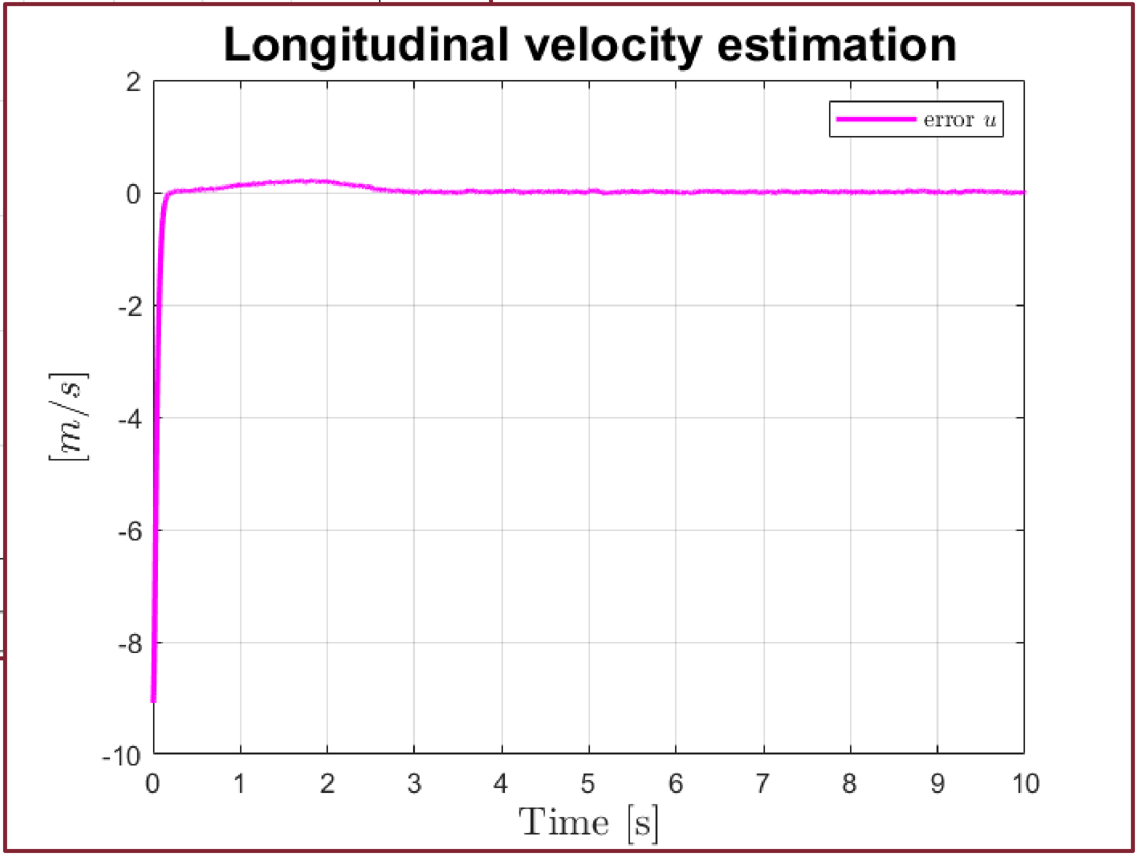
Estimation - Simulations





Estimation - Simulations





Conclusions

- Compared performance between active and passive anti roll bar.
- Presented CAD model for the designed system of 2 active anti roll bars on the Gajarda Formula SAE race car.
- Modeled the problem from a control point of view defining suitable approach to enhance performances.
- Applied Loop Shaping techniques on the electric motor and LQR for the optimal control action to minimize the body roll behaviour.
- Estimated the reduced state by mean of Extended Kalman Filter for feedback actions.

Further developements

- 14 DoF car model considering the Pitch and 4 suspension dynamics

- Delete flat surface assumption

- Introduce noise in the model and use Linear Quadratic Gaussian regulator

- Data fusion from the IMU

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MBOSTA-Racing-Team