Humanoid Robot Control using Orbital Energy of the Variable Height Inverted Pendulum

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Prof. G. Oriolo
Prof. L. Lanari
Supervisor: N. Scianca

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Leonardo Salustri (M1762556)
Paolo Sebeto (M1894763)
Francesco Scotti (M1758391)

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Introduction

- LIP is the simplest model for bipedal walking: $$\ddot{x} = \frac{g}{z_0} x$$

- All the mass of the robot is considered to be concentrated at its Center of Mass (CoM) with massless telescopic leg.

- It's not possible generate CoM height variations, as the hypothesis of constant CoM height is required to keep the model linear.

How to plan a gait introducing a variable CoM height trajectory ?

⬇

Variable height Inverted Pendulum, with Orbital Energy Control

Orbital Energy

- It is a conservative quantity.
- Can be seen as the sum of the kinetic energy and the potential one of a mass spring system with a negative elastic term.

$$\frac{1}{2}\dot{x}^2 - \frac{g}{2z_0}x^2 = E$$

- LIP is a restricting model, since it requires a constant height.
- A conserved orbital energy exists for any C^2 height trajectory f.
- The idea is to generate a trajectory for the CoM through the energy level of the system and consider a variable height profile for the CoM.

Variable Height Inverted Pendulum

- Consider a 2D motion in the xz-plane. $\boxed{m\ddot{\mathbf{q}} = m\mathbf{g} + \mathbf{f}_{gr}}$ with $\mathbf{q} = [x \;\; z]^T$

- It is assumed that the ZMP is on the foot contact point, always at the origin of the reference frame.

- We want the *ground reaction force* to be able to mantain the virtual holonomic constraint $z = f(x)$.

- This constraint is treated as a manifold in the system's 4-dimensional state space defined by: $\sigma = z - f(x) = 0$

$$\dot{\sigma} = \dot{z} - f'(x)\dot{x}$$
$$\ddot{\sigma} = \ddot{z} - f'(x)\ddot{x} - f''(x)\dot{x}^2$$

$$\Rightarrow$$

$$f_{gr} = m\sqrt{x^2 + z^2}\frac{g + f''(x)\dot{x}^2}{z - f'(x)x}$$

Variable Height Inverted Pendulum

- Parametrizing the ***ground reaction force*** as $\mathbf{f}_{gr} = m\mathbf{q}u$ where:

$$u = \frac{g + f''(x)\dot{x}^2}{z - f'(x)x}$$

- The unconstrained d.o.f. x and the complete dynamics are respectively:

$$\ddot{x} = ux, \qquad \ddot{\mathbf{q}} = \mathbf{g} + \mathbf{q}u \qquad \text{with } \mathbf{q} = [\mathrm{x} \;\; \mathrm{z}]^{\mathrm{T}}$$

- This single control input ($u \geqq 0$) is used to control both horizontal and vertical CoM position.

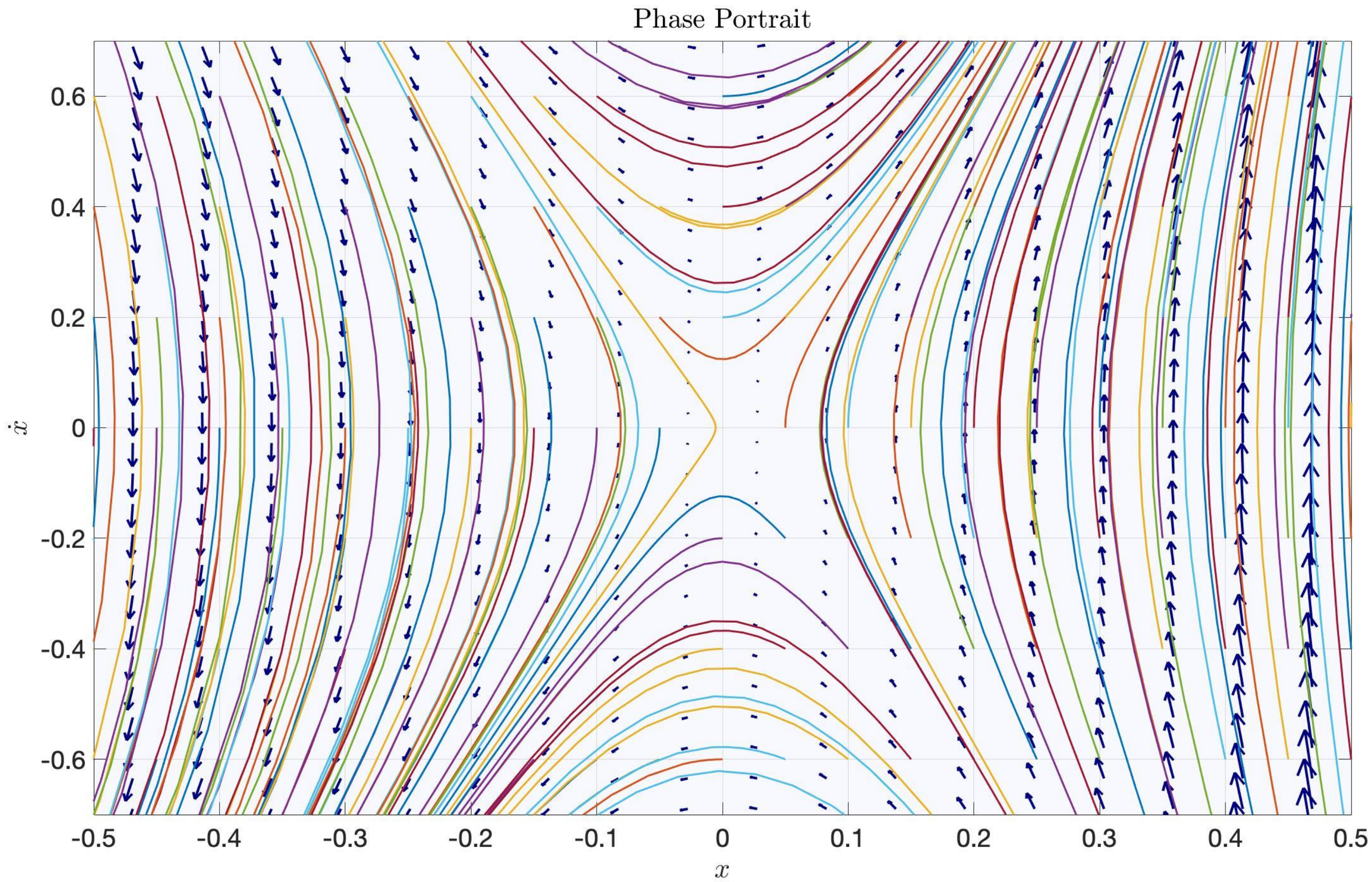
Conditions for Balance

- With balance, we refer to asymptotic convergence to a fixed point of the dynamics at a specified desired final height $z_f > 0$.

$$\dot{x} = \dot{z} = 0, \qquad x = 0$$
$$zu = g, \qquad z = z_f > 0$$

- Reaching balance practically means to stop the walking motion of the robot within a step, at a desired CoM height.

- Starting from any state $\mathbf{x} = (x, z, \dot{x}, \dot{z})$ convergence to a fixed point requires satisfying the condition $x\dot{x} < 0$


Phase Portrait
0.6
0.4
0.2
0
-0.2
-0.4
-0.6
$\dot{x}$
-0.5
-0.4
-0.3
-0.2
-0.1
0
0.1
0.2
0.3
0.4
0.5
x

Derivation of height trajectory

- First of all, given an initial state $\mathbf{x_0} = (x_0, z_0, \dot{x}_0, \dot{z}_0)$ we enforce the two initial conditions: $f(x_0) = z_0$ and $f'(x_0) = \frac{\dot{z}_0}{\dot{x}_0}$

- The balance conditions requires, for the *Orbital energy* associated to *f,* to satisfy:

$$E_f(x_0, \dot{x}_0) = E_f(0,0) = 0$$

$$E_f(x, \dot{x}) = \frac{1}{2}\dot{x}^2 \bar{f}^2 + gx^2 f(x) - 3g \int_0^x f(\xi)\xi d\xi$$

with: $\bar{f}(x) = f(x) - f'(x)x$

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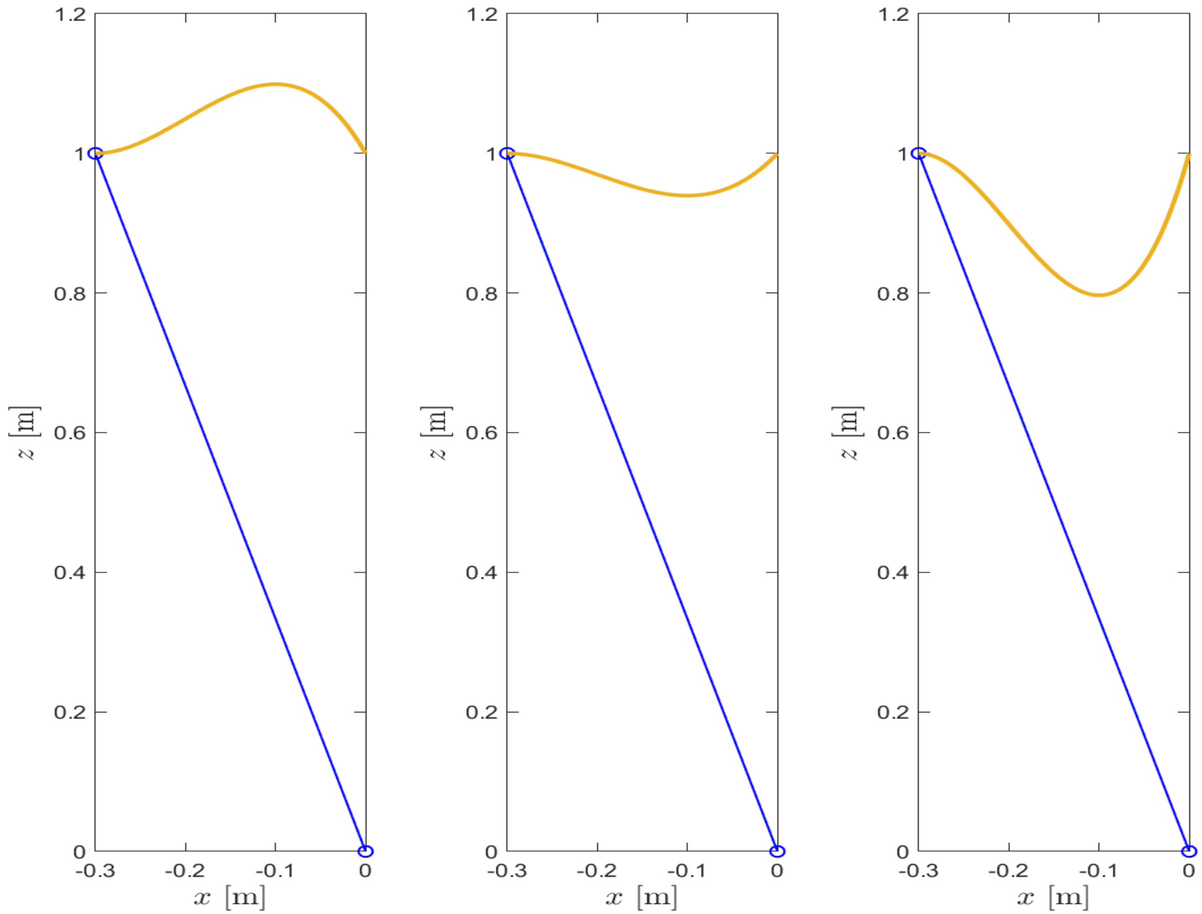
Cubic Polynomial *f(x)*

- Restricting the class of trajectories to polynomials, the integral in E_{des} is computable in closed form as:

$$3g\sum_{i=0}^{n}\frac{1}{i+2}c_i x_0^{i+2}=k$$

- Final desired height $f(0)=z_f$



The Controller u

- The general expression for the control *u* is: $$u = \frac{g + f''(x)\dot{x}^2}{z - f'(x)x}$$

- Given $f(x)$ as polynomial function and x, we can derive *u* in closed form as a rational function of x and x_0.

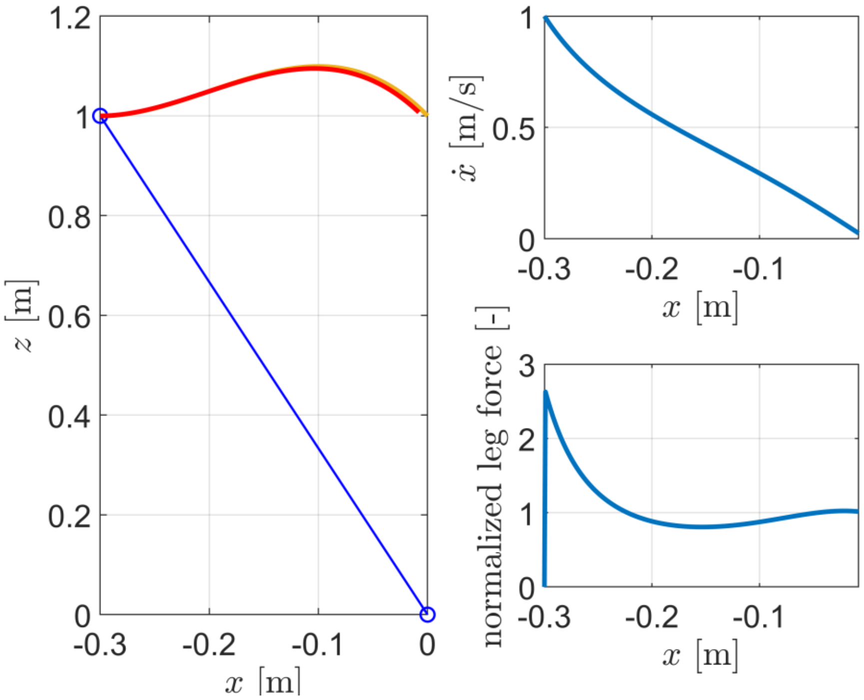
$$u = U\left(x, \mathbf{x}_0\right) = \frac{p\left(x, \mathbf{x}_0\right)}{q\left(x, \mathbf{x}_0\right)}$$

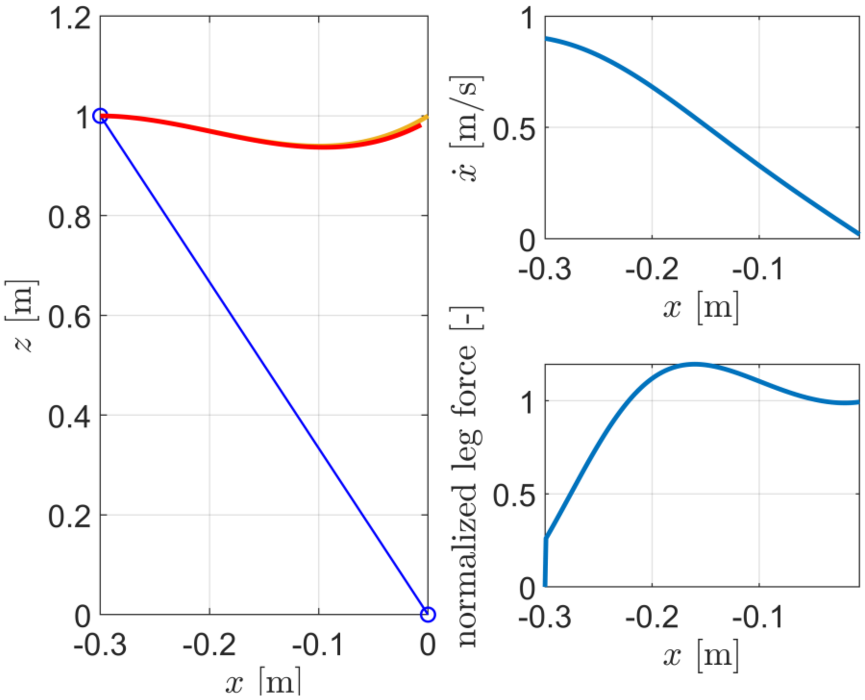
$$\Downarrow$$

$$u = U(x, \mathbf{x}) = -7a^2 + \frac{3z_f a^3 - ga}{b} - \frac{10a^3 b}{g}$$

with: $$a = \frac{\dot{x}}{x}$$ $$b = \dot{z} - az$$

Step Simulation with the Controller u

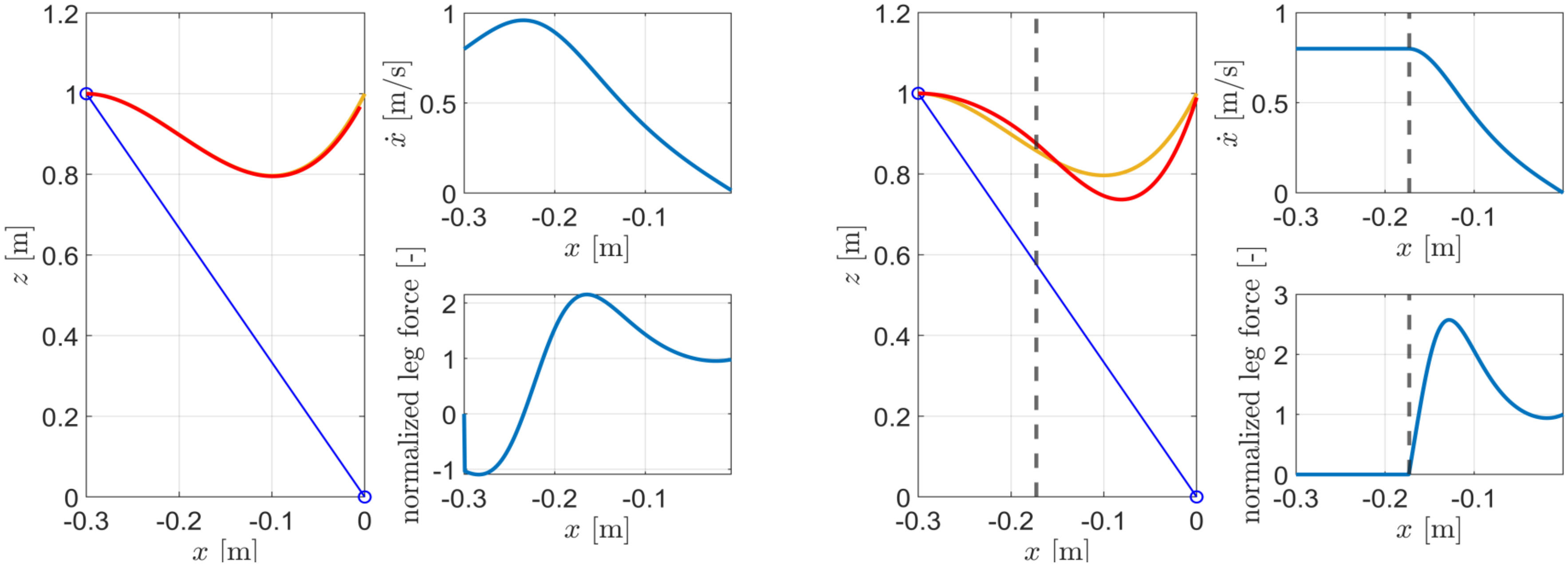




Clipped Variant Control

- The orbital energy controller u can not pull on the ground.

$$u = max(U(x, \mathbf{x}), 0)$$



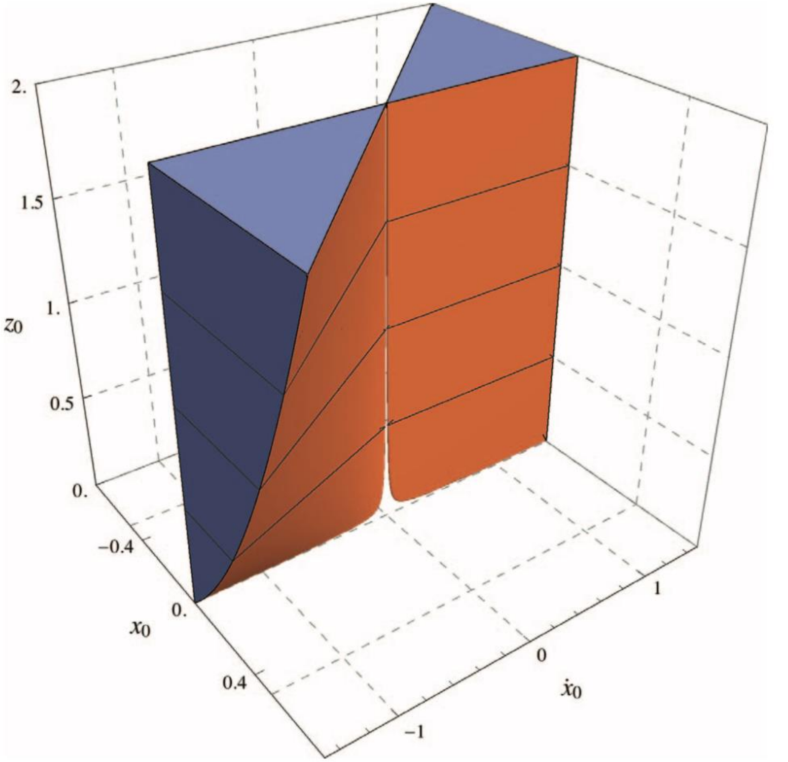
Region of attraction

$$\boxed{x\dot{x} < 0}$$

- Unilateral control $u \geqq 0$.

$$U\left(\delta x_0, \mathbf{x}_0\right) \geq 0 \quad \forall \delta \in [0, 1]$$

- Considering a *first-order formula*, i.e. an expression written combining a set of polynomial equations and inequalities (in variables y_1,...y_n) using the logical conjuction (∧), disjuction (∨), and negation (¬) operators, we obtained:



$$a_0 < 0 \wedge 7g + 20a_0b_0 + \sqrt{9g^2 + 120a_0^2 g z_f} \leq 0$$

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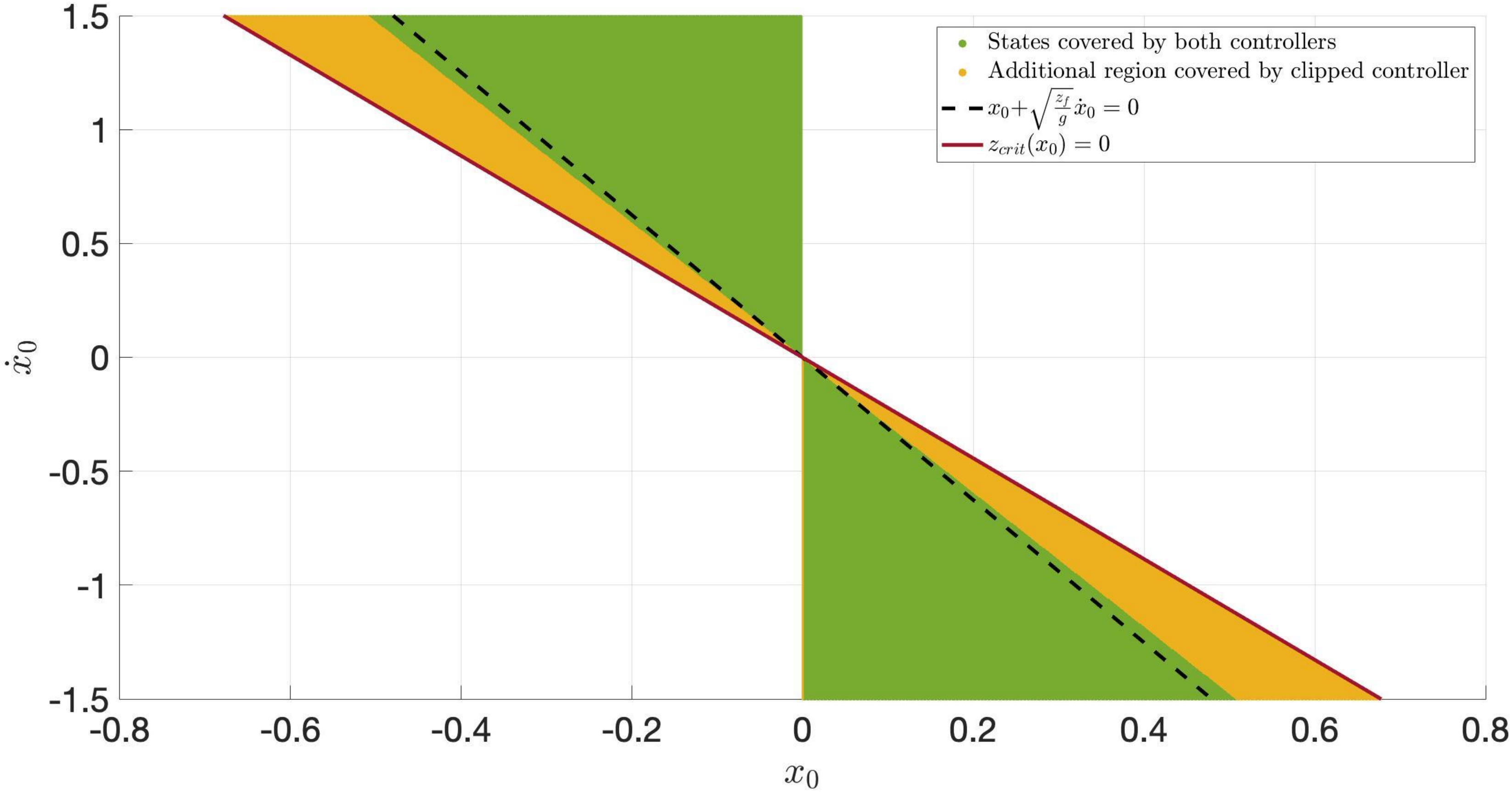
Region of attraction



Velocity Control

- Let v_{des} be the desired velocity at the top of the next stride (when $x = 0$), then:

$$E_{des} = \frac{1}{2} v_{des}^2 h^2(0)$$

$$E_{des} = \frac{1}{2} \dot{x}_0^2 h^2(x_0) + g x_0^2 f(x_0) - 3g \int_0^{x_0} f(\xi)\xi \, d\xi$$

- Equating these two expressions, and solving for x_0, we get the location to step in order to achieve the desired v_{des}

- It is important to have a smooth connection between consecutive functions $f(x)$ at each step, so:

$$f_{old}(x_{1old}) = f_{new}(x_{0new})$$
$$f'_{old}(x_{1old}) = f'_{new}(x_{0new}) \quad , \quad \dot{x}(t_{1old}) = \dot{x}(t_{0new})$$

Simulation settings

- The support leg is actuated to control the body's height as a function of the horizontal distance from the foot to the body mass.

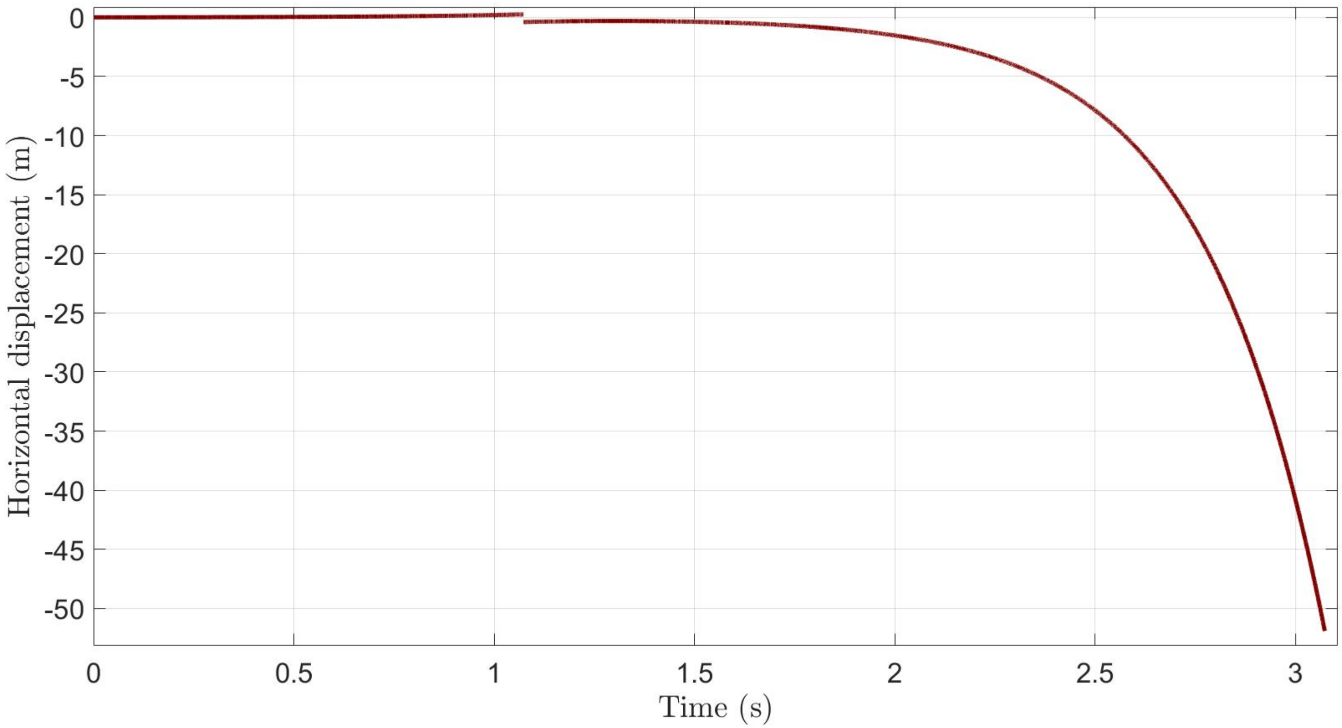
- PD control plus a feed-forward computed torque command:

$$f_{knee} = k_z(f(x) - z) + b_z(f^{'}(x)\dot{x} - \dot{z}) + m\sqrt{x^2 + z^2}\frac{g + f^{''}(x)\dot{x}^2}{z - f^{'}(x)x}$$

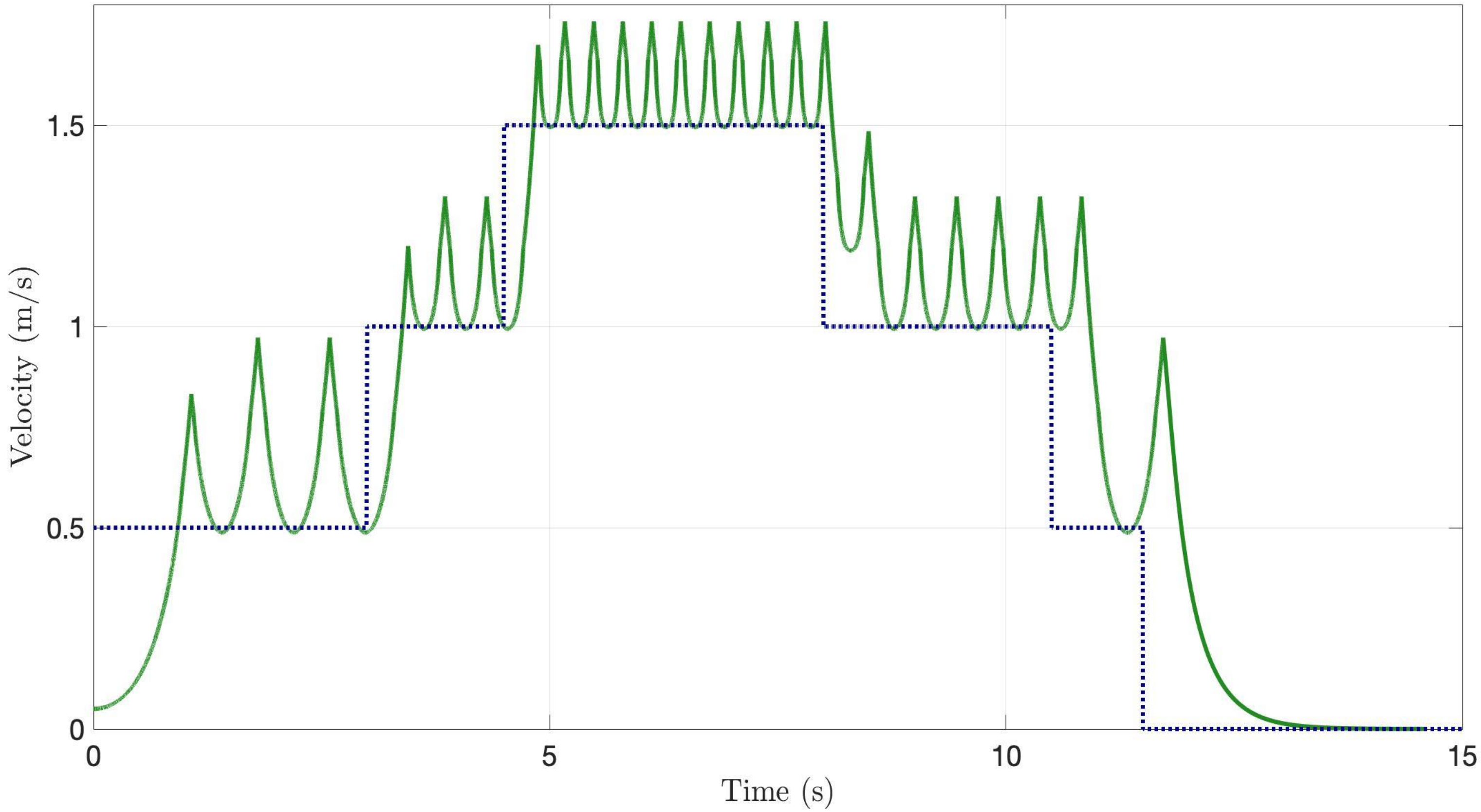
- We have used a symmetric polynomial $f(x) = a_0 + a_2x^2 + a_4x^4$, with $f(x) = z^*$ if $|x| \geqq x^*$ with $x^* = \sqrt{\frac{-a2}{a4}}$ corresponding to the point of zero slope of the polynomial. ($a_0 = 0.9375,\ a_2 =$ -2, $a_4 = 30.864$)

Multiple choices for x_0

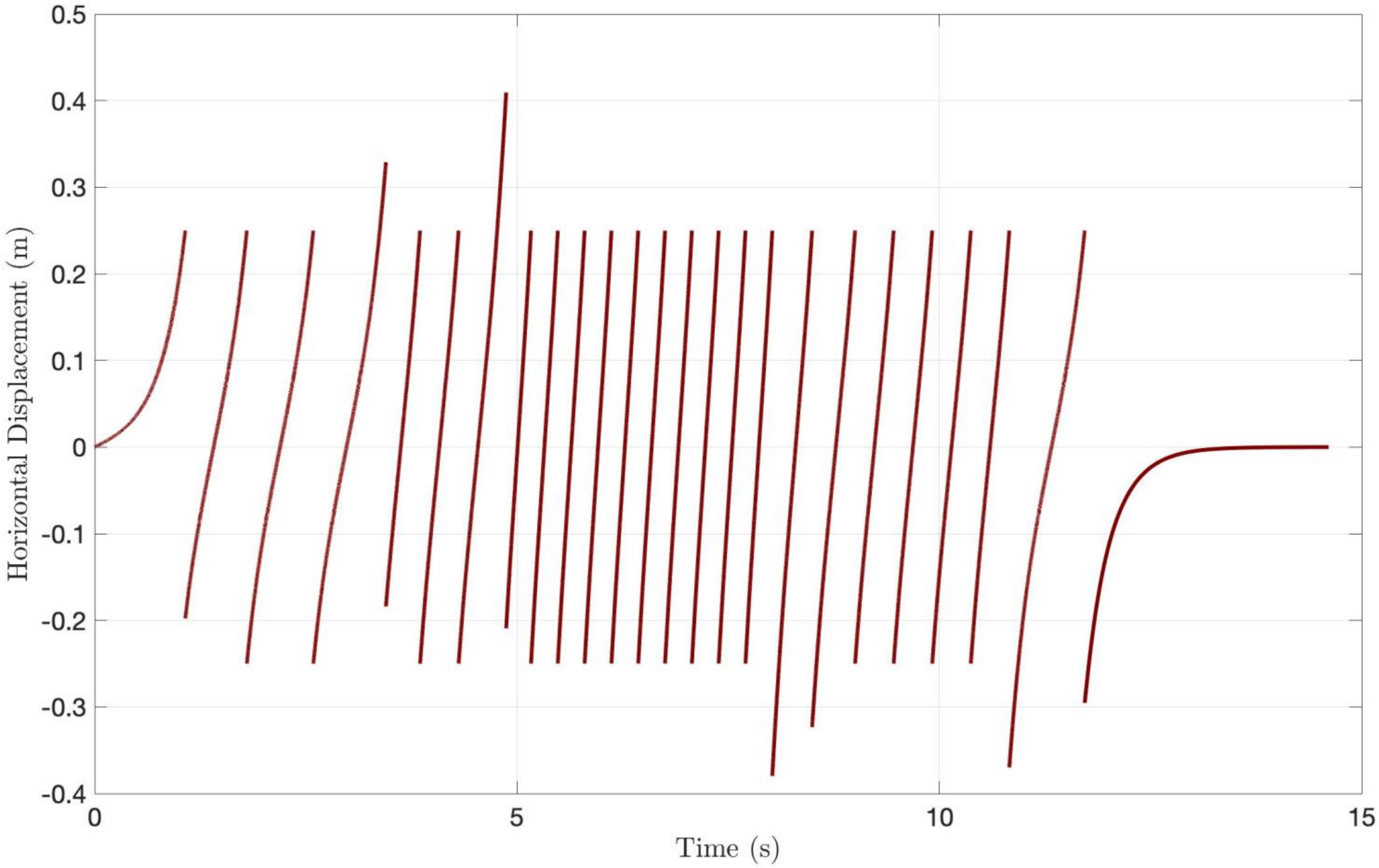
- Since the previous equation relative to E_{des} has multiple solutions w.r.t. x_0 if we choose a step sufficiently large considering a starting $\dot{x}_0$, the robot can't do it. So it falls down!



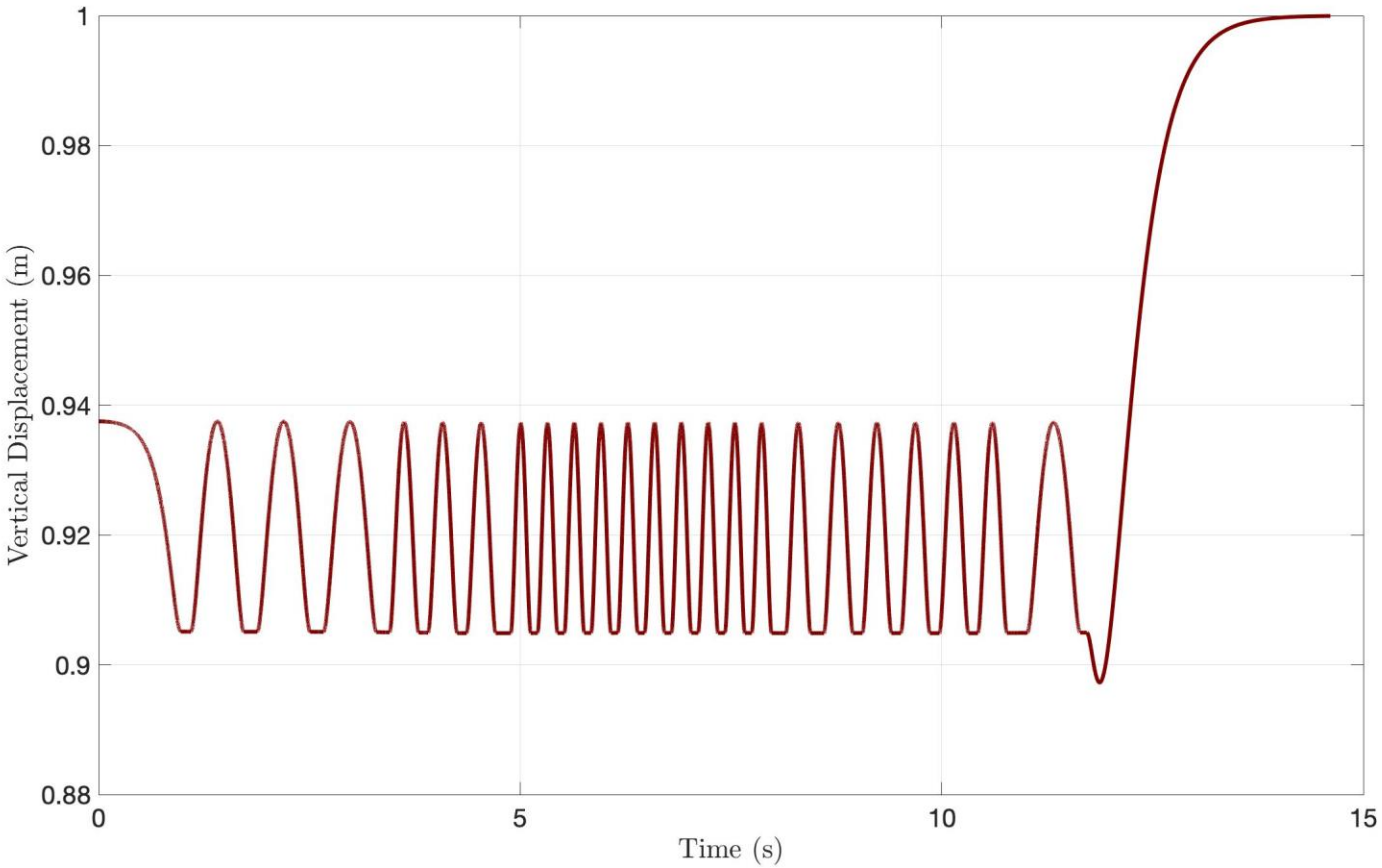
Simulation with variable v_{des}



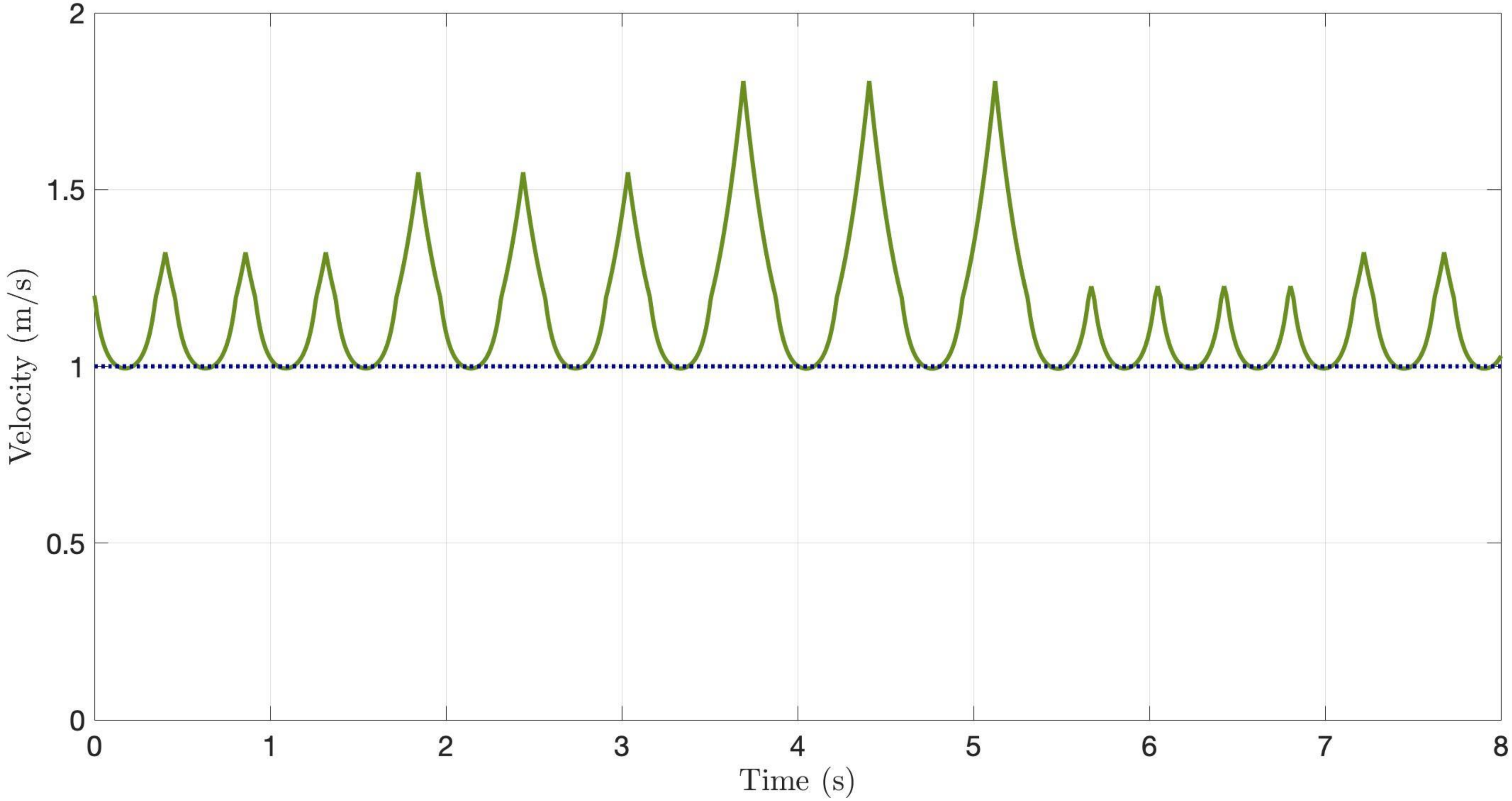
Simulation with variable v_{des}



Simulation with variable v_{des}



Simulation with variable step size



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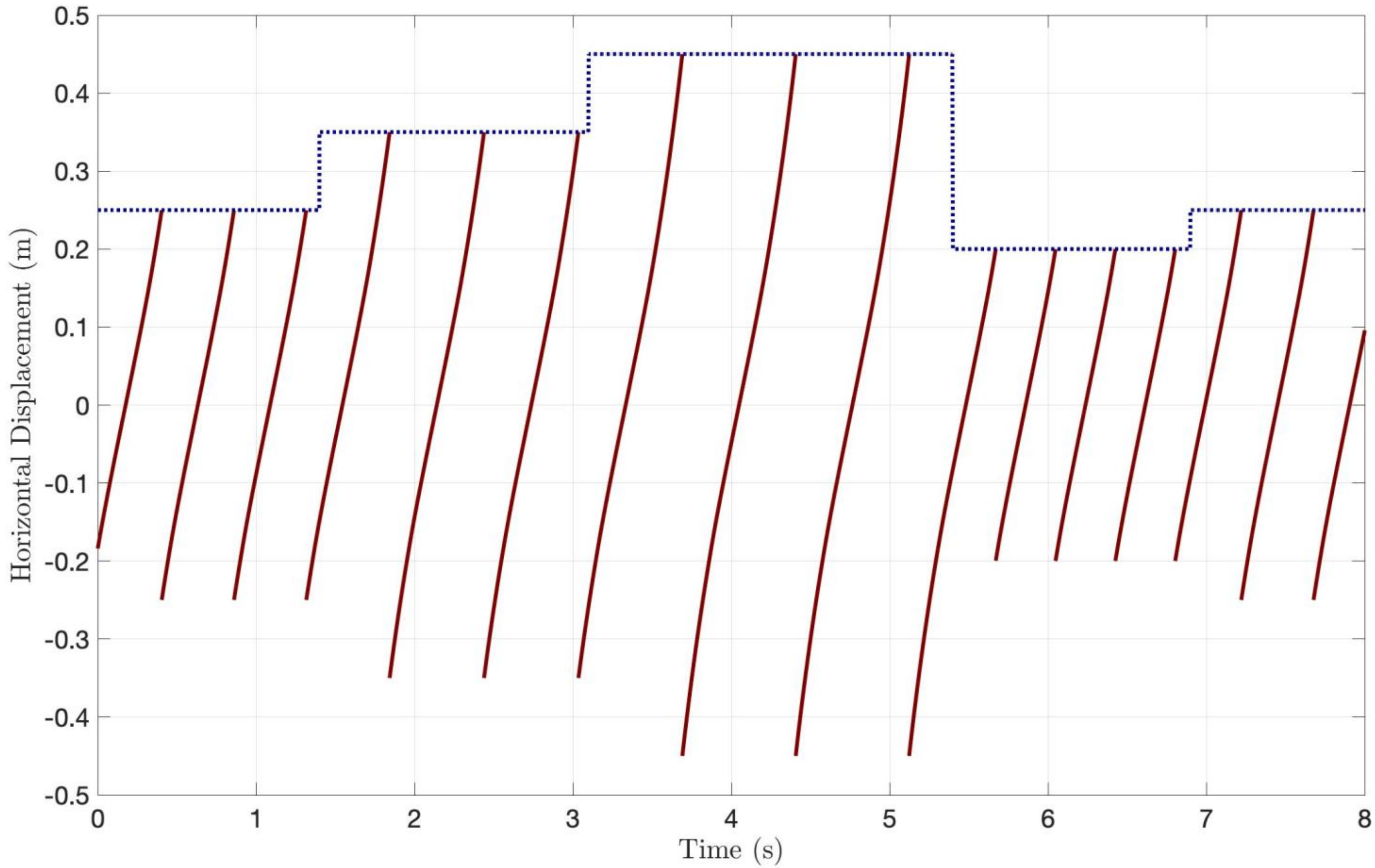
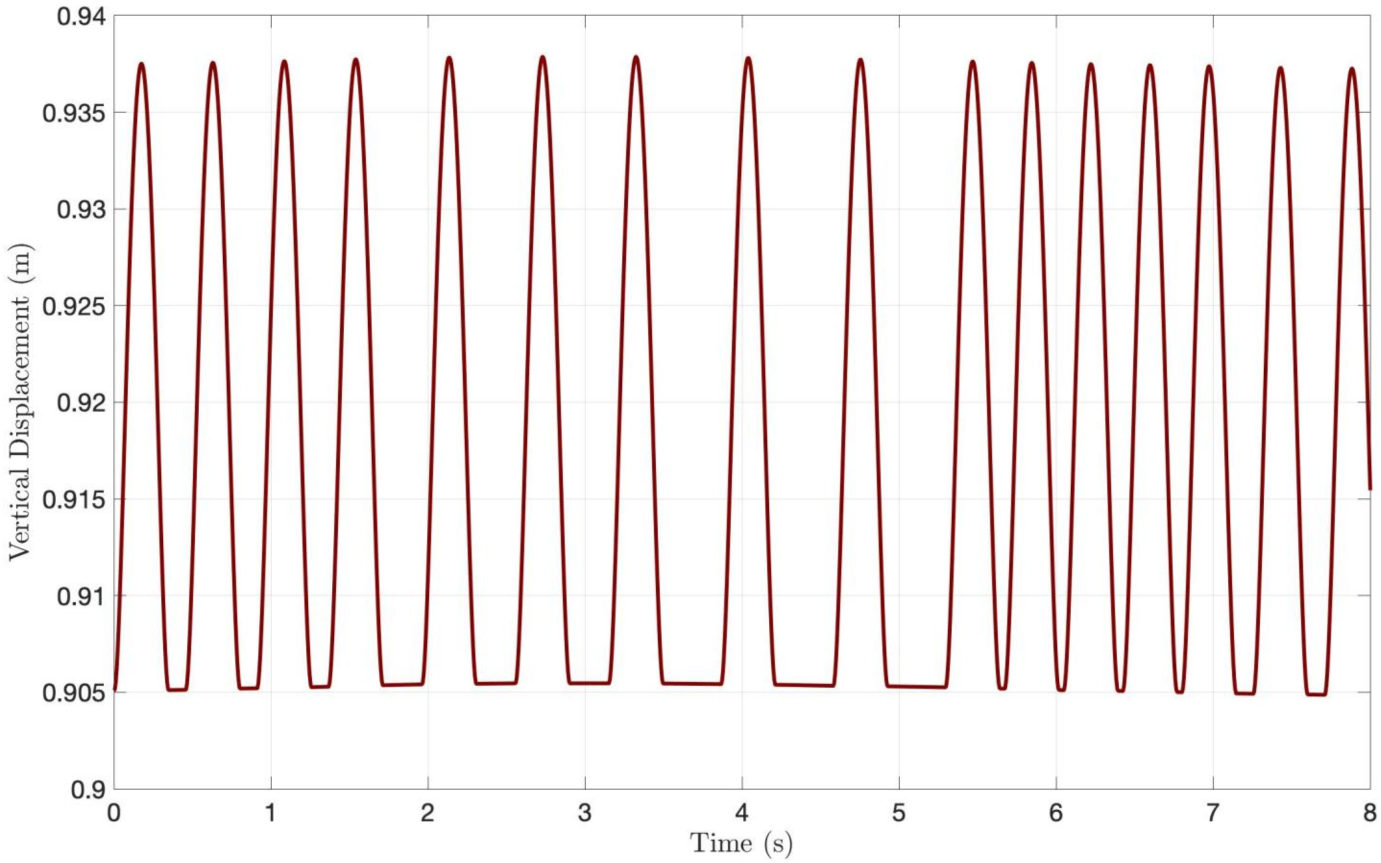
Simulation with variable step size



Simulation with variable step size



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MPC for variable CoM height

- The key idea is to control the robot through the definition of two relevant trajectories.
- The goal is to develop a trajectory for the zmp which guarantees a balance condition.
- It is imposed a virtual vertical dynamic such that the resultant dynamic is a 3D LIP.

$$
\begin{aligned}
\ddot{x}_c &= \omega^2(x_c - x_z) \\
\ddot{y}_c &= \omega^2(y_c - y_z) \\
\ddot{z}_c &= \omega^2(z_c - z_z) - g
\end{aligned}
$$

MPC for variable CoM height

- ZMP constraints:

$$-\frac{1}{2}\begin{bmatrix} d_x^z \\ d_y^z \\ d_z^z \end{bmatrix} \leq R_{k+i}^T \begin{bmatrix} x_z^{k+i} - x_f^{k+i} \\ y_z^{k+i} - y_f^{k+i} \\ z_z^{k+i} - z_f^{k+i} \end{bmatrix} \leq \frac{1}{2}\begin{bmatrix} d_x^z \\ d_y^z \\ d_z^z \end{bmatrix}$$

- Stability constraints considering anticipative tails:

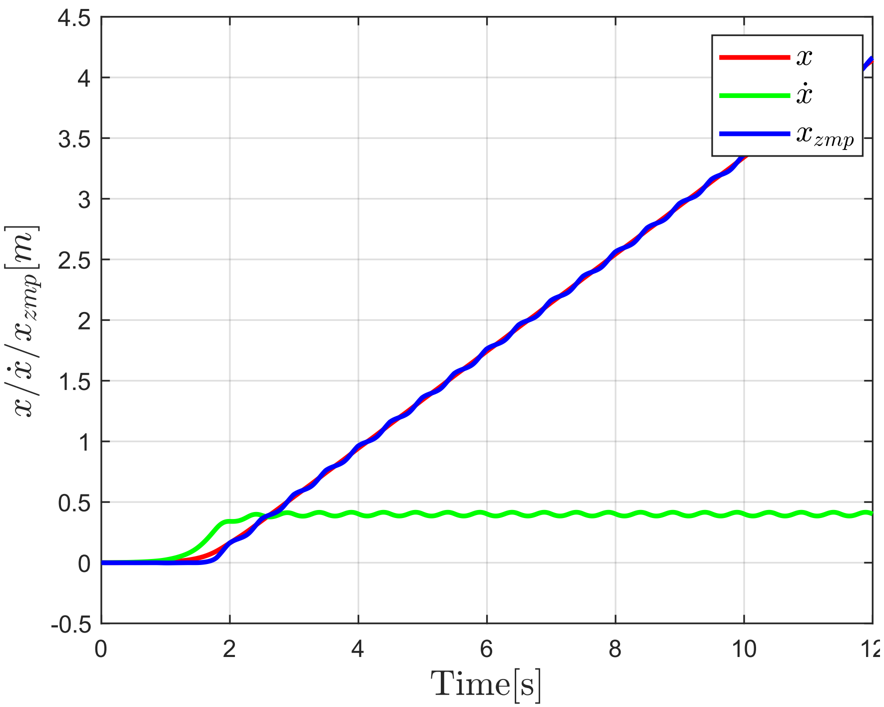
$$\frac{1-e^{-\delta\omega}}{\omega}\sum_{i=0}^{N-1} e^{-i\delta\omega}\dot{x}_z^{k+i} = x_c^k + \frac{\dot{x}_c^k}{\omega} - x^k$$

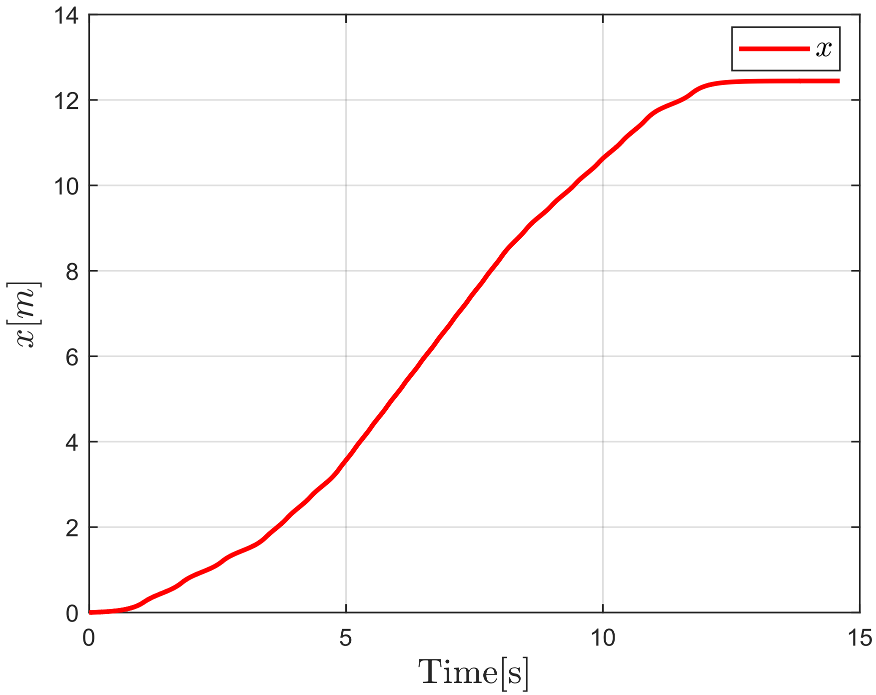
$$\frac{1-e^{-\delta\omega}}{\omega}\sum_{i=0}^{N-1} e^{-i\delta\omega}\dot{y}_z^{k+i} = y_c^k + \frac{\dot{y}_c^k}{\omega} - y^k$$

$$\frac{1-e^{-\delta\omega}}{\omega}\sum_{i=0}^{N-1} e^{-i\delta\omega}\dot{z}_z^{k+i} = z_c^k + \frac{\dot{z}_c^k}{\omega} - z^k - \frac{g}{\omega^2}$$

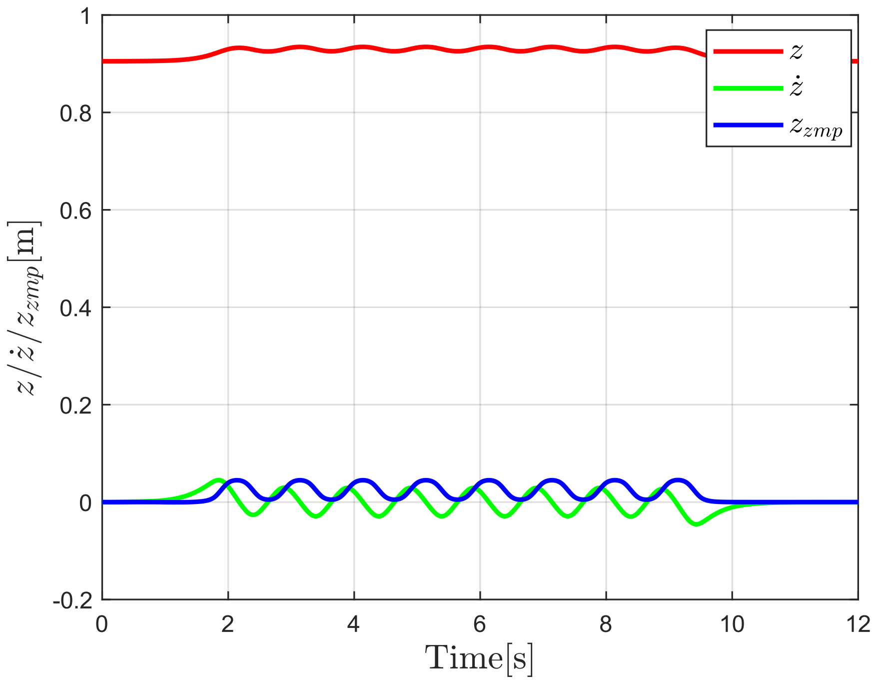
$$\boxed{\min_{\dot{X}_z^k,\dot{Y}_z^k,\dot{Z}_z^k} \sum_{i=1}^{N} \left((\dot{x}_z^{k+i})^2 + (\dot{y}_z^{k+i})^2 + (\dot{z}_z^{k+i})^2 + \beta(z_z^{k+i} - z_f^{k+i}) \right)}$$

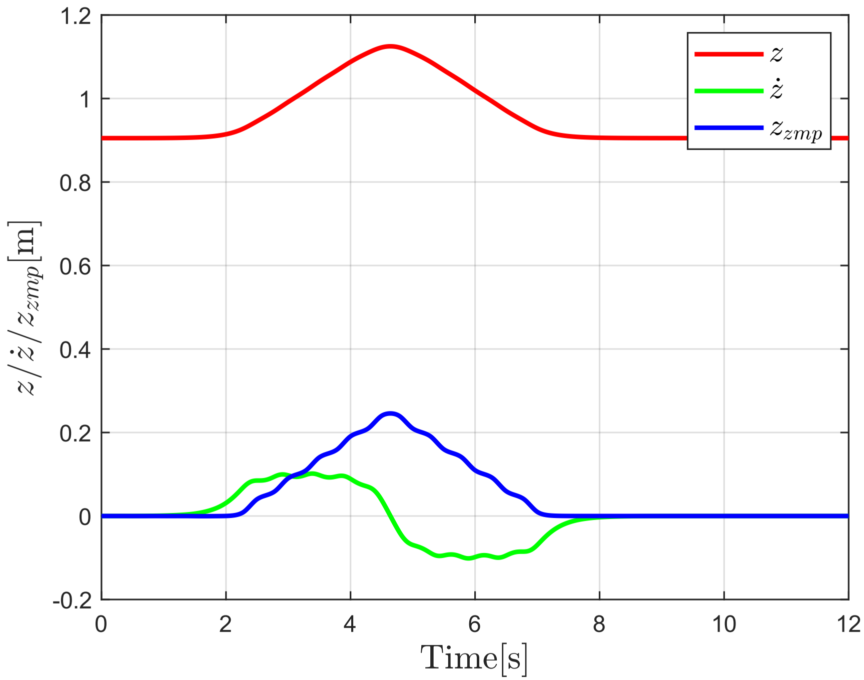
Simulation results





Simulation results





Conclusions

MPC

- Computationally more intensive
- Stability issues tackled directly

Orbital energy

- Lack of prediction
- Limitation of the point-foot assumption
- Possibility of imposing directly the CoM trajectory

Thank you for the attention!

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