Evolution over time of the structure of social graphs

Student Leonardo Serilli
Supervisors Frédéric Giroire, Nicolas Nisse, Malgorzata Sulkowska
Year 2021/2022

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INTRODUCTION

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Scope of the project
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CONTEXT

- **Scopus** Database

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Scopus ID

Total # collaborations (until 2014)

publications years

	ID	1990	1991	1992	1993	1994	1995	1996	1997	1998	...	2014	2015	2016	2017	2018	start_year	end_year
118063	26421678500	0	0	0	0	0	0	0	0	0	...	4	4	4	4	4	2014	2014
180546	56230251900	0	0	0	0	0	0	0	0	0	...	3	3	3	3	3	2013	2014
68772	7801413223	0	0	0	0	0	0	0	0	0	...	3	3	3	3	3	2011	2011
25152	6603158006	0	0	0	0	0	0	0	0	0	...	0	0	0	0	4	2018	2018
96494	20434297300	0	0	0	0	0	0	0	0	0	...	16	16	16	16	16	2013	2013
...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...
228654	57203927130	0	0	0	1	1	1	1	1	1	...	31	32	39	47	48	1991	2018
115362	25647427000	0	0	0	0	0	0	0	0	0	...	0	0	0	0	49	2018	2018
176446	56066133100	0	0	0	0	0	0	0	0	0	...	0	0	0	3	3	2017	2017
64352	7202888402	0	0	0	0	0	0	0	0	0	...	3	3	3	3	3	2006	2006
101801	23099287300	0	0	0	0	0	0	0	0	0	...	12	17	17	17	22	2009	2018

232838 rows × 35 columns

SCOPE OF THE PROJECT

- Build the **collaboration network G=(V,E)** **| V = {authors}, E = {collaborations}.**

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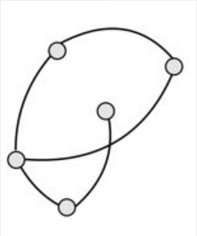
G_{1990}



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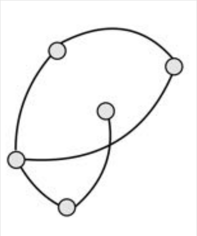

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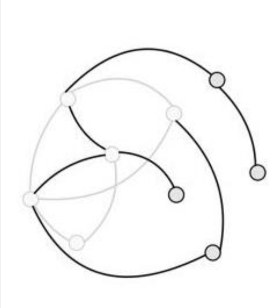

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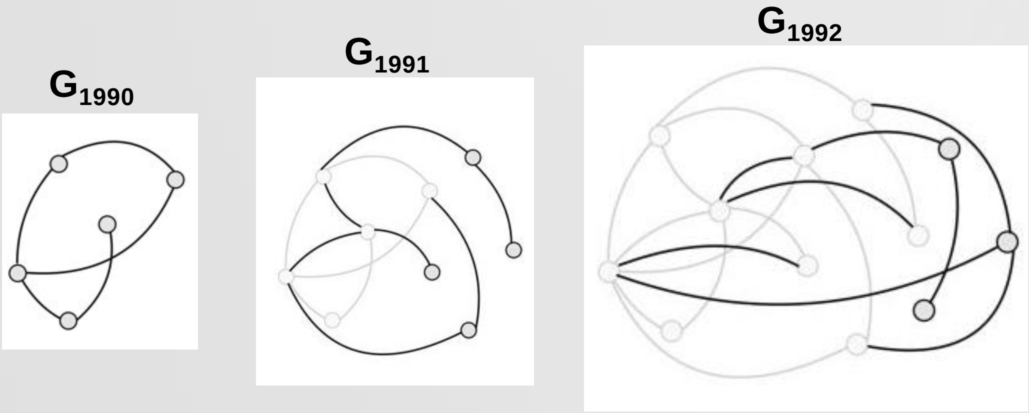

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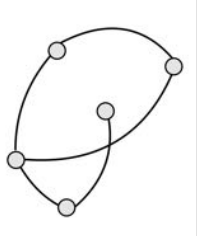

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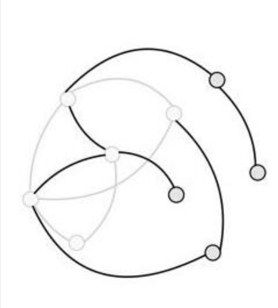

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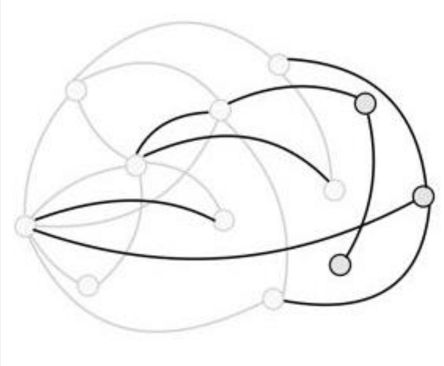

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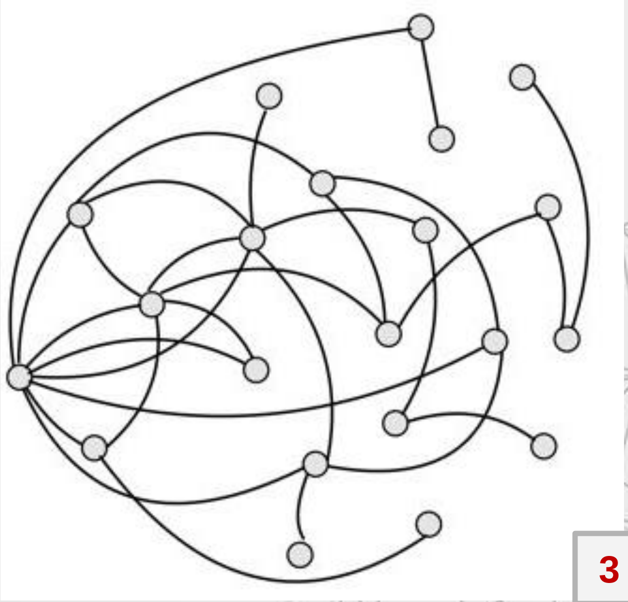

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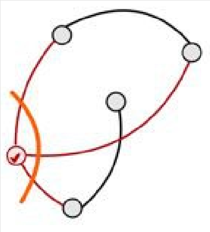

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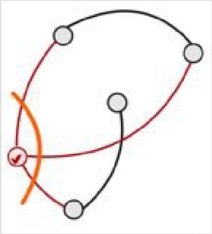

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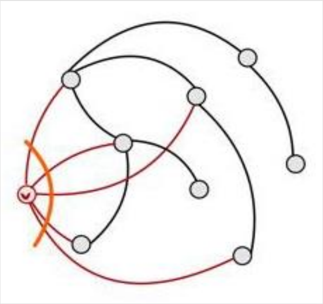

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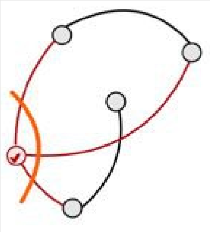

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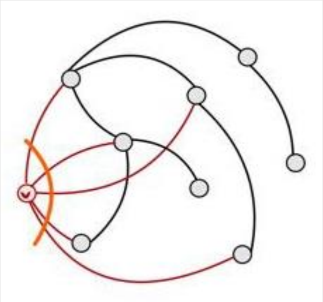

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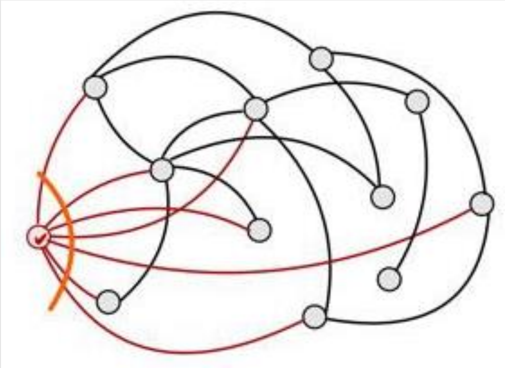

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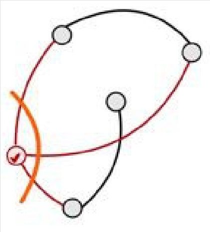

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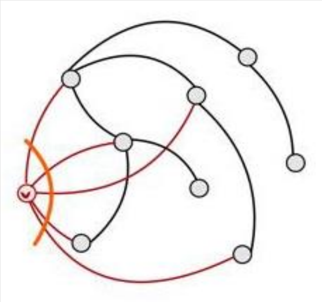

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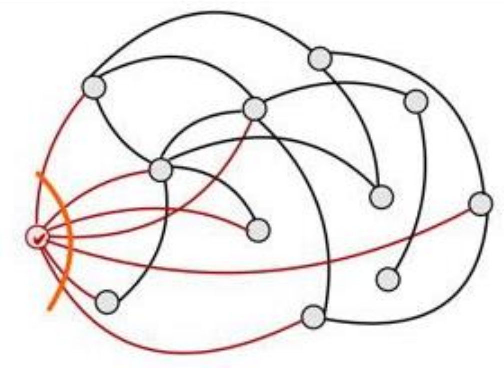
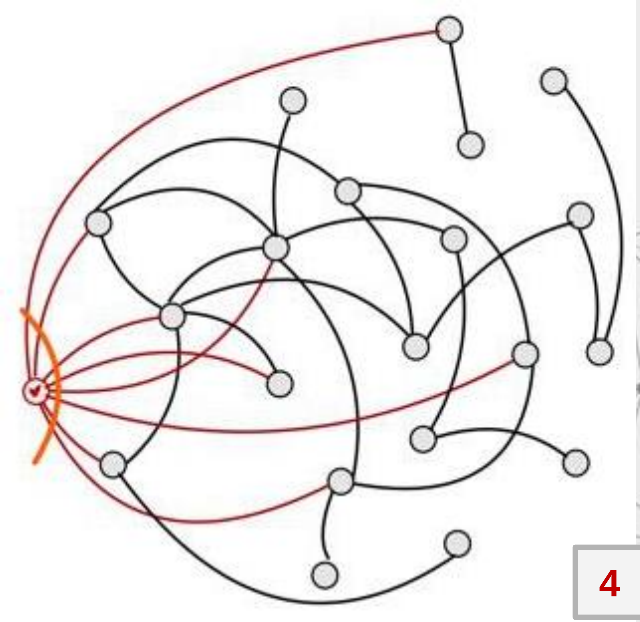


dv(2018)= 8



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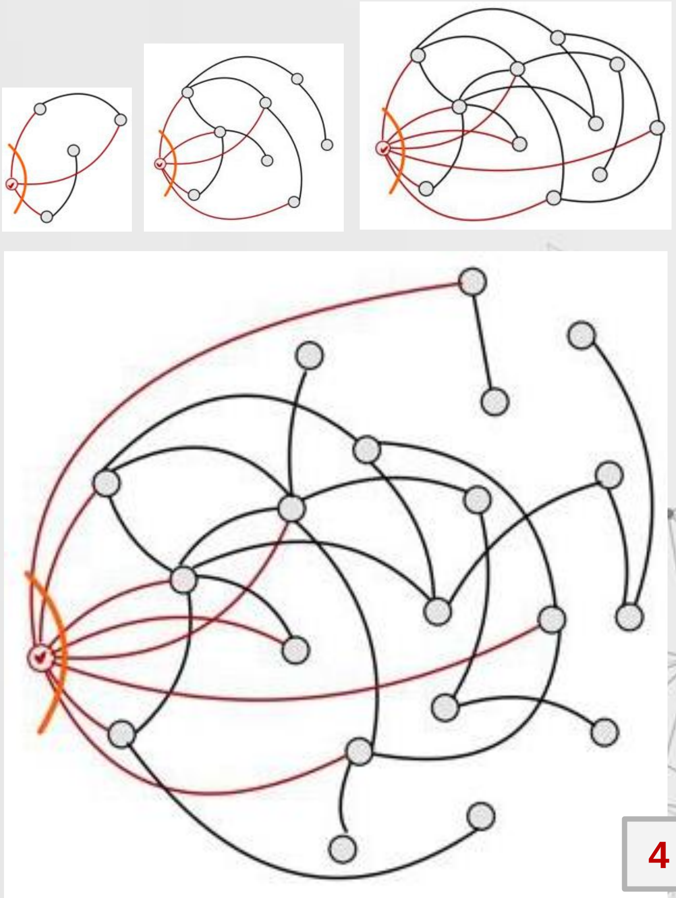

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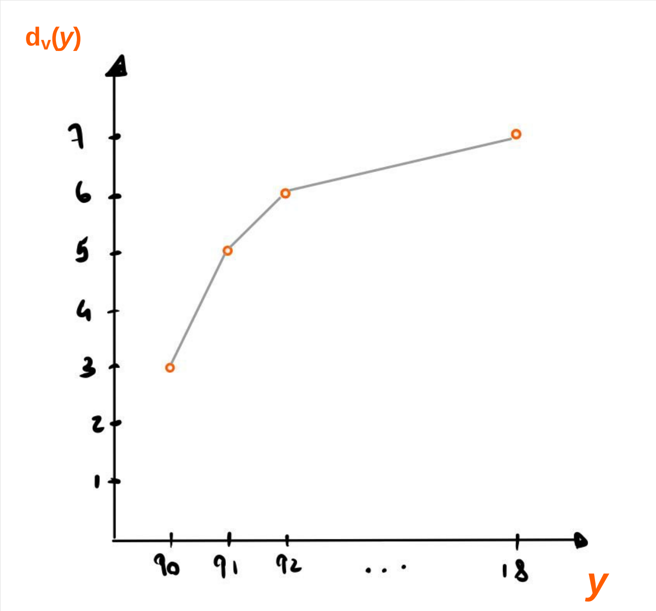

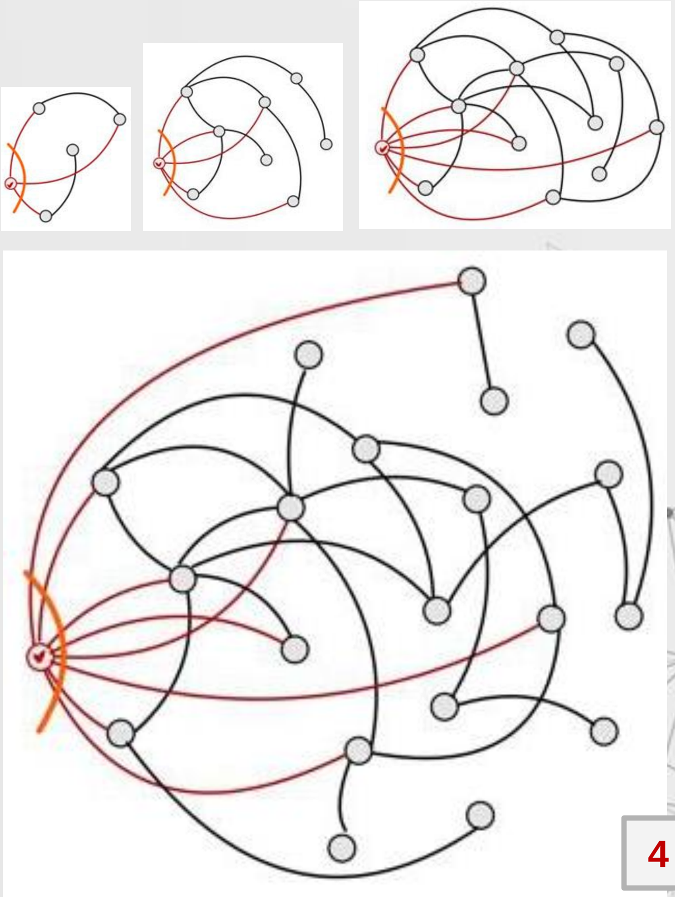

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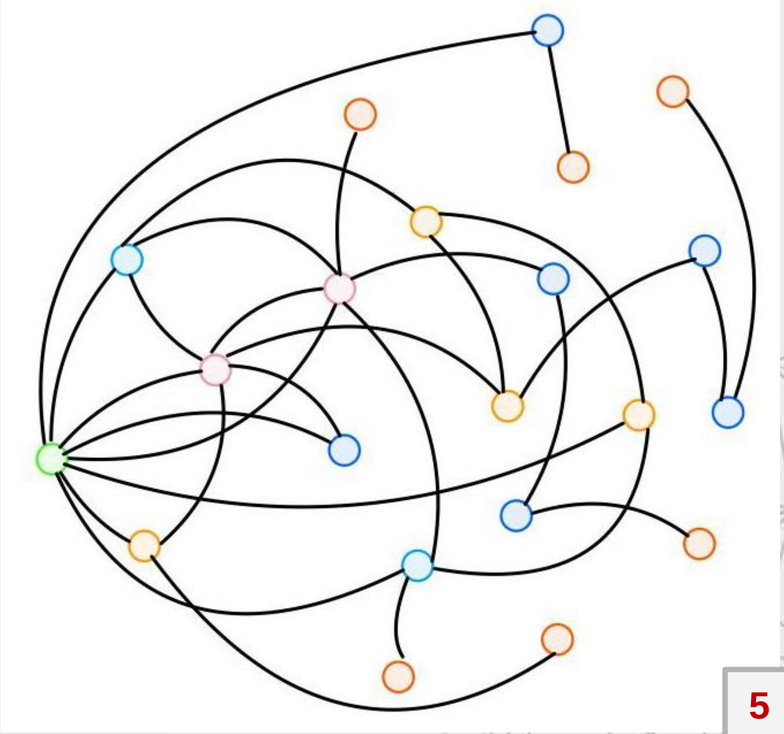

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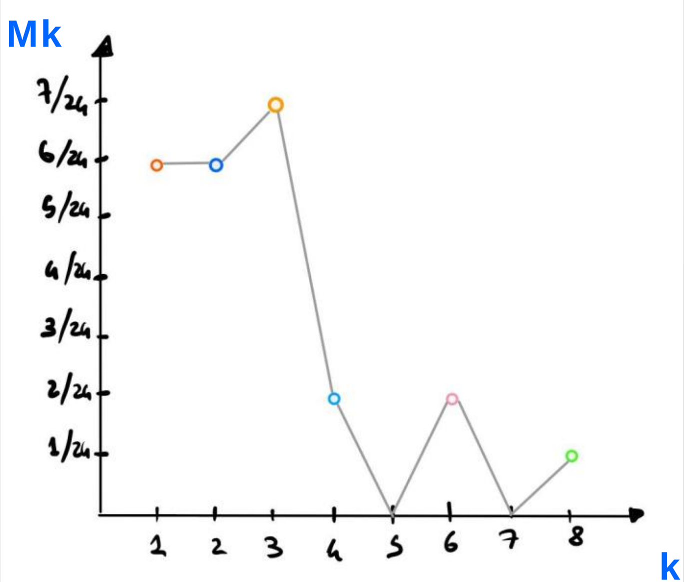

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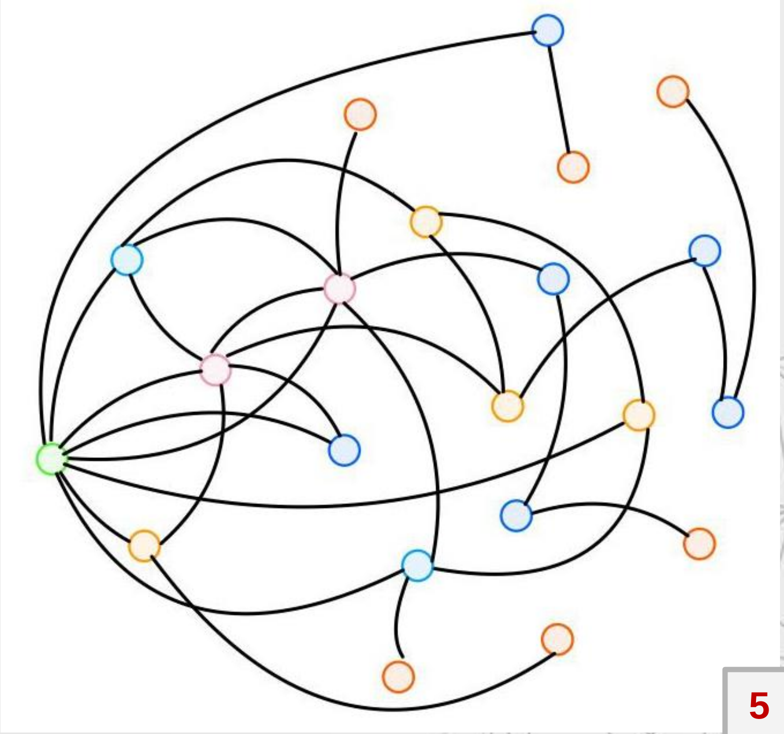

02

STATE OF THE ART

Network evolution over time
Power law and scale free networks
Power law in real world networks
Theoretical model
Matching model

NETWORK EVOLUTION OVER TIME

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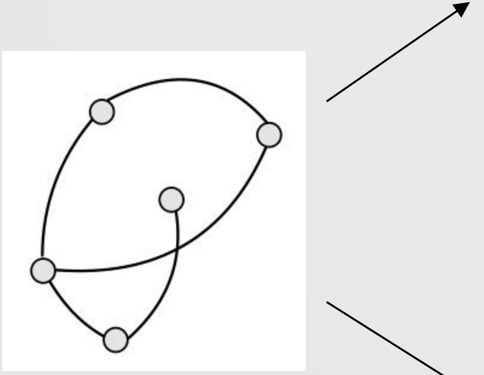
Two kinds of event:



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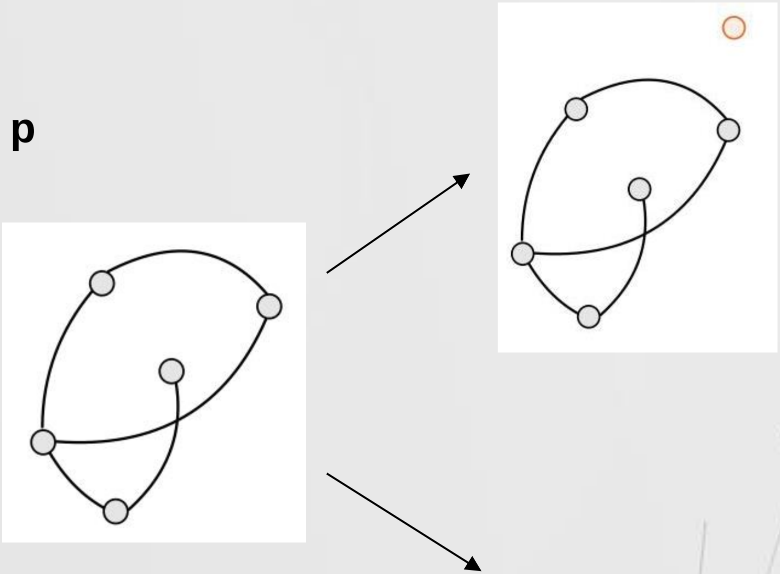

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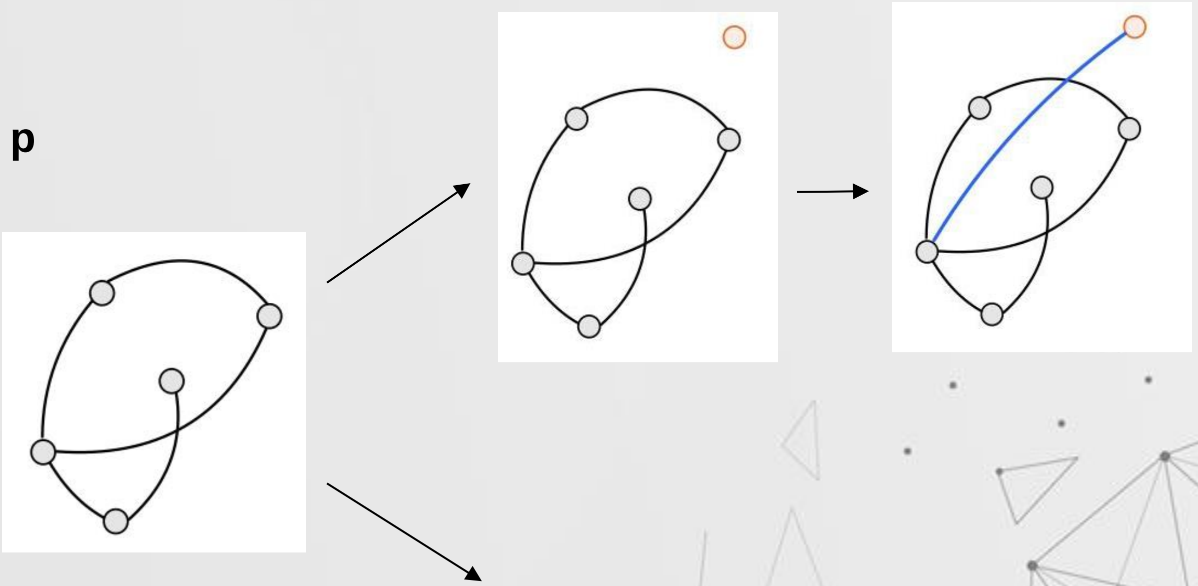

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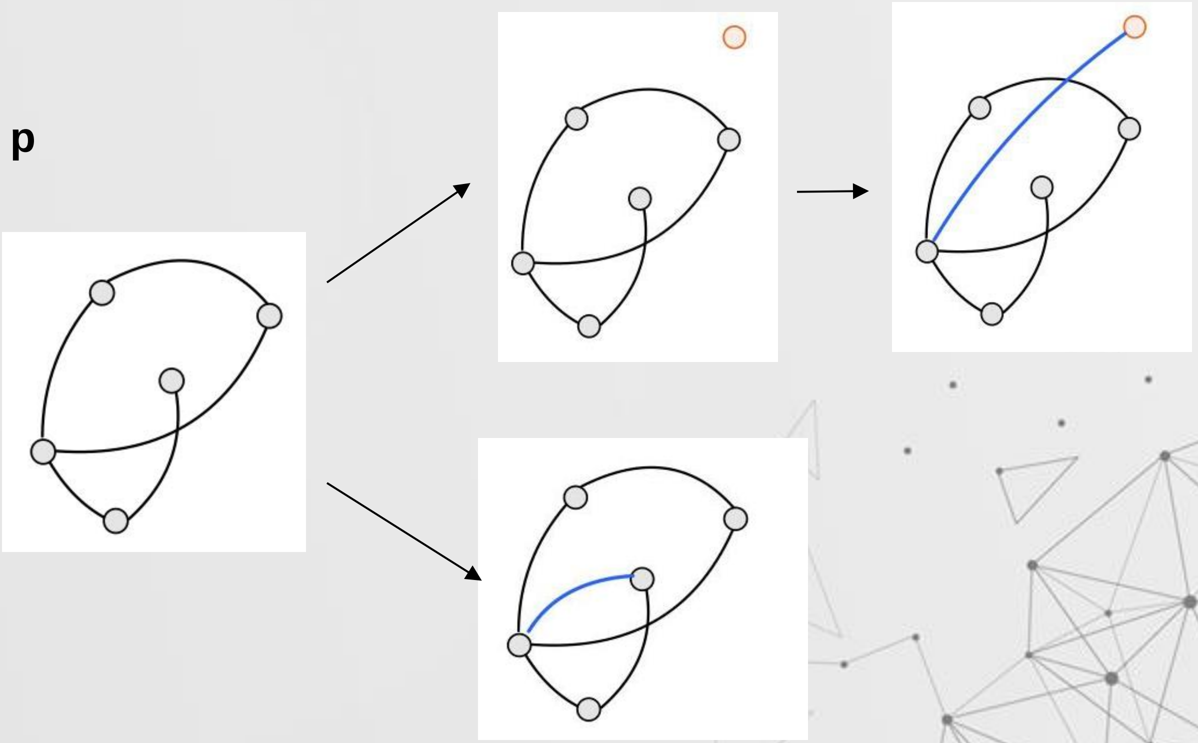

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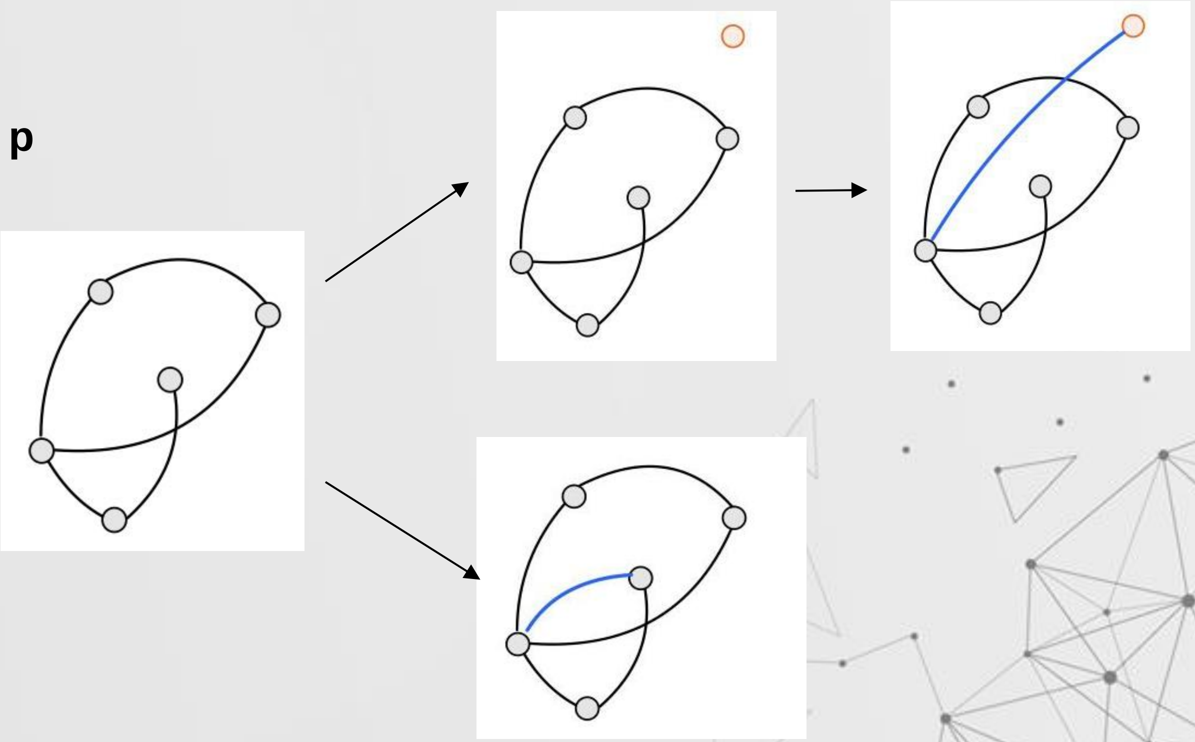

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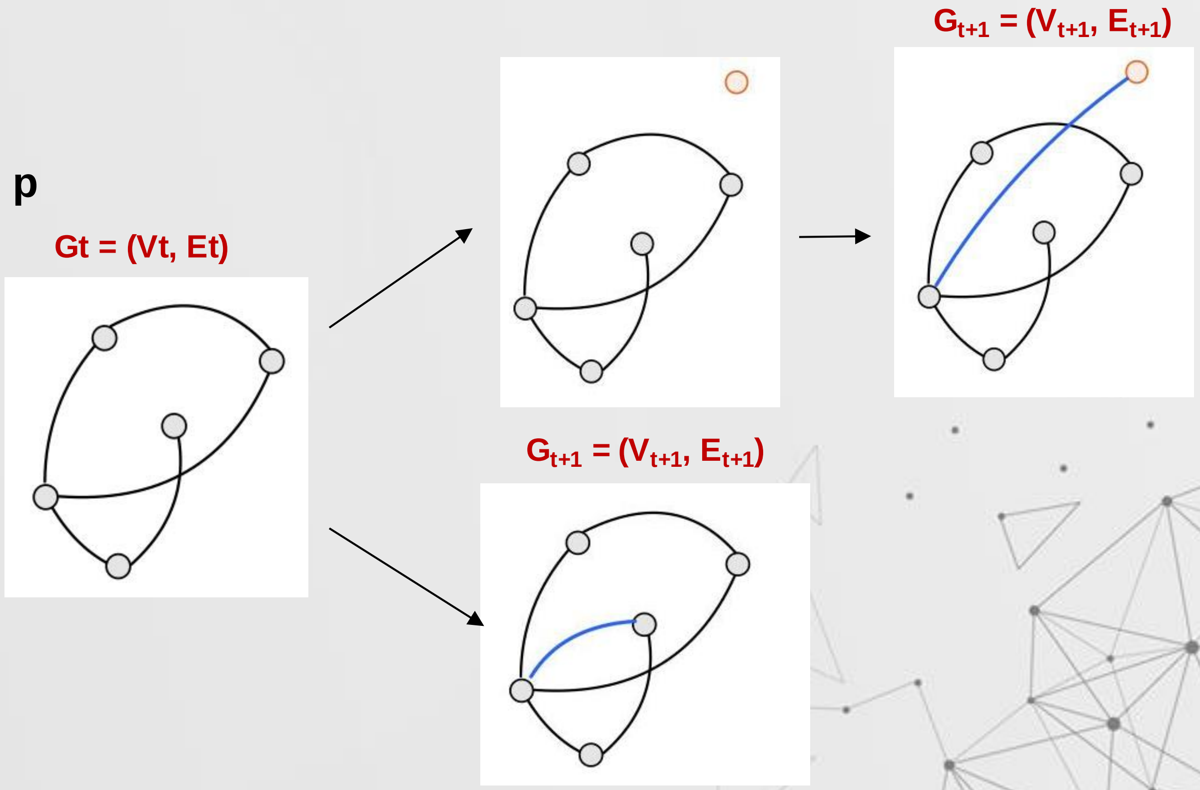

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POWER LAW AND SCALE-FREE NETWORKS

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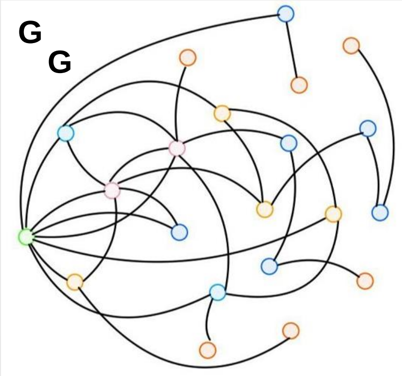

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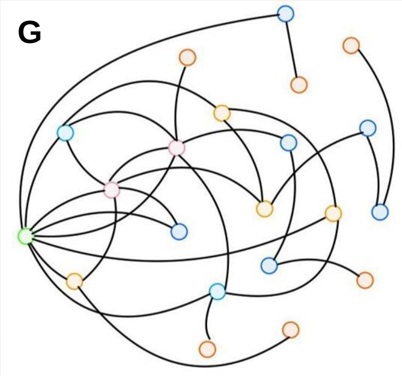

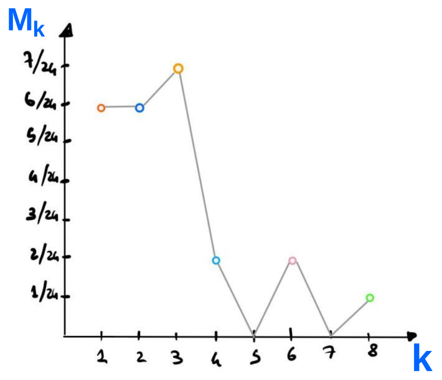

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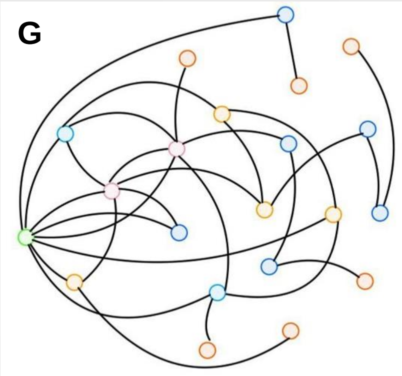

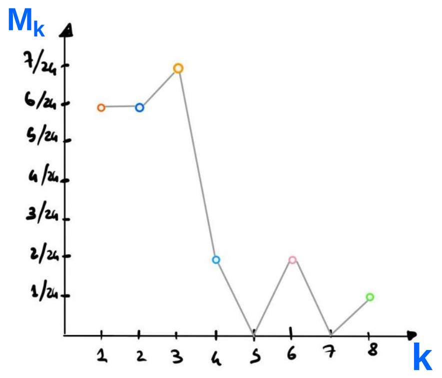

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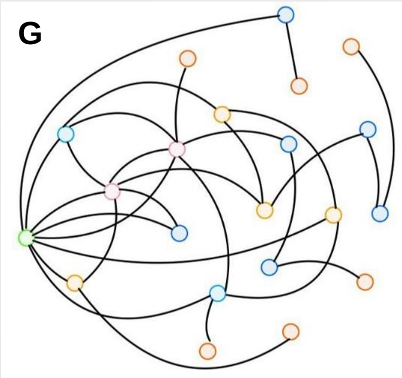

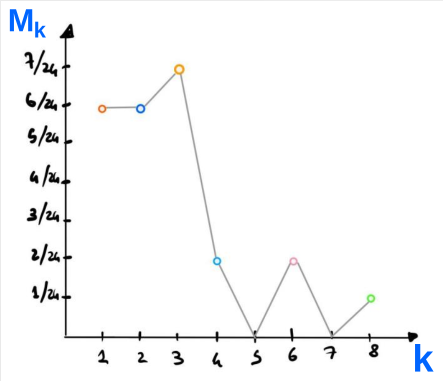

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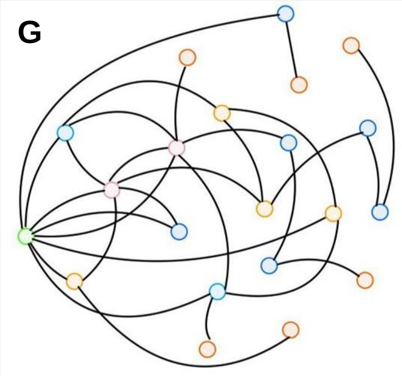

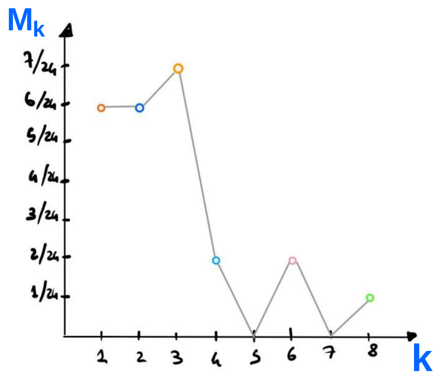

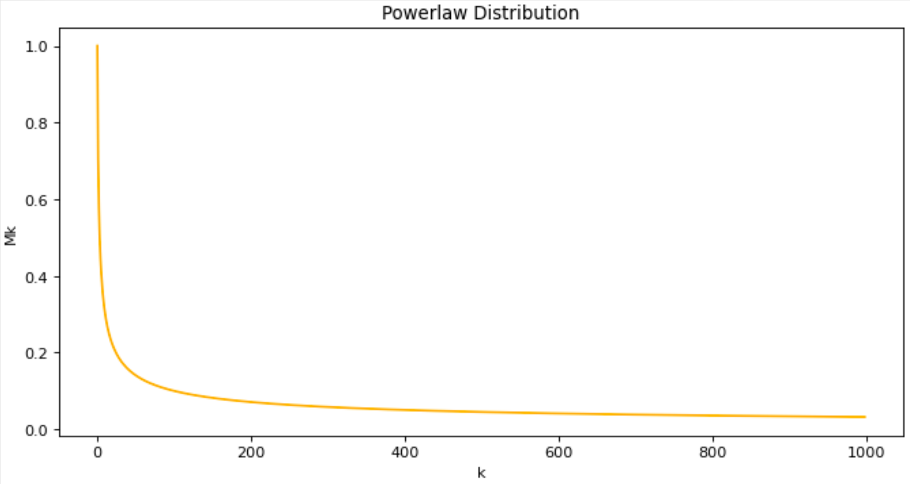

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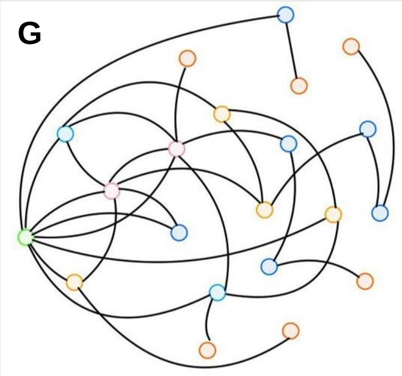

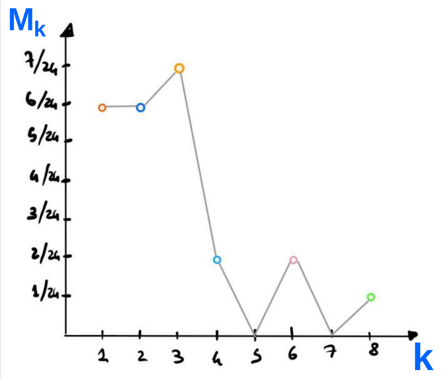

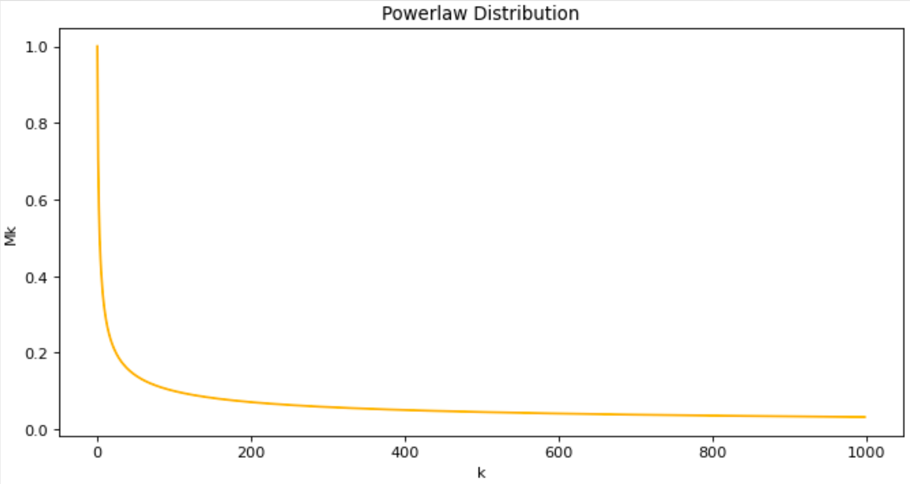

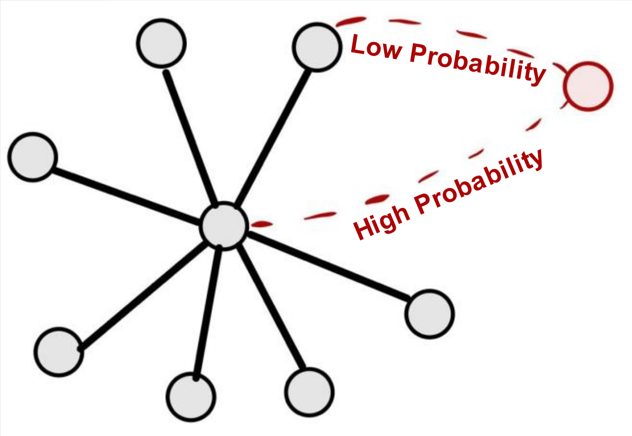

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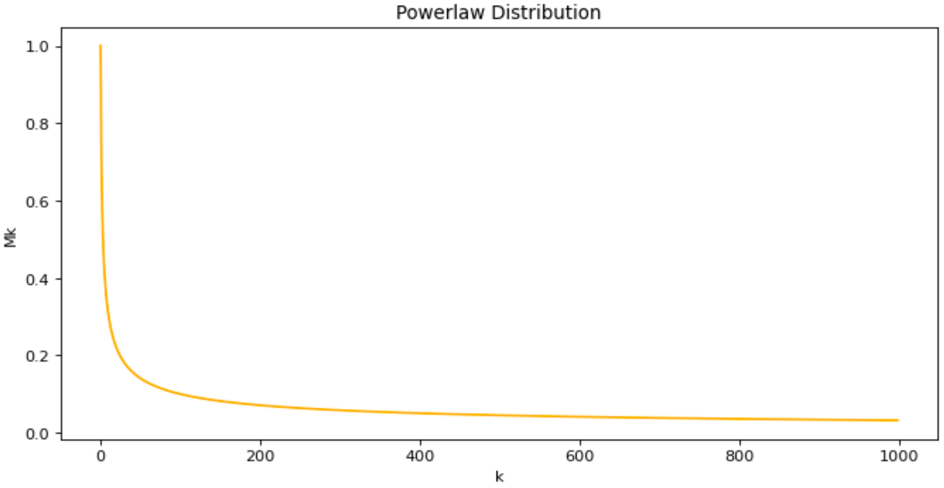

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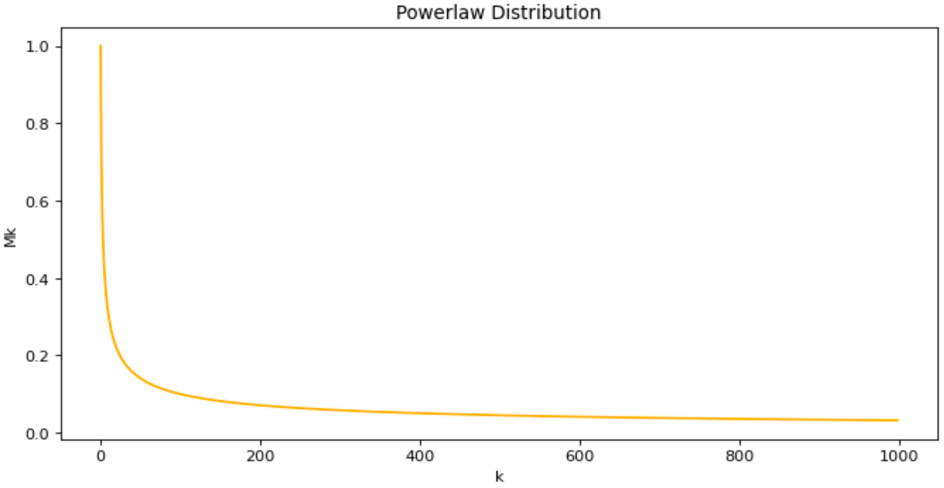

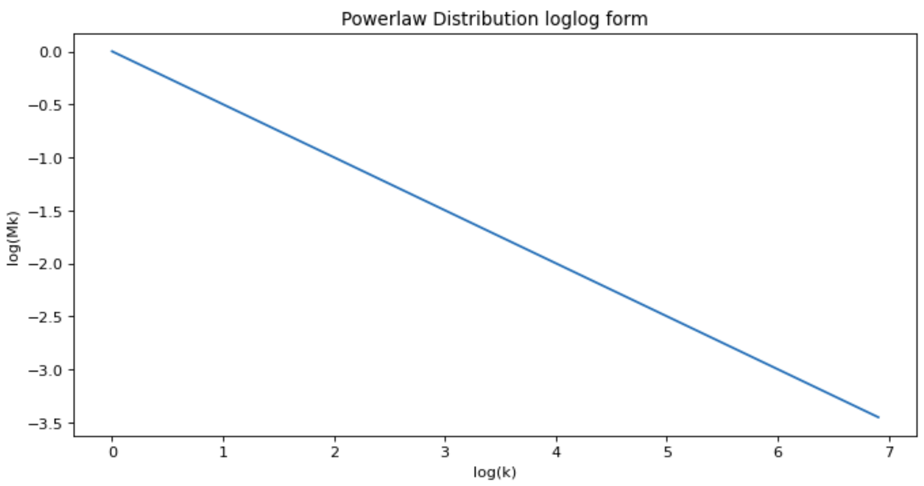

References:

- *[1] Derek J. de Solla P. **Networks of scientific papers**, 1965, doi:10.1126/science.149.3683.510.*
- *[2] Michalis F., Petros F., and Christos F. **On power-law relationships of the Internet topolog**y, 1999, doi:10.1145/316188.316229.*
- *[3] Bollobás B. and Riordan O**. Handbook of Graphs and Networks: From the Genome to the Internet,** 2003. Pages 1–34.*

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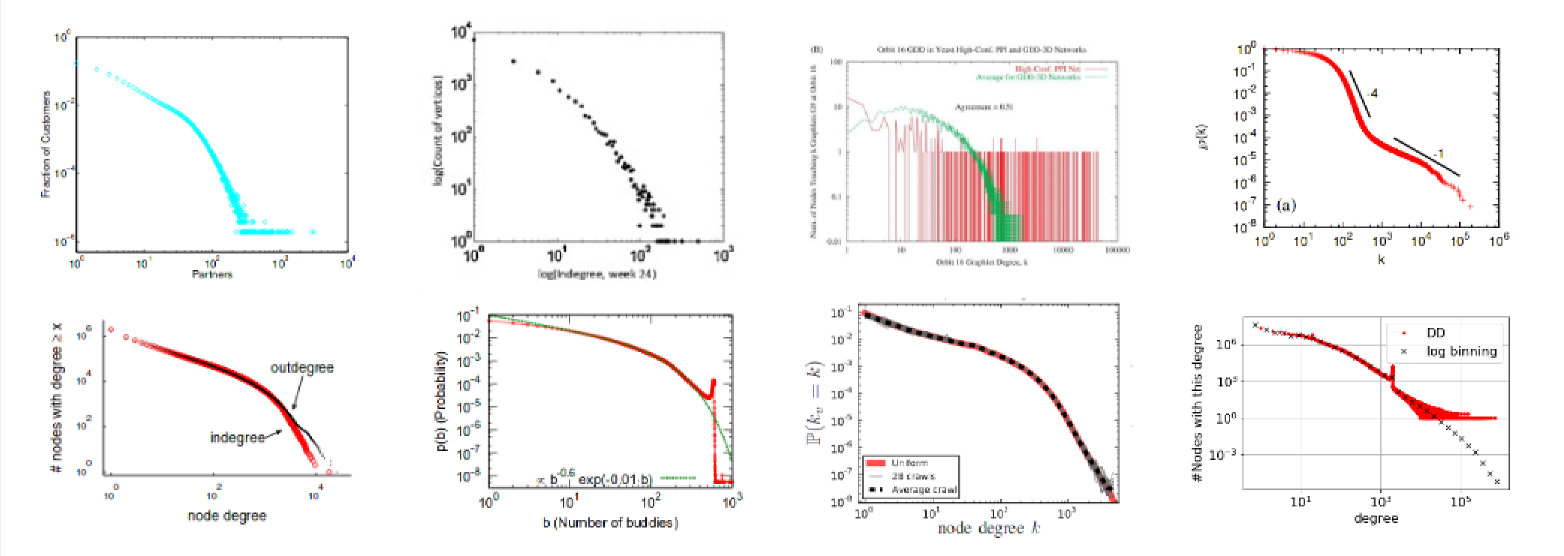

(biology, online market, social networks, ...)

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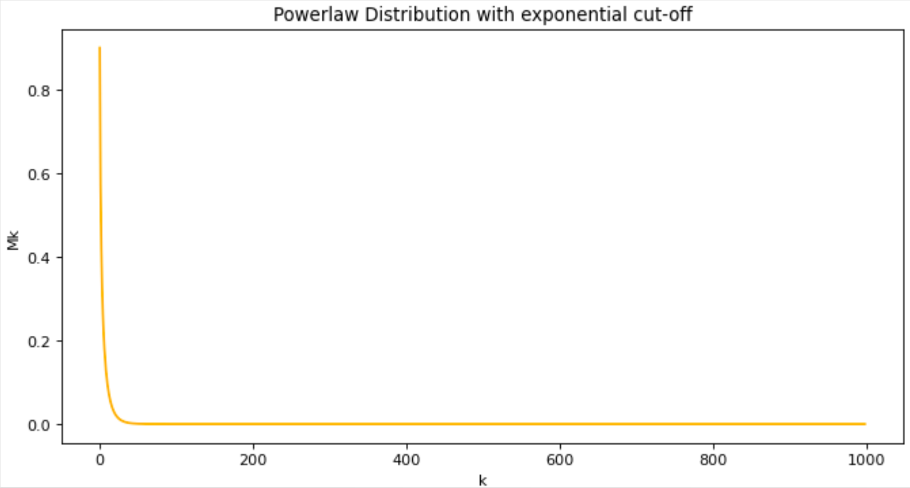

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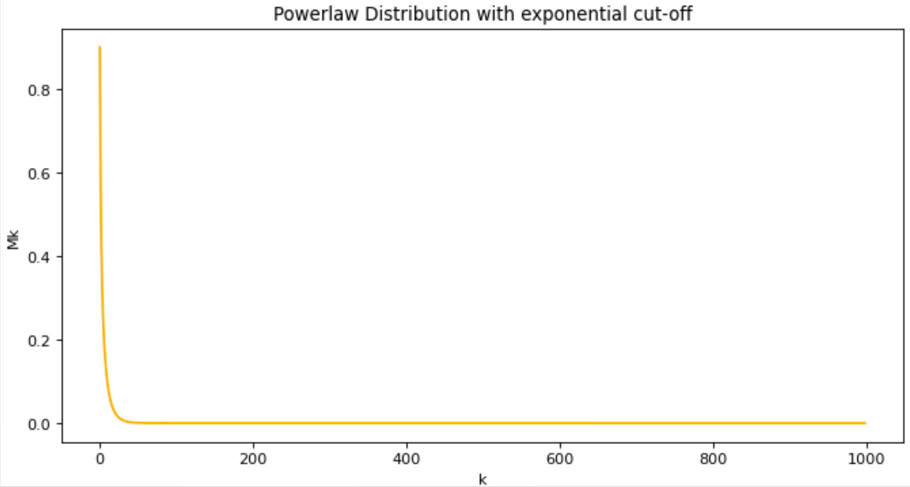

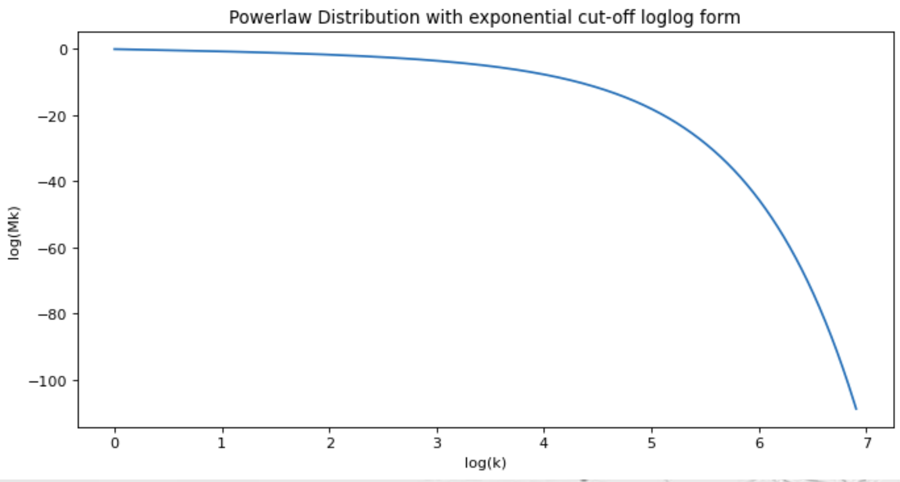

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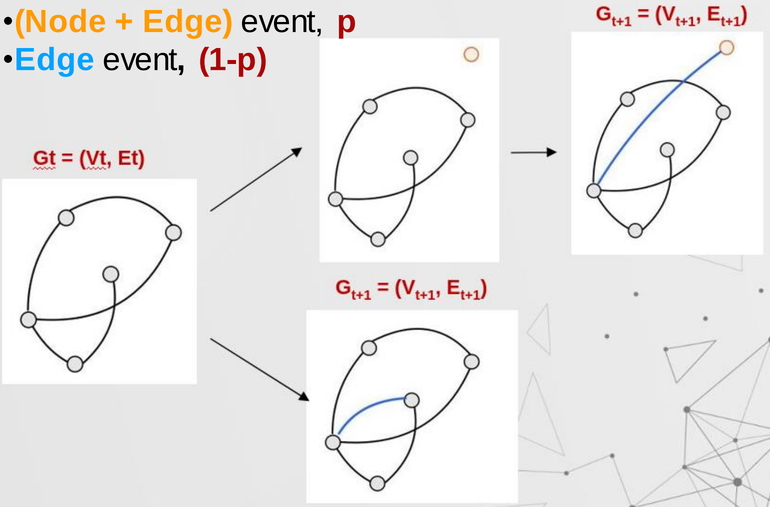
THEORETICAL MODEL

- (Node + Edge) event, p
- Edge event, (1-p)



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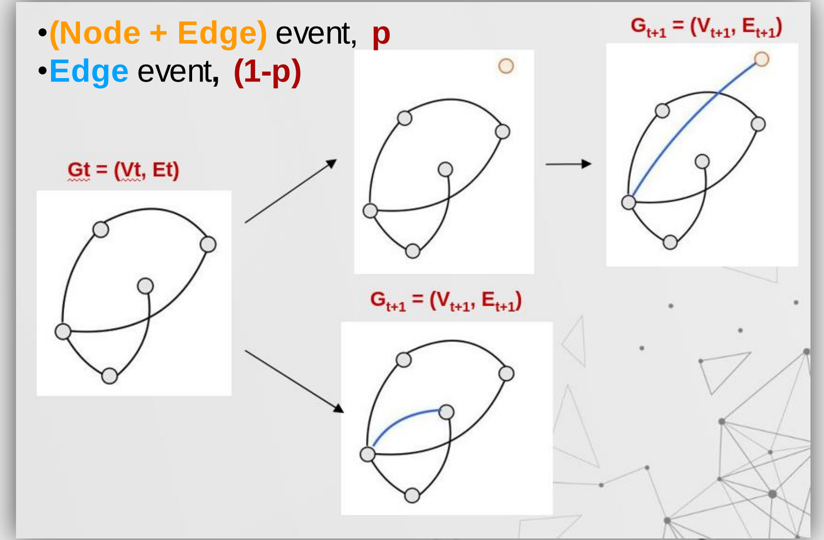
$\Pr[v \text{ is chosen at time step t}] =$



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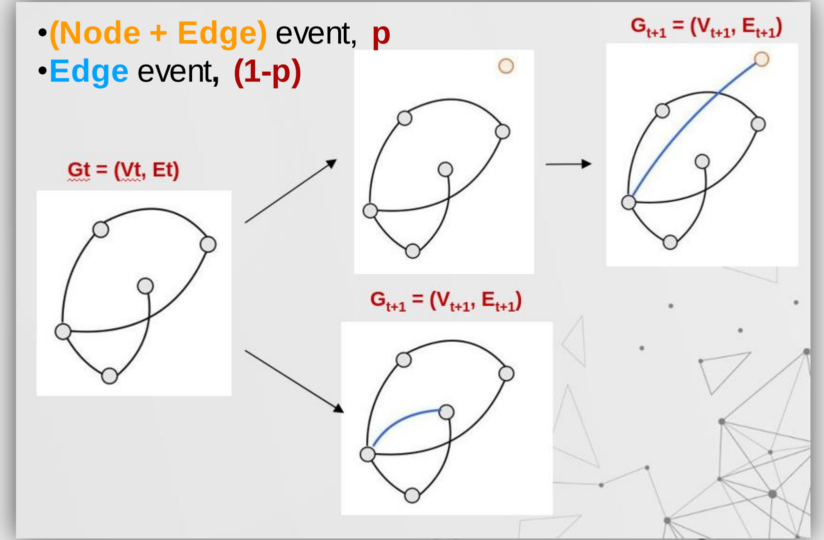

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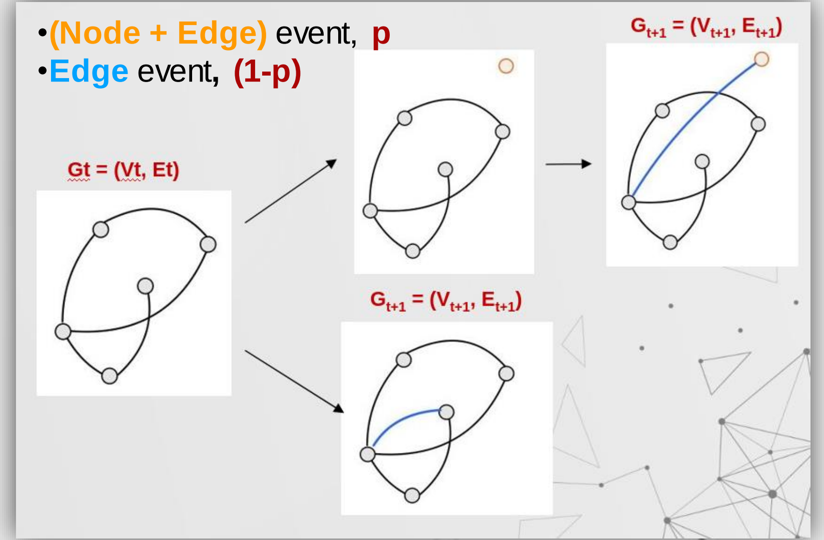

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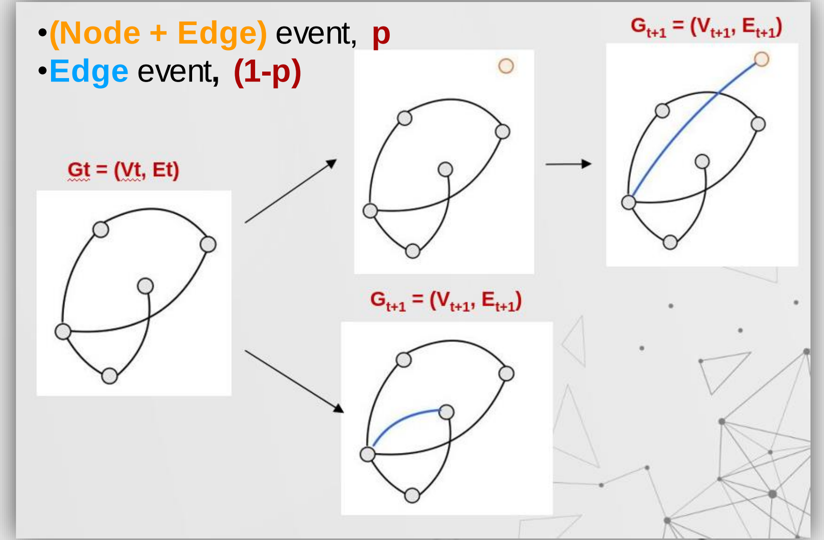

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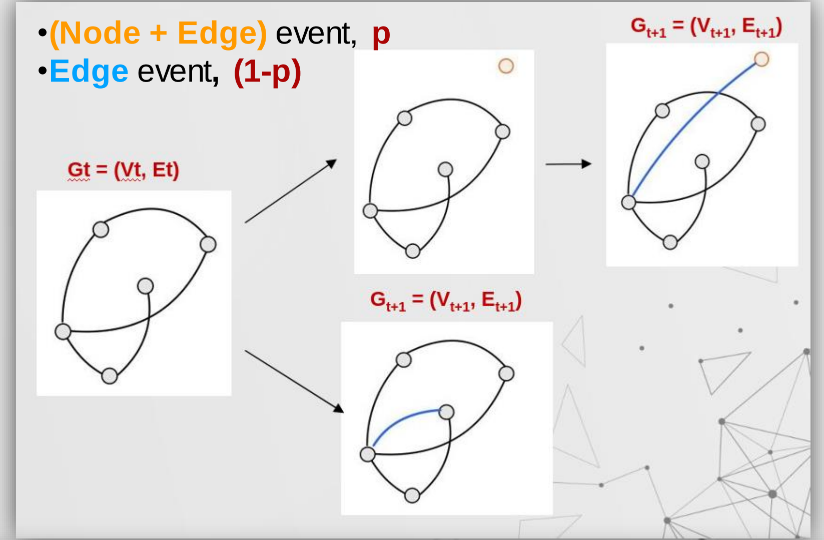

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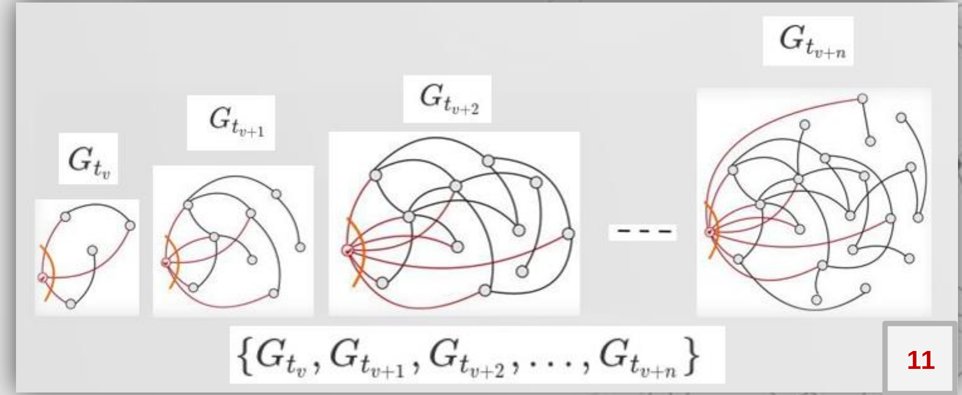

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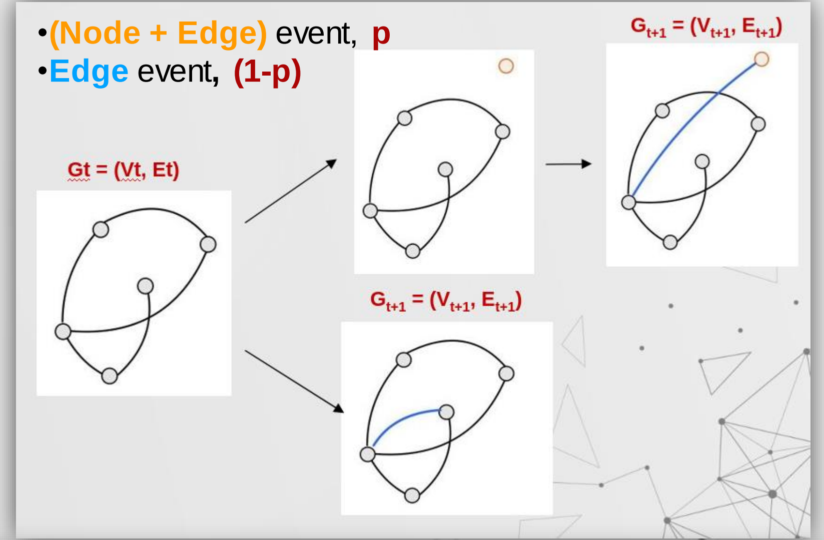

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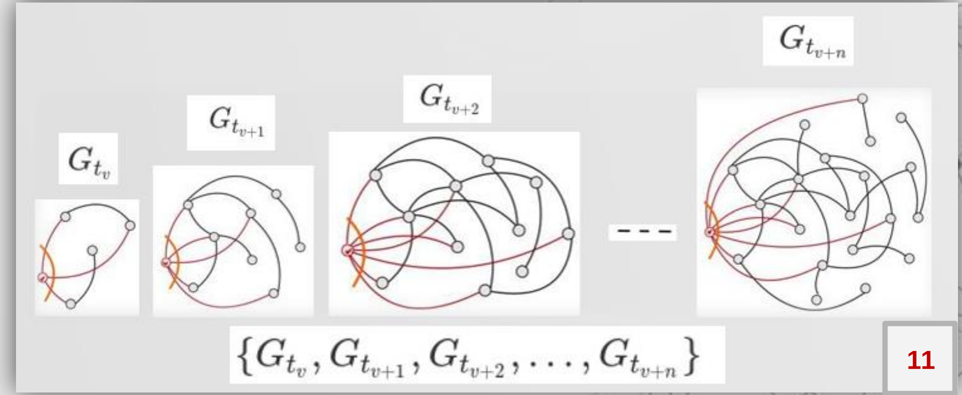

MATCHING MODEL

Barabasi Albert model [7]

Given:

- **P = 1**, so only **node+edge** event
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03

FACED PROBLEMS

(1) Which researchers should be taken into account?
(2) How to interpret a time step?

(1) Which researchers should be taken into account?

Problem

We faced the following issues trying to define **what an active author is**:

(1) Which researchers should be taken into account?

Problem

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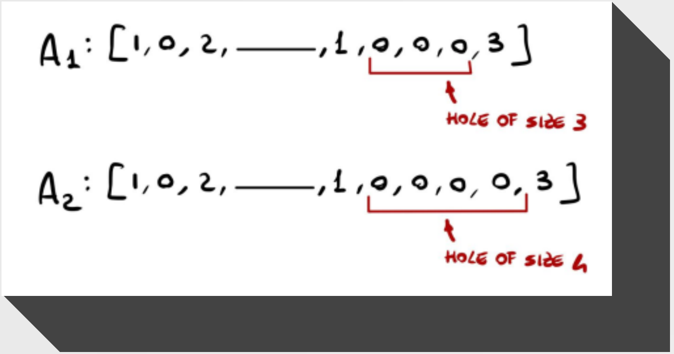

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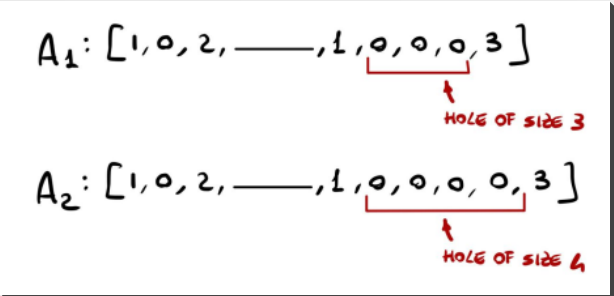

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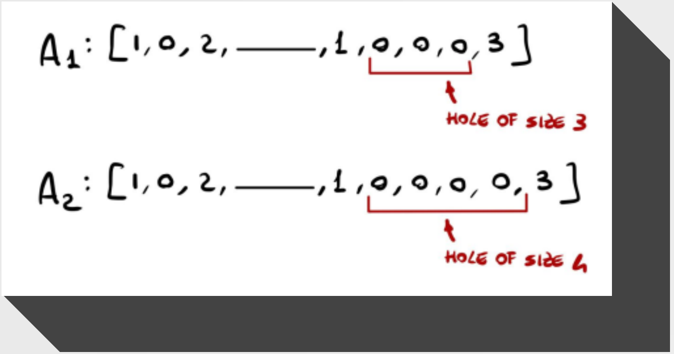

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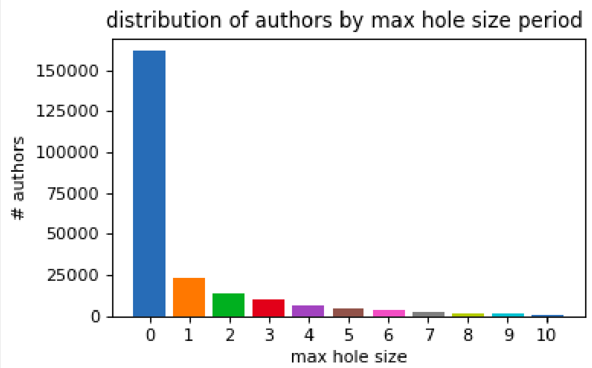

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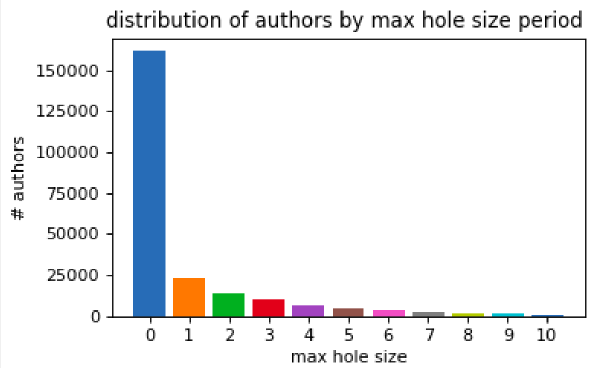

To include researcher who take a sabathical year from teaching every seven, to do research

≤ 7

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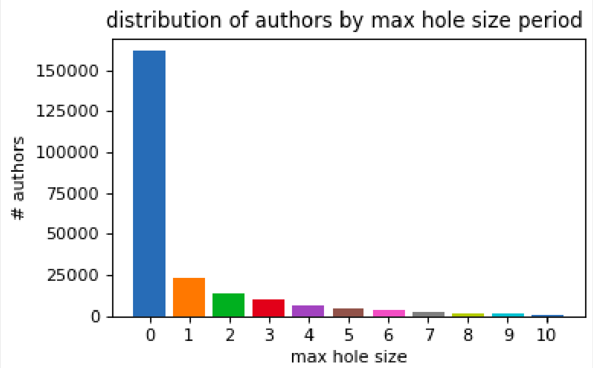

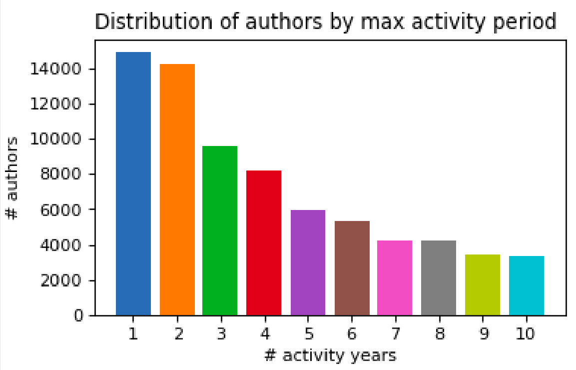

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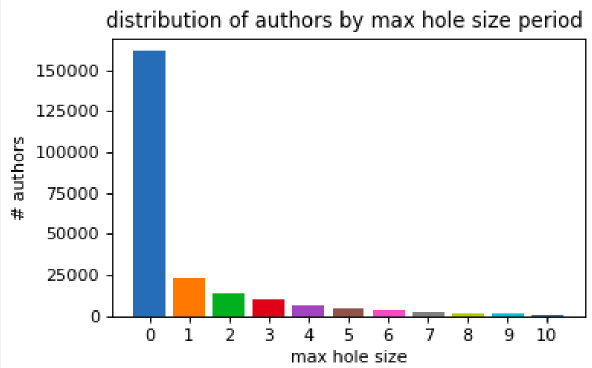

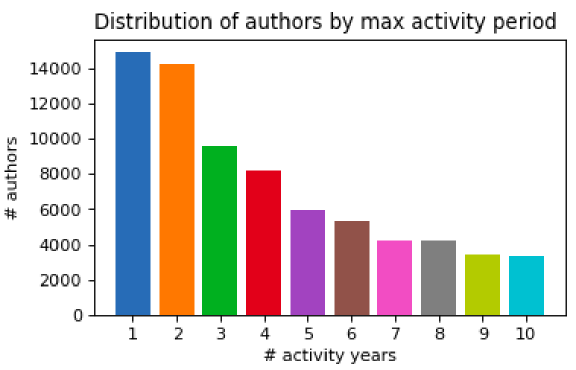

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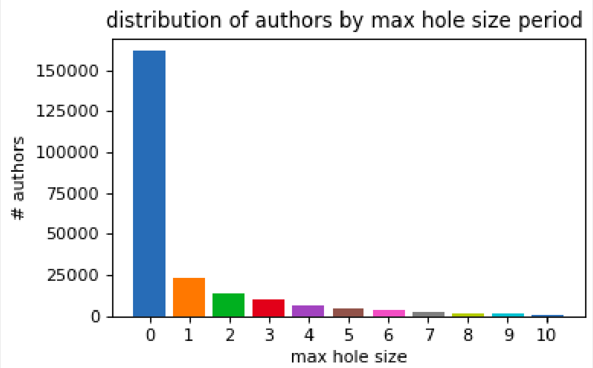

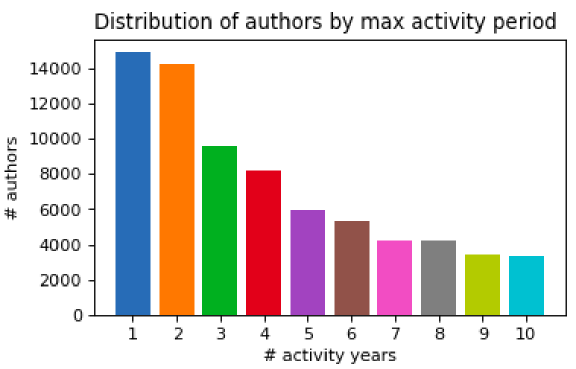

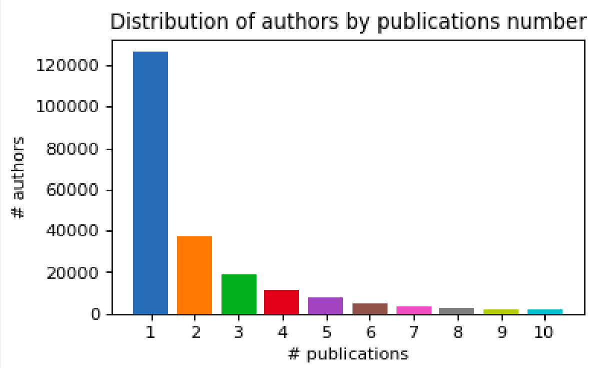

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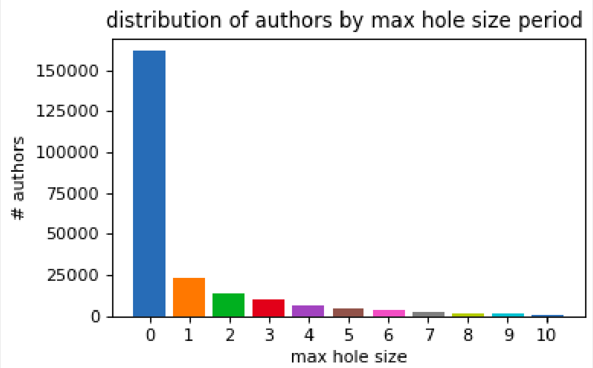

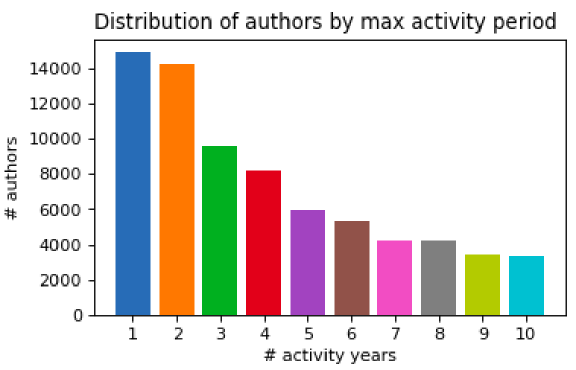

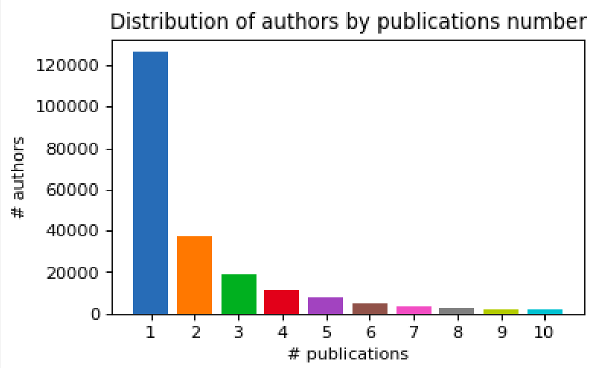

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≤ 7

To exlude PhD students

≥ 5

Good compromise between insight and #authors filtered

≥ 3

(1) Which researchers should be taken into account?

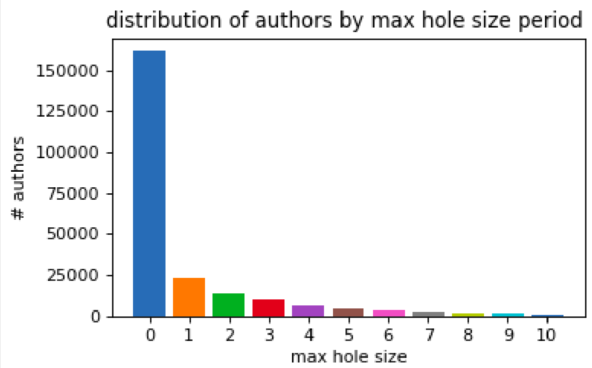
Hole size
(# consecutive years without publishing)

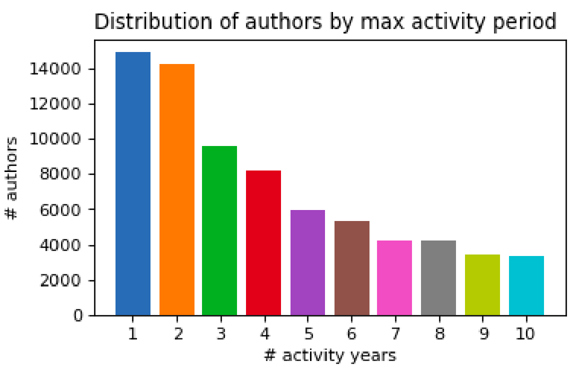
&

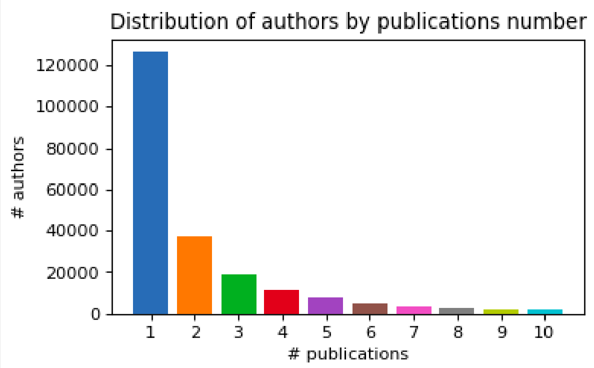
Activity period
(#years between 1st and last publication)

&

#publications







To include researcher who take a sabathical year from teaching every seven, to do research

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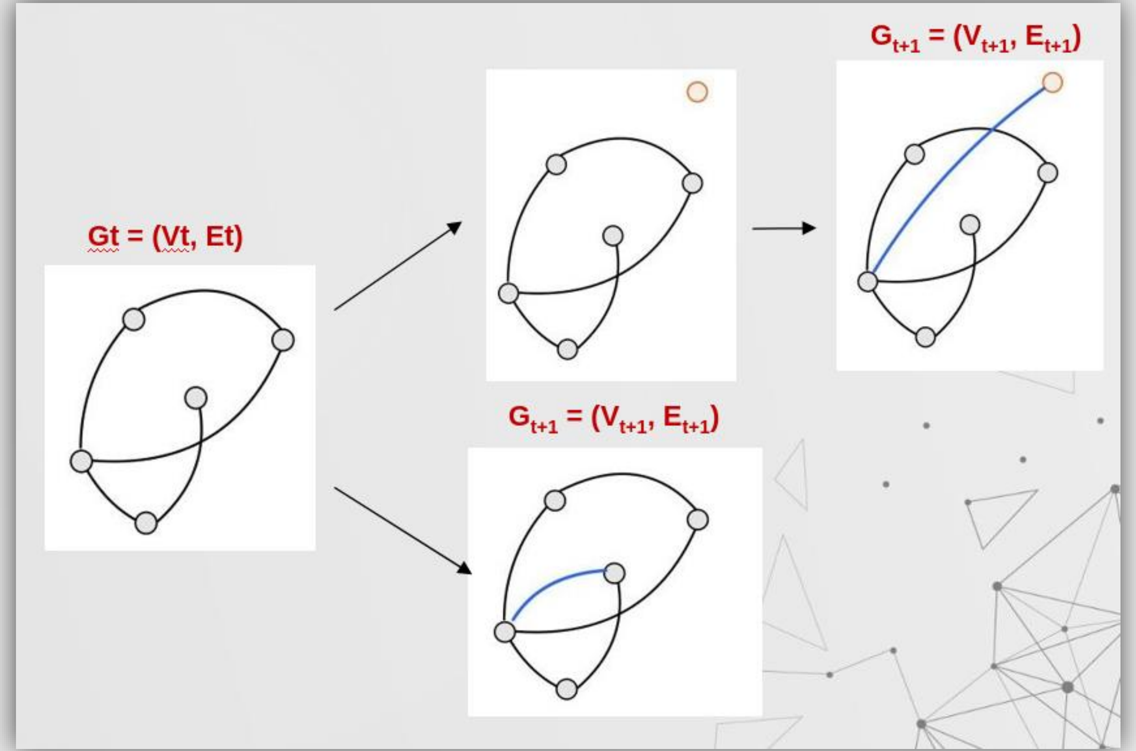
=> 16% of data left

(2) How to interpret a time step?



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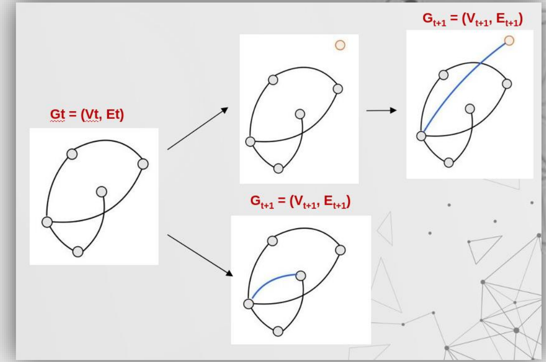

- Time step definition:

  $G_t = (V_t, E_t)$ graph after event **t | t∈N** - #events occurred (**(Node + Edge)** **and** **Edge** event)

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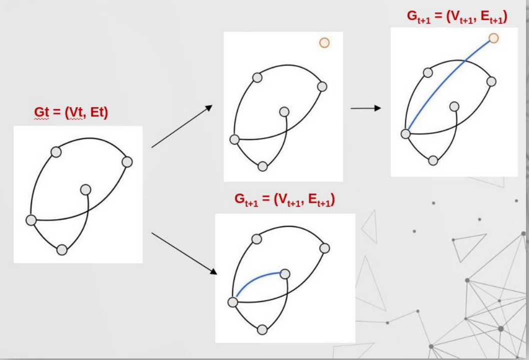

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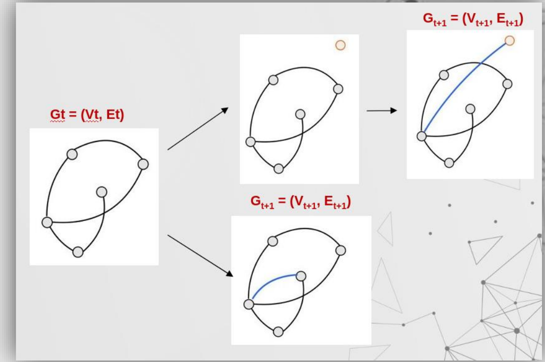

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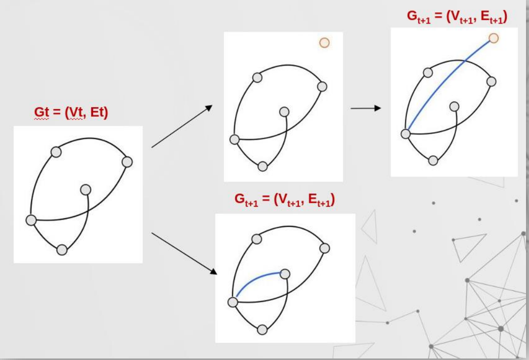

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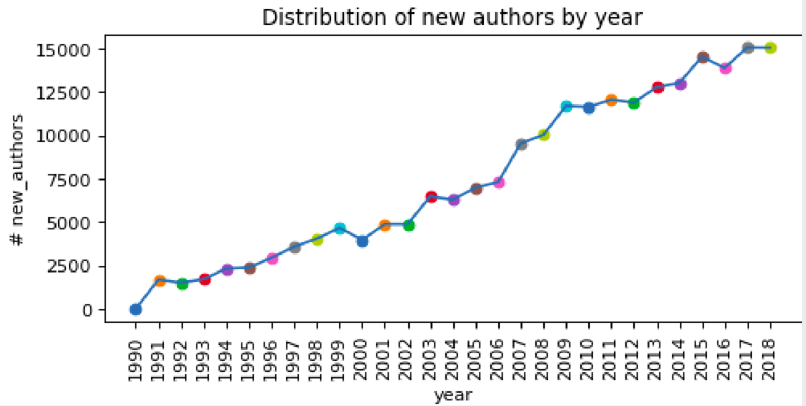

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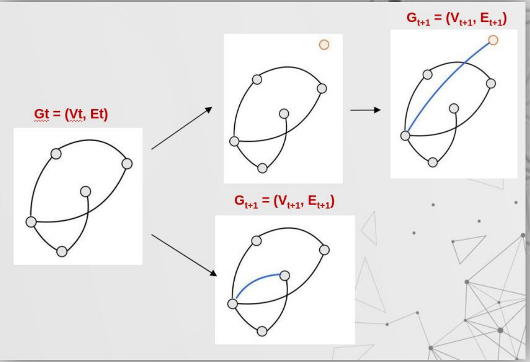

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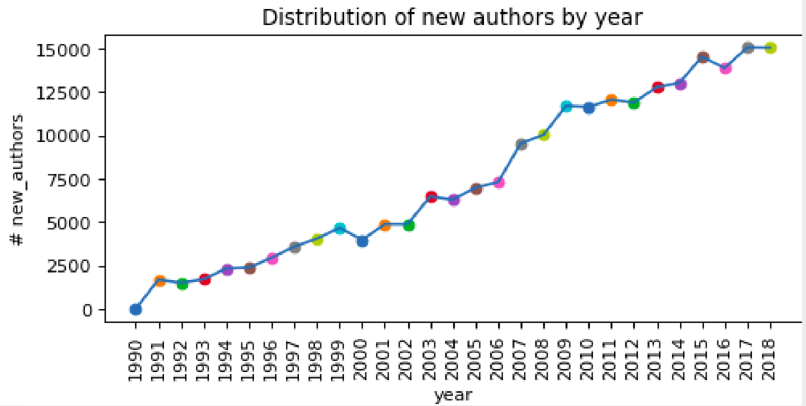

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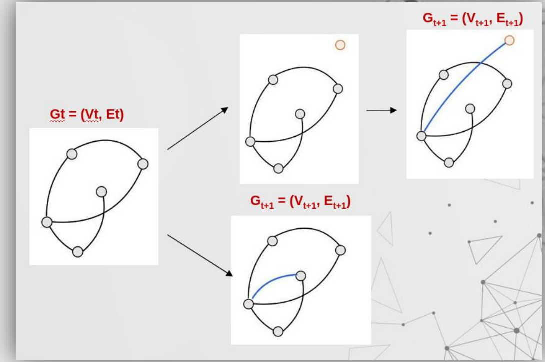

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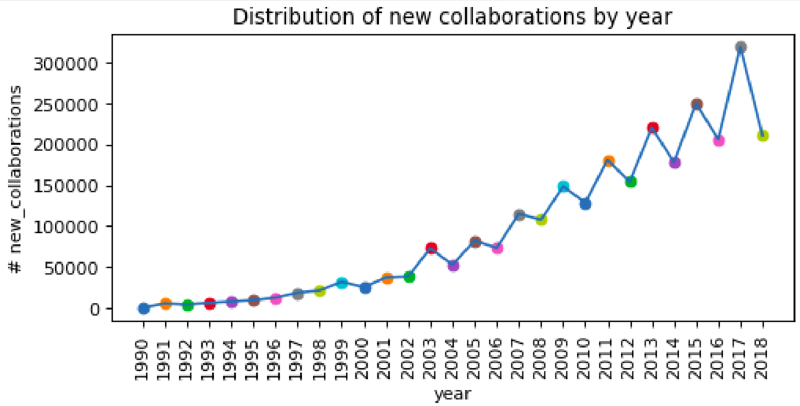

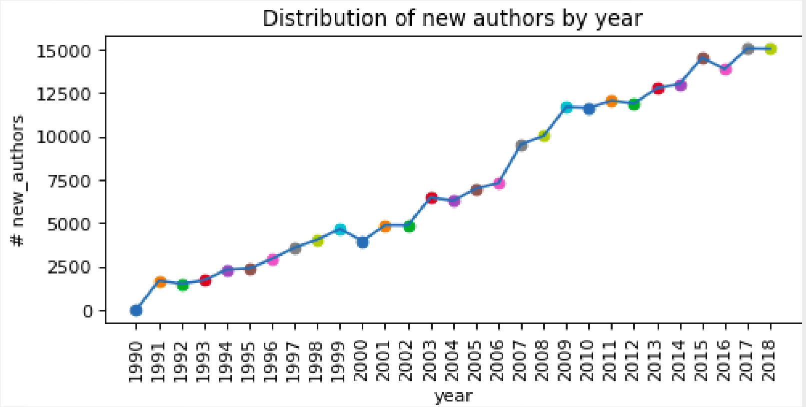

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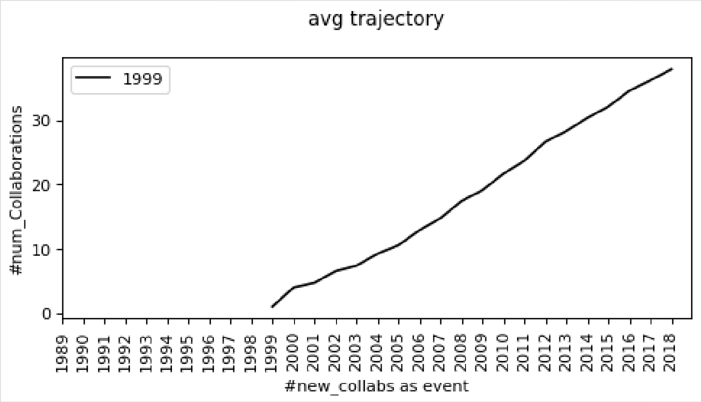
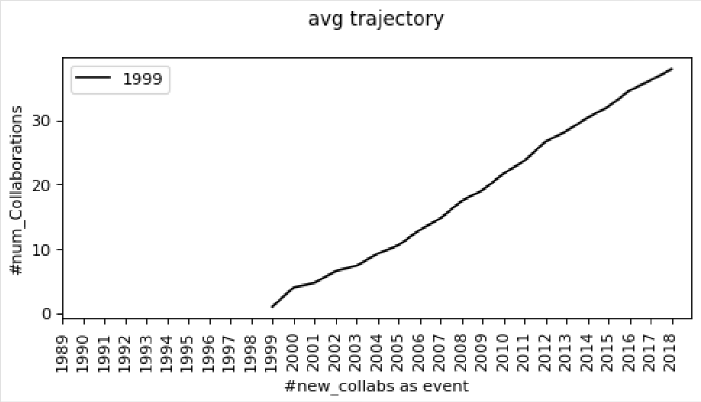


(2) How to interpret a time step?

- Vertex trajectories are **stretched on the x axis**

- Shown their **logarithmic** shape.



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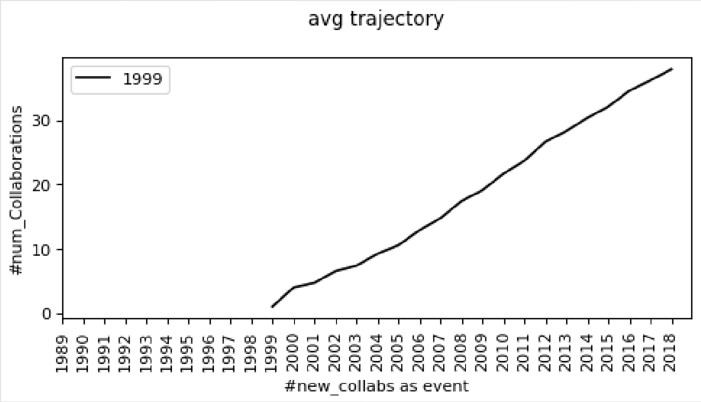
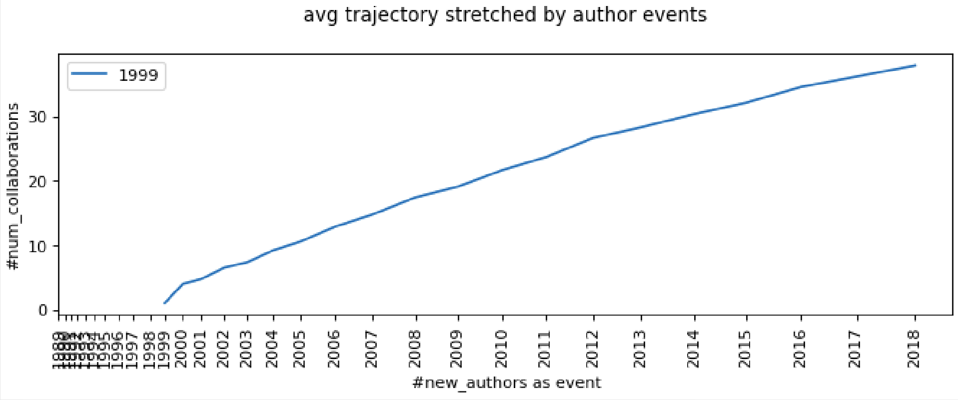
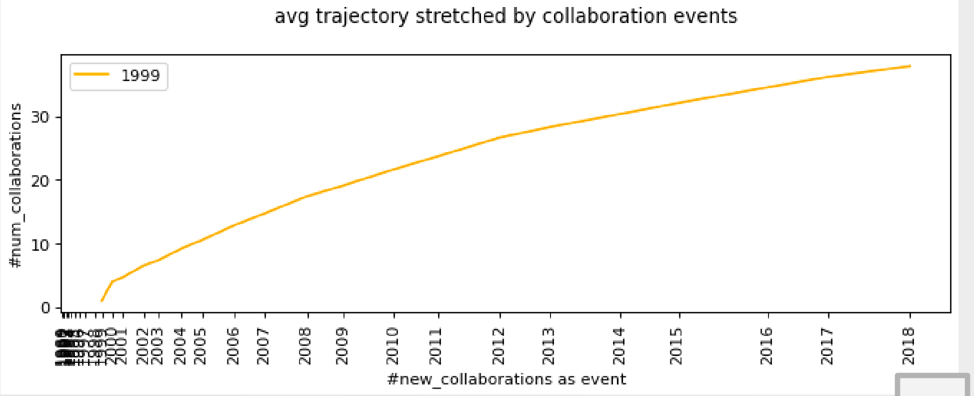






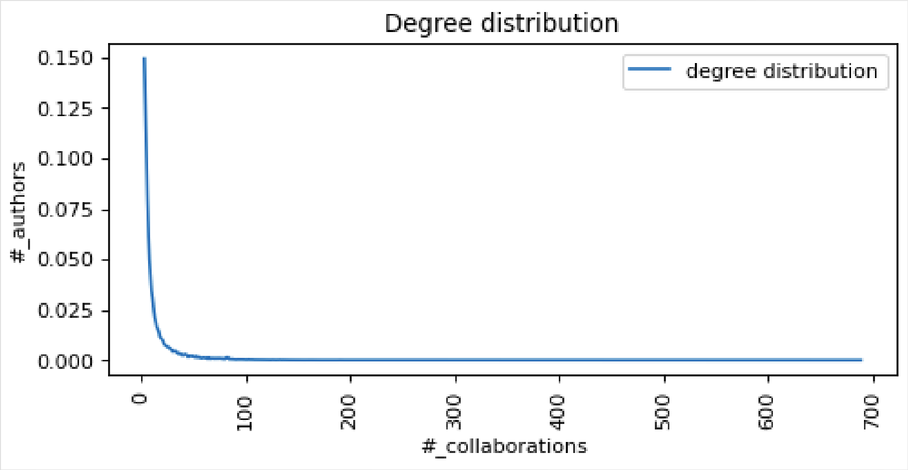
04

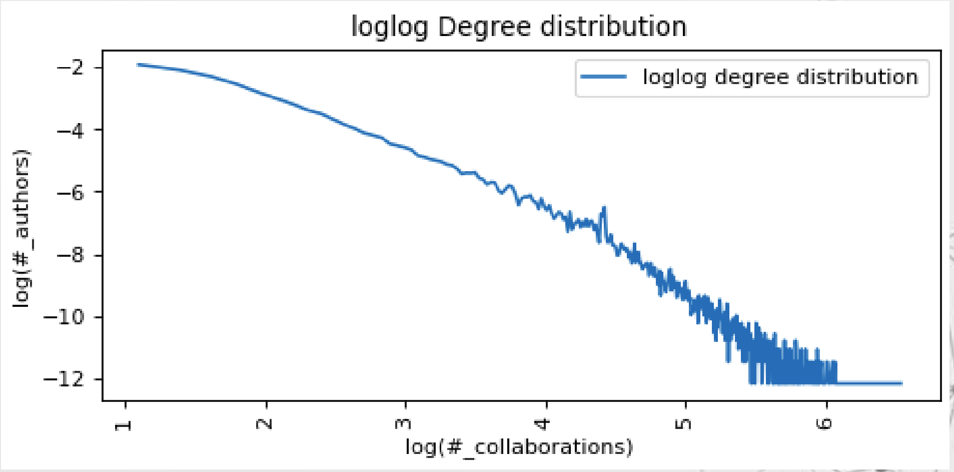
DEGREE DISTRIBUTION

Degree Distribution Retrieval
Degree Distribution Fitting

DEGREE DISTRIBUTION RETRIEVAL

To acquire more knowledge from the character of the data the **degree distribution of active authors** is found, taking the **#authors** with a given **#collaborations.**





DEGREE DISTRIBUTION FITTING

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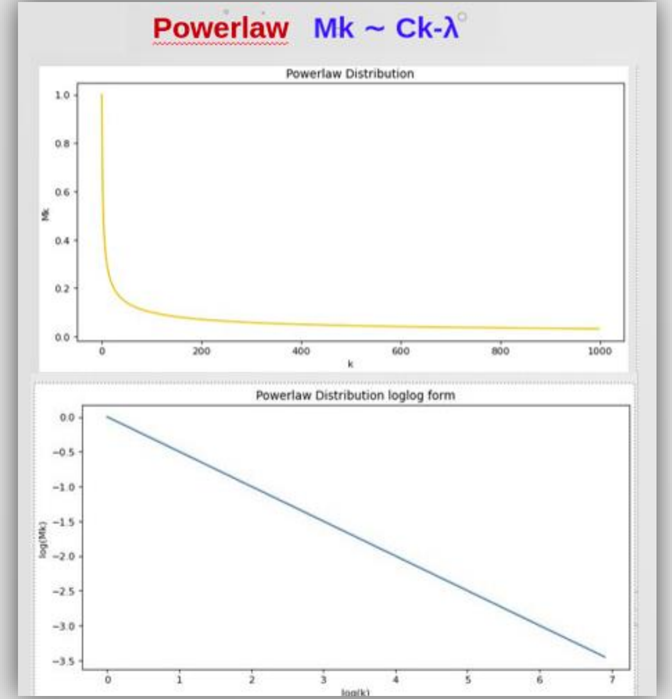
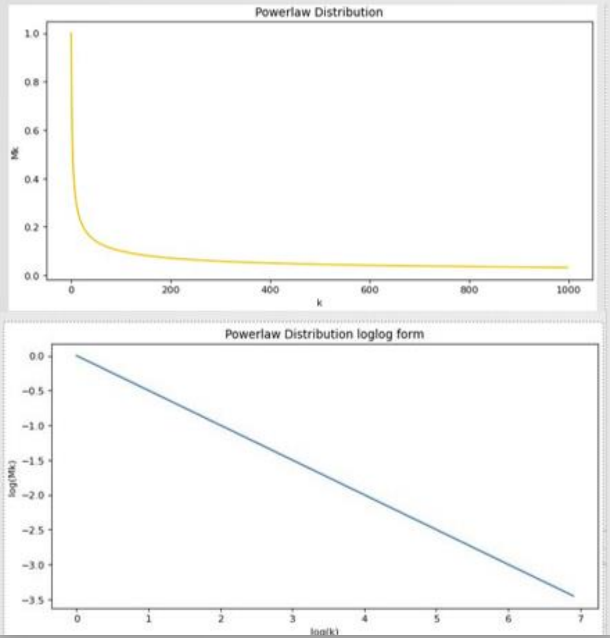


DEGREE DISTRIBUTION FITTING

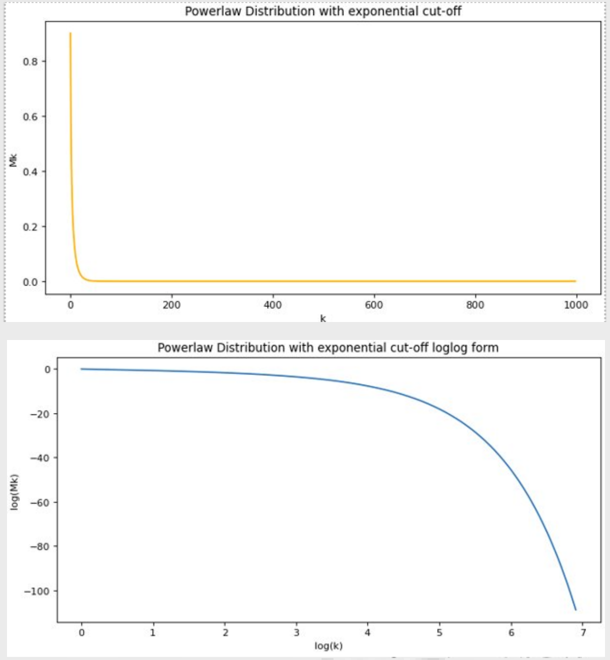
To acquire more knowledge from the character of the data the **degree distribution of active authors** is found, taking the **#authors** with a given **#collaborations.**

Powerlaw $Mk \sim Ck^{-\lambda}$



Powerlaw with exponential cut-off

$Mk \sim Ck^{-\lambda}\gamma^{-k} \mid 0 \leq \gamma < 1$



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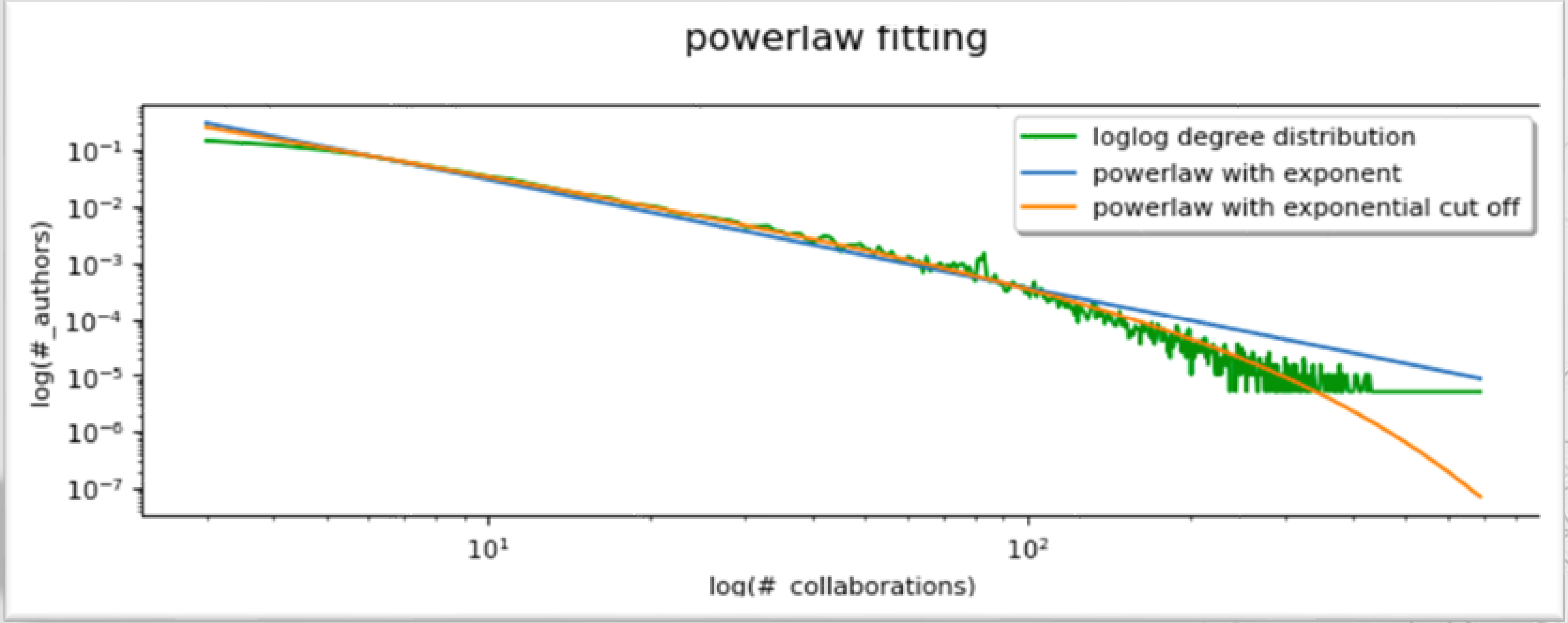


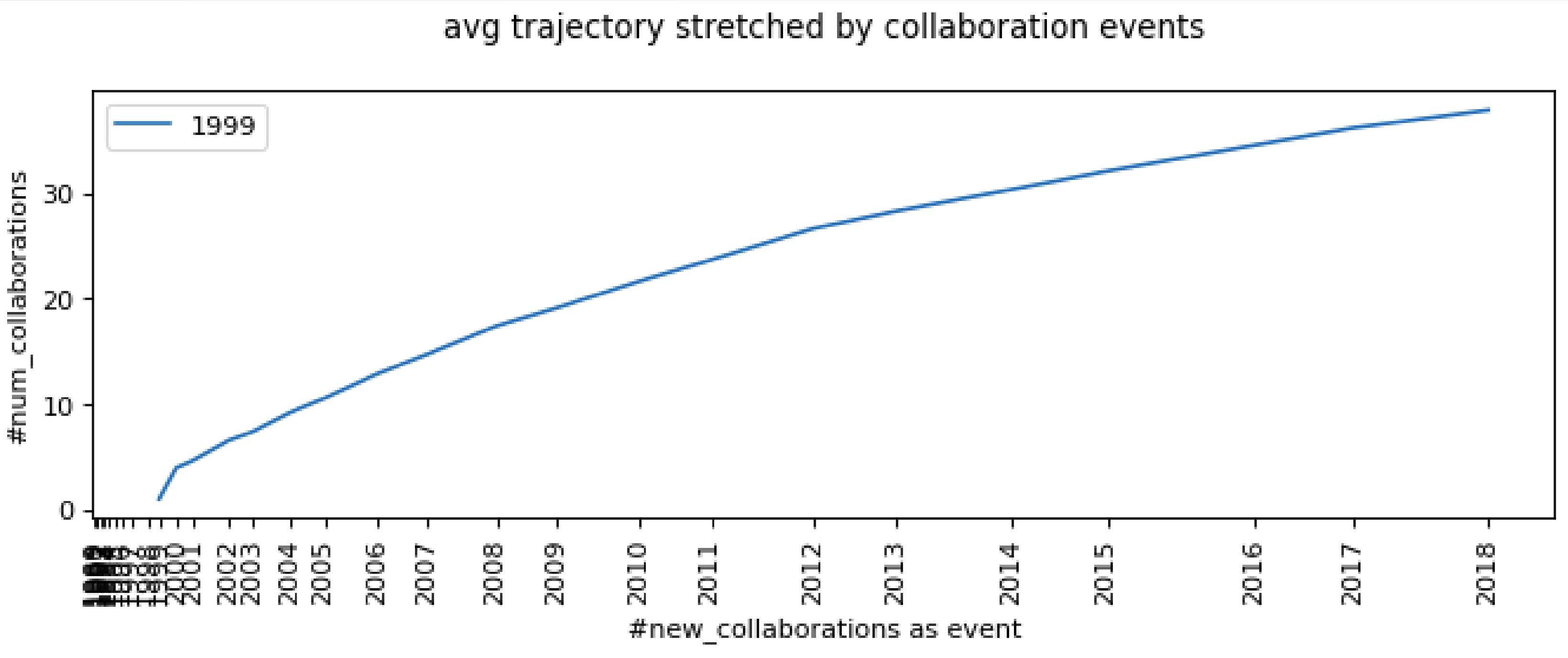
05

VERTEX TRAJECTORIES

Average Trajectories
Fitting Trajectories
General Fitting
Final Contraddiction

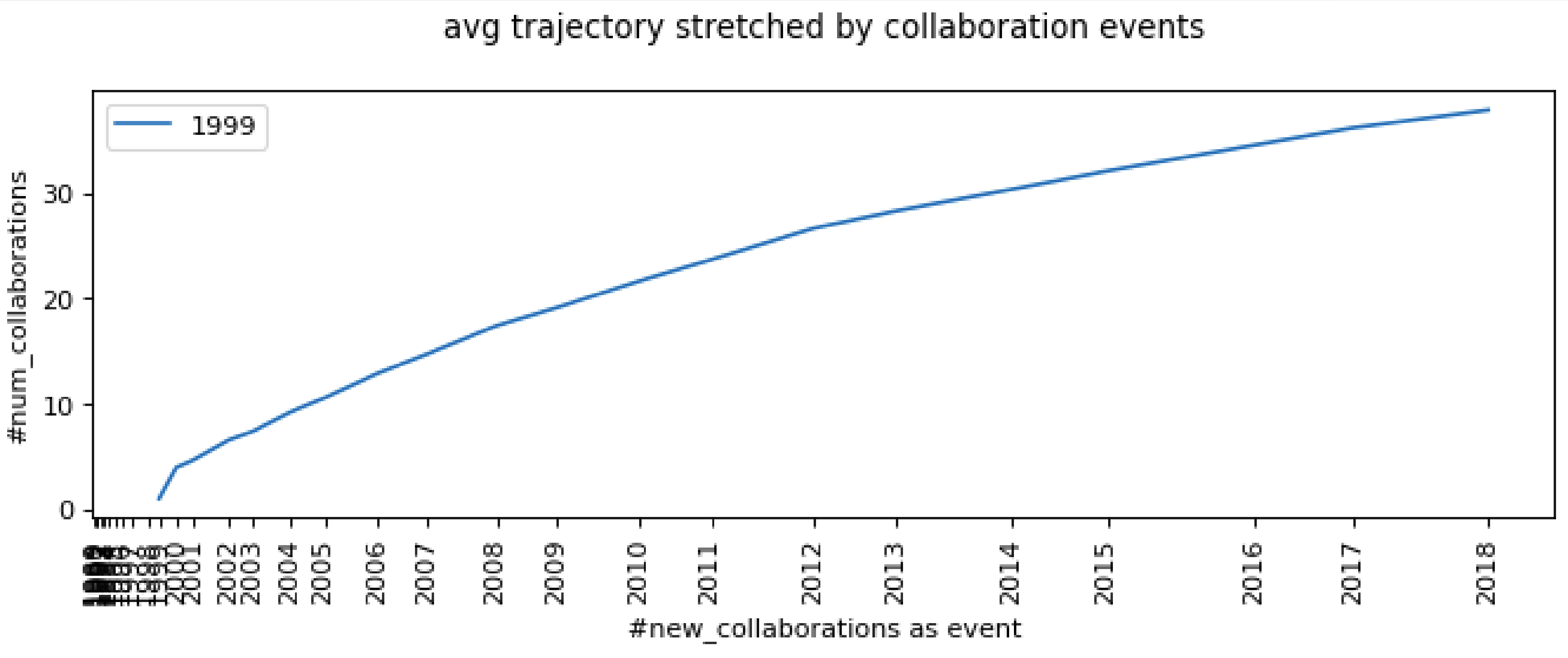
AVERAGE TRAJECTORIES

- Computed and plotted the **average trajectory of active authors** by starting year.



AVERAGE TRAJECTORIES

- Computed and plotted the **average trajectory of active authors** by starting year.
- The x-axis is stretched by the number of new collaborations as events.



FITTING TRAJECTORIES

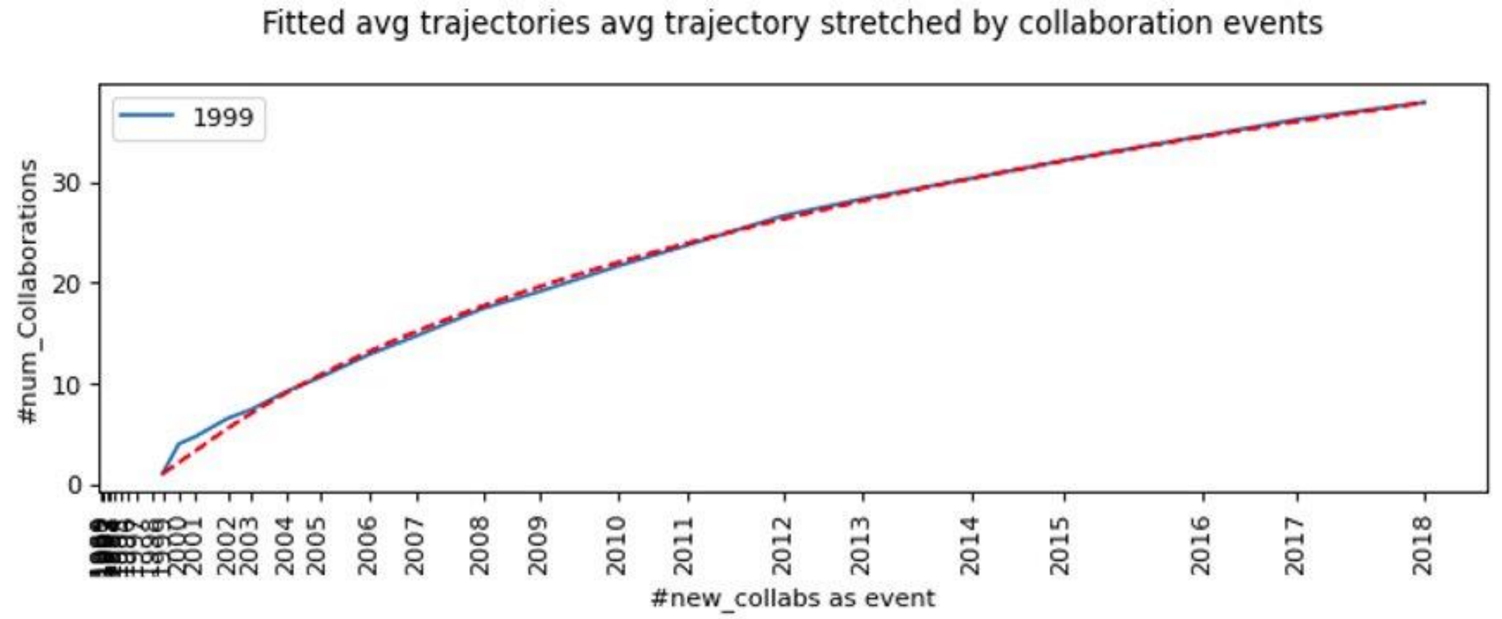
- Average trajectories are fitted one by one using the **logarithmic function** representing the theoretical vertex trajectory.

$$d_v(t) = (\alpha \ln(t/t_v) + 1)^{\beta}$$

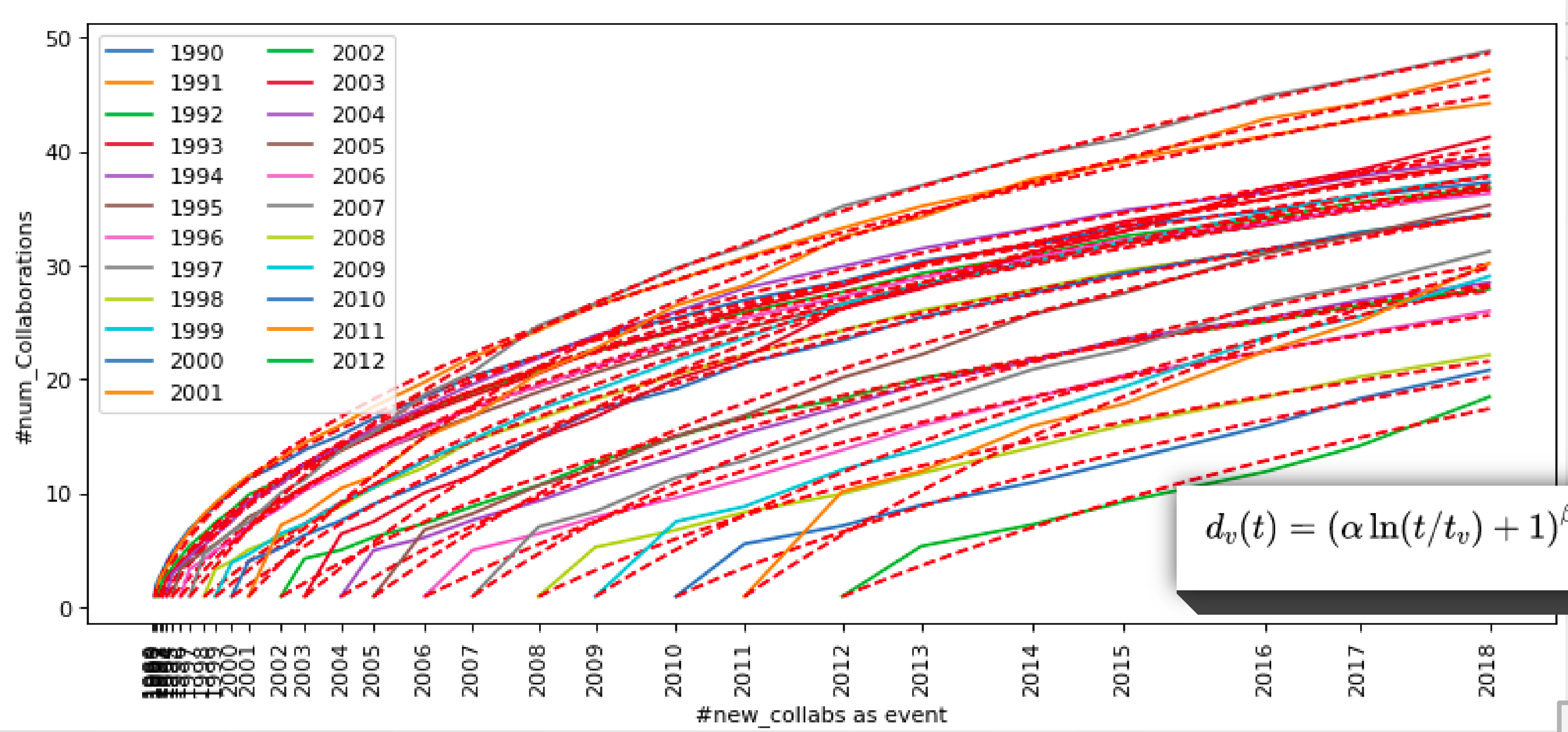
FITTING TRAJECTORIES

- Average trajectories are fitted one by one using the **logarithmic function** representing the theoretical vertex trajectory.
- Obtained a couple of individual parameter **α, β** for each.

$$d_v(t) = (\alpha \ln(t/t_v) + 1)^\beta$$



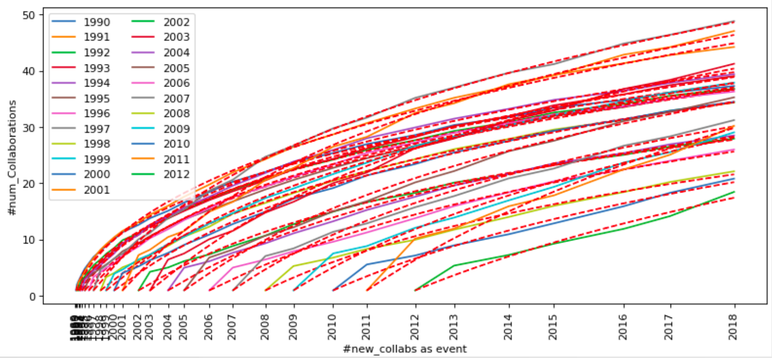
FITTING TRAJECTORIES



$$d_v(t) = (\alpha \ln(t/t_v) + 1)^\beta$$

FITTING TRAJECTORIES

$$d_v(t) = (\alpha^* \ln(t/t_v) + 1)^{\beta^*}$$



FITTING TRAJECTORIES

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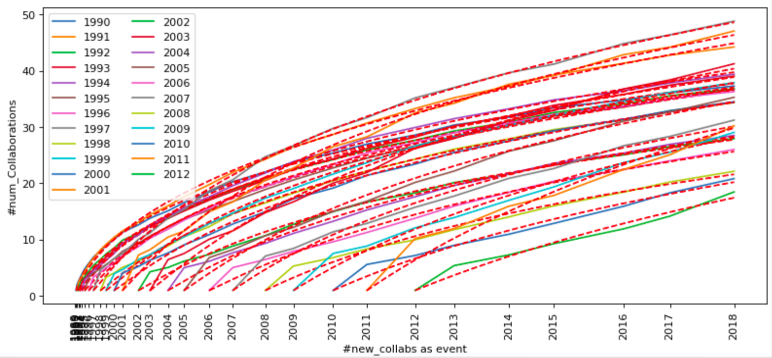

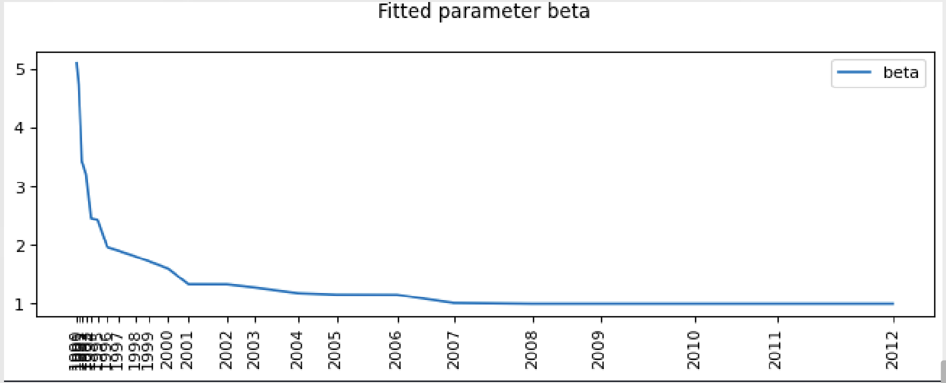

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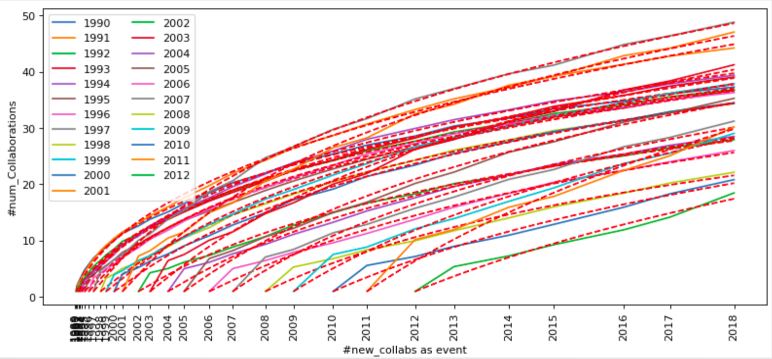

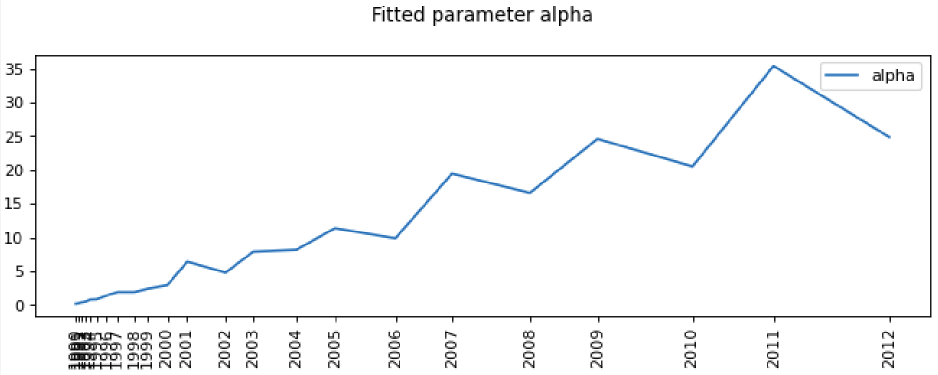

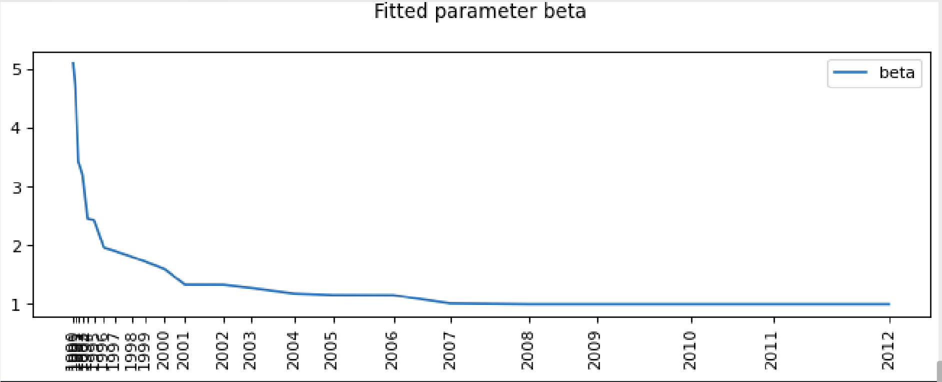

GENERAL FITTING

- Find $d_v(t)=(\alpha^* \ln(t/t_v) + 1)^{\beta^*}$ | α^* **and** β^* fit all curves minimizing the given error.
- Four different kinds of errors:

GENERAL FITTING

- Find **$d_v(t)=(\alpha^* \ln(t/t_v) + 1)^{\beta^*}$ | α^* and β^*** fit all curves minimizing the given error.

- Four different kinds of errors:

A) $min_{\alpha^*,\beta^*}\left(\sum_{t_v}\sum_{t\geq t_v}|d_{t_v}(t)^* - r_{t_v}(t)|^2\right)$

B) $min_{\alpha^*,\beta^*}\left(\sum_{t_v}\max_{t\geq t_v}|d_{t_v}(t)^* - r_{t_v}(t)|^2\right)$

C) $min_{\alpha^*,\beta^*}\left(\sum_{t_v}\sum_{t\geq t_v}|d_{t_v}(t)^* - d_{t_v}(t)|^2\right)$

D) $min_{\alpha^*,\beta^*}\left(\sum_{t_v}\max_{t\geq t_v}|d_{t_v}(t)^* - d_{t_v}(t)|^2\right)$

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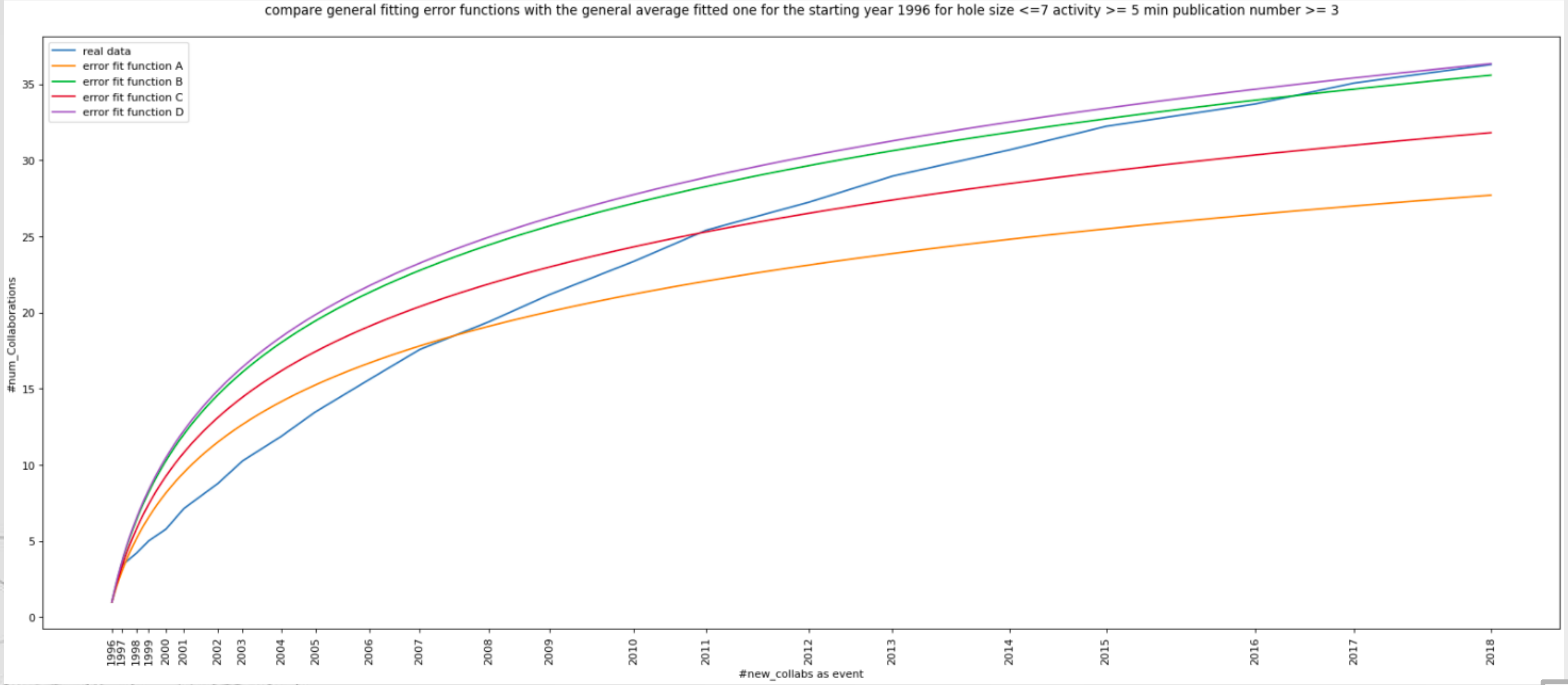
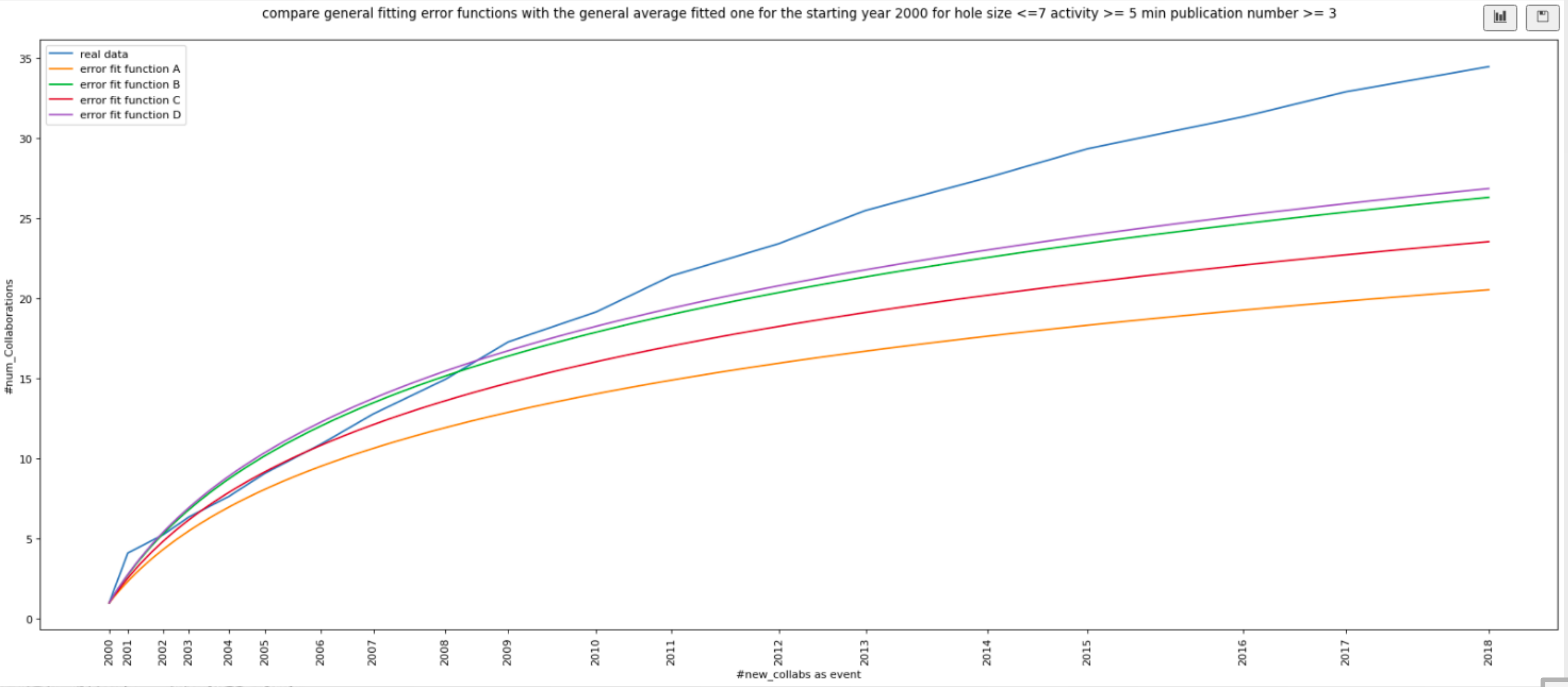
	ERROR	alpha	beta
err_f_A	27755.784282	6.929222	1
err_f_B	4301.973888	8.973418	1
err_f_C	444951.754961	7.993638	1
err_f_D	4714.507436	9.169267	1

GENERAL FITTING (1996)



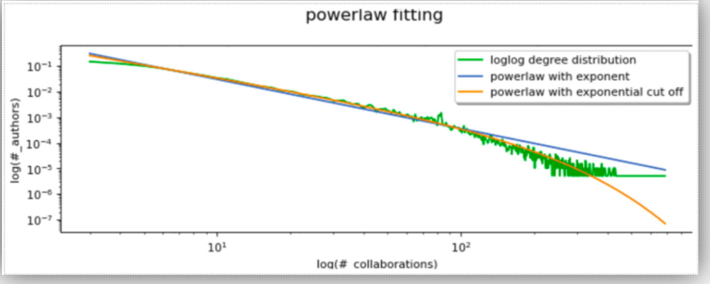
GENERAL FITTING (2000)



FINAL CONTRADDICTION

Real Data

M_k



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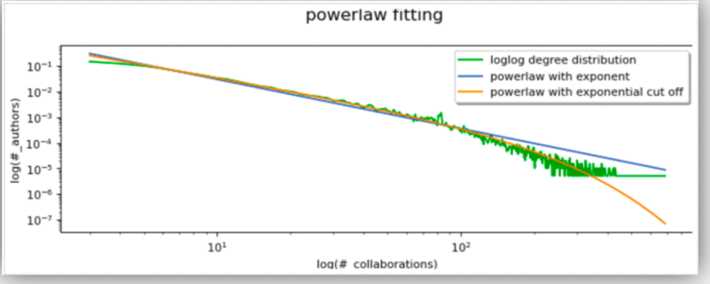


Theoretical model

- $0 \leq \gamma < 1$
- $f(d_v(t)) = dv(t)^{\gamma}$

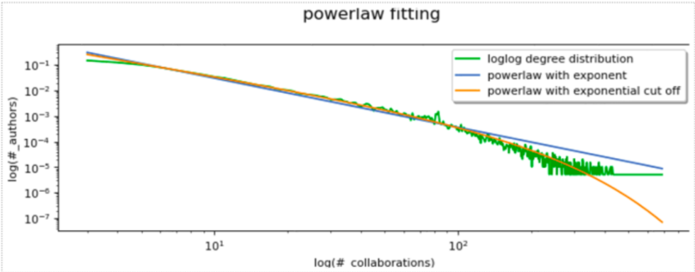
M_k

$$M_k \sim \alpha \cdot k^{-\gamma} \cdot \exp\left\{-\frac{\alpha}{1-\gamma} k^{1-\gamma}\right\}$$

FINAL CONTRADDICTION

Real Data

M_k



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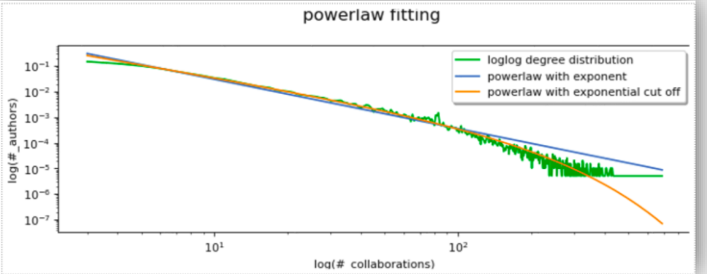

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=> • β seems to be 1

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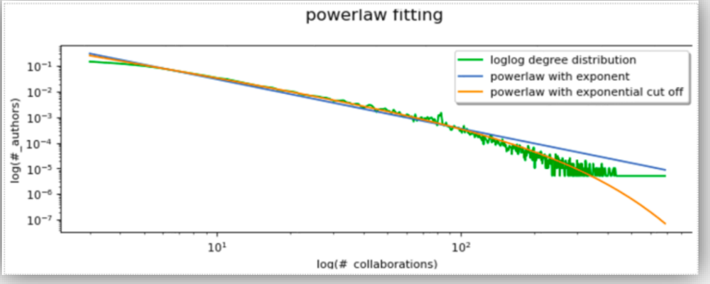

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FINAL CONTRADDICTION

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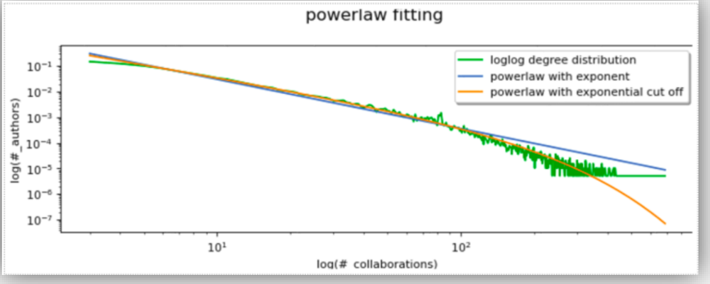
M_k



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- **β seems to be 1**

Theoretical model

- **$0 \leq \gamma < 1$**
- **$f(d_v(t)) = dv(t)^{\gamma}$**

M_k

$$M_k \sim \alpha \cdot k^{-\gamma} \cdot \exp\left\{-\frac{\alpha}{1-\gamma} k^{1-\gamma}\right\}$$

$d_v(t)$

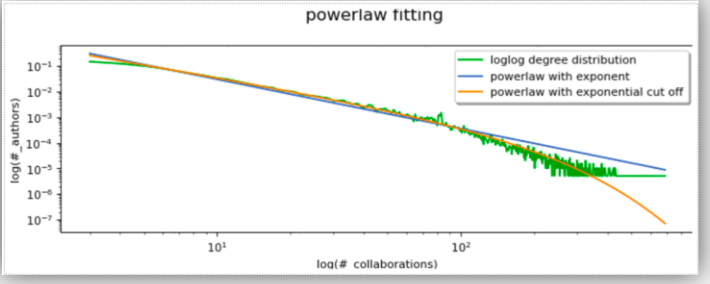
$$d_v(t) = \left(\frac{1-\gamma}{\alpha} \ln(t/t_v) + 1\right)^{1/(1-\gamma)}$$

=> $$\beta = \frac{1}{1-\gamma}$$

FINAL CONTRADDICTION

Real Data

M_k



$d_v(t)$

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- β seems to be 1

=> - $\gamma = 0$ and $f(d_v(t)) = 1$

Theoretical model

- $0 \leq \gamma < 1$
- $f(d_v(t)) = dv(t)^{\gamma}$

M_k

$$M_k \sim \alpha \cdot k^{-\gamma} \cdot \exp\left\{-\frac{\alpha}{1-\gamma} k^{1-\gamma}\right\}$$

$d_v(t)$

$$d_v(t) = \left(\frac{1-\gamma}{\alpha} \ln(t/t_v) + 1\right)^{1/(1-\gamma)}$$

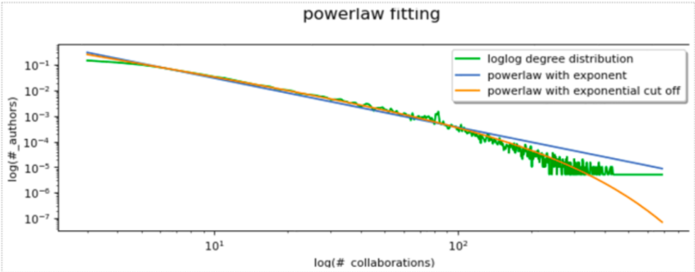
$$\beta = \frac{1}{1-\gamma}$$

=>

FINAL CONTRADDICTION

Real Data

M_k



$d_v(t)$ $d_v(t) = (\alpha \ln(t/t_v) + 1)^\beta$

- β seems to be 1
- γ = 0 and $f(d_v(t)) = 1$

Theoretical model

- $0 \leq \gamma < 1$
- $f(d_v(t)) = dv(t)^\gamma$

M_k

$$M_k \sim \alpha \cdot k^{-\gamma} \cdot \exp\left\{-\frac{\alpha}{1-\gamma} k^{1-\gamma}\right\}$$

$d_v(t)$

$$d_v(t) = \left(\frac{1-\gamma}{\alpha}\ln(t/t_v) + 1\right)^{1/(1-\gamma)}$$

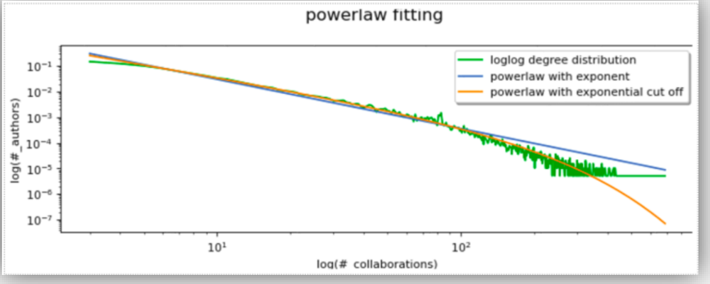
$$\beta = \frac{1}{1-\gamma}$$

=> $$\Pr[v \text{ is chosen at time step t}] = \frac{f\left(\mathrm{d}_t(v)\right)}{\sum_{w \in V_t} f\left(\mathrm{d}_t(w)\right)}$$

FINAL CONTRADDICTION

Real Data

M_k



$d_v(t)$

$$d_v(t) = (\alpha \ln(t/t_v) + 1)^{\beta}$$

- **β seems to be 1**
- **$\gamma = 0$ and $f(d_v(t)) = 1$**

Theoretical model

- **$0 \le \gamma < 1$**
- **$f(d_v(t)) = dv(t)^{\gamma}$**

M_k

$$M_k \sim \alpha \cdot k^{-\gamma} \cdot \exp\left\{-\frac{\alpha}{1-\gamma} k^{1-\gamma}\right\}$$

$d_v(t)$

$$d_v(t) = \left(\frac{1-\gamma}{\alpha} \ln(t/t_v) + 1\right)^{1/(1-\gamma)}$$

$$\beta = \frac{1}{1-\gamma}$$

$$\Pr[v \text{ is chosen at time step t}] = \frac{1}{n}$$ **<=**

06

NEXT STEPS

NEXT STEPS

The underlying theoretical model may be different from the one we assumed at the beginning:

Next steps:

1. **Generating a network** on a given theoretichal model **($f(d_v(t)=1)$** to fit our data;
2. Generating slightly different networks to check **which represent better** our case;
3. Defining a **new theorethical model** describing better the discovered behavior [11];
4. The analysis of data from **other research field**;

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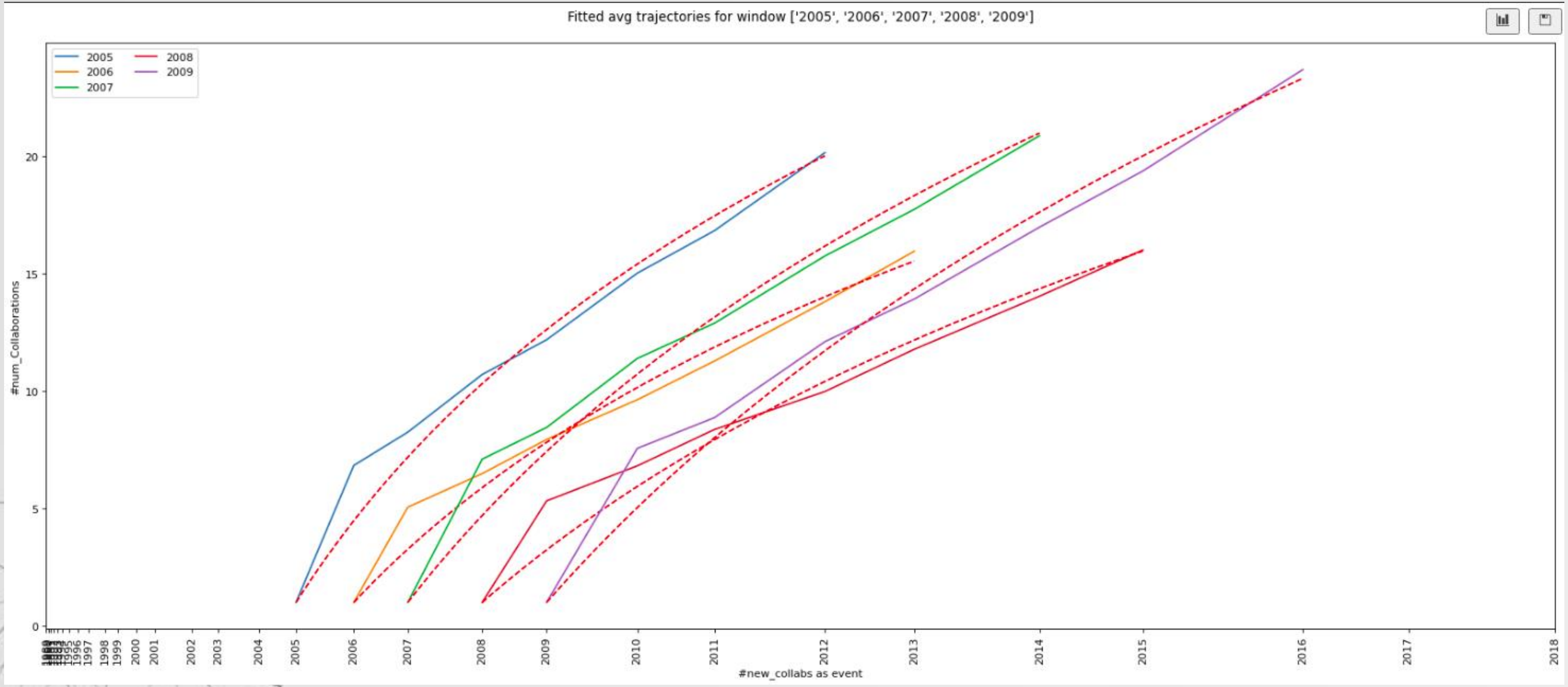
The End

WINDOW-SECTION FITTING

The same procedure is applied to **five year windows** and considering only the first **eight year section** of each trajectory.



WINDOW-SECTION FITTING

The general fitting this time is performed of the A and C errors type defined previously obtain the following parameters.

window	alpha	beta	error A
[1989, 1990, 1991, 1992, 1993]	0.642520	1.000000	315.834775
[1990, 1991, 1992, 1993, 1994]	2.479811	1.039867	45.397347
[1991, 1992, 1993, 1994, 1995]	3.227549	1.000000	38.597747
[1992, 1993, 1994, 1995, 1996]	3.733596	1.000000	48.907064
[1993, 1994, 1995, 1996, 1997]	4.625937	1.000000	113.231100
[1994, 1995, 1996, 1997, 1998]	5.258093	1.000000	88.733436
[1995, 1996, 1997, 1998, 1999]	6.043890	1.000000	86.015274
[1996, 1997, 1998, 1999, 2000]	6.907860	1.000000	59.308019
[1997, 1998, 1999, 2000, 2001]	8.611239	1.000000	221.401225
[1998, 1999, 2000, 2001, 2002]	8.964712	1.000000	213.152739
[1999, 2000, 2001, 2002, 2003]	10.635242	1.000000	272.652060
[2000, 2001, 2002, 2003, 2004]	11.634065	1.000000	210.052291
[2001, 2002, 2003, 2004, 2005]	13.394332	1.000000	177.619443
[2002, 2003, 2004, 2005, 2006]	13.390073	1.000000	174.522331
[2003, 2004, 2005, 2006, 2007]	15.449380	1.000000	144.535014
[2004, 2005, 2006, 2007, 2008]	15.642647	1.000000	144.264338
[2005, 2006, 2007, 2008, 2009]	17.902454	1.000000	210.980700
[2006, 2007, 2008, 2009, 2010]	18.600528	1.000000	208.848946
[2007, 2008, 2009, 2010, 2011]	21.851542	1.000000	414.262737

window	alpha	beta	error C
[1989, 1990, 1991, 1992, 1993]	1.680922	1.000000	116469.182190
[1990, 1991, 1992, 1993, 1994]	2.795480	1.117923	4744.905551
[1991, 1992, 1993, 1994, 1995]	3.054005	1.100979	3556.120018
[1992, 1993, 1994, 1995, 1996]	4.164666	1.000000	5641.857099
[1993, 1994, 1995, 1996, 1997]	4.857185	1.000000	16296.886017
[1994, 1995, 1996, 1997, 1998]	5.392140	1.000000	12396.764940
[1995, 1996, 1997, 1998, 1999]	5.968222	1.000000	14108.820976
[1996, 1997, 1998, 1999, 2000]	6.825733	1.000000	7909.599641
[1997, 1998, 1999, 2000, 2001]	8.262750	1.000000	26662.213671
[1998, 1999, 2000, 2001, 2002]	8.596449	1.000000	26030.916200
[1999, 2000, 2001, 2002, 2003]	10.192800	1.000000	29151.731420
[2000, 2001, 2002, 2003, 2004]	11.310162	1.000000	21234.077434
[2001, 2002, 2003, 2004, 2005]	13.104896	1.000000	13719.973595
[2002, 2003, 2004, 2005, 2006]	12.935749	1.000000	13522.175020
[2003, 2004, 2005, 2006, 2007]	15.101792	1.000000	6540.280182
[2004, 2005, 2006, 2007, 2008]	15.176833	1.000000	6653.254309
[2005, 2006, 2007, 2008, 2009]	1.961128	2.388138	22719.342323
[2006, 2007, 2008, 2009, 2010]	17.747342	1.000000	6585.673234
[2007, 2008, 2009, 2010, 2011]	20.972356	1.000000	9226.825981