Web Algorithms – Sponsored Search

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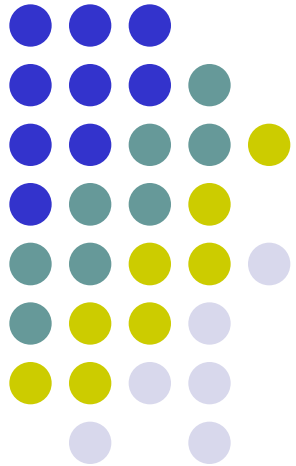
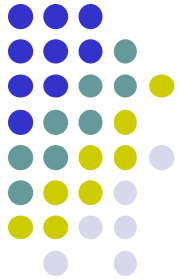
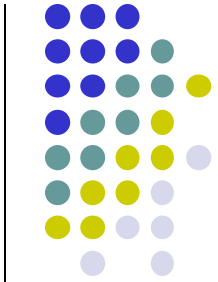


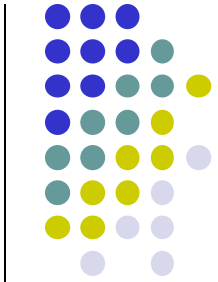


Auctions

Introduction



- Auction: ancient economic activity
- Used by governments to sell Treasure bills, timber, oil leases, or to buy services, infrastructures, ….
- Used by companies to sell wine goods, art, ...
- Brought into everyday life by the internet
- Examples: *eBay*, sponsored web search, …



- We first consider the simplified setting of a seller auctioning a single item to a set of buyers, who bid to get the item

- In the jargon of auctions:
  - seller ≡ auctioneer
  - buyers ≡ bidders

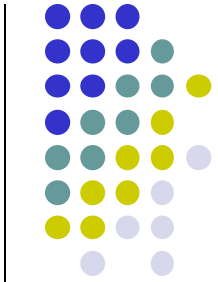
- Every bidder has an *intrinsic value* v_i for the item being auctioned

- He is interested in buying this item for a price up to this value

- Such a value is also called the bidder's *true value*

- Auctions useless if auctioneer knows the true values: it sells to the best bidder for a price close to her true value

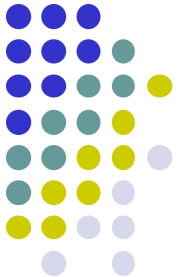
Types of Auctions



Four main types of auctions with single item sold:

- Ascending-bid (or English) auctions:
  - Carried interactively in real time
  - Auctioneer gradually raises the price
  - Bidders drop out, until finally only one remain
  - Such a bidder wins the item at this final price

- Descending-bid (or Dutch) auctions:
  - Carried interactively in real time
  - Auctioneer gradually lowers the price, starting from a high price
  - When a bidders accepts he wins at the current price
  - Used to sell flowers in the Netherlands (Dutch)



- First-price sealed-bid auctions:
  - Bidders simultaneously submit “sealed bids” to the auctioneer
  - Once implemented writing down bids and providing them in sealed envelops
  - The highest bidder wins
  - She pays her bid

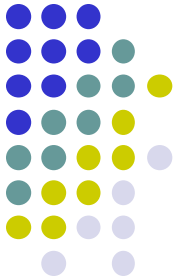
- Second-price sealed-bid (Vickrey) auctions:
  - Bidders simultaneously submit “sealed bids” to the auctioneer
  - The highest bidder wins
  - She pays the second highest bid
  - Vickrey wrote the first game-theoretical analysis of these auctions, getting the Nobel prize

Relationship between Types

Descending-bid and first-price auctions

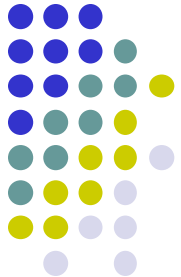
- In descending-bid auctions the price is lowered until someone accepts
- For each bidder i, there is a price b_i at which she will break the silence and accept at price b_i
- Thus, it is a equivalent to sealed-bid first-price auctions
- In fact, prices b_i of the descending auction play the role of bids in first-price auctions

Ascending-bid and second-price auctions

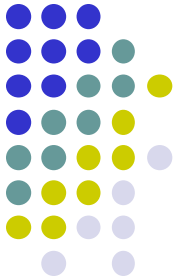


- In ascending-bid auctions the price is increased until only one remains
- For each bidder *i*, there is a price b_i at which she will drop out
- If she wins, she will pay the price of the last dropping out bidder
- So, the highest bidder gets the item at the price of the second highest bidder: it is a equivalent to sealed-bid second-price auctions
- Again, prices b_i of the descending auction play the role of bids in first-price auctions

Summarizing …



- First-price auctions can be seen as sealed-bid simulations of descending auctions
- Second-price auctions can be seen as sealed-bid simulations of ascending auctions
- But why should the auctioneer prefer the rather counter-intuitive second-price auctions with respect to first-price?
- After all, they seem to provide her less revenue, since they sell at a lower price, that is the second highest bid instead of the first

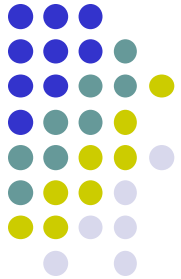


- In order to give an answer, let us investigate the bidders' behaviour of second-price auctions resorting on the equivalent ascending auctions

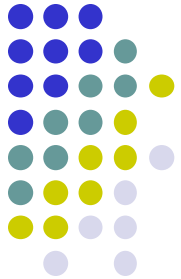
- When a bidder *i* in ascending auctions should decide to drop out?

- After the price reaches her true value v_i, staying in she either loses or wins paying more than v_i: better to drop out

- Before the price reaches her true value v_i, dropping out she gets nothing, but staying she might win at a price lower than v_i: better to stay

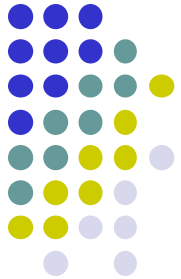


- In other words, she must drop out exactly when the price gets equal to her true value v_i
- From the point of view of the equivalent second-price auction, she should set her bid $b_i = v_i$
- Hence, *truthful bidding*, i.e. setting $b_i = v_i$, is the best solution for each bidder *i*

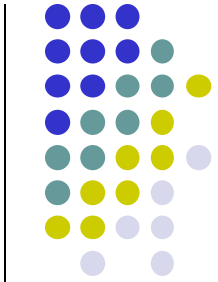


- But let's consider before first-price auctions
- A message coming from game-theory is that once you establish rules for people, people adapt …
- That is, in first-price auctions, they will tend to underbid, that is to set $b_i < v_i$, in order to strike a bargain
- The lowering of the bids offsets the difference between first and second price
- So only a superficial comparison would suggest that first-price is better than second-price
- In fact, it is possible to prove that under suitable conditions they provide the same expected revenue to the seller

Second-Price Auctions



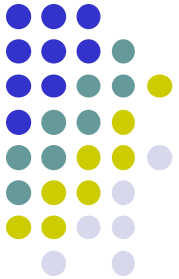
- Let us now give a closer look to second-price auctions
- Widely used:
  - Ebay (essentialy second-price)
  - Sponsored web search (a proper generalization)
- They are particularly relevant for their nice properties
- They are truthful: bidding the true value is a *dominant strategy*, that is it is always the most convenient choice



- Each bidder *i*
  - Has a true value v_i
  - Her strategy is a selection of an amount b_i
  - If b_i is not the winning bid, the payoff is *0*
  - If b_i is the winning bid and the second highest bid is b_j, the payoff is $v_i - b_j$

- If ties, that is two or more bidders submit the highest bid:
  - The bidder *i* among them with minimum *i* wins
  - The second highest price is the one of some other bidder in the same set, that is *i* gets payoff *0*

First-Price Auctions



- Let us now consider first-price auctions

- Each bidder i
  - Has a true value v_i
  - Her strategy is a selection of an amount b_i
  - If b_i is not the winning bid, the payoff is 0
  - If b_i is the winning bid the payoff is $v_i - b_i$

- They are not truthful: bidding the true value always provides payoff 0 !

- So bidders must underbid to get positive payoffs
- How much?
- Compromise between two factors:
  - Bidding closer to true value small payoff in case of win
  - Bidding lower reduces chances of winning
- Optimal trade-off difficult problem
  - It requires knowledge of the other bidders, that is of their expected valuation
  - With many bidders (with same properties) you should bid higher, as largest other bid tends to be higher