

**Question 1**

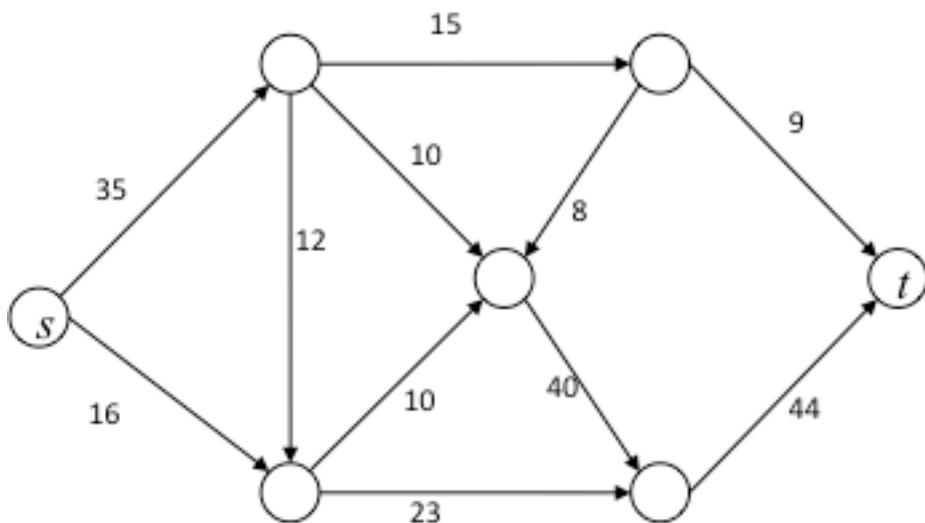
Given a directed graph  $G=(N,A)$  and two nodes  $s$  and  $t$ , propose an algorithm to find the maximum number of node disjoint  $s-t$  paths.

**Question 2**

A group of  $p$  families goes out to dinner together. The restaurant has  $q$  tables and each table has seating capacity  $b_1, \dots, b_j$ . Describe a model to find a seating arrangement (if any) such that no two members of the same family seat at the same table.

**Exercise 1**

Find the maximum flow and the minimum cut on the following graph:



**Exercise 2**

Given the following matrix:

$$\begin{vmatrix} 9.15 & 1.35 & 3.98 & 4.16 \\ 3.58 & 7.14 & 2.32 & 10.53 \\ 6.32 & 9.31 & 8.67 & 4.75 \end{vmatrix}$$

Let

$$r_i = \sum_{j=1}^4 a_{ij}, \text{ for } i \in 1, \dots, 3$$

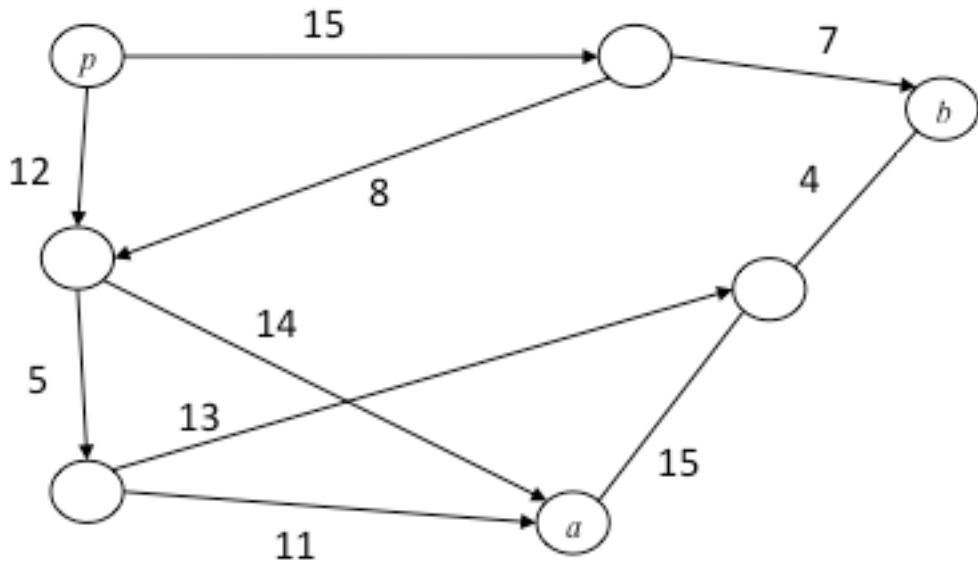
$$c_j = \sum_{i=1}^3 a_{ij}, \text{ for } j \in 1, \dots, 4$$

Round each element  $a_{ij}$ ,  $r_i$  and  $c_j$  up or down to integer so that the sum of the rounded elements in each row (column) equals row (column) sum.

**Exercise 3**

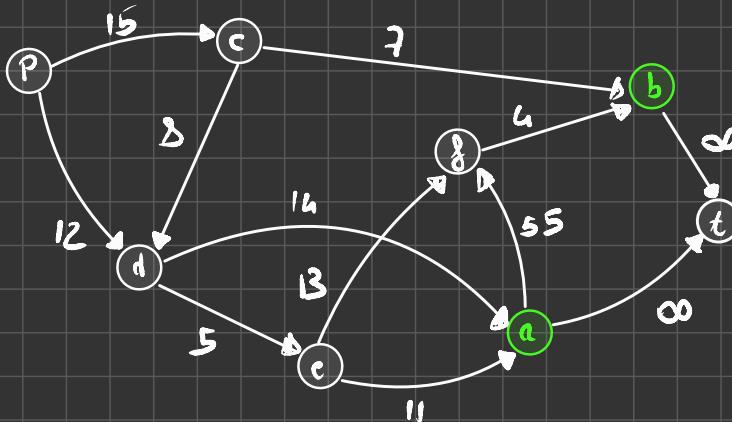
The following graph  $G=(N,A)$  represents a logistic distribution network. Node  $p$  is a manufacturing plant, origin of the goods and nodes  $a$  and  $b$  represent warehouses.

1. Evaluate the maximum quantity of goods that can be shipped from the plant to the warehouses
2. Suggest a method to increase goods shipping at warehouse  $b$  by at least 10% while keeping unchanged the quantity of goods shipped at the warehouse  $a$ .



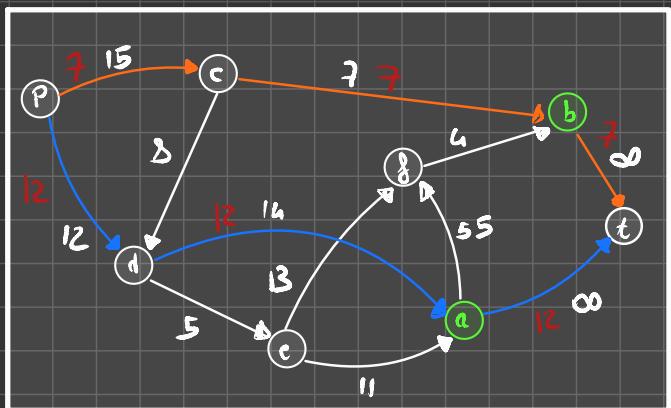
### Exercise 3

The following graph  $G=(N,A)$  represents a logistic distribution network. Node  $p$  is a manufacturing plant, origin of the goods and nodes  $a$  and  $b$  represent warehouses.

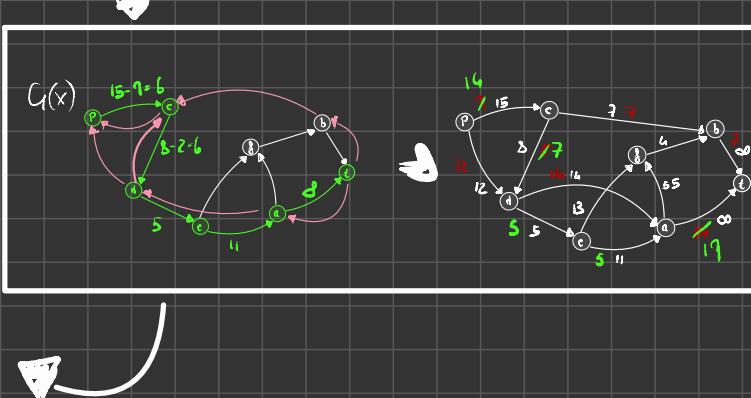
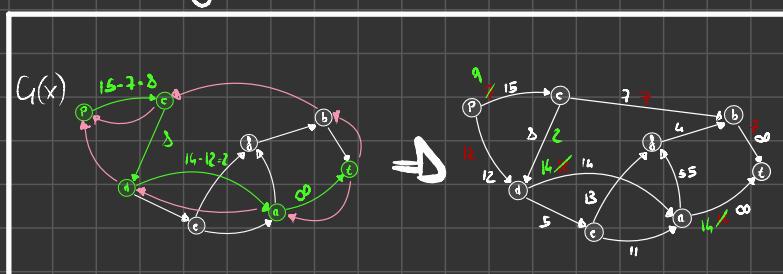


- Evaluate the maximum quantity of goods that can be shipped from the plant to the warehouses

- INITIAZI A FEAS Flow: Prendiamo un  $(P,t)$ -path a caso per ogni arco contrante in  $t$ , e settiamo il flow sul path per altra minima capacità degli archi che lo compongono



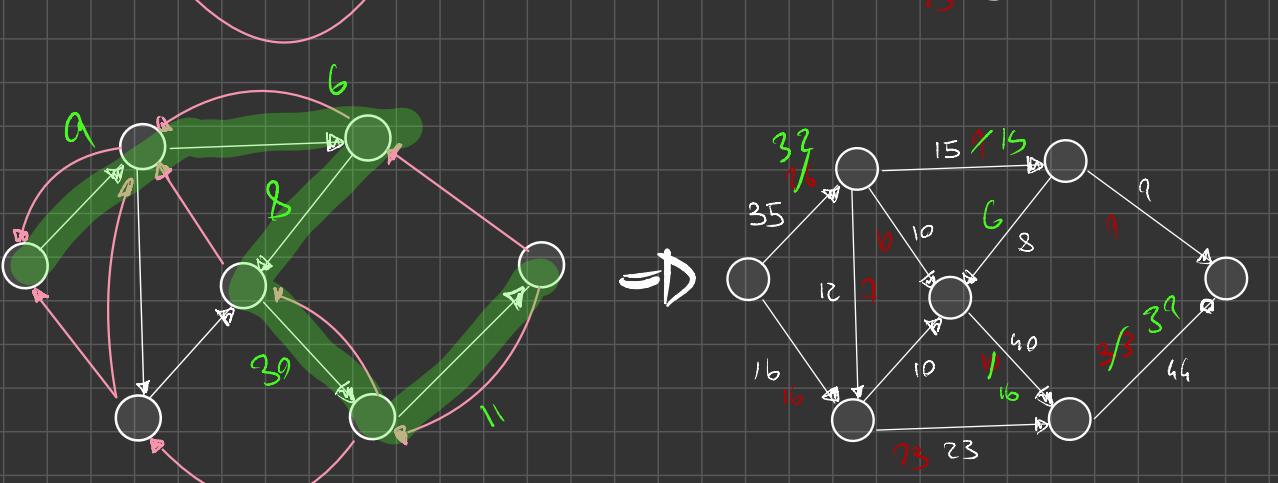
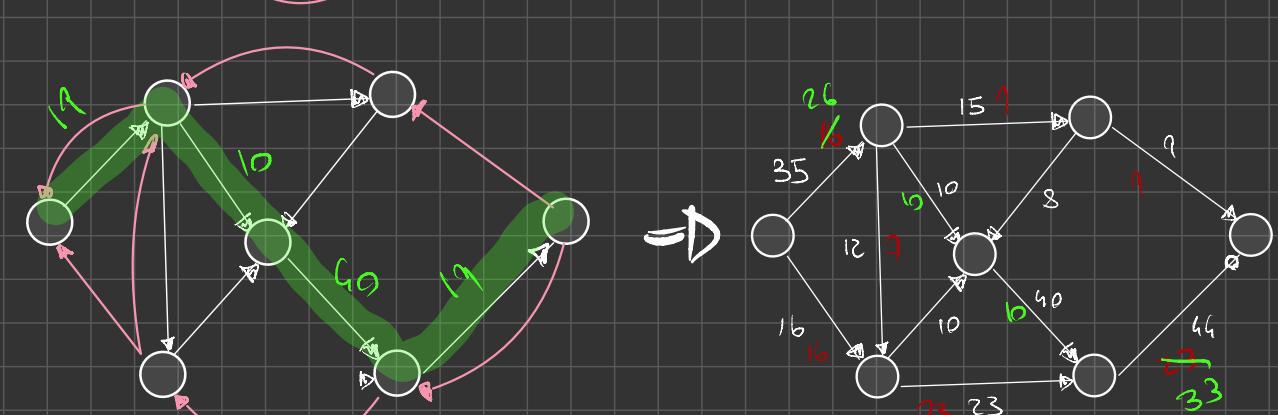
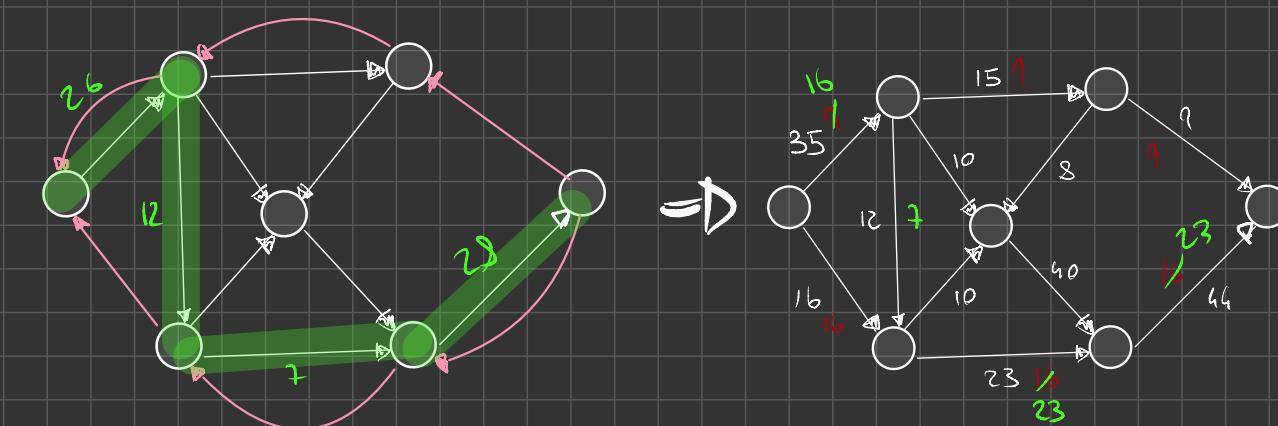
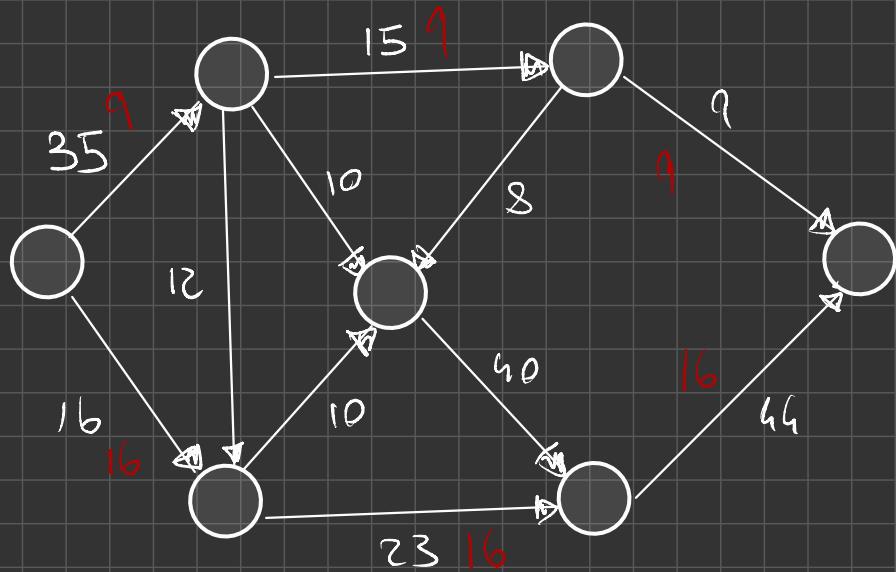
- F&F Algo per trovare il max-flow

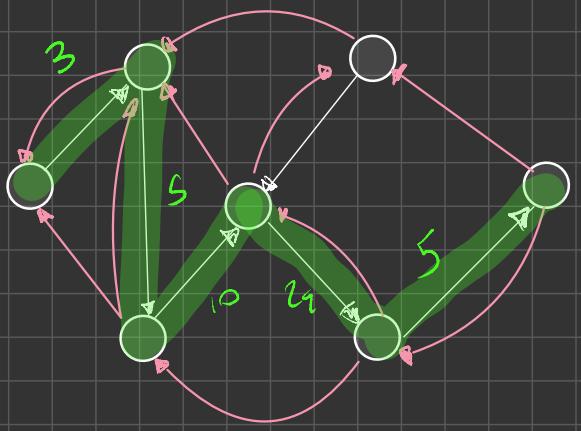


$\Rightarrow$  Il valore del flow equivale alla capacità del cut  $\Rightarrow f(P) = \mu(\delta(P)) = 7 + 14 + 5 = 26$

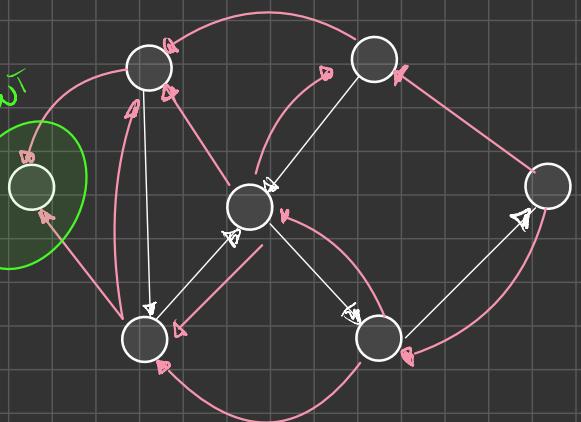
## Exercise 1

Find the maximum flow and the minimum cut on the following graph:

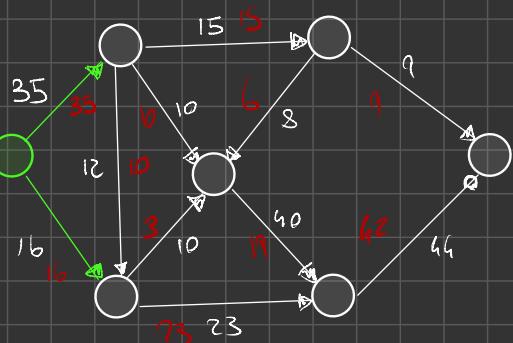
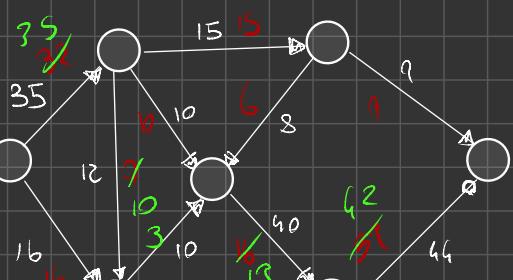




$\Rightarrow D$



$\Rightarrow D$



$$\Rightarrow 35 + 16 = \delta_x(S) = \mu(\delta(R)) = 35 + 16$$

## Question 2

A group of  $p$  families goes out to dinner together. The restaurant has  $q$  tables and each table has seating capacity  $b_1, \dots, b_j$ . Describe a model to find a seating arrangement (if any) such that no two members of the same family seat at the same table.

$$P = \# \text{FAMILIES}, \{a_i - a_p\} | a_i = \# \text{COMPONENTS FAMILIES}$$
$$Q = \# \text{TABLES}, \{b_i - b_q\} | b_i = \# \text{SEATS PER TABLES}$$

