Exercise on 0-1 knapsack problem

Let us suppose we have to catch a Lufthansa flight to Los Angeles. The baggage weight limit imposed by Lufthansa is 23 kg. Let us assume we are interested in bringing the following items.

Item	Weight a_i	Profit p_i	p_i/a_i		
I_0	6	10	5/3	→ 1,6	1
I_1	8	4	1/2	→ 0,5	6
I_2	8	9	9/8	→ 1,1	4
I_3	5	8	8/5	→ 1,6	2
I_4	10	9	9/10	→ 0,9	5
I_5	13	13	1	→ 1	3

1. Please apply the Greedy-Knapsack algorithm to solve this 0-1 knapsack problem instance. Please list and show all the different steps.

2. Does the solution change when applying the Modified-Greedy algorithm? NO

3. Please compute the solution applying the FPTAS-knapsack algorithm.

① $\{I_0, I_3, I_2\} = \emptyset$, $m_{GR} = 27$

② $P_{max} > m_{GR}$? $13 > 27$? No

$P = m \cdot P_{max} = 5 \cdot 13 = 65$

Item \ w_i	0	1	2	3	4	5	6	7	8	9	10	11	12	13
I_0	0													
I_1	0													
I_2	0													
I_3	0													
I_4	0													
I_5	0													

Solution

Item	Weight	Profit
I_0	6	10
I_1	8	4
I_2	8	9
I_3	5	8
I_4	10	9
I_5	13	13

$g_i = p_i/a_i$

$g_0 = 10/6 = 1.67$; $g_1 = 4/8 = 0.5$; $g_2 = 9/8 = 1.125$

$g_3 = 8/5 = 1.6$; $g_4 = 9/10 = 0.9$; $g_5 = 13/13 = 1$

Order: I_0, I_3, I_2, I_5, I_4, I_1

Step 1

I_0 -> $v + a_0 = 0 + 6 < b = 23$ -> $Q = \{I_0\}$; $v = 6$

Step 2

I_3 -> $v + a_3 = 6 + 5 = 11 < b = 23$ -> $Q = \{I_0, I_3\}$; $v = 11$

Step 3

I_2 -> $v + a_2 = 11 + 8 = 19 < b = 23$ -> $Q = \{I_0, I_3, I_2\}$; $v = 19$

Step 4

I_5 -> $v + a_5 = 19 + 13 = 32 < b = 23$ -> $Q = \{I_0, I_3, I_2\}$; $v = 19$; $m_{GR} = 27$

2. Since $p_{max} = 13 < 27 = m_{GR}$, the solution does not change when applying the modified-Greedy algorithm.

Exercise on 0-1 knapsack problem

Let us suppose to have the following instance of the 0-1 knapsack problem, with a knapsack capacity equal to 5 (b = 5).

Item	**Weight**	**Profit**
I_1	2	4
I_2	4	5
I_3	3	3
I_4	5	6

1. Please apply the *Progr-Dyn-Knapsack* algorithm to solve this 0-1 knapsack problem instance. Please fill out the table, return the solution with maximum profit, and list the items which are inserted into the knapsack in such a solution.

Solution

Item	Weight	Profit
I_1	2	4
I_2	4	5
I_3	3	3
I_4	5	6

i/w	0	1	2	3	4	5
0	0	0	0	0	0	0
1	0	0	4	4	4	4
2	0	0	4	4	5	5
3	0	0	4	4	5	7
4	0	0	4	4	5	7

- P(i=0, w=0) = P(i=1, w=0) = P(i=2, w=0) = P(i=3, w=0) = P(i=4, w=0) = P(i=0, w=1) = P(i=0, w=1) = P(i=0, w=2) = P(i=0, w=3) = P(i=0, w=4) = P(i=0, w=5) = 0.
- i = 1, w = 1; w_i = 2; w – w_i = -1; the i-th object exceeds the knapsack capacity -> P(i=1, w=1) = P(i-1, w) = 0
- i = 1, w = 2; w_i = 2; w – w_i = 0 (i-th object insertable into the knapsack); p_i = 4; P(i-1, w) = 0 < P(i-1, w-w_i) + p_i = 0 + 4 = 4 -> P(i=1, w=2) = 4
- i = 1, w = 3; w_i = 2; w – w_i = 1 (i-th object insertable into the knapsack); p_i = 4; P(i-1, w) = 0 < P(i-1, w-w_i) + p_i = 0 + 4 = 4 -> P(i=1, w=3) = 4
- i = 1, w = 4; w_i = 2; w – w_i = 2 (i-th object insertable into the knapsack); p_i = 4; P(i-1, w) = 0 < P(i-1, w-w_i) + p_i = 0 + 4 = 4 -> P(i=1, w=4) = 4
- i = 1, w = 5; w_i = 2; w – w_i = 3 (i-th object insertable into the knapsack); p_i = 4; P(i-1, w) = 0 < P(i-1, w-w_i) + p_i = 0 + 4 = 4 -> P(i=1, w=5) = 4
- i = 2, w = 1; w_i = 4; w – w_i = -3 the i-th object exceeds the knapsack capacity -> P(i=2, w=1) = P(i-1, w) = 0
- i = 2, w = 2; w_i = 4; w – w_i = -2 the i-th object exceeds the knapsack capacity -> P(i=2, w=2) = P(i-1, w) = 4
- i = 2, w = 3; w_i = 4; w – w_i = -1 the i-th object exceeds the knapsack capacity -> P(i=2, w=3) = P(i-1, w) = 4
- i = 2, w = 4; w_i = 4; w – w_i = 0 (i-th object insertable into the knapsack); p_i = 5; P(i-1, w) = 4 < P(i-1, w-w_i) + p_i = 0 + 5 = 5 -> P(i=2, w=4) = 5
- i = 2, w = 5; w_i = 4; w – w_i = 1 (i-th object insertable into the knapsack); p_i = 5; P(i-1, w) = 4 < P(i-1, w-w_i) + p_i = 0 + 5 = 5 -> P(i=2, w=5) = 5
- i = 3, w = 1; w_i = 3; w – w_i = -2 the i-th object exceeds the knapsack capacity -> P(i=3, w=1) = P(i-1, w) = 0

- $i = 3$, $w = 2$; $w_i = 3$; $w – w_i = -1$ the i-th object exceeds the knapsack capacity -> $P(i=3, w=2) = P(i-1, w) = 4$
- $i = 3$, $w = 3$; $w_i = 3$; $w – w_i = 0$ (i-th object insertable into the knapsack); $p_i = 3$; $P(i-1, w) = 4 > P(i-1, w-w_i) + p_i = 0 + 3 = 3$ -> $P(i=3, w=3) = 4$
- $i = 3$, $w = 4$; $w_i = 3$; $w – w_i = 1$ (i-th object insertable into the knapsack); $p_i = 3$; $P(i-1, w) = 5 > P(i-1, w-w_i) + p_i = 0 + 3 = 3$ -> $P(i=3, w=4) = 5$
- $i = 3$, $w = 5$; $w_i = 3$; $w – w_i = 2$ (i-th object insertable into the knapsack); $p_i = 3$; $P(i-1, w) = 5 < P(i-1, w-w_i) + p_i = 4 + 3 = 7$ -> $P(i=3, w=5) = 7$
- $i = 4$, $w = 1$; $w_i = 5$; $w – w_i = -4$ the i-th object exceeds the knapsack capacity -> $P(i=4, w=1) = P(i-1, w) = 0$
- $i = 4$, $w = 1$; $w_i = 5$; $w – w_i = -4$ the i-th object exceeds the knapsack capacity -> $P(i=4, w=1) = P(i-1, w) = 0$
- $i = 4$, $w = 2$; $w_i = 5$; $w – w_i = -3$ the i-th object exceeds the knapsack capacity -> $P(i=4, w=2) = P(i-1, w) = 4$
- $i = 4$, $w = 3$; $w_i = 5$; $w – w_i = -2$ the i-th object exceeds the knapsack capacity -> $P(i=4, w=3) = P(i-1, w) = 4$
- $i = 4$, $w = 4$; $w_i = 5$; $w – w_i = -1$ the i-th object exceeds the knapsack capacity -> $P(i=4, w=4) = P(i-1, w) = 5$
- $i = 4$, $w = 5$; $w_i = 5$; $w – w_i = 0$ (i-th object insertable into the knapsack); $p_i = 6$; $P(i-1, w) = 7 > P(i-1, w-w_i) + p_i = 0 + 6 = 6$ -> $P(i=4, w=5) = 7$

- $m = 7$
- S = the set of items inserted into the knapsack = $\{I_3, I_1\}$

Exercise on Min Multiprocessor Scheduling

Let us suppose to have the following instance of the Min Multiprocessor Scheduling problem, with 3 available CPUs (h = 3).

Process	**Running Time**
P_1	4
P_2	3
P_3	7
P_4	5
P_5	8
P_6	2
P_7	7
P_8	1

1. Please solve the problem instance by applying the *Greedy Algorithm of Graham*.

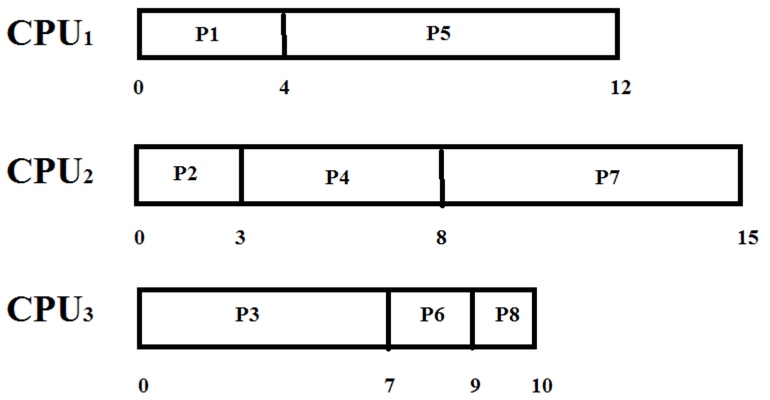
2. Please solve the problem instance by applying the *Ordered-Greedy Algorithm.*

3. Please solve the problem instance by applying the *PTAS-Scheduling Algorithm*, with different values of ε.

Solution

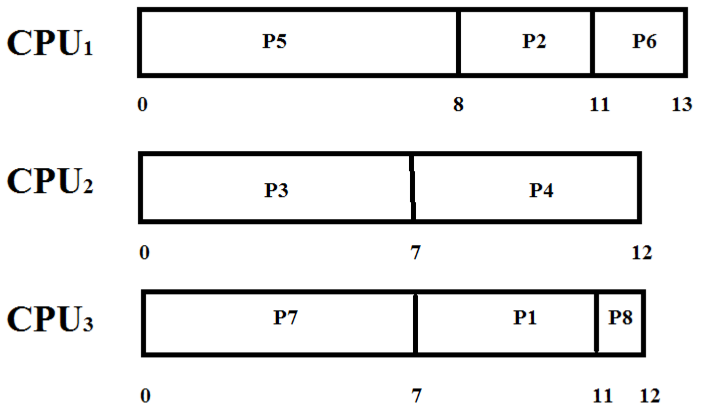
Process	Running Time
P_1	4
P_2	3
P_3	7
P_4	5
P_5	8
P_6	2
P_7	7
P_8	1

1. *Greedy Algorithm of Graham*



m = 15

2. *Ordered-Greedy Algorithm*



m = 13

Exercise on Min Weighted Set Cover

Let us consider the following instance of the Min-Weighted-Set-Cover problem:

- $U = \{1, 2, 3, 4, 5, 6\}$
- $S_1 = \{1, 6\}$; $S_2 = \{1, 2, 6\}$; $S_3 = \{1, 4\}$; $S_4 = \{1, 5\}$; $S_5 = \{1, 3, 5\}$; $S_6 = \{2, 6\}$
- $c_1 = 3$; $c_2 = 4$; $c_3 = 2$; $c_4 = 3$; $c_5 = 4$; $c_6 = 2$

1. Please apply the Greedy-Min-Weighted-Set-Cover algorithm to solve the problem instance. Please list and show all the different steps.

2. Please compute the optimal solution to the problem instance.

Solution

- $U = \{1, 2, 3, 4, 5, 6\}$
- $S_1 = \{1, 6\}$; $S_2 = \{1, 2, 6\}$; $S_3 = \{1, 4\}$; $S_4 = \{1, 5\}$; $S_5 = \{1, 3, 5\}$; $S_6 = \{2, 6\}$
- $c_1 = 3$; $c_2 = 4$; $c_3 = 2$; $c_4 = 3$; $c_5 = 4$; $c_6 = 2$

Step 0

$C = \phi$

$C\text{^} = \phi$

Step 1

$e_1 = 3/2$; $e_2 = 4/3$; $e_3 = 2/2$; $e_4 = 3/2$; $e_5 = 4/3$; $e_6 = 2/2$;

$C = \{1, 4\}$; $C\text{^} = \{S_3\}$

Step 2

$e_1 = 3/1$; $e_2 = 4/2$; $e_4 = 3/1$; $e_5 = 4/2$; $e_6 = 2/2$;

$C = \{1, 4, 2, 6\}$; $C\text{^} = \{S_3, S_6\}$

Step 3

$e_1 = 3/0$ (be careful when implementing it…); $e_2 = 4/0$; $e_4 = 3/1$; $e_5 = 4/2$;

$C = \{1, 4, 2, 6, 3, 5\}$; $C\text{^} = \{S_3, S_6, S_5\}$; $m = c_3 + c_6 + c_5 = 2 + 2 + 4 = 8$

Exercise n. 1

Given the matching market instance modeling the sponsored search described in the following:

- 3 slots (a, b, c);
- 3 advertisers (x, y, z);
- clickthrough rates: $r_a = 8$; $r_b = 4$; $r_c = 2$;
- revenues per click: $v_x = 4$; $v_y = 2$; $v_z = 1$

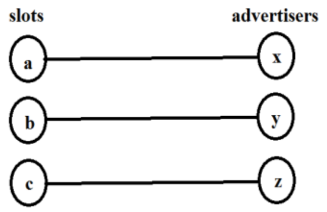
a. Please compute the best matching.
b. What is the best matching if the search engine associates advertisers with the following quality factors?
    - $q_x = 0.1$; $q_y = 0.6$; $q_z = 0.8$

Solution to Exercise n. 1

Point a)

clickthrough rates	slots	advertisers	revenues per click	valuations
8	a	x	4	**32**, 16, 8
4	b	y	2	16, **8**, 4
2	c	z	1	8, 4, **2**

Best Matching

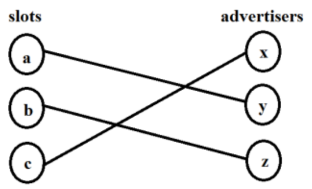


Total Valuation = 32 + 8 + 2 = 42

Point b)

clickthrough rates	slots	advertisers	revenues per click	valuations
$r_{ax} = 0.8$; $r_{ay} = 4.8$; $r_{az} = 6.4$	a	x	4	$v_{ax} = 3.2$; $v_{bx} = 1.6$; $\mathbf{v_{cx} = 0.8}$
$r_{bx} = 0.4$; $r_{by} = 2.4$; $r_{bz} = 3.2$	b	y	2	$\mathbf{v_{ay} = 9.6}$; $v_{by} = 4.8$; $v_{cy} = 2.4$
$r_{cx} = 0.2$; $r_{cy} = 1.2$; $r_{cz} = 1.6$	c	z	1	$v_{az} = 6.4$; $\mathbf{v_{bz} = 3.2}$; $v_{cz} = 1.6$

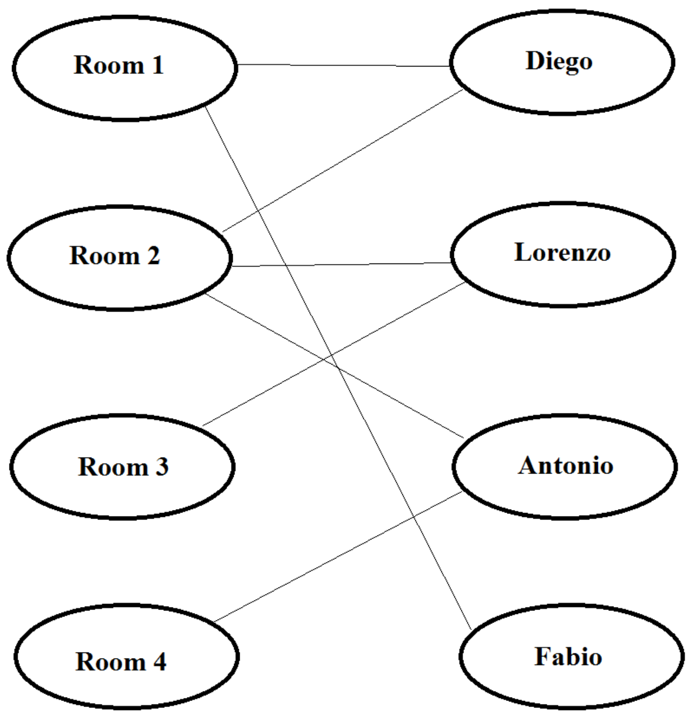
Best Matching



Total Valuation = 0.8 + 9.6 + 3.2 = 13.6

Exercise n. 2

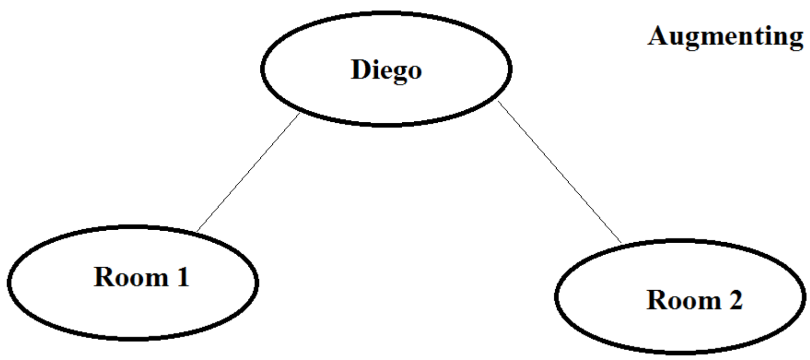
Given the following instance of the *room-assigning* problem, please compute the perfect matching by exploiting the *alternating breadth-first search* algorithm.



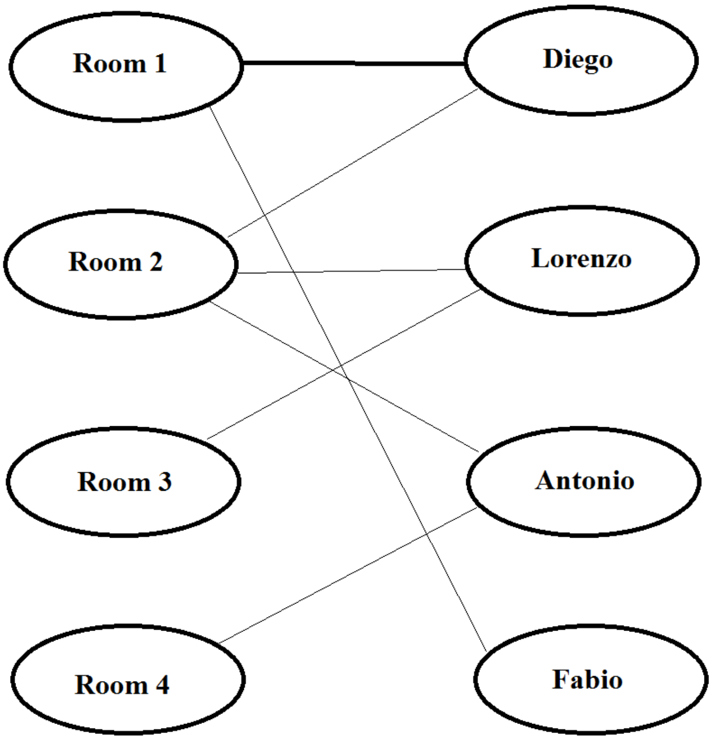
Solution to Exercise n. 2

First Step

W = Diego

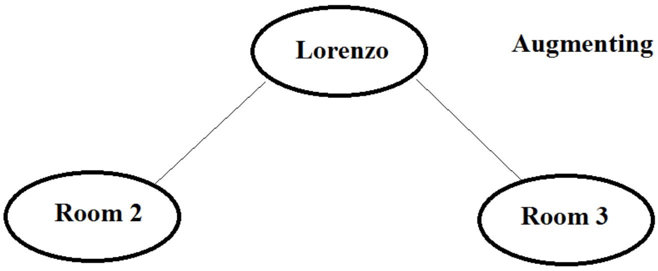


Augmenting Path chosen: Diego -> Room 1

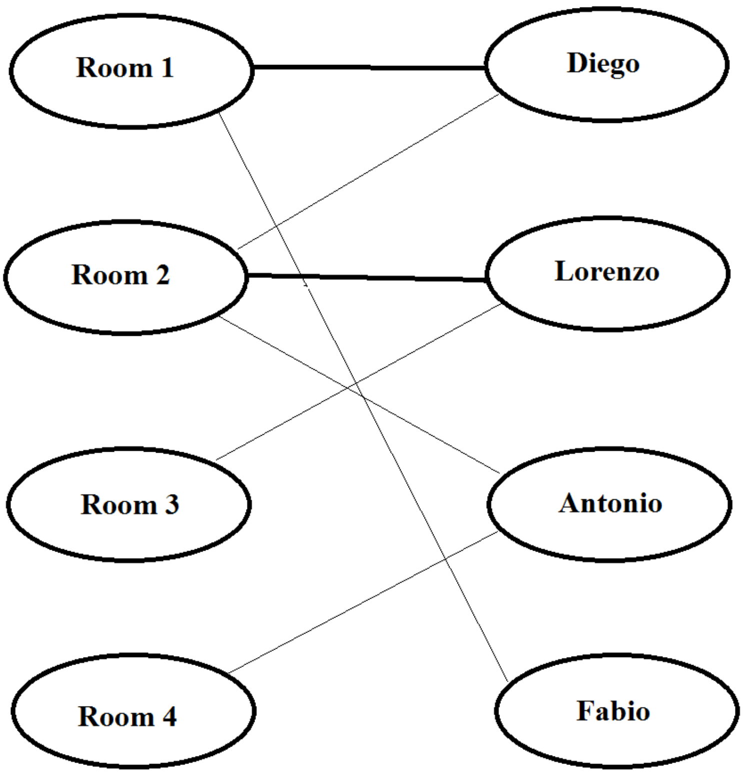


Second Step

W = Lorenzo

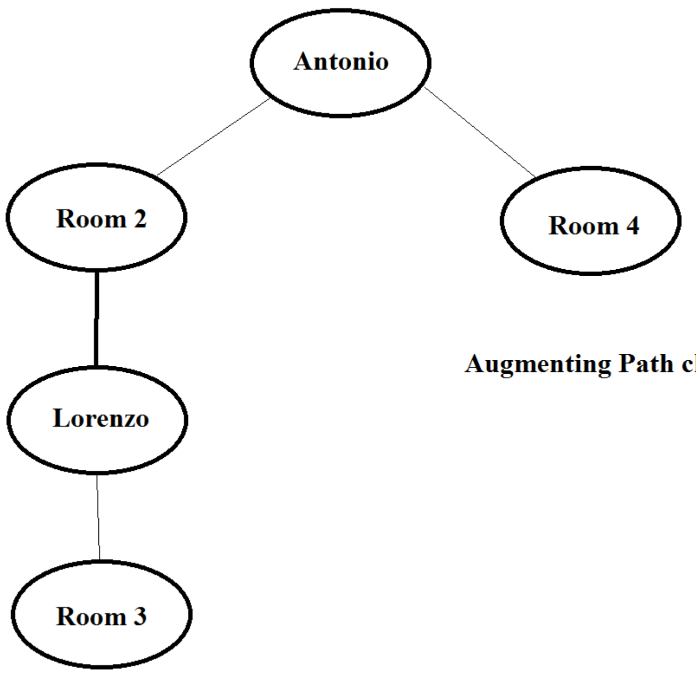


Augmenting Path chosen: Lorenzo -> Room 2

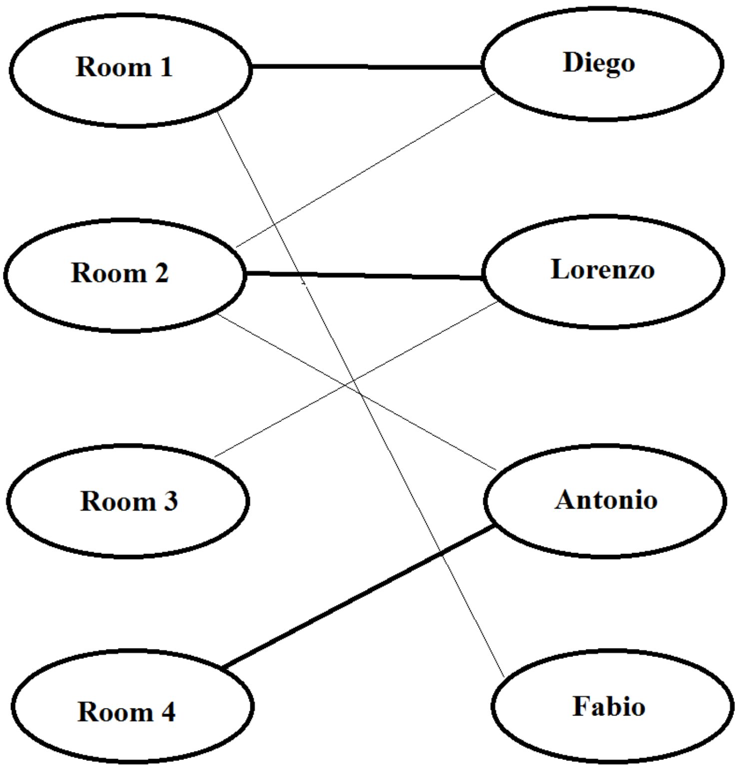


Third Step

W = Antonio



Augmenting Path chosen: Antonio -> Room 4

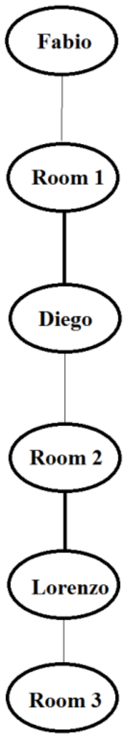


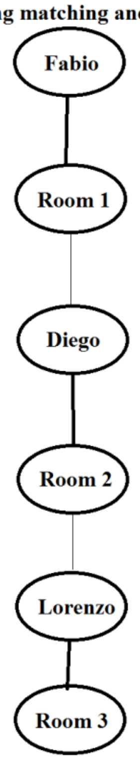
Fourth Step

W = Fabio

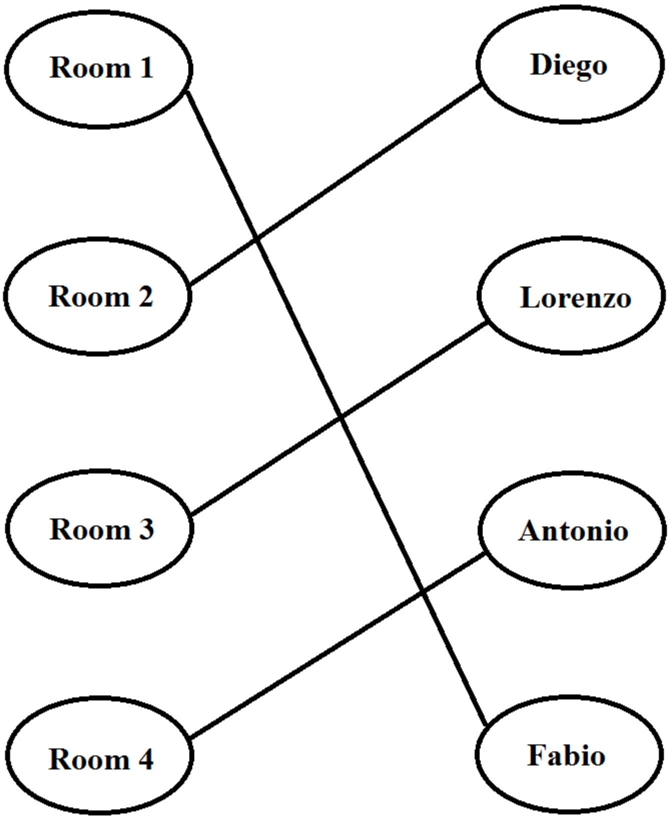
Augmenting Path: Fabio -> Room 1 -> Diego -> Room 2 -> Lorenzo -> Room 3

swapping matching and non-matching edges





Perfect Matching

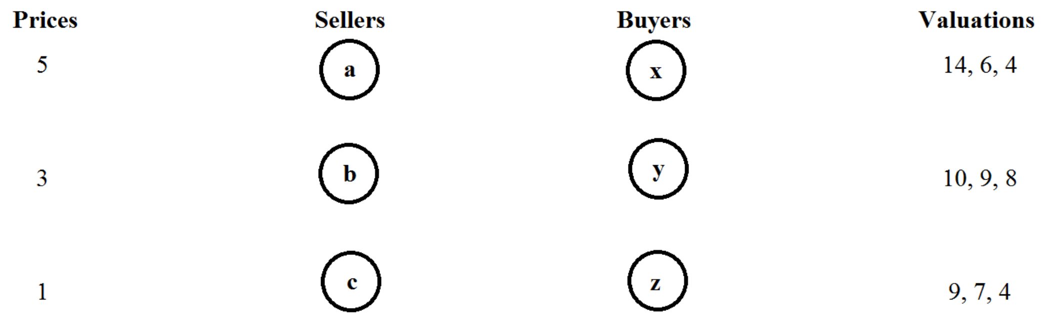


Exercise n. 3

Given the following instance of the *houses sales* problem, please compute the *preferred-seller graph*:

- 3 sellers (a, b, c);
- 3 buyers (x, y, z);
- $p_a = 5$; $p_b = 3$; $p_c = 1$
- $v_{ax} = 14$; $v_{bx} = 6$; $v_{cx} = 4$; $v_{ay} = 10$; $v_{by} = 9$; $v_{cy} = 8$; $v_{az} = 9$; $v_{bz} = 7$; $v_{cz} = 4$

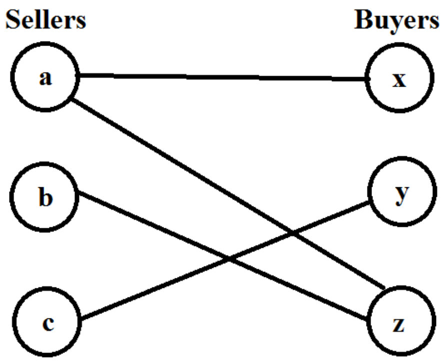
Solution to Exercise n. 3



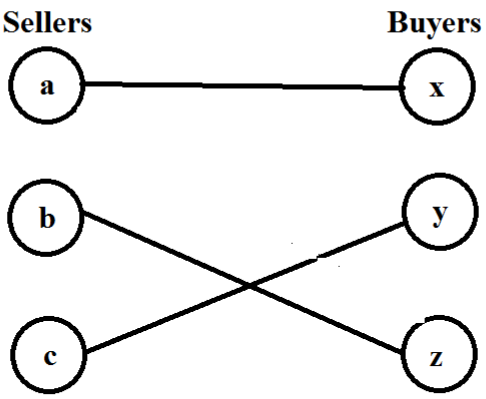
Payoff of each buyer for each house:

	a	b	c
x	9	3	3
y	5	6	7
z	4	4	3

Preferred-seller graph

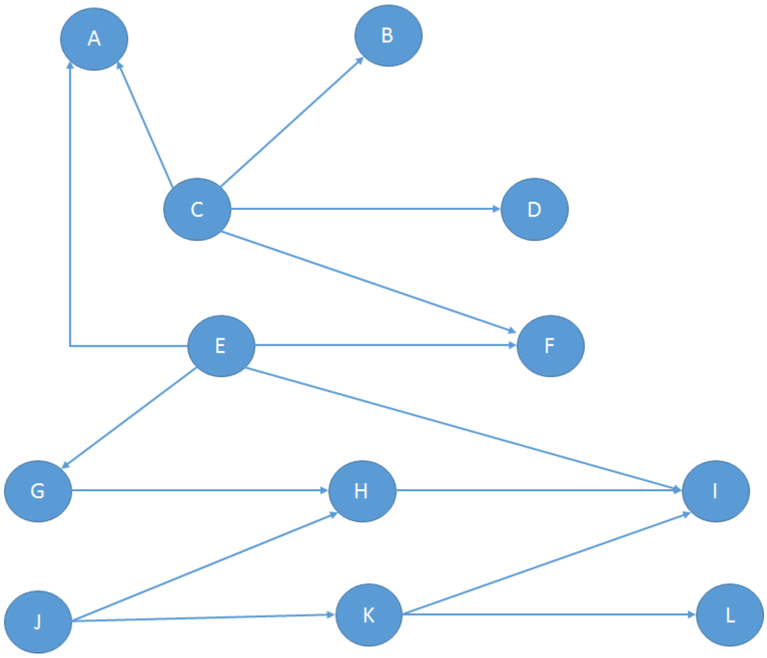


The prices clear the market, after breaking the tie (dropping the edge a->z)



Exercise n. 5

Given the graph shown in the following, where the nodes represent web pages, and the edges represent hyperlinks connecting web pages:



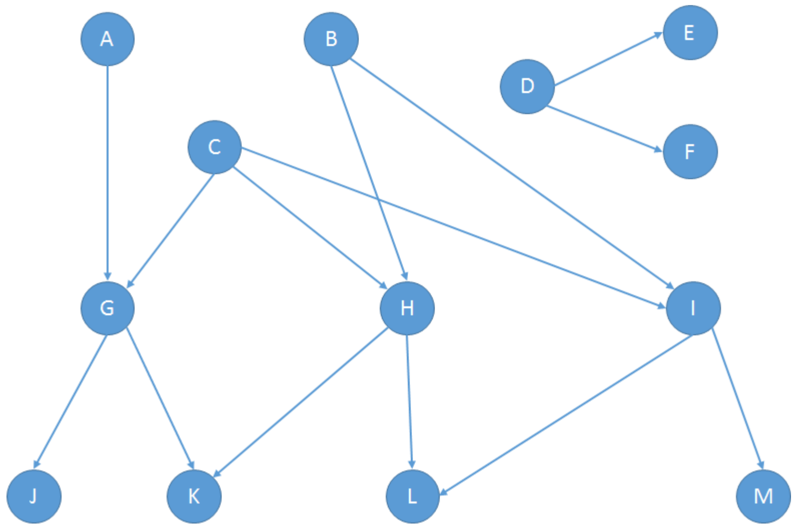
Let us assume:

- Root Set = {D, F, I, L}

a. Please apply the *HITS algorithm*, and return the sub-graph containing the *top-2 hubs* and *top-2 authorities*, obtained after *3 steps* of the algorithm.

Exercise n. 6

Given the graph shown in the following, where the nodes represent web pages, and the edges represent hyperlinks connecting web pages:



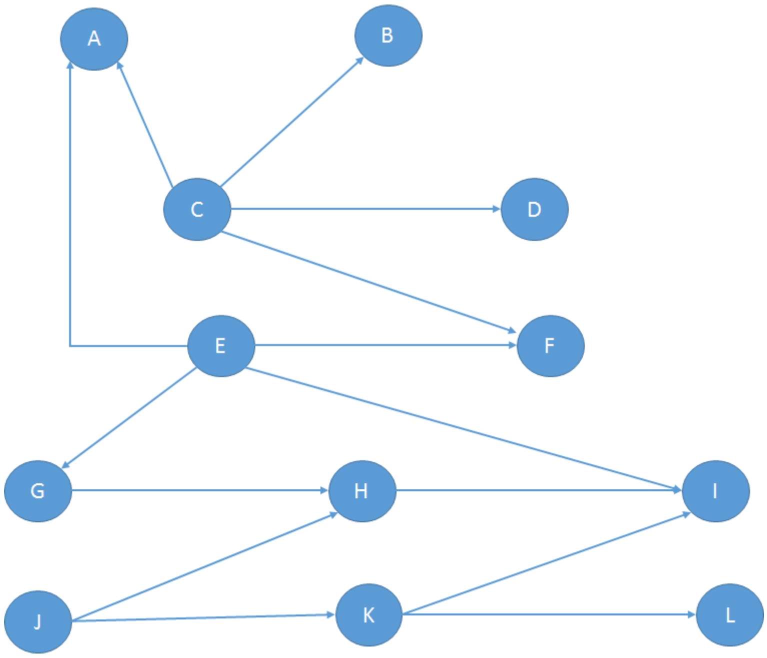
Let us assume:

- Seed Pages = {A, B, C, D}
- Trusted Pages = {A, B, C}
- Untrusted Pages = {D}
- β = Propagation Factor = 0.6
- θ = Trust Threshold = 0.3

a. Please identify the *spam pages*.

Exercise n. 5

Given the graph shown in the following, where the nodes represent web pages, and the edges represent hyperlinks connecting web pages:

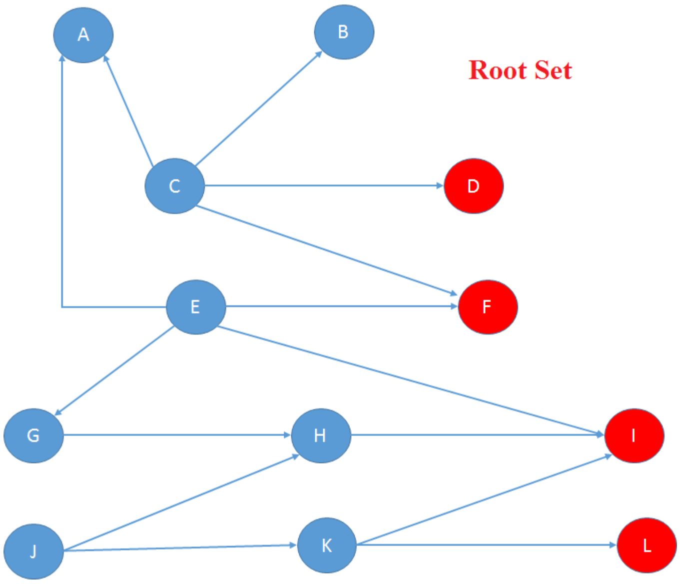


Let us assume:

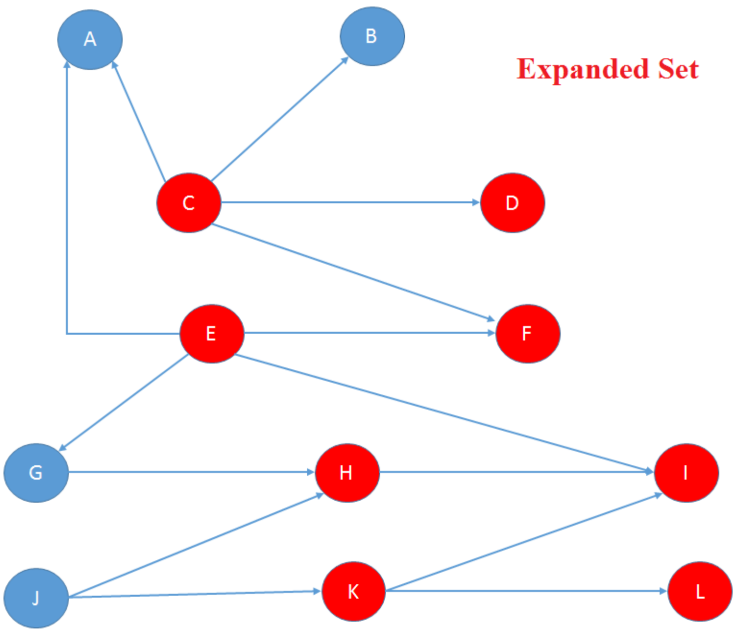
- Root Set = {D, F, I, L}

a. Please apply the *HITS algorithm*, and return the sub-graph containing the *top-2 hubs* and *top-2 authorities*, obtained after *3 steps* of the algorithm (without carrying out normalizations).

Solution to Exercise n. 5



- Expanded Set = {C, D, E, F, H, I, K, L}



Step 0

- h(C) = h(D) = h(E) = h(F) = h(H) = h(I) = h(K) = h(L) = 1
- a(C) = a(D) = a(E) = a(F) = a(H) = a(I) = a(K) = a(L) = 1

Step 1

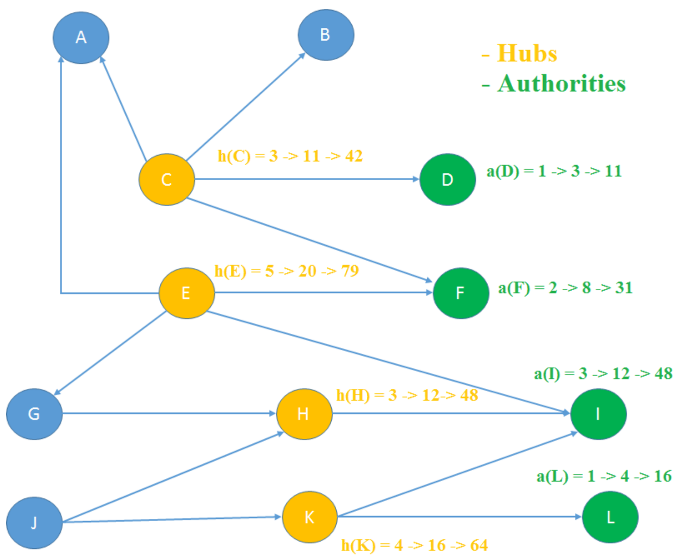
- a(C) = a(E) = a(H) = a(K) = 1, since these nodes do not have incoming links.
- a(D) = 1; a(F) = 2; a(I) = 3; a(L) = 1
- h(D) = h(F) = h(I) = h(L) = 1, since these nodes do not have outgoing links.
- h(C) = 3; h(E) = 5; h(H) = 3; h(K) = 4

Step 2

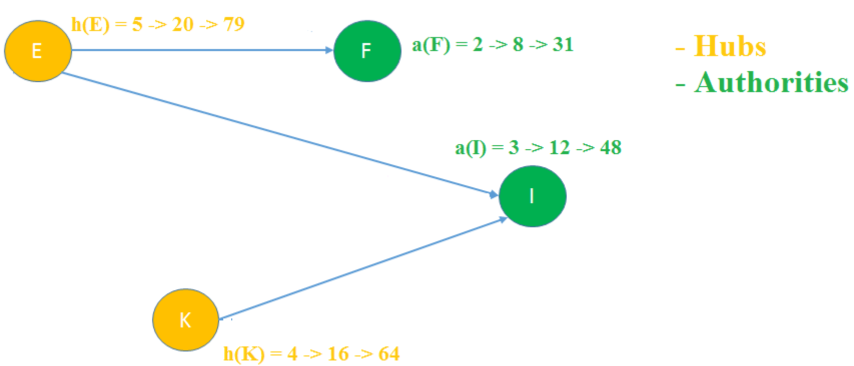
- a(D) = 3; a(F) = 8; a(I) = 12; a(L) = 4
- h(C) = 11; h(E) = 20; h(H) = 12; h(K) = 16

Step 3

- a(D) = 11; a(F) = 31; a(I) = 48; a(L) = 16
- h(C) = 42; h(E) = 79; h(H) = 48; h(K) = 64

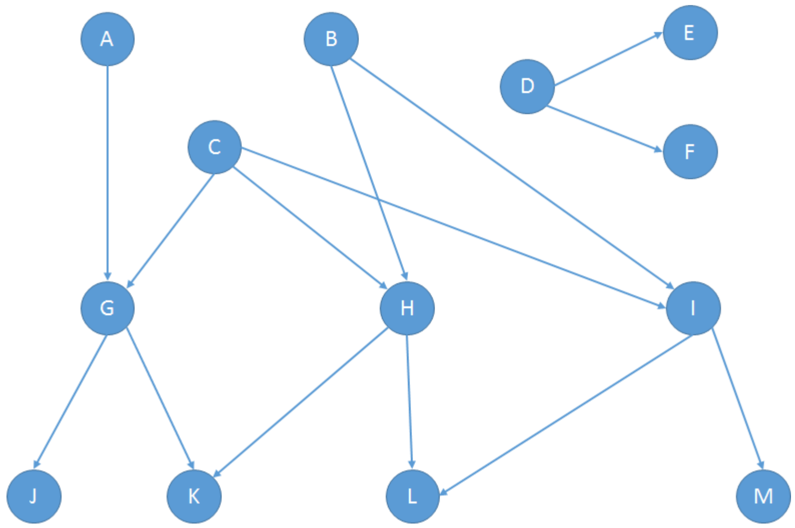


Final Sub-Graph



Exercise n. 6

Given the graph shown in the following, where the nodes represent web pages, and the edges represent hyperlinks connecting web pages:

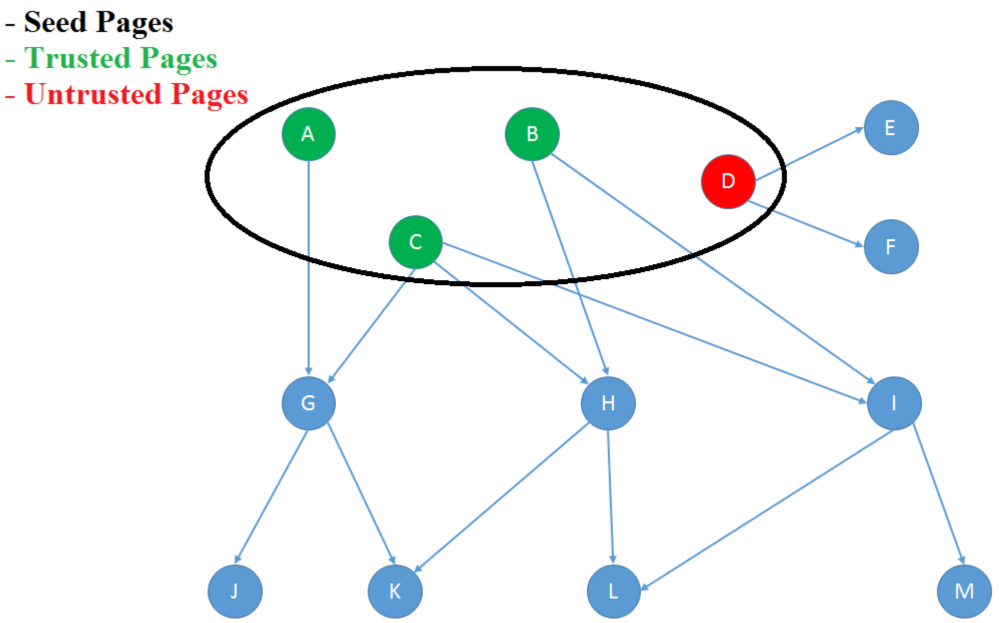


Let us assume:

- Seed Pages = {A, B, C, D}
- Trusted Pages = {A, B, C}
- Untrusted Pages = {D}
- β = Propagation Factor = 0.6
- θ = Trust Threshold = 0.3

a. Please identify the *spam pages*.

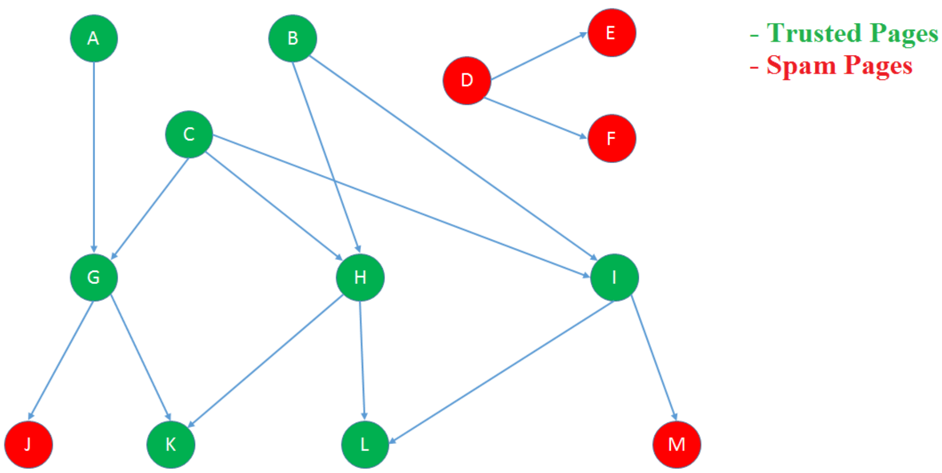
Solution to Exercise n. 6



- $|O_A| = 1$; $|O_B| = 2$; $|O_C| = 3$; $|O_D| = 2$; $|O_E| = 0$; $|O_F| = 0$; $|O_G| = 2$; $|O_H| = 2$; $|O_I| = 2$
- $t_D = t_E = t_F = 0$
- $t_A = t_B = t_C = 1$

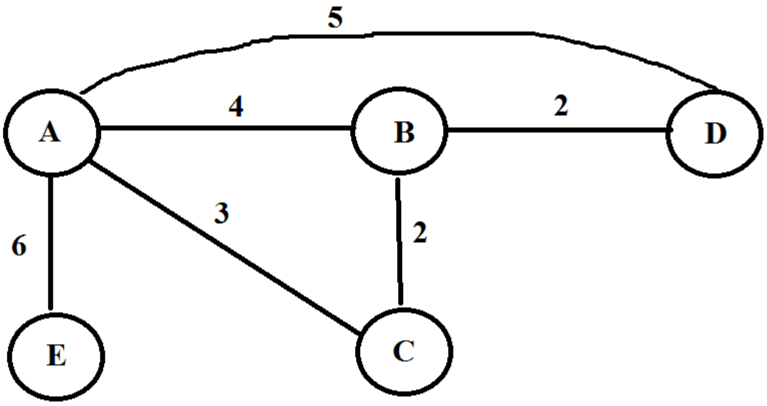
$$t_G = \beta \cdot \frac{t_A}{|O_A|} + \beta \cdot \frac{t_C}{|O_C|} = \frac{3}{5} \cdot \frac{1}{1} + \frac{3}{5} \cdot \frac{1}{3} = \frac{4}{5} = 0.8 > \theta \rightarrow \textbf{trusted page}$$
$$t_H = \beta \cdot \frac{t_C}{|O_C|} + \beta \cdot \frac{t_B}{|O_B|} = \frac{3}{5} \cdot \frac{1}{3} + \frac{3}{5} \cdot \frac{1}{2} = \frac{1}{2} = 0.5 > \theta \rightarrow \textbf{trusted page}$$
$$t_I = \beta \cdot \frac{t_C}{|O_C|} + \beta \cdot \frac{t_B}{|O_B|} = \frac{3}{5} \cdot \frac{1}{3} + \frac{3}{5} \cdot \frac{1}{2} = \frac{1}{2} = 0.5 > \theta \rightarrow \textbf{trusted page}$$
$$t_J = \beta \cdot \frac{t_G}{|O_G|} = \frac{3}{5} \cdot \frac{4}{5 \cdot 2} = \frac{12}{50} = 0.24 < \theta \rightarrow \textbf{spam page}$$
$$t_K = \beta \cdot \frac{t_H}{|O_H|} + \beta \cdot \frac{t_G}{|O_G|} = \frac{3}{5} \cdot \frac{1}{2 \cdot 2} + \frac{3}{5} \cdot \frac{4}{5 \cdot 2} = \frac{39}{100} = 0.39 > \theta \rightarrow \textbf{trusted page}$$
$$t_L = \beta \cdot \frac{t_H}{|O_H|} + \beta \cdot \frac{t_I}{|O_I|} = \frac{3}{5} \cdot \frac{1}{2 \cdot 2} + \frac{3}{5} \cdot \frac{1}{2 \cdot 2} = \frac{3}{10} = 0.3 = \theta \rightarrow \textbf{trusted page}$$
$$t_M = \beta \cdot \frac{t_I}{|O_I|} = \frac{3}{5} \cdot \frac{1}{2 \cdot 2} = \frac{3}{20} = 0.15 < \theta \rightarrow \textbf{spam page}$$

Spam Pages = {D, E, F, J, M}



Exercise n. 1

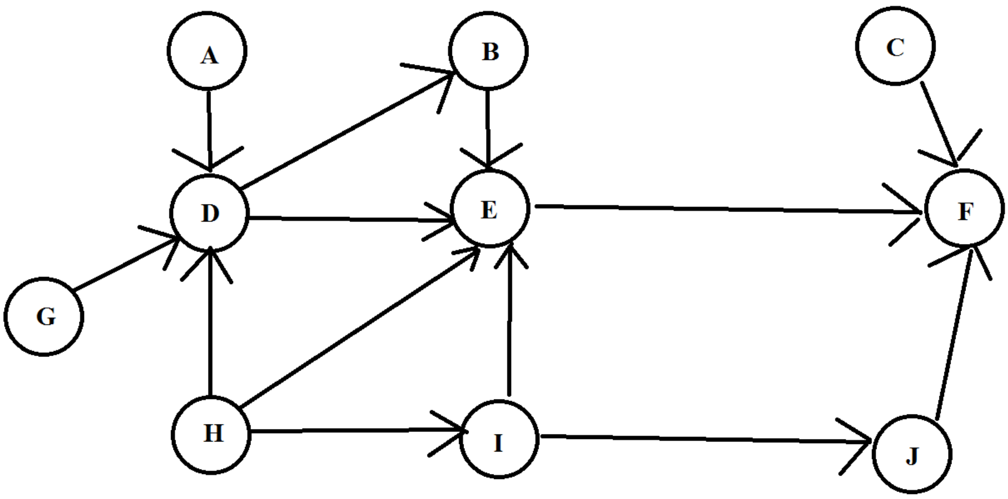
Given the graph shown in the following.



a. Compute the center, exploiting the concept of *radius* of a node.
b. Compute the center, exploiting the concept of *closeness centrality*.
c. Compute the center, exploiting the concept of *betweenness centrality*.

Exercise n. 2

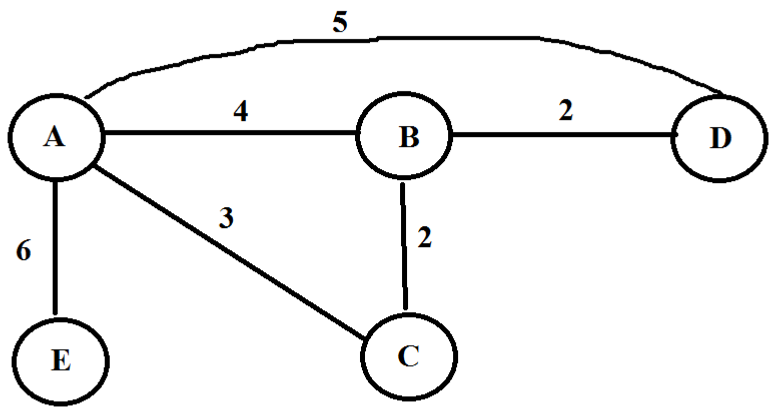
Given the graph shown in the following.



a. Compute the *in-degree centrality* for every node.
b. Compute the *out-degree centrality* for every node.

Exercise n. 1

Given the graph shown in the following.



a. Compute the center, exploiting the concept of *radius* of a node.
b. Compute the center, exploiting the concept of *closeness centrality*.
c. Compute the center, exploiting the concept of *betweenness centrality*.

Solution to Exercise n. 1

Point a)

d(A,B) = 4	d(B,A) = 4	d(C,A) = 3	d(D,A) = 5	d(E,A) = 6
d(A,C) = 3	d(B,C) = 2	d(C,B) = 2	d(D,B) = 2	d(E,B) = 10
d(A,D) = 5	d(B,D) = 2	d(C,D) = 4	d(D,C) = 4	d(E,C) = 9
d(A,E) = 6	d(B,E) = 10	d(C,E) = 9	d(D,E) = 11	d(E,D) = 11
r(A) = 6	**r(B) = 10**	**r(C) = 9**	**r(D) = 11**	**r(E) = 11**

Thus, A is the center of the graph, exploiting the concept of *radius* of a node.

Point b)

c(A) = 1/(4+3+5+6) = 1/18 = 0.056

c(B) = 1/(4+2+2+10) = 1/18 = 0.056

c(C) = 1/(3+2+4+9) = 1/18 = 0.056

c(D) = 1/(5+2+4+11) = 1/22 = 0.045

c(E) = 1/(6+10+9+11) = 1/36 = 0.028

Thus, exploiting the concept of *closeness centrality*, A, B, and C could be the center of the graph.

Point c)

Betweenness Centrality for Node A = 3

	$\sigma_{st}(v)$	σ_{st}	$\sigma_{st}(v)/\sigma_{st}$
(B,C)	0	1	0
(B,D)	0	1	0
(B,E)	1	1	1
(C,D)	0	1	0
(C,E)	1	1	1
(D,E)	1	1	1

Betweenness Centrality for Node B = 1

	$\sigma_{st}(v)$	σ_{st}	$\sigma_{st}(v)/\sigma_{st}$
(A,C)	0	1	0
(A,D)	0	1	0
(A,E)	0	1	0
(C,D)	1	1	1
(C,E)	0	1	0
(D,E)	0	1	0

Betweenness Centrality for Node C = 0

	$\sigma_{st}(v)$	σ_{st}	$\sigma_{st}(v)/\sigma_{st}$
(A,B)	0	1	0
(A,D)	0	1	0
(A,E)	0	1	0
(B,D)	0	1	0
(B,E)	0	1	0
(D,E)	0	1	0

Betweenness Centrality for Node D = 0

	$\sigma_{st}(v)$	σ_{st}	$\sigma_{st}(v)/\sigma_{st}$
(A,B)	0	1	0
(A,C)	0	1	0
(A,E)	0	1	0
(B,C)	0	1	0
(B,E)	0	1	0
(C,E)	0	1	0

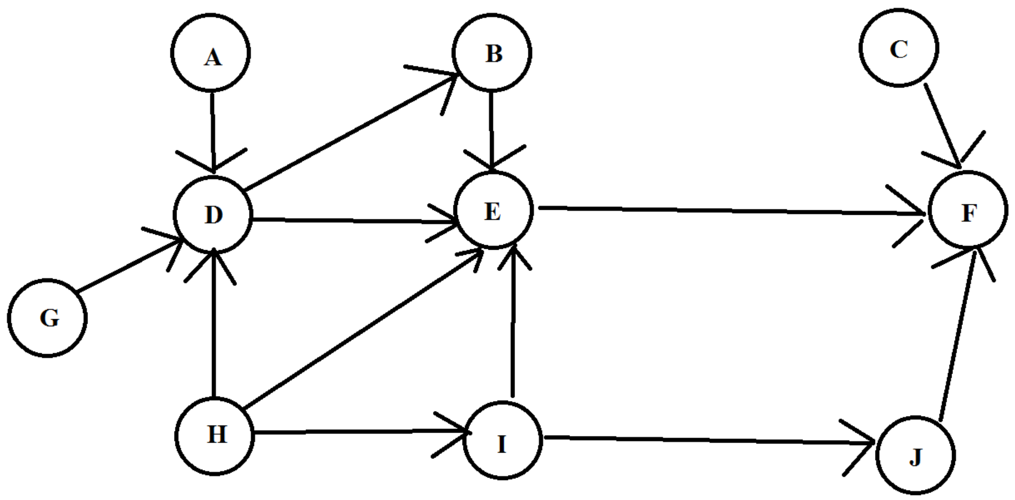
Betweenness Centrality for Node E = 0

	$\sigma_{st}(v)$	σ_{st}	$\sigma_{st}(v)/\sigma_{st}$
(A,B)	0	1	0
(A,C)	0	1	0
(A,D)	0	1	0
(B,C)	0	1	0
(B,D)	0	1	0
(C,D)	0	1	0

Thus, A is the center of the graph, exploiting the concept of *betweenness centrality* of a node.

Exercise n. 2

Given the graph shown in the following.



a. Compute the *in-degree centrality* for every node.
b. Compute the *out-degree centrality* for every node.

Solution to Exercise n. 2

Point a)

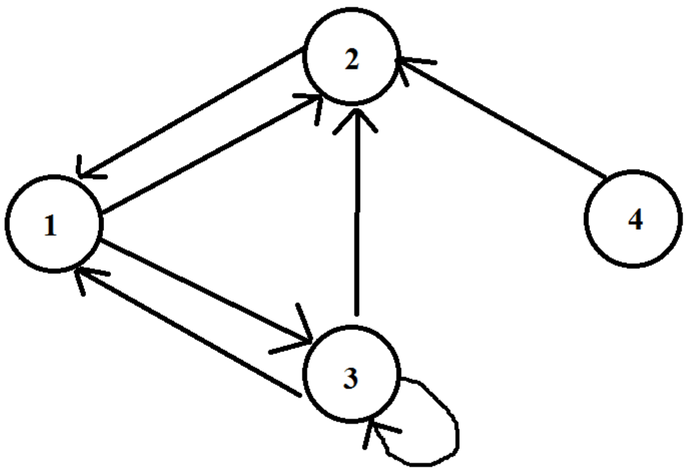
In(A) = 0; In(B) = 1; In(C) =0; In(D) = 3; In(E) = 4; In(F) = 3; In(G) = 0; In(H) = 0; In(I) = 1; In(J) = 1

Point b)

Out(A) = 1; Out(B) = 1; Out(C) = 1; Out(D) = 2; Out(E) = 1; Out(F) = 0; Out(G) = 1; Out(H) = 3; Out(I) = 2; Out(J) = 1

Exercise n. 3

Given the graph shown in the following.



a. Please compute the *co-citation index* for every couple of nodes.

Solution to Exercise n. 3

$$\mathrm{E} = \begin{pmatrix} 0 & 1 & 1 & 0 \\ 1 & 0 & 0 & 0 \\ 1 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix}$$

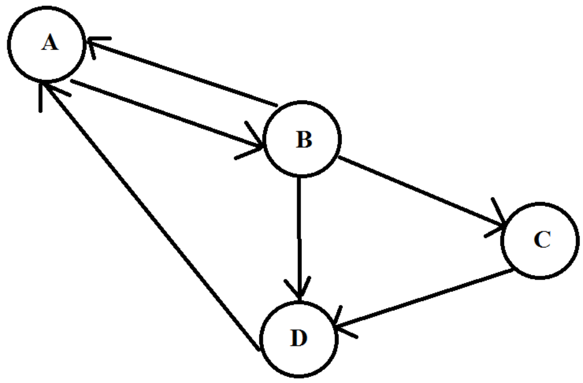
$$\mathrm{E}^T = \begin{pmatrix} 0 & 1 & 1 & 0 \\ 1 & 0 & 1 & 1 \\ 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

$$\mathrm{E}^T \cdot E = \begin{pmatrix} 2 & 1 & 1 & 0 \\ 1 & 3 & 2 & 0 \\ 1 & 2 & 2 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

The *co-citation index* for nodes *(i,j)* is the *(i,j)* item of the E^T*E matrix.

Exercise n. 4

Given the graph shown in the following.



a. Please compute the *prestige vector* after 3 steps of the algorithm.
b. What is the *ranking* imposed by the prestige vector?
c. Please compute the *ranking* imposed by the *PageRank* algorithm after 3 steps.
d. What happens in the computation of the PageRank steps if we also take into account a *damping factor d=0.2*? Please compute the new *PageRank* vector (after 2 steps).

Solution to Exercise n. 4

Point a)

$$\mathrm{E} = \begin{pmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \end{pmatrix}$$

$$\mathrm{E}^T = \begin{pmatrix} 0 & 1 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 \end{pmatrix}$$

Step 0

$p_A = p_B = p_C = p_D = 1$

Step 1

$$\begin{cases} p_A = p_B + p_D & = 2 \\ p_B = p_A & = 1 \\ p_C = p_B & = 1 \\ p_D = p_B + p_C & = 2 \end{cases} \qquad \|p\|_1 = 2 + 1 + 1 + 2 = 6 \qquad \begin{cases} p_A & = \frac{2}{6} \\ p_B & = \frac{1}{6} \\ p_C & = \frac{1}{6} \\ p_D & = \frac{2}{6} \end{cases}$$

Step 2

$$\begin{cases} p_A = \frac{1}{6} + \frac{2}{6} & = \frac{1}{2} \\ p_B & = \frac{2}{6} \\ p_C & = \frac{1}{6} \\ p_D = \frac{1}{6} + \frac{1}{6} & = \frac{2}{6} \end{cases}$$

$$\|p\|_1 = \frac{1}{2} + \frac{2}{6} + \frac{1}{6} + \frac{2}{6} = \frac{4}{3}$$

$$\begin{cases} p_A = \frac{1}{2} \cdot \frac{3}{4} & = \frac{3}{8} \\ p_B = \frac{2}{6} \cdot \frac{3}{4} & = \frac{1}{4} \\ p_C = \frac{1}{6} \cdot \frac{3}{4} & = \frac{1}{8} \\ p_D = \frac{2}{6} \cdot \frac{3}{4} & = \frac{1}{4} \end{cases}$$

Step 3

$$\begin{cases} p_A = \frac{1}{4} + \frac{1}{4} & = \frac{1}{2} \\ p_B & = \frac{3}{8} \\ p_C & = \frac{1}{4} \\ p_D = \frac{1}{4} + \frac{1}{8} & = \frac{3}{8} \end{cases}$$

$$\|p\|_1 = \frac{1}{2} + \frac{3}{8} + \frac{1}{4} + \frac{3}{8} = \frac{3}{2}$$

$$\begin{cases} p_A = \frac{1}{2} \cdot \frac{2}{3} & = \frac{1}{3} \\ p_B = \frac{3}{8} \cdot \frac{2}{3} & = \frac{1}{4} \\ p_C = \frac{1}{4} \cdot \frac{2}{3} & = \frac{1}{6} \\ p_D = \frac{3}{8} \cdot \frac{2}{3} & = \frac{1}{4} \end{cases}$$

Point b)

Ranking = <A, B, D, C>

Point c)

Step 0

$p_A = p_B = p_C = p_D = \frac{1}{4}$

$N_A = 1;\ N_B = 3;\ N_C = 1;\ N_D = 1;\ N = 4$

Step 1

$P'_A = \frac{P_B}{N_B} + \frac{P_D}{N_D} = \frac{P_B}{3} + \frac{P_D}{1} = \frac{1}{4 \cdot 3} + \frac{1}{4} = \frac{1}{3}$

$P'_B = \frac{P_A}{N_A} = \frac{1}{4}$

$P'_C = \frac{P_B}{N_B} = \frac{1}{4 \cdot 3} = \frac{1}{12}$

$P'_D = \frac{P_B}{N_B} + \frac{P_C}{N_C} = \frac{1}{3 \cdot 4} + \frac{1}{4 \cdot 1} = \frac{1}{3}$

Step 2

$P'_A = \frac{P_B}{N_B} + \frac{P_D}{N_D} = \frac{1}{4 \cdot 3} + \frac{1}{3 \cdot 1} = \frac{5}{12}$

$P'_B = \frac{P_A}{N_A} = \frac{1}{3 \cdot 1} = \frac{1}{3}$

$P'_C = \frac{P_B}{N_B} = \frac{1}{4 \cdot 3} = \frac{1}{12}$

$P'_D = \frac{P_B}{N_B} + \frac{P_C}{N_C} = \frac{1}{4 \cdot 3} + \frac{1}{12 \cdot 1} = \frac{1}{6}$

Step 3

$P'_A = \frac{P_B}{N_B} + \frac{P_D}{N_D} = \frac{1}{3 \cdot 3} + \frac{1}{6 \cdot 1} = \frac{5}{18} = 0.278$

$P'_B = \frac{P_A}{N_A} = \frac{5}{12 \cdot 1} = \frac{5}{12} = 0.417$

$P'_C = \frac{P_B}{N_B} = \frac{1}{3 \cdot 3} = \frac{1}{9} = 0.111$

$P'_D = \frac{P_B}{N_B} + \frac{P_C}{N_C} = \frac{1}{3 \cdot 3} + \frac{1}{12 \cdot 1} = \frac{7}{36} = 0.194$

Ranking = <B, A, D, C>

Point d)

Step 0

$p_A = p_B = p_C = p_D = \frac{1}{4}$

$N_A = 1;\ N_B = 3;\ N_C = 1;\ N_D = 1;\ N = 4$

Step 1

$P'_A = (1-d) \cdot (\frac{P_B}{N_B} + \frac{P_D}{N_D}) + \frac{d}{N} = \frac{4}{5} \cdot \frac{1}{3} + \frac{1}{5 \cdot 4} = \frac{19}{60}$

$P'_B = (1-d) \cdot \frac{P_A}{N_A} + \frac{d}{N} = \frac{4}{5} \cdot \frac{1}{4} + \frac{1}{5 \cdot 4} = \frac{1}{4}$

$P'_C = (1-d) \cdot \frac{P_B}{N_B} + \frac{d}{N} = \frac{4}{5} \cdot \frac{1}{12} + \frac{1}{5 \cdot 4} = \frac{7}{60}$

$P'_D = (1-d) \cdot (\frac{P_B}{N_B} + \frac{P_C}{N_C}) + \frac{d}{N} = \frac{4}{5} \cdot \frac{1}{3} + \frac{1}{5 \cdot 4} = \frac{19}{60}$

Step 2

$P'_A = (1-d) \cdot (\frac{P_B}{N_B} + \frac{P_D}{N_D}) + \frac{d}{N} = \frac{4}{5} \cdot (\frac{1}{4 \cdot 3} + \frac{19}{60}) + \frac{1}{5 \cdot 4} = \frac{111}{300} = 0.37$

$P'_B = (1-d) \cdot \frac{P_A}{N_A} + \frac{d}{N} = \frac{4}{5} \cdot \frac{19}{60} + \frac{1}{5 \cdot 4} = \frac{91}{300} = 0.303$

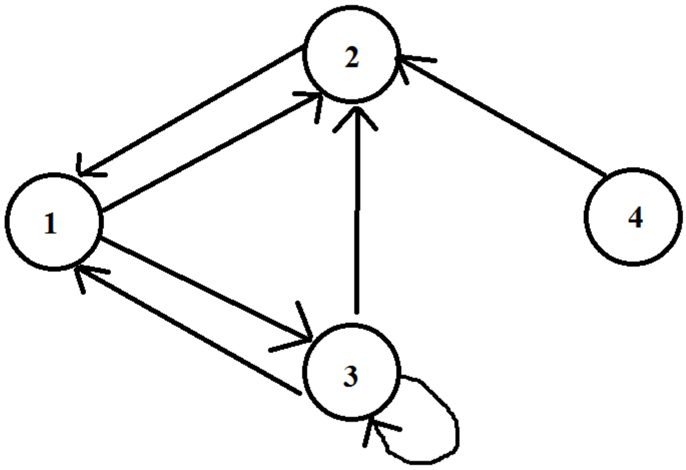
$P'_C = (1-d) \cdot \frac{P_B}{N_B} + \frac{d}{N} = \frac{4}{5} \cdot \frac{1}{4 \cdot 3} + \frac{1}{5 \cdot 4} = \frac{7}{60} = 0.117$

$P'_D = (1-d) \cdot (\frac{P_B}{N_B} + \frac{P_C}{N_C}) + \frac{d}{N} = \frac{4}{5} \cdot (\frac{1}{4 \cdot 3} + \frac{7}{60}) + \frac{1}{5 \cdot 4} = \frac{63}{300} = 0.21$

Ranking = <A, B, D, C>

Exercise n. 3

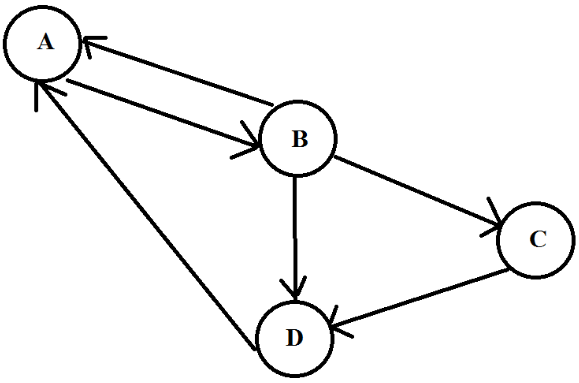
Given the graph shown in the following.



a. Please compute the *co-citation index* for every couple of nodes.

Exercise n. 4

Given the graph shown in the following.



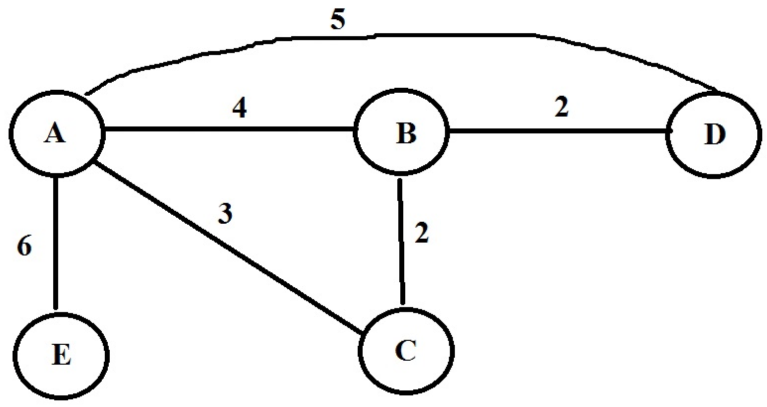
a. Please compute the *prestige vector* after 3 steps of the algorithm.
b. What is the *ranking* imposed by the prestige vector?
c. Please compute the *ranking* imposed by the *PageRank* algorithm after 3 steps.
d. What happens in the computation of the PageRank steps if we also take into account a *damping factor d=0.2*? Please compute the new *PageRank* vector (after 2 steps).

GREEDY KNAPSACK

Item	Weight	Profit	P_i/a_i
I_0	6	10	10/6
I_1	8	4	4/8
I_2	8	9	9/8
I_3	5	8	5/8
I_4	10	9	10/9
I_5	13	13	13/13

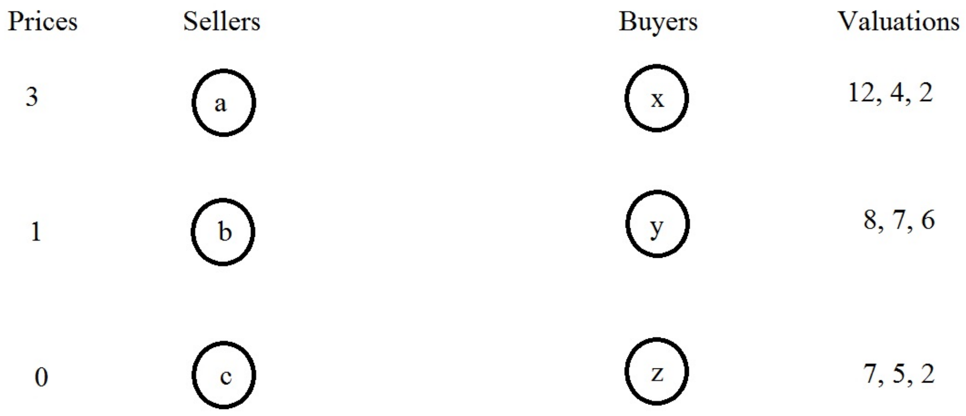
$$\frac{\sigma(B)}{\sigma}$$

BETWENESS CENTRALITY = ∀ per B

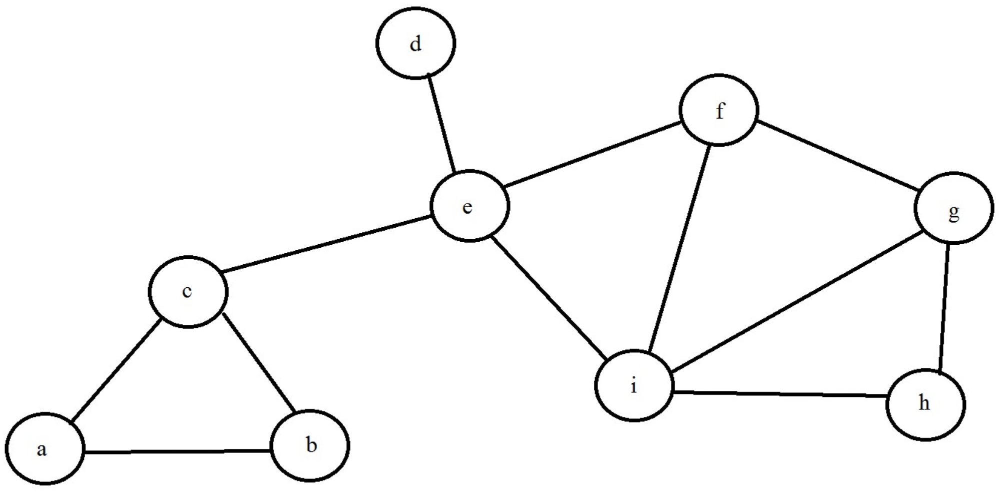


	$\sigma_{u,v}$	$\sigma_{u,v}(B)$	
A, B	1	1	1
A, C	1	1	1
A, D	1	1	1
A, E	1	0	0
B, C	1	1	1
B, D	1	1	1
B, E	1	1	1
C, D	1	1	1
C, E	1	1	1
D, E	1	1	1

NON NOMINATO



MIN VERTEX COVER



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