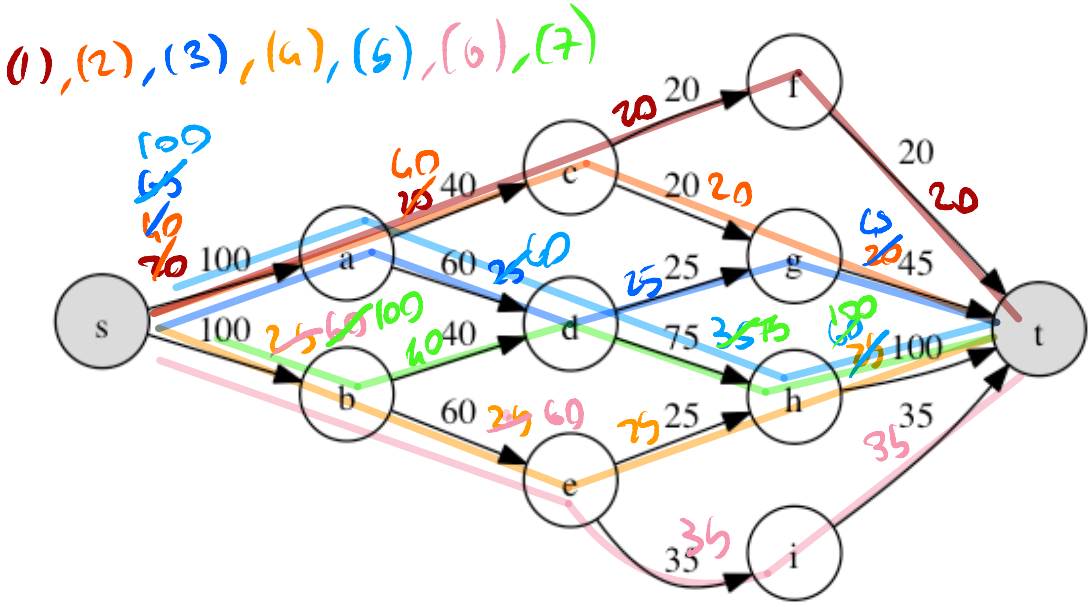
Network Design [Network Flows]
First Midterm March, 28 2019

Cognome _____
Nome _____

Question 1
Compare the complexity of the Ford and Fulkerson algorithm on the following graph with the worst-case analysis.



Question 2
Given a directed graph $G=(N, A)$ and two nodes s and t, propose an algorithm to find the maximum number of **node disjoint** s-t paths.

Exercise 1
Given the following matrix:

$$\begin{vmatrix} 8.34 & 1.35 & 3.85 & 2.35 \\ 4.37 & 7.02 & 1.52 & 8.63 \\ 6.78 & 9.23 & 8.36 & 2.65 \end{vmatrix}$$

Let

$$r_i = \sum_{j=1}^{4} a_{ij}, \text{ for } i \in 1, \ldots, 3$$

$$c_j = \sum_{i=1}^{3} a_{ij}, \text{ for } j \in 1, \ldots, 4$$

Round each element a_{ij}, r_i and c_j **up** or **down** to integer so that the sum of the rounded elements in each row (column) equals row (column) sum.

Exercise 2
The following graph represents an optical fiber network in which each arc represents a link with capacity u_{ij} Gigabit/sec. In each node is located a multiplexer with capacity 3.0 Gigabit/sec.

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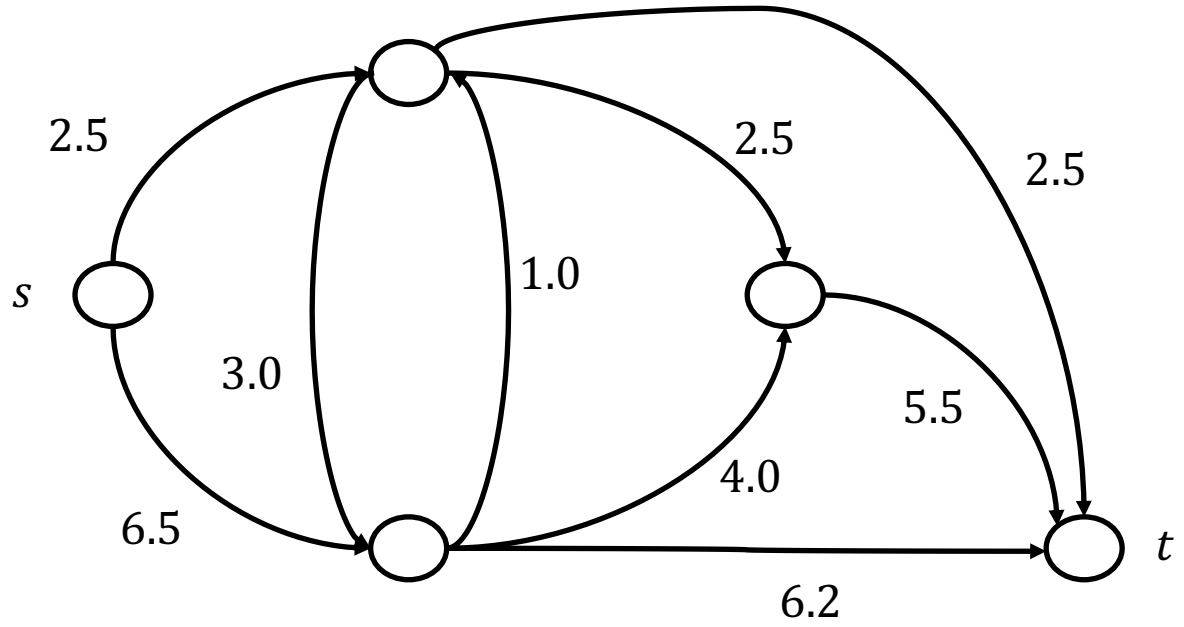

1. Evaluate the maximum transmission capacity from *s* to *t*
2. Is it convenient to increase multiplexers capacity to 4.5 Gigabit/sec the maximum capacity from *s* to *t*? (discussion required)

Exercise 3

A manufacturing process consists of two consecutive stages. The plant has 3 production bays for the first stage and 4 production bays for the second stage. Bay 1 in the first stage can feed only bays 2 and 4 in the second stage, bay 2 in the first stage can feed only bay 4 in the second stage, finally, bay 3 in the first stage can feed only bays 1,2,3 in the second stage. The following table reports the daily capacity of each production bay:

	Capacity
Stage 1	
Bay 1	6200
Bay 2	4400
Bay 3	5100

Stage 2	
Bay 1	4500
Bay 2	5700
Bay 3	3200
Bay 4	4300

1. Evaluate the maximum daily production capacity of the plant
2. Is it useful to add a fourth bay in stage 1 with capacity 5000 connected to bays 1 and 3 in the second stage? (Discussion required)

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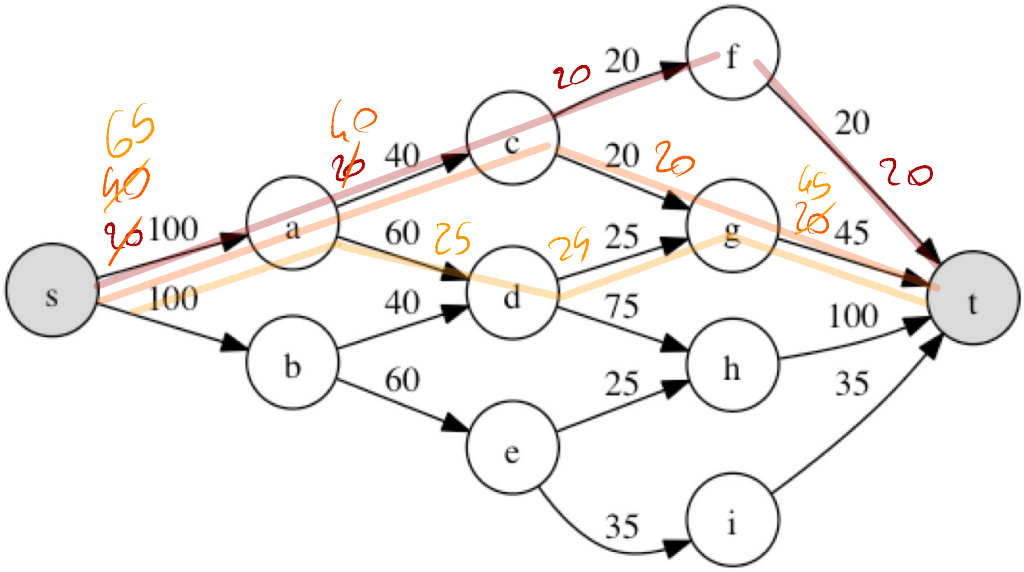
Q 1

Question 1
Compare the complexity of the Ford and Fulkerson algorithm on the following graph with the worst-case analysis.



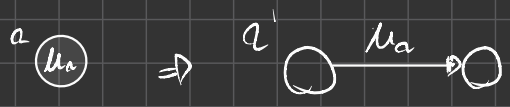
- La complessità di FBF è $O(n \cdot m)$
- Il caso peggiore è quello in cui prendi i Path nell'ordine:
  (1), (2), (3), (4), (5), (6)

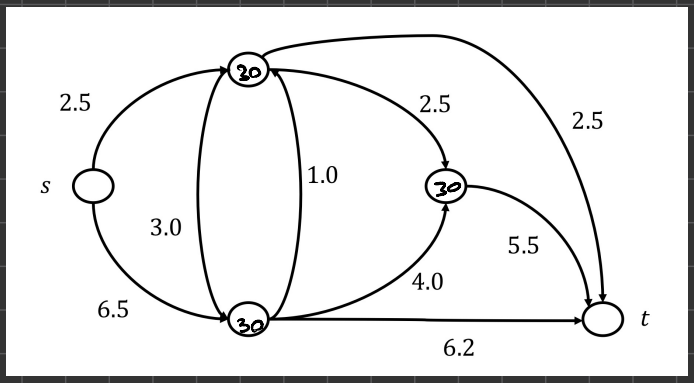
#ES 1

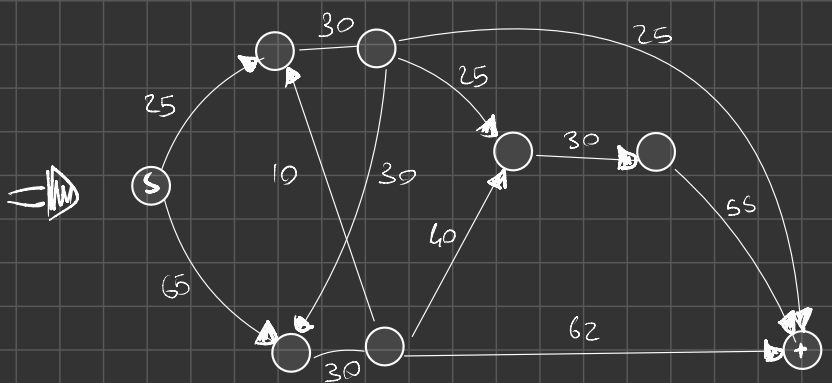
Exercise 2

The following graph represents an optical fiber network in which each arc represents a link with capacity u_{ij} Gigabit/sec. In each node is located a multiplexer with capacity 3.0 Gigabit/sec.

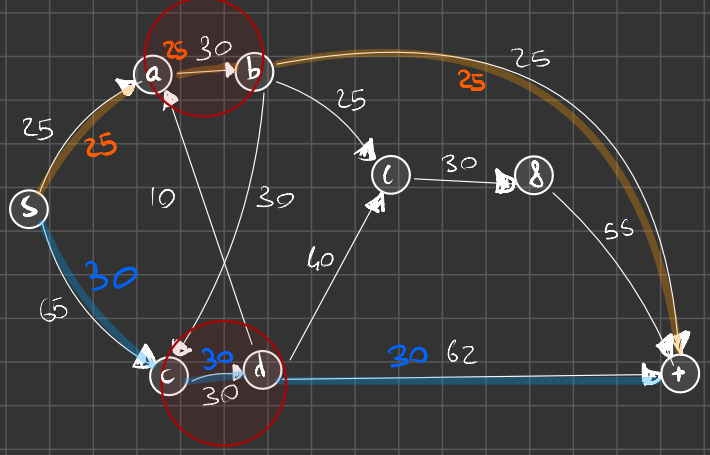
- Quando hai CAPACITÀ sui NODI applichi il NODE SPLITTING







1. Evaluate the maximum transmission capacity from s to t



- L'idea è che il massimo flow che può usare dagli archi (a,b), (c,d) è 60, quindi se riusciamo a farli usare 60 unità di flow da s siamo apposto
- sopra riusciamo partire 25
- sotto al più riusciamo a mandare 30

2. Is it convenient to increase multiplexers capacity to 4.5 Gigabit/sec the maximum capacity from s to t? (discussion required)

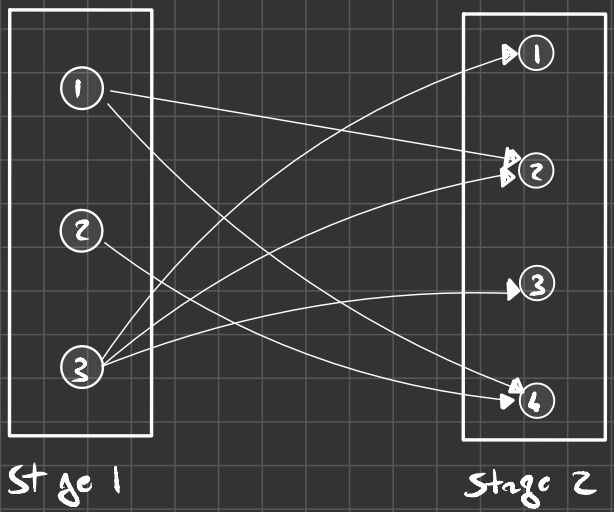
Sì, così facciamo passare più flow, perché il limite su (a,b), (c,d) diventa 90 anziché 60

#ES 3

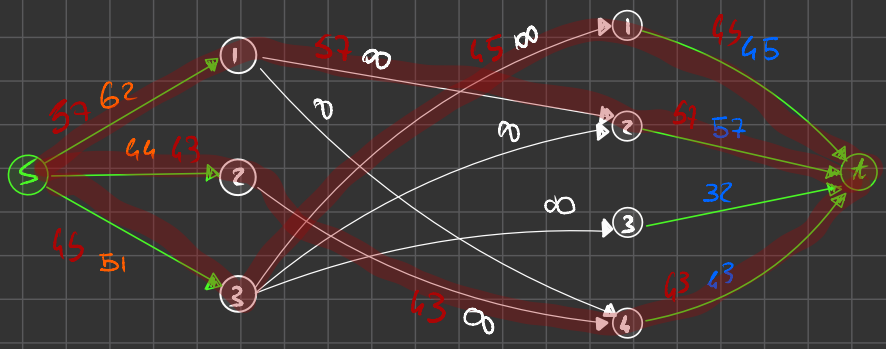
Exercise 3
A manufacturing process consists of two consecutive stages. The plant has 3 production bays for the first stage and 4 production bays for the second stage. Bay 1 in the first stage can feed only bays 2 and 4 in the second stage, bay 2 in the first stage can feed only bay 4 in the second stage, finally, bay 3 in the first stage can feed only bays 1,2,3 in the second stage. The following table reports the daily capacity of each production bay:

S_1	Prod. capacity
1	6200
2	4400
3	5100

S_2	Prod. capacity
1	4500
2	5700
3	3200
4	4300

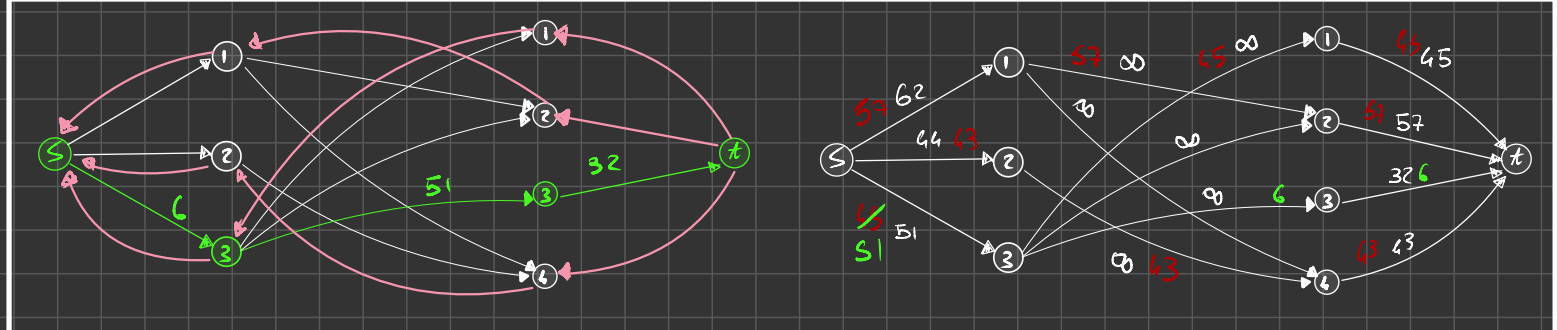


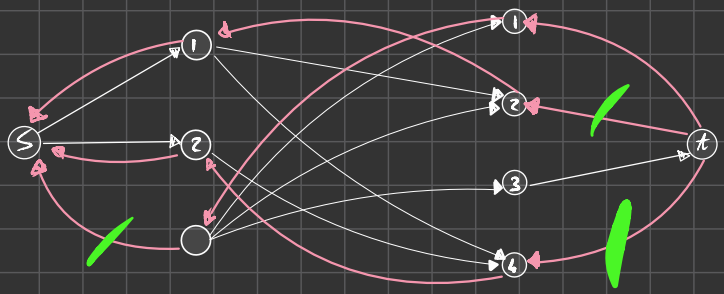
1. Evaluate the maximum daily production capacity of the plant

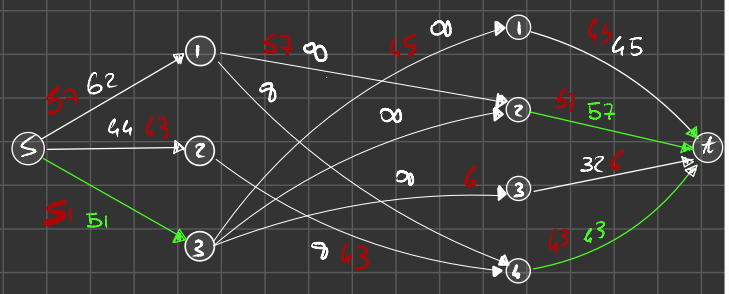


- Troviamo un Path per ogni arco uscente da (s) e inizializziamo il flow pari alla minima capacity sul path

- Applichiamo ora F&F

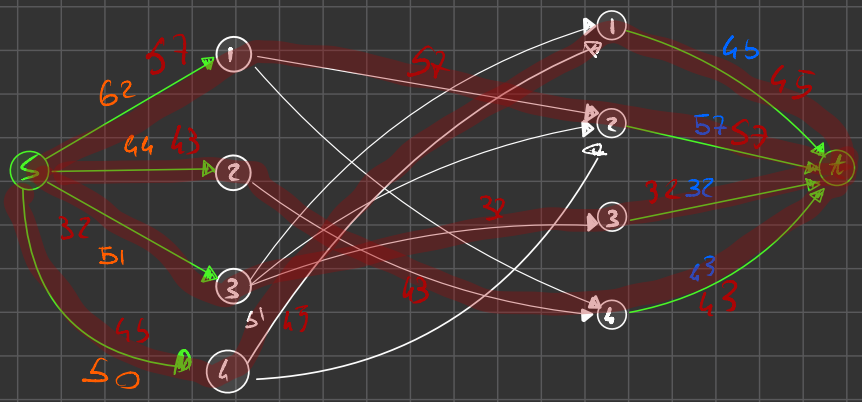






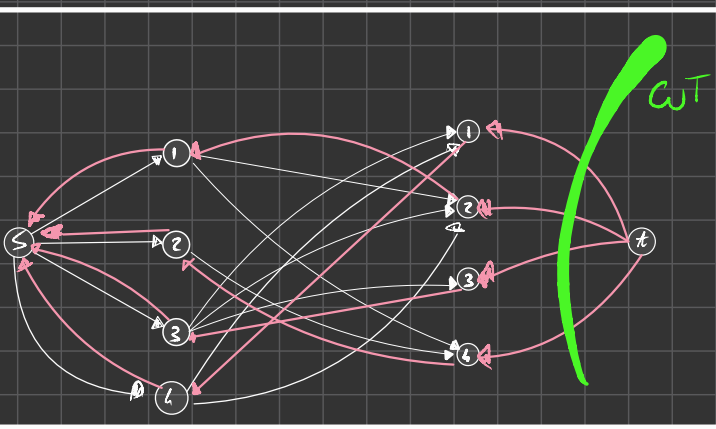
$\Rightarrow \delta_x(S) = u(\delta(R)) = 57+43+51 = 57+43+51 = \boxed{151}$

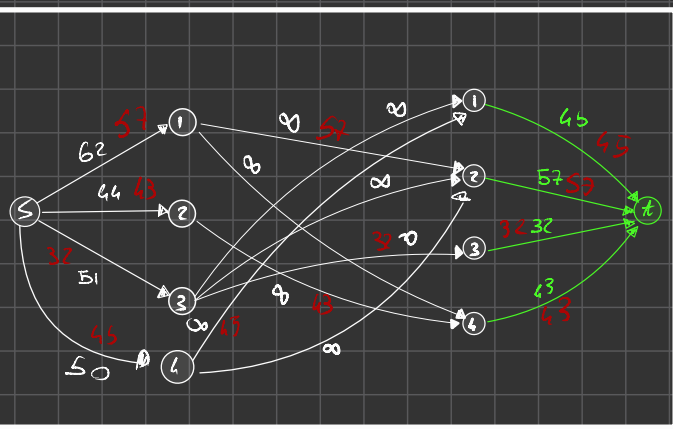
2. Is it useful to add a fourth bay in stage 1 with capacity 5000 connected to bays 1 and 2 in the second stage? (Discussion required)



• INIT. FLOW FEAS.

• F&F





$\delta_x(S) = u(\delta(R)) = 57+43+32+45 = 57+43+32+45 = \boxed{177}$