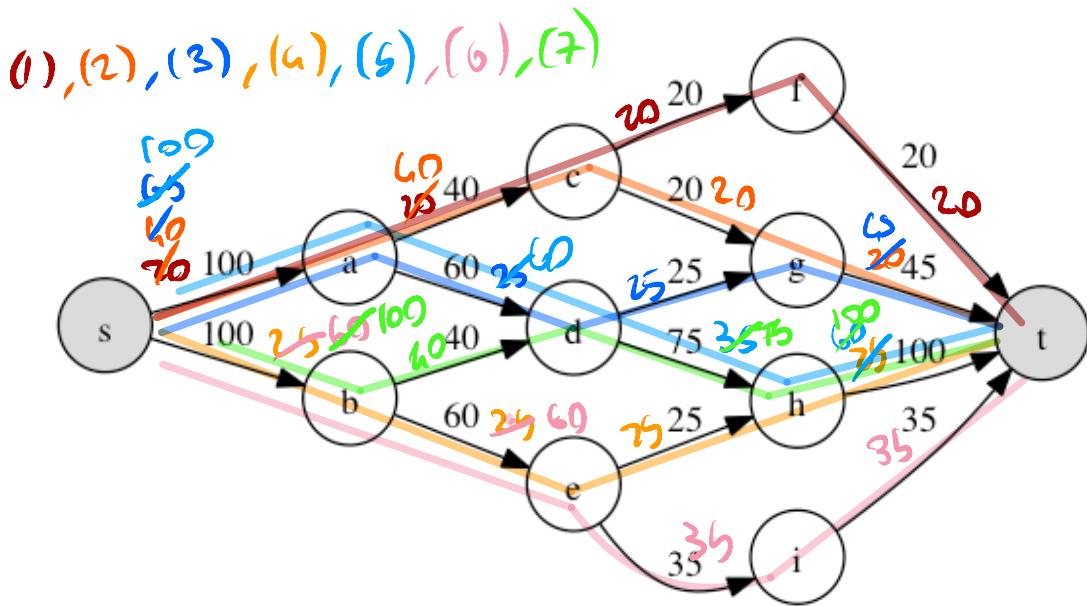


Question 1

Compare the complexity of the Ford and Fulkerson algorithm on the following graph with the worst-case analysis.



Question 2

Given a directed graph $G=(N, A)$ and two nodes s and t , propose an algorithm to find the maximum number of **node disjoint** $s-t$ paths.

Exercise 1

Given the following matrix:

$$\begin{vmatrix} 8.34 & 1.35 & 3.85 & 2.35 \\ 4.37 & 7.02 & 1.52 & 8.63 \\ 6.78 & 9.23 & 8.36 & 2.65 \end{vmatrix}$$

Let

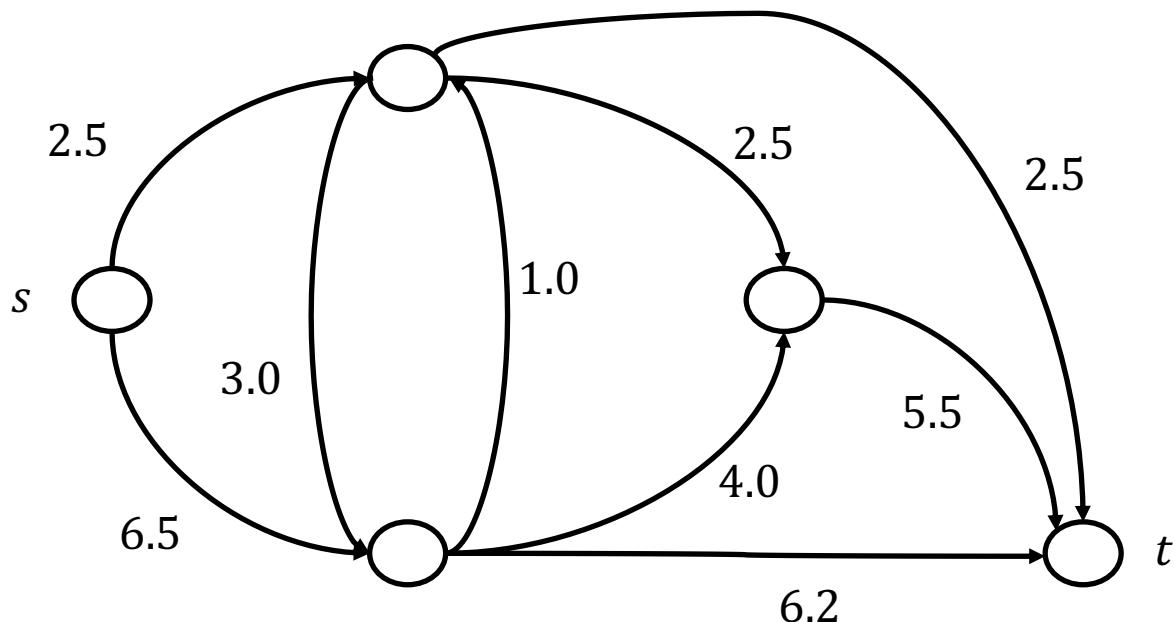
$$r_i = \sum_{j=1}^4 a_{ij}, \text{ for } i \in 1, \dots, 3$$

$$c_j = \sum_{i=1}^3 a_{ij}, \text{ for } j \in 1, \dots, 4$$

Round each element a_{ij} , r_i and c_j **up or down** to integer so that the sum of the rounded elements in each row (column) equals row (column) sum.

Exercise 2

The following graph represents an optical fiber network in which each arc represents a link with capacity u_{ij} Gigabit/sec. In each node is located a multiplexer with capacity 3.0 Gigabit/sec.



1. Evaluate the maximum transmission capacity from s to t
2. Is it convenient to increase multiplexers capacity to 4.5 Gigabit/sec the maximum capacity from s to t ? (discussion required)

Exercise 3

A manufacturing process consists of two consecutive stages. The plant has 3 production bays for the first stage and 4 production bays for the second stage. Bay 1 in the first stage can feed only bays 2 and 4 in the second stage, bay 2 in the first stage can feed only bay 4 in the second stage, finally, bay 3 in the first stage can feed only bays 1,2,3 in the second stage. The following table reports the daily capacity of each production bay:

Capacity	
Stage 1	
Bay 1	6200
Bay 2	4400
Bay 3	5100

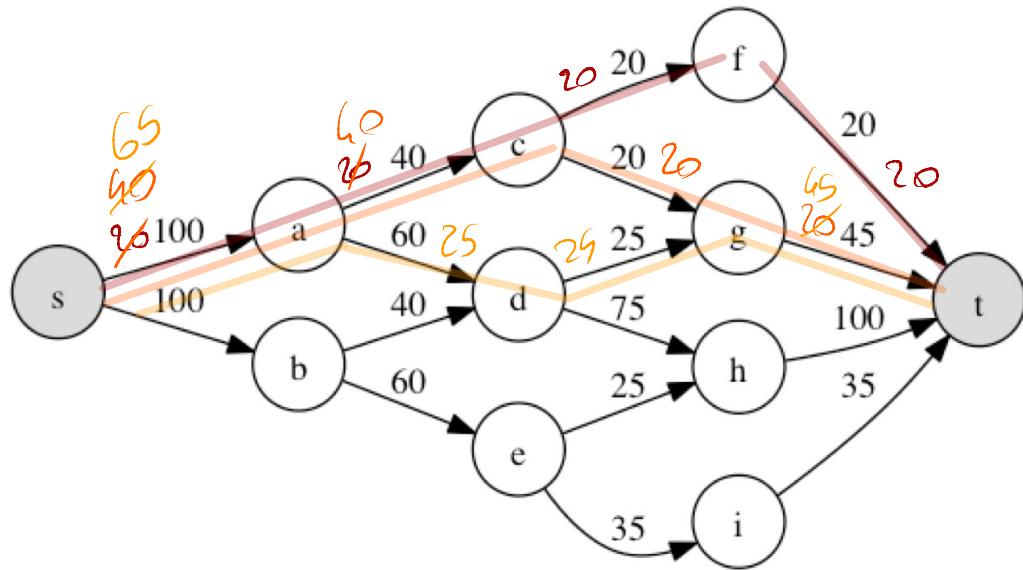
Stage 2	
Bay 1	4500
Bay 2	5700
Bay 3	3200
Bay 4	4300

1. Evaluate the maximum daily production capacity of the plant
2. Is it useful to add a fourth bay in stage 1 with capacity 5000 connected to bays 1 and 3 in the second stage? (Discussion required)

Q1

Question 1

Compare the complexity of the Ford and Fulkerson algorithm on the following graph with the worst-case analysis.



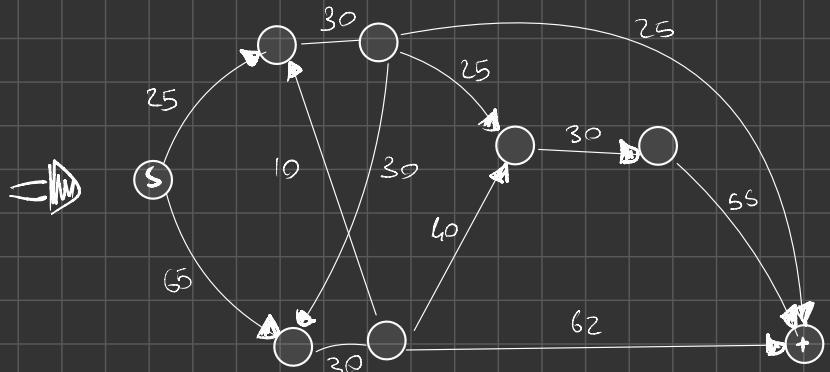
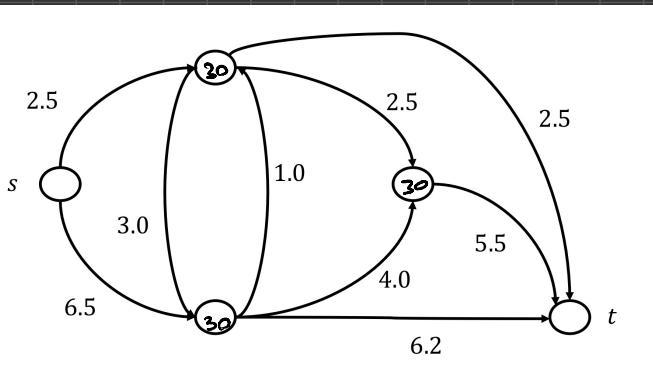
- La complessità di F&F è $O(m \cup m)$
- In caso peggiore se quello in cui prende i Path one by one:
(1), (2), (3), (4), (5), (6)

#ES 1

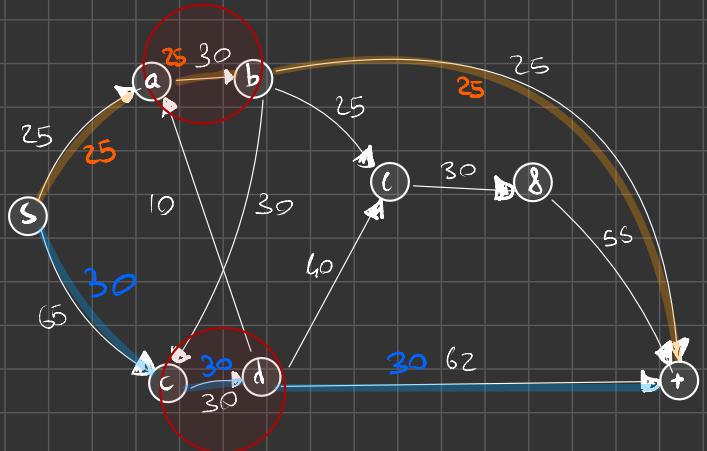
Exercise 2

The following graph represents an optical fiber network in which each arc represents a link with capacity u_{ij} Gigabit/sec. In each node is located a multiplexer with capacity 3.0 Gigabit/sec.

- Quando hai CAPACITÀ sui nodi Applich: i) NODE SPLITTING



- Evaluate the maximum transmission capacity from s to t



- L'idea è che il massimo flow che puoi usare dagli archi (a,b) , (c,d) è 60, quindi se riusciamo a far usare 60 unità di flow da s siamo appunto
- Sopra facciamo partire 25
- Sotto al più riusciamo a mandare 30

- Is it convenient to increase multiplexers capacity to 4.5 Gigabit/sec the maximum capacity from s to t ? (discussion required)

Se così facciamo passeremo più flow, perché i limiti su (a,b) , (c,d) diventano anche 60

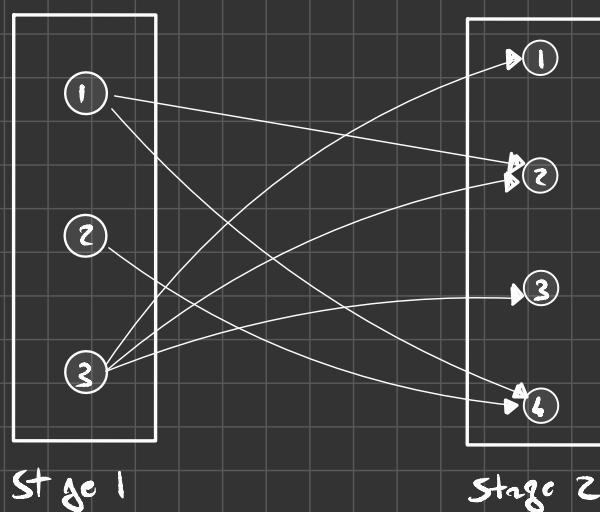
#ES 3

Exercise 3

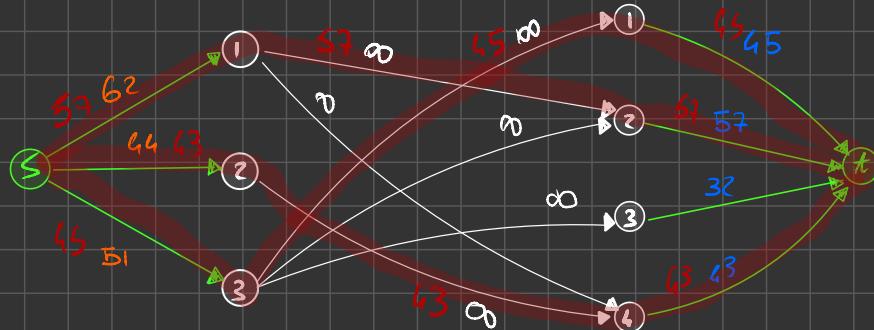
A manufacturing process consists of two consecutive stages. The plant has 3 production bays for the first stage and 4 production bays for the second stage. Bay 1 in the first stage can feed only bays 2 and 4 in the second stage, bay 2 in the first stage can feed only bay 4 in the second stage, finally, bay 3 in the first stage can feed only bays 1,2,3 in the second stage. The following table reports the daily capacity of each production bay:

S_1	PROD. CAPACITY
1	6200
2	4400
3	5100

S_2	PROD. CAPACITY
1	4500
2	5700
3	3200
4	4300

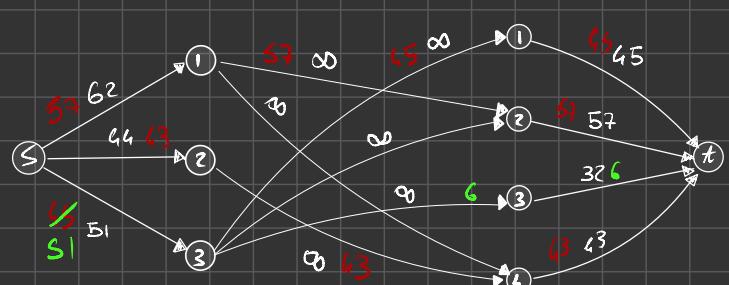


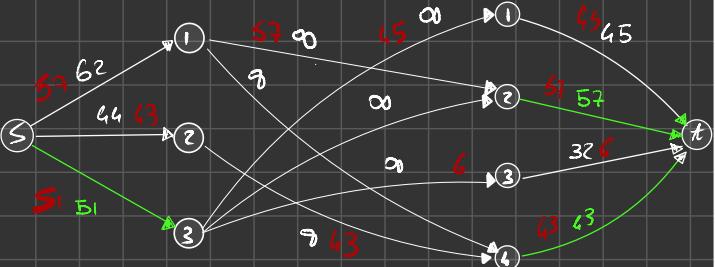
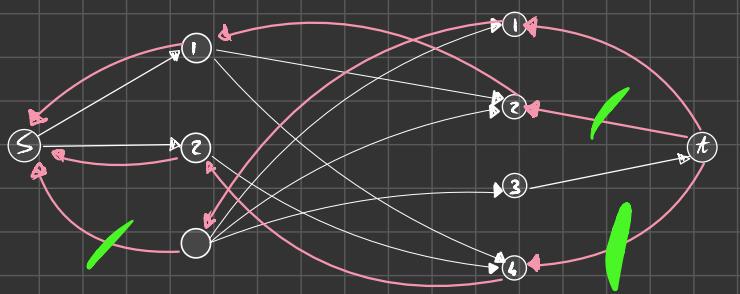
- Evaluate the maximum daily production capacity of the plant



• Traciamo un Path per ogni arco uscente da S e imballiamoci i flow più alla minima capacità sul path

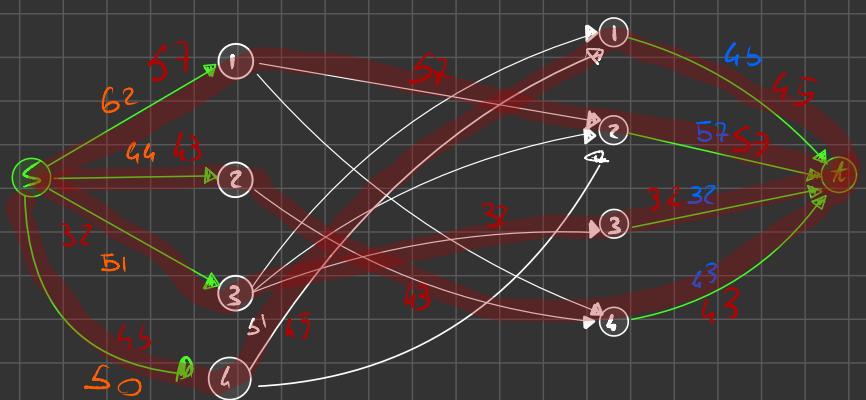
• APPROVVISO DI T&F





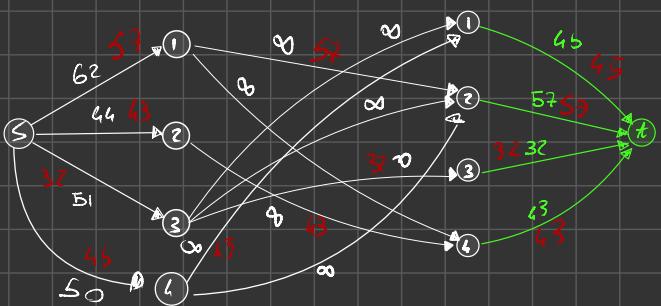
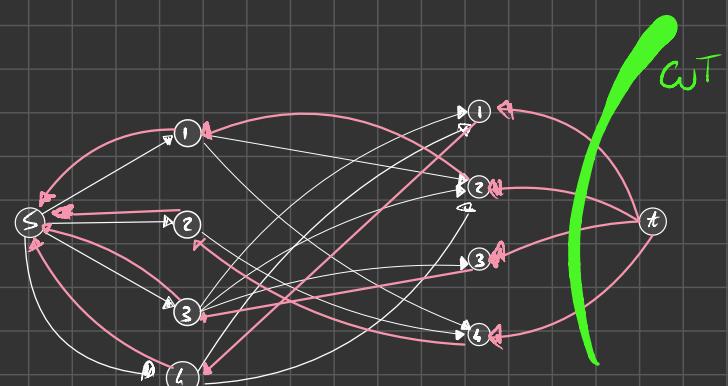
$$\Rightarrow \delta_X(S) = \mu(\delta(R)) = 57 + 43 + 51 = 57 + 43 + 51 = 151$$

2. Is it useful to add a fourth bay in stage 1 with capacity 5000 connected to bays 1 and 2 in the second stage? (Discussion required)



INIT. FLOW FEAS.

F&F



$$\delta_X(S) = \mu(\delta(R)) = 57 + 43 + 50 + 45 = 57 + 43 + 50 + 45 = 177$$