Graph Theory and Optimization
Why is it useful?

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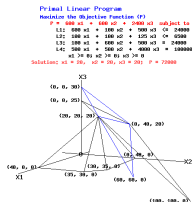October 2018

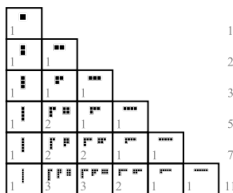http://www-sop.inria.fr/members/Nicolas.Nisse/lectures/

Outline

Combinatorics

Branch of mathematics concerning the study of finite or countable objects (existence, enumeration, structure).
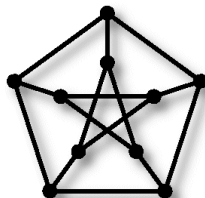
Integer Linear Programming
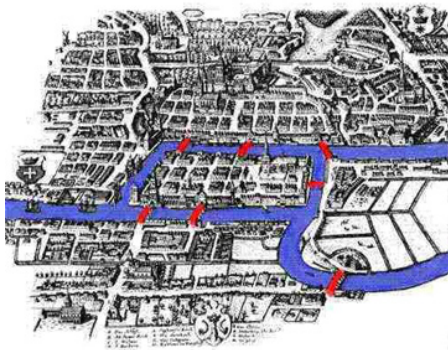
Integer partition

Graphs

Graph Theory   an old story

Euler 1735: **Koenisberg bridges**.

Existe-t-il un parcours empruntant tous les ponts une fois et une seule ?
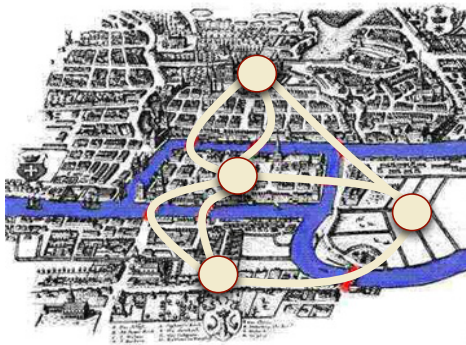
Is there a cycle going through each bridge exactly once?

Graph Theory   an old story

Modeling: city = graph, island = vertex, bridge = edge

Graph Theory   an old story

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Question: can we find an eulerian cycle in this graph?

Cycle going through all edges once and only once

Graph Theory   an old story

Modeling: city = graph, island = vertex, bridge = edge

Question: can we find an eulerian cycle in this graph?

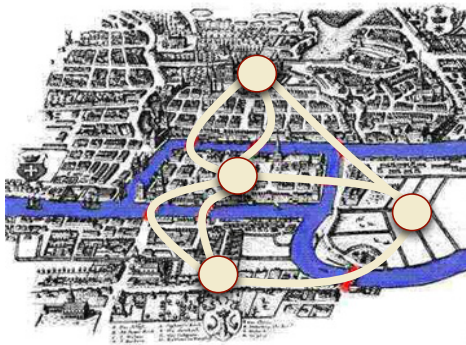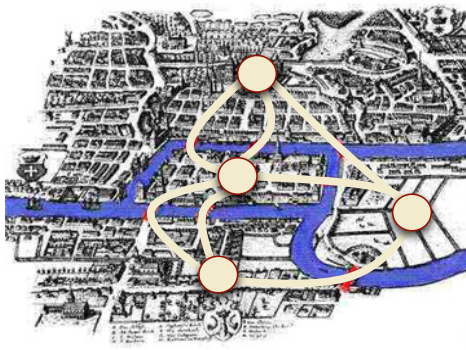**Solution:** Such cycle exists if and only if all nodes have even degree

Graph Theory   an old story

Modeling: city = graph, island = vertex, bridge = edge

Question: can we find an eulerian cycle in this graph?

Solution: Such cycle exists if and only if all nodes have even degree

An intriguing variant: find a cycle going through all vertices once and only once (Hamiltonian cycle) is very difficult

One million dollar (Clay price) !
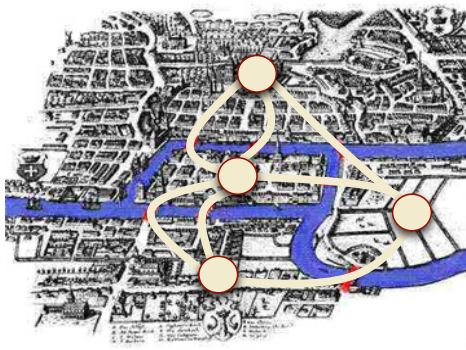
Outline

1. Combinatoric and Graph theory

2. Examples of applications

3. Objectives of this school

1st Example: roads' network

What is the "best" road for reaching Oulu from Helsinki?

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Model geographical netowrk by a graph

1st Example: roads' network

What is the "best" road for reaching Oulu from Helsinki?

Model geographical netowrk by a graph

Use powerful tools that deal with graphs

1st Example: roads' network

More difficult setting

- traffic jam
- bus/subway schedule
- no-left, no-right and no U-turn signs at intersections.

Again, graph algorithm tools may help
That is how your GPS work !!

2nd Example: the Internet
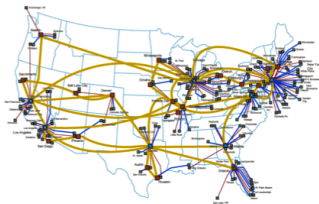
Internet network (Autonomous Systems)
Optical networks (WDM)

- node= IP routers
- links= optical fiber
- capacity on links

- How to compute "best" routes?
- Where to put Amplificators?
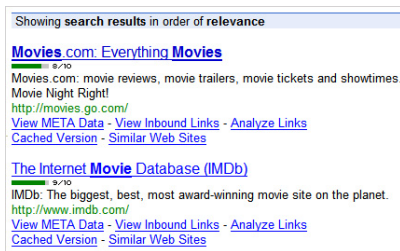- Which links to be turned off to limit energy consumption?

...

3rd Example: Social Network

Model of social interaction
a user = a node
two friends = an edge

- structure of social networks?

- communities?

- how to do advertisement?

- how to prevent advertisement?

More Example: Web (google)

Google PageRank:
    sort search results
node= web page
link = hyperlink

Showing **search results** in order of **relevance**

Movies.com: Everything **Movies**
▬ 8/10
Movies.com: movie reviews, movie trailers, movie tickets and showtimes.
Movie Night Right!
http://movies.go.com/
View META Data - View Inbound Links - Analyze Links
Cached Version - Similar Web Sites

The Internet **Movie** Database (IMDb)
▬ 9/10
IMDb: The biggest, best, most award-winning movie site on the planet.
http://www.imdb.com/
View META Data - View Inbound Links - Analyze Links
Cached Version - Similar Web Sites

1. finding pages with the word movies in it
2. determining the importance of a page.

More Example: Web (google)
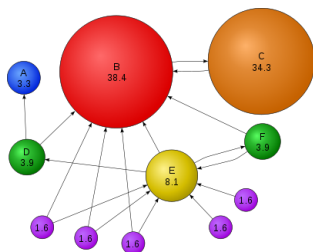
Google PageRank:
> sort search results

node= web page

link = hyperlink

1. finding pages with the word movies in it

2.
   - build the graph of the Web
   - do a random walk on a the graph or compute the eigenvector of a matrix

Outline

Objectives of this school

1. learn how to model problems (from many domains) using graphs
2. know the available tools to handle these problem
   - classical algorithms
   - Linear Programming
3. decide if a problem is "easy" or "difficult"
4. know what to do when facing a "difficult" problem
   - exact exponential algorithms
   - parameterized algorithms
   - approximation algorithms
   - heuristics