

Graph Theory and Optimization

Why is it useful?

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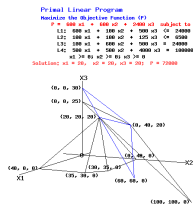
Outline

- 1 Combinatoric and Graph theory
- 2 Examples of applications
- 3 Objectives of this school

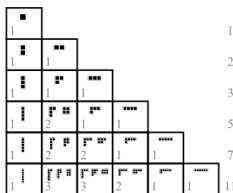
Combinatorics

Branch of mathematics concerning the study of **finite or countable** objects (existence, enumeration, structure).

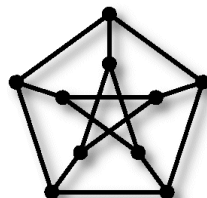
Integer Linear Programming



Integer partition



Graphs



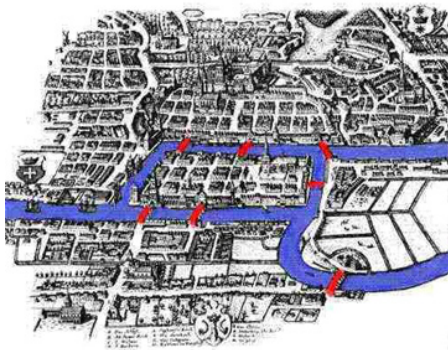
Graph Theory

an old story

Euler 1735: Koenisberg bridges.

Existe-t-il un parcours empruntant tous les ponts une fois et une seule ?

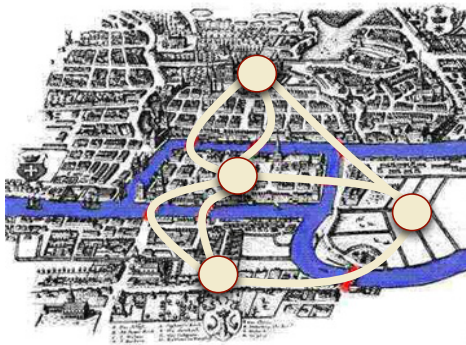
Is there a **cycle** going through each bridge exactly once?



Graph Theory

an old story

Modeling: city = **graph**, island = **vertex**, bridge = **edge**



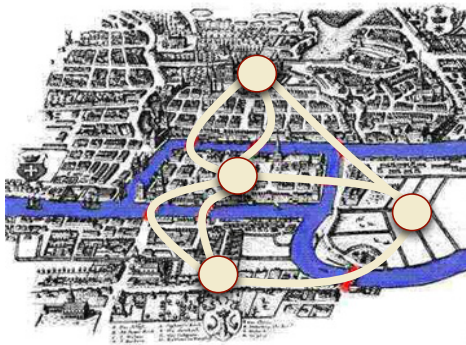
Graph Theory

an old story

Modeling: city = **graph**, island = **vertex**, bridge = **edge**

Question: can we find an **eulerian cycle** in this graph?

Cycle going through all edges once and only once



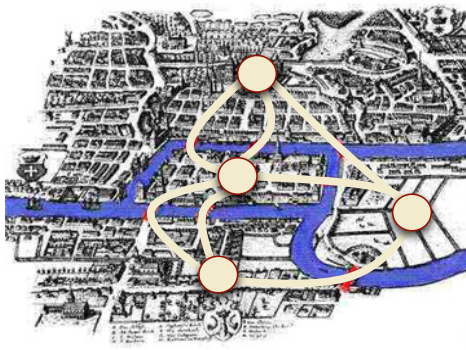
Graph Theory

an old story

Modeling: city = **graph**, island = **vertex**, bridge = **edge**

Question: can we find an **eulerian cycle** in this graph?

Solution: **Such cycle exists if and only if all nodes have even degree**



Graph Theory

an old story

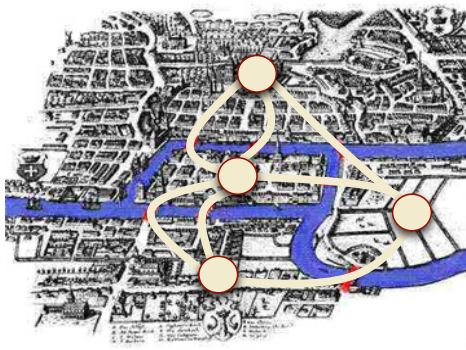
Modeling: city = **graph**, island = **vertex**, bridge = **edge**

Question: can we find an **eulerian cycle** in this graph?

Solution: **Such cycle exists if and only if all nodes have even degree**

An intriguing variant: find a cycle going through all vertices once and only once (**Hamiltonian cycle**) is **very** difficult

One million dollar (Clay price) !



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1st Example: roads' network

What is the “best” road for reaching Oulu from Helsinki?



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Model geographical network by a **graph**



1st Example: roads' network



What is the “best” road for reaching Oulu from Helsinki?

Model geographical network by a **graph**

Use powerful tools that deal with graphs

1st Example: roads' network

More difficult setting

- traffic jam
- bus/subway schedule
- no-left, no-right and no U-turn signs at intersections.

Again, graph algorithm tools may help

That is how your GPS work !!



2nd Example: the Internet



Internet network (Autonomous Systems)

Optical networks (WDM)

- node= IP routers
- links= optical fiber
- capacity on links

- How to compute “best” routes?
- Where to put Amplificators?
- Which links to be turned off to limit energy consumption?

...



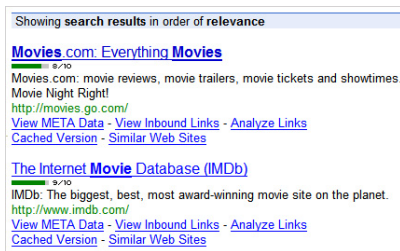
3rd Example: Social Network



Model of social interaction
a user = a node
two friends = an edge

- structure of social networks?
- communities?
- how to do advertisement?
- how to prevent advertisement?

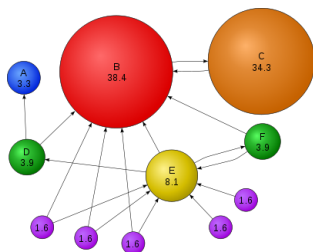
More Example: Web (google)



Google **PageRank**:
sort search results
node= web page
link = hyperlink

- 1 finding pages with the word movies in it
- 2 determining the importance of a page.

More Example: Web (google)



Google **PageRank**:

sort search results

node= web page

link = hyperlink

- 1 finding pages with the word movies in it
- 2
 - build the graph of the Web
 - do a **random walk** on a the graph or compute the eigenvector of a matrix

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Objectives of this school

- ① learn how to **model** problems (from many domains) using graphs
- ② know the available tools to handle these problem
 - **classical algorithms**
 - **Linear Programming**
- ③ decide if a problem is "**easy**" or "**difficult**"
- ④ know what to do when facing a "difficult" problem
 - exact exponential algorithms
 - parameterized algorithms
 - **approximation algorithms**
 - heuristics