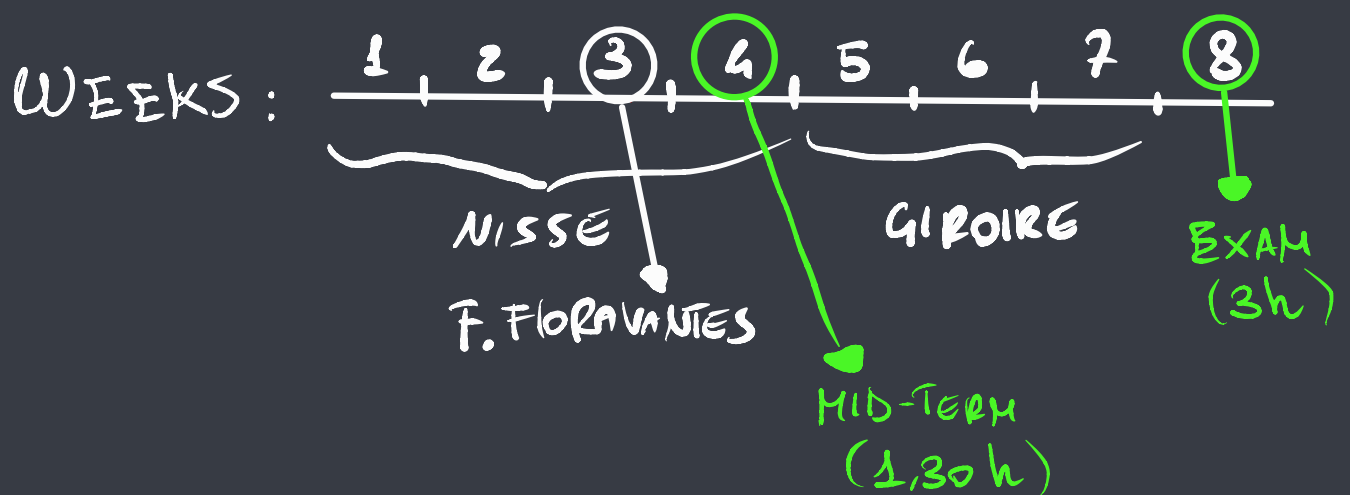


# GRAPH ALGOS & COMB. OPT



2021

# INTRO TO GRAPHS

TH

- $T = (V, E)$  is a Tree  $\Leftrightarrow T$  is CONNECTED and  $|V| = |E| + 1$

PROOF ( $\Leftarrow$ )

( $\Rightarrow$  IS TRIVIAL)

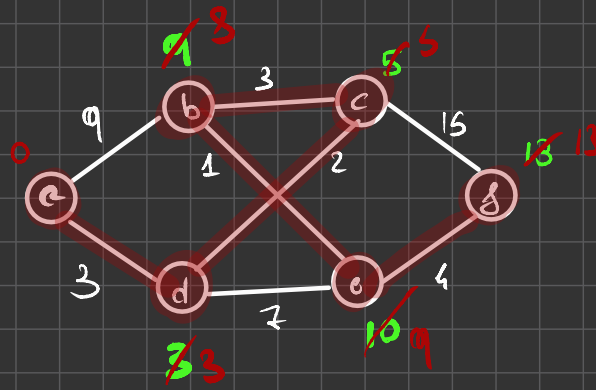
- (RIC  $T$  is a TREE if CONNECTED and NO CYCLES)
- Suppose  $T$  not a tree  $\Rightarrow \exists$  cycle  $(v_1, \dots, v_p)$
- remove  $\{v_1, v_p\} \in E$  obtain  $T'$  (connected)
- $|E'| = |E| - 1 = |V| - 2 = |V'| - 2$
- $|E'| < |V'| - 1 \Rightarrow T'$  NOT CONNECTED  $\perp$



- $G = (V, E)$  is **connected** if, for every two vertices  $x \in V$  and  $y \in V$ , there exists a path from  $x$  to  $y$ .

**Exercise:** Prove that if  $|E| < |V| - 1$  then  $G = (V, E)$  is NOT connected

|    | a | b | c | d | e  | f  |
|----|---|---|---|---|----|----|
| 1° | 0 | ∞ | ∞ | ∞ | ∞  | ∞  |
| 2° |   | 9 | " | 3 | "  | "  |
| 3° |   | " | 5 |   | 10 | "  |
| 4° |   | 3 |   |   | "  | 20 |
| 5° |   |   |   |   | 9  | "  |
| 6° | 0 | 3 | 5 | 3 | 9  | 13 |



Dijkstra