ALGORITHMS 4 TÉLECOMUNICATION

2022

MIN-MIN FORMULATION

MIN-SUM · Given directed $G=(V,E)$ find 2 edge disjoint $(s\text{-}t)$-paths s.t. the sum of the weights of the paths is minimized $\left(O(m+n\log n)\right)$

MIN-MIN · Same but we want to minimize weights only of the first path (NP-COMP.)

MIN-MAX · Minimize the weight of the longest

MIN-MIN FORMULATION

· $x^i_{uv} \begin{cases} 1 & \text{if } uv \in \text{PATH}_i \mid i\in\{1,2\} \\ \varnothing \end{cases}$

· $\min \sum_{uv\in E} x^1_{uv}\cdot w_{uv}$

$$\text{s.t.} \quad \sum_{u\in N^-(v)} x^i_{uv} - \sum_{u\in N^+(v)} x^i_{uv} = \begin{cases} -1 & \text{if } v=s \\ 1 & \text{if } v=t \\ \varnothing \end{cases} \quad ,\forall v\in V \wedge \forall i\in\{1,2\} \quad // \text{ flow BALANCE}$$

$$x^1_{uv} + x^2_{uv} \le 1 \quad ,\forall uv\in E \quad // \ uv\in \text{PATH}_1 \text{ OR } \text{PATH}_2$$

we assumed capacities 1

$$x^i_{uv} \in \{0,1\} \quad ,\forall i\in\{1,2\} \wedge \forall uv\in E$$

MIN-MAX FORMULATION

· The same of MIN-MIN but the PATH_1 must be the heaviest:

$$\hookrightarrow \sum_{uv\in E} x^1_{uv}\cdot w_{uv} - \sum_{uv\in E} x^2_{uv}\cdot w_{uv} \ge \varnothing$$

(s_1,t_1)-(s_2,t_2) · If I want 2 disjoint paths (s_1,t_1) and (s_2,t_2) I just remove the objective and modify the BALANCE CONSTRAINT

$$\sum_{u\in N^-(v)} x^i_{uv} - \sum_{u\in N^+(v)} x^i_{uv} = \begin{cases} -1 & \text{if } v=s_i \\ 1 & \text{if } v=t_i \\ \varnothing \end{cases} \quad ,\forall v\in V \wedge \forall i\in\{1,2\} \quad // \text{ flow BALANCE}$$

generalized...

MULTI COMMODITY FLOW

· k paths from s_k to t_k, we want to minimize the total weight

$$\min \sum_{k\in K}\sum_{uv\in E} f^k_{uv}\cdot w_{uv}$$

$$\text{s.t.} \quad \sum_{u\in N^-(v)} f^k_{uv} - \sum_{u\in N^+(v)} f^k_{uv} = \begin{cases} -1 & \text{if } v=s_k \\ 1 & \text{if } v=t_k \\ \varnothing \end{cases} \quad ,\forall v\in V \wedge \forall k\in K \quad // \text{ flow BALANCE}$$

$$\sum_{k\in K} f^k_{uv} \le c_{uv} \quad ,\forall uv\in E \quad // \ uv\in \text{PATH}_k$$

capacity

$$f^k_{uv} \ge 0 \quad ,\forall k\in K \wedge \forall uv\in E$$

· POLYNOMIAL if $f^k_{uv}\in\mathbb{N}$

How to minimize the maximum load?

· Let z be the MAXIMUM LOAD (the load of an arc is the total flow on it)

$$\begin{aligned} \min\ & z \\ \text{s.t. } & \sum_{k\in K} f^k_{uv} \le z \\ & \vdots \end{aligned}$$

How to minimize #arcs used?

· Introduce $x_{uv}=1$ if $uv\in E$ is used

$$\begin{aligned} \min\ & \sum_{uv\in E} x_{uv} \\ \text{s.t. } & f^k_{uv} \le b_k x_{uv} \quad \forall uv\in E \wedge k\in K \\ & \vdots \end{aligned}$$

b_k: flow demand for commodity b_k

How to maximize the # granted requests?

· Let $y_k=1$ if REQUEST k is accepted

$$\begin{aligned} \max\ & \sum_{k\in K} y_k \\ \text{s.t. } & \sum_{u\in N^-(v)} f^k_{uv} - \sum_{u\in N^+(v)} f^k_{uv} = \begin{cases} -b_k y_k & \text{if } v=s_k \\ b_k y_k & \text{if } v=t_k \\ 0 \end{cases} \quad \forall v\in V \wedge \forall k\in K \\ & \vdots \end{aligned}$$

ROUTING (s_k-t_k)-FLOW

MULTI-ROUTING · The (s_k-t_k)-flow can use several paths

MONO-ROUTING · Unique path for (s_k-t_k)-flow

How to solve multi-commodity flow w/ mono-routing?

· $x^k_{uv}=1$ if $uv\in E$ is in PATH_k

FLOW CONSERVATION

$$\sum_{u\in N^-(v)} x^k_{uv} - \sum_{u\in N^+(v)} x^k_{vu} = \begin{cases} 1 & \text{if } v=s_k \\ -1 & \text{if } v=t_k \\ 0 \end{cases} \quad \forall uv\in E \wedge \forall k\in K$$

CAPACITY

$$\sum_{k\in K} x^k_{uv}\cdot b_k \le c_{uv} \quad , \forall uv\in E$$

DOMAIN

$$x^k_{uv}\in\{0,1\}$$

OBJ

...

BALANCED COLORING

Let $G=(V,E)$ and $C=\{1,\dots,k\}$

Let $x_{ic} = \begin{cases} 1 & \text{, if node } i\in V \text{ is colored with } c\in C \\ \emptyset & \text{, else} \end{cases}$

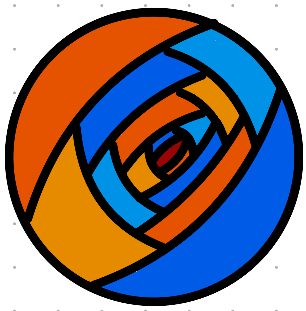
Let also α be the maximum number of same colored nodes, and β the minimum

Our objective is to minimize: $z(G,k) = \min\limits_{\pi\in\Pi_k} B(G,\pi) = \alpha-\beta$

Then the ILP is the following

$$
\begin{array}{lll}
\min & \alpha-\beta & \\
\text{s.t.} & \alpha \geq \sum\limits_{i\in V} x_{ic} & ,\forall c\in C \\
& \beta \leq \sum\limits_{i\in V} x_{ic} & ,\forall c\in C \\
& x_{ic} + x_{sc} \leq 1 & ,\forall is\in E \wedge \forall c\in C \\
& \sum\limits_{i\in V} x_{ic} \geq 1 & ,\forall c\in C \\
& \sum\limits_{c\in C} x_{ic} = 1 & \forall i\in V \\
& k \leq |V| & \\
& \alpha \leq |V| & \\
& \beta \geq \emptyset & \\
& x_{ic} \in \{0,1\} & ,\forall i\in V \wedge \forall c\in C
\end{array}
$$

- Imagine we want that $\alpha-\beta\leq 2$:
- Introduce $C_c = \begin{cases} 1 & \text{, if color } c \text{ is used} \\ 0 & \end{cases}$



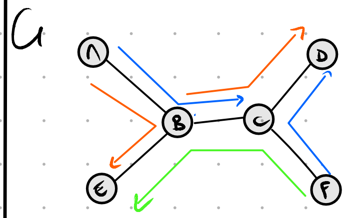
$$
\begin{array}{lll}
\min & \sum\limits_{i=1}^{m} c_i & \\
\text{s.t.} & \sum\limits_{c=1}^{m} x_{ic} = 1 & \forall i\in V \\
& x_{ic}+x_{sc} \leq 1 & ,\forall (i,s)\in E \wedge \forall c\in\{1,\dots,m\} \\
& \sum\limits_{i\in V} x_{ic} \leq C_c\cdot m & ,\forall c\in\{1,\dots,m\} \\
& x_{ic} \leq C_c & ,\forall c\in\{1,\dots,m\} \wedge \forall i\in V \\
& C_{c+1} \leq C_c & ,\forall c\in\{1,\dots,m-1\}
\end{array}
$$

$\alpha - \beta \le 2$

$\beta \le \sum_{i \in V} x_{i,c} + m(1-c_c) \quad \forall i \in V \wedge \forall c \in \{1..m\}$

OPTICAL FIBER

• LOAD: #PATHS on an arc that goes in same direction

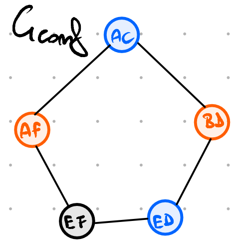


BUILD CONFLICT GRAPH

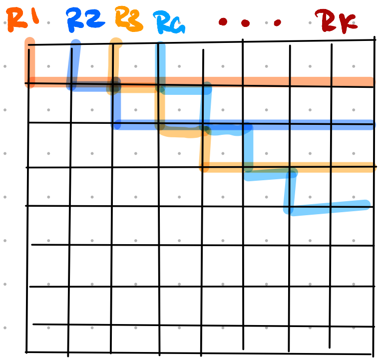
$\Rightarrow$

• Two nodes in G cannot have same colour if they be 2 conflict request

// 1 NODE FOREACH REQUEST
// 1 ARC FOR EACH CONFLICT



MAXIMIZE CONFLICTS

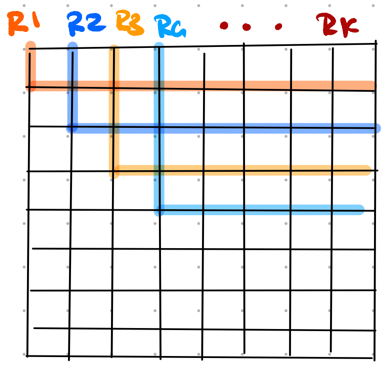


Always complete conflict graph

$LOAD(G,K,R) = 2$

$\delta(G,K,R) = |K|$

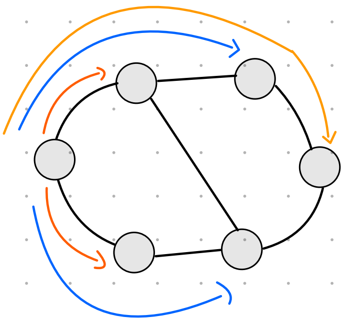
MINIMIZE CONFLICTS



$LOAD(G,K,R) \le \delta(G,K)$

OTA

• $\forall G \quad L(G, OTA) = \delta(G, OTA)$ and can be computed in polynomial time!



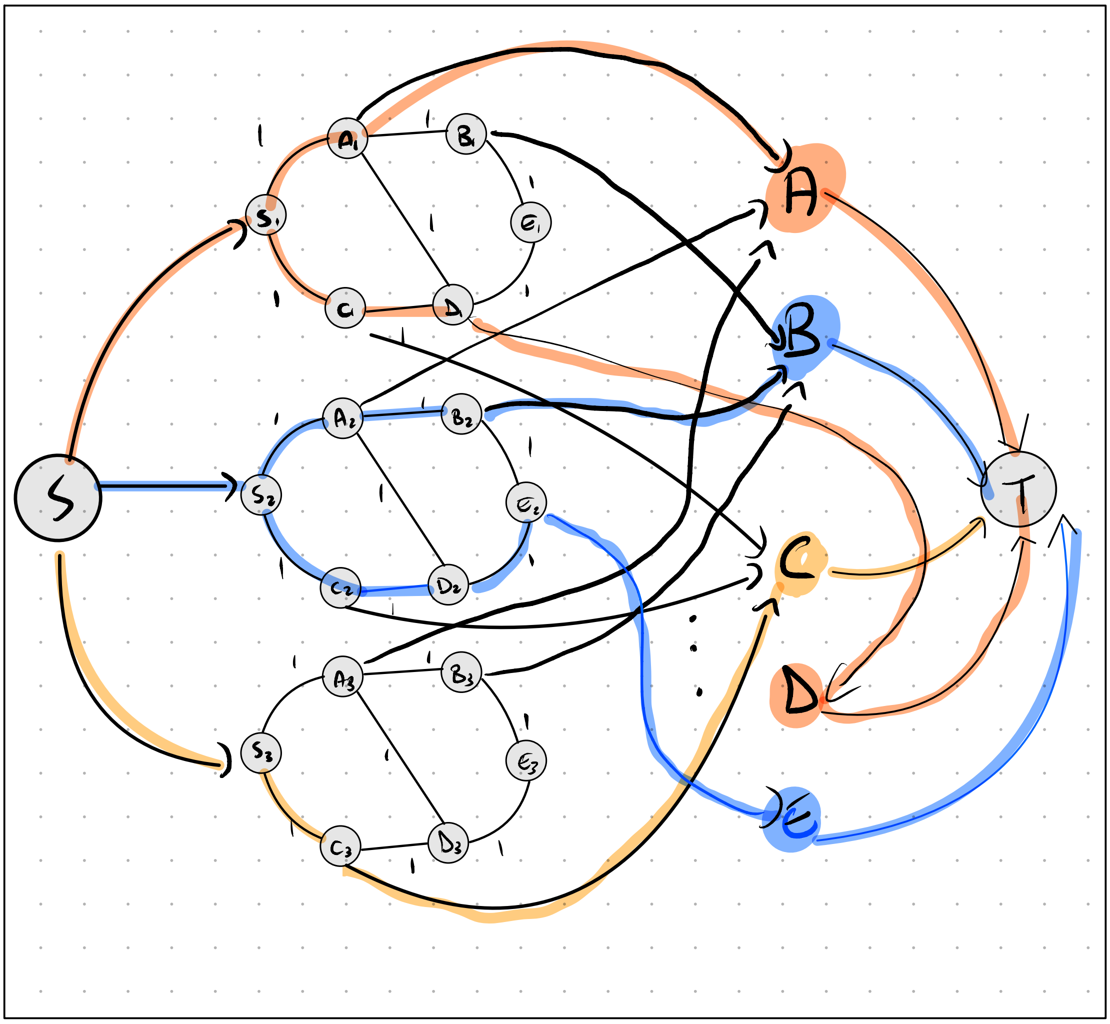
How to find those paths?

=)

· To solve the problem Just get an headache

• One copy for each color, is now a flow Problem

(Try with one copy, then 2, ... until you solve)



OTM -> Just put capacity ∅ in arcs i->T you don't want

ILP for RWA

(1) Compact

(2) Path formulation

(3) Configuration

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(1) COMPACT FORMULATION (Node/Arc) CONSTRS VARS

$G = (V, E)$

$K = \{(s_k, t_k, 1)\}$

$\Lambda = \{\lambda_1, \lambda_2, \ldots, \lambda_{|\Lambda|}\}$ // Requests

- Use the multi commodity flow formulation:

VARS

- $x^k_{uv,\lambda} = 1$ if request k has arc uv with color λ
- $y^k_\lambda = 1$ if the path of request k has color λ
- $z_\lambda = 1$ if wavelength λ is used so $\sum_{k\in K} y^k_\lambda \le |K|\cdot z_\lambda$

CONSTS

- $\sum_{v\in N^-(u)} x^k_{uv,\lambda} - \sum_{v\in N^+(u)} x^k_{uv,\lambda} = \begin{cases} -y^k_\lambda & \text{if } u=s_k \\ +y^k_\lambda & \text{if } v=t_k \\ 0 & \text{else} \end{cases}$ $,\forall u\in V \wedge \forall k\in K \wedge \forall \lambda\in\Lambda$ // FLOW CONS
- $\sum_{\lambda\in\Lambda} y^k_\lambda = 1$ $,\forall k\in K$ // EACH REQ IS ROUTED
- $\sum_{k\in K} x^k_{uv,\lambda} \le 1$ $,\forall uv\in E \wedge \forall\lambda\in\Lambda$ // CAPACITY CONSTR
- $\sum_{k\in K} y^k_\lambda \le |K|\cdot z_\lambda$ $,\forall\lambda\in\Lambda$ // is λ used?

OBJECTIVE

- $\min \sum_{\lambda\in\Lambda} z_\lambda$

COMPLEXITY

- #VAR $O(|K|\cdot|\Lambda|\cdot|E|)$
- # CONSTRS $O(|V|\cdot|K|\cdot|\Lambda| + |E|\cdot|\Lambda|)$

(2) PATH FORMULATION (ARC/PATH)

CONSTRS. VARS.

- Π^k : set of all (s_k, t_k)-paths in G
- $x^k_{\pi,\lambda} = 1$ if req. $k \in K$ use path $\pi \in \Pi^k$ with wavelength $\lambda \in \Lambda$
- $y^k_\lambda = 1$ if $k \in K$ ose $\lambda \in \Lambda$
- $z_k = 1$ if $\lambda \in \Lambda$ is used

$\Rightarrow$ Here path are given so the ILP looks better than before.

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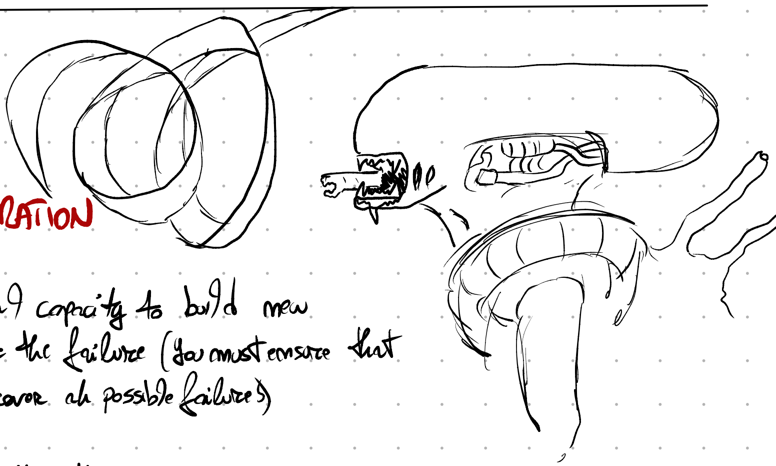
(3) CONFIGURATION FORMULATION

- Configuration : reqs with arc disjoint routing
- $\otimes$: set of all configuration.

. . .



PROTECTION & RESTAURATION

PROTECTION: Already have backup paths for link failure

RESTAURATION: Use residual capacity to build new paths after the failure (you must ensure that you can cover all possible failures)

MODELS

- $\boxed{1+1}$: send traffic in both path
- $\boxed{1:1}$: use the backup path only after the failure
- $\boxed{\text{SHARED PROTECTION}}$

Master 2 IFI/UbiNet : Algorithms for telecommunications

Final exam, March 2021

– 3 hours –

Course and manuscript notes allowed. Not computers, cellphones, calculators, books.

Instruction and comments: the points awarded will be based on the correctness of your answers as well as on the clarity of the main steps in your reasoning. ***All proposed solutions must be proved. All the exercises are independent.*** *The points are indicated so you may adapt your effort.*

Exercise 1 (Traffic monitoring, 4 points, 30 minutes)

The goal of this exercise is to write an integer linear program (ILP) for placing as few sensors as possible on the links of a network in order to monitor its traffic (e.g., to perform deep packet inspection on each connection request). We use the following definitions and notations.

- The network is modeled by a directed graph $G = (V, E)$;
- All links of the network have the same capacity c;
- K is the set of connection requests, and request $k \in K$ is characterized by its source s_k, its destination t_k, and its bandwidth requirement b_k;
- A connection request $k \in K$ from node s_k to node t_k is routed along a unique path;
- A connection request $k \in K$ is *monitored* if there is a sensor on one of the links of its path;
- If there is a sensor on link ab, it monitors all the requests routed on that link. So we place at most one sensor per directed link, and there is no need to place a sensor on a link that carries no traffic (i.e., that is not used);
- A sensor can monitor the traffic of a single directed link. Therefore, if there is a directed link ab from node a to node b and a directed link ba from node b to node a, we may have to place one sensor on link ab and a second sensor on link ba.

The objective is to monitor the traffic of all the requests using as few sensors as possible.

Question: (define all variables, explain all contraints, give number of variables and constraints)

→ Write an ILP that computes simultaneously the routing of the connection requests and the placement of the smallest possible number of sensors.
Recall that a connection request must be routed on a single path and that links have capacity c.

Hint: use a variable $x_{k,e}$ that is set to 1 if request k is monitored on link e and a variable y_e that indicates whether a sensor is placed on link e or not.

Exercise 2 (Flow & Connectivity, 3 points, 20 minutes)
A directed graph $G = (V, E)$ is strongly connected if for every pair of vertices u and v in V, there is a directed path from u to v and a directed path from v to u.

Question: (define all variables, explain all contraints, give number of variables and constraints)

→ Write an ILP to find the smallest subset $F \subseteq E$ of edges of G such that the subgraph $H = (V, F)$ is strongly connected.

Exercise 3 (IoT, sensors, drones, 6 points, 50 minutes)
A set S of sensors are deployed on the floor, and for each sensor $s \in S$, we know its position p_s. We use a drone to regularly collect the data generated by the sensors.

- Let P be the set of all possible positions of the drone.
- When the drone is at position $p \in P$, it can collect the data generated by all the sensors at distance at most r. So it can collect the data generated by any sensor $s \in S$ such that $dist(p, p_s) \leq r$, where p_s is the position of sensor s.
- We assume that the sensors are deployed at positions that are close enough to the positions in P (i.e., $\forall s \in S, \exists p \in P$ such that $dist(p_s, p) \leq r$).
- Between time steps t and $t+1$, the drone can move from its current position p to any position $p' \in P$ such that $dist(p, p') \leq f$.

  We can define the directed graph $G = (P, E)$ with nodes the possible positions of a drone, and there is an arc $e = (p, p') \in E$ from position p to position p' if $dist(p, p') \leq f$. The length of arc $e = (p, p')$ is $dist(p, p')$. We assume that this graph is connected.
- To collect the data from the sensors, the drone starts at time $t = 0$ from position $p_0 \in P$, then follows a path in G to collect data from all the sensors in S and returns to position p_0.

The objective of this exercise is to determine the path followed by the drone to collect the data from the sensors in as few time steps as possible.

Question: (define all variables, explain all contraints, give number of variables and constraints)

1. Write an ILP to determine the path followed by the drone with as few steps as possible.

   Recall that at any time t, the drone can be at a unique position $p \in P$.

   Hint: use a variable $x_{p,t}$ that is set to 1 if the drone is at position $p \in P$ at time t.
2. Given a number T of steps (sufficient to collect all the data, e.g., $T = |P|$), explain how to modify your ILP to minimize the length of the path followed by the drone.

Exercise 4 (Frequency assignment, 6 points, 50 minutes)

The goal of this exercise is to write an Integer Linear Program (ILP) for assigning integer values to the vertices of an undirected graph subject to special constraints.

We use the following definitions and notations.

- $G = (V, E)$ is an undirected graph with $|V|$ vertices and $|E|$ edges;
- $\text{dist}(u, v)$ is the shortest path distance in G between u and v;
- $F = \{1, 2, \cdots, p\}$ is a set of integer values and p is a large enough integer.

Questions (recall to explain each variable and each constraint)

4.1 Using one binary variable $x_{u,f}$ for each vertex $u \in V$ and each integer value $f \in F$, write an ILP to assign an integer value to each vertex of the graph subject to the following constraints:

(a) If $\text{dist}(u, v) = 1$, then $|f_u - f_v| \geq 2$. In other words, if u is assigned value f, then its neighbor $v \in N(u)$ cannot be assigned values $f - 1$, f, or $f + 1$;

(b) If $\text{dist}(u, v) = 2$, then $f_u \neq f_v$.

The objective is to minimize the number of different integer values assigned to the vertices of the graph.

4.2 Explain how to modify your ILP in order to minimize the largest assigned integer value.

In your answers, you can reuse the variables and constraints of Question 4.1.

4.3 We now want to assign k integer values to each vertex with the extra constraint

(c) If $f, f' \in F$ are assigned to vertex u, then we have $|f - f'| \geq 3$.

Write an ILP to assign k integer values to each vertex of the graph subject to constraints (a), (b) and (c). The objective is to minimize the number of different integer values assigned to the vertices of the graph.

For your information. In this exercise, you have assigned frequencies to base stations of a wireless network. In fact, each vertex of the graph is a base station, and there is an edge between two vertices if the corresponding base stations are at physical distance less than d_T (transmission distance). The frequencies are modeled by integer values. Constraints (a), (b) and (c) model the fact that close frequencies interfere and the level of interference decreases with the physical distance between base stations.

Exercise 5 (Sudoku, 4 points, 30 minutes)

5	3	4	6	7	8	9	1	2
6	7	2	1	9	5	3	4	8
1	9	8	3	4	2	5	6	7
8	5	9	7	6	1	4	2	3
4	2	6	8	5	3	7	9	1
7	1	3	9	2	4	8	5	6
9	6	1	5	3	7	2	8	4
2	8	7	4	1	9	6	3	5
3	4	5	2	8	6	1	7	9

$\sum_k \sum_{r,c \in Block(i,j)} x_{r,c,k} \leq 1 \quad \forall 0 \leq i,j \leq 2$

Figure 1: Example of partially completed sudoku grid.

The objective of the sudoku game is to fill a 9×9 grid with digits so that each column, each row, and each of the nine 3×3 subgrids that compose the grid (also called "boxes" or "blocks") contain all of the digits from 1 to 9. A starting partially completed grid is given, as shown in the example of Figure 1.

- $F = \{f_{r,c,k} \mid 1 \leq r, c, k \leq 9\}$ is the initial partially completed grid (input);
- $f_{r,c,k}$: constant set to 1 if the cell at row r and column c has digit k, and 0 otherwise;
- block (i, j), with $0 \leq i, j \leq 2$, contains cells $\{((r = 3i+i', c = 3j+j') \mid 1 \leq i', j' \leq 3\}$

Question. (explain each constraint)
Using one binary variable $x_{r,c,k}$ per cell of the grid and possible digit, write a ILP to solve any sudoku problem as defined by its partially completed initial grid F.

$f_{r,c,k} \leq x_{r,c,k} \quad \forall r,c,k$

$\sum_c x_{r,c,k} = 1 \quad , \forall r,k$

$\sum_r x_{r,c,k} = 1 \quad , \forall c,k$

$\sum_k x_{r,c,k} = 1 \quad , \forall r,c$

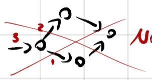
$\sum_{\{r,c\} \in Block(i,j)} x_{r,c,k} \leq 1 \quad , \forall 0 \leq i,j \leq 2 \wedge \forall k$

Var:

$$X_{ij} \begin{cases} 1 & \text{if sensor in } (i,j) \in E \\ 0 \end{cases}$$

$$f_{ij}^{k} \in \mathbb{N} \qquad m_{ij}^{k} \begin{cases} 1 & \text{if } k \text{ monitor } ij \\ 0 \end{cases}$$

UNIQUE PATH



$$\min \sum_{(i,j)\in E} X_{ij}$$

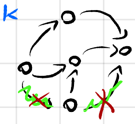
$$\text{s.t} \sum_{i\in N^-(j)} f_{ij}^{k} - \sum_{i\in N^+(j)} f_{ji}^{k} = \begin{cases} -1 & \text{if } i = s_k \\ +1 & \text{if } i = t_k \\ 0 \end{cases} \quad \forall k\in K, \forall i \in V$$

$$\sum f_{ij}^{k} b_k \leq c \qquad \forall i,j \in E$$

$$m_{ij}^{k} \leq f_{ij}^{k} \qquad \forall k \in K \wedge \forall ij \in E$$

$$\sum_{(ij)\in E} m_{ij}^{k} \geq 1 \qquad \forall k \in K$$

at least one sensor foreach k



$$\sum_{k\in K} m_{ij}^{k} \leq X_{ij} \cdot |K| \qquad \forall ij \in E$$

sensor $\leq |K|$

$$X_{ij} \in \{0,1\} \qquad \forall (i,j) \in E$$

$$m_{ij}^{k} \in \{0,1\} \qquad \forall (i,j) \in E \wedge \forall k \in K$$

$$f_{ij}^{k} \geq 0 \qquad \forall (i,j) \in E \wedge \forall k \in K$$

$$K = \bigcup_{i,j \in V | i \neq j} \{(i,j), (j,i)\}$$

$$x_{ij} = \begin{cases} 1 & \text{if } (i,j) \in F \subseteq E \\ 0 \end{cases}$$

$$\min \sum_{(i,j)\in E} x_{ij}$$

$$\text{s.t.} \quad \sum_{s\in N^-(i)} f_{si}^{k} - \sum_{s\in N^+(i)} f_{is}^{k} = \begin{cases} -1 & \text{if } i = s_k \\ 1 & \text{if } i = t_k \\ 0 \end{cases} \qquad \forall k \in K$$

$$f_{ij}^{k} \leq x_{ij} \qquad \forall (i,j) \in E \wedge k \in K$$

$$x_{ij} \in \{0,1\} \qquad \forall (i,j) \in E$$

$$f_{ij}^{k} \geq 0 \qquad \forall (i,j) \in E \wedge k \in K$$

$N(s)$ = Set of positions p such that $dist(p, p^s) \leq r$

$x_{uv} = 1$ if edge uv is selected

$$\sum_{p \in N^+(b)} x_{bp} = 1 \longrightarrow \text{leaves base station only once}$$

$$\sum_{q \in N^+(p)} x_{pq} - \sum_{q \in N^-(p)} x_{qp} = 0 \quad \forall p \in P \longrightarrow \text{Ensures a cycle}$$

$$\sum_{p \in N(s)} \sum_{q \in N^+(p)} x_{pq} \geq 1 \quad \forall s \in S$$

→ For each sensor s the drone must visit at least one position $p \in N(s)$

$$\min \sum_{uv \in P} x_{uv}$$

EX 2

$$k = \bigcup_{i,j \in V | i \neq j} \{(i,j), (j,i)\}$$

$x_{ij} = 1$ if $(i,j) \in F \subseteq E$

$$\min \sum_{(i,j) \in E} x_{ij}$$

$$\text{s.t.} \quad \sum_{s \in N^-(i)} f_{si}^k - \sum_{s \in N^+(i)} f_{is}^k = \begin{cases} -1 & \text{if } i = s_k \\ 1 & \text{if } i = t_k \\ 0 & \end{cases} \quad \forall k \in K$$

$$f_{ij}^k \leq x_{ij} \quad \forall (i,j) \in E \wedge k \in K$$

$$x_{ij} \in \{0,1\} \quad \forall (i,j) \in E$$

$$f_{ij}^k \geq 0 \quad \forall (i,j) \in E \wedge k \in K$$

SUDOKU

$$f_{r,c,k} \leq x_{r,c,k} \quad , \forall r,c,k$$

$$\sum_c x_{r,c,k} = 1 \quad , \forall r,k$$

$$\sum_r x_{r,c,k} = 1 \quad , \forall c,k$$

$$\sum_k x_{r,c,k} = 1 \quad , \forall r,c$$

$$\sum_{\{r,c\} \in Block(i,j)} x_{r,} \quad 1 \quad , \forall\, 0 \leq i,j \leq 2 \wedge \forall k$$

SENSOR MONITORING

VARIABLES

$x_{ij} = 1$ if $\exists$ sensor in $(i,j) \in E$

$\delta_{ij}^k \geq 0$

$m_{ij}^k = 1$ if $k \in K$ monitored in $(i,j) \in E$

$P_i^k \in \mathbb{R}$

ILP

$$\min \sum_{(i,j)\in E} x_{ij}$$

$$\sum_{s\in N^-(i)} x_{si} - \sum_{s \in N^+(i)} x_{ij} = \begin{cases} -1 & \text{if } i = s_k \\ 1 & \text{if } i = t_k \\ 0 \end{cases} \quad , \forall k \in K \wedge \forall i \in V$$

$$\sum_{k\in K} \delta_{ij}^k \cdot b_k \leq C \quad , \forall (i,j) \in E$$

MNR

$$m_{ij}^k \leq \delta_{ij}^k \quad , \forall (i,j) \in E \wedge k \in K$$

$$\sum_{(i,j)\in E} m_{ij}^k \geq 1 \quad , \forall k \in K$$ // every k must be monitored

$$\sum_{k\in K} m_{ij}^k \leq |K| \cdot x_{ij} \quad , \forall (i,j) \in E$$ // number of monitored reqs cannot exced the number of requests (OBV $\frac{0}{0}$)

$$\delta_{ij}^k + x_{ij} \leq m_{ij}^k + 1 \quad , \forall (i,j) \in E \wedge \forall k \in K$$ // to monitor k there must be a sensor on the path and the path

$$P_j^k \geq P_i^k + 1 - n(1 - \delta_{ij}^k) \quad , \forall (i,j) \in E \wedge k \in K$$ // AVOID CYCLES

$$x_{ij} \in \{0,1\} \quad , \forall (i,j) \in E$$

$$m_{ij}^k \in \{0,1\} \quad , \forall (i,j) \in E \wedge k \in K$$

$$\delta_{ij}^k \geq 0 \quad , \forall (i,j) \in E \wedge k \in K$$

$$P_i^k \in \mathbb{R} \quad , \forall i \in V \wedge k \in K$$

FREQ. ASSIGNMENT

SETS

A: set of antennas $a \in A$
F: set of frequencies $f \in F$
U: set of users $u \in U$

VARIABLES

$x_{a,f} = 1$ if $a \in A$ uses $f \in F$

$y_f = 1$ if $f \in F$ is used

$P_{u,a} = 1$ if $u \in U$ communicate with $a \in A$

$z_{u,f} = 1$ if $u \in U$ uses $f \in F$

LP

min $\sum_{f \in F} y_f$

s.t.

$\sum_{f \in F} x_{a,f} = 1 \quad \forall a \in A$ // an antenna uses one freq.

$\sum_{a \in A} x_{a,f} \leq |A| y_f \quad \forall f \in F$ // if f is used then at least one antenna must use it

$\sum_{a' \in A \setminus \{a\} \mid (a,a') \leq d_t} x_{a',f} \leq (1 - x_{a,f})|A| \quad , \forall a \in A \wedge \forall f \in F$ // if a uses f then no other a' in it's
// d_t can use the same f

$$\left.\begin{aligned} x_{a,f} &\leq z_{u,f} && , \forall u \in U \wedge \forall a \in A \wedge \forall f \in F \\ x_{a,f} &\leq P_{u,a} && , \forall u \in U \wedge \forall a \in A \wedge \forall f \in F \\ P_{u,a} + z_{u,f} &\leq x_{a,f} + 1 && , \forall u \in U \wedge \forall a \in A \wedge \forall f \in F \end{aligned}\right\} x_{a,f} \leq z_{u,f} \cdot P_{u,a}$$

// if a uses f then every u that communicate
// with a must use f

$\sum_{a \in A \mid d(u,a) \leq d_t} P_{u,a} = 1 \quad , \forall u \in U$ // u can communicate with only one a in his d_t range

$\sum_{a \in A \mid d(u,a) > d_t} P_{u,a} = 0 \quad , \forall u \in U$ // u cannot communicate with no a out of his d_t range

$\sum_{f \in F} z_{u,f} = 1 \quad , \forall u \in U$ // u can use only one f

$\sum_{a \in A \mid d(u,a) \leq d_I} x_{a,f} \leq 1 + (1 - z_{u,f})|A| \quad , \forall u \in U \wedge \forall f \in F$ // if u uses f there can be only one a in his d_I range

$\sum_{u \in U} P_{u,a} \cdot b_u \leq \sum_{f \in F} x_{a,f} \cdot c_f \quad , \forall a \in A$ // sended data cannot exced capacity

Ex 4

if $d_t = 2$

$$\sum_{a' \in A \mid d(a,a') = 2} x_{a',f} \leq (1 - x_{a,f})|A|$$

if $d_t = 1$

$$\sum_{a' \in A \mid d(a,a') = 1} x_{a',f} + x_{a',f-1} + x_{a',f+1} \leq (1 - x_{a,f})|A|$$