University of Oulu
Finland

School on Graph and Optimization
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Exercises : Flows

Exercise 1 [Flows]

Consider the elementary network flow N depicted in Figure 1 (left) and the initial flow f from s to t in Figure 1 (right).

- What must be checked to show that f is a flow ? What is the value of the flow f ?
- Apply the Ford-Fulkerson Algorithm to N starting from the flow f. The first two steps (in particular, the auxiliary digraphs) of the execution of the algorithm must be detailed.
- Give the flow and the cut obtained. Conclusion ?

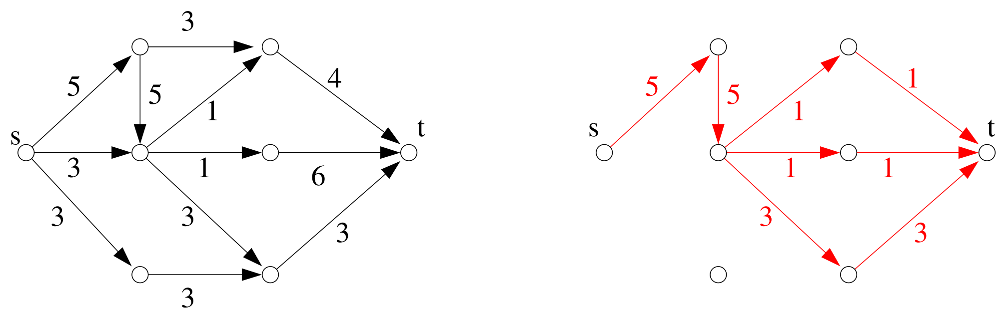
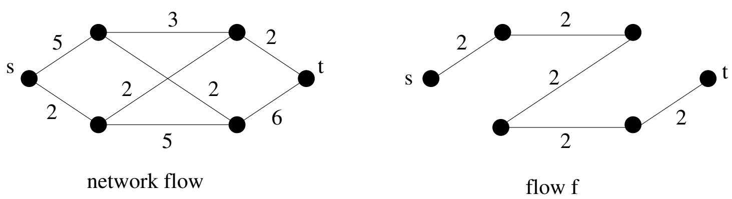


FIGURE 1 – (left) Elementary network flow with arcs' capacity in black. (right) A flow f from s to t : a number close to an arc indicates the amount of flow along it. Arcs that are not represented have no flow.

Exercise 2 Let us consider the (undirected) flow network and the flow f as below :



Apply the "push" algorithm to compute a maximum flow with f as initial flow (detail the first steps). Give the value of the maximum flow (between s and t) and a cut with minimum size.

Exercise 3 There are 3 production sites A, B, C and 5 consumption sites p_1, p_2, p_3, p_4, p_5 ; their production and consumption, respectively, are given in the following tables.

production site		A	B	C	
production		5	4	7	
consumption site	p_1	p_2	p_3	p_4	p_5
consumption	3	4	5	2	1

Finally, each production sites can only serve the consumption sites as summarized in the following table.

A	B	C
p_1, p_3	p_2, p_4	p_3, p_4, p_5

The problem is to satisfy the consumption sites. Model the following problem in terms of flows and give a solution to the problem or explain why it could not exist.

Exercise 4 Suppose we are in the middle of a baseball season where each team T_i , $1 \leq i \leq n$ has won $w(i)$ games so far and thus has $w(i)$ points (recall that in baseball each game has one point and we cannot have a tie.). Let $G_1, G_2, \cdots, G_k$ be the schedule of the remaining games, where each G_i is an unordered pair of teams.

Given T_i , $w(i)$, $1 \leq i \leq n$, and $G_1, G_2, \cdots, G_k$, can we predict that T_1 does or not does not have a chance to have the top score at the end of the season ? if T_1 has a chance of being champion, how can we find a sequence of outcomes (i.e. results of $G_1, G_2, \cdots, G_k$) such that T_1 reaches the top rank at the end of the season ? (of course, model this problem as a flow problem)

As an example, consider the following instance of the problem, where $g(i)$ is the number of remaining games to be played by team i, and $g(i,j)$ is the number of remaining games to be played by team i against team j. Is Harvard eliminated or not ?

	$w(i)$	$g(i)$	$g(i,j)$			
Team	*Wins*	*To play*	*Yale*	*Harward*	*Corneil*	*Brown*
Yale	33	8		1	6	1
Harvard	29	4	1		0	3
Cornell	28	7	6	0		1
Brown	27	5	1	3	1	

hint : consider the bipartite graph with vertex set $A \cup B$ where A consists of the remaining games, and B consists of each of the teams.

Exercise 5 The goal of this exercise is to show an application of flows to organize the defense of the projects of some students.

Assume that the students $\{S_1, \cdots, S_n\}$ have to present their work to some professors at the end of their projects. There are q professors $\mathcal{P} = \{P_1, \cdots, P_q\}$. Each student S_i has a project Q_i, $i \leq n$. For any project, each professor is either a specialist of the subject or not. That is, for any $i \leq n$, $\mathcal{P}$ is partitionned into Sp_i and NSp_i, respectively the subset of the professors that are specialist of the project Q_i, and the professors that are not. Finally, each professor P_j, $j \leq q$, can attend at most a_j defenses.

Each student S_i must present his work to x professors, y of them are specialists of P_i and $z = x - y$ of them are not.

Use a flow-model to organize the juries (which professor will attend which presentation).