

## Exercise 1

$$f(x) = 2x^4 - 4x^3 + 3x^2 + 4x - 3$$

$$f'(x) = 8x^3 - 12x^2 + 6x + 4$$

$$x_0 = 0 \Rightarrow f(0) = -3 \Rightarrow f'(0) = 4$$

$$\alpha = \frac{1}{10}$$

### STEP 1

$$x_1 = x_0 - \alpha f'(x_0) = -\frac{2}{5} = -0.4$$

$$f(x_1) = -\frac{19}{5} = -3.8$$

$$f'(x_1) = -\frac{4}{5} = -0.8$$

### STEP 2

$$x_2 = x_1 - \alpha f'(x_1) = -\frac{2}{5} + \frac{4}{50} = -\frac{16}{50} = -\frac{8}{25} = -0.32$$

$$f(x_2) = 2\left(-\frac{8}{25}\right)^4 - 4\left(-\frac{8}{25}\right)^3 + 3\left(-\frac{8}{25}\right)^2 + 4\left(-\frac{8}{25}\right) - 3 = -\frac{19}{5} = -3.8$$

$$f'(x_2) = 8\left(-\frac{8}{25}\right)^3 - 12\left(-\frac{8}{25}\right)^2 + 6\left(-\frac{8}{25}\right) + 4 = \frac{3}{5} = 0.6$$

### STEP 3

$$x_3 = x_2 - \alpha f'(x_2) = -\frac{8}{25} - \frac{3}{50} = -\frac{19}{50} = -0.38$$

$$f(x_3) = 2\left(-\frac{19}{50}\right)^4 - 4\left(-\frac{19}{50}\right)^3 + 3\left(-\frac{19}{50}\right)^2 + 4\left(-\frac{19}{50}\right) - 3 = -\frac{19}{5} = -3.8$$

$$f'(x_3) = 8\left(-\frac{19}{50}\right)^3 - 12\left(-\frac{19}{50}\right)^2 + 6\left(-\frac{19}{50}\right) + 4 = -\frac{9}{20} = -0.45$$

### COMMENT

- The starting solution is already approximation and with the given learning rate, at each step  $i$  of the algorithm we keep getting almost the same solution for  $f(x_i)$ .
- With a bit smaller learning rate we can improve a bit the value of the solution.

