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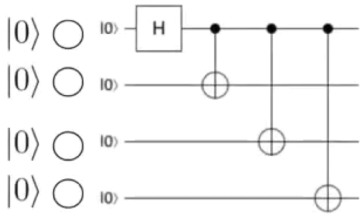
Ex. 1 — What are observables in quantum mechanics? How can you represent them? If a quantum system is in state $|q\rangle$, what can you observe?

They are quantities that can be measured hermitian linear operators in the Hilbert space $L \in \mathbb{C}^{n\times n} \mid L = (L^T)^*$. Their measurement is a real number that corresponds to an EIGENVALUE of L.
A system in the state $|q\rangle$ is a linear combination of it's basis, for example $\{|0\rangle, |1\rangle\}$

$$|q\rangle = \alpha|0\rangle + \beta|1\rangle$$

where $\alpha, \beta \in \mathbb{C}$, so we can observe $|0\rangle$ or $|1\rangle$ with probability respectively
$\mathbb{P}(|0\rangle) = \|\alpha\|^2$, $\mathbb{P}(|1\rangle) = \|\beta\|^2$.

Ex. 2 — Do the following circuits produce the same output?



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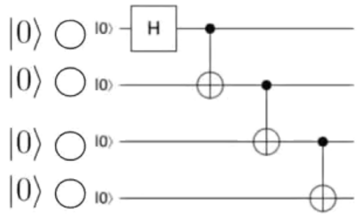

- first qubit goes to $|+\rangle$
- for each of the other we have 50% of probability of flipping to $|1\rangle$
- The possible output are so

$$\{|+000\rangle, |+001\rangle, |+010\rangle, |+011\rangle, |+100\rangle, |+101\rangle, |+110\rangle, |+111\rangle\}$$

$\neq$

- first qubit goes to $|+\rangle$
- we ha 50% of probability that the first qubit flip to $|1\rangle$. If so all other will do the same
- So all possible outputs are:

$$\{|+000\rangle, |+111\rangle\}$$

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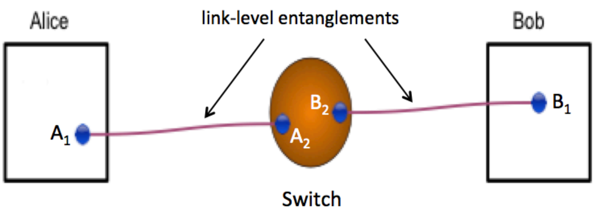
Ex. 3 — When Alice and Bob are located far apart from each other quantum switches (or repeaters) are used to generate a two-qubit Bell state between a qubit owned by Alice (qubit A_1) and a qubit owned by Bob (qubit B_1).

Consider the situation where there is one quantum switch between Alice and Bob. In a first step, link-level entanglements are performed, namely, a two-qubit Bell state between Alice's qubit A_1 and qubit A_2 located at the switch is generated and a two-qubit Bell state between Bob's qubit B_1 and qubit B_2 located at the switch is also generated. See figure below.

Propose a procedure using a protocol seen in class to create a two-qubit Bell state between Alice's qubit A_1 and Bob's qubit B_1.



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Ex. 4 — Consider the coined quantum walk on a line starting at position zero with the Hadamard coin and with the initial state:

$$|\psi(0)\rangle = |0\rangle\,|0\rangle\,.$$

At what time step, if measured, the distribution of the walker position starts to be non-symmetric? Please justify your answer.

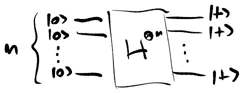
The procedure consist in apply Hadamard to the coined state an then a shift operator, then you can both measure it or repeat

$$|c\rangle \otimes |k\rangle = |0\rangle \otimes |0\rangle \xrightarrow{H \otimes I} \frac{|0\rangle \otimes |1\rangle}{\sqrt{2}} \otimes |0\rangle \xrightarrow{S} \frac{1}{\sqrt{2}} (|0\rangle \otimes |1\rangle + |1\rangle \otimes |-1\rangle)$$

After the first step $k=1$ or $k=-1$ with same probability. Delaying the measurement will change those probabilities.

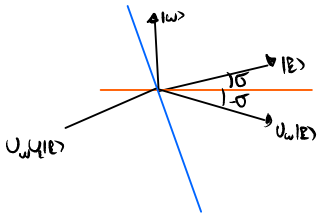
Ex. 5 — What is the goal of the Grover's algorithm? What is its advantage? What do you remember about how it works?

- The Grover's algorithm solves the problem of searching in an unstructured search space of n elements. The advantage is that it solves it in a $O(\sqrt{n})$ instead of $O(n)$.
- It works by creating the so called "EVERYWHERE STATE" $|E\rangle$, which contains all possible states. Of course it contains also the one we are looking for $|w\rangle$.
- $|E\rangle$ is built as follow.

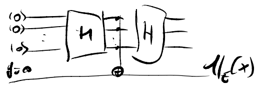


$$\Rightarrow |E\rangle = |+\rangle^{\otimes n} = \left(\tfrac{1}{\sqrt{2}}\right)^n (\underbrace{|0\ldots0\rangle + \_\_\_\_ + |11\ldots1\rangle}_{|w\rangle \text{ is somewhere here.}})$$

- A oracle is needed such that $f(x) = \begin{cases} 1 & \text{if } x \text{ is } w \\ 0 & \text{else} \end{cases}$
- The vectors $|E\rangle, |w\rangle$ are almost ortogonal because $|w\rangle$ is in $|E\rangle$, it's so possible to recursively rotate $|E\rangle$ w.r.t. the plane ortogonal to $|w\rangle$ and then to $|E\rangle$ while the angle between $|E\rangle$ and $|w\rangle$ is smaller than θ, the angle between $|E\rangle$ and the plane ortogonal to $|w\rangle$

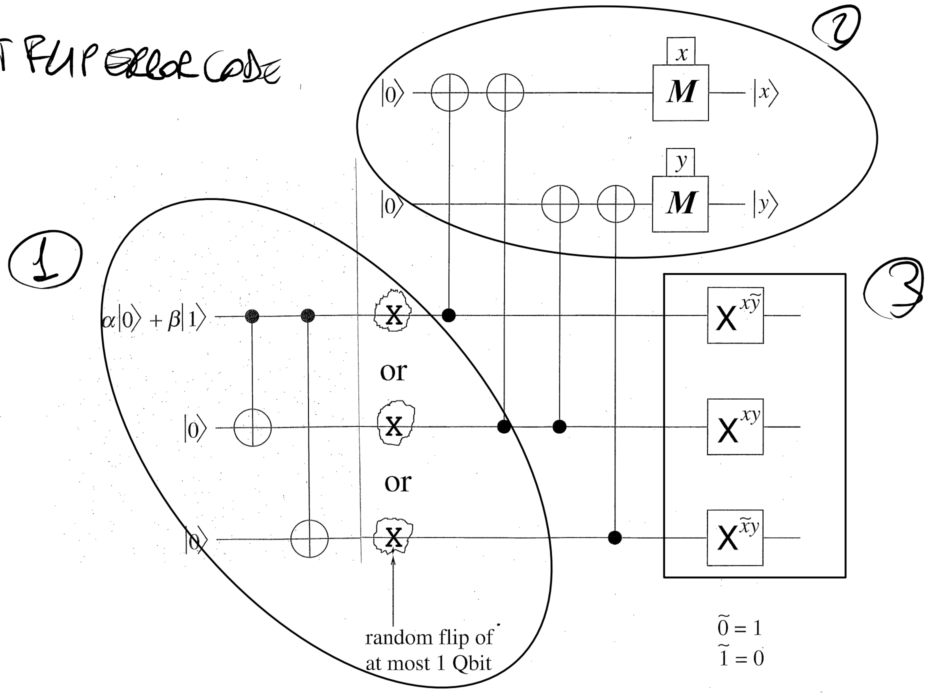


- In order to rotate we can take advantage of the uncomputation circuit, putting an oracle in it associated to the state associated with the rotation
- for U_w we already have $1_w(x)$
- for U_E we can use



Ex. 6 — What quantum system is the figure below representing? Can you describe its operation? Note: the square with an "M" denotes a measurement.

BIT FLIP ~~ERROR~~ CODE



① • Here we end up in the following scheme

$(1-P)^3(\alpha|000\rangle+\beta|111\rangle) + P^3(\alpha|001\rangle+\beta|110\rangle + \alpha|010\rangle+\beta|101\rangle + \alpha|100\rangle+\beta|110\rangle + \dots)$ } CASES IN WHICH AT MOST ONE BIT FLIPS CAN BE CORRECTED

② • We can take advan the following:

	Z_1Z_2	Z_2Z_3
$\alpha\|000\rangle+b\|111\rangle$	+1	+1
$\alpha\|001\rangle+b\|110\rangle$	+1	-1
$\alpha\|010\rangle+b\|101\rangle$	-1	-1
$\alpha\|100\rangle+b\|011\rangle$	-1	+1

measuring the 2 qubit we can apply the right X gate to flip the right qubit

③ • Flip the corrupted qubit.

⇒ unfortunately this code is able to correct only 1 bit flipping errors and not even the phase flipping (but we can reduce the phase flipping problem to the bit flip one)

↳ $\alpha|0\rangle+\beta|1\rangle \to \alpha|0\rangle-\beta|1\rangle$
becomes
$\alpha|+++\rangle+\beta|---\rangle$

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Ex. 7 — Explain why decoherence plays a critical role in quantum technologies. You can base your discussion on the case of a qubit $|0\rangle + \exp^{i\theta} |1\rangle$, with $|0\rangle$ and $|1\rangle$ eigenvector of σ_z and on the probability of obtaining a given measurement result when measuring σ_x.

- Decoherence is the process of the spontaneous change of a qubit state $|q\rangle$ to $|q'\rangle$ with the passing of the time
- If a qubit can be in the state $|q\rangle$ w.p. p and in $|q'\rangle$ w.p. $(1-p)$ then their mixed state can be represented by mixing their density matrix obtaining

$$\rho = p|q\rangle\langle q| + (1-p)|q'\rangle\langle q'|$$

- p is the [illegible] $= \langle q|\rho|q\rangle$

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