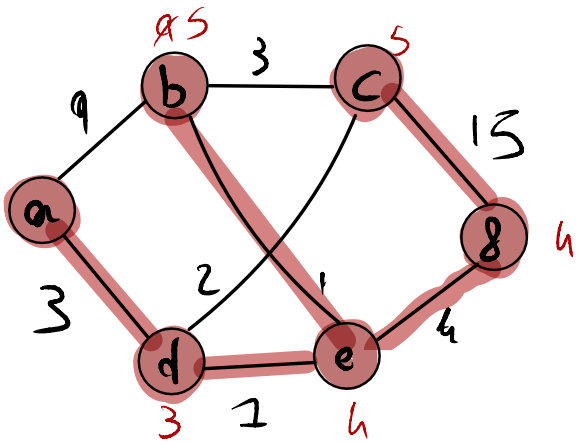
MIDTERM

Exercise 4 (Dijkstra. 4 points, 15 minutes) Consider the graph H depicted in Figure 1.

- Give the definition of a shortest-path tree rooted in a.
- Apply the Dijkstra algorithm on H to compute a shortest-path tree rooted in a and the distance between any vertex and the vertex a.

  You must explain the execution of the algorithm (you may write the table as seen during the lecture). In particular, indicate the order in which vertices are considered during the execution of the algorithm.
- Give the obtained shortest-path tree rooted in a.

- A shortest-path Tree T rooted in a is a Tree s.t. the path in T from a to a generic node i is the shortest $\forall i \in V$



CHOSEN NODE \ NODES	a	b	c	d	e	f
INIT	0	∞	∞	∞	∞	∞
a	0	9	∞	3	∞	∞
d	0	9	5	3	4	∞
e	0	5	5	3	4	4
f	0	5	5	3	4	4
b	0	5	5	3	4	4
c	0	5	5	3	4	4

DIJKSTRA

INPUT: $G=(V,E)$, $r \in V$

OUTPUT: $d(v)=d(r,v)\ \forall v \in V$

INIT: $d(r)=0$, $d(v)=\infty\ \forall v \in V \setminus \{r\}$,
S set of seen node, $T=(V_T=\{r\}, E_T=\{\emptyset\})$
$Parent(v)=\emptyset\ \forall v \in V$, $w_{ij}\ \forall (i,j) \in E$

ALGO: WHILE $(|S| \neq |V|)$
- $v = v \in V \setminus S \mid d(v) = \min_{u \in V \setminus S} d(u)$
- $V_T = V_T \cup \{v\}$
- $E_T = E_T \cup (v, Parent(v))$
- $S = S \cup \{v\}$
- FOR $(u \in N(v) \setminus S)$
  - IF $(d(v,u) < d(u))$
    - $d(u) = d(v) + w_{vu}$
    - $Parent(u) = v$

SHORTEST PATH TREE