University of Oulu
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School on Graph and Optimization
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Exercises : Modeling as Linear Programs

Exercise 1 [Maximum clique (NP-complete)] A clique of a graph G = (V, E) is a subset C of V, such that every two nodes in V are joined by an edge of E. The maximum clique problem consists of finding the largest cardinality of a clique.

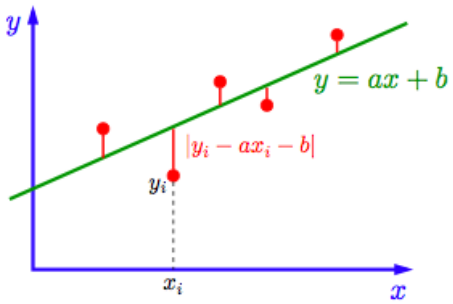
Express the maximum clique problem as an integer linear program.
Hint : think of a constraint for nodes not linked by an edge.

Exercise 2 [French newspaper enigma] What is the maximum size of a set of integers between 1 and 100 such that for any pair (a,b), the difference a-b is not a square ?

1. Model this problem as a graph problem.
2. Write a linear program to solve it.

Exercise 3 [Modelling] A university class has to go from Marseille to Paris using buses. There are some strong inimities inside the group and two people that dislike each other cannot share the same bus. What is the minimum number of buses needed to transport the whole group ? Write a LP that solve the problem. (We suppose that a bus does not have a limitation on the number of places.)

Exercise 4 [Chebyshev approximation]
Data : m measures of points $(x_i, y_i) \in \mathbb{R}^{n+1}$, $i = 1, ..., m$.
Objective : Determine a linear approximation $y = ax + b$ minimizing the largest error of approximation.



The decision variables of this problem are $a \in \mathbb{R}^n$ and $b \in R$. The problem may be formulated as :

$$\min z = \max_{i=1,...,m} \{|y_i - ax_i - b|\}.$$

It is not under the form of a linear program. Let us do some transformations.
Questions :

1. We call *Min-Max problem* the problem of minimizing the maximum of a set of numbers :

$$\min z = \max\{c_1 x, ..., c_k x\}.$$

How to write a Min-Max problem as a LP ?

2. Can we express the following constraints : $|x| \leq b$ or $|x| \geq b$ using a LP (that is without absolute values) ? If yes, how ?
3. Rewrite the problem of finding a linear approximation as an LP ?

Exercise 5 [Modelling as a linear programme]

You have 1000 euros to invest. At the beginning of each year, you can invest your available money in three different plans, Plan 1, Plan 2 and Plan 3 (available money means not blocked in a plan).

- Money put in Plan 1 is blocked for one year. After one year, you total interest rate is 5% (if you invested 100 euros, you will have 105 euros after one year).
- Money put in Plan 2 is blocked for two years. After two years, you total interest rate is 12% (if you invested 100 euros, you will have 112 euros after two years).
- Money put in Plan 3 is blocked for three years. After three years, you total interest rate is 19%.

Your planning horizon is 7 years. You want to maximize your available money at the end of the 7th year (again, available money means not blocked in a plan).

Write a linear programme solving the problem.

Hint : You may use the following variables. Let x_{it} be the amount of money that you invest in Plan i at the beginning of year t.