Università Bocconi

Microeconomics

Prof. Elisa Borghi &
Prof. Maristella Botticini

Lecture T4

Chapter 5: Choice

1. Constrained optimal choice (max *U* s.t. *BC*)
2. Interior and corner solutions
   A. Cobb-Douglas preferences (interior)
   B. Perfect complements preferences (interior)
   C. 1. Preferences for neutral goods. 2. Concave preferences (corner)
   D. Quasi-linear prefererences (interior and corner)
   E. Perfect substitutes preferences (interior and corner)
3. Deriving direct demand functions (Marshallian)
   A. Cobb-Douglas preferences
   B. Perfect complements preferences
   C. Perfect substitutes preferences
   D. Quasi-linear preferences
   E. Neutral goods
   F. Concave preferences

From Chapter 2 …
Rationality in economics

Behavioral postulate

- A decision maker always chooses his or her **most preferred** alternative from his or her set of **available** alternatives.

From Chapter 2 …
How does Jannik Sinner choose?

Economic theory tells us that individuals choose in a «rational way», that is, they maximize their well-being subject to scarcity and trade-offs. How?

1. They first identify the set of affordable alternatives (**budget set!**) among those available

2. Given their **preferences (!)**, they choose «the best» alternative among those affordable

Constrained optimal choice: definition

A bundle of commodities $x^* = (x^*_1 , x^*_2)$ is an optimal bundle if, given the individual's income and the prices of the commodities,

1) the individual can afford to purchase this bundle [«**constrained choice**»]
2) the individual will not change this bundle with any other bundles because this bundle maximizes his/her utility [«**optimal choice**»]

Constrained optimal choice: Interior and corner solutions

Definition: **interior solutions**

An optimal bundle x^* is an interior solution if the individual chooses both commodities

$$x_1^* > 0 \quad \text{and} \quad x_2^* > 0$$

Definition: **corner solutions**

An optimal bundle x^* is a corner solution if the individual chooses only one of the two goods

$$x_1^* > 0 \quad \text{and} \quad x_2^* = 0$$

or

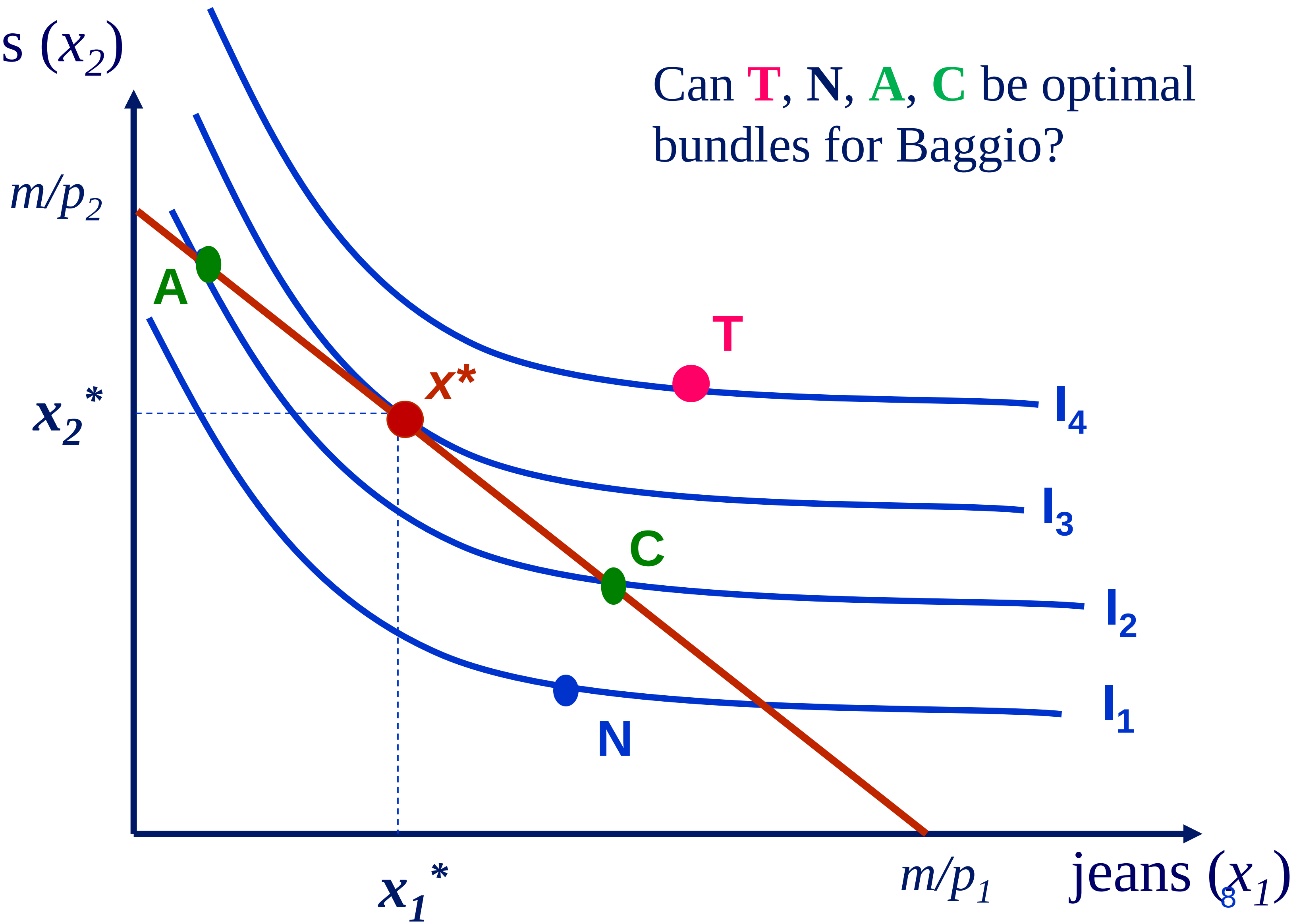
$$x_1^* = 0 \quad \text{and} \quad x_2^* > 0$$

… economic intuition

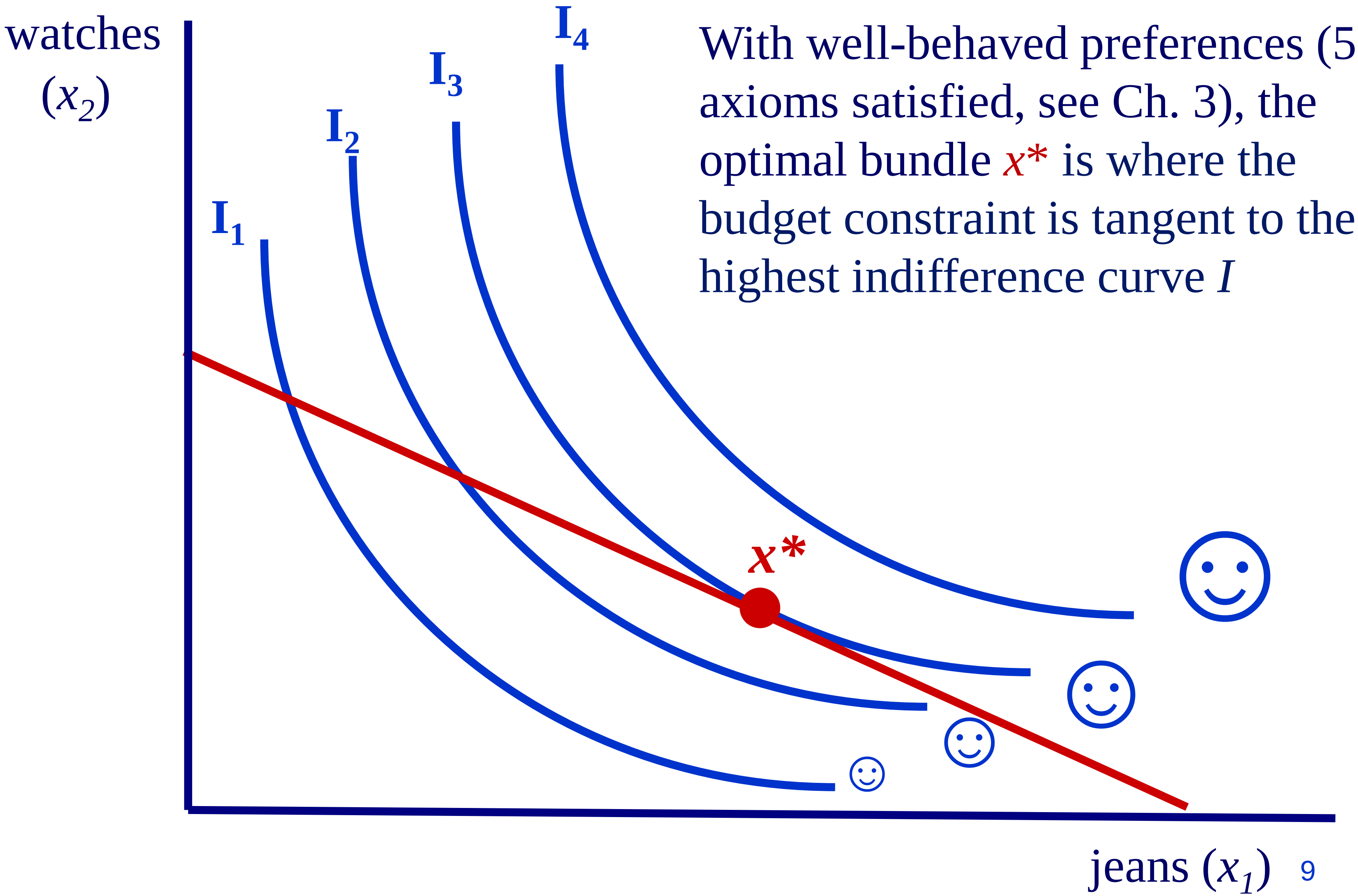
- Before stating the mathematical conditions to find a (constrained) optimal bundle, we look at some graphs that provide us with the intuition for what we will see later using calculus.

Well-behaved preferences: optimal bundle, I

Can T, N, A, C be optimal bundles for Baggio?



Well-behaved preferences: optimal bundle, II



With well-behaved preferences (5 axioms satisfied, see Ch. 3), the optimal bundle x^* is where the budget constraint is tangent to the highest indifference curve *I*

CASE A: Optimality conditions for well-behaved preferences with interior solutions

If preferences satisfy the 5 axioms (Ch. 3) and if the optimal bundle $x^* = (x_1^*, x_2^*)$ is an interior solution, then x^* satisfies these conditions:

Feasibility condition: x^* lies on the budget constraint

$$\boxed{p_1 x_1^* + p_2 x_2^* = m}$$

Optimality condition: at x^*, the IC and the budget constraint are tangent

$$\boxed{MRS(x^*) = \frac{p_1}{p_2}}$$

At the optimal bundle x^*

Budget constraint	**Indifference curves**
tells the individual how many units of commodity 2 **he/she must give up** in order to consume one more unit of commodity 1	tell the individual how many units of commodity 2 **he/she is willing to give up** to obtain one more unit of commodity 1 and be «equally happy»
$$\frac{p_1}{p_2}$$	$$MRS(x^*) \equiv \frac{MU_{x_1}}{MU_{x_2}}$$

At the optimal bundle “how much I MUST trade off” coincides with “how much I WANT to trade off”

Math + economic intuition

(che bello !!!)

- Let's write again the optimality condition that an interior bundle satisfies for well-behaved preferences:

$$MRS \equiv \frac{MU_{x_1}}{MU_{x_2}} = \frac{p_1}{p_2}$$

- Rearranging (trick & track):

$$\frac{MU_{x_1}}{p_1} = \frac{MU_{x_2}}{p_2}$$

we can see that, at the optimal bundle x^* the marginal utility per euro spent is equated across commodities.

CASE A: **example of well-behaved preferences ?**

Cobb-Douglas!!!

$$u(x_1, x_2) = x_1^a x_2^b \qquad \text{with } a, b > 0$$

- With Cobb-Douglas preferences, the optimal bundle is ALWAYS an **interior bundle**.
- With Cobb-Douglas preferences, the **tangency condition** for optimality ALWAYS hold.

- Remark: any monotonic transformation of the Cobb-Douglas above maintain these same properties for the optimal bundles.

CASE B: optimality conditions for interior solutions when the utility function is not differentiable

Feasibility condition: x^* lies on the budget constraint

$$p_1 x_1^* + p_2 x_2^* = m$$

Optimality condition: ?????????

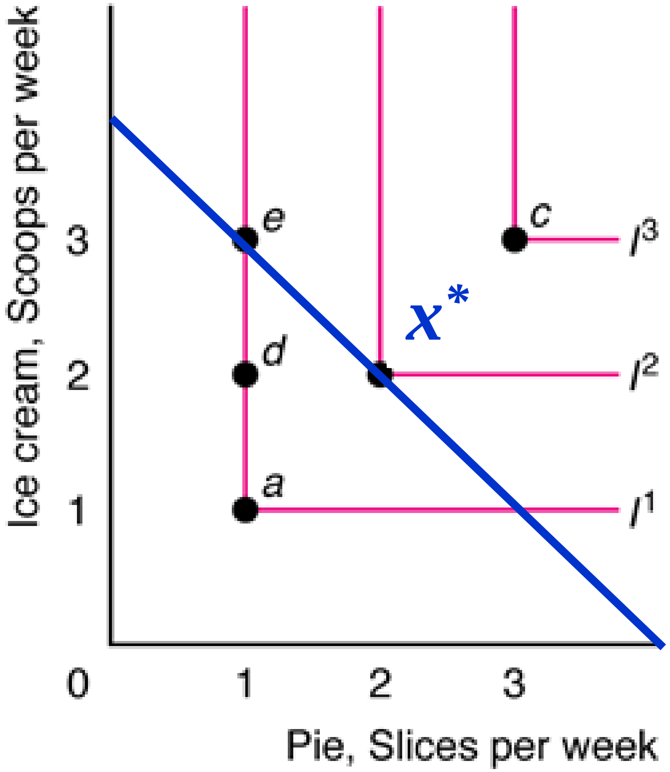
Example:

Perfect complements

$$u(x_1, x_2) = \min(x_1, x_2)$$

$$MRS(x^*) \neq \frac{p_1}{p_2}$$

because MRS is undefined at the kink !!!



CASE B: optimality conditions for interior solutions when the utility function is not differentiable

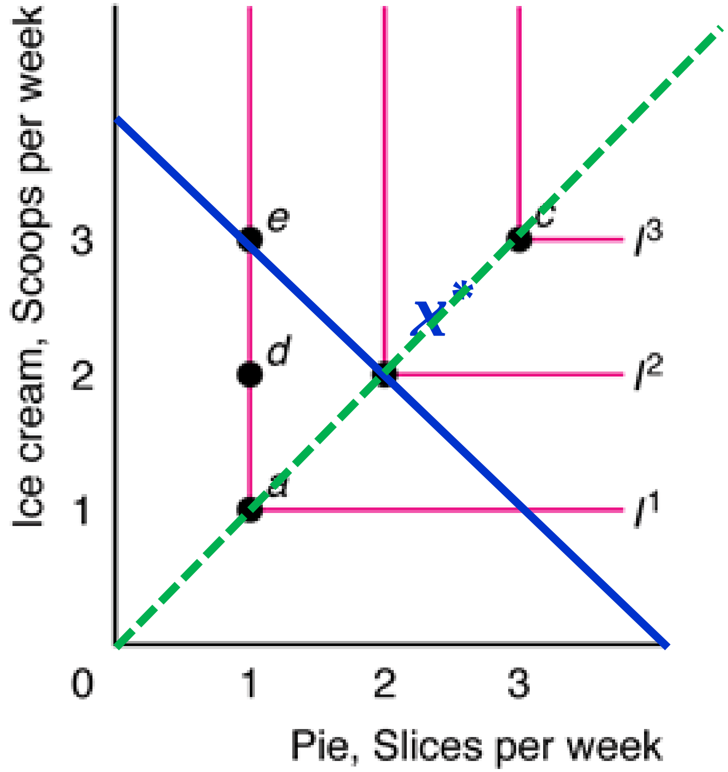
Feasibility condition: x^* lies on the budget constraint

$$p_1 x_1^* + p_2 x_2^* = m$$

Optimality condition:

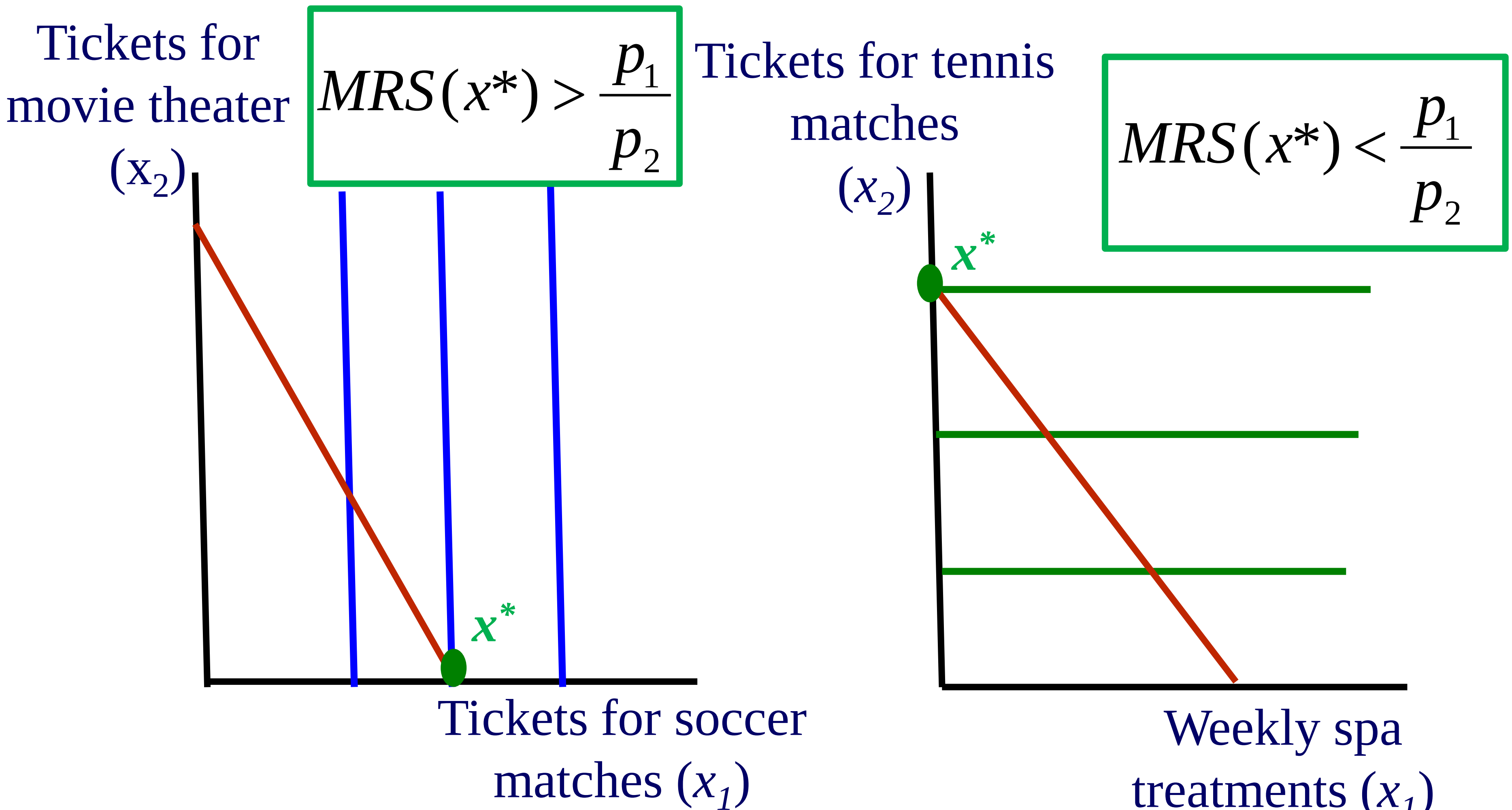
$$x_1^* = x_2^*$$

Perfect complements



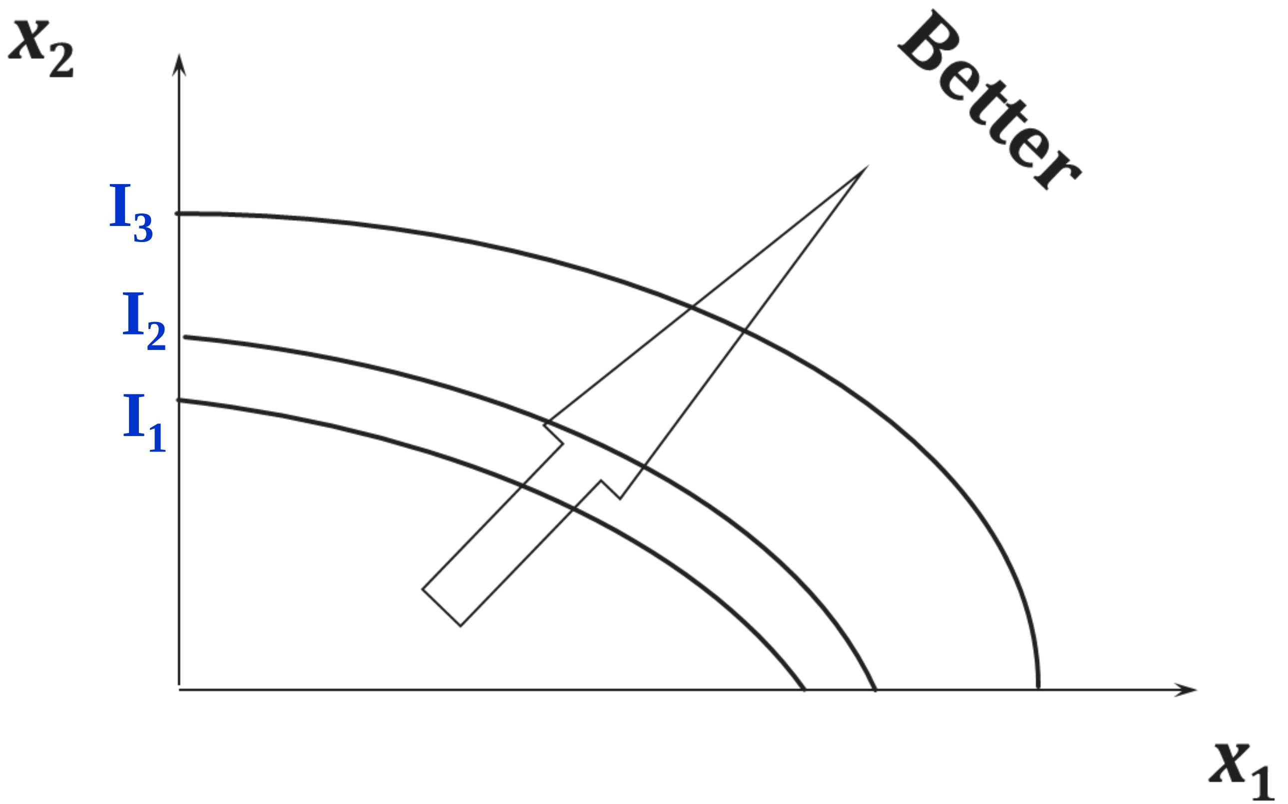
CASE C: corner solutions

(example I: neutral goods)



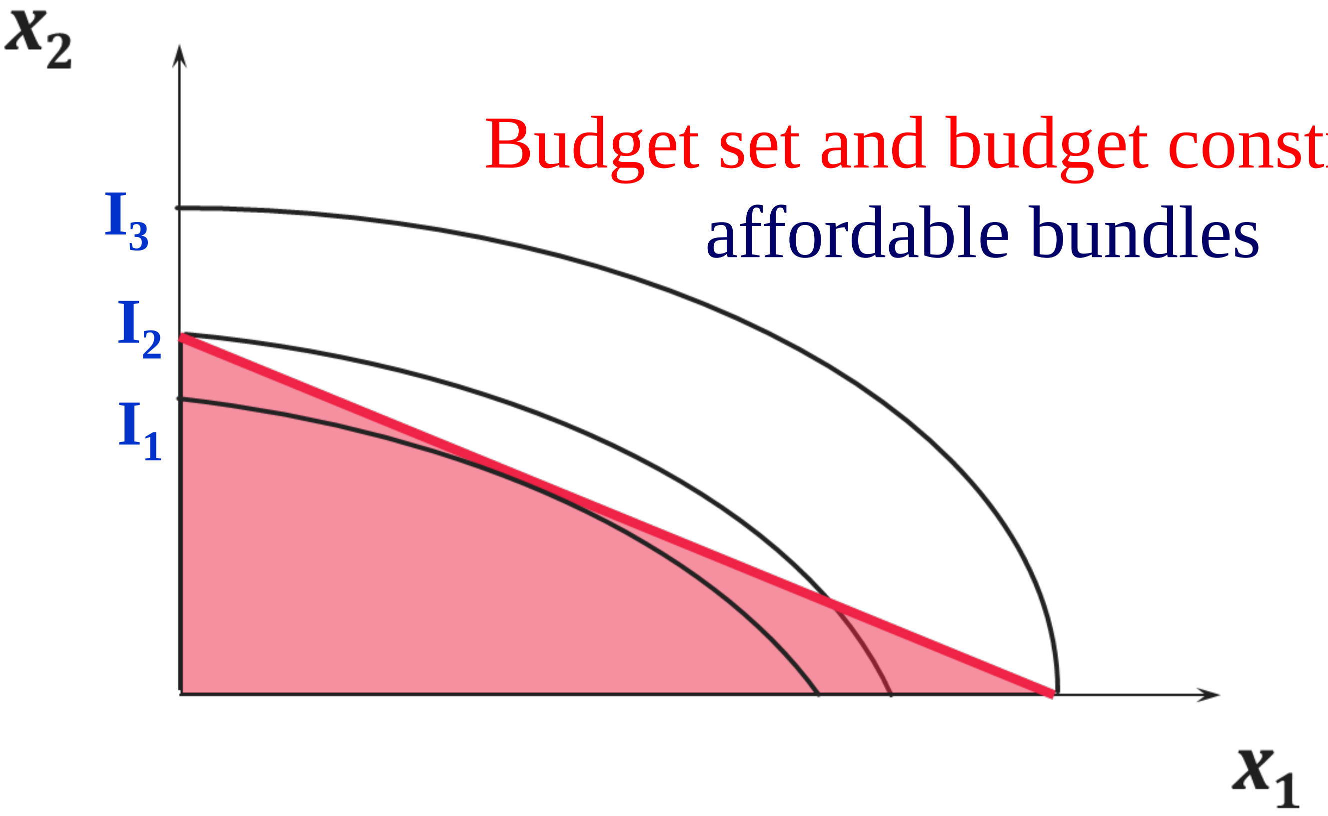
CASE C: corner solutions

(example II: concave preferences, «addiction»)



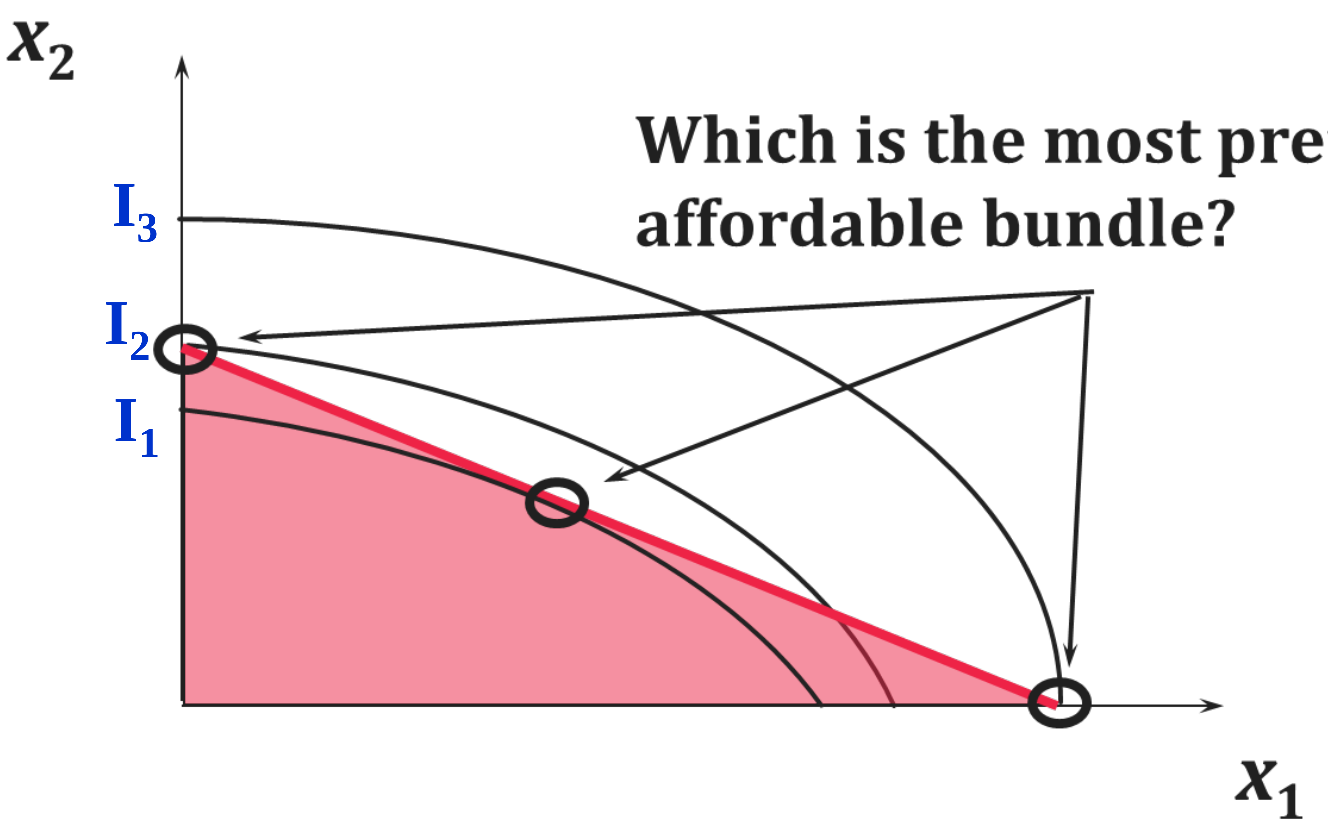
CASE C: corner solutions

(example II: concave preferences, «addiction»)



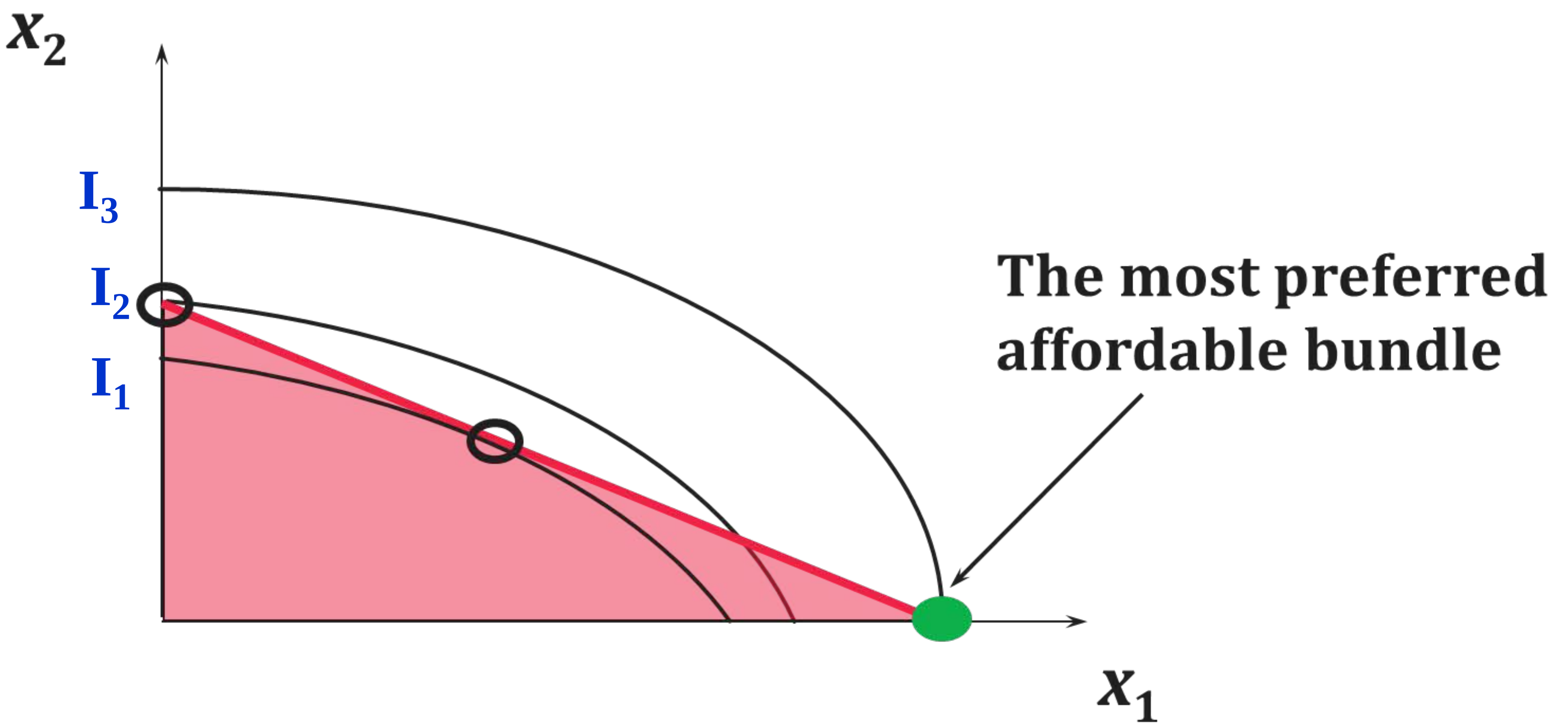
CASE C: corner solutions

(example II: concave preferences, «addiction»)



CASE C: corner solutions

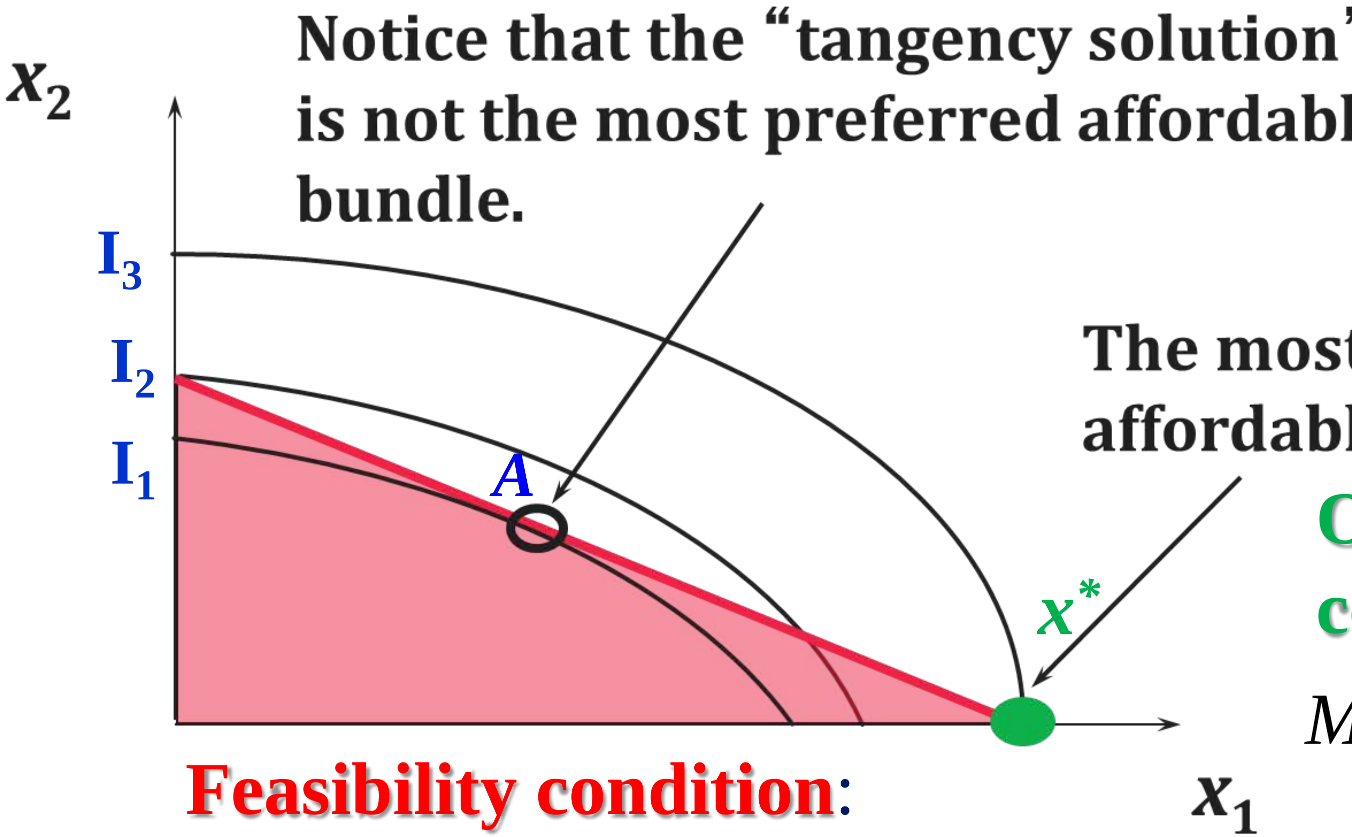
(example II: concave preferences, «addiction»)



CASE C: corner solutions

(example II: concave preferences, «addiction»)

$$MRS(A) = \frac{p_1}{p_2}$$



Optimality condition:

$$MRS(x^*) > \frac{p_1}{p_2}$$

Feasibility condition:

$$p_1 x_1^* + p_2 x_2^* = m$$

CASE D

Preferences with «mixed» solutions

(che bello !!!)

- Are there preferences whose optimal bundles can be both interior and corner solutions??? YES !!!

1. **Quasi-linear preferences** $u(x_1, x_2) = af(x_1) + bx_2$

   or

   $u(x_1, x_2) = ax_1 + bf(x_2)$

2. **Perfect substitutes preferences** $u(x_1, x_2) = ax_1 + bx_2$

CASE D1: quasi-linear preferences (i) tangency & interior solution

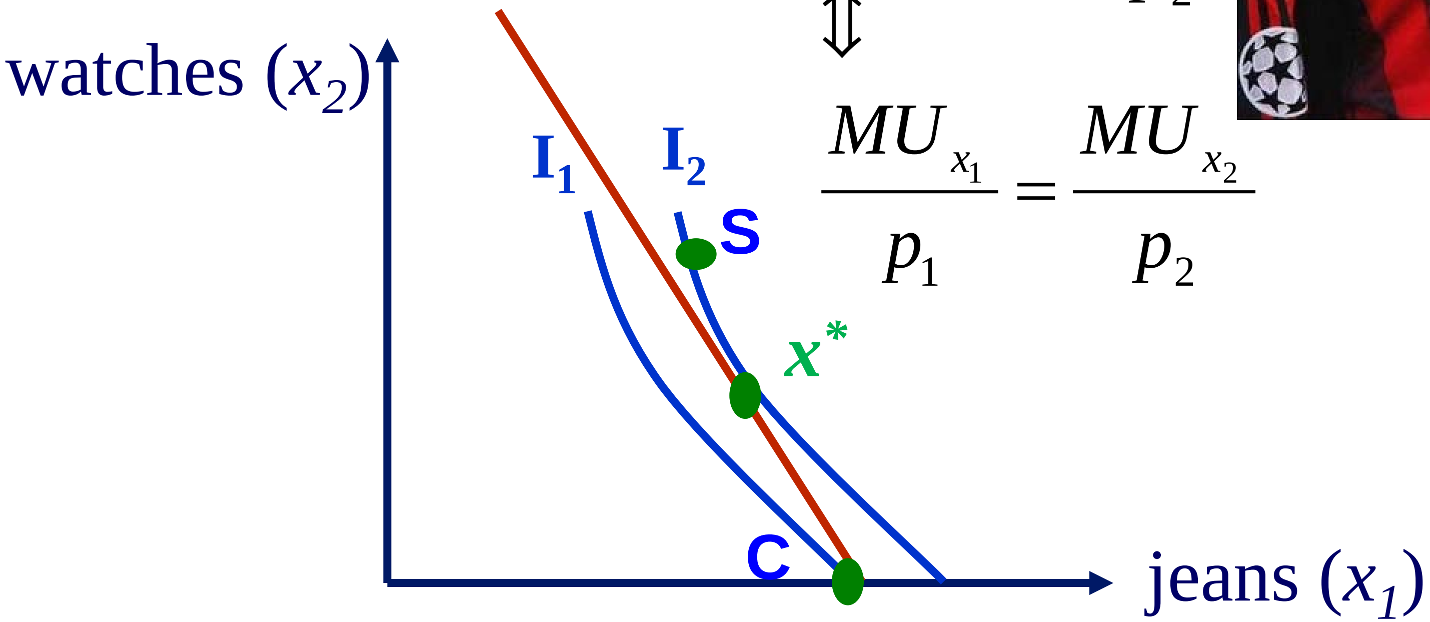
x^* is Pirlo's optimal bundle

Feasibility condition: $p_1 x_1^* + p_2 x_2^* = m$

Optimality condition: at x^*, $MRS(x^*) = \dfrac{p_1}{p_2}$

$$\Updownarrow$$

$$\frac{MU_{x_1}}{p_1} = \frac{MU_{x_2}}{p_2}$$



$x^*_1 > 0$ and $x^*_2 > 0$, Pirlo buys both goods: **interior solution**

CASE D1: quasi-linear preferences (ii) no tangency & corner solution

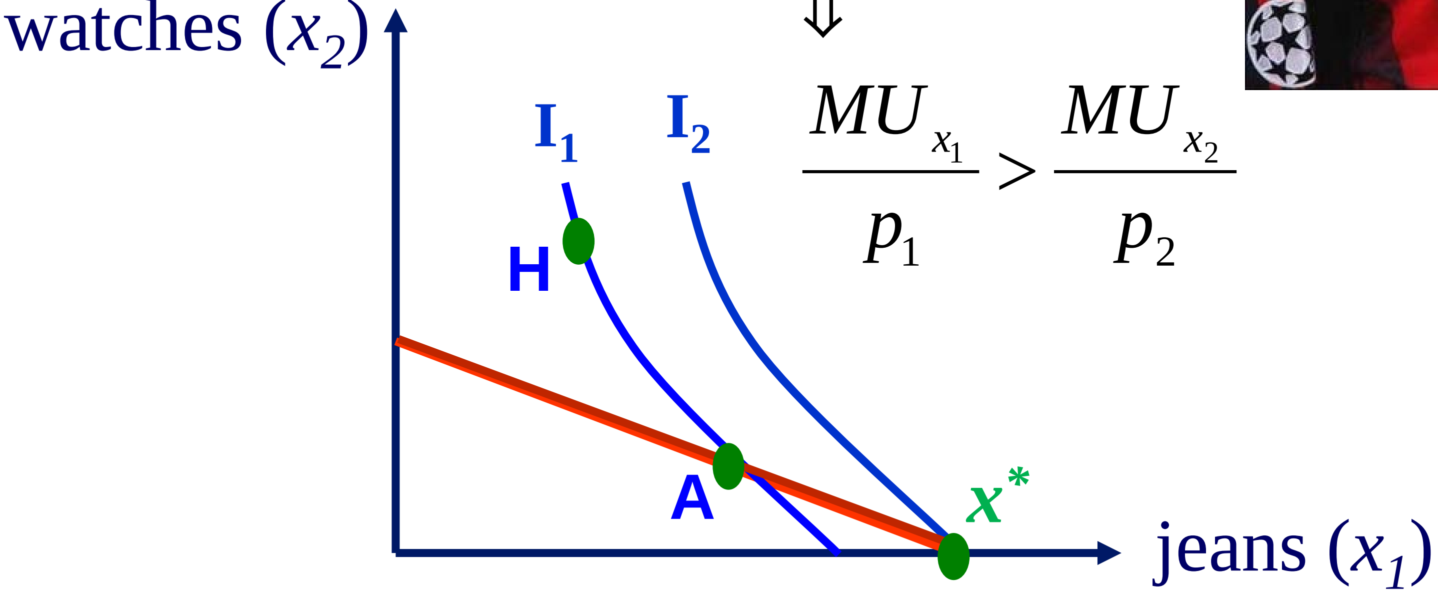
x^* is Pirlo's optimal bundle

Feasibility condition: $p_1 x_1^* + p_2 x_2^* = m$

Optimality condition: at x^*, $MRS(x^*) > \frac{p_1}{p_2}$

$$\Updownarrow$$

$$\frac{MU_{x_1}}{p_1} > \frac{MU_{x_2}}{p_2}$$



$x^*_1 > 0$ and $x^*_2 = 0$, Pirlo buys only jeans: **corner solution**

CASE D1: quasi-linear preferences (iii) tangency + corner solution

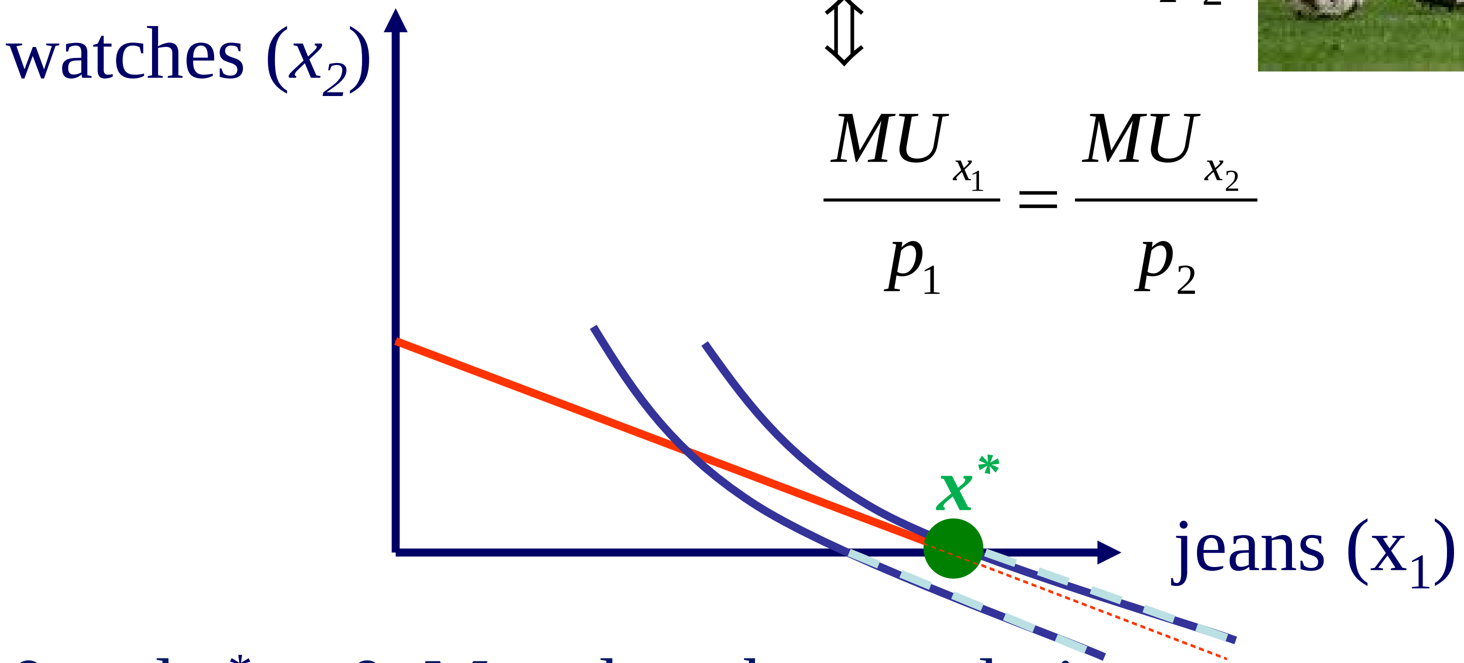
x^* is Maradona's optimal bundle

Feasibility condition: $p_1 x_1^* + p_2 x_2^* = m$

Optimality condition: at x^*, $MRS(x^*) = \frac{p_1}{p_2}$

$$\Updownarrow$$

$$\frac{MU_{x_1}}{p_1} = \frac{MU_{x_2}}{p_2}$$

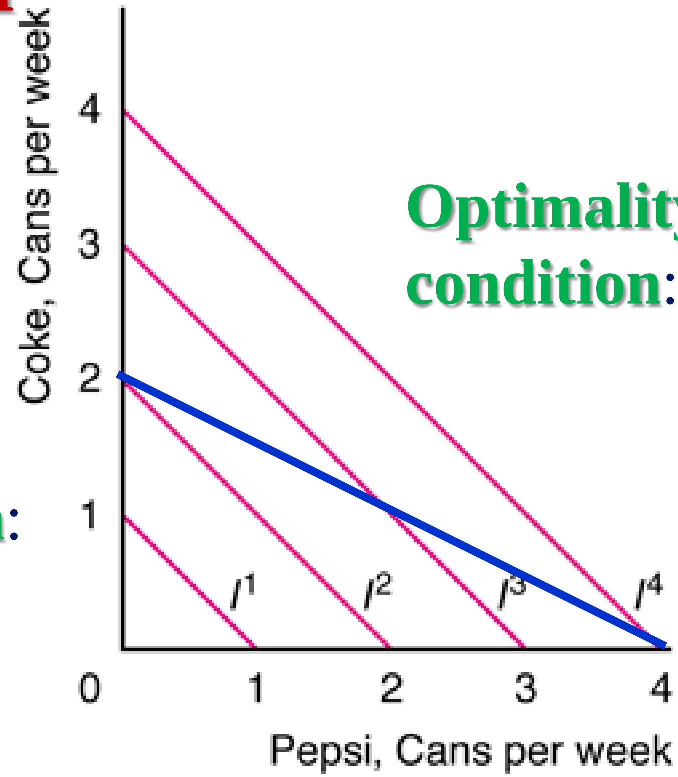


$x^*_1 > 0$ and $x^*_2 = 0$, Maradona buys only jeans: **corner solution**

CASE D2: perfect substitutes preferences

$x_1 = Pepsi$

$x_2 = Coca - Cola$



Feasibility condition:

$p_1 x_1^* + p_2 x_2^* = m$

Optimality condition:

Case i.

$$\boxed{MRS > \frac{p_1}{p_2}} \Leftrightarrow$$

$$\frac{MU_{x_1}}{MU_{x_2}} > \frac{p_1}{p_2} \Leftrightarrow$$

$$\frac{MU_{x_1}}{p_1} > \frac{MU_{x_2}}{p_2}$$

If $MRS > \frac{p_1}{p_2}$, the optimal bundle is : $\boxed{x_1^* = \frac{m}{p_1} \quad x_2^* = 0}$

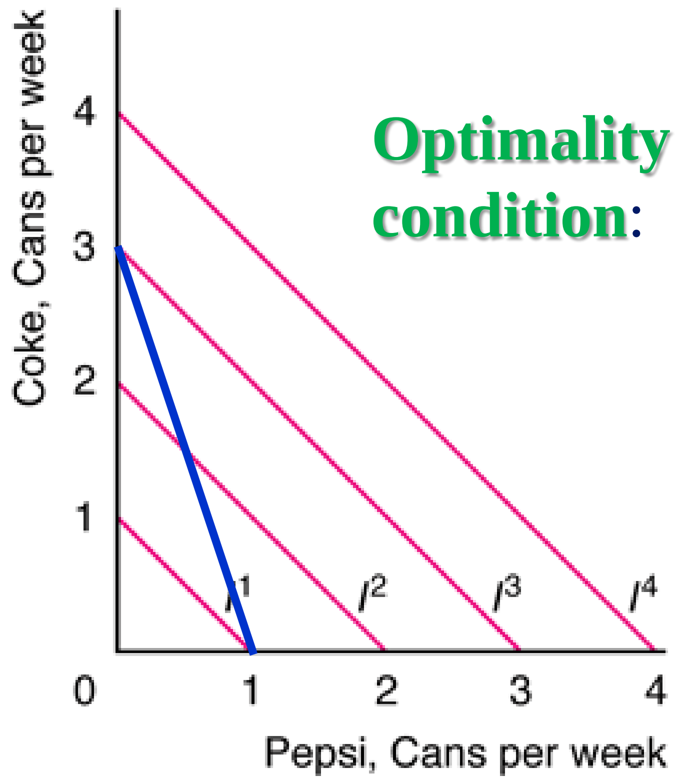
If Pepsi is relatively cheaper than Coca-Cola,

the individual consumes only Pepsi $\rightarrow$ **corner solution**

CASE D2: perfect substitutes preferences

$x_1 = Pepsi$

$x_2 = Coca - Cola$



Optimality condition:

Case ii.

$$\boxed{MRS < \frac{p_1}{p_2}} \Leftrightarrow$$

$$\frac{MU_{x_1}}{MU_{x_2}} < \frac{p_1}{p_2} \Leftrightarrow$$

$$\frac{MU_{x_1}}{p_1} < \frac{MU_{x_2}}{p_2}$$

Feasibility condition:

$$p_1 x_1^* + p_2 x_2^* = m$$

If $MRS < \frac{p_1}{p_2}$, the optimal bundle is : $\boxed{x_1^* = 0 \quad x_2^* = \frac{m}{p_2}}$

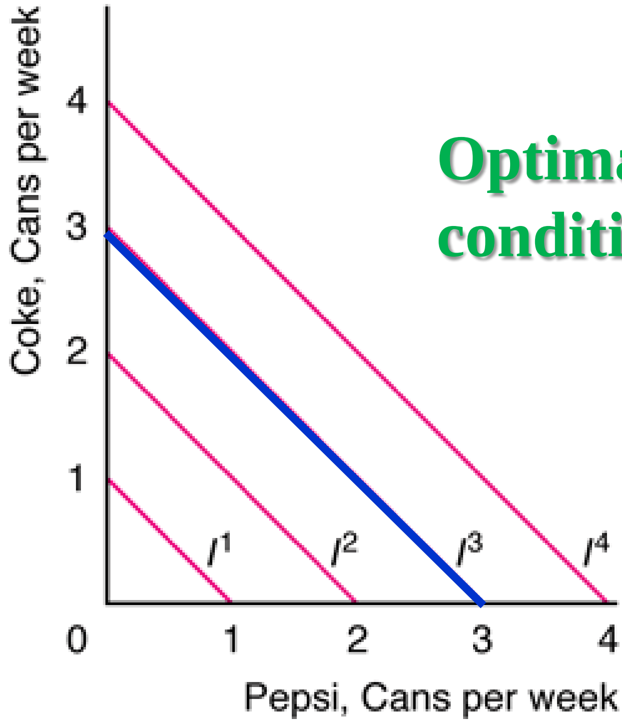
If Pepsi is relatively more expensive than Coca - Cola,

the individual consumes only Coca - Cola $\rightarrow$ **corner solution**

CASE D2: perfect substitutes preferences

$x_1 = Pepsi$

$x_2 = Coca - Cola$



Case iii.

Optimality condition:

$$\boxed{MRS = \frac{p_1}{p_2}} \Leftrightarrow$$

$$\frac{MU_{x_1}}{MU_{x_2}} = \frac{p_1}{p_2} \Leftrightarrow$$

$$\frac{MU_{x_1}}{p_1} = \frac{MU_{x_2}}{p_2}$$

Feasibility condition:

$$p_1 x_1^* + p_2 x_2^* = m$$

If $MRS = \frac{p_1}{p_2}$, optimal bundle: $\boxed{\text{any bundle } x^* \text{ s.t. } p_1 x_1^* + p_2 x_2^* = m}$

If $MRS = \frac{p_1}{p_2}$, the individual is indifferent between consuming only Pepsi or only Coca-Cola or any combination $\rightarrow$ **corner or interior solution**

Interior and corner solutions

- Before we introduce the demand function, let’s write an «executive summary» of what we have learned so far (so we do not get lost…).

If x^* is an interior solution

these two conditions must hold

Feasibility condition	Optimality condition
$p_1 x_1^* + p_2 x_2^* = m$	$MRS(x^*) \boxed{=} \frac{p_1}{p_2}$ (tangency condition) Exception: kinked preferences (e.g., perfect complements)

Preferences with ALWAYS interior solutions

Feasibility condition	Optimality condition
$p_1 x_1^* + p_2 x_2^* = m$	**With** tangency condition $MRS(x^*) = \frac{p_1}{p_2}$ • **Cobb-Douglas** **Without** tangency condition • **Perfect complements**

If x^* is a corner solution

these two conditions <u>must</u> hold

Feasibility condition

$$p_1 x_1^* + p_2 x_2^* = m$$

Optimality condition

$$MRS(x^*) \boxed{\geq} \frac{p_1}{p_2} \Leftrightarrow$$

$$\boxed{x_1^* > 0 \text{ and } x_2^* = 0}$$

or

$$MRS(x^*) \boxed{\leq} \frac{p_1}{p_2} \Leftrightarrow$$

$$\boxed{x_1^* = 0 \text{ and } x_2^* > 0}$$

Preferences with ALWAYS corner solutions

Feasibility condition	Optimality condition
$p_1 x_1^* + p_2 x_2^* = m$	• Neutral goods • Concave preferences

Preferences with both interior and corner solutions

Feasibility condition	Optimality condition
$p_1 x_1^* + p_2 x_2^* = m$	• Quasi-linear • Perfect substitutes

Demand Curve: definition

- Demand functions express the quantities demanded in terms of the prices of both commodities and income:

$$x_1(p_1, p_2, m)$$

$$x_2(p_1, p_2, m)$$

- That is, demand functions tell us the quantities demanded by the individual for any prices and incomes **such that these quantities maximize his/her utility subject to the budget constraint**.

Deriving demand functions: solving the constrained optimization problem

- An individual's constrained optimization problem (= maximizing utility subject to the budget constraint) can be stated mathematically as follows:

$$\max_{x_1, x_2} U(x_1, x_2)$$

$$s.t.\ p_1 x_1 + p_2 x_2 = m$$

- Solution of this optimization problem delivers utility-maximizing values of x_1 and x_2 as functions of prices, p_1 and p_2, and income, m:

$$x_1(p_1, p_2, m)$$

$$x_2(p_1, p_2, m)$$

How to solve the optimization problem

- **For preferences that satisfy the 5 axioms**, the set of optimal bundles x_1, x_2 is found by solving a system of two equations corresponding to:

(1) the **optimality condition** (= tangency between IC and BC)

$$MRS = \frac{p_1}{p_2} \quad \text{which can be written as} \quad \frac{MU_{x_1}}{MU_{x_2}} = \frac{p_1}{p_2}$$

(2) the **feasibility condition** (the bundles chosen must satisfy the BC)

$$p_1 x_1 + p_2 x_2 = m$$

where we take prices and income as given and **we solve for the two unknown variables, x_1, x_2**. Once we solve this system, we find the quantities demanded as functions of prices and income.

CASE A: deriving demand functions for **Cobb-Douglas preferences** (example I)

- Cobb-Douglas utility function

$$U(x_1, x_2) = x_1^{\frac{1}{2}} x_2^{\frac{1}{2}}$$

- Budget constraint:

$$p_1 x_1 + p_2 x_2 = m$$

- The demand functions that result from this constrained optimization problem are:

$$x_1(p_1, p_2, m) = \frac{1}{2}\frac{m}{p_1} \qquad x_2(p_1, p_2, m) = \frac{1}{2}\frac{m}{p_2}$$

REMARK

With Cobb-Douglas utility functions, the quantity demanded of each good is a function of only the good’s own-price and income.

OK! Now let’s derive these functions solving the optimization problem.

$$\max_{x_1, x_2} x_1^{\frac{1}{2}} x_2^{\frac{1}{2}}$$

$$\text{s.t.} \quad p_1 x_1 + p_2 x_2 = m$$

$$\text{Let's calculate} \quad MU_{x_1} = \frac{1}{2} x_1^{-\frac{1}{2}} x_2^{\frac{1}{2}}$$

$$\text{Let's calculate} \quad MU_{x_2} = \frac{1}{2} x_1^{\frac{1}{2}} x_2^{-\frac{1}{2}}$$

Taking the ratio of the two marginal utilities

$$\text{we get the } MRS = \frac{MU_{x_1}}{MU_{x_2}} = \frac{\frac{1}{2} x_1^{-\frac{1}{2}} x_2^{\frac{1}{2}}}{\frac{1}{2} x_1^{\frac{1}{2}} x_2^{-\frac{1}{2}}} = \frac{x_2}{x_1}$$

(1) optimality condition $MRS = \dfrac{x_2}{x_1} = \dfrac{p_1}{p_2}$

(2) feasibility condition $p_1 x_1 + p_2 x_2 = m$

$$\begin{cases} \dfrac{x_2}{x_1} = \dfrac{p_1}{p_2} \\ p_1 x_1 + p_2 x_2 = m \end{cases}$$

Taking p_1, p_2 and m as given, we solve for x_1 and x_2 as functions of prices and income. With some easy algebraic steps...

The demand function for x_1 is

$$x_1(p_1, p_2, m) = \frac{1}{2}\frac{m}{p_1}$$

The demand function for x_2 is

$$x_2(p_1, p_2, m) = \frac{1}{2}\frac{m}{p_2}$$

… How about these other Cobb – Douglas preferences? (example II)

- Cobb-Douglas utility function:

$$U(x_1, x_2) = x_1\, x_2$$

- Budget constraint:

$$p_1 x_1 + p_2 x_2 = m$$

- The demand functions that result from this constrained optimization problem are …. ???

$$\max_{x_1, x_2} x_1 x_2$$

$$\text{s.t.} \quad p_1 x_1 + p_2 x_2 = m$$

Let's calculate $MU_{x_1} = x_2$

Let's calculate $MU_{x_2} = x_1$

Taking the ratio of the two marginal utilities

we get the $MRS = \dfrac{MU_{x_1}}{MU_{x_2}} = \dfrac{x_2}{x_1}$

(1) optimality condition $MRS = \frac{x_2}{x_1} = \frac{p_1}{p_2}$

(2) feasibility condition $p_1 x_1 + p_2 x_2 = m$

$$\begin{cases} \frac{x_2}{x_1} = \frac{p_1}{p_2} \\ p_1 x_1 + p_2 x_2 = m \end{cases}$$

Taking p_1, p_2 and m as given, we solve for x_1 and x_2 as functions of prices and income. With some easy algebraic steps...

The demand function for x_1 is

$$x_1(p_1, p_2, m) = \frac{1}{2}\frac{m}{p_1}$$

The demand function for x_2 is

$$x_2(p_1, p_2, m) = \frac{1}{2}\frac{m}{p_2}$$

Some remarks….

… How about these other Cobb – Douglas preferences? (example III)

- Cobb-Douglas utility function:

$$U(x_1, x_2) = \ln x_1 + \ln x_2$$

- Budget constraint:

$$p_1 x_1 + p_2 x_2 = m$$

- The demand functions that result from this constrained optimization problem are …. ???

$$\max_{x_1, x_2} \ln x_1 + \ln x_2$$

$$\text{s.t.} \quad p_1 x_1 + p_2 x_2 = m$$

Let's calculate $MU_{x_1} = \dfrac{1}{x_1}$

Let's calculate $MU_{x_2} = \dfrac{1}{x_2}$

Taking the ratio of the two marginal utilities

we get the $MRS = \dfrac{MU_{x_1}}{MU_{x_2}} = \dfrac{x_2}{x_1}$

(1) optimality condition $MRS = \dfrac{x_2}{x_1} = \dfrac{p_1}{p_2}$

(2) feasibility condition $p_1 x_1 + p_2 x_2 = m$

$$\begin{cases} \dfrac{x_2}{x_1} = \dfrac{p_1}{p_2} \\ p_1 x_1 + p_2 x_2 = m \end{cases}$$

Taking p_1, p_2 and m as given, we solve for x_1 and x_2 as functions of prices and income. With some easy algebraic steps...

The demand function for x_1 is

$$x_1(p_1, p_2, m) = \frac{1}{2}\frac{m}{p_1}$$

The demand function for x_2 is

$$x_2(p_1, p_2, m) = \frac{1}{2}\frac{m}{p_2}$$

Food for thought (important!)

- Look at cases I, II, and III in the previous slides. We get the same demand functions although the utility functions we started from were different.
- Did we make any mistakes, maybe? NOOOO!!!
- The three utility functions

$$U(x_1, x_2) = x_1^{\frac{1}{2}} x_2^{\frac{1}{2}} \qquad U(x_1, x_2) = x_1 x_2$$

$$U(x_1, x_2) = \ln x_1 + \ln x_2$$

- are a monotonic transformation one of the other, so they represent the same identical preferences, and hence they deliver the same demand functions.
- This means that you do **<u>not</u>** need to redo the calculations if they tell you that three different individuals have these utility functions. Once you derive the demand function for one of them, the other two will have the same demand functions. GREAT!!! YEAH!!!!

… How about these other Cobb – Douglas preferences? (example IV)

- Cobb-Douglas utility function:

$$U(x_1, x_2) = x_1^{\frac{1}{3}} x_2^{\frac{2}{3}}$$

- Budget constraint:

$$p_1 x_1 + p_2 x_2 = m$$

- The demand functions that result from this constrained optimization problem are …. ???

$$\max_{x_1, x_2} x_1^{\frac{1}{3}} x_2^{\frac{2}{3}}$$

$$\text{s.t.} \quad p_1 x_1 + p_2 x_2 = m$$

$$\text{Let's calculate} \quad MU_{x_1} = \frac{1}{3} x_1^{-\frac{2}{3}} x_2^{\frac{2}{3}}$$

$$\text{Let's calculate} \quad MU_{x_2} = \frac{2}{3} x_1^{\frac{1}{3}} x_2^{-\frac{1}{3}}$$

Taking the ratio of the two marginal utilities

$$\text{we get the } MRS = \frac{MU_{x_1}}{MU_{x_2}} = \frac{\frac{1}{3} x_1^{-\frac{2}{3}} x_2^{\frac{1}{3}}}{\frac{2}{3} x_1^{\frac{1}{3}} x_2^{-\frac{1}{3}}} = \frac{1}{2} \frac{x_2}{x_1}$$

(1) optimality condition $MRS = \frac{1}{2}\frac{x_2}{x_1} = \frac{p_1}{p_2}$

(2) feasibility condition $p_1x_1 + p_2x_2 = m$

$$\begin{cases} \frac{1}{2}\frac{x_2}{x_1} = \frac{p_1}{p_2} \\ p_1x_2 + p_2x_2 = m \end{cases}$$

Taking p_1, p_2 and m as given, we solve for x_1 and x_2 as functions of prices and income. With some easy algebraic steps...

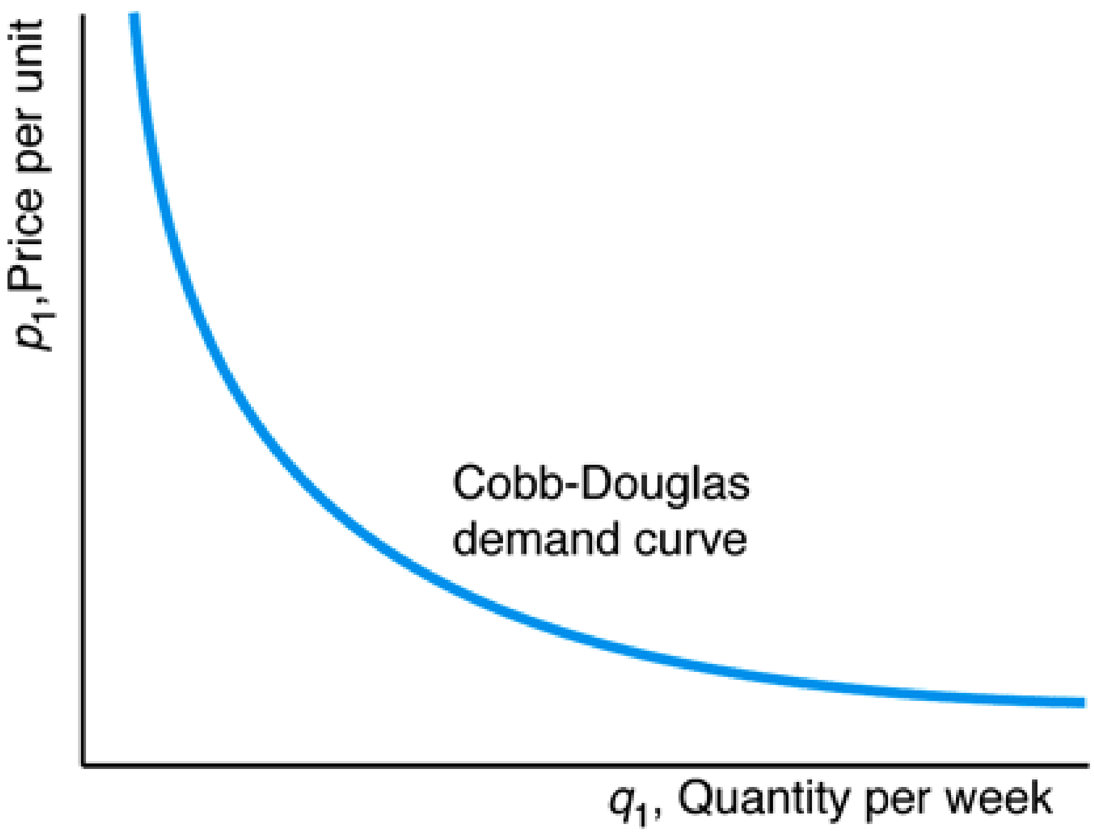
The demand function for x_1 is

$$x_1(p_1, p_2, m) = \frac{1}{3}\frac{m}{p_1}$$

The demand function for x_2 is

$$x_2(p_1, p_2, m) = \frac{2}{3}\frac{m}{p_2}$$

Plotting the demand curve for Cobb-Douglas utility functions



CASE B: deriving demand functions for perfect complements

- Utility function for perfect complements:

$$U(x_1, x_2) = \min(x_1, x_2)$$

- Budget constraint:

$$p_1 x_1 + p_2 x_2 = m$$

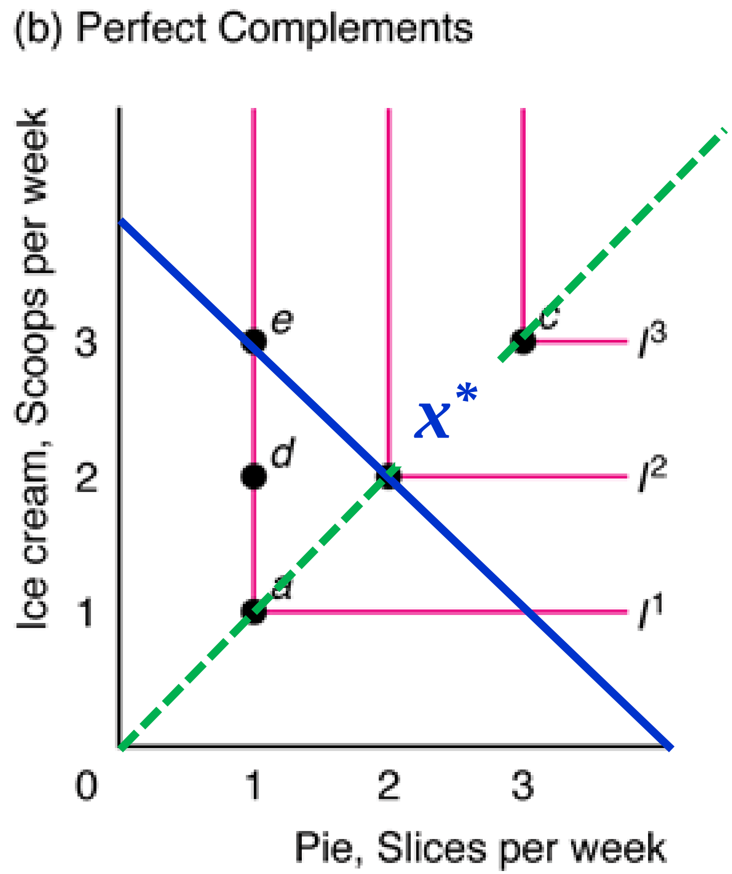
- The demand functions that result from this constrained optimization problem are …. ???

- REMARK: for the case in which the ratio between the two goods is not 1 to 1, please watch Prof. Borghi's video in which she explains how to solve the utility maximization problem when the ratio is not 1 to 1.

$$\max_{x_1, x_2} \min x_1 x_2$$

$$\text{s.t.} \quad p_1 x_1 + p_2 x_2 = m$$

Let's draw the indifference curves and the budget constraint.



(1) optimality condition $x_1 = x_2$

because with perfect complements we want to consume them in the same proportion

(2) feasibility condition $p_1 x_1 + p_2 x_2 = m$

$$\begin{cases} x_1 = x_2 \\ p_1 x_2 + p_2 x_2 = m \end{cases}$$

Taking p_1, p_2 and m as given, we solve for x_1 and x_2 as functions of prices and income. With some easy algebraic steps...

The demand function for x_1 is

$$x_1(p_1, p_2, m) = \frac{m}{p_1 + p_2}$$

The demand function for x_2 is

$$x_2(p_1, p_2, m) = \frac{m}{p_1 + p_2}$$

Some remarks….

Graphically … ?

CASE C: deriving demand functions for perfect substitutes

- Utility function for perfect substitutes:

$$U(x_1, x_2) = x_1 + x_2$$

- Budget constraint:

$$p_1 x_1 + p_2 x_2 = m$$

- The demand functions that result from this constrained optimization problem are …. ??? Let's draw three graphs.

- REMARK: for the case in which the ratio between the two goods is not 1 to 1, please watch Prof. Borghi's video in which she explains how to solve the utility maximization problem when the ratio is not 1 to 1.

$$\text{Let's calculate} \quad MU_{x_1} = 1$$

$$\text{Let's calculate} \quad MU_{x_2} = 1$$

Taking the ratio of the two marginal utilities

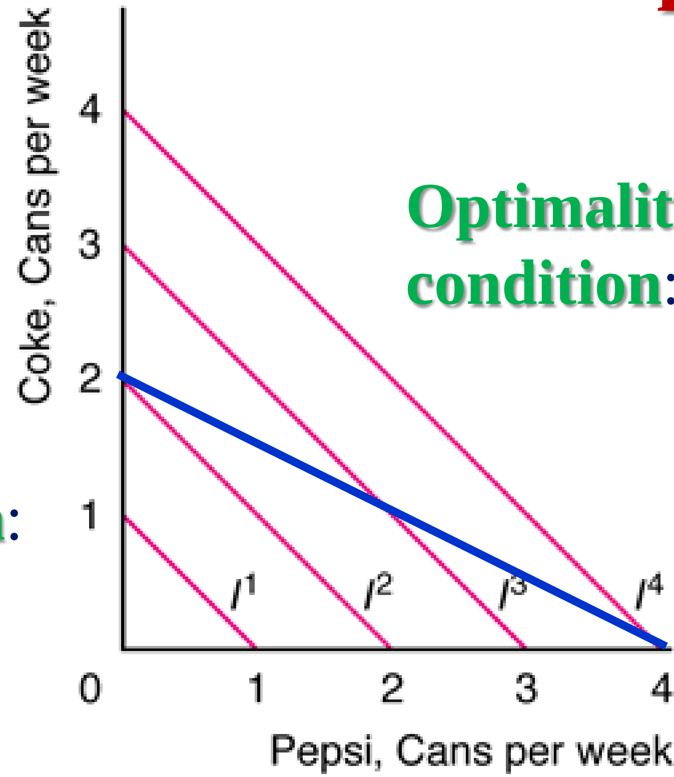
we get the $MRS = \dfrac{MU_{x_1}}{MU_{x_2}} = 1$

The individual is willing to exchange 1 Coca - Cola for 1 Pepsi because he / she considers them perfect substitutes in the ratio 1:1.

CASE C: perfect substitutes preferences

$x_1 = Pepsi$

$x_2 = Coca - Cola$



Feasibility condition:

$p_1 x_1^* + p_2 x_2^* = m$

Optimality condition:

Case i.

$$\boxed{MRS > \frac{p_1}{p_2}} \Leftrightarrow$$

$$\frac{MU_{x_1}}{MU_{x_2}} > \frac{p_1}{p_2} \Leftrightarrow$$

$$\frac{MU_{x_1}}{p_1} > \frac{MU_{x_2}}{p_2}$$

If $MRS > \frac{p_1}{p_2}$, the optimal bundle is : $\boxed{x_1^* = \frac{m}{p_1} \quad x_2^* = 0}$

If Pepsi is cheaper than Coca - Cola, the individual consumes only Pepsi $\rightarrow$ $\boxed{\textbf{corner solution}}$

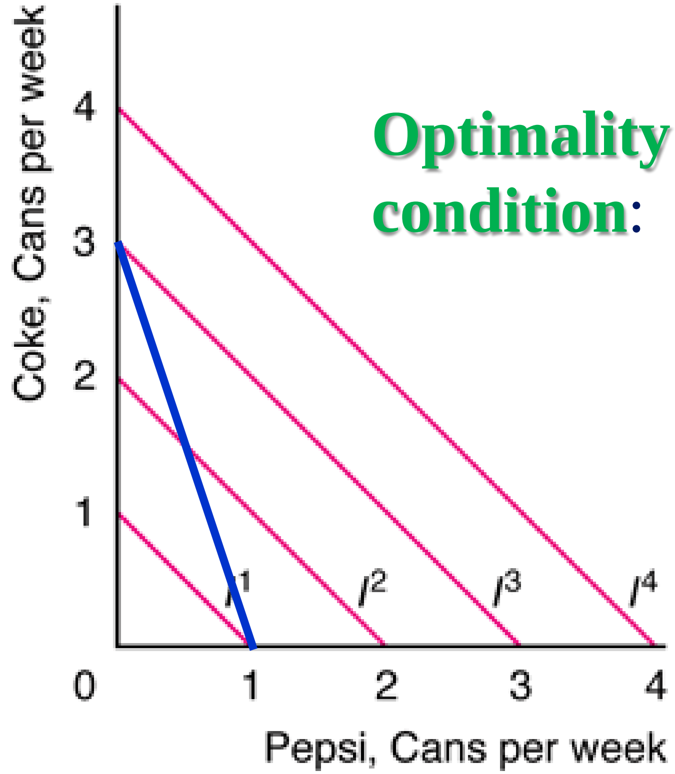
CASE C: perfect substitutes preferences

$x_1 = Pepsi$

$x_2 = Coca - Cola$

Feasibility condition:

$p_1 x_1^* + p_2 x_2^* = m$



Optimality condition:

Case ii.

$$\boxed{MRS < \frac{p_1}{p_2}} \Leftrightarrow$$

$$\frac{MU_{x_1}}{MU_{x_2}} < \frac{p_1}{p_2} \Leftrightarrow$$

$$\frac{MU_{x_1}}{p_1} < \frac{MU_{x_2}}{p_2}$$

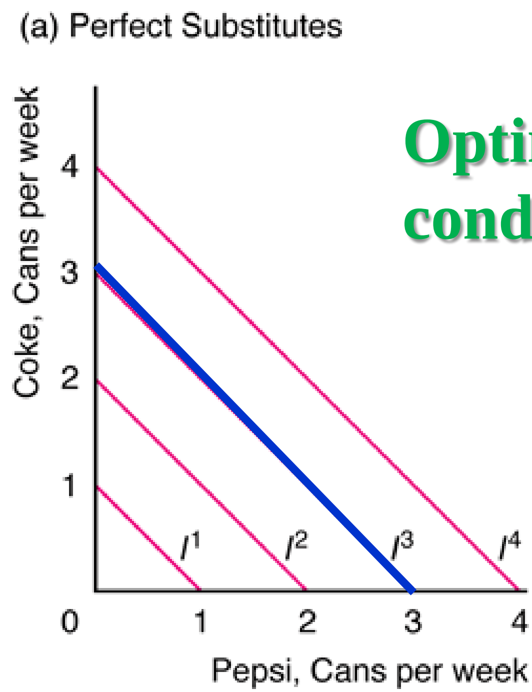
If $MRS < \dfrac{p_1}{p_2}$, the optimal bundle is : $\boxed{x_1^* = 0 \quad x_2^* = \dfrac{m}{p_2}}$

If Pepsi is more expensive than Coca - Cola, the individual consumes only Coca - Cola $\rightarrow$ **corner solution**

CASE C: perfect substitutes preferences

$x_1 = Pepsi$

$x_2 = Coca - Cola$



Feasibility condition:

$$p_1 x_1^* + p_2 x_2^* = m$$

Case iii.

Optimality condition:

$$\boxed{MRS = \frac{p_1}{p_2}} \Leftrightarrow$$

$$\frac{MU_{x_1}}{MU_{x_2}} = \frac{p_1}{p_2} \Leftrightarrow$$

$$\frac{MU_{x_1}}{p_1} = \frac{MU_{x_2}}{p_2}$$

If $MRS = \frac{p_1}{p_2}$, optimal bundle : $\boxed{\text{any bundle } x^* \text{ s.t. } p_1 x_1^* + p_2 x_2^* = m}$

If $MRS = \frac{p_1}{p_2}$, the individual is indifferent between consuming only Pepsi or only Coca-Cola or any combination $\rightarrow$ **corner or interior solution**

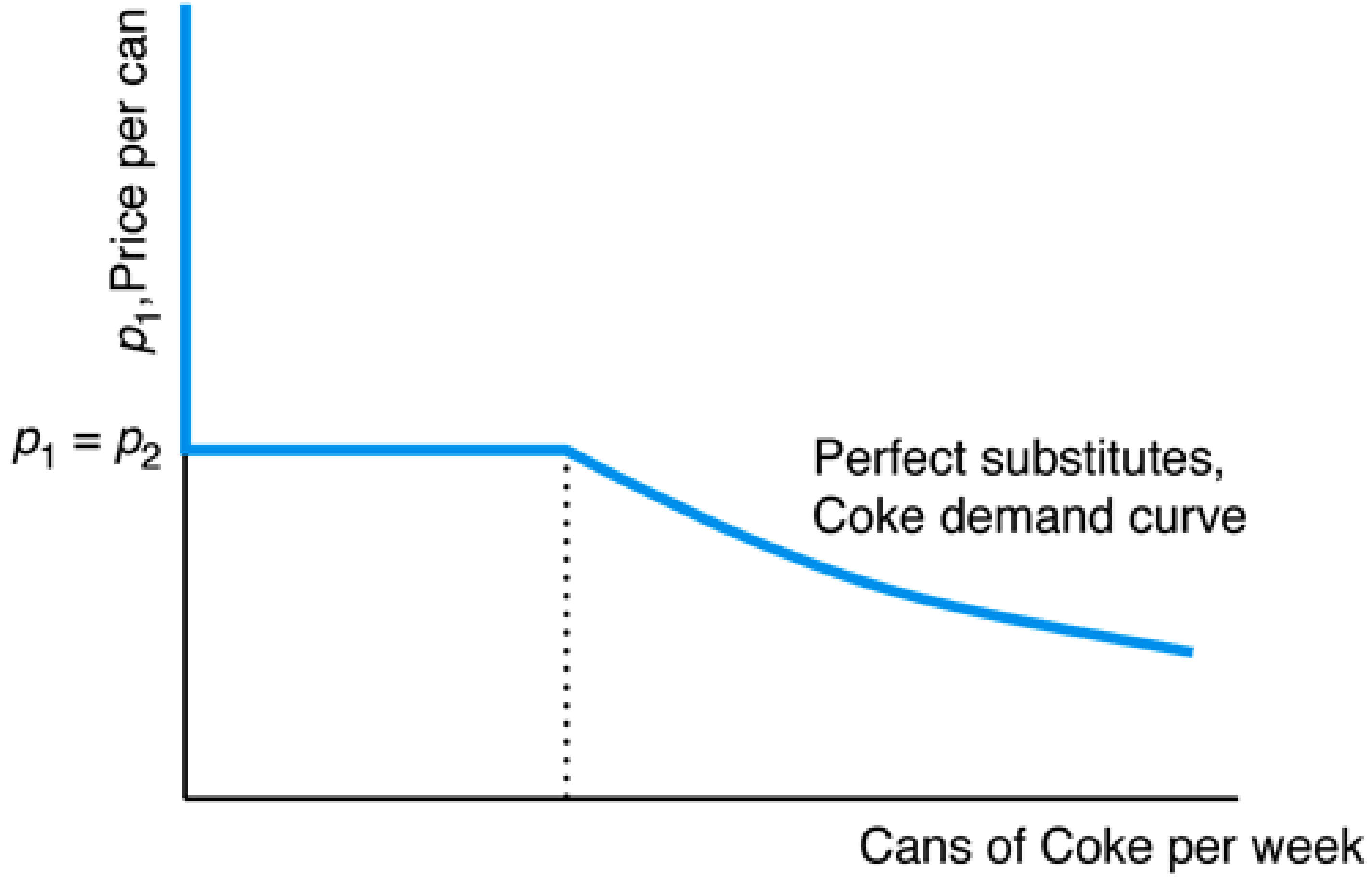
Therefore, the demand functions are as follows:

$$\text{If } MRS > \frac{p_1}{p_2} \quad \text{then } x_1 = \frac{m}{p_1} \quad x_2 = 0$$

$$\text{If } MRS < \frac{p_1}{p_2} \quad \text{then } x_1 = 0 \quad x_2 = \frac{m}{p_2}$$

$$\text{If } MRS = \frac{p_1}{p_2} \quad \text{then } p_1 x_1 + p_2 x_2 = m$$

Plotting the demand curve for perfect substitutes

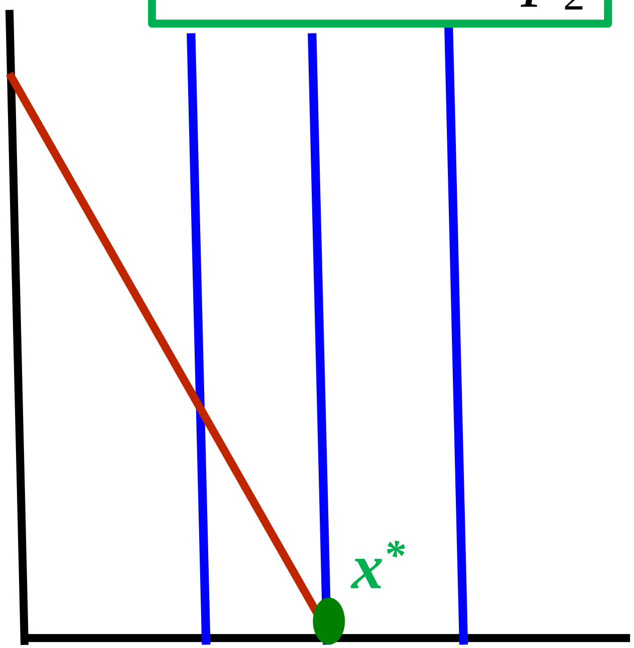


CASE D: deriving demand functions for quasi-linear preferences

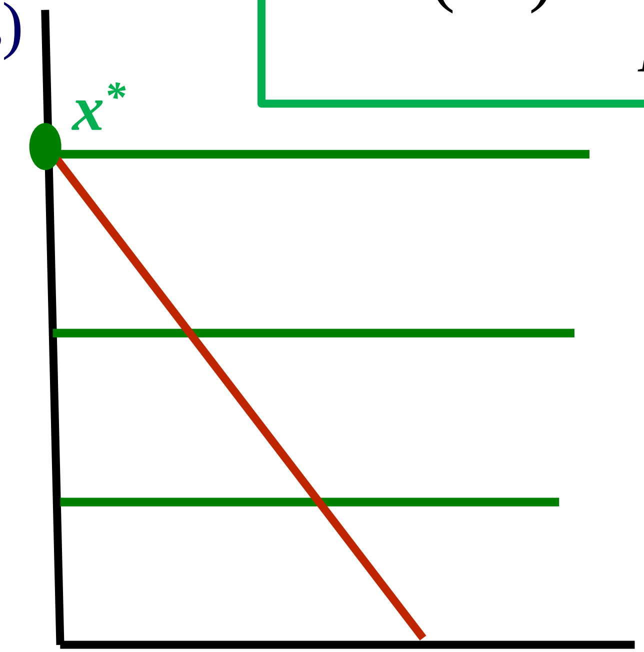
September 17 lecture, review session R1: Professor Borghi will solve one problem to illustrate in detail Case D

CASE E: deriving demand functions for neutral goods

$$MRS(x^*) > \frac{p_1}{p_2}$$



$$MRS(x^*) < \frac{p_1}{p_2}$$



Therefore, the demand functions are as follows:

Case i: vertical indifference curves

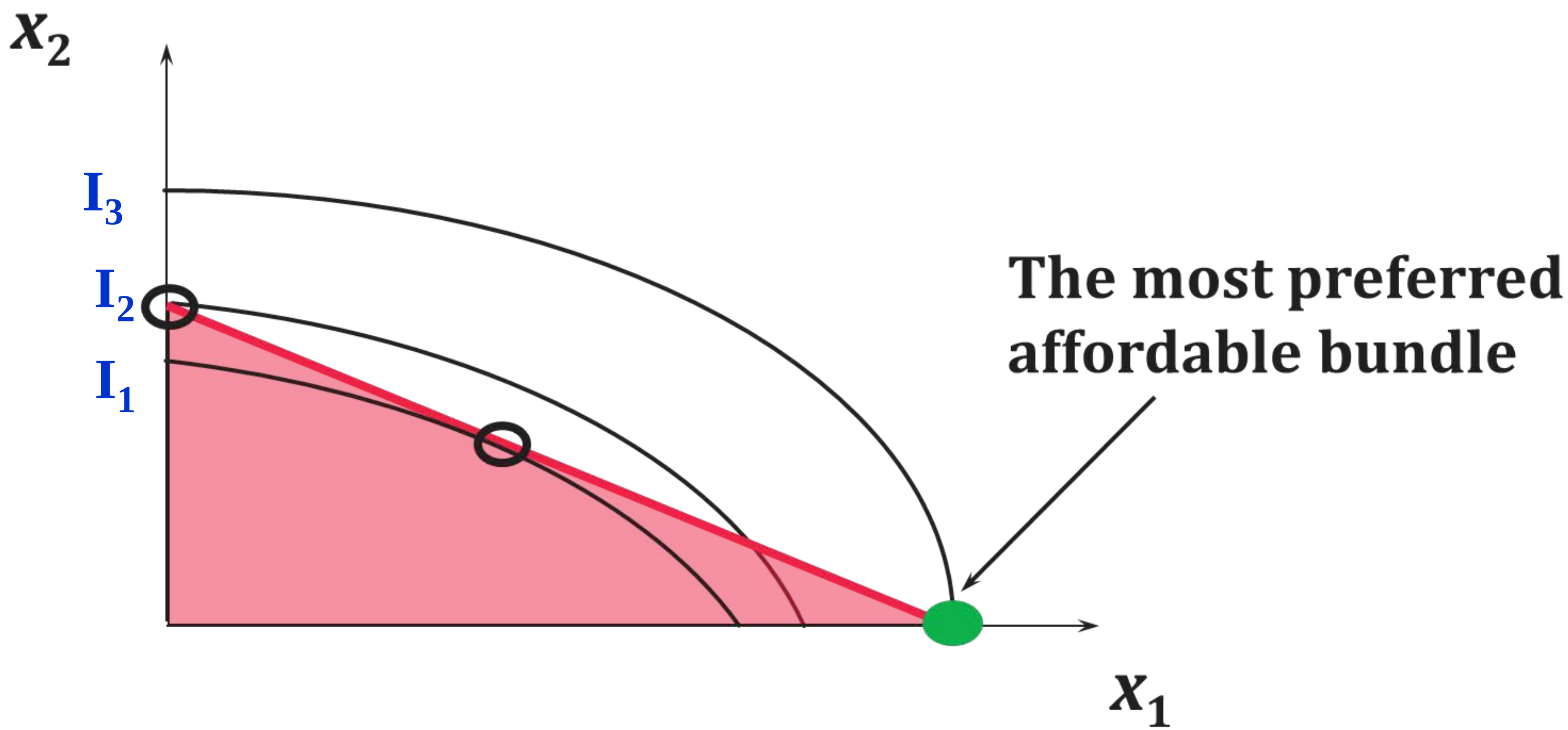
$$\text{If } MRS(x^*) = \infty \qquad \text{then } x_1 = \frac{m}{p_1} \qquad x_2 = 0$$

Case ii: horizontal indifference curves

$$\text{If } MRS(x^*) = 0 \qquad \text{then } x_1 = 0 \qquad x_2 = \frac{m}{p_2}$$

Graphically … ?

CASE F: deriving the demand functions for concave preferences (addiction»)



Therefore, the demand functions are as follows:

$$\text{If } MRS(x^*) > \frac{p_1}{p_2} \qquad \text{then } x_1 = \frac{m}{p_1} \qquad x_2 = 0$$

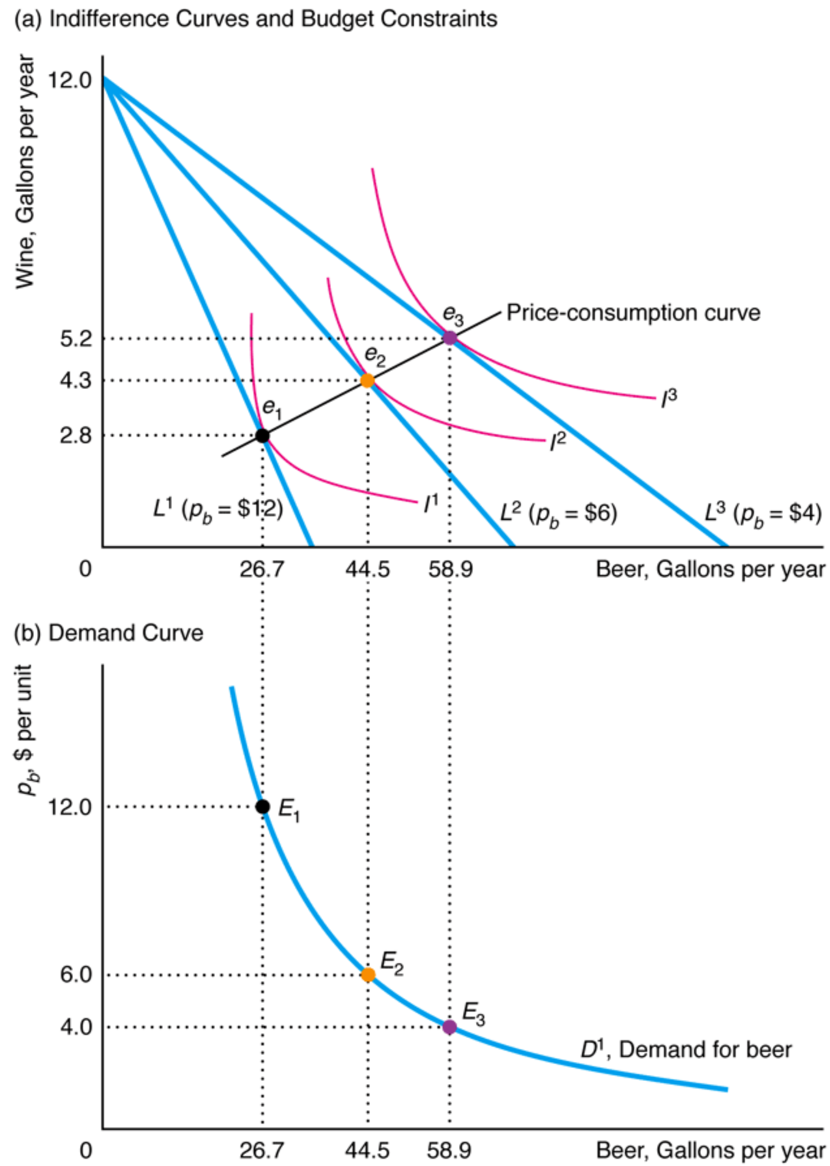
$$\text{If } MRS(x^*) < \frac{p_1}{p_2} \qquad \text{then } x_1 = 0 \qquad x_2 = \frac{m}{p_2}$$

Graphically … ?

Deriving demand curves graphically

- Allowing the price of the good on the x-axis to fall, the budget constraint rotates out and shows how the optimal quantity of the x-axis good purchased increases.

- This traces out points along the demand curve.



Chapter 5:
Take home message

- Which is the key concept we learned in this chapter?