

**Università Bocconi**

# **Microeconomics**

**Prof. Elisa Borghi &  
Prof. Maristella Botticini**

Lecture T4



# Chapter 6: Demand

---

## Road map

1. Comparative statics analysis
2. **When the own price changes**
  - i. price-offer curve
  - ii. (direct) demand function ( $x$  is function of  $p$ )
    - a) ordinary goods
    - b) Giffen goods
  - iii. inverse demand function ( $p$  is function of  $x$ )
3. **When the price of the other good changes**
  - i. (gross) complements
  - ii. (gross) substitutes
  - iii. unrelated goods
4. **When the income changes**
  - i. income-offer curve
  - ii. shifts of the demand function
  - iii. Engel curve

# 1. Properties of Demand Functions --- Comparative statics

---

- Comparative statics analysis of ordinary demand functions is the study of how ordinary demands

$$x_1^*(p_1, p_2, m)$$

$$x_2^*(p_1, p_2, m)$$

change as prices  $(p_1, p_2)$  and income  $(m)$  change.

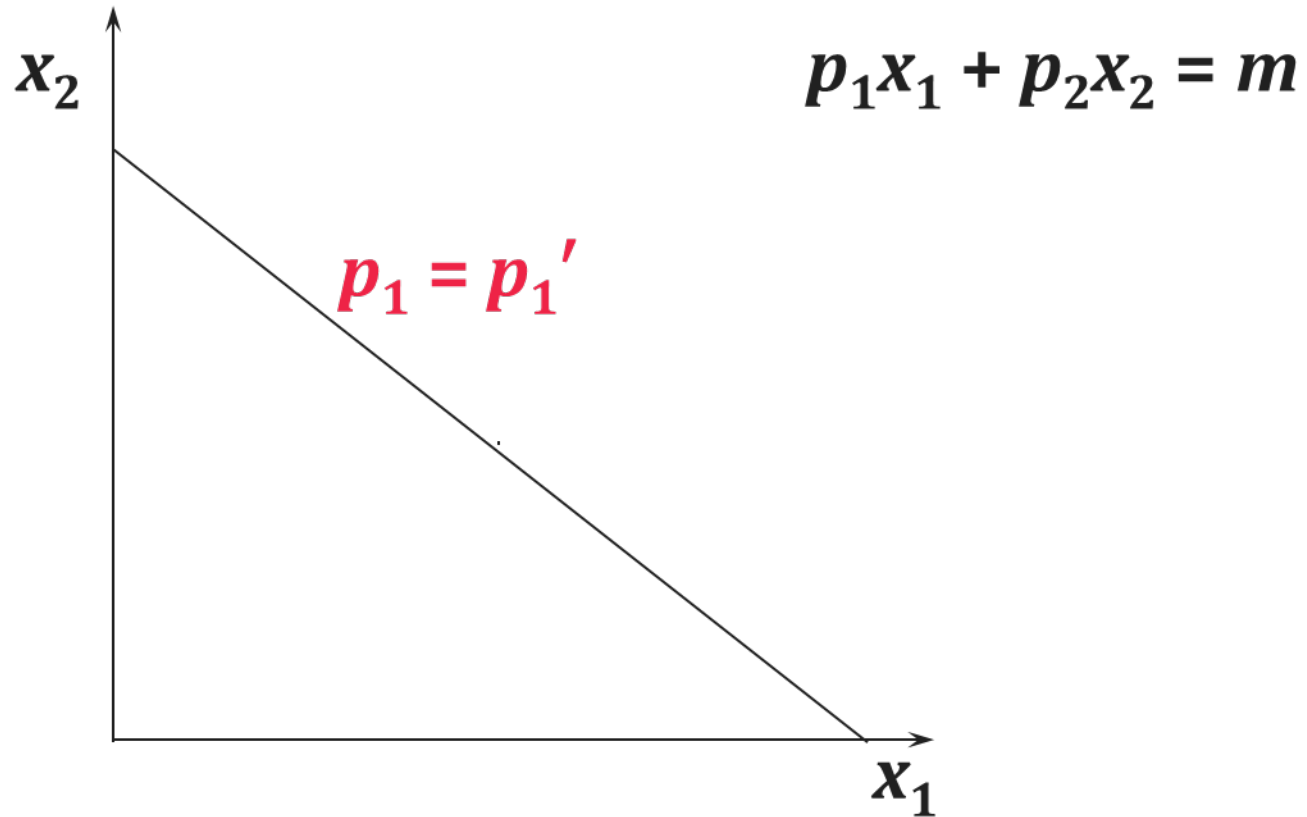
## 2. Own-Price Changes

---

- QUESTION: How does  $x_1^*(p_1, p_2, m)$  change **as  $p_1$  changes**, holding  $p_2$  and  $m$  constant?
- Suppose only  $p_1$  increases, from  $p_1'$  to  $p_1''$  and then to  $p_1'''$ .

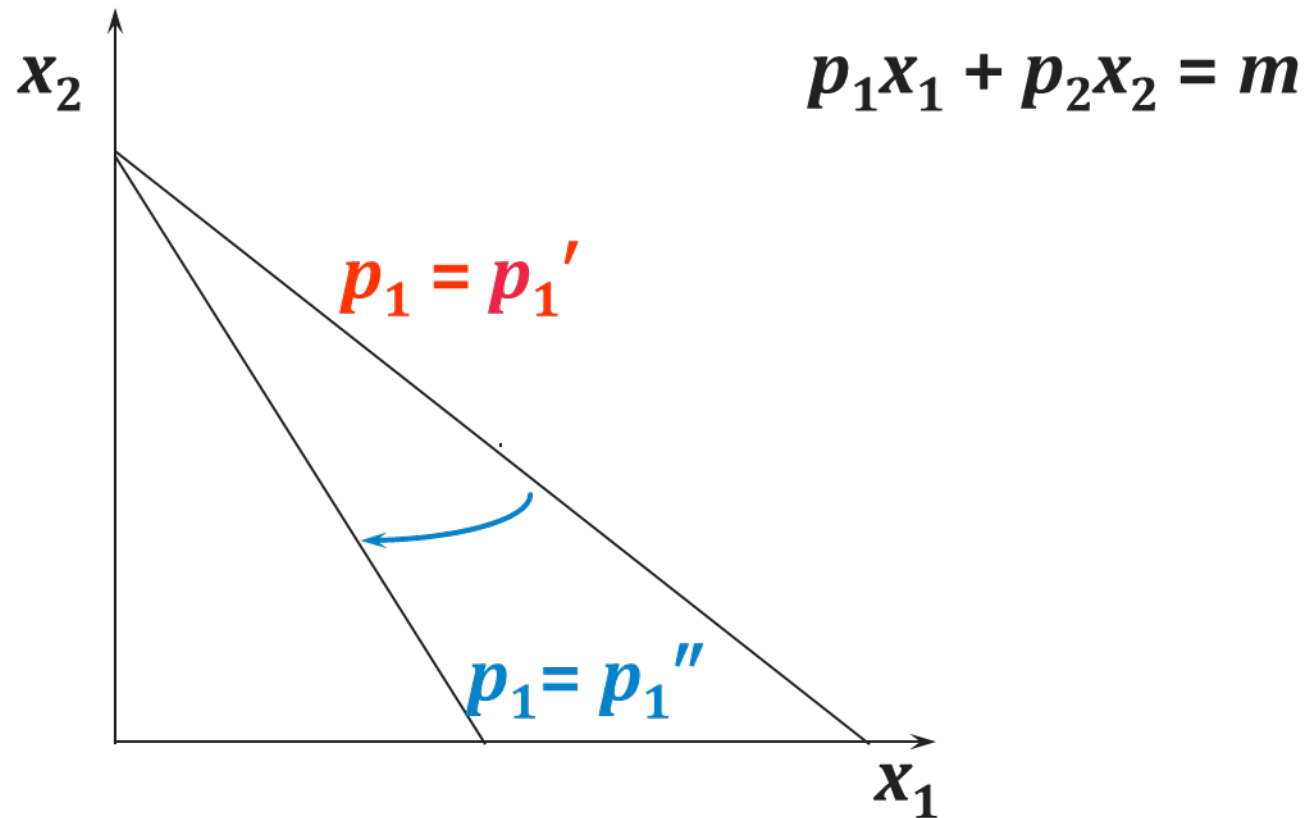
## 2. Own-Price Changes (fixed $p_2$ and $m$ ) --- initial budget set

---



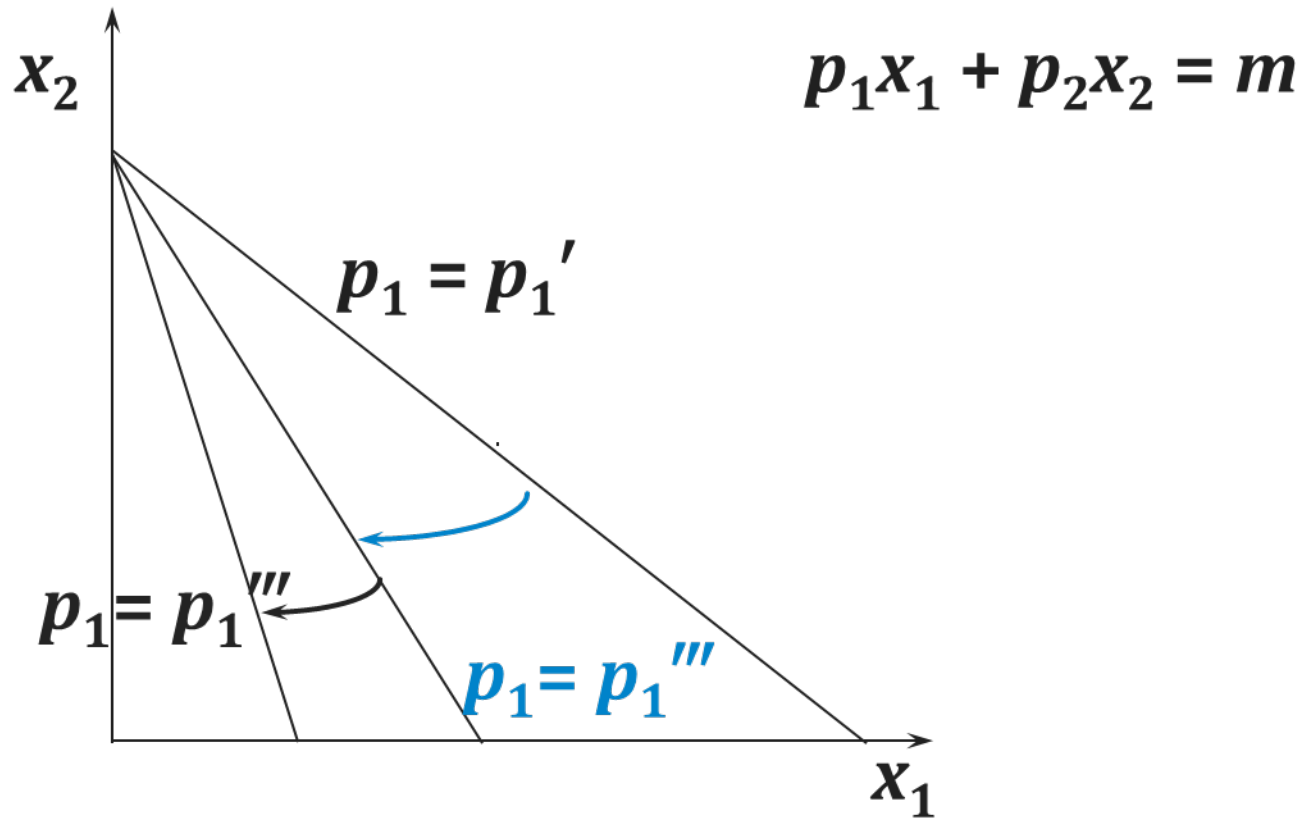
## 2. Own-Price Changes (fixed $p_2$ and $m$ ) --- new budget set

---



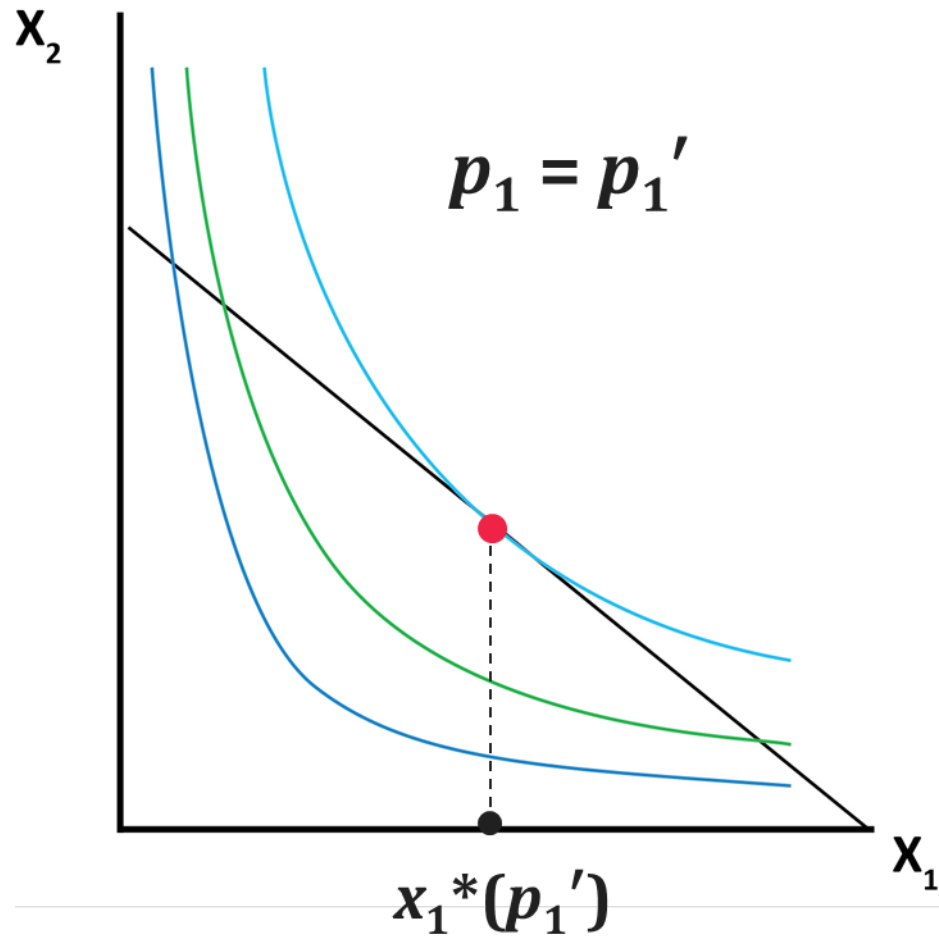
## 2. Own-Price Changes (fixed $p_2$ and $m$ ) --- new budget set

---



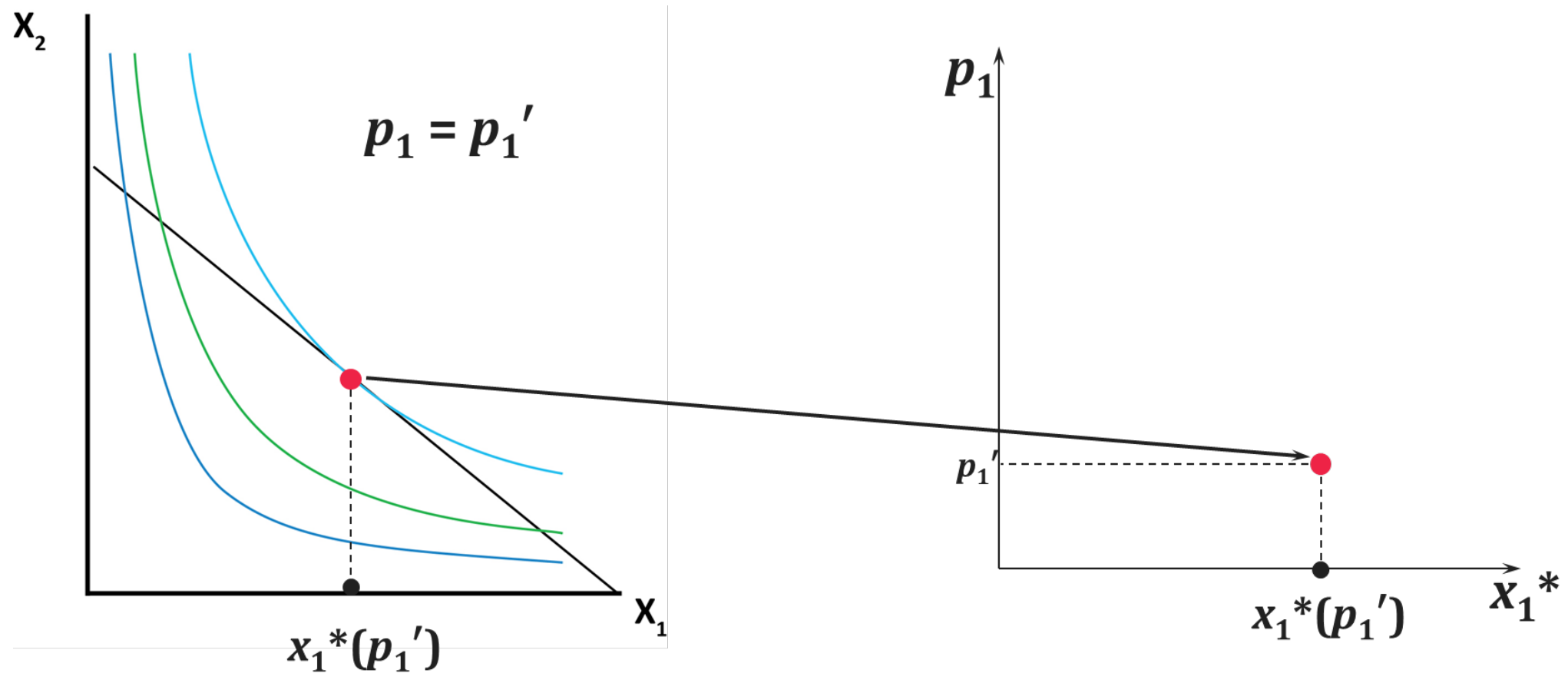
## 2. Own-Price Changes (fixed $p_2$ and $m$ ) --- optimal choice

---

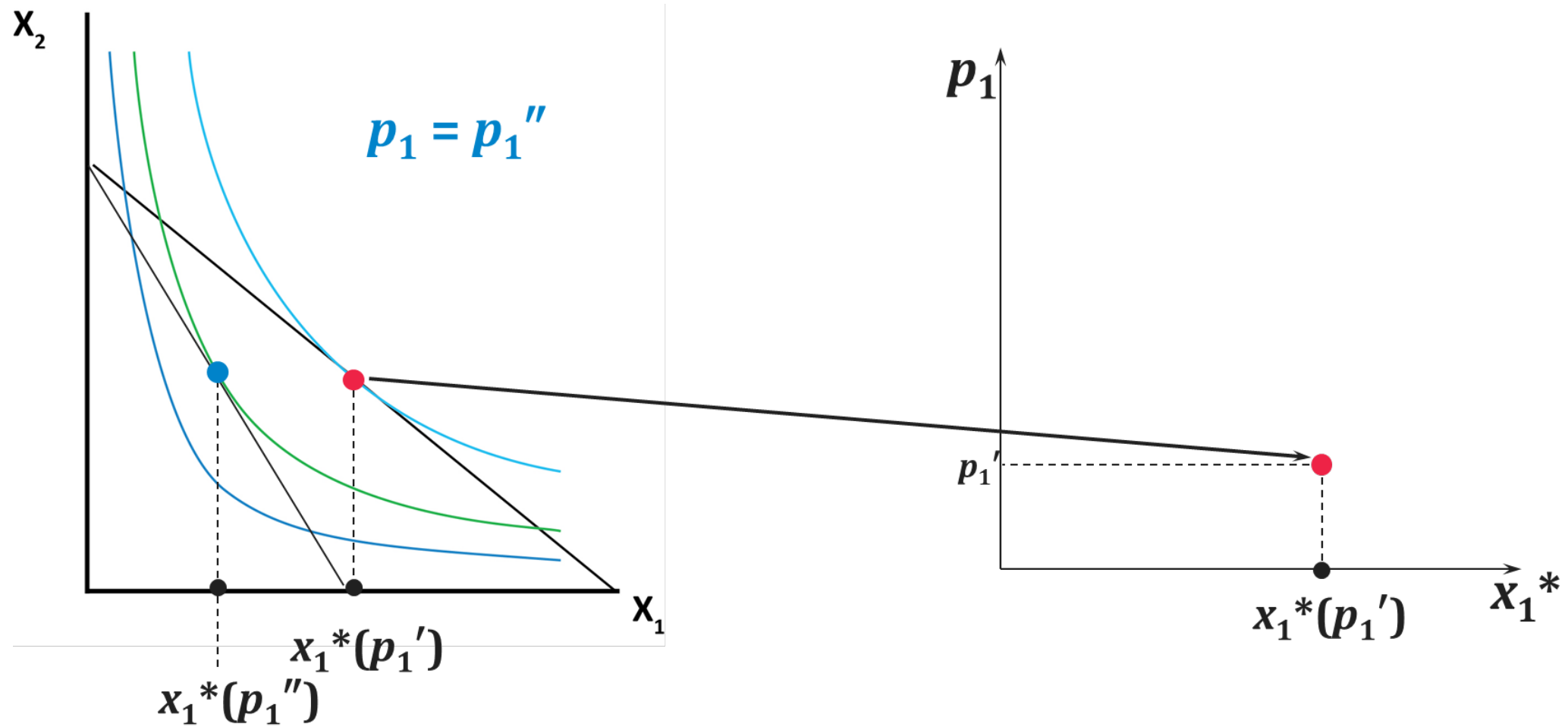




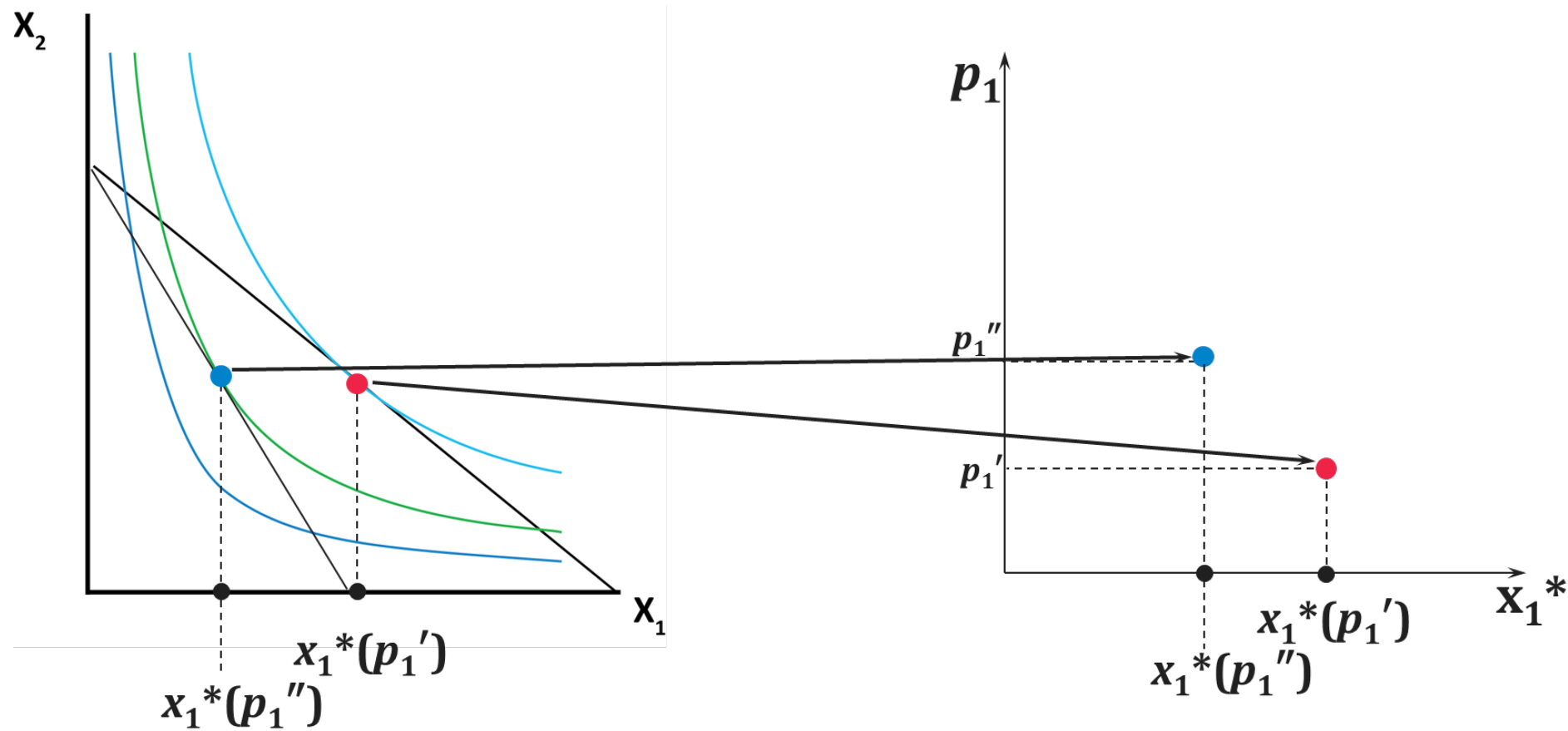
## 2. Own-Price Changes (Fixed $p_2$ and $m$ ) --- demand curve



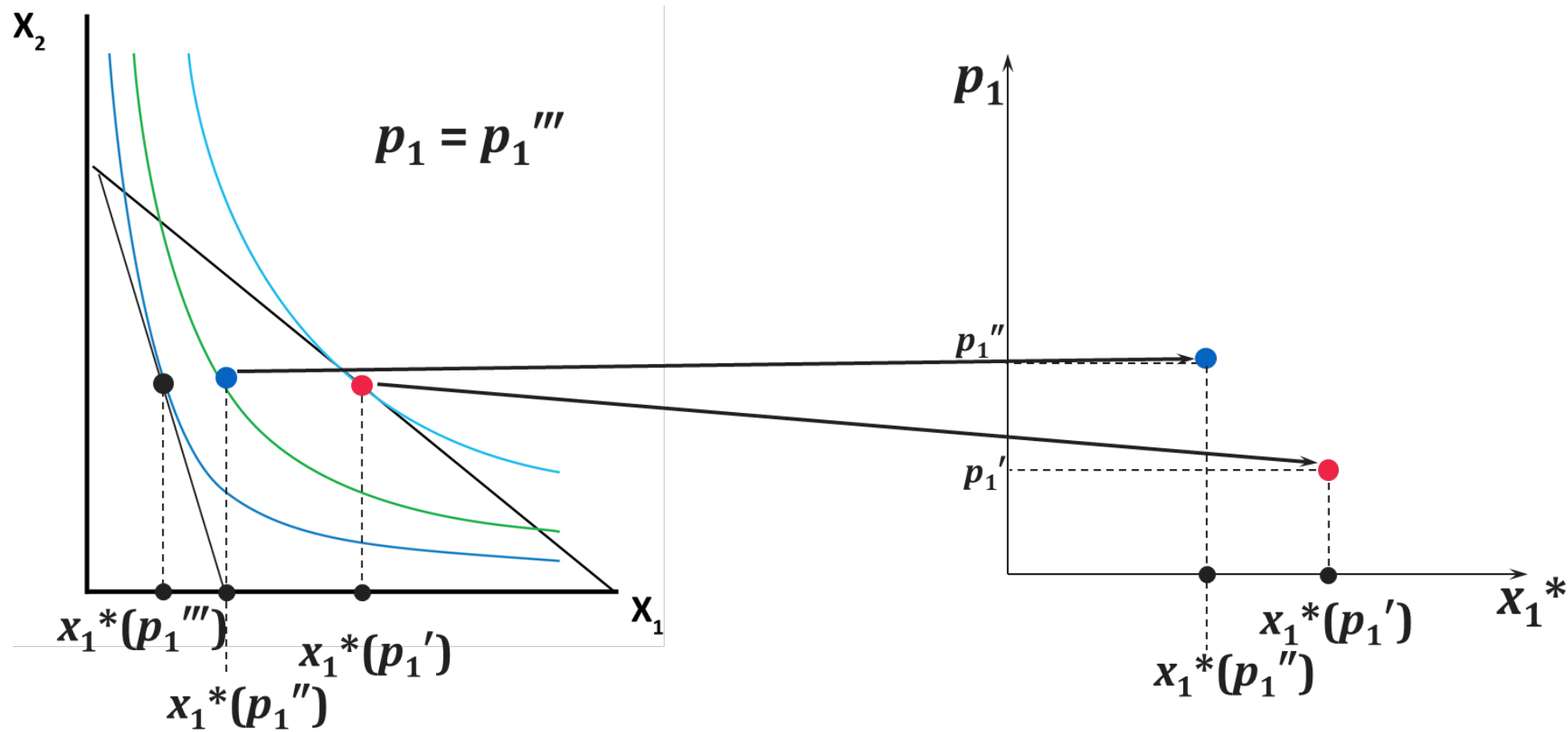
## 2. Own-Price Changes (fixed $p_2$ and $m$ ) --- new optimal choice



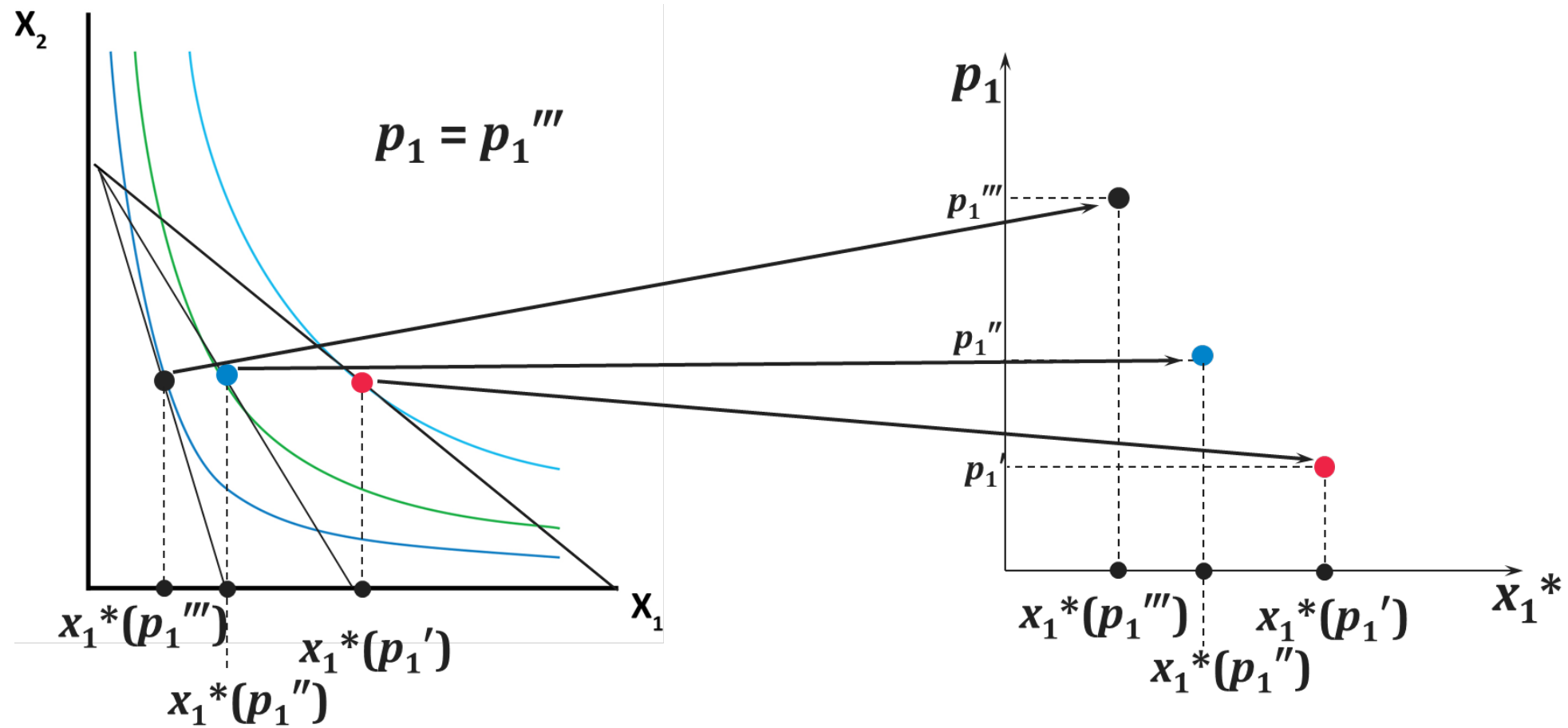
## 2. Own-Price Changes (fixed $p_2$ and $m$ ) --- demand curve



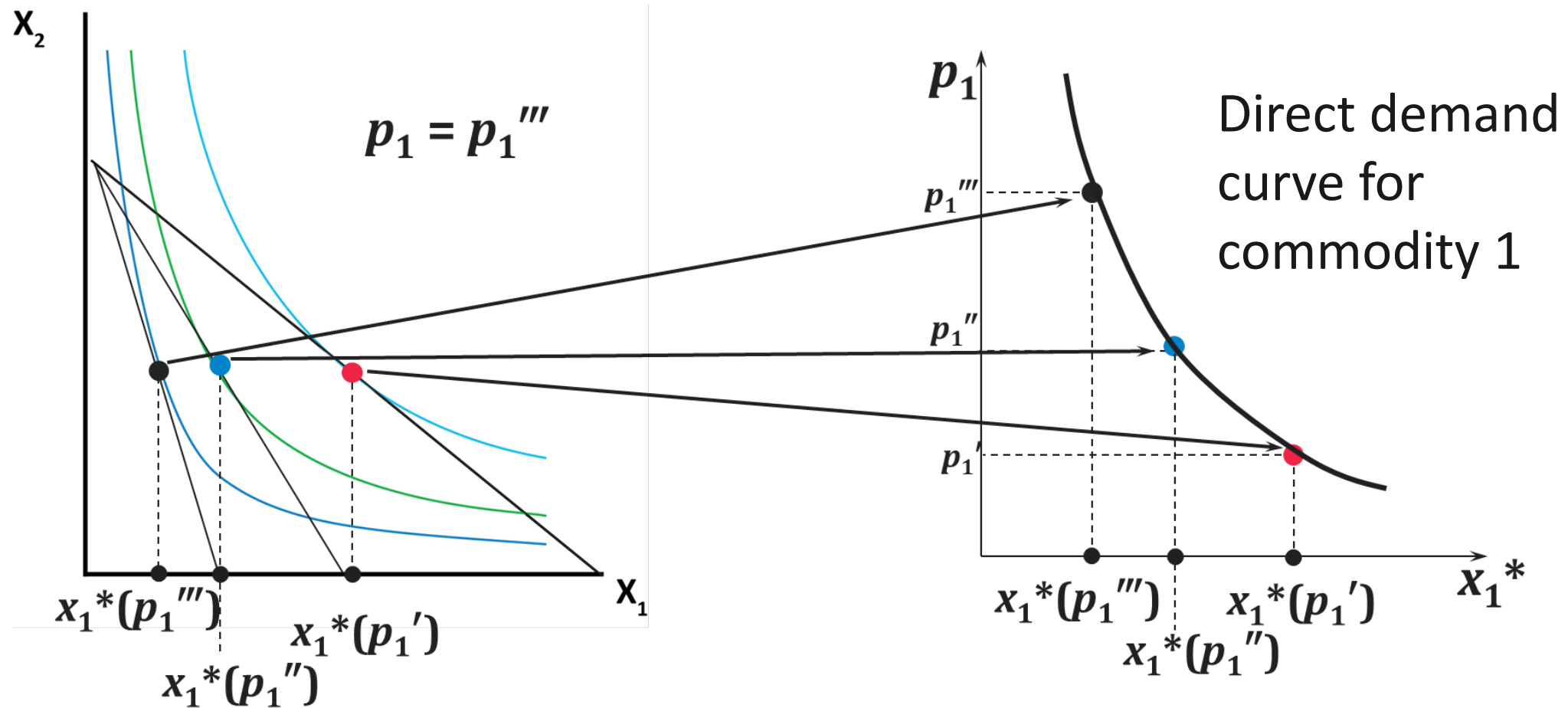
## 2. Own-Price Changes (fixed $p_2$ and $m$ ) --- --- new optimal choice



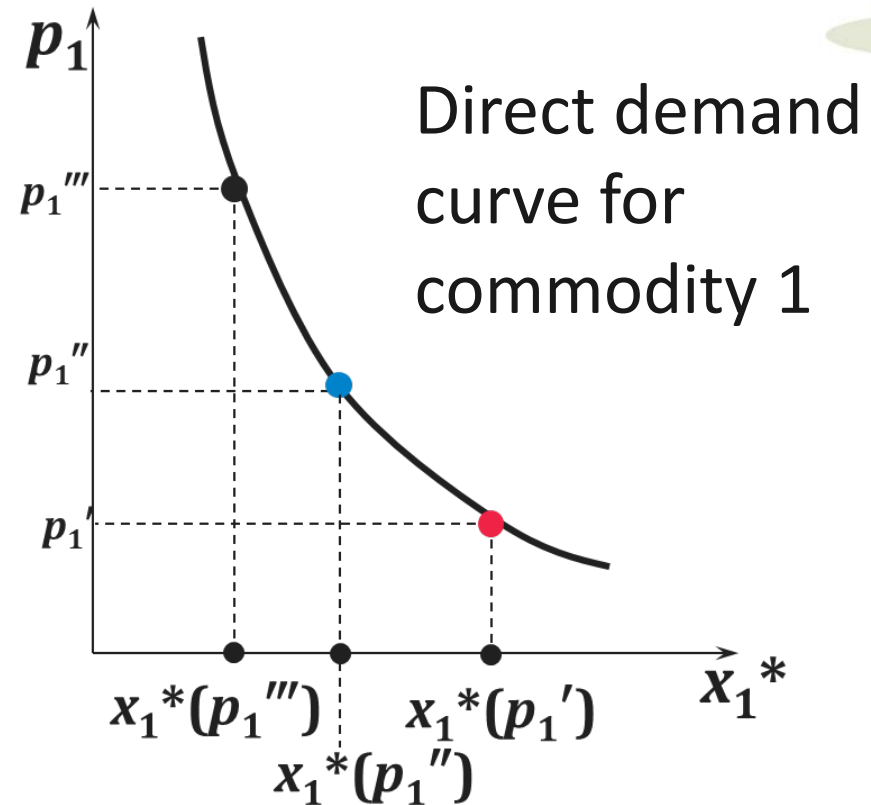
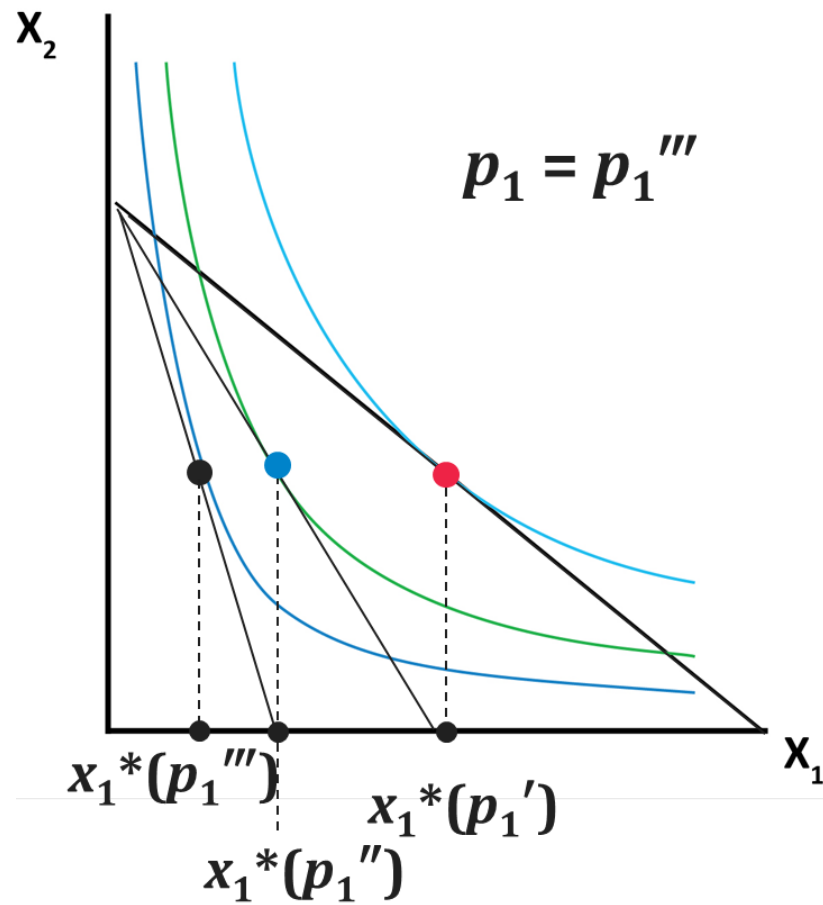
## 2. Own-Price Changes (fixed $p_2$ and $m$ ) --- demand curve



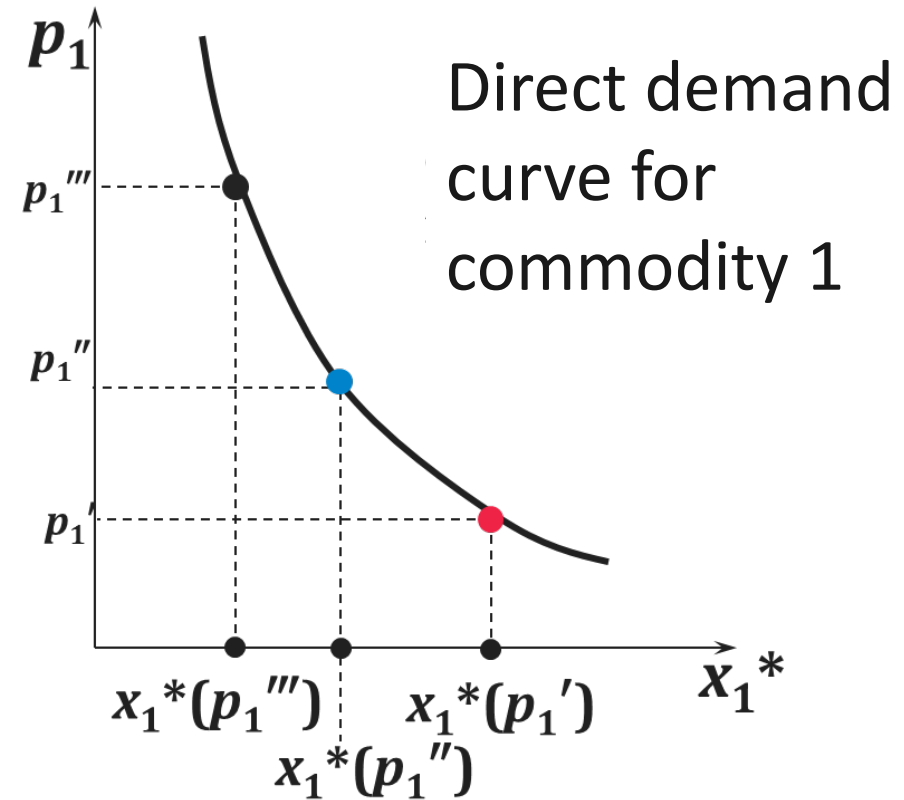
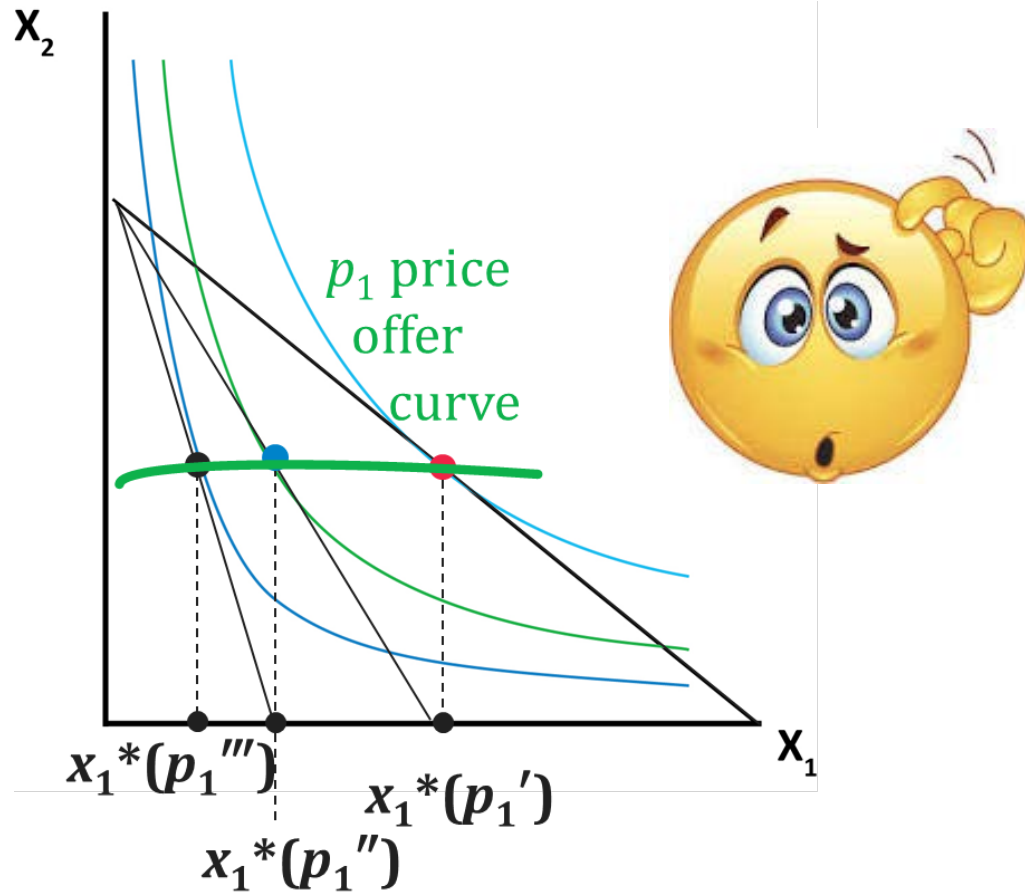
## 2. Own-Price Changes (fixed $p_2$ and $m$ ) --- demand curve !!!



## 2. Own-Price Changes (fixed $p_2$ and $m$ )



## 2. Own-Price Changes (fixed $p_2$ and $m$ ) --- price offer curve





## 2. Own-Price Changes --- Price offer curve and demand curve

---

### DEFINITION

- The curve containing all the utility-maximizing bundles traced out as  $p_1$  changes, with  $p_2$  and  $m$  constant, is the  **$p_1$ -price offer curve**.

### DEFINITION

- The plot of the  $x_1$ -coordinate of the  $p_1$ -price offer curve against  $p_1$  is the **direct demand curve** for commodity 1.



## 2. Own-Price Changes --- Direct and inverse demand functions

---

- Usually we ask “Given the price for commodity 1, what is the quantity demanded of commodity 1?”

DIRECT DEMAND FUNCTION ( $x$  is function of  $p$ )

- But we could also ask the **inverse** question “At what price for commodity 1 would a given quantity of commodity 1 be demanded?”

INVERSE DEMAND FUNCTION ( $p$  is function of  $x$ )

## 2. Own-Price Changes --- Ordinary goods

---



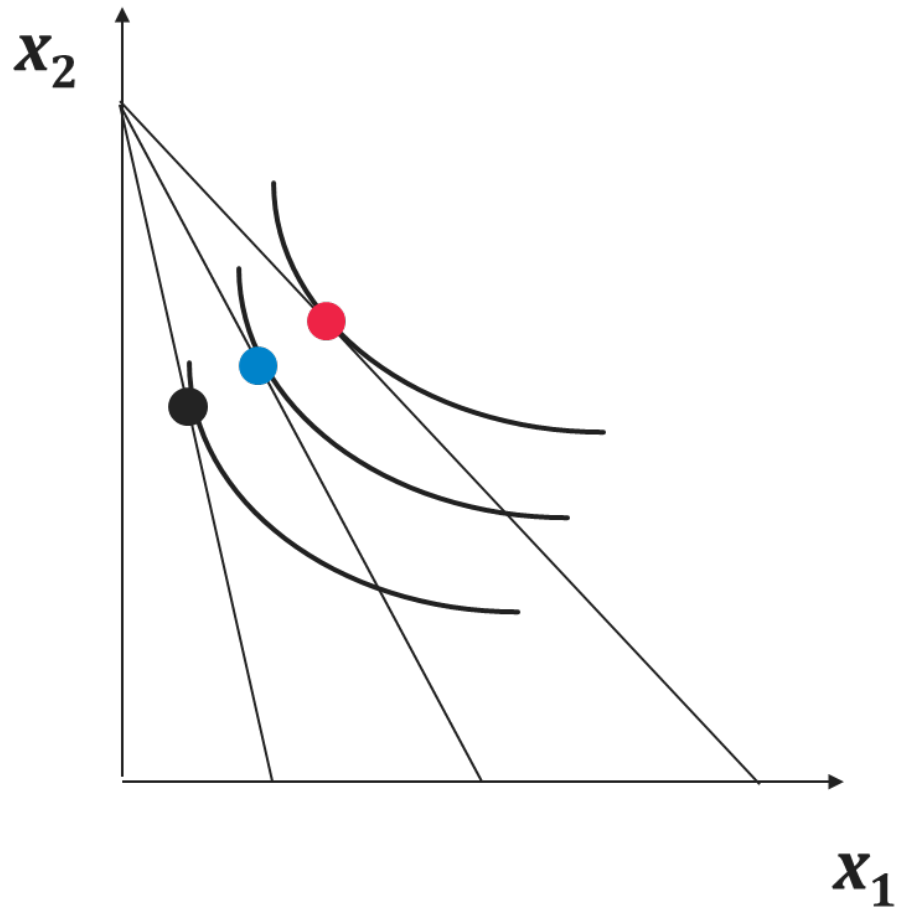
### DEFINITION

- A good is called **ordinary** if the quantity demanded of it always increases as its own price decreases.
- ... and vice versa.

## 2. Own-Price Changes --- Ordinary goods: optimal bundles

---

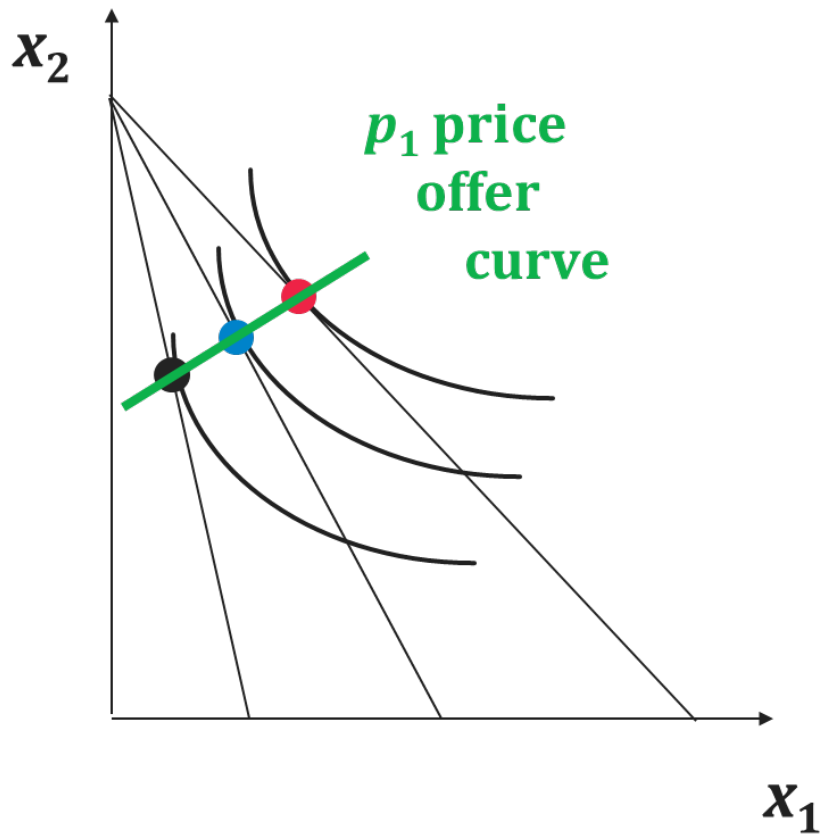
- Fixed  $p_2$  and  $m$ .



## 2. Own-Price Changes --- Ordinary goods: the price offer curve

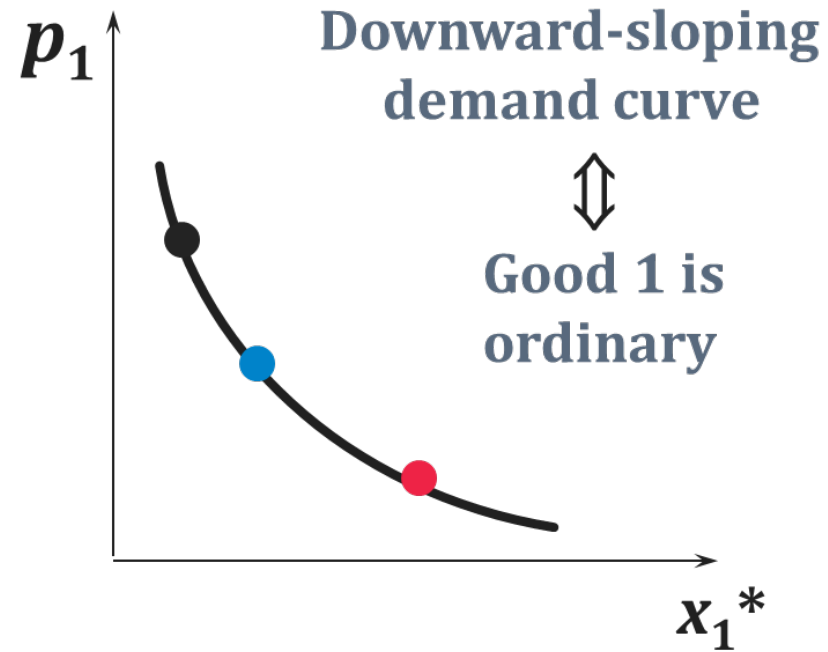
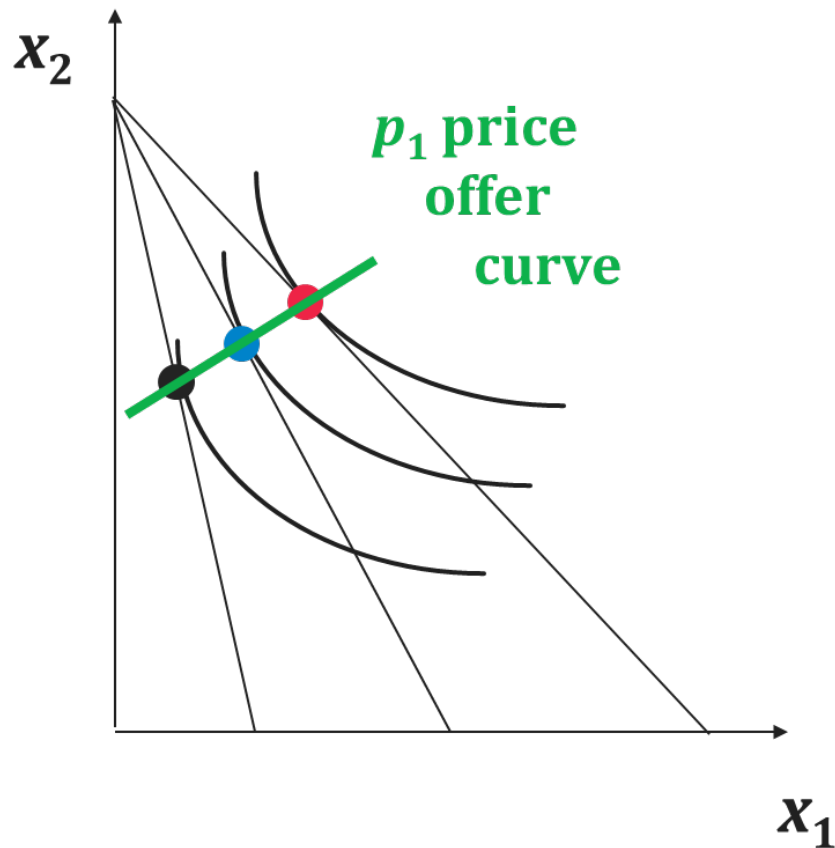
---

- Fixed  $p_2$  and  $m$ .



## 2. Own-Price Changes --- Ordinary goods --- demand curve

- Fixed  $p_2$  and  $m$ .



## 2. Own-Price Changes --- Giffen goods

---

### DEFINITION

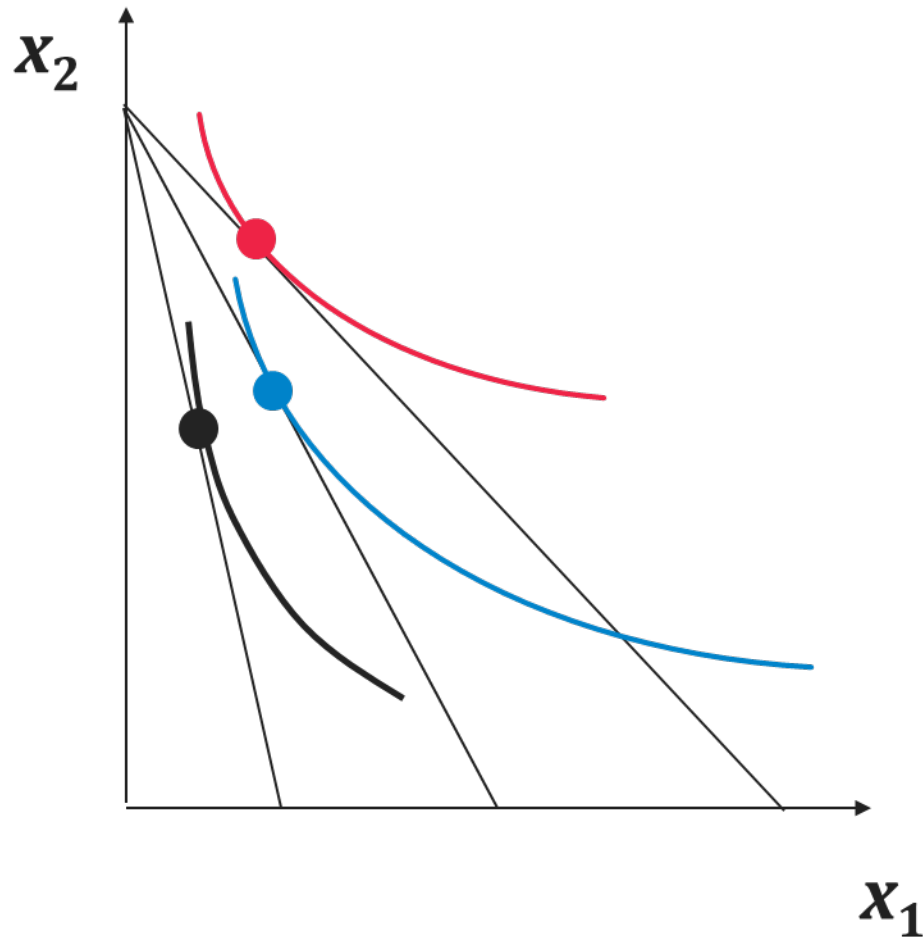
- If, for *some* values of its own price, the quantity demanded of a good rises as its own price increases, then the good is called **Giffen good**.



## 2. Own-Price Changes --- Giffen goods: optimal bundles

---

- Fixed  $p_2$  and  $m$ .

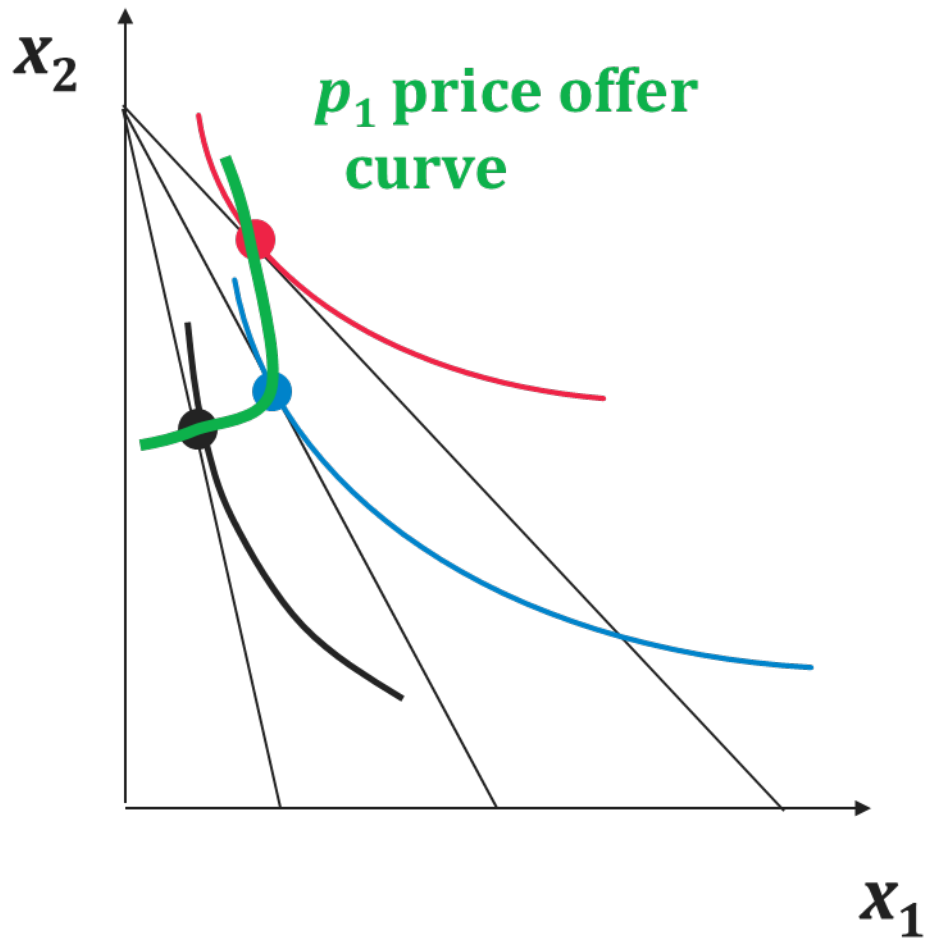




## 2. Own-Price Changes --- Giffen goods: price offer curve

---

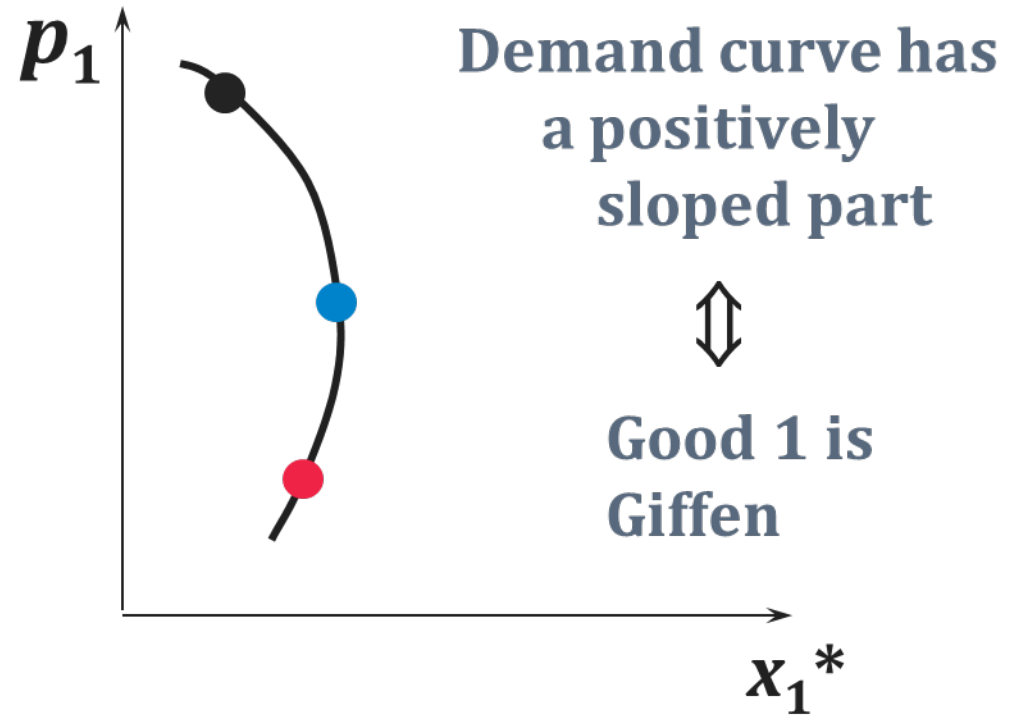
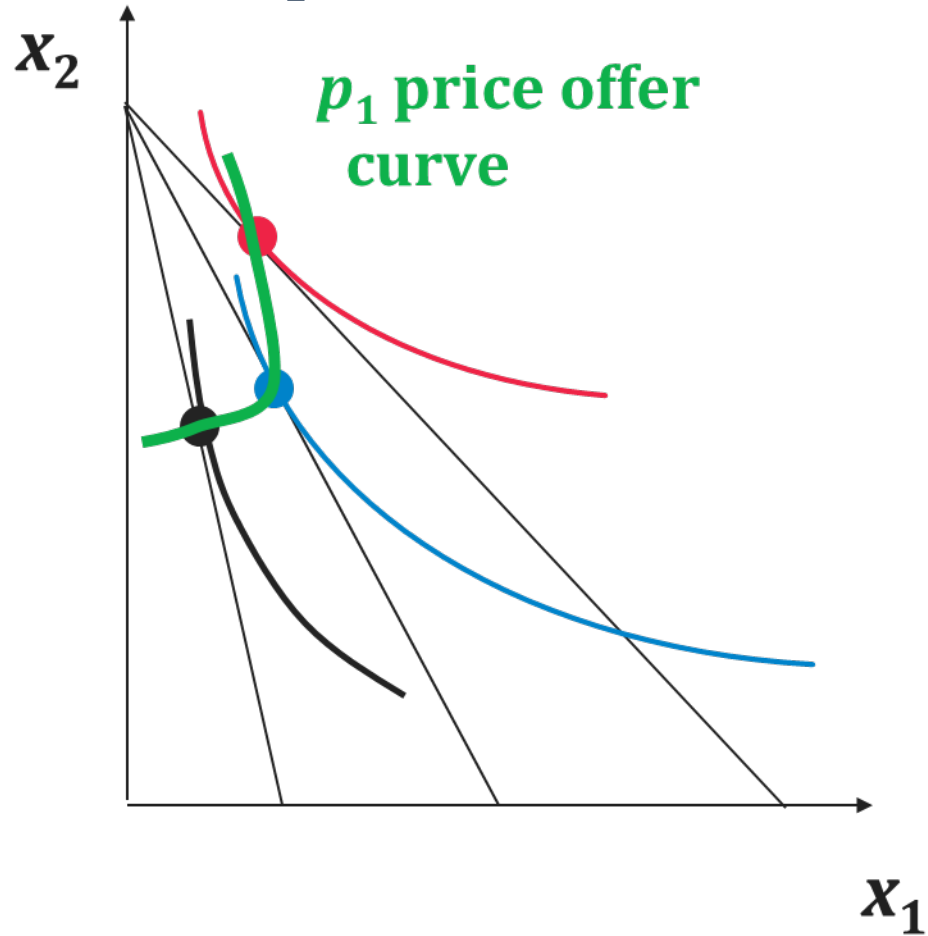
- Fixed  $p_2$  and  $m$ .



## 2. Own-Price Changes --- Giffen goods: demand curve



- Fixed  $p_2$  and  $m$ .



### 3. Cross Price Changes

---

#### QUESTION

- How does the value of  $x_1^*(p_1, p_2, m)$  change as  $p_2$  changes, holding both  $p_1$  and  $m$  constant?

### 3. Cross Price Changes --- (gross) substitutes and complements

---

#### DEFINITION

If an increase in  $p_2$

- *increases* demand for commodity 1, then commodity 1 is a **gross substitute** for commodity 2.
- *reduces* demand for commodity 1, then commodity 1 is a **gross complement** for commodity 2.

(Symmetric definition if  $p_1$  increases.)

### 3. Cross Price Changes --- example

---

- Suppose

$$x_1^* = \frac{m}{p_1 + p_2}$$

- so ...

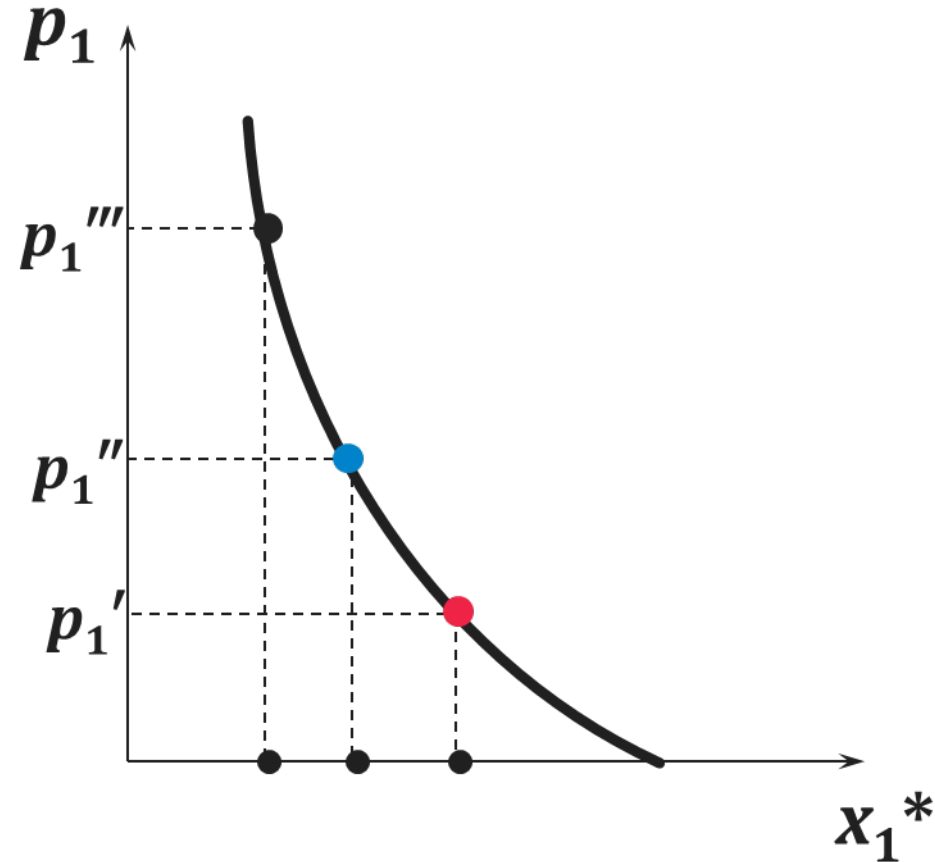
$$\frac{\partial x_1^*}{\partial p_2} = -\frac{m}{(p_1 + p_2)^2} < 0.$$

Therefore commodity 2 is a gross complement for commodity 1.

### 3. Cross Price Changes --- example

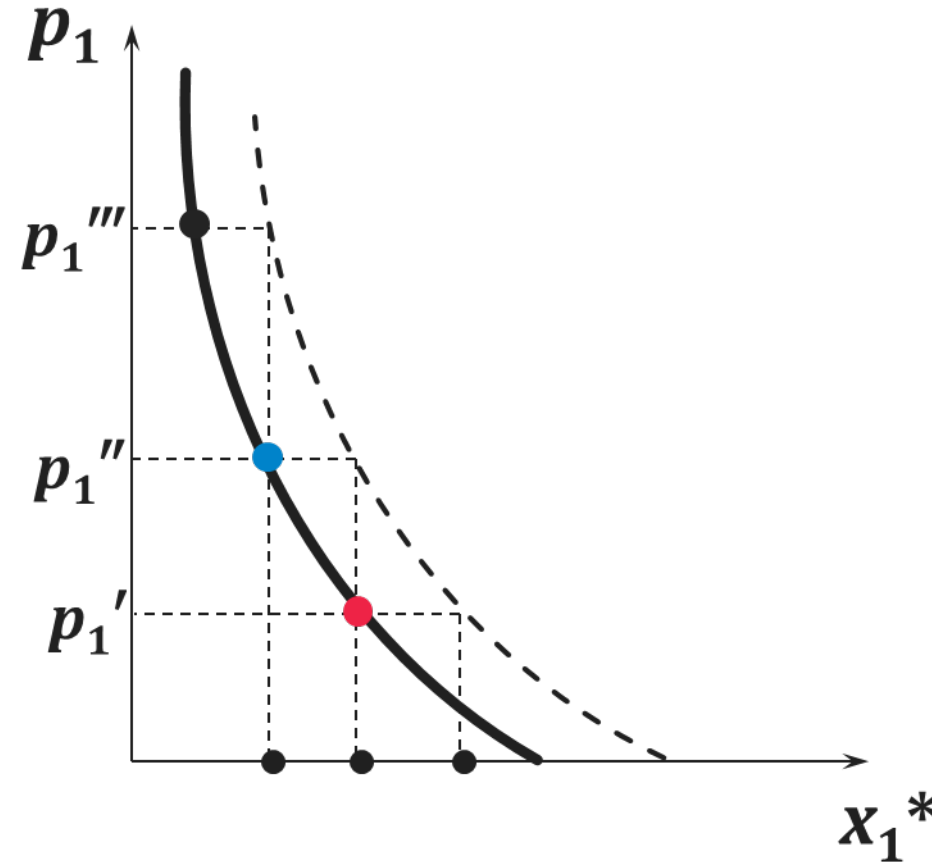
---

- Consider the demand function of commodity 1.
- Suppose the price of commodity 2 decreases.
- Then the individual will demand more of  $x_2$  AND of  $x_1$  given that the two commodities are gross complements.



### 3. Cross Price Changes --- example

- If the price of good 2 decreases, the demand curve for good 1 shifts outward.
- Hence, good 2 is a gross complement for good 1.



## 4. Income Changes

---

### QUESTION

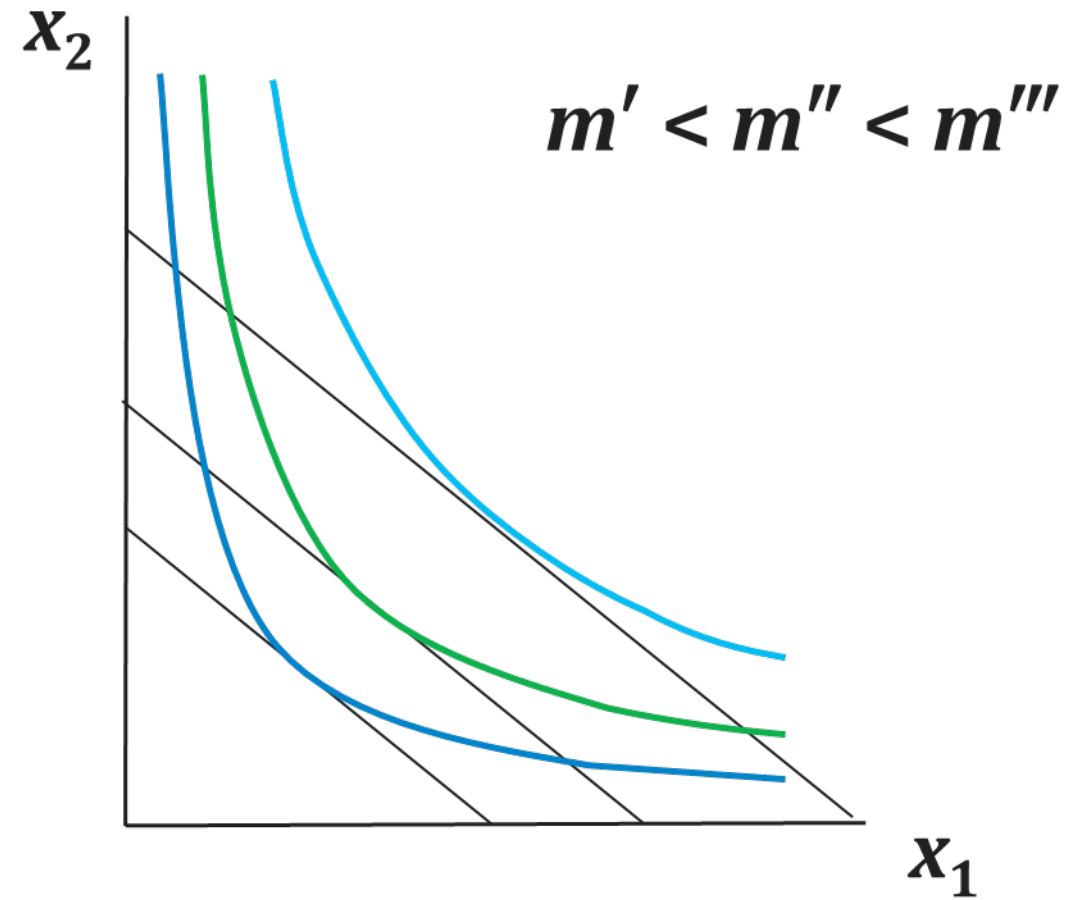
- How does the value of  $x_1^*(p_1, p_2, m)$  change as  $m$  changes, holding both  $p_1$  and  $p_2$  constant?



## 4. Income Changes

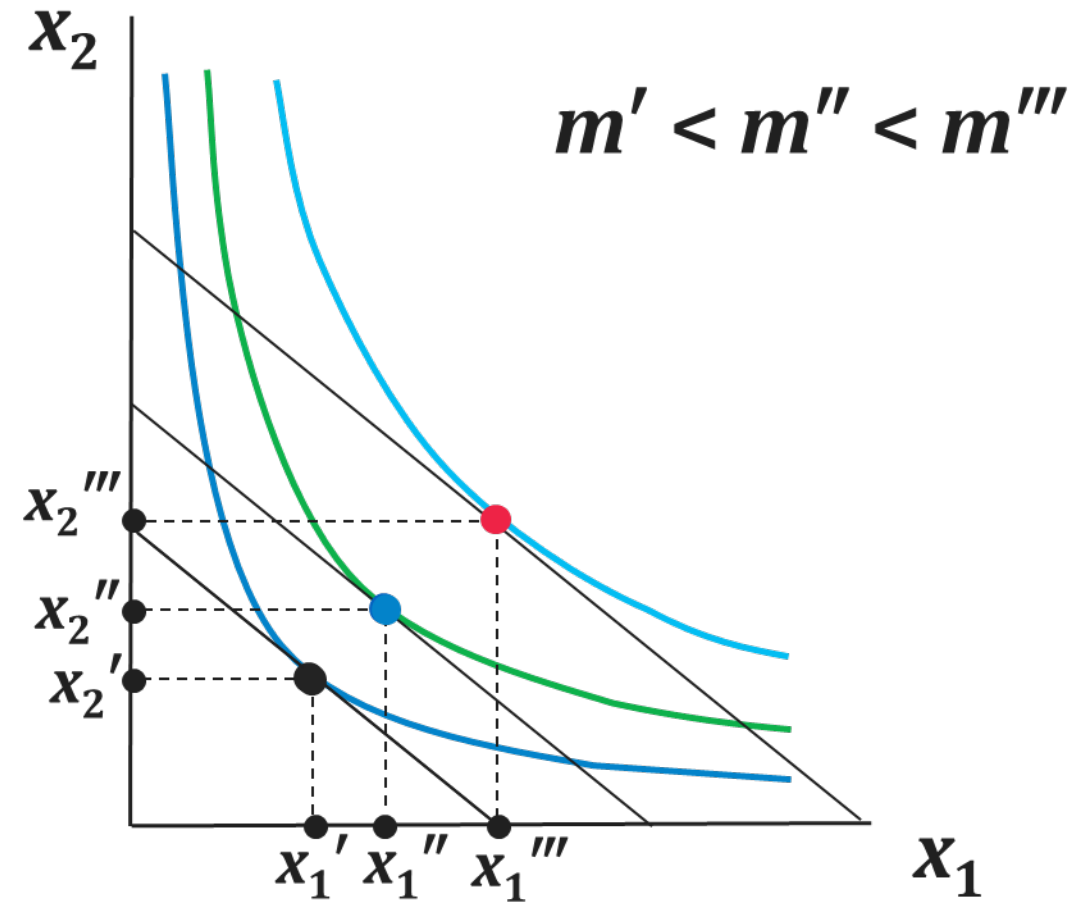
---

Fixed  $p_1$  and  $p_2$ .



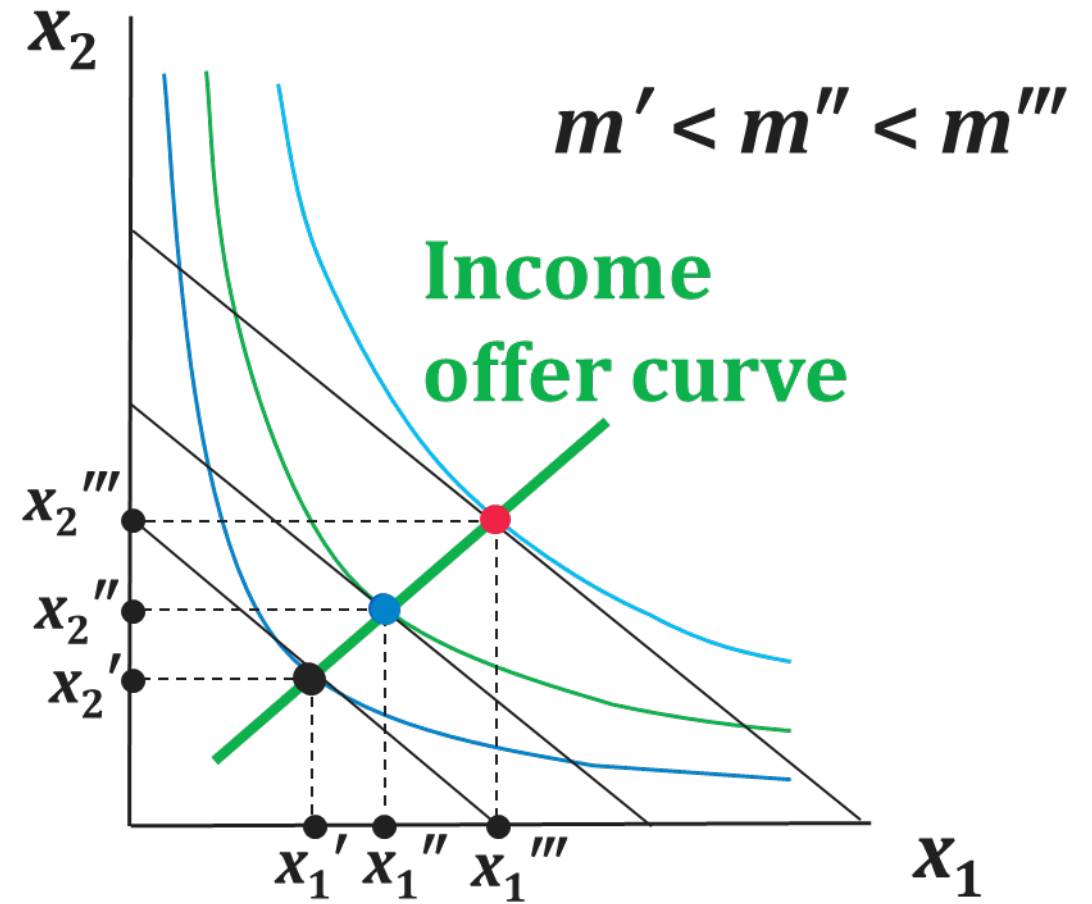
## 4. Income Changes: optimal bundles

Fixed  $p_1$  and  $p_2$ .



## 4. Income Changes: income offer curve

Fixed  $p_1$  and  $p_2$ .



## 4. Income Changes: Engel curve

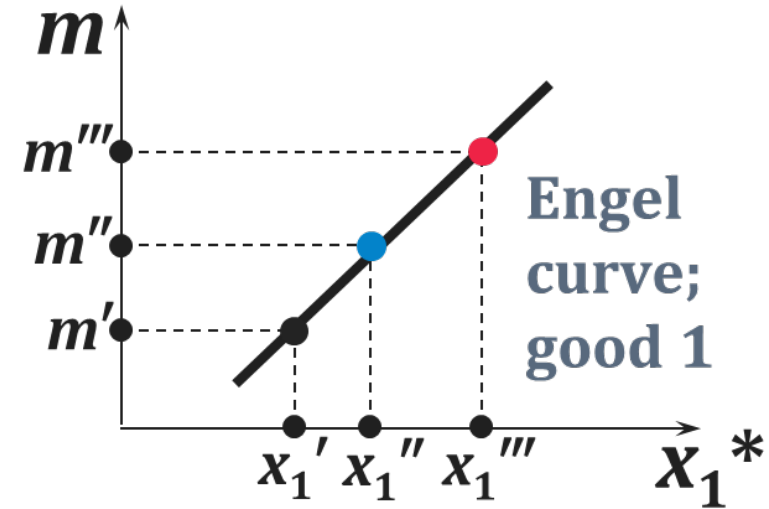
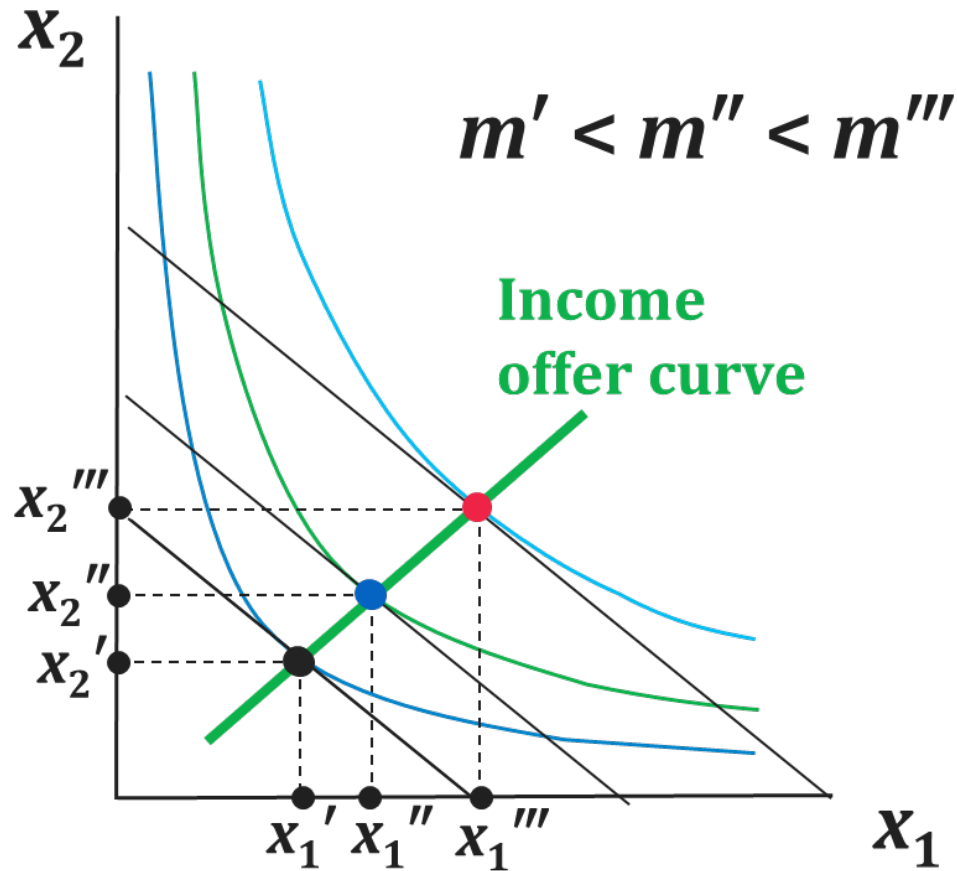
---



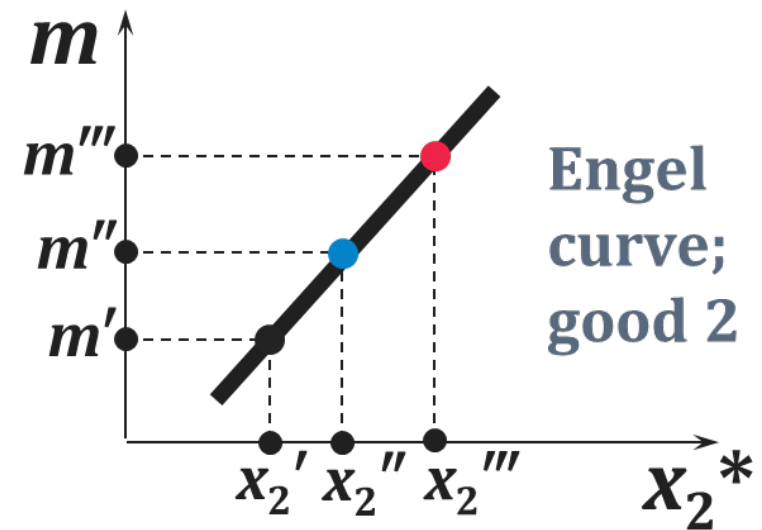
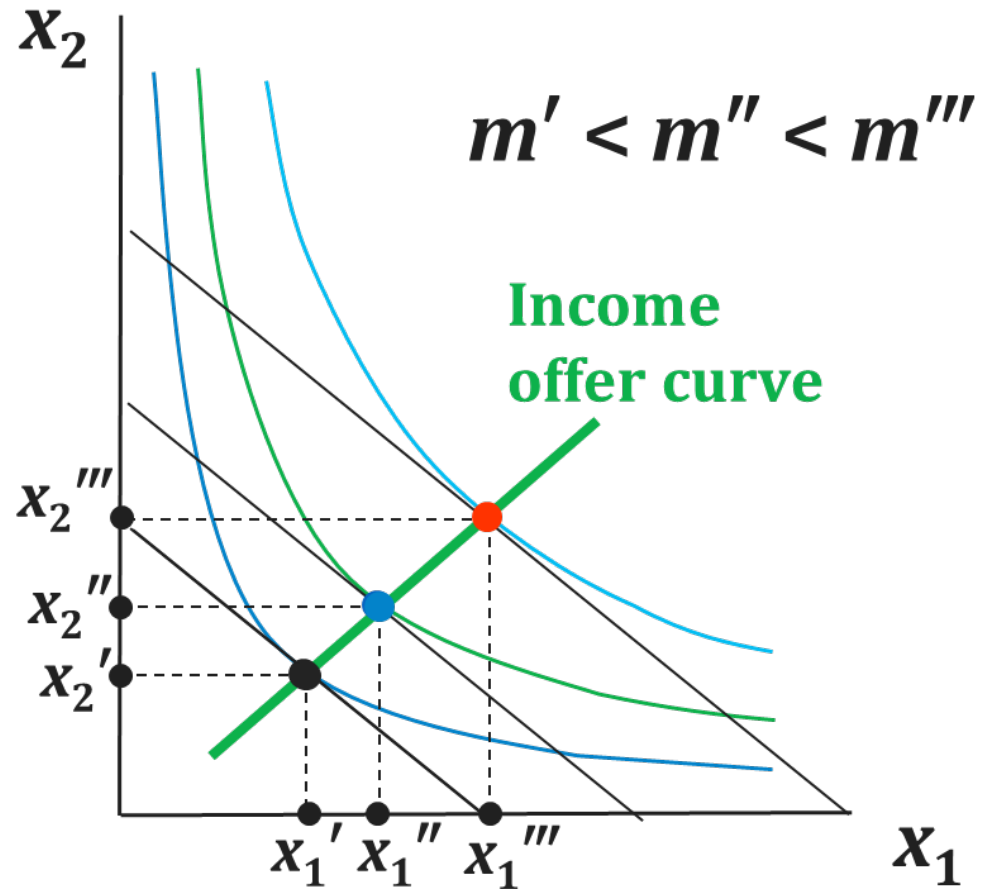
### DEFINITION

A plot of quantity demanded against income is called an **Engel curve**.

## 4. Income Changes: Engel curve for good 1



## 4. Income Changes: Engel curve for good 2



## 4. Income Changes --- Example: Engel curve for Cobb Douglas preferences

---

- Let  $u(x_1, x_2) = x_1^a x_2^b$  be the Cobb Douglas utility function.
- The ordinary demand equations are

$$x_1^* = \frac{am}{(a+b)p_1} \qquad x_2^* = \frac{bm}{(a+b)p_2}$$

## 4. Income Changes --- Example: Engel curve for Cobb Douglas preferences

---

$$x_1^* = \frac{am}{(a+b)p_1}; x_2^* = \frac{bm}{(a+b)p_2}$$

- Rearranging to isolate  $m$ , these are

- Engel curve for good 1: 
$$m = \frac{(a+b)p_1}{a} x_1^*$$

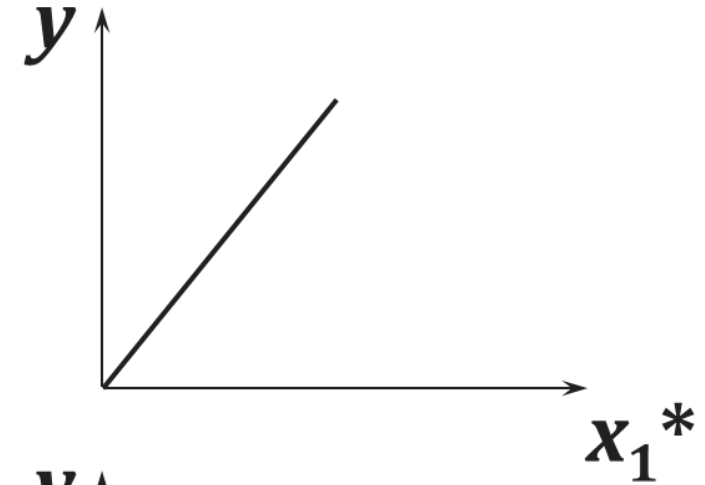
- Engel curve for good 2: 
$$m = \frac{(a+b)p_2}{b} x_2^*$$



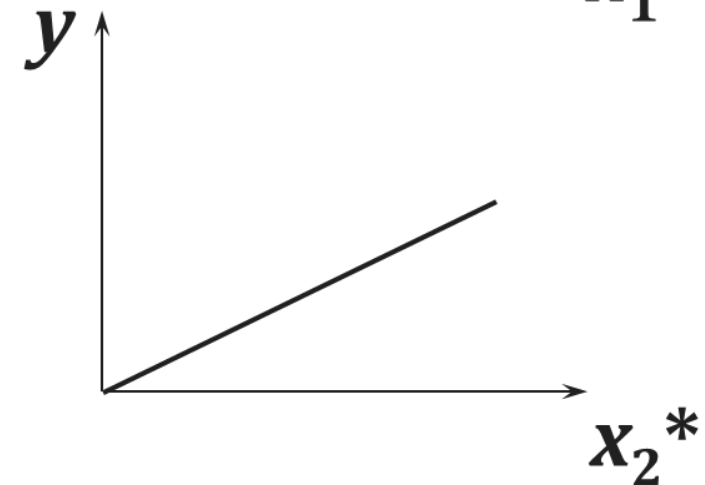
## 4. Income Changes --- Example: Engel curve for Cobb Douglas preferences

---

- Engel curve for good 1  $m = \frac{(a+b)p_1}{a} x_1^*$



- Engel curve for good 2  $m = \frac{(a+b)p_2}{b} x_2^*$



## 4. Income Changes --- Normal goods

---

### DEFINITION

- A good for which quantity demanded rises with income is called **normal**.
- Therefore a normal good's Engel curve is positively sloped.

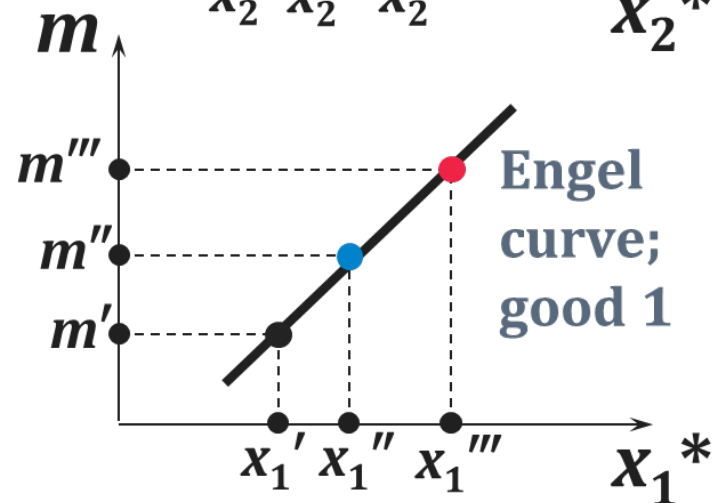
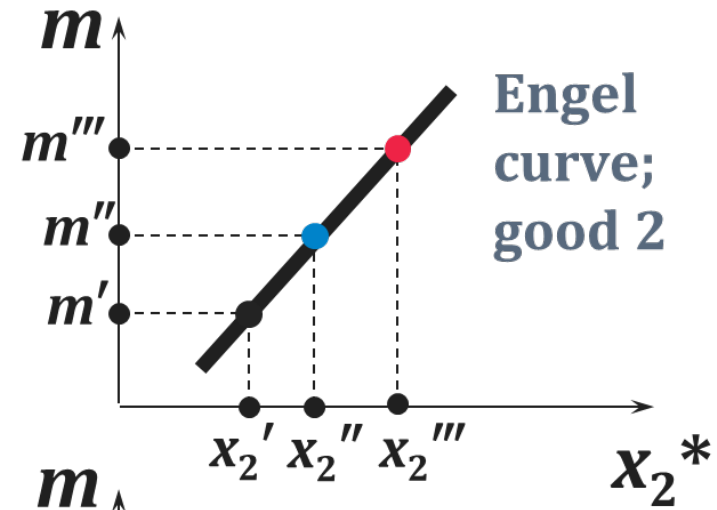
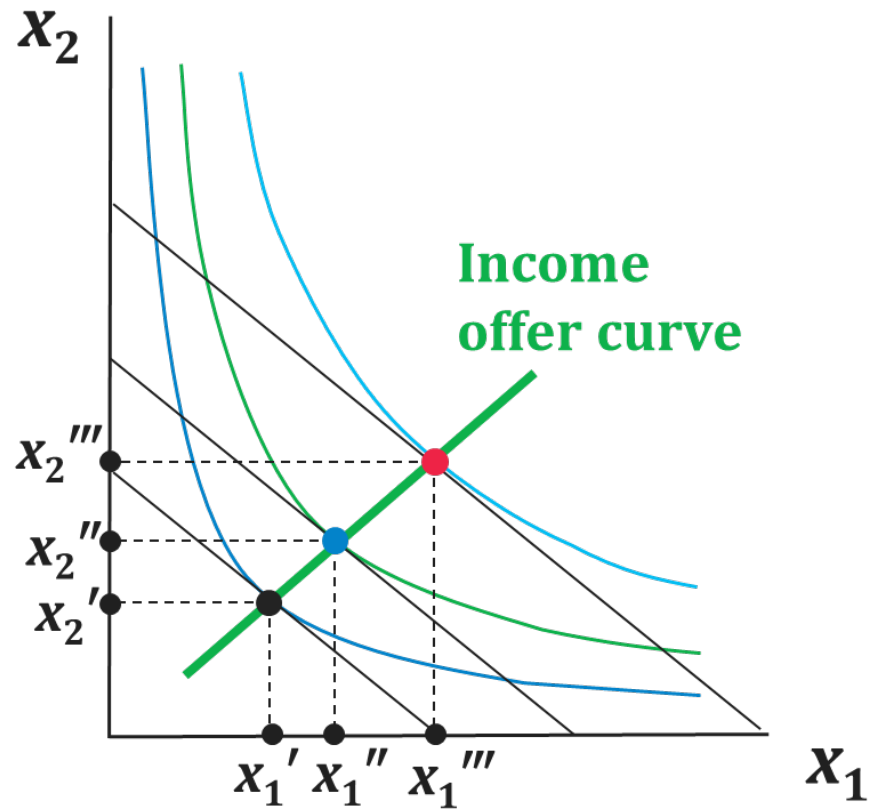
## 4. Income Changes --- Inferior goods

---

### DEFINITION

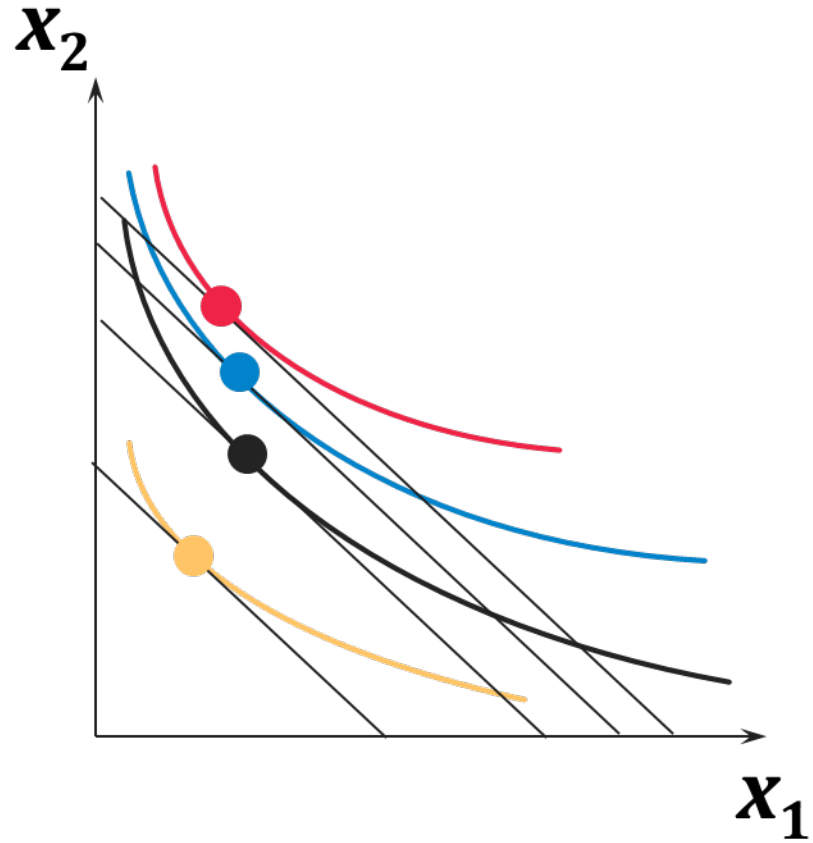
- A good for which quantity demanded decreases with income is called **inferior**.
- Therefore an inferior good's Engel curve is negatively sloped.

## 4. Income Changes --- Normal goods: income offer curve and Engel curves



## 4. Income Changes --- Inferior ( $x_1$ ) and normal ( $x_2$ ) goods: optimal bundles

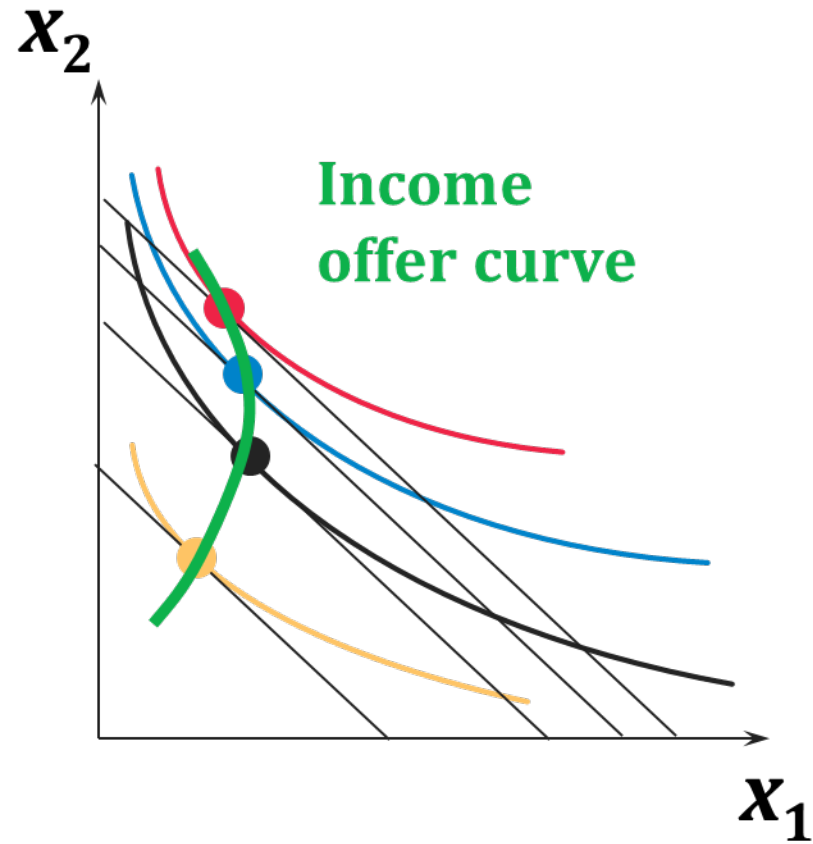
---



## 4. Income Changes --- Inferior ( $x_1$ ) and normal ( $x_2$ ) goods: optimal bundles

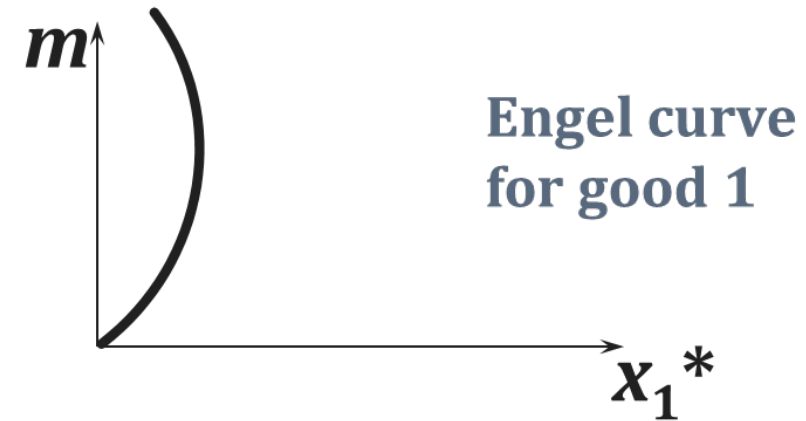
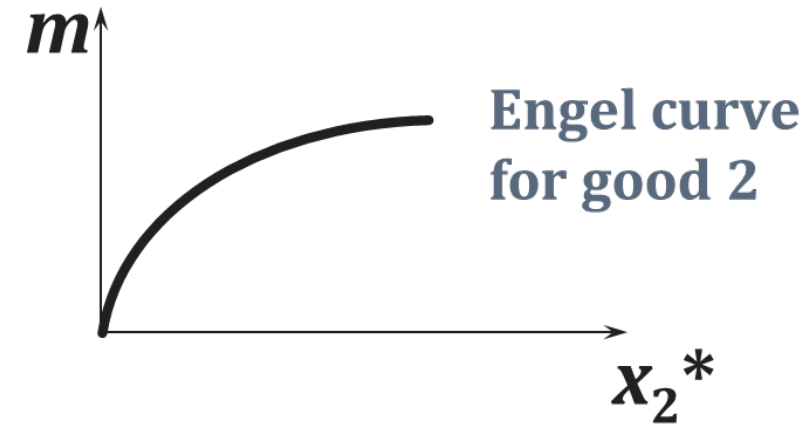
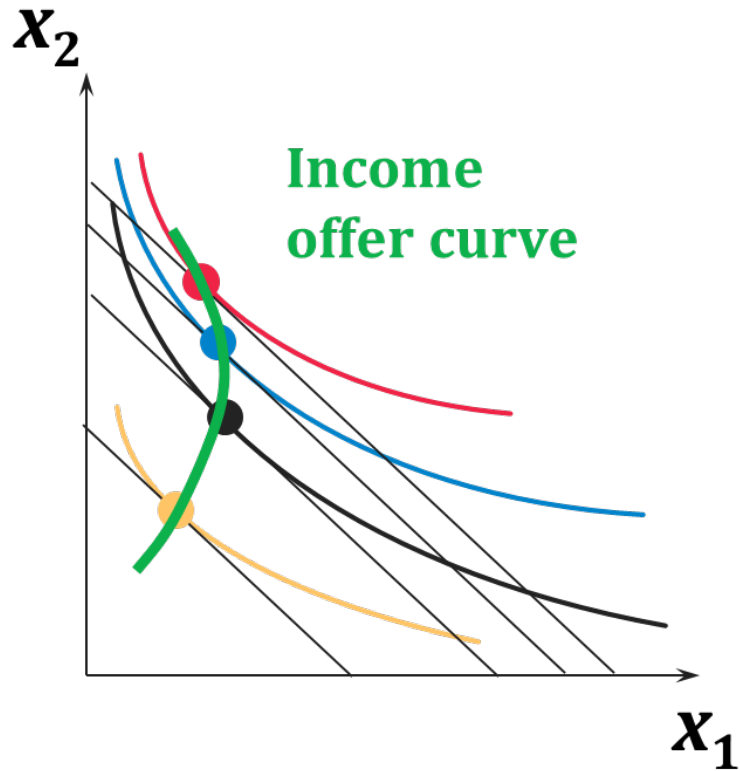
--- Income offer curve

---



## 4. Income Changes --- Inferior ( $x_1$ ) and normal ( $x_2$ ) goods: optimal bundles --- Engel curves

---



---

## **Chapter 6:**

### **Take home message**

Which is the key concept we learned in this chapter?

**Comparative statics analysis**

