Coursework: Serial and Parallel Robot Kinematics

This document explains requirements for the assessed coursework for the UFMF4X-15-M Robotic Fundamentals module.

The module is assessed in one report that is worth 50% of your overall mark. The assessment will be based on a series of simulated experiments using a series and a parallel manipulator.

The coursework also includes suggestions of topics you may want to work on in order to demonstrate a fuller understanding of the more advanced parts and attain a higher mark. This additional work is expressed here in an open-ended way, as appropriate for Masters level study.

The deadline for submission is **11th of January 2024, Thursday, 2pm on UWE Blackboard.**

PART I:

A. From material covered during lectures, lab exercises and examples, complete the following tasks:

1) Derive a DH representation of forward kinematics for the Lynxmotion arm[1]. Use MATLAB, lecture material and further reading. Include all your investigations and report this.
2) Analyse the workspace of the centre of the wrist (5th joint) when each preceding joint moves through its range of motion and plot the 2D and 3D views of the workspace.
3) Derive the inverse kinematics model for the manipulator (analytical solution).

B. Complete the following:

1) Plan a task* in MATLAB with **at least** 5 positions. This process should give you at least 5 sets of Cartesian coordinates specifying the end-effector position and orientation in 3D space.
2) Solve the Inverse Kinematics for these positions in 3D space and obtain sets of Joint Coordinates. Create an appropriate plot/animation in MATLAB for the motion of the robot.
3) Implement 3 different trajectories between the Cartesian Points identified above and create an appropriate plot to demonstrate them):
    a. Implement a free motion between the points
    b. Implement a straight line trajectory between the points
    c. Set an obstacle between any two points (e.g. a cylinder between point 3 and 4) and implement an object avoidance trajectory.

---

[1] www.lynxmotion.com

*The task you choose to do for section B is up to you. Example tasks of appropriate complexity include; pick and place of a small object, using the arm to draw a simple figure, a collaborative task between two instances of the robot.

PART II:

Consider a **planar parallel robot** used in a surgical procedure to position a needle above the patient, see Figure 1.

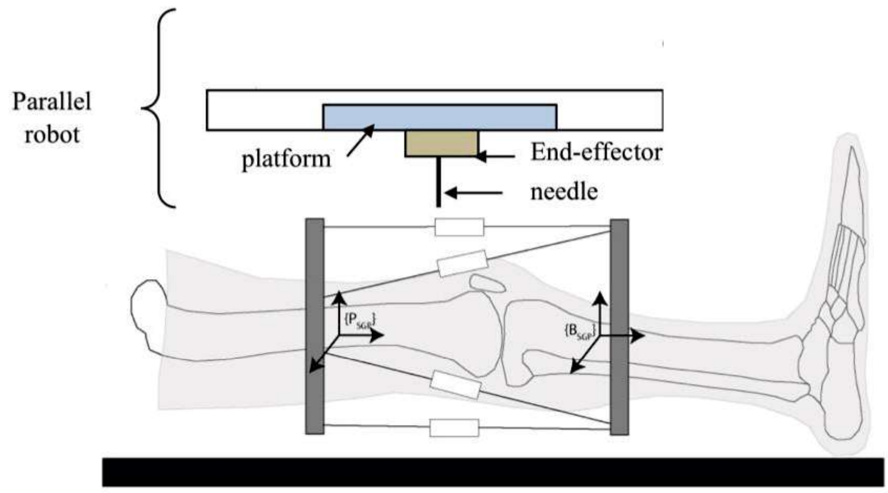


Figure 1. Needle positioning Parallel Robot.

The parallel robot consists of a fixed Base Plate, a Moving Platform, six passive revolute joints (M_i and PP_i where $i=1:3$) and three active revolute joints (PB_i where $i=1:3$). The i^{th} strut of the robot consists therefore of three joints that are subdivided in the upper section s_A, and the lower section L. For further information to the robot see its kinematic model in Figure 2.

Develop a kinematic simulation of the parallel robot in MATLAB to:

1) Solve and implement parallel robot Inverse Kinematics. Calculate the joint coordinates θ_i where $i=1:3$ from the Cartesian parameters of the Platform's centre {C} (where the needle is based). These parameters are the (X_c, Y_c) coordinates and the orientation of the platform (a). Plot the kinematic model in two different positions. Changing the Cartesian input parameters should adapt each leg to the new Cartesian position. An example plot is shown in Figure 3.
2) Plot the Parallel robots workspace for a given orientation a. This is crucial for the mechanism synthesis analysis.

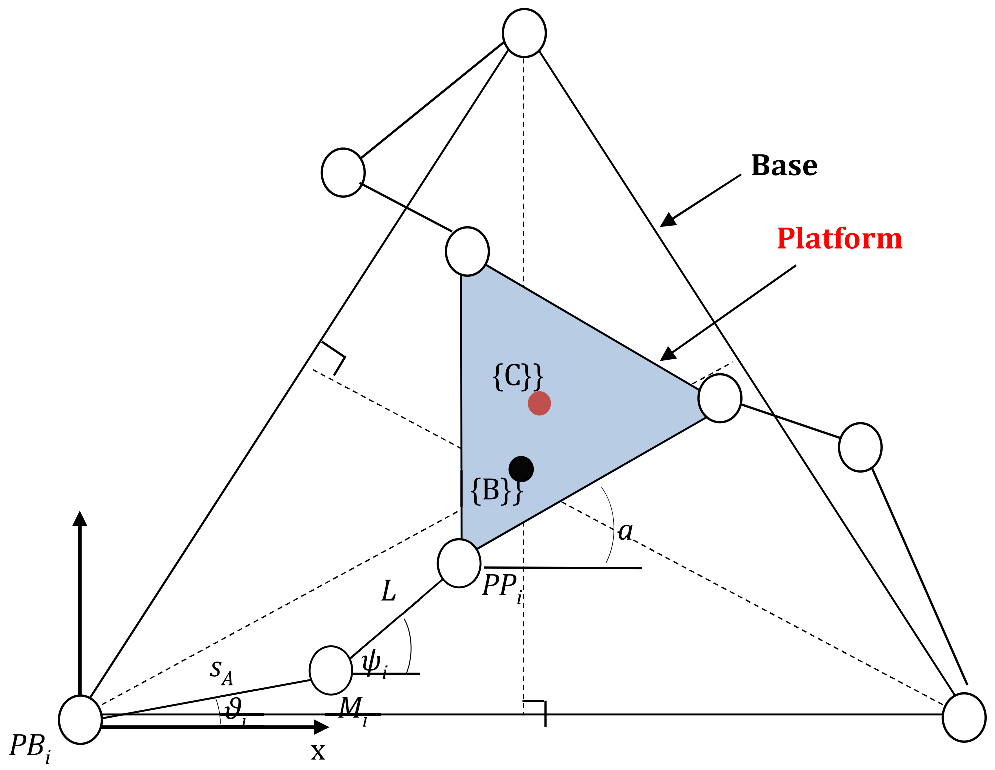


Figure 2: Planar Parallel Robot Kinematic Model

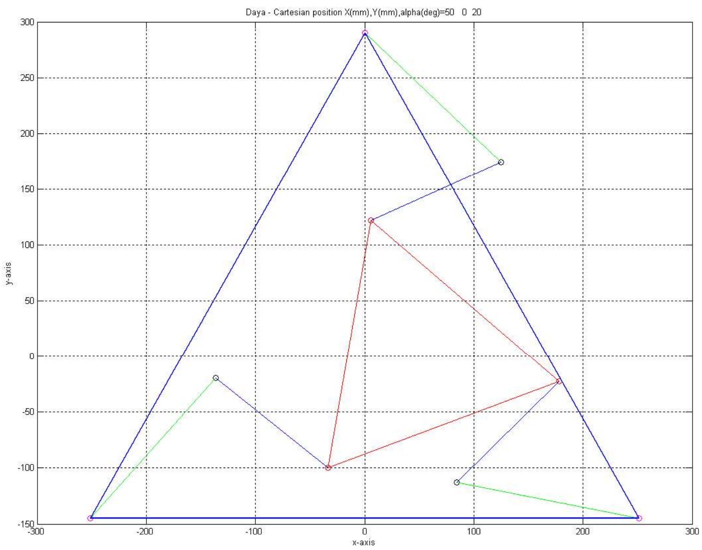


Figure 3: Simulated robot motion

PART III:

TO ATTAIN A HIGHER MARK CARRY OUT THE FOLLOWING:

Choose a relevant topic that is not mentioned here, but relevant to the coursework subject domain, and carry out theoretical or experimental work on that topic. For example, this work could be analysing alternative methods of inverse kinematics calculation such as motor babbling, optimisation, screw theory, etc., investigating methods and algorithms that can be used to avoid problems when operating close to a singularity in the robot's workspace etc. or something else. **Discuss your chosen topic with the module tutors before you start.**

Reporting Details

You have to write a report, using not more than 3000 words to describe your investigations and results. You need to:

1) Demonstrate that you understand the theory behind the approaches you use to solve a problem.
2) Critical assessment and analysis of the relative merits of the approaches you have used
3) Show that you have appreciation of issues and principles used to establish safe operation of manipulators in the human environment
4) Present your conclusions.
5) Provide any references using the Harvard system[2]
6) Provide any code you have written in **an appendix AND the actual .m files** (Matlab) as attachments.

[2] http://www1.uwe.ac.uk/students/studysupport/studyskills/referencing/uweharvard.aspx

Assessment weighting:

The following table is given for clarity of the marking process per question.

Task		Mark
Part I	A1 - FK	10%
	A2 - workspace	10%
	A3 - IK	15%
	B1 - FK test	5%
	B2 - IK test + animation	10%
	B3 - trajectories	10%
Part II	1 - IK	15%
	2 - workspace	15%
Part III		10.0%
total		100%

However, for **each** of the question you should keep in mind that your report will be marked based on analysis of your results, sufficient evidence and references that support your claims and clarity and relevance of your discussion, as shown below:

Element	Mark
Results (calculations, simulation)	45%
Analysis & Discussion	40%
Running code	10%
Quality of report/clarity/presentation	5%
Total	**100%**

Appendix

Calculations Flowchart:

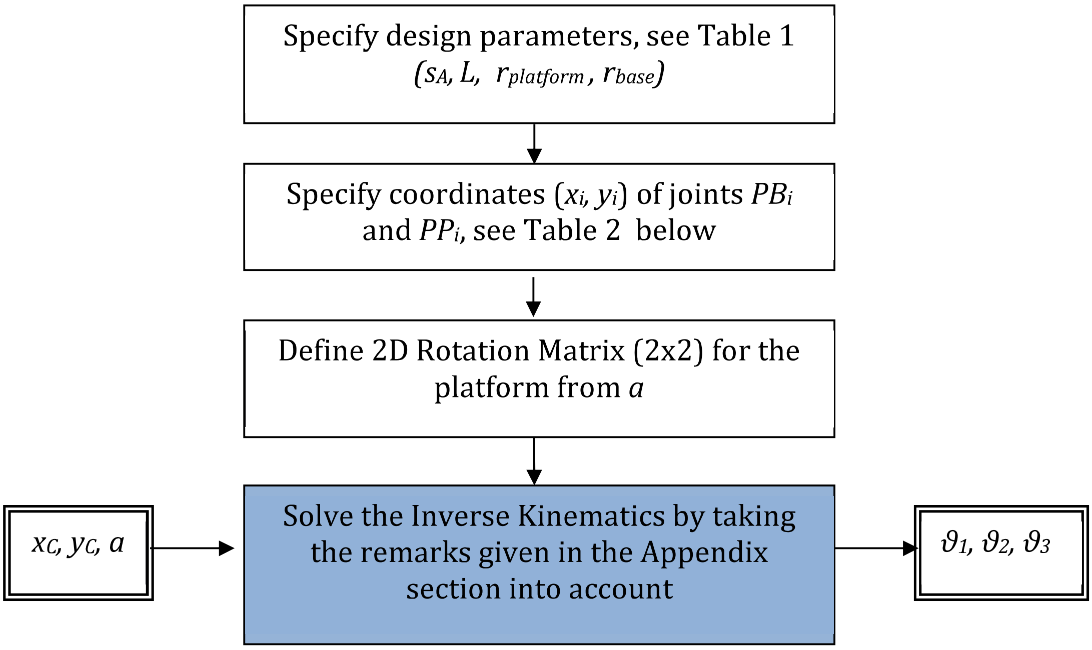
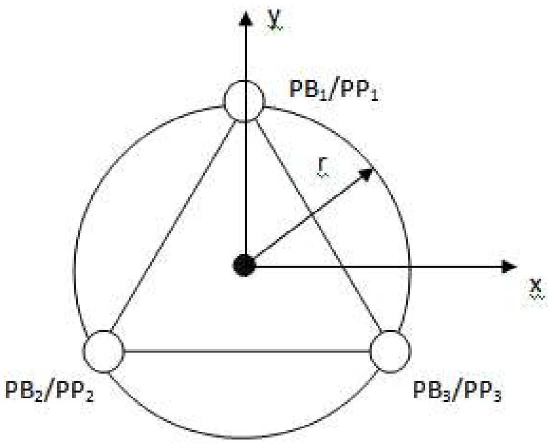


Table 1 Design parameters of planar parallel robot

Parameter – Data Geometry	Value
s_A	170mm
L	130mm
$r_{platform}$ *(joint circle radius)*	130mm
r_{base} *(joint circle radius)*	290mm

Table 2 Angular joint positions

PB_1	pi/2
PB_2	pi+pi/6
PB_3	2pi-pi/6
PP_1	pi/2
PP_2	pi+pi/6
PP_3	2pi-pi/6



From Figure 2, the loop closure equation of one vector chain is:

$$\overrightarrow{BPP} = \overrightarrow{BPB} + \overrightarrow{PBM} + \overrightarrow{MPP} \qquad \textbf{(B.1)}$$

The known parameters are $X_c\ , Y_c, \alpha$, which are the position and rotation of frame $\{C\}$ with regards to frame $\{B\}$ (world frame).

How to calculate a relationship between the base and the platform points for each leg

For leg *i* the coordinated for point PP_i, w.r.t to frame {PB} are:

$$x_{PP_i} = r_{s_A} c_\theta + L c_\psi \qquad \textbf{(B.2a)}$$

$$y_{PP\_i} = r_{s_A} s_\theta + L s_\psi \qquad \textbf{(B.2b)}$$

Using the above two equations, $(\mathrm{B.2a})^2 + (\mathrm{B.2b})^2 \xrightarrow{yields}$

$$x_{PP_i}^2 + y_{PP_i}^2 - 2r_{s_A}\left(x_{PP_i} c_\theta + y_{PP_i} s_\theta\right) + r_{S_A}^2 - L^2 = 0 \qquad \textbf{(B.3)}$$

To reduce the complexity of equation (B.3), we define three intermediate parameters:

$$e_1 = -2 y_{PP_i} r_{s_A} \qquad \textbf{(B.4a)}$$

$$e_2 = -2 x_{PP_i} r_{s_A} \qquad \textbf{(B.4b)}$$

$$e_3 = x_{PP_i}^2 + y_{PP_i}^2 + r_{S_A}^2 - L^2 \qquad \textbf{(B.4c)}$$

Using the equations (B.4a,b,c), (B.3) becomes:

$$e_1 s_\theta + e_2 c_\theta + e_3 = 0 \qquad \textbf{(B.5)}$$

If we define a parameter t as t= $tan\left(\frac{\theta}{2}\right)$, then:

$$s_\theta = \frac{2t}{1+t^2} \qquad \text{and} \qquad c_\theta = \frac{1-t^2}{1+t^2}$$

And replacing in (B.5), we get the following quadratic equation:

$$(e_3 - e_2)t^2 + 2e_1 t + e_2 + e_3 = 0$$

The solutions of the equation are:

$t_{1,2} = \dfrac{-e_1 \pm \sqrt{e_1^2 + e_2^2 - e_3^2}}{e_3 - e_2}$	$\theta = 2atan(t_{1,2})$ (two solutions)

How to calculate the coordinates of the base and platform points w.r.t the world reference frame.

We need this to use in the vector relationship described in the previous step:

$$\overrightarrow{PB_iPP_i} = \begin{bmatrix} x_{PP_i} \\ y_{PP_i} \\ 0 \end{bmatrix} = \overrightarrow{BPP_i} - \overrightarrow{BPB_i}$$

For the platform we know the transformation matrix of the centre C w.r.t to frame $\{B\}$:

$$T_{BC} = \begin{bmatrix} R_{BC} & \overrightarrow{BC} \\ 0_{3x3} & 1 \end{bmatrix} = \begin{matrix} & Rot(z, \alpha) & & X_c \\ & & & Y_c \\ & & & 0 \\ 0 & 0 & 0 & 1 \end{matrix}$$

Also,

$$\overrightarrow{BPP_i} = R_{BC}\overrightarrow{CPP_i} + \overrightarrow{BC},$$

where $\overrightarrow{CPP_i} = \begin{bmatrix} -r_{plat}\cos(\frac{\pi}{6}) \\ -r_{plat}\sin(\frac{\pi}{6}) \\ 0 \end{bmatrix}$ from the triangle.

In a similar way you can calculate $\overrightarrow{BPB_i}$.

The other two leg coordinates can be calculated using the same procedure. Having obtained these coordinates, the position of the M_i joints can easily be obtained.