UFU 45 ANOS

Transformada de Laplace

Sistemas de Controle

Leonardo Vecchi Meirelles

12011ECP002

Setembro 2023

1) a) $F(s) = \int_0^{\infty} e^{-st}\,dt = -\frac{1}{s} e^{-st} \Big|_0^{\infty} = \frac{1}{s}$

b) $F(s) = \int_0^{\infty} t e^{-st}\,dt = \frac{e^{-st}}{s^2}(-st-1) \Big|_0^{\infty} = \frac{-(st+1)}{s^2 e^{st}} \Big|_0^{\infty} = \frac{1}{s^2}$

c) $F(s) = \int_0^{\infty} \operatorname{sen} wt\, e^{-st}\,dt = \frac{e^{-st}}{s^2+w^2}(-s \operatorname{sen} wt - w \cos wt) \Big|_0^{\infty}$

$= \frac{w}{s^2+w^2}$

d) $F(s) = \int_0^{\infty} \cos wt\, e^{-st}\,dt = \frac{e^{-st}}{s^2+w^2}(-s \cos wt + w \operatorname{sen} wt) \Big|_0^{\infty}$

$= \frac{s}{s^2+w^2}$

2) $F(s) = \dfrac{w}{(s+a)^2 + w^2}$

a)

b) $F(s) = \dfrac{(s+a)}{(s+a)^2 + w^2}$

c) $\int dt = t \; ; \; \int t\,dt = \dfrac{t^2}{2} \; ; \; \int \dfrac{t^2}{2} dt = \dfrac{t^3}{6} \; ;$

$\therefore F(s) = \dfrac{6}{s^4}$

3) a) $(s+7)X(s) = \dfrac{5s}{s^2+2^2}$

$$\frac{5s}{(s+7)(s^2+4)} = \frac{-35}{53}\cdot\frac{1}{s+7} + \frac{5}{53}\cdot\frac{7s+4}{s^2+4}$$

$$\frac{5s}{(s+7)(s^2+4)} = \frac{-35}{53}\frac{1}{s+7} + \frac{5}{53}\frac{7s+2\sqrt{4}}{s^2+4}$$

$$\therefore x(t) = \frac{-35}{53}e^{-7t} + \left(\frac{35}{53}\cos 2t + \frac{10}{53}\operatorname{sen} 2t\right)$$

4) $s^2X(s) - 4s + 4 + 2sX(s) - 8 + 2X(s) = \dfrac{2}{s^2+2^2}$

$$X(s) = \frac{4s^3+4s^2+16s+18}{(s^2+4)(s^2+2s+2)}$$

$$X(s) = -\left(\frac{1}{5}\right)\frac{s+\frac{1}{2}2}{s^2+2^2} + \left(\frac{1}{5}\right)\frac{21(s+1)+2}{(s+1)^2+1}$$

$$\therefore x(t) = \frac{1}{5}\left[21e^{-t}\cos t + \frac{2}{21}e^{-t}\operatorname{sen} t - \frac{1}{2}\operatorname{sen} 2t - \cos 2t\right]$$

Ex 5)

```
syms t
%% letra a)
theta=45*pi/180;
f = 8*t^2*cos(3*t+theta);
pretty(f);
F = laplace(f);
F = simplify(F);
pretty(F);

%% letra b)
theta = 60*pi/180;
f = 3*t*exp(-2*t)*sin(4*t+theta);
pretty(f);
F = laplace(f);
F = simplify(F);
pretty(F);
```

```
>> Ex5Nise
 2    /        pi \
t  cos| 3 t + -- | 8
      \        4 /

                  3        2
  sqrt(2) (- 8 s   + 72 s   + 216 s - 216)
- --------------------------------------
                     2     3
                   (s   + 9)

     /        pi \
t sin| 4 t + -- | exp(-2 t) 3
     \        3 /

                                           2
(8 s + 4 sqrt(3) s - 12 sqrt(3) + sqrt(3) s   + 16) 3
----------------------------------------------------
                     2            2
                 2 (s   + 4 s + 20)

>>
```

Ex 6)

```
syms s
%% letra a)
G = (s^2+3*s+10)*(s+5)/((s+3)*(s+4)*(s^2+2*s+100));
pretty(G);
g = ilaplace(G);
pretty(g);

%% letra b)
G = (s^3+4*s^2+2*s+6)/((s+8)*(s^2+8*s+3)*(s^2+5*s+7));
pretty(G);
g = ilaplace(G);
pretty(g);
```

```
>> Ex6Nise
                   2
     (s + 5) (s  + 3 s + 10)
--------------------------------
                      2
(s + 3) (s + 4) (s  + 2 s + 100)

                                            /                        sqrt(11) sin(3 sqrt(11) t) \
                                    exp(-t) | cos(3 sqrt(11) t) - -------------------------- | 5203
exp(-3 t) 20   exp(-4 t) 7                  \                                57233            /
------------ - ----------- + ---------------------------------------------------------------
    103            54                                          5562

          3      2
         s  + 4 s  + 2 s + 6
-------------------------------------
          2               2
(s + 8) (s  + 8 s + 3) (s  + 5 s + 7)

          /                      4262 sqrt(13) sinh(sqrt(13) t) \
exp(-4 t) | cosh(sqrt(13) t) - ------------------------------ | 1199
          \                              15587                 /
--------------------------------------------------------------------
                                  417

                  /                              / sqrt(3) t \      \
                  |                    sqrt(3) sin| --------- | 131 |
        /  5 t \ |    / sqrt(3) t \             \     2     /      |
     exp| - --- | | cos| --------- | + ---------------------------- | 65
        \   2  / \    \     2     /                15                /       exp(-8 t) 266
   - ------------------------------------------------------------------- - -------------
                                      4309                                          93

>>
```

7) $(s^3+3s^2+5s+1)Y(s)=(s^3+4s^2+6s+8)X(s)$

$$\frac{Y(s)}{X(s)}=\frac{s^3+4s^2+6s+8}{s^3+3s^2+5s+1}$$

8) a) $(s^2+5s+10)X(s)=7F(s)$

$$\mathcal{L}^{-1}\{\ \} \Rightarrow \frac{d^2x}{dt^2}+5\frac{dx}{dt}+10x=7f$$

9) $$\frac{C(s)}{R(s)}=\frac{s^5+2s^4+4s^3+s^2+4}{s^6+7s^5+3s^4+2s^3+s^2+5}$$

$\Rightarrow (s^6+7s^5+3s^4+2s^3+s^2+5)C(s)=(s^5+2s^4+4s^3+s^2+4)R(s)$

$$\Rightarrow \frac{d^6c}{dt^6}+7\frac{d^5c}{dt^5}+3\frac{d^4c}{dt^4}+2\frac{d^3c}{dt^3}+\frac{d^2c}{dt^2}+5c=\frac{d^5r}{dt^5}+2\frac{d^4r}{dt^4}+4\frac{d^3r}{dt^3}+\frac{d^2r}{dt^2}+4r$$

10) $\dfrac{C(s)}{R(s)} = \dfrac{s^4+2s^3+5s^2+s+1}{s^5+3s^4+2s^3+4s^2+5s+2}$

$$\Rightarrow \underbrace{\frac{d^5c}{dt^5}+3\frac{d^4c}{dt^4}+2\frac{d^3c}{dt^3}+4\frac{d^2c}{dt^2}+5\frac{dc}{dt}+2c}_{} = \frac{d^4r}{dt^4}+2\frac{d^3r}{dt^3}+5\frac{d^2r}{dt^2}+\frac{dr}{dt}+r$$

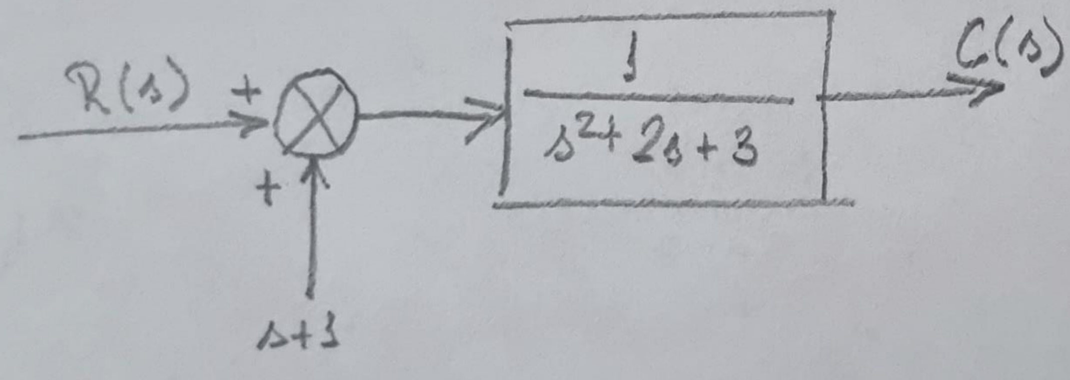
Substituindo $r(t)=t^3$ $*$

$\Rightarrow * = 18\,\delta(t)+(36+90t+9t^2+3t^3)u(t)$

11) $s^2X(s)-s+1+2sX(s)-2+3X(s)=R(s)$

$\Rightarrow (s^2+2s+3)X(s) = R(s)+s+1$

$\Rightarrow X(s) = \dfrac{R(s)}{s^2+2s+3} + \dfrac{s+1}{s^2+2s+3}$



Exercício 15)

```
syms s
F = (((10^4) * (s + 5) * (s + 70)) / (s * (s + 45) * (s + 55) * (s^2 + 7*s + 110) * (s^2 + 6*s + 95)));

% Simplify the expression
simplified_F = simplify(F);
disp(simplified_F);

[numerator, denominator] = numden(F);
[nums, dens] = numden(simplifyFraction(numerator / denominator));
[r, p, k] = residue(sym2poly(nums), sym2poly(dens));

disp('Residues:');
disp(r);

disp('Poles:');
disp(p);

disp('Direct Terms:');
disp(k);
```

```
>> Ex15Nise
((10000*s + 50000)*(s + 70))/(s*(s + 45)*(s + 55)*(s^2 + 6*s + 95)*(s^2 + 7*s + 110))

Residues:
  -0.0018 + 0.0000i
   0.0066 + 0.0000i
   0.9513 + 0.0896i
   0.9513 - 0.0896i
  -1.0213 - 0.1349i
  -1.0213 + 0.1349i
   0.1353 + 0.0000i

Poles:
 -55.0000 + 0.0000i
 -45.0000 + 0.0000i
  -3.5000 + 9.8869i
  -3.5000 - 9.8869i
  -3.0000 + 9.2736i
  -3.0000 - 9.2736i
   0.0000 + 0.0000i

Direct Terms:
>>
```

Interpretando os resultados, temos a seguinte expressão parcial:

$$F(s) = \frac{10000\,s}{(s\,+\,45)(s\,+\,55)(s^2+\,6\,s\,+\,95)(s^2+\,7\,s\,+\,110)} + \frac{750000}{(s\,+\,45)(s\,+\,55)(s^2+\,6\,s\,+\,95)(s^2+\,7\,s\,+\,110)} + \frac{3500000}{(s\,+\,45)(s\,+\,55)(s^2+\,6\,s\,+\,95)(s^2+\,7\,s\,+\,110)s}$$