



UFU 45 ANOS

Prova 1

Sistemas de Controle

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Questão 1)

Letra a)

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$$1) a) f(t) = 3 + 7t + t^2 + \delta(t)$$

$$\mathcal{L}\{f(t)\} = \mathcal{L}\{3 + 7t + t^2 + \delta(t)\} = 3 \cdot \frac{1}{s} + 7 \cdot \frac{1}{s^2} + \frac{2}{s^3} + 1$$

$$\therefore F(s) = \frac{3}{s} + \frac{7}{s^2} + \frac{2}{s^3} + 1 = \frac{s^3 + 3s^2 + 7s + 2}{s^3}$$

Letra b)

$$b) f(t) = t \cos 3t = -(-t) \cos 3t$$

$$\mathcal{L}\{f(t)\} = \mathcal{L}\{-(-t) \cos 3t\} = -\frac{d}{ds} \left(\frac{s}{s^2 + 9} \right) = -\left(\frac{(s^2 + 9) - s \cdot 2s}{(s^2 + 3^2)^2} \right)$$

$$\therefore F(s) = \frac{s^2 - 9}{(s^2 + 9)^2}$$

Letra c)

$$c) F(s) = \frac{1}{s(s+2)^2} = \frac{A}{s} + \frac{B}{s+2} + \frac{C}{(s+2)^2} = \frac{A(s+2)^2 + Bs(s+2) + Cs}{s(s+2)^2}$$

$$1 = A(s+2)^2 + Bs(s+2) + Cs$$

$$\bullet s \rightarrow 0: 1 = 4A \Rightarrow A = \frac{1}{4}$$

$$\bullet s \rightarrow -2: 1 = -2C \Rightarrow C = -\frac{1}{2}$$

$$\bullet s \rightarrow -1: 1 = A - B - C \Rightarrow \frac{1}{4} - B + \frac{1}{2} = 1 \Rightarrow B = -\frac{1}{4}$$

$$F(s) = \frac{1}{4s} - \frac{1}{4(s+2)} - \frac{1}{2(s+2)^2}$$

$$\mathcal{L}^{-1}\{F(s)\} = \frac{1}{4} \cdot 1 - \frac{1}{4} \cdot e^{-2t} - \frac{1}{2} \cdot e^{-2t} \cdot t$$

$$\therefore f(t) = \frac{e^{-2t}}{4} \cdot (-2t + e^{2t} - 1)$$

Letra d)

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$$d) F(s) = \frac{3s+2}{s(s+1)(s^2+4s+10)} = \frac{A}{s} + \frac{B}{s+1} + \frac{Cs+D}{s^2+4s+10}$$

$$3s+2 = A(s+1)(s^2+4s+10) + Bs(s^2+4s+10) + s(Cs+D)(s+1)$$

$$\bullet s \rightarrow 0: A = 1/5$$

$$\bullet s \rightarrow -1: B = 1/7$$

$$\therefore 3s+2 = \frac{1}{5}(s+1)(s^2+4s+10) + \frac{1}{7}s(s^2+4s+10) + (Cs+D)s(s+1)$$

$$\Rightarrow 3s+2 = s^3\left(C + \frac{12}{35}\right) + s^2\left(D + C + \frac{11}{7}\right) + s\left(D + \frac{148}{35}\right) + 2$$

$$\begin{cases} D + 148/35 = 3 \\ D + C + 11/7 = 0 \\ C + 12/35 = 0 \end{cases} \xrightarrow[\text{Resolvendo o sistema}]{} C = -12/35; D = -43/35$$

$$\therefore F(s) = \frac{1/5}{s} + \frac{1/7}{s+1} + \frac{(-12/35)s - 43/35}{s^2+4s+10} = \frac{1}{5s} + \frac{1}{7(s+1)} + \frac{-12s-43}{35(s^2+4s+10)}$$

$$= \frac{1}{5s} + \frac{1}{7(s+1)} - \frac{12}{35} \left(\frac{s+2}{(s+2)^2+6} \right) - \frac{19}{35} \left(\frac{1}{(s+2)^2+6} \right)$$

$$\therefore \mathcal{L}^{-1}\{F(s)\} = \mathcal{L}^{-1}\left\{\frac{1}{5s}\right\} + \mathcal{L}^{-1}\left\{\frac{1}{7(s+1)}\right\} - \frac{12}{35} \mathcal{L}^{-1}\left\{\frac{s+2}{(s+2)^2+6}\right\} - \frac{19}{35} \mathcal{L}^{-1}\left\{\frac{1}{(s+2)^2+6}\right\}$$

$$\therefore f(t) = \frac{1}{5} + \frac{1}{7}e^{-t} - \frac{12}{35}e^{-2t}\cos(\sqrt{6}\cdot t) - \frac{19}{35}e^{-2t}\frac{1}{\sqrt{6}}\sin(\sqrt{6}\cdot t)$$

$$f(t) = \frac{1}{210} \cdot e^{2t} (30e^t + 42e^{2t} - 19\sqrt{6}\sin(\sqrt{6}\cdot t) - 72\cos(\sqrt{6}\cdot t))$$

Questão 2)

Letra a)

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2) a) O sistema possui três graus de liberdade. Para M_1 , temos:

$$\left[\begin{array}{c} \text{Soma das impedâncias} \\ \text{conectadas ao movimento} \\ \text{em } X_1 \end{array} \right] X_1(s) - \left[\begin{array}{c} \text{Soma das impedâncias} \\ \text{entre } X_1 \text{ e } X_2 \end{array} \right] X_2(s)$$

$$- \left[\begin{array}{c} \text{Soma das impedâncias} \\ \text{entre } X_1 \text{ e } X_3 \end{array} \right] X_3(s) = \left[\begin{array}{c} \text{Soma das forças} \\ \text{aplicadas em } X_1 \end{array} \right]$$

Segue-se analogamente para M_2 e M_3 e nos próximos exercícios.

Logo, tem-se:

M_1 :

$$[M_1 s^2 + D_1 s + K_1] X_1(s) - [D_1 s] X_2(s) - [0] X_3(s) = 0$$

$$\Rightarrow (4s^2 + 2s + 6) X_1(s) - 2s X_2(s) = 0 \quad (\text{I})$$

M_2 :

$$[M_2 s^2 + (D_1 + D_2)s + K_2] X_2(s) - [D_1 s] X_1(s) - [K_2] X_3(s) = F(s)$$

$$\Rightarrow (4s^2 + 4s + 6) X_2(s) - 2s X_1(s) - 6 X_3(s) = F(s) \quad (\text{II})$$

M_3 :

$$[M_3 s^2 + D_3 s + K_2] X_3(s) - [K_2] X_2(s) - [0] X_1(s) = 0$$

$$\Rightarrow (4s^2 + 2s + 6) X_3(s) - 6 X_2(s) = 0 \quad (\text{III})$$

Em (I):

$$X_2(s) = \frac{4s^2 + 2s + 6}{2s} X_1(s) \quad (\text{IV})$$

Em (III):

$$X_2(s) = \frac{(4s^2 + 2s + 6) X_3(s)}{6} \quad (\text{V})$$

Por (IV) e (V), igualando:

$$\frac{(4s^2 + 2s + 6)X_1(s)}{2s} = \frac{(4s^2 + 2s + 6)X_3(s)}{6}$$

$$X_3(s) = \frac{6X_1(s)}{2s} \text{ (VI) e analogamente: } X_1(s) = \frac{2sX_3(s)}{6} \text{ (VII)}$$

Finalmente, substituindo (IV) e (VI) em (II):

$$(4s^2 + 4s + 6) \cdot \frac{(4s^2 + 2s + 6)X_1(s)}{2s} - 2sX_1(s) - 6 \cdot \frac{6X_1(s)}{2s} = F(s)$$

$$X_1(s) \left(\frac{(4s^2 + 4s + 6) \cdot (4s^2 + 2s + 6)}{2s} - 2s - \frac{36}{2s} \right) = F(s) \Rightarrow X_1(s) \cdot (8s^3 + 12s^2 + 26s + 18) = F(s)$$

$$\therefore \frac{X_1(s)}{F(s)} = \frac{1}{8s^3 + 12s^2 + 26s + 18}$$

A partir de (VII), tem-se:

$$\frac{2s \cdot X_3(s)}{6F(s)} = \frac{1}{8s^3 + 12s^2 + 26s + 18}$$

$$\therefore \frac{X_3(s)}{F(s)} = \frac{6}{2s \cdot (8s^3 + 12s^2 + 26s + 18)} = \frac{3}{8s^4 + 12s^3 + 26s^2 + 18s}$$

Letra b)

2) b) A partir dos passos descritos no item a), temos as equações de movimento:

Em θ_1 :

$$[Js^2 + Ds + K]\theta_1(s) - [Ds + K]\theta_2(s) = 0$$

$$\Rightarrow (5s^2 + 9s + 9)\theta_1(s) - (s + 9)\theta_2(s) = 0 \quad (I)$$

Em θ_2 :

$$(3s^2 + s + 12)\theta_2(s) - (s + 9)\theta_1(s) = T(s) \quad (II)$$

Isolando em (I):

$$\theta_2(s) = \frac{(5s^2 + 9s + 9)\theta_1(s)}{s + 9} \quad \text{Analogamente: } \theta_1(s) = \frac{(s + 9)\theta_2(s)}{5s^2 + 9s + 9} \quad (III)$$

Substituindo em (II)

$$\frac{(3s^2 + s + 12)(5s^2 + 9s + 9)\theta_1(s)}{s + 9} - (s + 9)\theta_1(s) = T(s)$$

$$\theta_1(s) \left(\frac{(3s^2 + s + 12)(5s^2 + 9s + 9)}{s + 9} - (s + 9) \right) = T(s) = \theta_1(s) \cdot \frac{15s^4 + 32s^3 + 95s^2 + 99s + 27}{s + 9}$$

$$\therefore \frac{\theta_1(s)}{T(s)} = \frac{s + 9}{15s^4 + 32s^3 + 95s^2 + 99s + 27}$$

Com (III), tem-se:

$$\frac{(s + 9)\theta_2(s)}{(5s^2 + 9s + 9)T(s)} = \frac{s + 9}{15s^4 + 32s^3 + 95s^2 + 99s + 27} \Rightarrow \frac{\theta_2(s)}{T(s)} = \frac{5s^2 + 9s + 9}{15s^4 + 32s^3 + 95s^2 + 99s + 27}$$

Letra c)

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2) c) Refletindo as impedâncias para θ_2 :

$$\theta_2(s) \{ [J_e] s^2 + [D_e] s + [K_e] \} = F(s)$$

$$\theta_2(s) \left\{ \left[150 + 3 \cdot \left(\frac{50}{5} \right)^2 + 100 \cdot \left(\frac{5}{25} \cdot \frac{50}{5} \right)^2 \right] s^2 + \left[500 \cdot \left(\frac{5}{25} \cdot \frac{50}{5} \right)^2 \right] s + \left[300 + 3 \cdot \left(\frac{50}{5} \right)^2 \right] \right\} = \left(\frac{50}{5} \right) T(s)$$

Portanto:

$$\theta_2(s) \cdot [850s^2 + 2000s + 600] = \left(\frac{50}{5} \right) T(s)$$

$$\therefore \frac{\theta_2(s)}{T(s)} = \frac{1}{85s^2 + 200s + 60}$$

Questão 3)

Letra a)

$$3) a) \frac{C(s)}{R(s)} = \frac{G(s)}{1 + G(s)H(s)} = \frac{\frac{\omega_n^2}{s(s+2\zeta\omega_n)}}{1 + \frac{\omega_n^2}{s(s+2\zeta\omega_n)}} = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$

Assim, sabendo que $\zeta = 0,8$ e $\omega_n = 25 \text{ rad/s}$, os parâmetros de desempenho são:

$$\omega_d = \omega_n \sqrt{1 - \zeta^2} = 25 \sqrt{1 - 0,8^2} = \underline{15 \text{ rad/s}}$$

$$\sigma_d = \zeta \omega_n = 0,8 \cdot 25 = \underline{20}$$

$$T_r \approx \frac{1,8}{\omega_n} = \frac{1,8}{25} = \underline{0,072 \text{ s}}$$

$$T_p = \frac{\pi}{\omega_d} = \frac{\pi}{20} = \underline{0,157 \text{ s}}$$

$$T_s = \frac{4}{\zeta \omega_n} = \frac{4}{\sigma_d} = \frac{4}{20} = \underline{0,2 \text{ s}}$$

$$M_p = e^{-\left(\frac{\pi \zeta}{\sqrt{1 - \zeta^2}}\right)} \cdot 100\% = e^{-\left(\frac{0,8\pi}{\sqrt{0,36}}\right)} \cdot 100\% = \underline{1,516\%}$$

$$\theta = \cos^{-1}(\zeta) = \cos^{-1}(0,8) \approx \underline{36,87^\circ}$$

Letra b)

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$$3) b) \sum F = m \cdot a(t)$$

$$F(s) - f_v \cdot sX(s) - KX(s) = m \cdot s^2 X(s)$$

$$F(s) = m \cdot s^2 X(s) + f_v \cdot sX(s) + KX(s) \Rightarrow \frac{F(s)}{m} = X(s) \left[s^2 + \frac{f_v}{m} s + \frac{K}{m} \right]$$

$$\frac{X(s)}{F(s)} = \frac{1/m}{s^2 + f_v/m \cdot s + K/m} \Rightarrow \frac{X(s)}{F(s)} = \frac{0,2}{s^2 + 0,4s + 4}$$

Com isso, os parâmetros de desempenho são:

$$\omega_n^2 = 4 \Rightarrow \omega_n = \sqrt{4} = 2 \text{ rad/s}$$

$$2 \cdot \zeta \cdot \omega_n = 0,4 \Rightarrow \zeta = \frac{0,4}{4} = 0,1$$

$$\sigma_d = \zeta \omega_n = 0,1 \cdot 2 = 0,2$$

$$\omega_d = \omega_n \sqrt{1 - \zeta^2} = 2 \cdot \sqrt{0,99} = 1,989 \text{ rad/s}$$

$$T_r \approx \frac{1,8}{\omega_n} = \frac{1,8}{2} = 0,9 \text{ s}$$

$$T_p = \frac{\pi}{\omega_d} = \frac{\pi}{1,989} = 1,579 \text{ s}$$

$$T_s = \frac{4}{\sigma_d} = \frac{4}{0,2} = 20 \text{ s}$$

$$M_p = e^{-\left(\frac{\pi \zeta}{\sqrt{1 - \zeta^2}}\right)} \cdot 100\% = e^{-\left(\frac{\pi \cdot 0,1}{\sqrt{0,99}}\right)} \cdot 100\% = 72,92\%$$

$$\theta = \cos^{-1} \zeta = \cos^{-1} 0,1 \approx 84,26^\circ$$

Letra c)

3) c) Em \mathcal{I} , considerando $\theta_1(s) = \theta_2(s)$, tem-se:

$$T(s) - \mathcal{I}s^2\theta_1(s) - \mathcal{D}s\theta_1(s) - K\theta_1(s) = 0$$

Substituindo valores:

$$\frac{T(s)}{2} = \theta_1(s) \left[s^2 + \frac{1}{2}s + \frac{1}{2} \right] \Rightarrow \frac{\theta_1(s)}{T(s)} = \frac{\theta_2(s)}{T(s)} = \frac{\frac{1}{2}}{s^2 + \frac{1}{2}s + \frac{1}{2}}$$

Dessa forma, os parâmetros de desempenho serão:

$$\omega_n^2 = \frac{1}{2} \Rightarrow \omega_n = 0,707 \text{ rad/s}$$

$$2. \zeta \omega_n = \frac{1}{2} \Rightarrow \zeta = \frac{1}{2} \cdot \frac{1}{2 \cdot 0,707} = 0,354$$

$$\sigma_d = \zeta \omega_n = 0,354 \cdot 0,707 = 0,25$$

$$\omega_d = \omega_n \sqrt{1 - \zeta^2} = 0,707 \sqrt{1 - 0,354^2} = 0,661 \text{ rad/s}$$

$$T_n = \frac{1,8}{\omega_n} = \frac{1,8}{0,707} = 2,546 \text{ s}$$

$$T_p = \frac{\pi}{\omega_d} = \frac{\pi}{0,661} = 4,753 \text{ s}$$

$$T_s = \frac{4}{\sigma_d} = \frac{4}{0,25} = 16 \text{ s}$$

$$M_p = e^{-\left(\frac{\pi \zeta}{\sqrt{1 - \zeta^2}}\right)} \cdot 100\% = e^{-\left(\frac{0,354 \pi}{\sqrt{0,935}}\right)} \cdot 100\% = 30,993\%$$

$$\theta = \cos^{-1} \zeta = \cos^{-1} 0,354 = 69,3^\circ$$

Questão 4)

Letra a)

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$$4) a) H(s) = \frac{G(s)}{1+G(s)H(s)} = \frac{\frac{450}{s^4+3s^3+10s^2+30s+150}}{1+\frac{450}{s^5+3s^4+10s^3+30s^2+150s}} = \frac{450s}{s^5+3s^4+10s^3+30s^2+150s+450}$$

Encontrando os polos:

$$s^5 + 3s^4 + 10s^3 + 30s^2 + 150s + 450 = 0 \Rightarrow (s+3)(s^4 + 10s^2 + 150) = 0$$

$$\bullet (s+3) = 0 \Rightarrow \underline{s_0 = -3}$$

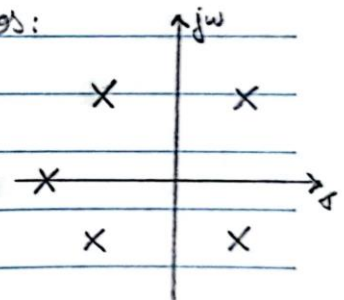
$$\bullet s^4 + 10s^2 + 150 = 0 \Rightarrow s^2 = X \Rightarrow X^2 + 10X + 150 = 0$$

$$\Delta = 10^2 - 4 \cdot 1 \cdot 150 = -500$$

$$X = \frac{-10 \pm \sqrt{-500}}{2} = \frac{-10 \pm j \cdot 10\sqrt{5}}{2} = -5 \pm j5\sqrt{5}$$

$$s^2 = X \Rightarrow s = \pm \sqrt{X} \Rightarrow \underline{s = \pm \sqrt{-5 \pm j5\sqrt{5}}}$$

Polos:



Dessa forma, são 3 polos presentes nos reais negativos

$(-3; -\sqrt{-5+j5\sqrt{5}}; -\sqrt{-5-j5\sqrt{5}})$ e 2 polos nos reais positivos $(\sqrt{-5+j5\sqrt{5}} \text{ e } \sqrt{-5-j5\sqrt{5}})$. Portanto, o sistema é instável por causa de seus polos com parte real positiva.

Letra b)

$$4) b) H(s) = \frac{G(s)}{1-G(s)H(s)} = \frac{\frac{18}{s^5+s^4-7s^3-7s^2-18s}}{1-\frac{18}{s^5+s^4-7s^3-7s^2-18s}} = \frac{18}{s^5+s^4-7s^3-7s^2-18s-18}$$

Encontrando os polos:

$$s^5 + s^4 - 7s^3 - 7s^2 - 18s - 18 = 0 \Rightarrow (s+1)(s^2+2)(s-3)(s+3)$$

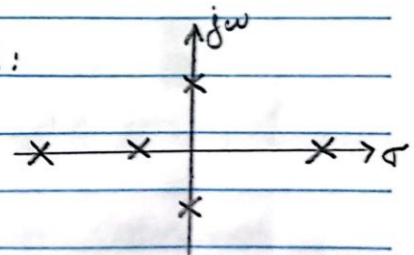
$$\bullet s+1=0 \Rightarrow \underline{s=-1}$$

$$\bullet s^2+2=0 \Rightarrow s = \pm \sqrt{-2} \Rightarrow \underline{s = \pm j\sqrt{2}}$$

$$\bullet s-3=0 \Rightarrow \underline{s=3}$$

$$\bullet s+3=0 \Rightarrow \underline{s=-3}$$

Polos:



Devido ao polo $s=3$, por estar no semi-plano dos reais positivos, o sistema é dito instável.

Questão 5)

Letra a)

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5) a) $T_s = 4s$; $T_p = 1s$; $f_v = 1,5$

- Modelando o sistema:

$$\sum F(t) = m \cdot a(t)$$

$$f(t) - f_v \dot{x}(t) - Kx(t) = m \cdot \ddot{x}(t)$$

$$\ddot{x}(t) + \frac{f_v}{m} \dot{x}(t) + \frac{K}{m} x(t) = \frac{f(t)}{m}$$

$$s^2 X(s) + \frac{f_v}{m} s X(s) + \frac{K}{m} X(s) = \frac{F(s)}{m}$$

$$X(s) \left[s^2 + \frac{f_v}{m} s + \frac{K}{m} \right] = \frac{F(s)}{m}$$

$$\therefore \frac{X(s)}{F(s)} = \frac{1/m}{s^2 + \frac{f_v}{m} s + \frac{K}{m}}$$

$$\bullet T_p = \frac{\pi}{\omega_d} \Rightarrow \omega_d = \frac{\pi}{T_p} \Rightarrow \underline{\omega_d = \pi}$$

$$\bullet T_s \approx \frac{4}{\sigma_d} \Rightarrow \sigma_d \approx \frac{4}{T_s} \Rightarrow \underline{\sigma_d \approx 1}$$

$$\bullet \sigma_d = \frac{f_v}{2m} = \frac{1,5}{2m} = \frac{0,75}{m} \Rightarrow m = \frac{0,75}{\sigma_d} = \underline{0,75 \text{ Kg}}$$

$$\bullet \sigma_d = \zeta \omega_n = \frac{f_v}{2m} \Rightarrow \zeta \omega_n = \frac{1,5}{2 \cdot 0,75} \Rightarrow \zeta = \frac{1}{\omega_n}$$

$$\bullet \omega_d = \omega_n \sqrt{1 - \zeta^2} \Rightarrow \pi = \omega_n \sqrt{1 - \left(\frac{1}{\omega_n}\right)^2} \Rightarrow \underline{\omega_n = \sqrt{1 + \pi^2}}$$

$$\bullet \zeta = \frac{1}{\omega_n} = \frac{1}{\sqrt{1 + \pi^2}} \approx \underline{0,303}$$

$$\bullet \theta = \cos^{-1} \zeta = \cos^{-1} 0,303 = \underline{72,36^\circ}$$

$$\bullet T_r \approx \frac{1,8}{\omega_n} = \frac{1,8}{\sqrt{1 + \pi^2}} = \underline{0,545s}$$

$$\bullet \omega_n^2 = \frac{K}{m} \Rightarrow \omega_n^2 m = K \Rightarrow K = 8,1522$$

$$\bullet M_p = e^{-\left(\frac{\pi \zeta}{\sqrt{1 - \zeta^2}}\right)} \cdot 100\% = e^{-\left(\frac{\pi \cdot 0,303}{\sqrt{1 - 0,303^2}}\right)} \cdot 100\% = \underline{36,83\%}$$

Respostas:

$$\omega_d = \pi$$

$$\sigma_d = 1$$

$$m = 0,75 \text{ Kg}$$

$$\omega_n = \sqrt{1 + \pi^2} \approx 3,29 \text{ rad/s}$$

$$\theta = 72,36^\circ$$

$$\zeta = 0,303$$

$$T_r = 0,545s$$

$$K = 8,1522 \text{ N/m}$$

$$M_p = 36,83\%$$

$$T_p = 1s$$

$$T_s = 4s$$

Letra b)

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5)b)

A modelagem será feita a partir da função:

$$\frac{X(s)}{F(s)} = \frac{10,75}{s^2 + 2s + 10,8696}$$

, em Python, fazendo uso das bibliotecas:

numpy, matplotlib e scipy.

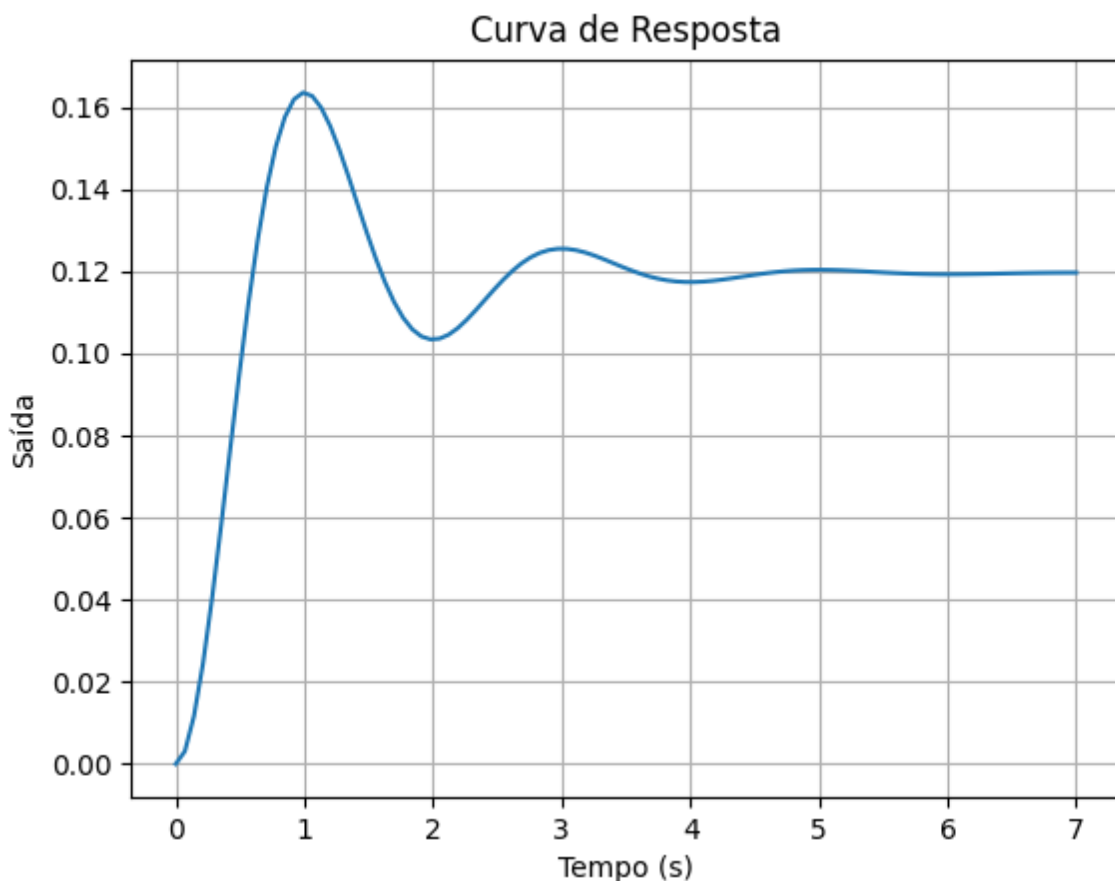
Código:

```
import numpy as np
import matplotlib.pyplot as plt
from scipy import signal

#Função de Transferência
num = [1.3]
den = [1, 2, 10.8696]
system = signal.TransferFunction(num, den)

#Função de entrada
t, y = signal.step(system)

#Plotar resposta
plt.plot(t, y)
plt.title('Curva de Resposta')
plt.xlabel('Tempo (s)')
plt.ylabel('Saída')
plt.grid(True)
plt.show()
```



Questão 6)

Letra a)

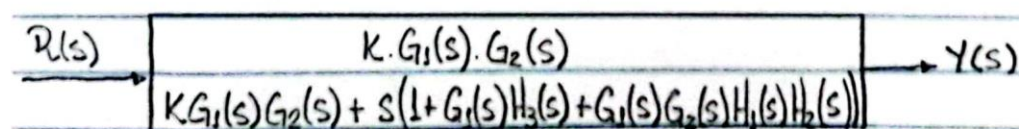
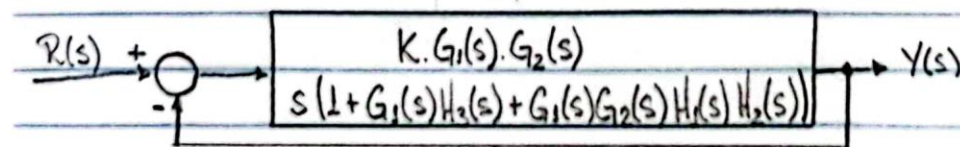
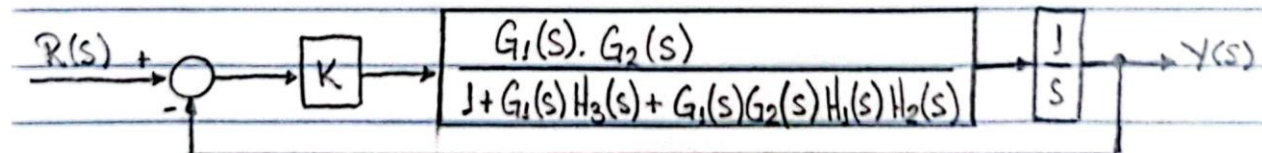
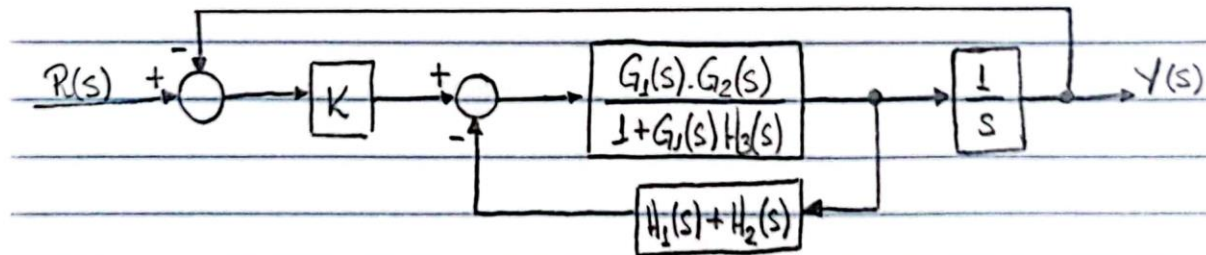
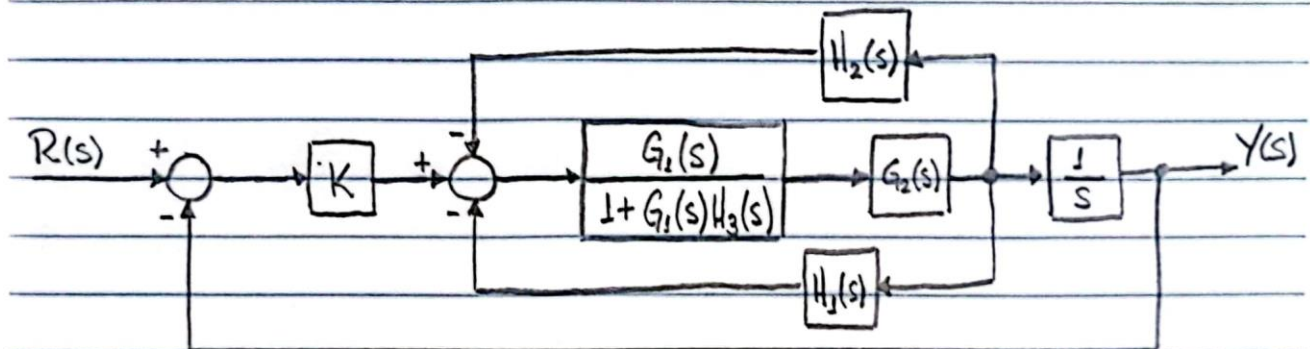
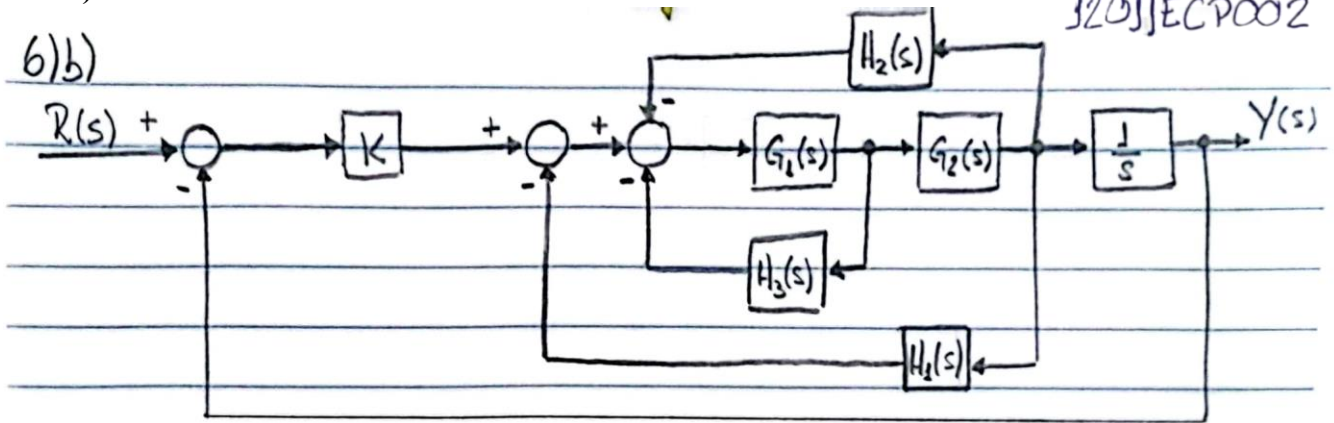
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6) a) No loop interno, $G_e(s) = \frac{1}{s+12} \cdot \frac{20}{s} = \frac{20}{s(s+12)}$, e $H_e(s) = 0,2s$.
Portanto, $T_e(s) = \frac{G_e(s)}{1 + G_e(s)H_e(s)} = \frac{20}{s(s+16)}$. Combinando com a função

de transferência equivalente do por paralelo, $G_p(s) = 20$, o sistema é
reduzido a um sistema de feedback unitário equivalente com $G(s) = G_p(s)T_e(s)$
 $= 20 \cdot \frac{20}{s(s+16)} = \frac{400}{s(s+16)}$. Portanto, $T(s) = \frac{C(s)}{R(s)} = \frac{G(s)}{1 + G(s)} = \frac{400}{s^2 + 16s + 400}$.

Letra b)

6)b)



Logo, a Função de Transferência $H(s) = \frac{Y(s)}{R(s)}$ é dada por:

$$H(s) = \frac{K \cdot G_1(s) \cdot G_2(s)}{K G_1(s) G_2(s) + s(1 + G_1(s) H_3(s) + G_1(s) G_2(s) H_1(s) H_2(s))}$$