UFU 45 ANOS

Roteiro 7 - Sistemas de 1ª e 2ª ordem

Sistemas de Controle

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12011ECP002

Novembro 2023

2) $C(s) = R(s) \cdot G(s) = \frac{1}{s} \cdot \frac{5}{s+5}$

Exponsão em Froções Porciais:

$$C(s) = \frac{A}{s} + \frac{B}{s+5}$$

$$\cancel{s} \cdot \frac{1}{\cancel{s}} \cdot \frac{5}{s+5} = \frac{A}{\cancel{s}} \cdot \cancel{s} + \frac{Bs}{s+5} \xRightarrow{s=0} A + 0 = \frac{5}{5} = \underline{1}$$

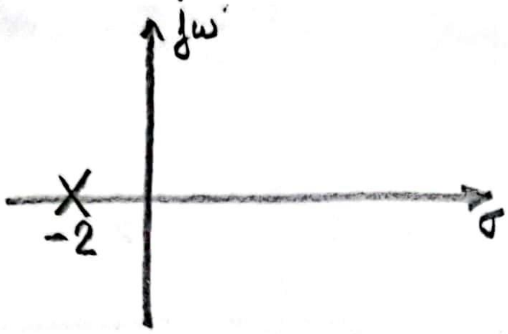
$$\frac{1}{s} \cdot \frac{5}{\cancel{s+5}} \cdot \cancel{(s+5)} = \frac{A}{s}(s+5) + \frac{B\cancel{(s+5)}}{\cancel{s+5}} \xRightarrow{s=-5} B = \frac{5}{-5} = \underline{-1}$$

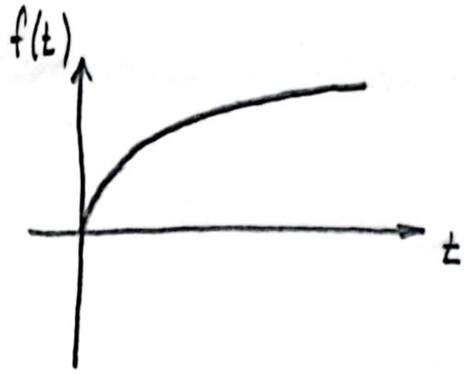
$$C(s) = \frac{1}{s} - \frac{1}{s+5}$$

Aplicando a Inversa:

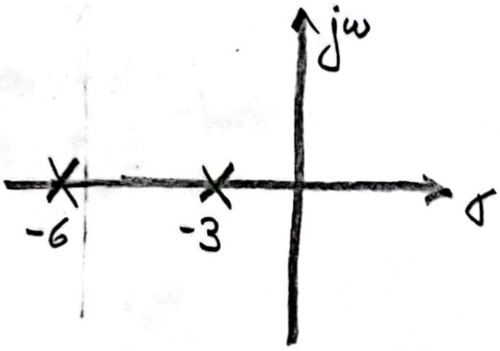
$$\boxed{c(t) = u(t) - \frac{1}{5} \cdot e^{-5t}}$$

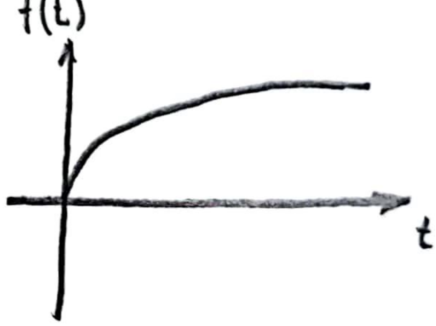
8) a)



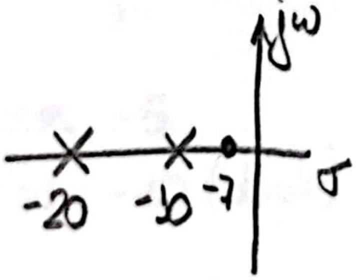


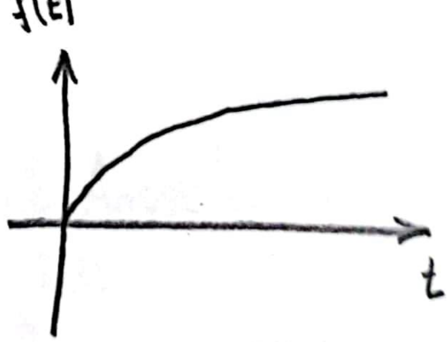
b) Dois polos reais - Superamortecido



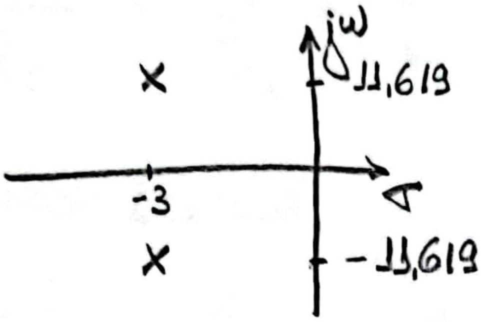


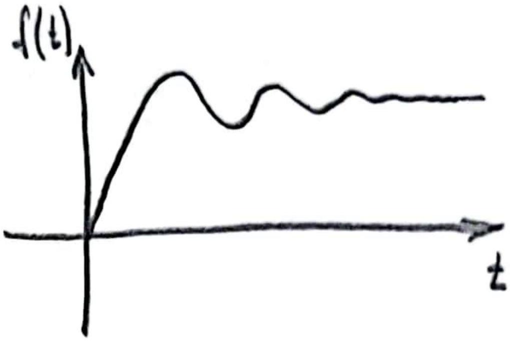
c) Zeros: -7 - Superamortecido
Polos: -10 e -20



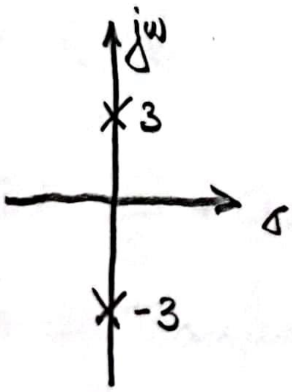


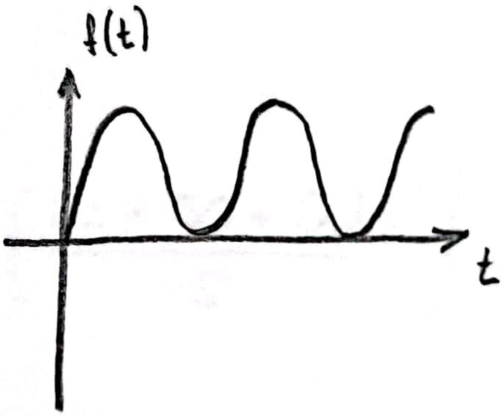
d) Polos: $-3 \pm j11.619$ (Complexo Conjugado) - Subamortecido



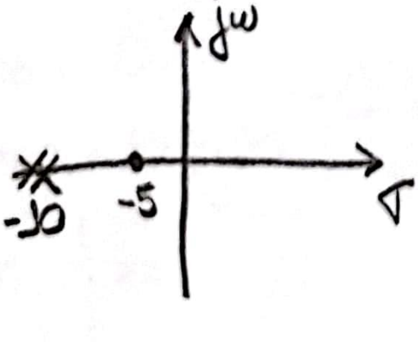


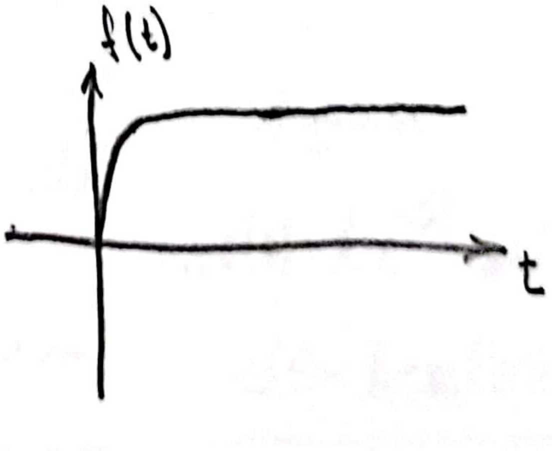
e) Zeros: -2
Polos: $\pm j3$ - Não Amortecido





f) Zeros: -5
Polos: -10 (duplo) - Criticamente Amortecido





12) LKC em V_1:

$$\frac{(V_1 - V)}{R_1} + \frac{V_1}{R_2} + \frac{1}{L}\int V_1 \, d\tau + C \cdot \frac{dv_1}{dt} = 0$$

Aplicando Laplace:

$$\frac{(V_1 - V)}{10^4} + \frac{V_1}{10^4} + \frac{V_1}{200s} + 10\mu s V_1 = 0$$

$$V_1 \left[\frac{2}{10K} + \frac{1}{200s} + 10\mu s \right] = \frac{V}{10K}$$

- Como $V_1 = V_0$

$$V = V_0 \left[2 + \frac{50}{s} + 0{,}1s \right] = V_0 \left[\frac{2s + 50 + 0{,}1s^2}{s} \right]$$

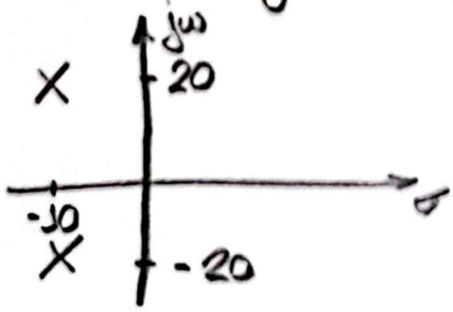
Função de Transferência

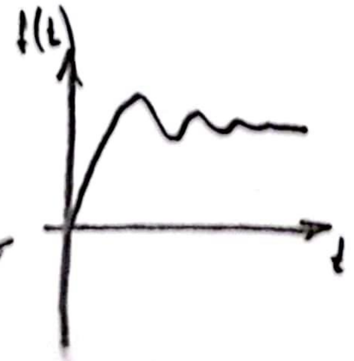
$$\frac{V_0}{V} = \frac{s}{2s + 50 + 0{,}1s^2}$$

Polos e Zeros:

Zero: 0

Polos: $-10 \pm j20$ - Subamortecido





$$\therefore c(t) = A \cdot e^{-0t} \cdot \cos(20t - \phi)$$

$$c(t) = A\cos(20t - \phi)$$

19)

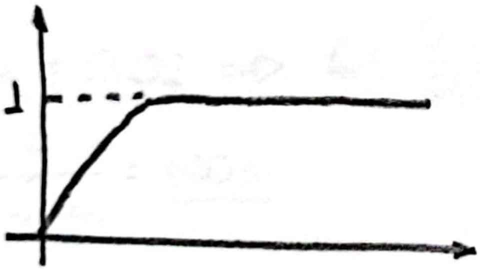
$$c(t) = 1 - \frac{1}{\sqrt{1-0{,}0225}} \cdot e^{-0{,}15 \cdot 20t} \cdot \cos\left(\omega_n \sqrt{1-0{,}0225t} - \phi\right)$$

$$c(t) = 1 - 1{,}0114\, e^{-3t} \cdot \cos\left(20\sqrt{1-0{,}0225t} - 0{,}15\right)$$

$\hookrightarrow *$

$* \ \phi = \tan^{-1}\left(\zeta / \sqrt{1-\zeta^2}\right)$

$\therefore \phi = \tan^{-1}\left(\dfrac{0{,}15}{\sqrt{1-0{,}15^2}}\right) = \underline{0{,}151}$



$\omega_n = 20$ rad/s

$\zeta = 0{,}15$

20) a) $\omega_n = \sqrt{16} = 4$

$2\zeta\omega_n = 3 \Rightarrow \zeta = \dfrac{3}{2\omega_n} = \dfrac{3}{8} = 0{,}375$

$T_s = \dfrac{4}{\zeta\omega_n} = \dfrac{8}{3}s$

$T_p = \dfrac{\pi}{\omega_n\sqrt{1-\zeta^2}} = 0{,}847s$

$\%OS = e^{-(\zeta\pi/\sqrt{1-\zeta^2})}\cdot 100 = e^{-1{,}271}\cdot 100 = 28{,}06\%$

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b) $\omega_n = \sqrt{0{,}04} = 0{,}2$

$2\zeta\omega_n = 0{,}02 \Rightarrow \zeta = \dfrac{0{,}01}{0{,}2} = 0{,}05$

$T_s = \dfrac{4}{\zeta\omega_n} = 400s$

$T_p = \dfrac{\pi}{\omega_n\sqrt{1-\zeta^2}} = 15{,}73s$

$\%OS = e^{-(\zeta\cdot\pi/\sqrt{1-\zeta^2})}\cdot 100 = e^{-0{,}157}\cdot 100 = 85{,}47\%$

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c) $\omega_n = \sqrt{1{,}05\cdot 10^7} = 3240{,}37$

$2\zeta\omega_n = 1600 \Rightarrow \zeta = \dfrac{1600}{2\cdot 3240{,}37} = 0{,}2469$

$T_s = \dfrac{4}{\zeta\omega_n} = 4{,}99\cdot 10^{-3}s$

$T_p = \dfrac{\pi}{\omega_n\sqrt{1-\zeta^2}} = 10^{-3}s$

$\%OS = e^{-(\zeta\pi/\sqrt{1-\zeta^2})}\cdot 100 = e^{-0.8}\cdot 100 = 44{,}93\%$

23) a) $\zeta = \dfrac{-\ln(M_p/100)}{\sqrt{\pi^2 + \ln^2(M_p/100)}} = \dfrac{2,12}{\sqrt{\pi^2 + 4,49}} = \underline{0,5593}$

$T_s = \dfrac{-\ln(0,02 \cdot \sqrt{1-\zeta^2})}{\zeta \omega_n} \cong \dfrac{4}{\zeta \omega_n} \Rightarrow \omega_n = \dfrac{4}{\zeta . 0,6} = \underline{11,9197}$

$F(s) = \dfrac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2} = \dfrac{142,08}{s^2 + 13,33s + 142,08}$

Determinando os polos:

$s_{1,2} = -\zeta\omega_n \pm \omega_n\sqrt{\zeta^2 - 1}$

$\underline{s_{1,2} = -6,667 \pm j\,9,881}$

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b) As formulas utilizadas são os mesmas dos apresentados no item a com exceção de T_p.

$\zeta = \dfrac{-\ln(0,1)}{\sqrt{\pi^2 + \ln^2(0,1)}} = \underline{0,1518}$

$T_p = \dfrac{\pi}{\omega_n\sqrt{1-\zeta^2}} \Rightarrow \omega_n = \dfrac{\pi}{5\sqrt{1-(0,1518)^2}} = \underline{0,63568\,s}$

Determinando os polos:

$\underline{s_{1,2} = -0,0964 \pm j\,0,62862}$

c)

$$T_s = \frac{4}{\zeta \omega_n} \Rightarrow \omega_n = \frac{4}{7\zeta} \quad (\text{I})$$

$$T_p = \frac{\pi}{\omega_n \sqrt{1-\zeta^2}} \Rightarrow \omega_n = \frac{\pi}{3\sqrt{1-\zeta^2}} \quad (\text{II})$$

Igualando (I) e (II):

$$\frac{4}{7\zeta} = \frac{\pi}{3\sqrt{1-\zeta^2}} \Rightarrow \frac{\zeta}{\sqrt{1-\zeta^2}} = \frac{12}{7\pi} \Rightarrow \frac{\zeta^2}{1-\zeta^2} = 0{,}2978$$

$$\underline{\zeta = 0{,}479}$$

$$\therefore \omega_n = \frac{4}{7 \cdot 0{,}479} = \underline{1{,}193}$$

Determinando os polos:

$$s_{1,2} = -\zeta\omega_n \pm \omega_n\sqrt{\zeta^2-1}$$

$$\underline{s_{1,2} = 0{,}571 \pm j1{,}047}$$

25) a) Pela Segunda Lei de Newton:

$$f(t) - c\dot{x} - Kx = m.\ddot{x}$$

$$m\ddot{x} + c\dot{x} + Kx = f(t)$$

Aplicando Laplace:

$$X(s)\left[ms^2 + cs + K\right] = F(s)$$

Função de Transferência:

$$G(s) = \frac{X(s)}{F(s)} = \frac{1}{ms^2 + cs + K} = \frac{1}{5s^2 + 2s + 20}$$

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b) $$G(s) = \frac{1}{20} \cdot \frac{4}{s^2 + \frac{2}{5}s + 4}$$

$$\omega_n = \sqrt{4} = 2$$

$$2\zeta\omega_n = \frac{2}{5} \Rightarrow \zeta = \frac{\cancel{2}}{5} \cdot \frac{1}{\cancel{2}\omega_n} = \frac{1}{10}$$

$$\%OS = e^{-(\pi\zeta/\sqrt{1-\zeta^2})} . 100 = e^{-(\pi . 0,1/\sqrt{0,99})} . 100 = 72,92\%$$

$$T_s = \frac{4}{\zeta\omega_n} = \frac{4}{0,1 . 2} = 20\,s$$

$$T_p = \frac{\pi}{\omega_n\sqrt{1-\zeta^2}} = 1,58\,s$$

$$T_r = \frac{1,8}{\omega_n} = 0,9\,s$$

26) a) Função de Transferência

$$\theta_2\left[Js^2+Ds+K\right]=T(s) \Rightarrow \theta_2(s)\left[2s^2+s+1\right]=T(s)$$

$$G(s)=\frac{\theta_2(s)}{T(s)}=\frac{1}{2s^2+s+1}=\frac{1/2}{s^2+1/2\,s+1/2}$$

b) Encontrando os valores característicos:

$$\%OS=e^{-\left(\frac{\pi\zeta}{\sqrt{1-\zeta^2}}\right)}.100=e^{-\left(\frac{1,112}{0,935}\right)}.100=\underline{30,44\%}$$

$$T_s=\frac{4}{\omega_n}=\frac{4}{0,707}=\underline{5,658\,s}$$

$$T_p=\frac{\pi}{\omega_n\sqrt{1-\zeta^2}}=\frac{\pi}{0,661}=\underline{4,753\,s}$$

Obs.

$$\omega_n=\sqrt{1/2}=\underline{0,707}$$

$$2\zeta.\omega_n=1/2 \Rightarrow \zeta=\frac{0,5}{2.0,707}=\underline{0,354}$$