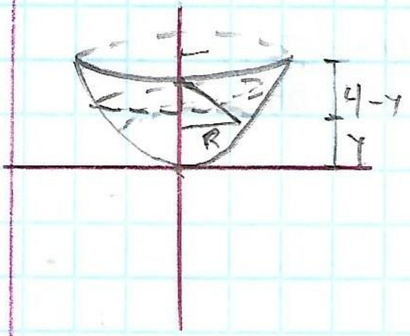
Leonel Antonio González García. 24/11/202
201709088

Primera Retrasada.

Pregunta 1.



$R = 4ft$

$r = 1.5\,plg \rightarrow 1.5\,\cancel{plg} * \frac{1ft}{12\,\cancel{plg}} = \frac{1}{8}\,ft$

$(4-y)^2 + R^2 = 16$

$\cancel{16} - 8y + y^2 + R^2 = \cancel{16}$

$R^2 = 8y - y^2$

$A(y) = \pi R^2$

$A(y) = \pi(8y - y^2) \quad \Rightarrow \quad Ay\,\frac{dy}{dt} = -a\sqrt{2gy}$

Entonces $\rightarrow \quad \pi(8y-y^2)\,\frac{dy}{dt} = -a\sqrt{2(32)y}$

$\pi(8y-y^2)\,\frac{dy}{dt} = -a\sqrt{64y}$

$\pi(8y-y^2)\,\frac{dy}{dt} = -a\,8\sqrt{y}$

$\int \frac{\pi(8y-y^2)}{\sqrt{y}}\,dy = \int -8a\,dt$

$-\frac{2}{15}\,y^{3/2}\,\pi(-40+3y) = -8ta + C$

Si $t=0$ entonces $y = 8ft$

$-\frac{2}{15}(8)^{3/2}\,\pi(-40+3(8)) = -8\cancel{(0)}^{0}a + C$

$-\frac{2}{15}\,16\sqrt{2}\,\pi(-16) = C$

$\frac{512}{15}\sqrt{2}\,\pi = C$

Se tiene entonces

$$\frac{2}{15} y^{3/2} \pi (\cancel{-40} + 3y) = -8\,ta + \frac{512}{15}\sqrt{2}\,\pi$$

$Y = 0 \quad t = ?$

$a = \pi r^2$

$a = \pi \left(\frac{1}{8}\right)^2$

$a = \frac{\pi}{64}$

$0 = -8\,ta + \frac{512}{2}\sqrt{2}\,\pi$

$8\,ta = \frac{512}{15}\sqrt{2}\,\pi$

$t = \frac{151.6504}{8a}$

$t = \frac{151.6504}{\cancel{8}\frac{\pi}{\cancel{64}}} = \frac{151.6504}{\frac{\pi}{8}} = 386.17$ segundos

$t = 386.17\,s.$

$t = 386.17\,s * \frac{1\,min}{60\,s} = 6.43\,min$

Pregunta 2. $y'' + y = \cot x$.

$y'' + y = 0$

$Y_1 = \operatorname{sen} x$

$Y_2 = \cos x$

$Y_h = A \operatorname{sen} x + B \cos x$

$Y_p = U_1 \operatorname{sen} x + U_2 \cos x$

$U_1 = \int \frac{Y_2 f x}{W} dx$

$U_2 = \int \frac{Y_1 f x}{W} dx$

$W = \begin{vmatrix} \operatorname{sen} x & \cos x \\ \cos x & -\operatorname{sen} x \end{vmatrix} = -\operatorname{sen}^2 x - \cos^2(x) = -1$

$U_1 = -\int \frac{\cos x \cdot \cot x}{-1} dx$

$U_1 = \int \cos(x) \cot(x)\, dx$

$U_1 = \ln \left| \tan\left(\frac{x}{2}\right) + \cos(x) \right| + C$

$$U_2 = \int \frac{\operatorname{Sen}(x)\cot(x)}{-1}\,dx$$

$$U_2 = \int \operatorname{Sen}x \cot x\,dx = -\operatorname{Sen}x + C$$

Solución Particular $\rightarrow Y_p = U_1 + U_2$

$$Y_p = \operatorname{Sen}x \ln|\tan(x/2) + \cos(x) + \cos(x)(-\operatorname{Sen}x)$$

$$Y_p = \operatorname{Sen}x \ln|\tan\frac{x}{2}| + \cos x - \operatorname{Sen}(x)\cos(x)$$

$$Y_g = A\operatorname{Sen}x + B\cos x + \operatorname{Sen}x\ln|\tan(\tfrac{x}{2})| + \cos x - \operatorname{Sen}x\cos x.$$

$$Y_g = A\operatorname{Sen}x + B\cos x - \frac{\operatorname{Sen}(2x)}{2} + \operatorname{Sen}x\left[\ln|\tan(\tfrac{x}{2})| + \cos x\right] //$$

Pregunta 3. $Y(\ln y^2 - \ln x^2 + 1)dx - xdy = 0.$ $\quad Y(1) = e^{\pi}$

$y(\ln y^2 - \ln x^2 + 1)dx - xdy = 0 \quad // \div dx$

$Y(\ln y^2 - \ln x^2 + 1) - xy' = 0.$

$$Y' = \frac{y(2\ln(y) + 1 - 2\ln(x))}{x}$$

$Y = vx \rightarrow v = y/x$

$xv' + v = v(2\ln|v| + 1)$

$v = e^{c_1 x^2}$

$$\frac{y}{x} = e^{c_1 x^2}$$

$y = e^{c_1 x^2} x.$

$Y(1) = e^{\pi}$

$e^{\pi} = e^{c_1(1)}(1)$

$\ln e^{\pi} = \ln e^{c_1}$

$\pi = c_1$

$\pi = c_1.$

$Y = xe^{\pi x^2}$

Pregunta 5. $\frac{dx}{dt} = y$ $\frac{dy}{dt} = x.$

$x' = y$ (I)
$y' = x$ (II)

despejando y de I

$-y = x'$

derivar respecto t

$-y' = x''$

Sustituir II en II

$x'' = x$

$x'' - x = 0$

$x = c_1 e^{t} + c_2 e^{-t}$

$x'' = c_1 e^{t} - c_2 e^{-t}$

Sustituyendo

$-y = x'$ $\quad -y = c_1 e^{t} - c_2 e^{-t}$

$x = c_1 e^{t} + c_2 e^{-t}$
$y = c_1 e^{t} - c_2 e^{-t}.$

$x = c_1 e^{t} + c_2 e^{-t}$
$y = -c_2 e^{-t} + c_1 e^{t}$

Pregunta 4. $f(t) = 8\cos 2t.$
$m = 32 lb.$

$m\frac{d^2x}{dt^2} + kx = 0$

$\frac{d^2x}{dt^2} + \frac{kx}{m} = 0$

$\frac{d^2x}{dt^2} + w^2x = 0$

$m^2 + w^2 = 0.$
$m = \pm w.$
$x = c_1 \cos w_0 t + c_2 \operatorname{Sen} w_0 t.$

$F = k(x).$
$32 = k(2)$
$k = \frac{32}{2} = 16.$

$m = 1$

$\frac{d^2x}{dt^2} + \frac{16}{1}x = 0 \Rightarrow \frac{d^2x}{dt^2} + 16x = 0$

$x' + 16x = 0$

$x = c_1 \cos \cancel{(2t)}^{0} + c_2 \cancel{\operatorname{Sen} w_0 t}^{0}.$

$1 = c_1$

$x(t) = \cos 4t - \frac{1}{12}\cos 2t.$