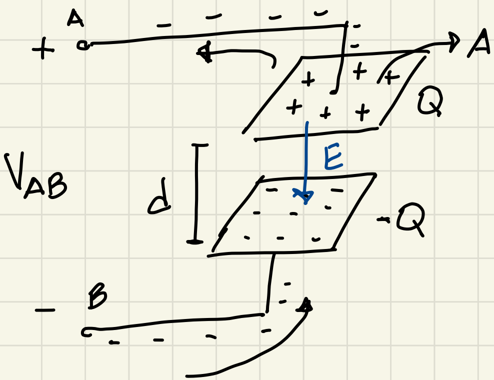
1. · capacitancia ·

- definición
- serie - paralelo

física II

ing. Claudia Contreras

Definición de Capacitancia



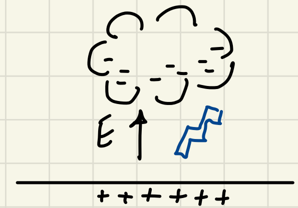
$$\boxed{|\vec{E}| = \frac{|\sigma|}{\varepsilon_0}}$$

$$\boxed{C = \frac{Q}{\Delta V}}$$

$$1\,\frac{C}{V} = 1 \text{ Faradio}$$

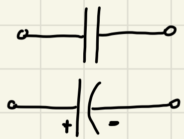
$V_{MAX} \rightarrow$ Voltaje de ruptura

$E_{MAX} \rightarrow$ resistencia dieléctrica.



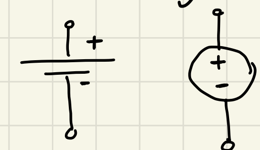
Simbología

C



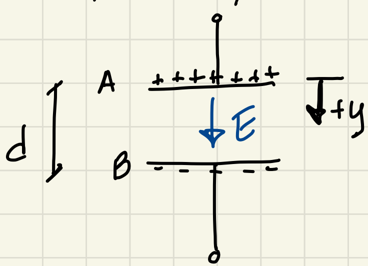
fuentes de voltaje co.

Simbología



batería fem

placas paralelas



$$\Delta V = V_{AB} = \int_A^B \vec{E} \cdot \vec{dy}$$

$$= \int_0^d \frac{\sigma}{\varepsilon_0}\, dy = \frac{\sigma y}{\varepsilon_0}\Big|_0^d = \frac{\sigma d}{\varepsilon_0}$$

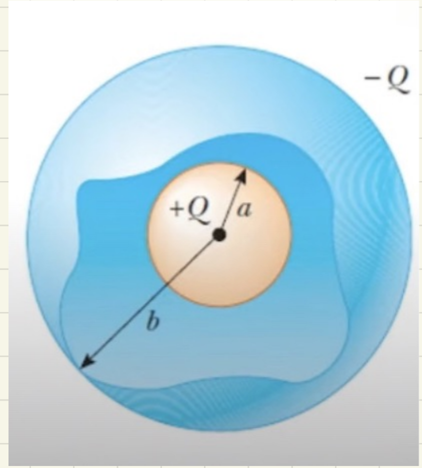
$$\Delta V = \frac{Q d}{A \varepsilon_0}$$

$$\boxed{\Delta V = E d}$$

$$C = \frac{Q}{\Delta V} = \frac{\cancel{Q}}{\frac{\cancel{Q} d}{A \varepsilon_0}} = \frac{A \varepsilon_0}{d}$$

$$\boxed{C = \frac{\varepsilon_0 A}{d}}$$

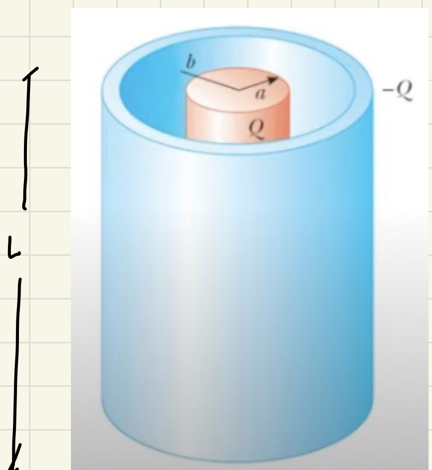
capacitor esférico



$$k = \frac{1}{4\pi\varepsilon_0}$$

$$\boxed{C = \frac{ab}{k(b-a)}}$$

capacitor cilíndrico

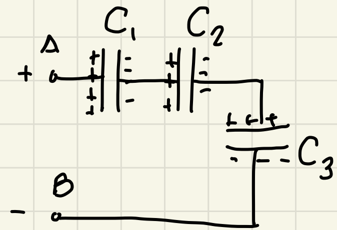


$$\boxed{C = \frac{L}{2k\ln\left(\frac{b}{a}\right)}}$$

Conexión de Capacitores

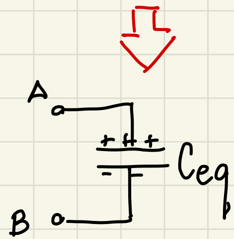
$$C = \frac{Q}{\Delta V}$$

Serie



$Q_1 = Q_2 = Q_3$ ⇨ MISMA CARGA

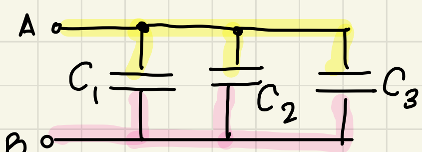
$$\Delta V_1 = \frac{Q_1}{C_1}$$



$$Q_{eq} = Q_1 = Q_2 = Q_3$$

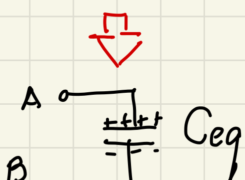
$$\boxed{C_{eq} = \left(\frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}\right)^{-1}}$$

Paralelo



MISMO ΔV

$$\Delta V_1 = \Delta V_2 = \Delta V_3 = V_{AB}$$

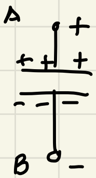


$$\boxed{C_{eq} = C_1 + C_2 + C_3}$$

$$\Delta V_{eq} = \Delta V_1 = \Delta V_2 = \Delta V_3 = V_{AB}$$

Problema 1. Un capacitor lleno de aire está formado por dos placas paralelas, cada una de ellas con un área de $7.6\,cm^2$, separadas una distancia de 1.8mm. A estas placas se les aplica una diferencia de potencial de 20V. Calcule: a) el campo eléctrico entre las placas; b) la densidad de carga superficial; c) la capacitancia y d) la carga sobre cada placa.

$$C = \frac{Q}{\Delta V}$$



$V_{AB} = 20V$

$\frac{1V}{m} = \frac{1N}{C}$

$A = 7.6cm^2$

$A = 7.6 \times 10^{-4} m^2$

$d = 1.8 \times 10^{-3} m$

$\Delta V = 20V$

a) $\Delta V = Ed \rightarrow E = \frac{\Delta V}{d} = \frac{20}{1.8 \times 10^{-3}} = \underline{11{,}111\, V/m}$

b) $E = \frac{\sigma}{\varepsilon_0} \rightarrow \sigma = E\varepsilon_0 = \underline{98.33 \frac{nC}{m^2}}$

c) $C = \frac{\varepsilon_0 A}{d} = \frac{8.85 \times 10^{-12}(7.6 \times 10^{-4})}{1.8 \times 10^{-3}} = \underline{3{,}7367\, pF}$

d) $Q = C\Delta V = \underline{74.73\, pC}$

$\sigma = \frac{Q}{A}$ $\quad Q = \sigma A$

Energía potencial de un capacitor

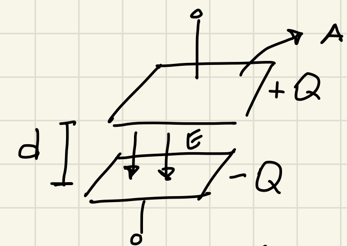
$\Delta V = V$

$C = \dfrac{Q}{\Delta V}$

$$\boxed{\begin{aligned} U &= \frac{1}{2} C \Delta V^2 \\ U &= \frac{1}{2} Q V \\ U &= \frac{1}{2} \frac{Q^2}{C} \end{aligned}}$$

$= \frac{1}{2} C V^2$

Joules



para placas paralelas

densidad de energía (u) (J/m^3)

$$\boxed{\begin{aligned} u &= \frac{U}{Ad} \\ u &= \frac{1}{2} \varepsilon_0 E^2 \end{aligned}}$$

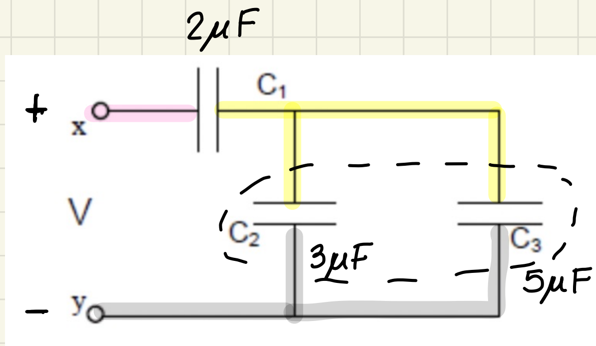
Problema 2. En el circuito que se muestra en la figura $C_1 = 2\mu F, C_2 = 3\mu F, C_3 = 5\mu F$, calcule:

a) La capacitancia equivalente del circuito;

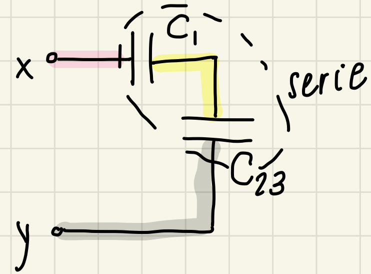
b) Si la carga en el capacitor C_3 es $15\mu C$, calcule el voltaje y la carga de cada capacitor.

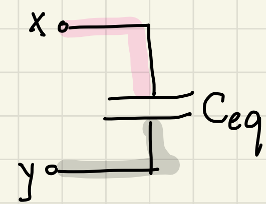
c) La diferencia de potencial entre los puntos xy

$$C = \frac{Q}{\Delta V}$$



$$C_{23} = C_2 + C_3 = 8\mu F \qquad V_{23} = V_2 = V_3$$





como $C_2 \parallel C_3 \Rightarrow$ mismo ΔV

$$C_{eq} = \left(\frac{1}{C_{23}} + \frac{1}{C_1}\right)^{-1} = \underline{1.6\mu F}$$

Para C_3

$Q_3 = 15\mu C \qquad C_3 = 5\mu F$

$$\Delta V_3 = \frac{Q_3}{C_3} = \frac{15\mu C}{5\mu F} = 3V$$

$$U_3 = \frac{1}{2} C_3 V_3^2 = \frac{1}{2}(5\times 10^{-6})(3)^2 = 22{,}5\mu J$$

Para C_2

$V_2 = V_3 = 3V$

$C_2 = 3\mu F$

$Q_2 = C_2 V_2 = 9\mu C$

$$U_2 = \frac{1}{2} C_2 V_2^2 = 13{,}5\mu J$$

Para C_1

$Q_1 = Q_{23} \qquad V_{23} = 3V$

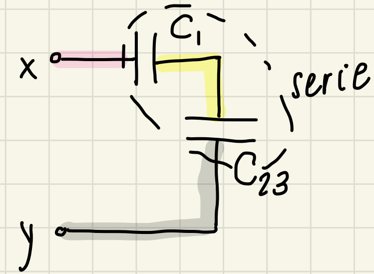
$Q_{23} = C_{23} V_{23}$

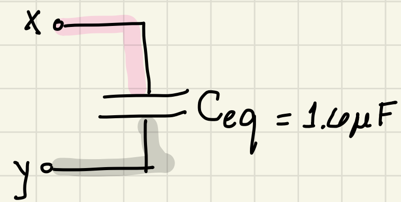
$Q_{23} = 8\mu F * 3V$

$Q_{23} = 24\mu C$

	Q	ΔV	U
C_1	$24\mu C$	$12V$	$144\mu J$
C_2	$9\mu C$	$3V$	$13{,}5\mu J$
C_3	$15\mu C$	$3V$	$22{,}5\mu J$
			$\Sigma\ 180\mu J$

$U_{sist} = \underline{180\mu J}$





$Q_1 = Q_{23} = 24\mu C$

$\Delta V_1 = \dfrac{Q_1}{C_1} = \dfrac{24\mu C}{2\mu F} = 12V$

$U_1 = \frac{1}{2} C_1 V_1^2 = \frac{1}{2}(2\mu F)(12)^2 = 144\mu J$

c)

$V_{xy} = V_x - V_y$

$V_{xy} = V_{eq}$

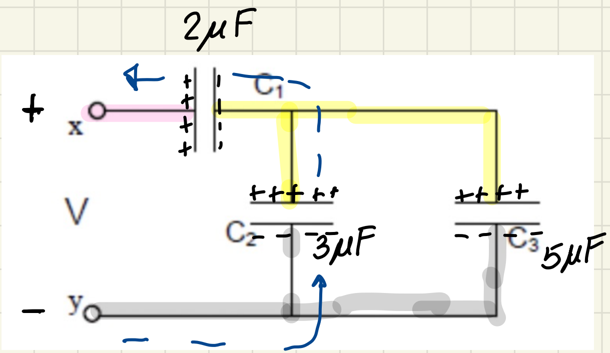
$Q_{eq} = Q_1 = Q_{23} = 24\mu C$

$V_{eq} = \dfrac{Q_{eq}}{C_{eq}} = \dfrac{24\mu C}{1.6\mu F} = \underline{15V}$

Otra forma de encontrar la energía de un sistema capacitores

$$U_{eq} = U_{sist.} = \frac{1}{2} C_{eq} V_{eq}^2 = \frac{1}{2}(1.6\times10^{-6})(15)^2 = \underline{180\,\mu J}$$

Otra forma de encontrar el voltaje entre los puntos x y

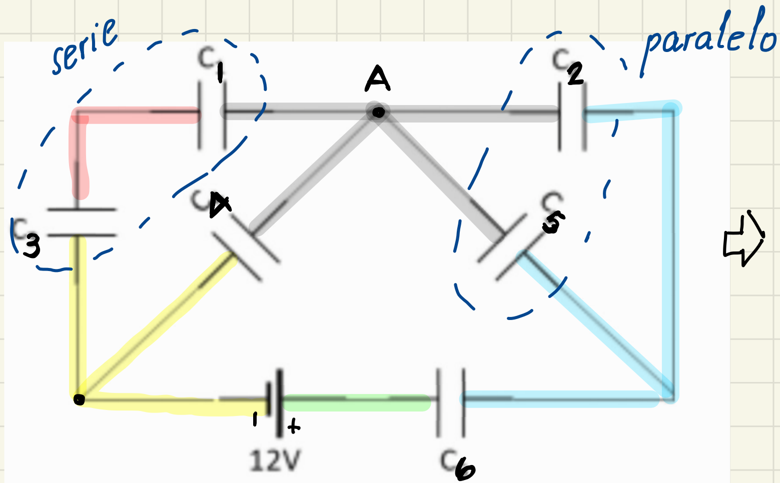


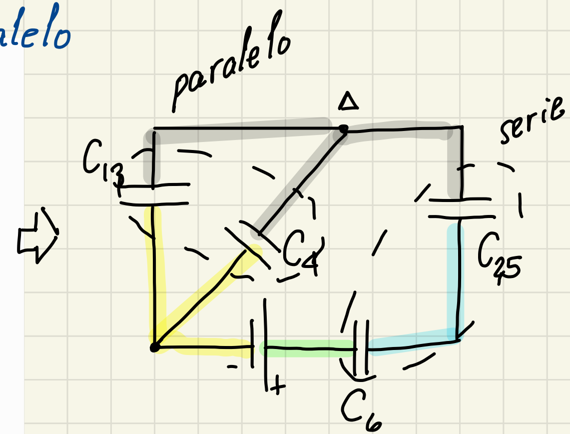
$$V_y + \Delta V_2 + V_1 = V_x$$

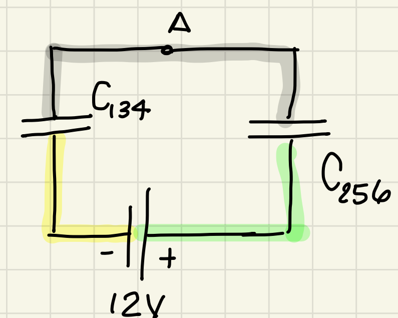
$$V_x - V_y = \Delta V_2 + \Delta V_1$$

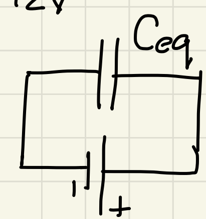
$$= 3 + 12 = 15\,V$$

Problema 3. En el circuito que se muestra en la figura todos los capacitores tienen una capacitancia $C = 2\mu F$ y están conectados a una batería de 12V. Calcule la capacitancia equivalente entre los terminales de la batería y la energía que almacena el capacitor C_6.









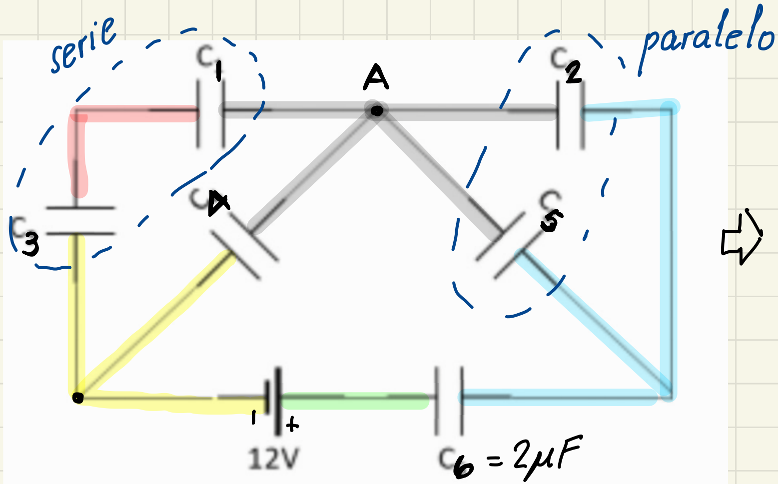
$$C_{25} = C_2 + C_5 = 4\mu F$$

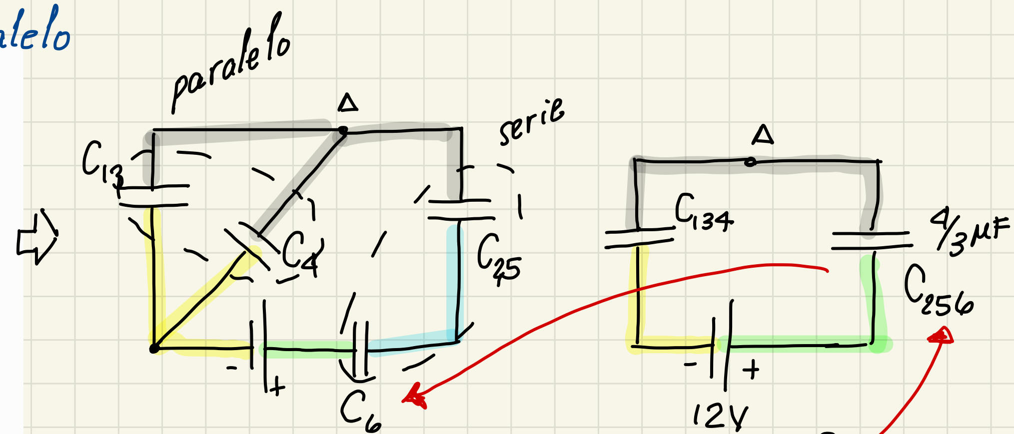
$$C_{13} = \left(\frac{1}{C_1} + \frac{1}{C_3}\right)^{-1} = 1\mu F$$

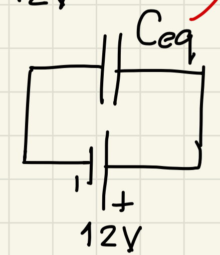
$$C_{134} = C_{13} + C_4 = 3\mu F$$

$$C_{256} = \left(\frac{1}{C_{25}} + \frac{1}{C_6}\right)^{-1} = \frac{4}{3}\mu F$$

$$C_{eq} = \left(\frac{1}{C_{134}} + \frac{1}{C_{256}}\right)^{-1} = \frac{12}{13}\mu F \approx \underline{0{,}9231\mu F}$$







b) $U_6 = \frac{1}{2} C_6 V_6^2 = \frac{1}{2} \frac{Q_6^2}{C_6}$

$= \frac{1}{2} \frac{\left(\frac{144}{13} \times 10^{-6}\right)^2}{2 \times 10^{-6}}$

$$\boxed{U_6 = 30.47 \mu J}$$

$C_{eq} = \frac{12}{13} \mu F \quad \Delta V_{eq} = 12V$

$Q_{eq} = C_{eq} \Delta V_{eq} = \frac{144}{13} \mu C$

$Q_{eq} = Q_{256} = \frac{144}{13} \mu C$

$V_{256} = \frac{Q_{256}}{C_{256}} = 8.3077V$

$Q_{256} = Q_6 = Q_{25}$

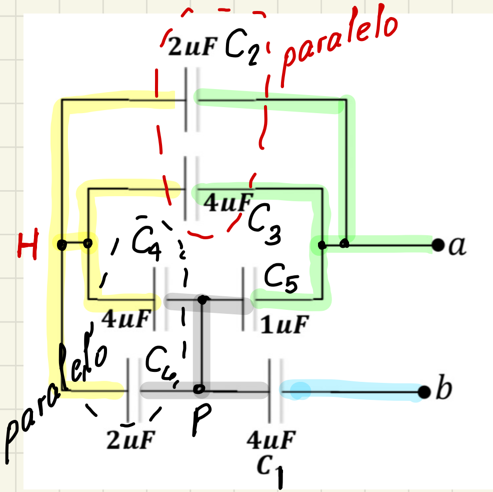
$Q_6 = \frac{144}{13} \mu C$

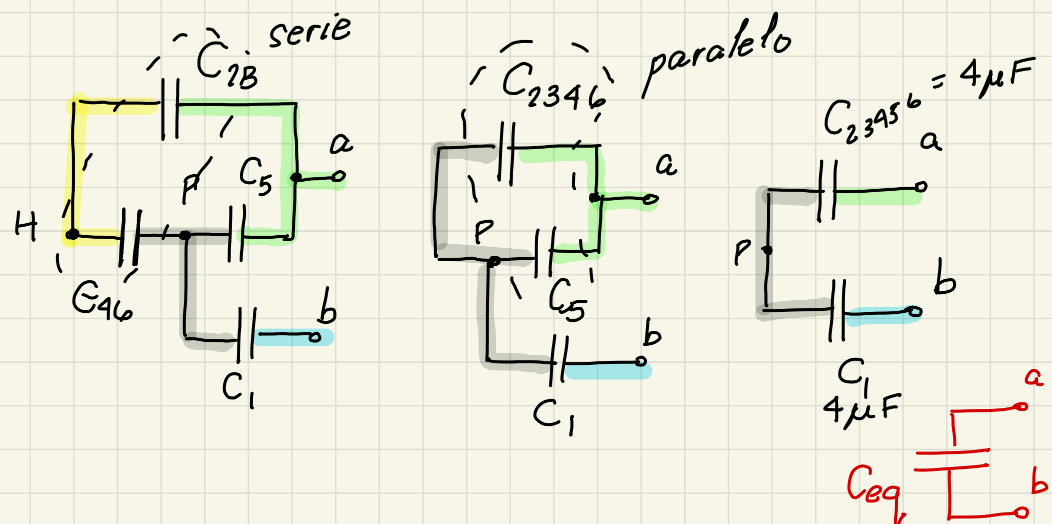
Problema 4. Para el circuito que se muestra:

(a) Encuentre la capacitancia equivalente entre los puntos a y b del circuito que se muestra. $R: 2\mu F$

(b) Si entre los puntos a y b se conecta una fuente de 18V, ¿Cuál es la energía del sistema de capacitores? $R: 324\mu J$

(c) ¿Cuál será la diferencia de potencial entre los bornes del capacitor uno C_1? $R: 9V$





$$C_{46} = C_4 + C_6 = 4\mu F + 2\mu F = 6\mu F$$

$$C_{23} = C_2 + C_3 = 2\mu F + 4\mu F = 6\mu F$$

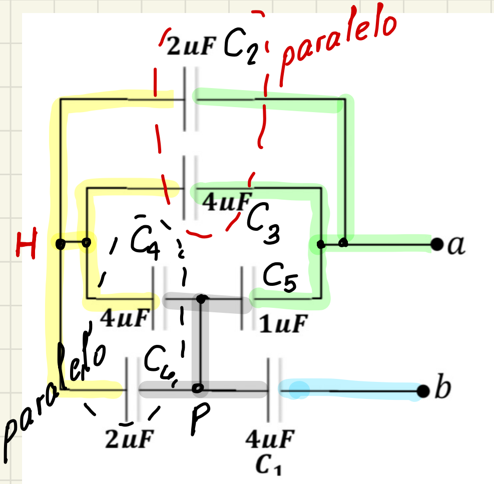
$$C_{2346} = \left(\frac{1}{C_{23}} + \frac{1}{C_{46}}\right)^{-1} = 3\mu F$$

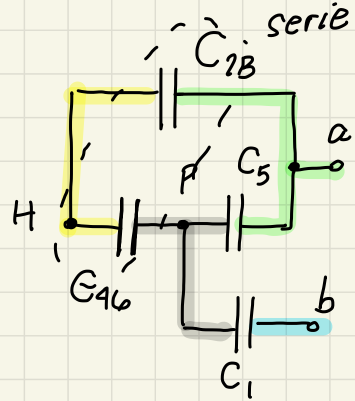
$$C_{23456} = C_{2346} + C_5 = 3\mu F + 1\mu F = 4\mu F$$

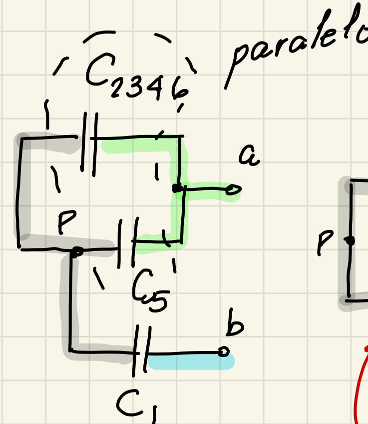
$$C_{eq} = \left(\frac{1}{C_1} + \frac{1}{C_{23456}}\right)^{-1} = 2\mu F$$

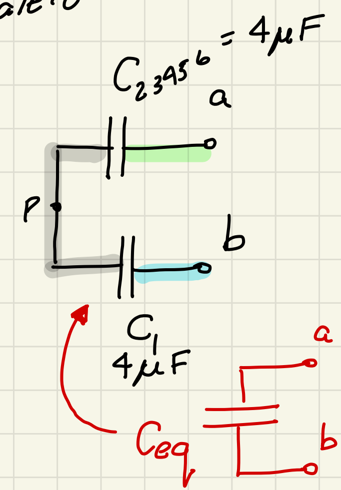
b) $U_{sist} = \frac{1}{2} C_{eq} V_{eq}^2 = \frac{1}{2}(2\times10^{-6})(18)^2 = \underline{324\,\mu J}$

$C = \frac{Q}{\Delta V}$









Como C_{eq} sust. a C_1 y C_{23456} en serie

→ $Q_{eq} = Q_1 = Q_{23456}$

$Q_{eq} = V_{eq} C_{eq} = 18(2\times10^{-6}) = 36\,\mu C$

$Q_1 = 36\mu C$ → $V_1 = \frac{Q_1}{C_1} = \frac{36\mu C}{4\mu F} = \underline{9V}$