capacitancia

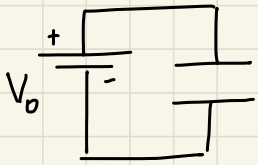
DIELÉCTRICOS

Física II

ing. Claudia Contreras

Problema 1. Un capacitor al vacío de placas paralelas tiene una energía de 8.38J almacenada en él. La separación entre las placas es de 2.30mm. Si se reduce la separación a 1.15mm. ¿Cuál será la energía si a) se desconecta el capacitor de la fuente de potencial? b) si permanece conectado a la fuente.

$C = \frac{Q}{V}$



$d_0 = 2.3mm$

$C_0 = \frac{\varepsilon_0 A}{d}$

V_0

$Q_0 = C_0 V_0$

$U_0 = \frac{1}{2} C_0 V_0^2 = 8.38J$

$d_f = 1.15mm = d_0/2$

$C_f = \frac{\varepsilon_0 A}{d_f} = \frac{\varepsilon_0 A}{d_0/2} = 2\frac{\varepsilon_0 A}{d}$

$C_f = 2C_0$

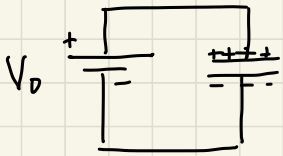
$Q_f = Q_0$

$V_f = \frac{Q_f}{C_f} = \frac{Q_0}{2C_0}$

$V_f = \frac{V_0}{2}$

$U_f = \frac{1}{2} C_f V_f^2 = \frac{1}{2}(2C_0)\left(\frac{V_0}{2}\right)^2$

$U_f = \frac{1}{2}\left(\frac{C_0 V_0^2}{2}\right) = 4.19J$



$V_f = V_0$

$C_f = 2C_0$

$Q_f = C_f V_f$

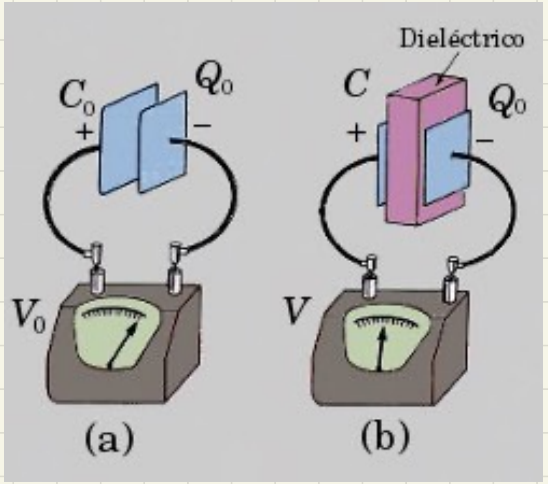
$Q_f = 2C_0 V_0$

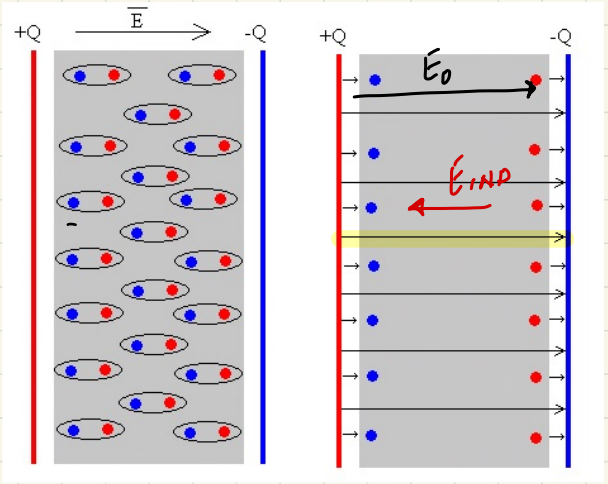
$Q_f = 2Q_0$

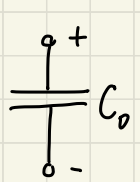
$U_f = \frac{1}{2} C_f V_f^2 = \frac{1}{2}(2C_0) V_0^2$

$U_f = 2\left[\frac{1}{2} C_0 V_0^2\right] = 16.76J$

Dieléctricos







V_0

E_0

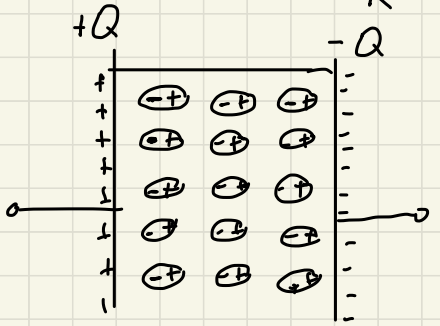
Q_0

$Q_f = Q_0$

$C = KC_0$

$V_f = \frac{V_0}{K}$

$E_f = \frac{E_0}{K}$



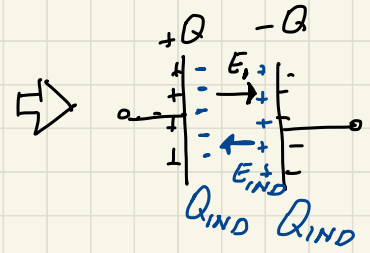
K ⇒ constante dieléctrica

placas paralelas

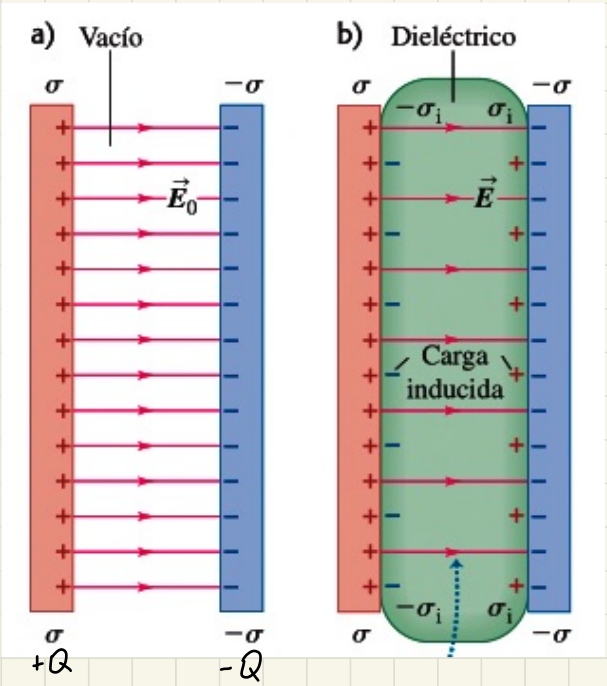
$C = \frac{K \varepsilon_0 A}{d}$

ε = permitividad

$\varepsilon = K\varepsilon_0$



$\vec{E} = \vec{E_0} + \vec{E_{IND}}$



$$\vec{E} = \vec{E}_0 + \vec{E}_{IND}$$

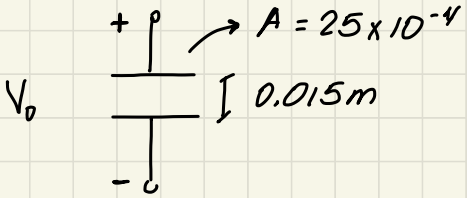
$$\frac{E_0}{K} = E_0 - E_{IND}$$

$$\frac{\sigma}{\varepsilon_0 K} = \frac{\sigma}{\varepsilon_0} - \frac{\sigma_{IND}}{\varepsilon_0}$$

$$\sigma_{IND} = \sigma\left(1 - \frac{1}{K}\right)$$

$$Q_{IND} = Q\left(1 - \frac{1}{K}\right)$$

Problema 2. Un capacitor en el aire tiene una separación d=1.5cm y un área de 25 cm^2, las placas se conectan a una batería de 250V. Se desconecta la batería y se sumerge en agua destilada, K=80. Determine la carga, el voltaje y capacitancia antes y después de la inmersión.

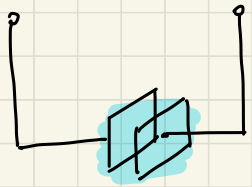


$V_0 = 250V$

$C_0 = \frac{\varepsilon_0 A}{d} = \frac{8.85\times10^{-12} * 25\times10^{-4}}{0.015}$

$C_0 = 1.475\ pF$

$Q_0 = C_0 V_0 = 368.75\ pC$



C_f , Q_f , V_f , Q_{IND}

$Q_f = Q_0 = 368.75pC$

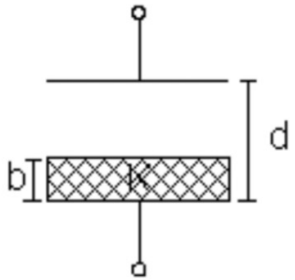
$C_f = KC_0 = 118\ pF$

$V_f = \frac{V_0}{K} = \frac{250}{80} = 3.125\ V$

$Q_{IND} = Q\left(1 - \frac{1}{K}\right) = 368.75\times10^{-12}\left(1 - \frac{1}{K}\right)$

$Q_{IND} = 364.14\ pC$

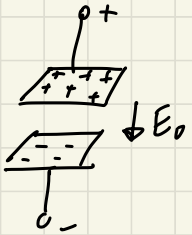
Problema 3. Un capacitor de placas paralelas con aire entre sus placas tiene un área de $0.2\ m^2$ y una separación entre placas de $d = 2cm$. Una batería carga las placas a una diferencia de potencial de $120V$, y luego se desconecta. Se introduce un trozo de dieléctrico con constante $K = 4.2$ como se muestra en la figura; $b = 1cm$. Determine: a) C_o, C_f; Q_o, Q_f; V_o, V_f. b) el trabajo requerido para insertar el trozo de dieléctrico.



$A = 0.2m^2$

$d = 0.02m$

$V_o = 120V$



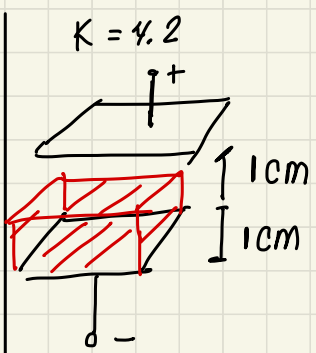
$$C_o = \frac{\varepsilon_0 A}{d} = \frac{8.85\times10^{-12} * 0.2}{0.02}$$

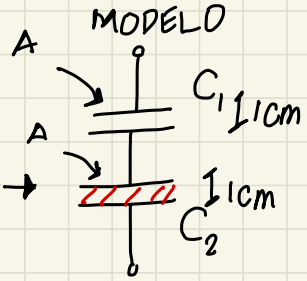
$$C_o = 88.5\ pF$$

$$V_o = 120V$$

$$Q_o = C_o V_o = 10,62\ nC$$

$$U_o = \frac{1}{2} C_o V_o^2 = 637.2nJ$$





$$Q_o = Q_f = 10.62nC$$

$$C_f = \left(\frac{1}{C_1} + \frac{1}{C_2}\right)^{-1} = 142.96pF$$

$$V_f = \frac{Q_f}{C_f} = 74.287V$$

$$U_f = \frac{1}{2} C_f V_f^2 = 394.46nJ$$

$$C_1 = \frac{\varepsilon_0 A}{0.01} = \frac{8.85\times10^{-12} * 0.2}{0.01}$$

$$C_1 = 177pF$$

$$C_2 = \frac{K\varepsilon_0 A}{0.01} = \frac{4.2 * 8.85\times10^{-12} * 0.2}{0.01}$$

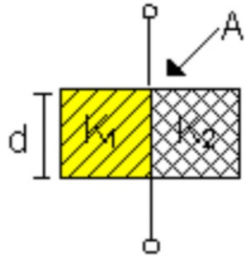
$$C_2 = 743.4pF$$

$$W = +\Delta U = U_f - U_0$$

$$= 394.46nJ - 637.2nJ$$

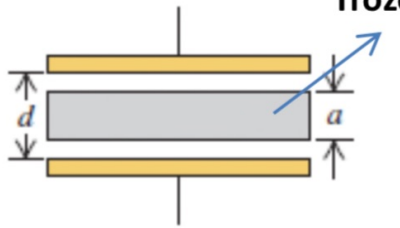
$$W = -242.74nJ$$

Problema 4. Determine la capacitancia de los siguientes capacitores.

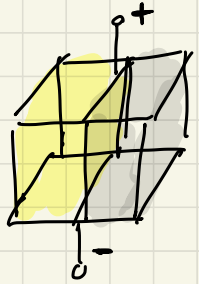


Área A

Trozo de metal

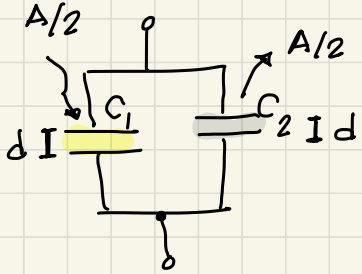


$$d_1 = d_2 = \frac{d-a}{2}$$



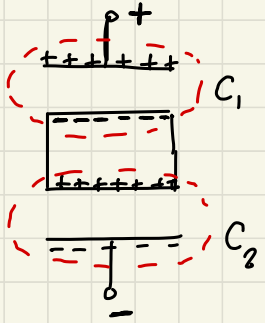
$$C = C_1 + C_2$$

$$C = \frac{\varepsilon_0 A}{2d}(K_1 + K_2)$$



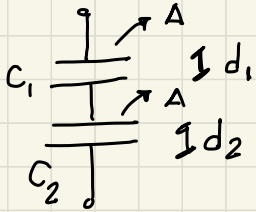
$$C_1 = \frac{K_1 \varepsilon_0 A/2}{d} = \frac{K_1 \varepsilon_0 A}{2d}$$

$$C_2 = \frac{K_2 \varepsilon_0 (A/2)}{d} = \frac{K_2 \varepsilon_0 A}{2d}$$



$$C_{eq} = \left(\frac{1}{C_1} + \frac{1}{C_2}\right)^{-1}$$

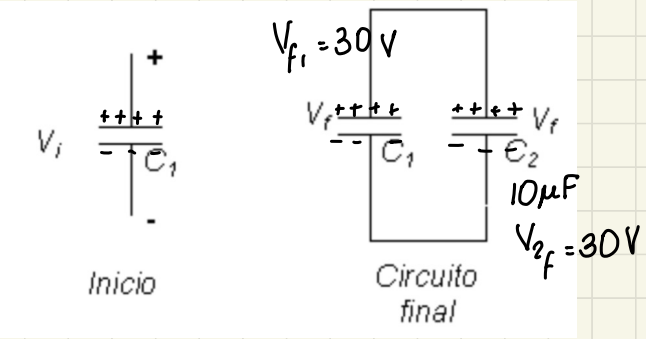
$$C_{eq} = \frac{\varepsilon_0 A}{d-a}$$

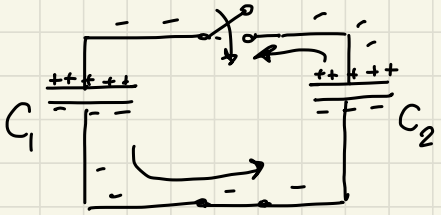


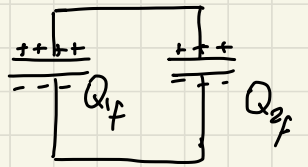
$$C_1 = \frac{\varepsilon_0 A}{\left(\frac{d-a}{2}\right)} = \frac{2\varepsilon_0 A}{d-a}$$

$$C_2 = \frac{2\varepsilon_0 A}{d-a}$$

Problema 5. Un capacitor aislado de capacitancia no conocida ha sido cargado a una diferencia de potencial de 100V. Cuando el capacitor con carga es conectado en paralelo con un capacitor sin carga de $C_2 = 10\mu F$, la diferencia de potencial de la combinación es 30V. ¿Cuál es el valor de la capacitancia desconocida?







C_1

$V_{i_1} = 100\,V$

$Q_{i_1} = C_1 V_{i_1} = 100 C_1$

$$Q_{i_1} + Q_{i_2} = Q_{f_1} + Q_{f_2}$$

$$100 C_1 = C_1 V_{f_1} + C_2 V_{f_2}$$

$$100 C_1 = 30 C_1 + (10 \times 10^{-6})(30)$$

$$70 C_1 = 300 \times 10^{-6}$$

$$C_1 = 4.286\,\mu F$$