APLICACIONES Ley de Gauss

--- simetría ---

ESFÉRICA Y CILÍNDRICA

2

EJERCICIOS
• PARA RESOLVER •
EN CLASE

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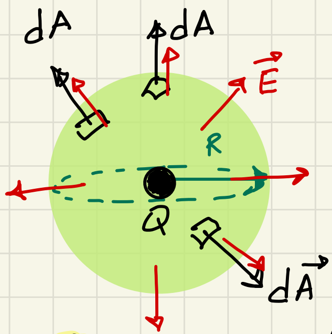
Ley de Gauss

$$\Rightarrow dE = \frac{k\,dq}{r^2}$$

$$\Phi_E = \oint \vec{E}\cdot d\vec{A} = \oint E\,dA \cancel{\cos 0^\circ}^{\,1}$$

$$= \oint E\,dA = \oint \frac{kQ}{R^2}\,dA = \frac{kQ}{R^2}\oint dA$$

$$= \frac{kQ}{\cancel{R^2}} * 4\pi \cancel{R^2} = \frac{1}{\cancel{4\pi}\varepsilon_0} Q * \cancel{4\pi} = \frac{Q}{\varepsilon_0}$$



$$\Phi_E = \frac{Q}{\varepsilon_0} \qquad |E| = \frac{kQ}{R^2}$$

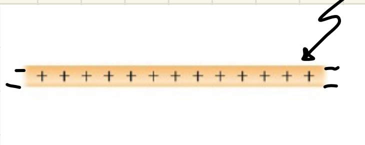
$$\boxed{\oint \vec{E}\cdot d\vec{A} = \frac{q_{enc}}{\varepsilon_0}} \longrightarrow \text{ley de gauss}$$

	Esfera.	Cilindro
Área	$4\pi r^2$	$2\pi r h$
Volumen	$\frac{4}{3}\pi r^3$	$\pi r^2 h$

Tipos de distribuciones de carga

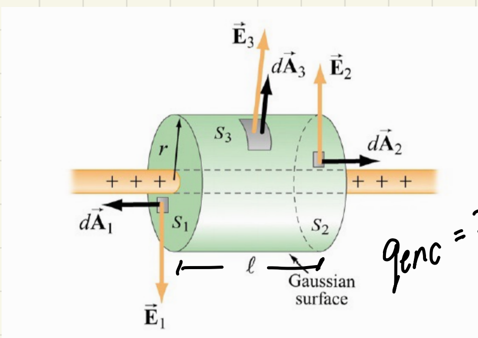
Lineales

Q, L



$\lambda = \frac{Q}{L} \quad \left(\frac{C}{m}\right)$

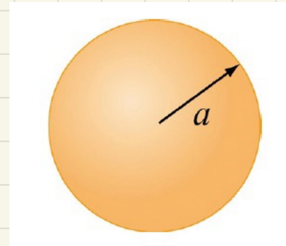
simetría cilíndrica



$q_{enc} = \lambda l$

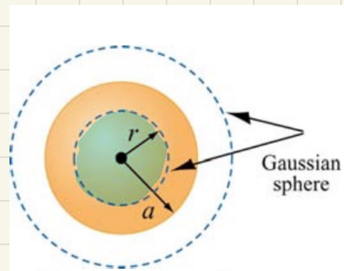
$$q_{enc} = \lambda \text{ longitud}$$

Volumétricas (aislantes)



$\rho \rightarrow$ densidad volumétrica de carga

$$\rho = \frac{Q}{\text{Volumen}} = \frac{Q}{\frac{4}{3}\pi a^3} \quad \frac{C}{m^3}$$

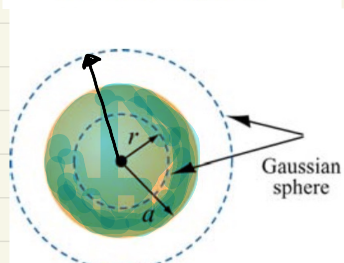


Concentric sphere

$q_{enc} = \rho \text{ Volumen}$

$\rightarrow$ para $r < a$

$$q_{enc} = \rho \frac{4}{3}\pi r^3$$

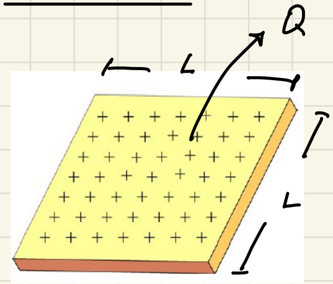


Concentric sphere

$\rightarrow$ para $r > a$

$$q_{enc} = \rho \frac{4}{3}\pi a^3 = Q$$

Superficiales

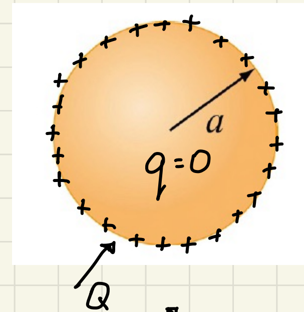


$\sigma \rightarrow$ densidad superficial de carga

$$\sigma = \frac{Q}{\text{Área}} = \frac{Q}{L^2} \qquad \left(\frac{C}{m^2}\right)$$

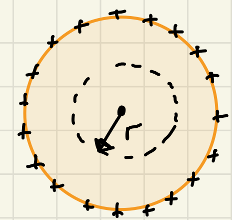
$$q_{enc} = \sigma \, \text{Área}$$

conductor



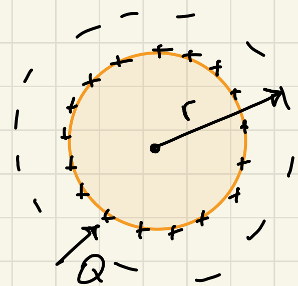
$$\sigma = \frac{Q}{4\pi a^2}$$

$\Downarrow$



para $r < a$

$$q_{enc} = \emptyset$$



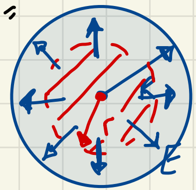
para $r > a$

$$q_{enc} = Q = \sigma \, 4\pi a^2$$

Problema 1.

Una esfera aislante con un radio de $0.15m$, tiene una densidad uniforme de carga de $\rho = 7.50nC/m^3$. Calcule el campo eléctrico a las siguientes distancias de su centro: a) inmediatamente afuera de su superficie. b) a $0.30m$ c) en el interior a $0.075m$

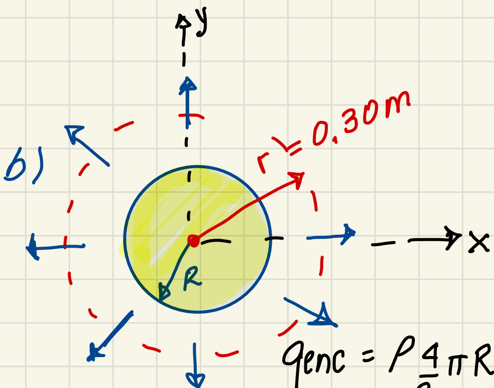
$\rho = 7.5\ nC/m^3$



$R = 0.15m$

$q_{enc} = \rho \frac{4}{3}\pi r^3$

b)



$q_{enc} = \rho \frac{4}{3}\pi R^3$

c) $\vec{E}\,(r = 0.075m)$

$$\oint \vec{E}\cdot d\vec{A} = \frac{q_{enc}}{\varepsilon_0}$$

$$E(\cancel{4}\pi r^{\cancel{2}}) = \frac{\rho \frac{\cancel{4}}{3}\pi r^{\cancel{3}}}{\varepsilon_0}$$

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$$E = \frac{\rho r}{3\varepsilon_0}$$

$$E = \frac{7.5\times 10^{-9}(0.075)}{3(8.85\times 10^{-12})}$$

$$\vec{E} = 21.186\ \frac{N}{C}\ \hat{r}$$

b) $\vec{E}\,(r = 0.30m)$

$$\oint \vec{E}\cdot d\vec{A} = \frac{q_{enc}}{\varepsilon_0}$$

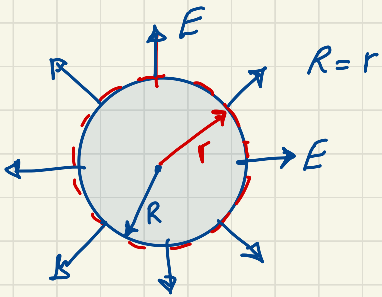
$$E(\cancel{4}\pi r^2) = \frac{\rho \frac{\cancel{4}}{3}\pi R^3}{\varepsilon_0}$$

$$E = \frac{\rho R^3}{3\varepsilon_0 r^2}$$

$$\vec{E} = \frac{7.5\times 10^{-9}(0.15)^3}{3\,\varepsilon_0\,(0.3)^2}\ \hat{r}$$

$$\vec{E} = 10.593\ \frac{N}{C}\ \hat{r}$$

a) $\vec{E}\,(r = 0.15\,m)$

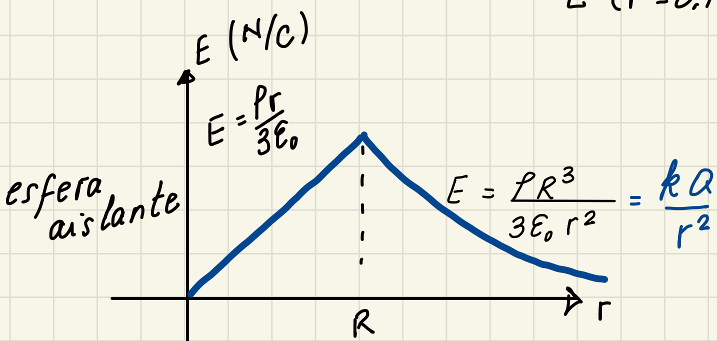


$$\oint \vec{E}\cdot d\vec{A} = \frac{q_{enc}}{\varepsilon_0}$$

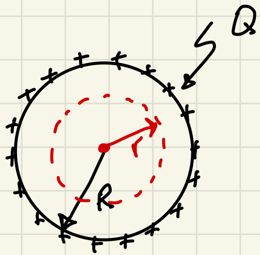
$$E(\cancel{4\pi} r^2) = \frac{\rho\, \frac{\cancel{4}}{3}\cancel{\pi} R^3}{\varepsilon_0}$$

$$E = \frac{\rho R^3}{3\varepsilon_0 r^2} \Rightarrow \text{como } r = R \Rightarrow E = \frac{\rho R}{3\varepsilon_0} = \frac{7.5\times10^{-9}(0.15)}{3(8.85\times10^{-12})}$$

$$\underline{\vec{E}(r = 0.15\,m) = 42.373\,\frac{N}{C}\,\hat{r}}$$

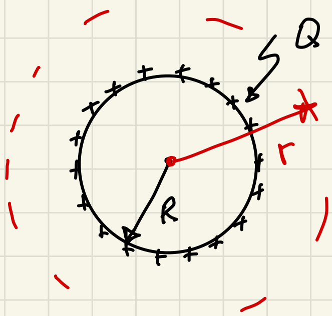


¿Qué sucede si tengo una esfera solamente cargada superficialmente?



$$\sigma = \frac{Q}{4\pi R^2}$$

a) $E(r<R) \Rightarrow q_{enc} = 0 \Rightarrow \underline{E = \emptyset}$



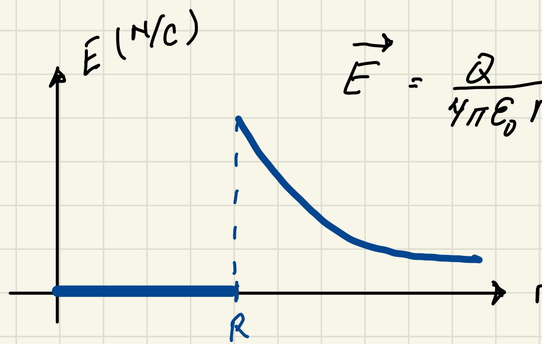
$$q_{enc} = Q$$

b) $E(r>R)$

$$\oint \vec{E} \cdot d\vec{A} = \frac{q_{enc}}{\varepsilon_0}$$

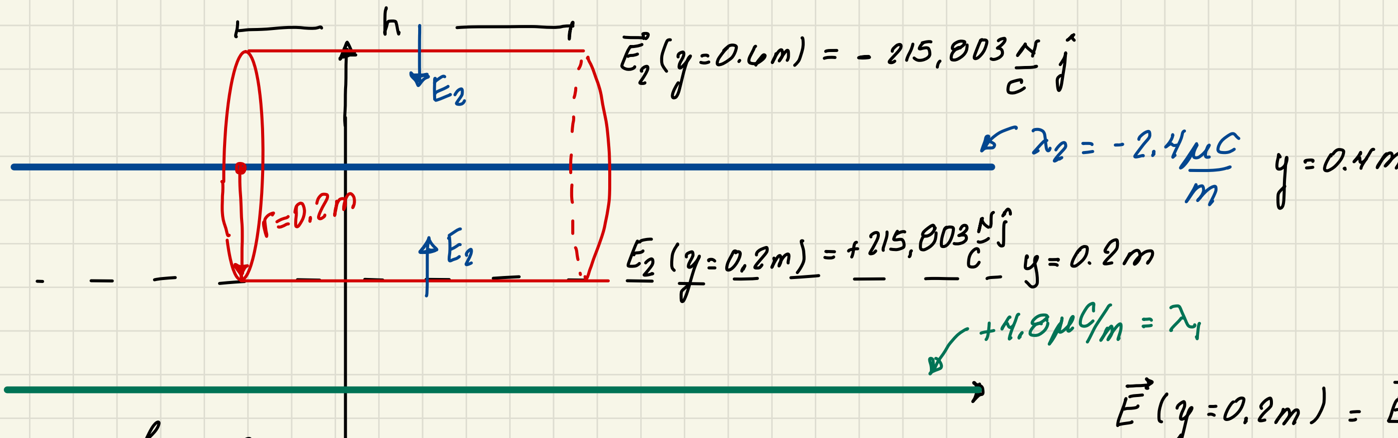
$$E(4\pi r^2) = \frac{Q}{\varepsilon_0}$$

$$\vec{E} = \frac{Q}{4\pi\varepsilon_0 r^2} = \frac{kQ}{r^2}\,\hat{r}$$



Problema 2.

Una línea con carga uniforme y muy larga tiene una carga por unidad de longitud de $+4.8\mu C/m$ yace a lo largo del eje "x". Una segunda línea con carga tiene una carga por unidad de longitud de $-2.4\mu C/m$ y es paralela al eje "x" en $y = 0.4m$. ¿Cuál es el campo eléctrico producido por ambas líneas en $y = 0.2m$?



para línea 2:

$$\oint \vec{E}\cdot d\vec{A} = \frac{q_{enc}}{\varepsilon_0} \qquad q_{enc} = \lambda_2 h$$

$$E(2\pi r \cancel{h}) = \frac{\lambda_2 \cancel{h}}{\varepsilon_0}$$

$$\vec{E}_2 = \frac{\lambda_2}{2\pi\varepsilon_0 r}\hat{r} = \frac{-2.4\times10^{-6}}{2\pi(8.85\times10^{-12})(0.2)} = -215{,}803\frac{N}{C}\hat{r}$$

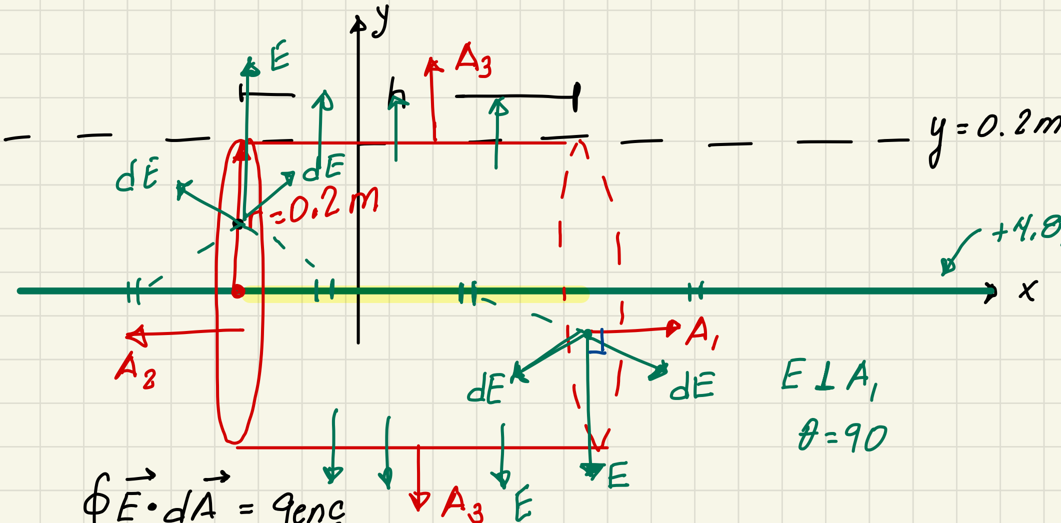
$$\vec{E}(y=0.2m) = \vec{E}_1 + \vec{E}_2$$

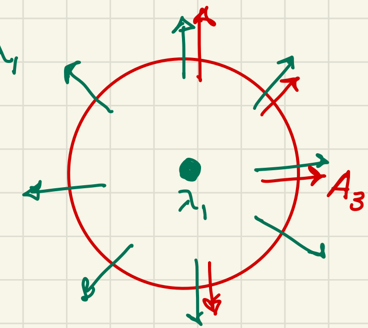
$$\vec{E}_1 = 431{,}606 \frac{N}{C}\hat{j}$$

$$E_2 = +215{,}803 \frac{N}{C}\hat{j}$$

$$\boxed{\vec{E} = 647{,}409 \frac{N}{C}\hat{j}}$$

$$q_{enc} = \lambda h$$





$$\oint \vec{E} \cdot d\vec{A} = \frac{q_{enc}}{\varepsilon_0}$$

$$\cancel{\int \vec{E} \cdot d\vec{A}_1} + \cancel{\int \vec{E} \cdot d\vec{A}_2} + \int \vec{E} \cdot dA_3 = \frac{\lambda_1 h}{\varepsilon_0}$$

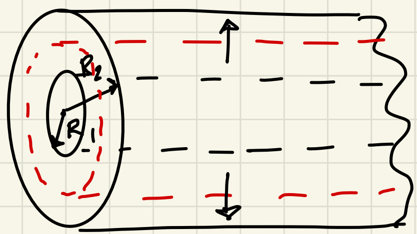
$$\vec{E}_1 = 431{,}606 \ \frac{N}{C} \ \hat{j}$$

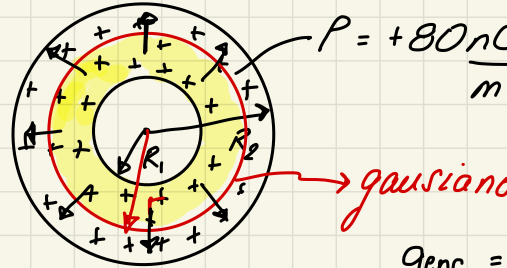
$$E(2\pi r \cancel{h}) = \frac{\lambda_1 \cancel{h}}{\varepsilon_0}$$

$$\boxed{\vec{E} = \frac{\lambda_1}{2\pi r \varepsilon_0} \hat{r}} = \frac{4.8 \times 10^{-6}}{2\pi (0.2)(8.85 \times 10^{-12})} = 431{,}406 \ \frac{N}{C} \ \hat{r}$$

Problema 3.

Un cilindro hueco de radio interior $1mm$ y radio exterior $3mm$ tiene una densidad volumétrica de carga de 80 nC/m^3, distribuida uniformemente en todo su volumen. a) Determine la magnitud del campo eléctrico (en N/C) en un punto localizado a 2 mm del eje del cilindro. b) Determine el campo eléctrico en un punto ubicado a 5 mm del eje del cilindro. c) Determine el campo eléctrico en un punto ubicado a 0.5 mm del eje del cilindro.





$\rho = +80 \frac{nC}{m^3}$

$R_1 = 1mm$

$q_{enc} = \rho\left[\pi r^2 h - \pi R_1^2 h\right]$

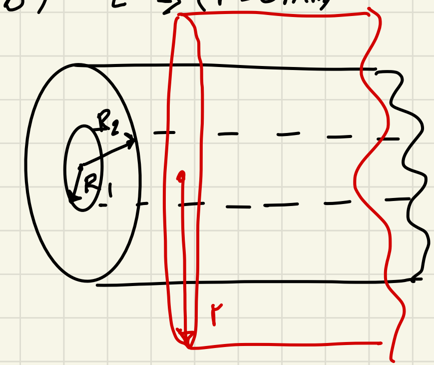
a) $E(r=2mm)$

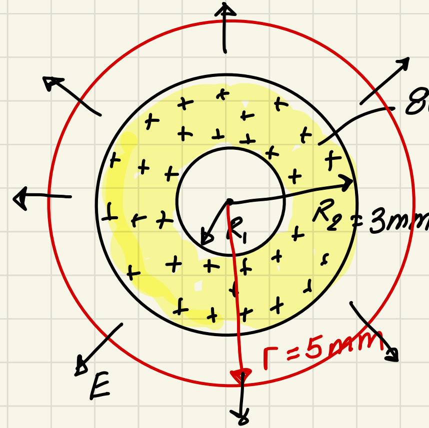
$$\oint \vec{E}\cdot d\vec{A} = \frac{q_{enc}}{\varepsilon_0}$$

$$E(2\cancel{\pi} r \cancel{h}) = \frac{\rho\left[\cancel{\pi} r^2 \cancel{h} - \cancel{\pi} R_1^2 \cancel{h}\right]}{\varepsilon_0}$$

$$\vec{E}(r=2mm) = \frac{\rho\,(r^2 - R_1^2]}{2\varepsilon_0 r} = \frac{80\times10^{-9}\left[(2\times10^{-3})^2 - (1\times10^{-3})^2\right]}{2\,(8.85\times10^{-12})(2\times10^{-3})} = \underline{6.78\ \frac{N}{C}\ \hat{r}}$$

b) $\vec{E} \Rightarrow (r = 5mm)$





$$q_{enc} = \rho\left[\pi R_2^2 h - \pi R_1^2 h\right]$$

$$\oint \vec{E}\cdot d\vec{A} = \frac{q_{enc}}{\varepsilon_0}$$

$$E(2\cancel{\pi} r \cancel{h}) = \frac{\rho \cancel{\pi} \cancel{h}\left[R_2^2 - R_1^2\right]}{\varepsilon_0}$$

$$E(r = 5mm) = \frac{\rho\left[R_2^2 - R_1^2\right]}{2\varepsilon_0 r}\,\hat{r}$$

$$= \frac{80\times10^{-9}\left(0.003^2 - 0.001^2\right)}{2\,\varepsilon_0\,(0.005)}$$

$$= 7.23\,\frac{N}{C}\,\hat{r}$$

c) $E(r = 0.5mm) \Rightarrow q_{enc} = 0 \Rightarrow E(r = 0.0005) = 0$