Intermediate report

Diagonalizing Spin Hamiltonians with the Lanczos Algorithm

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3rd December 2024

This report demonstrates the application of the *Lanczos algorithm* for approximating a hermitian matrix by a lower-dimensional tri-diagonal one. Given a tri-diagonal approximation on a low-dimensional subspace, standard methods for fully diagonalizing or computing matrix functions such as the exponential become feasible and usually give approximations for the corresponding objects of the original matrix. This method is especially efficient when applied to sparse matrices as typically arise in spin Hamiltonians. Here, we consider a system of many non-interacting two-level systems that couple equally to a harmonic oscillator—the Dicke model. The Dicke model features a so-called *superradiant phase*, where there are states that spontaneously emit more photons than expected. This phenomenon is typically described by a transition amplitude of spontaneous emission scaling up to quadratically with the number of two-level systems. Here, we apply Lanczos algorithm in order to simulate dynamical properties of the Dicke model. In particular, the expectation value for the number of photons depending on time is compared for different initial states and different coupling strengths. Prior to its application to this problem, the algorithm itself is being discussed in detail and is implemented in the programming language `python`.

1 Introduction

- motivation and background for spin systems
  - study of matter and light-matter interaction
  - look into QFTCM for justification of the Hubbard model
- motivation and background for the Lanczos method
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The time evolution for a duration $t \in \mathbb{R}$ of a quantum system modeled by a time-independent self-adjoint operator H, the *Hamiltonian*, acting on some Hilbert space $\mathcal{H}$ is given by the unitary

operator[1] $U(t) = e^{-\mathrm{i}tH}$.

[1] Here, and in the following we work in natural units where $\hbar = 1$.