Intermediate report

Diagonalizing Spin Hamiltonians with the Lanczos Algorithm

Leonhard Richter

15th November 2024

This report demonstrates the application of an efficient method for approximately diagonalizing sparse hermitian matrices—the *Lanczos method*. An espacially suitable usecase for this method is the application to spin Hamiltonians with only local interactions. Here, an Ising model coupled to a monochromatic cavity field—the Dicke-Ising model— is being considered and the time evolution of the expected number of photons in the cavity gets approximated using the Lanczos method. This method itself is being discussed in detail and is implemented in the programming language `julia`.

1 Introduction

- motivation and background for spin systems
  - study of matter and light-matter interaction
  - look into QFTCM for justification of the Hubbard model
- motivation and background for the Lanczos method
  -

The time evolution for a duration $t \in \mathbb{R}$ of a quantum system modeled by a time-independent self-adjoint operator H, the *Hamiltonian*, acting on some Hilbert space $\mathcal{H}$ is given by the unitary operator[1] $U(t) = e^{-\mathrm{i}tH}$.

[1] Here, and in the following we work in natural units where $\hbar = 1$.