How Does the Primordial Binary Fraction Affect the Survival Time of an Open Cluster in the Galactic Disk?

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1 Abstract

This study investigates how primordial binary star populations influence the long-term stability of star clusters. Using a custom-built 2D N-body simulation, clusters with varying initial binary fractions (0%, 8%, 16%, 24%) were modeled over extended periods. Results show that clusters with binaries consistently retained more mass and exhibited slower core contraction compared to those without, supporting the theory that binary interactions act as an internal energy source that delays core collapse. The stabilizing effect increased steadily with binary fraction, with no observed threshold where binaries became detrimental. These findings suggest that the size of the primordial binary fraction has a significant positive effect on cluster longevity.

2 Introduction

Star clusters are dense, gravitationally bound systems composed of hundreds to thousands of stars. Due to their many-body interactions, they evolve dynamically over time, gradually losing mass and structure. A key process driving this evolution is two-body relaxation, in which repeated weak gravitational encounters cause some stars to sink toward the center while others gain energy and escape.

As more massive stars concentrate in the core, gravitational interactions become more frequent and intense, increasing the central density. This leads to the preferential ejection of lower-mass stars, further accelerating the contraction. Eventually, a dense core composed of stellar remnants forms—a process known as core collapse. In many cases, the remaining massive stars have evolved into neutron stars or black holes, creating a "dark remnant core."

Primordial binaries—binary systems that formed early in the cluster's history—play an important role in this evolution. These binaries serve as internal energy sources, gradually releasing energy through gravitational interactions. This energy can inhibit core collapse by maintaining pressure support in the core, or in some cases, accelerate mass loss by ejecting lighter stars.

While the influence of binary stars on cluster dynamics is well established, the exact relationship between primordial binary fraction and cluster lifetime remains unclear. Though higher binary fractions are expected to increase stability, they may also lead to increased dynamical interactions and ejections. The computational cost of high-resolution N-body simulations, which scale at $O(N^2)$, limits the ability to explore this question fully.

This study models the effect of varying primordial binary fractions on cluster stability using a custom 2D N-body simulation. By comparing structural evolution and mass retention across different binary fractions, the aim is to establish a correlation between binary content and cluster survival time.

3 Literature Review

Globular clusters are among the most tightly bound stellar systems, making them ideal candidates for dynamical modeling via N-body simulations [Heggie and Hut(2003)].

A key mechanism in their evolution is two-body relaxation, where stars exchange energy and angular momentum through small repeated gravitational encounters [Binney and Tremaine(2008)]. This leads to energy equipartition, with massive stars moving inward and low-mass stars drifting outward or escaping. Relaxation timescales in real clusters span 10^8 to 10^9 years [Spitzer(1987)]. In comparison a small star system takes around a minute to run using my simulation.

The resulting mass segregation causes a dense core of massive stars to form. If unchecked, this leads to core collapse—an observed feature in both simulations and real clusters [Meylan and Heggie(1997)]. In advanced stages, the core is often dominated by compact remnants such as white dwarfs, neutron stars, or black holes [Baumgardt et al.(2003)Baumgardt, Makino, and Hut] hence is called a dark remnant core.

Primordial binaries help counteract collapse by injecting energy into the system. During interactions, hard binaries (those with tight orbits) transfer kinetic energy to surrounding stars, becoming even more tightly bound in the process—a phenomenon known as binary heating [Heggie(1975)]. This mechanism delays core contraction and stabilizes the system [Goodman and Hut(1989)].

Studies such as Hurley et al. [Hurley et al.(2007)] and Fregeau et al. [Fregeau et al.(2003)] show that higher binary content slows core contraction and improves mass retention. However, the long-term impact of different binary fractions on cluster survival remains uncertain, especially in simplified models.

Despite their limitations, low-N simulations are still valuable for identifying general trends in dynamical behavior [Portegies Zwart et al.(2010)]. This work contributes to that effort by quantifying the effect of primordial binaries on cluster decay and structure.

4 Theoretical Background

The evolution of star clusters is driven by gravitational interactions. Key mechanisms include two-body relaxation, mass segregation, and heating via binary stars.

4.1 Two-Body Relaxation

Although stars can be estimate to follow straight trajectories over short timescales, weak gravitational interactions gradually alter their paths. This cumulative effect—two-body relaxation—results in energy exchange and orbital changes. Over time, this leads to energy equipartition: low-energy stars migrate outward while high-energy stars sink inward.

The timescale for relaxation depends on particle number and density and typically spans hundreds of millions to billions of years.

4.2 Mass Segregation and Core Collapse

As energy is redistributed, massive stars lose energy and move toward the core, while lighter stars are pushed outward or ejected. This process, known as mass segregation, increases central density and raises the likelihood of strong gravitational interactions. Eventually, the core contracts—a runaway process leading to collapse. At this stage, the cluster core is typically dominated by stellar remnants (dark remnant core).

4.3 Primordial Binary Stars and Binary Heating

Primordial binaries significantly influence cluster evolution. During close encounters, binary systems can transfer kinetic energy to nearby stars, a process known as binary heating. This energy input counters the tendency for the core to collapse.

Hard binaries become more tightly bound while transferring energy outward, functioning as internal heat sources. Clusters with high binary fractions tend to maintain stability longer and lose mass more gradually. However, wide or unbound binaries can behave differently, potentially destabilizing the system through high-energy interactions.

4.4 Theoretical Evolution

Typical evolutionary stages for a star cluster include:

1. Widely spaced stars with low average kinetic energy.
2. Energy exchange via two-body relaxation leading to mass ejection.
3. Inward migration of massive stars due to mass segregation.
4. Gradual core contraction until collapse or stabilization via binary heating.

5 Methodology

5.1 Simulation Overview

To investigate the impact of primordial binaries, I developed a custom N-body simulation to measure the longevity of star clusters with varying binary fractions.

5.2 Program Overview

A custom Python simulation was developed using the NumPy library to handle mathematical operations efficiently. Due to the $O(N^2)$ complexity, performance scaled poorly with higher particle counts.

Accurate time integration was essential. Small time steps were used to reduce numerical errors and maintain energy conservation below $10^{-6}\%$. Simulations employed an individual time-step scheme and a direct high-precision N-body integrator. No softening was used, ensuring accurate modeling of close encounters.

Initial particle positions were randomized while controlling total mass and velocity distributions. Each scenario was run ten times to reduce the influence of stochastic effects.

Gravity was modeled using Newton's law of gravity:

$$F_g = \frac{G m_a m_b}{r^2}$$

A scaled value of G was used to shorten simulation timescales while maintaining realistic dynamical behavior.

According to Neuton's 3rd law every action has an equal and opposite reaction, which can be expressed as

$$F_1 = -F_2$$

which is consequently equivalent to

$$m_1 a_1 = -m_2 a_2$$

When the simulation is running, each particle calculates the force vector magnitude using Neuton's law of gravity and the vector's direction using

$$\theta = tan^{-1}(\frac{y_1 - y_1}{x_1 - x_2})$$

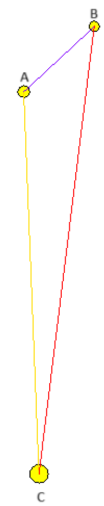

Figure 1: Visualized force vectors for the N-Body simulations

As previously said, the computational resources required scale quadratically with the number of particles. For example a system with 400 particles will require 16 times more resources than a system with 100 particles. To help optimize the simulation, Neuton's 3rd law of motion was used to apply forces to pairs of particles, essentially halving the computational cost.

5.3 Initial Conditions

Simulation parameters are summarized in Table 1. Units are abstract and internally consistent.

Table 1: Simulation Parameters

Parameter	**Value**
G	500
Mass scale	500
Collision radius	0.01
Initial speed	5
Cluster radius	400
Total mass	7000

5.4 Analysis Metrics

Clusters were analyzed using Half-Mass Radius The radius enclosing half the cluster mass. The radius in which the total mass was measured in was controlled with the variable Cluster radius. Mass was kept consistent across all simulations at 7000 units. The average particle speed was set to 5 units however the average velocity of all particles was equal to zero. This was done by randomly assigning velocities of a component, working out the kinetic energy of the component using:

$$KE = \frac{1}{2}mv^2$$

and then using the SoftMax function on this particular component vector of all particles to make the sum of all component forces to equal one.

$$\alpha(z) = \frac{e^{z_i}}{\Sigma_{j=1}^{K} e^{z_j}}$$

Where K is the size of the input vector This process was done four times on each 2D direction, then added together and multiplied by the number of particles multiplied by the initial speed variable.

A cluster was considered decayed when less than 50% of its original mass remained bound.

5.5 Investigation

I have decided to investigate four separate binary fractions: 0%, 8%, 16%, and 24%. The total mass of the system and the total Kinetic energy of the system was equal for all situations, which helped avoid randomized data. Furthermore, each simulation was repeated 10 times and the data averaged into one line on the graph. Despite my efforts to completely remove the random aspects of the investigation, a relatively small particle number (250) as well as randomized position initialization caused data to be noisy as shown on Figure 2 because of different gravitational potential energy stores. Nevertheless, it showed a clear trend:

6 Results

Clusters were simulated with binary fractions of 0%, 8%, 16%, and 24%, each averaged over ten runs.

6.1 Mass Retention Over Time

Figure 2 shows retained mass over time. The 0% binary case exhibited the fastest decay. Higher binary fractions showed improved mass retention, with 24% binaries performing best.

Clusters with 24% binary content survived approximately 30,000 simulation units longer than those with no binaries.

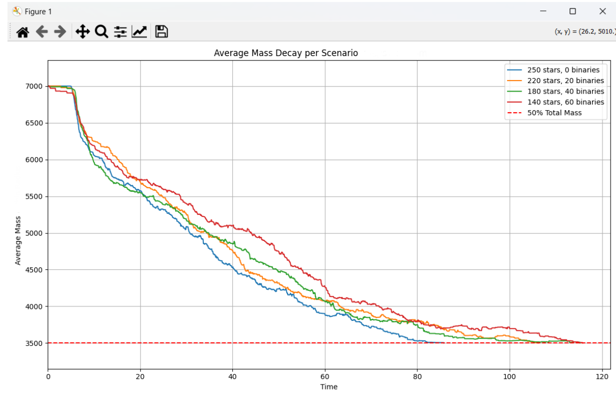

Figure 2: Mass retention vs. time for varying binary fractions.

6.2 Core Radius Evolution

Mass within the core radius (user-definer radius from the center of mass) was used to assess collapse. Figures 3 and 4 illustrate the difference between binary-free and binary-rich clusters.

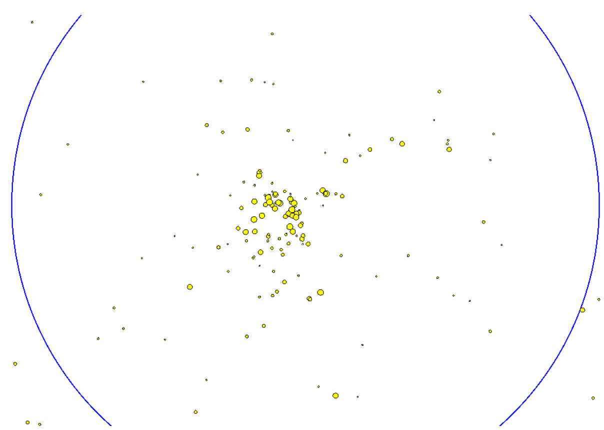

Figure 3: Core structure for 250 non-binary stars.

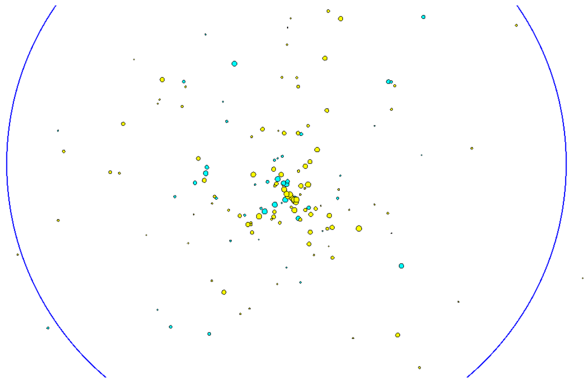

Figure 4: Core structure for 180 non-binary stars and 35 binary pairs.

The binary-rich system retained more off-center stars, indicating that binary heating helped prevent full collapse by sustaining peripheral orbits. Nevertheless, the process of core collapse is present in both Figure 3 and Figure 4, demonstrating that high binary fractions only delay the process not avoid it.

7 Discussion

Results confirm that primordial binaries enhance cluster stability by injecting energy into the system, delaying collapse, and reducing mass loss. Clusters with more binaries survived significantly longer.

No diminishing returns were observed up to 24% binary content. The stabilizing effect of binaries outweighed the increased dynamical interactions and associated ejections.

While the simulations were 2D and used abstract units, the qualitative trends align with theoretical predictions and prior studies.

8 Conclusion

This study investigated how varying primordial binary fractions affect the longevity of star clusters. Simulations demonstrated that higher binary content consistently improved mass retention and delayed core collapse. This happened because primordial binaries act as internal energy sources, extending the life of the cluster and help slow down two-body relaxation. Despite simplifications like 2-Dimensional space, results support established theory and provide a basis for further study. On average a 8% increase in primordial star fraction increases the half-life of the cluster by 5%.

Future work should explore 3D models, higher particle counts, and external tidal fields to better approximate real clusters.

9 Code Repository

The full Python source code and LaTeX document are available at:

`https://leonid-elkin.github.io/`

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