Getting in Between People and Their Measurement
Psychometric Evaluation of (Intensive) Longitudinal Data

TILBURG UNIVERSITY

Understanding Society

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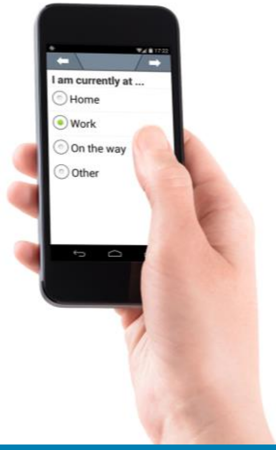
Experience Sampling

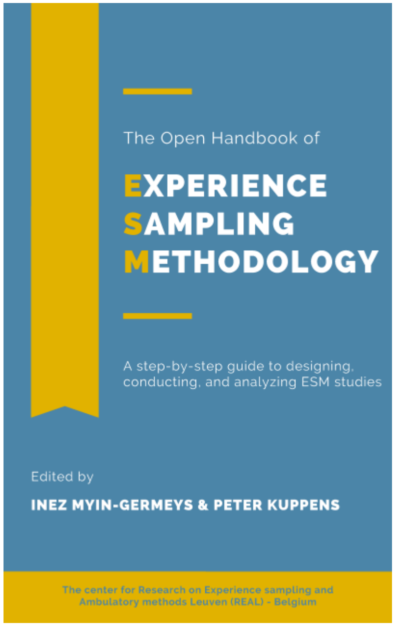
- Experience Sampling and related methods (Ambulatory Assessment, Ecological Momentary Assessment) are becoming more and more common on social sciences.
- Methods aim to study processes as they unfold in real-time and in the real world.
- As such many measurements are taken (>20) that are close together in time (every couple of hours instead of every couple of months) resulting in Intensive Longitudinal Data (ILD).



Experience Sampling

- Experience Sampling and related methods often include self-report measures in the form of questionnaires.
- This means: we have the to worry about the same things we always have to worry about with questionnaires (i.e., reliability, validity, measurement invariance, response styles).
- Unlike for cross-sectional data we don’t really have validated questionnaires for ILD yet:
  - And existing questionnaires can’t just be used in ESM studies.
  - The frequent measurements cause all kinds of new challenges.

Experience Sampling



- Chapter 4 from the “Free ESM Handbook” gives guidelines on how to construct questionnaires for ESM studies (think of questionnaire length, wording, response scale, order of questions…).

- In this lecture, we’ll focus on how to evaluate your measurements after the data has been collected.

- We’ll start by recapping some of the basics of proper measurement in cross-sectional and panel data.

- We’ll then extend the insights from “traditional” data types to ILD and will highlight what additional steps and checks you need to do with this new type of data.

Outline

- Introduction to Different Data Types
- Psychometrics for Cross-Sectional Data
- Lab 1: Invariance & Reliability for Cross-Sectional Data

- Psychometrics for Panel and Intensive Longitudinal Data
- Lab 2: Reliability & Invariance for Longitudinal Data

- Exploring Non-Invariance for Intensive Longitudinal Data
- Lab3: Latent Markov Factor Analysis

Introduction to Different Data Types

Measuring Psychological Constructs

	Yes	No
I often have cold feet in bed.		

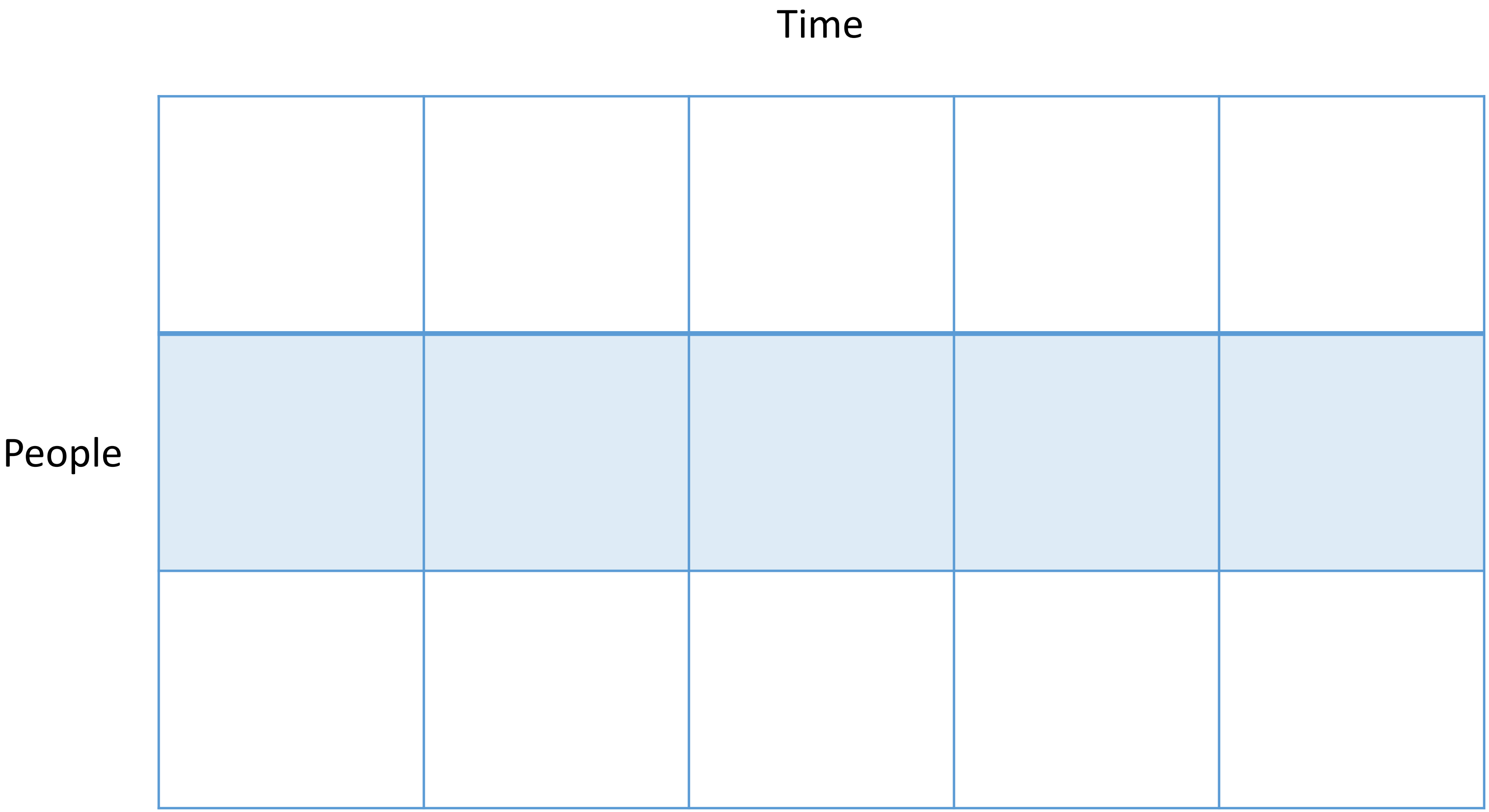
Measuring Psychological Constructs

	Yes	No
I bump my head often.		
The seat of my bike is very low.		
My shoes are too big for most people.		
I often have cold feet in bed.		
My shoe size is one of the larger children's sizes.		
I literally look up to most of my friends.		
I think I would do well on a basketball team.		
In the bus, I have trouble reaching the grips hanging from the ceiling.		
When walking up the stairs, I usually take two or more steps at a time.		
Other people will avoid being aggressive to me.		
If I were a policeman, I wouldn’t be very imposing.		
I’m uncomfortable sitting in the seats of lecture rooms.		
While walking, I often take more steps than the person I’m with.		
I often have to stand on my toes to look in the mirror.		
People often literally look over me.		
I have to pay extra attention to not bump my head against doorposts.		

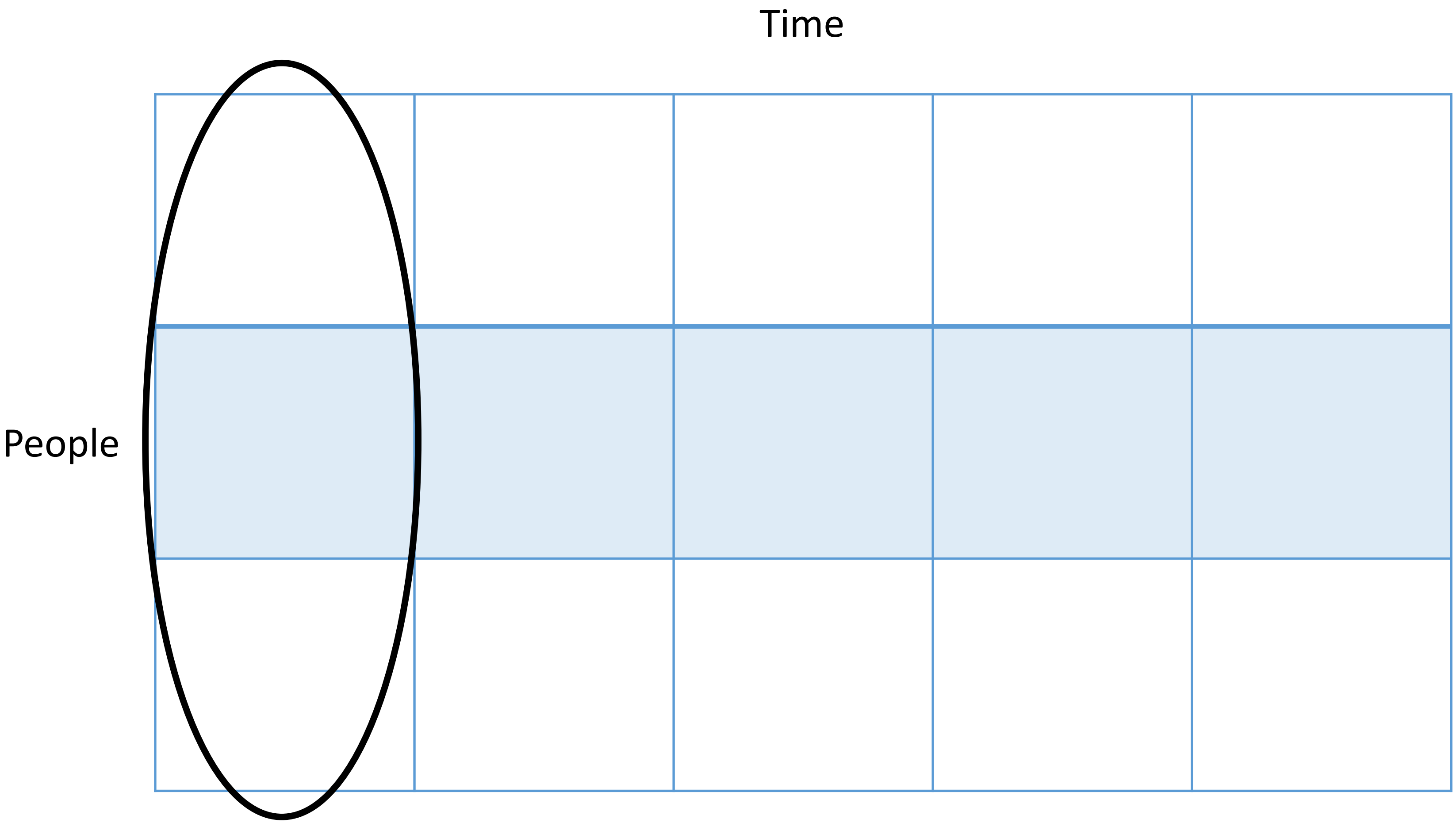
Measuring Psychological Constructs

- Questionnaires applied to three types of data (mainly)
  - Cross-sectional
  - Panel
  - Intensive Longitudinal

Three Types of Data



Three Types of Data



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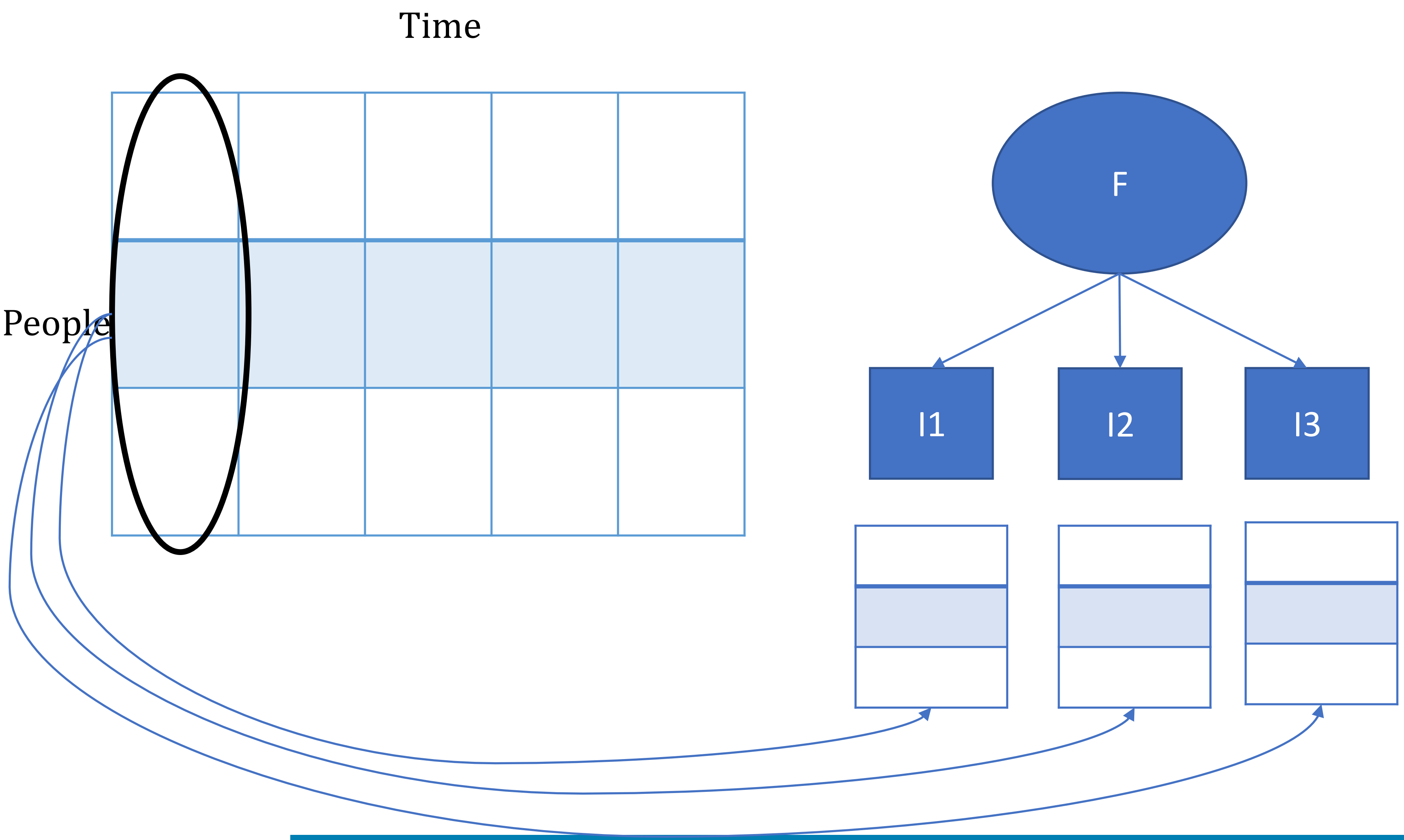

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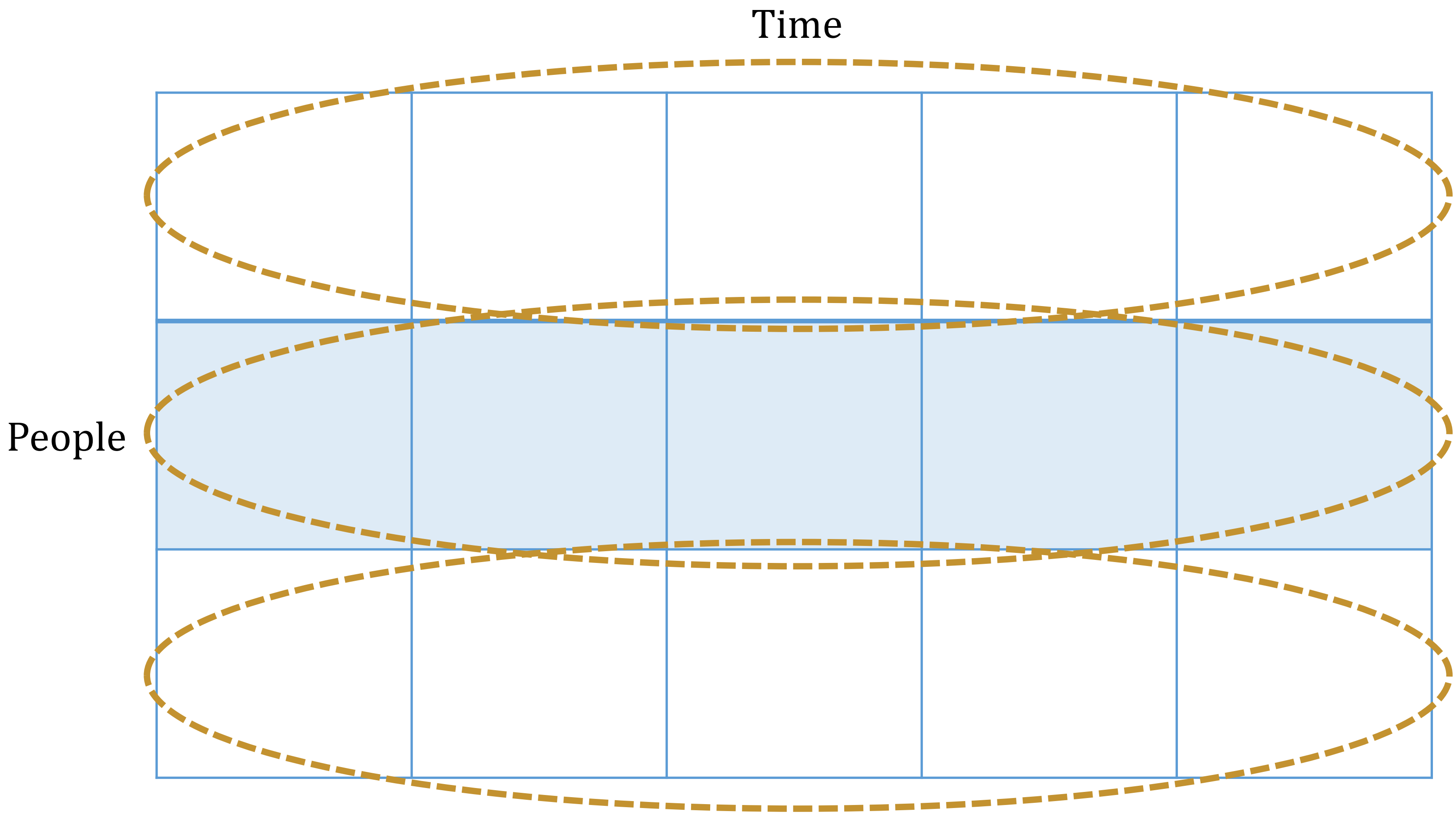

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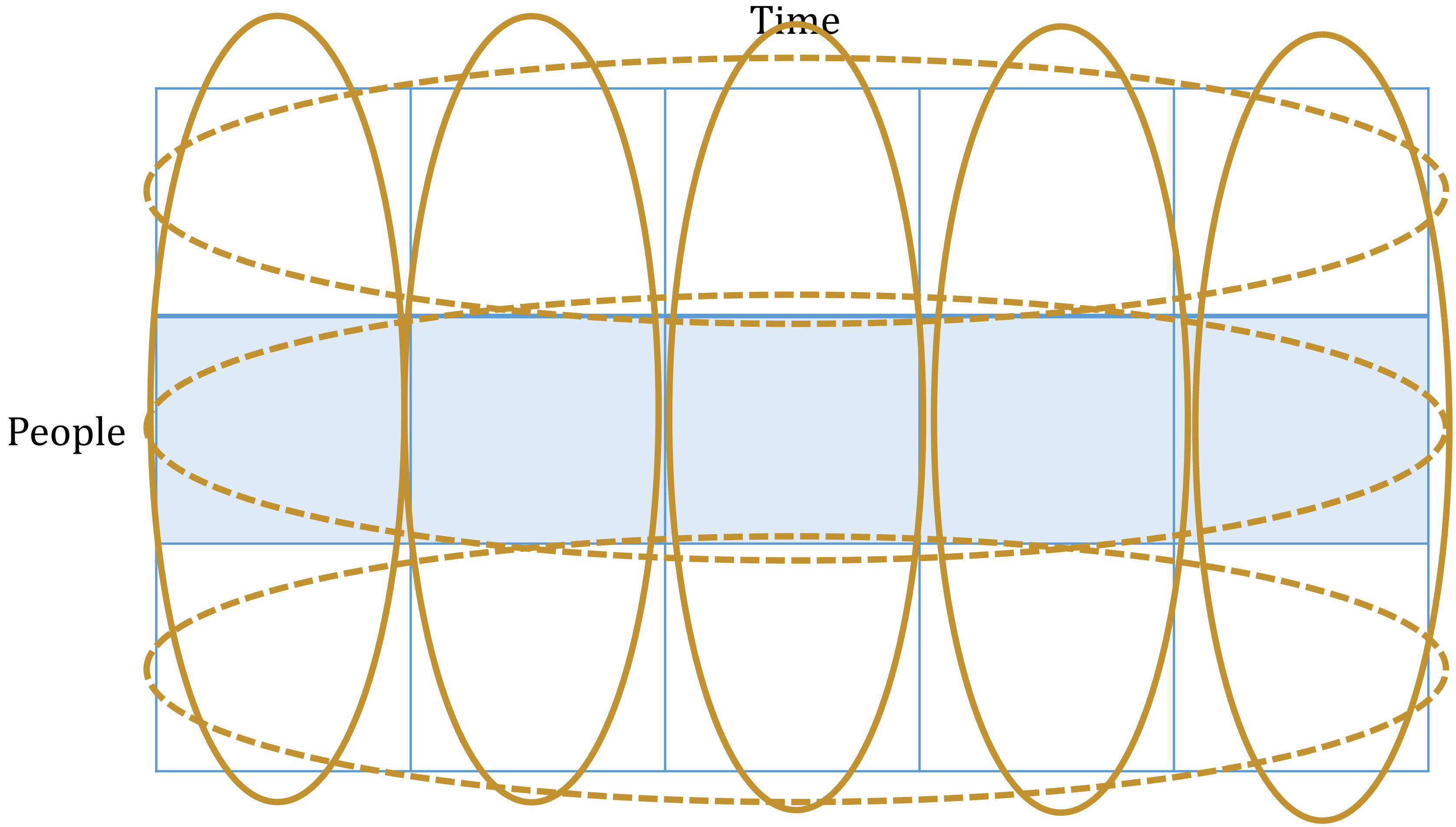

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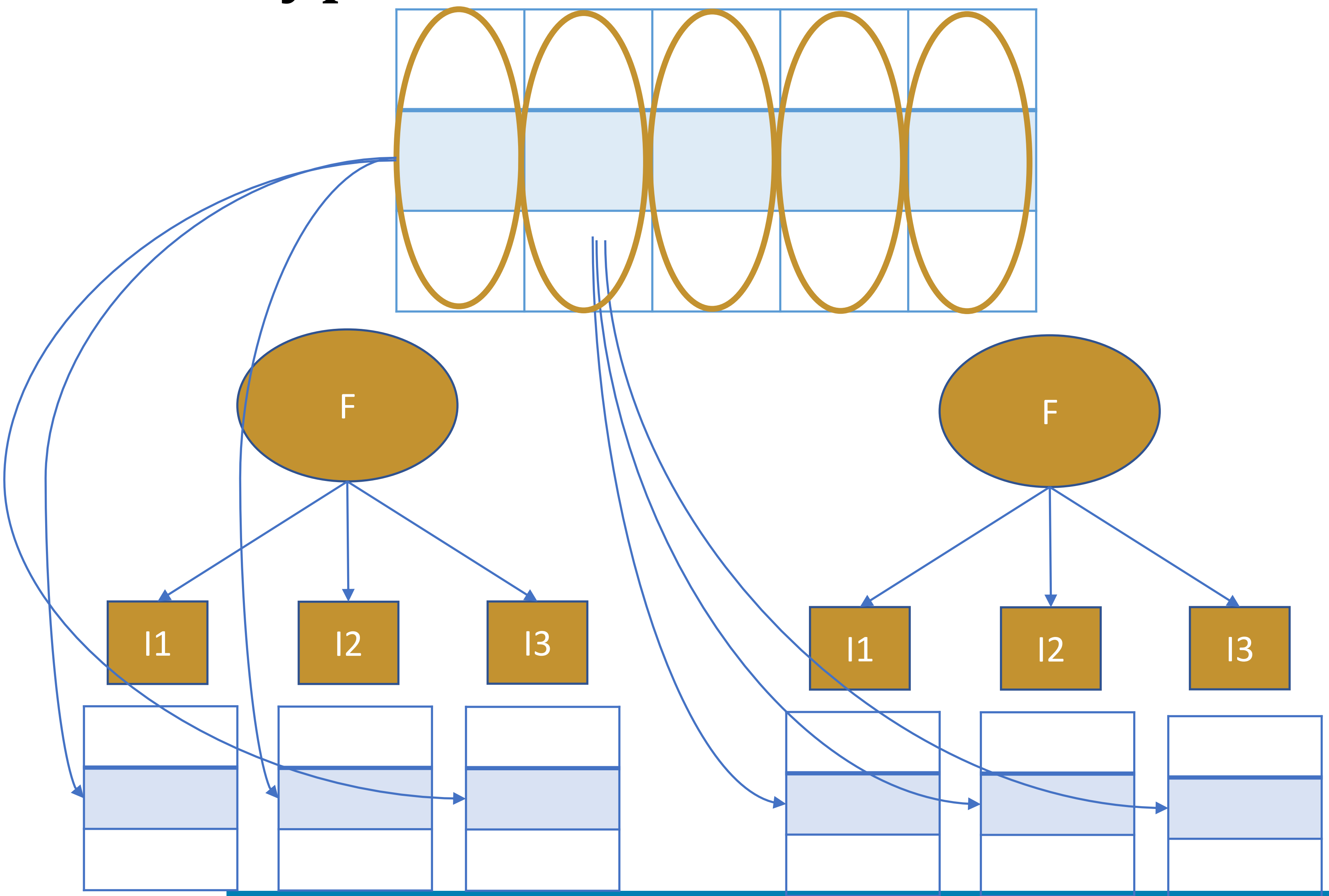

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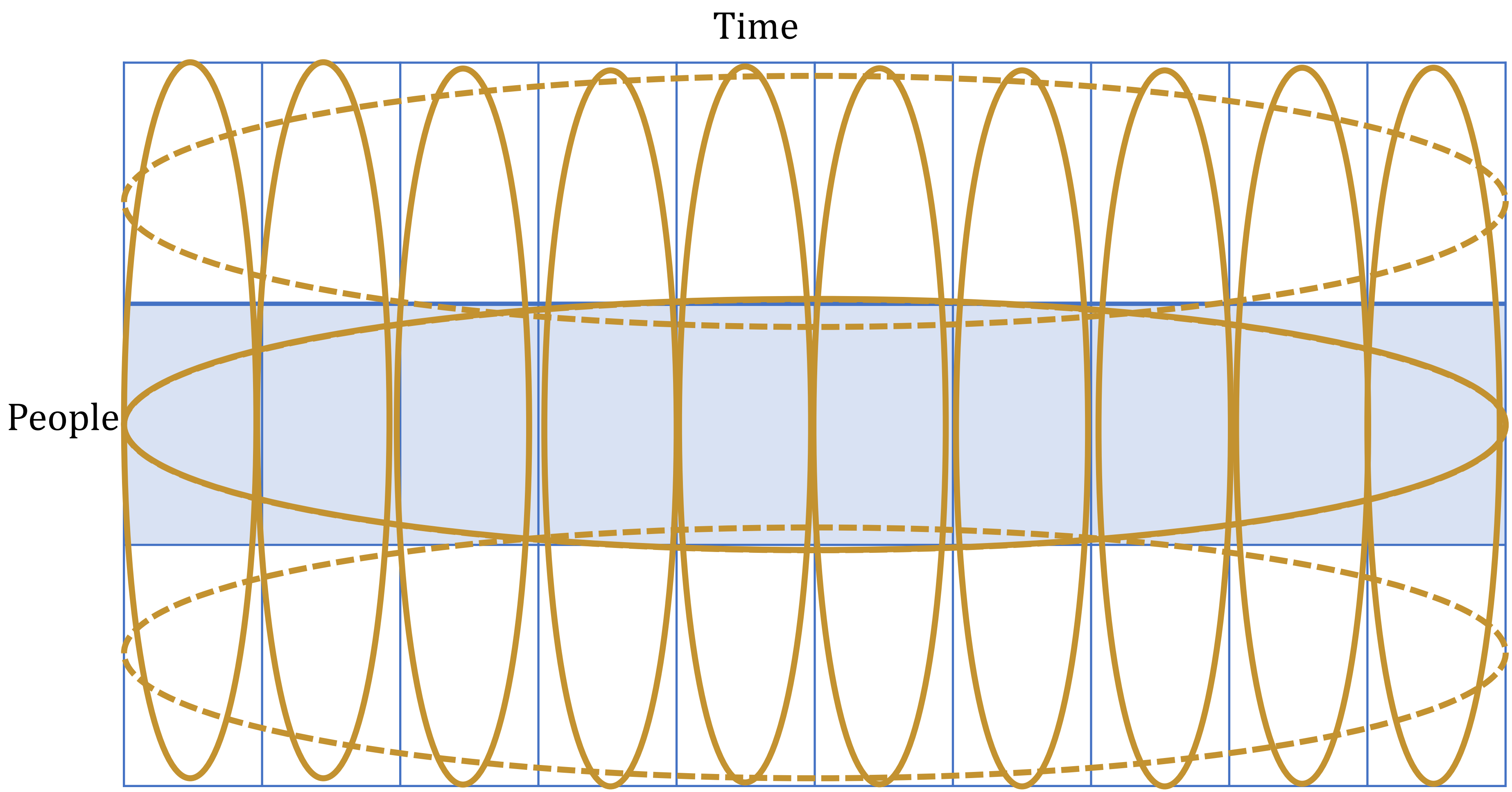


Introduce Cross-Sectional & Panel Data

To what extent do the following statements apply to you?	1-7
I often check the location of security cameras.	
I count the number of security guards.	
I sense whether someone is an easy target.	
I enjoy getting others in trouble.	
I enjoy other people's pain.	
I dream about stealing money.	
I do not sympathize with others.	
I root for the bad guy in movies.	
I have been told that I am not a good roommate.	
I am more important than others.	
I am bored by other people.	
I often walk around naked outside.	
I often bully/bullied my siblings.	
I don't have many friends.	
Money is more important than relationships.	
I would hurt others to get what I want.	

Psychometrics for Cross-Sectional Data

Psychometrics for Cross-Sectional Data

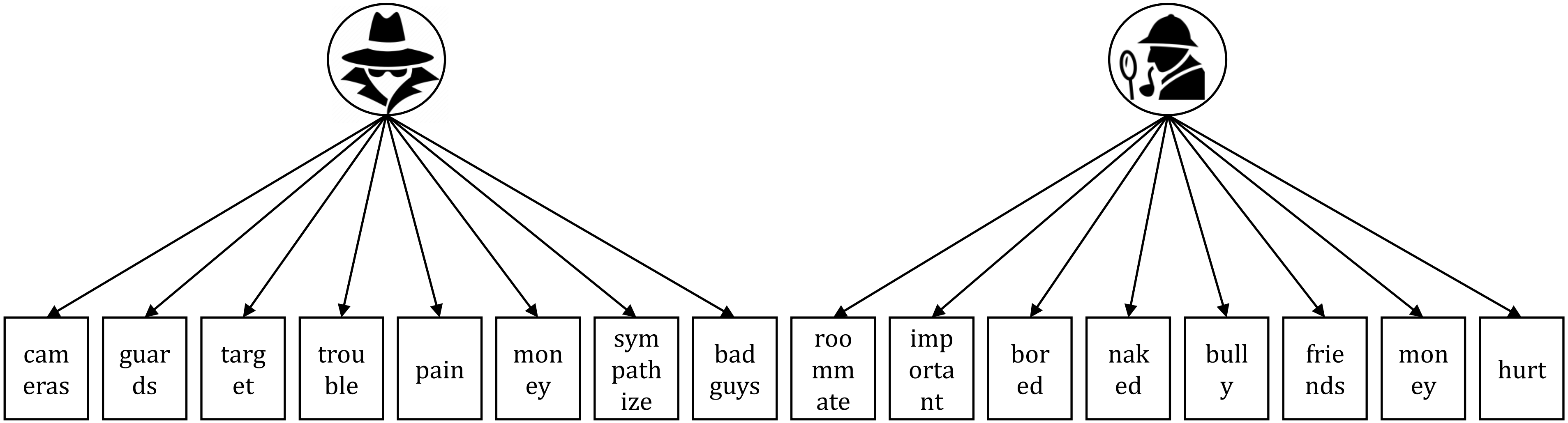
- Factor model
  - Do items measure what they are expected to measure?
  - It allows distinguishing the amount of variance in each item that is due to the common factor and the remaining error variance.

- Invariance
  - Is all systematic variability in item scores attributable to the psychological construct of interest?
  - Or is there variability due to group membership?
    - Does the factor model differ across groups?

- Reliability
  - Determining the reliability of assessing between-person differences.
  - When we have multiple items that measure the same underlying construct, internal consistency can be used as a measure of reliability.

Factor Model

Factor Model



- Allows to investigate whether items that are expected to measure a factor indeed measure the factor.
- Allows distinction between item variance that is due to the common factor and unique item variance.
- You can think of factor analysis as regressing item scores on the latent factors. Regression weights are called loadings.

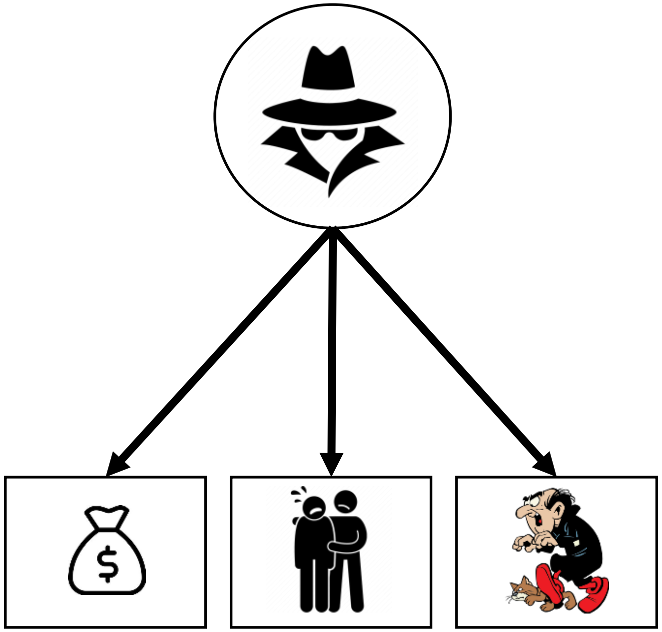
Factor Model Cross-Sectional Data

Regression notation:

$$\boldsymbol{y}_i = \tau + \boldsymbol{\Lambda}\boldsymbol{\eta}_i + \epsilon_i$$

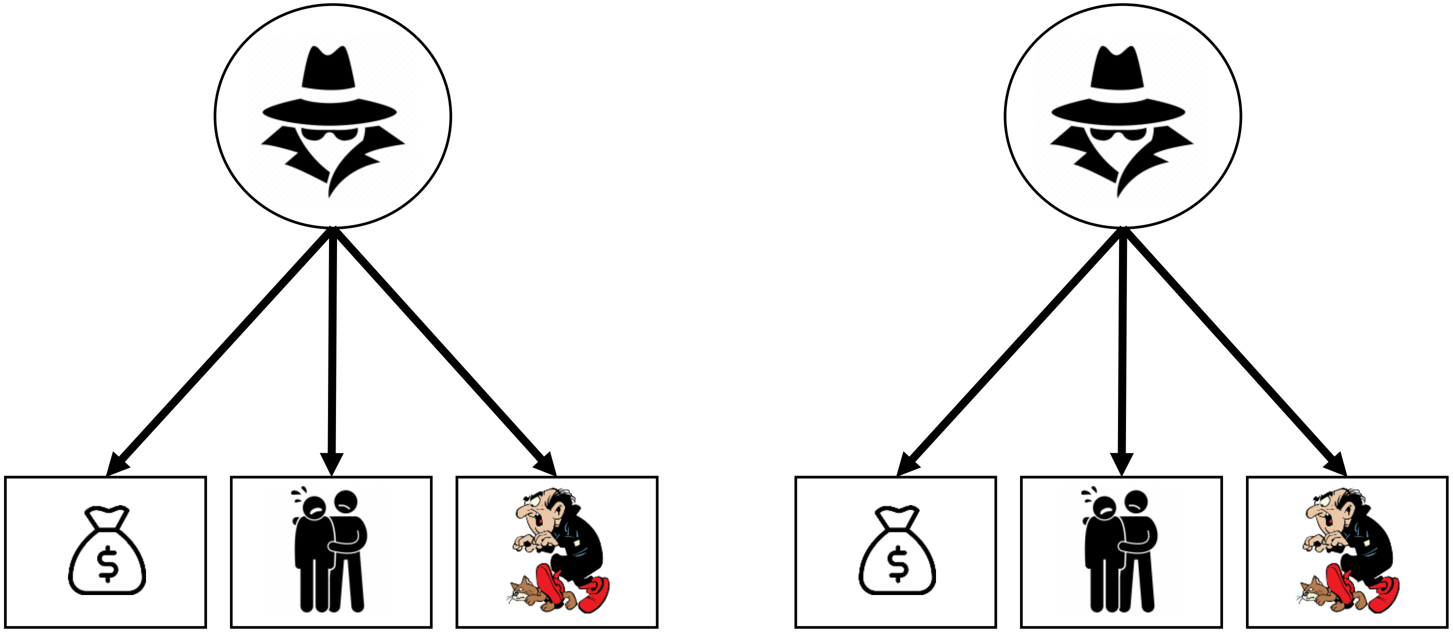
Measurement Invariance

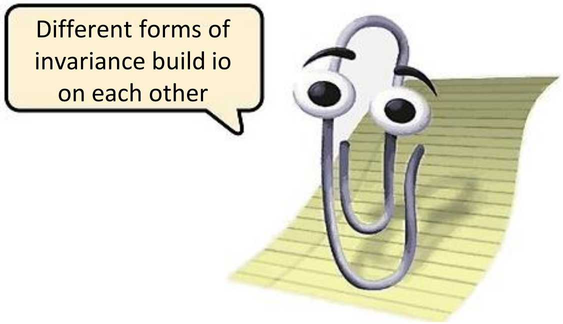
Measurement invariance



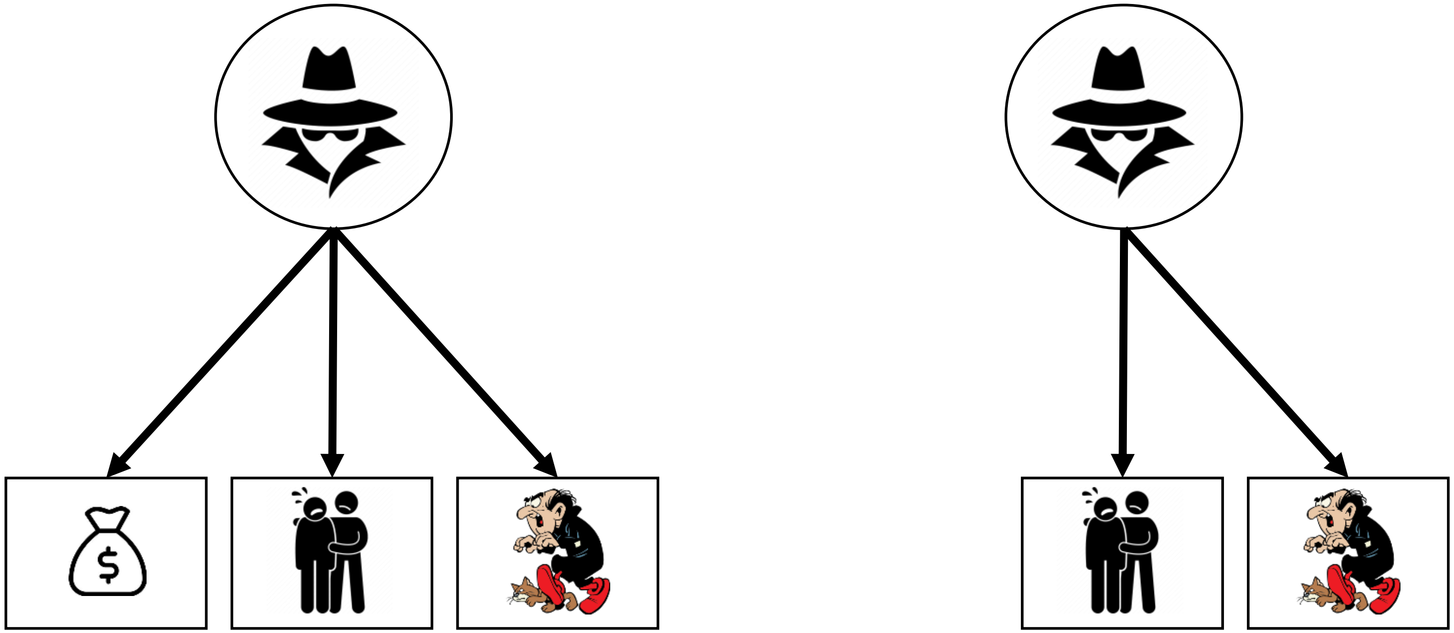
I dream about stealing money.
I do not sympathize with others.
I root for the bad guy in movies.

Configural invariance



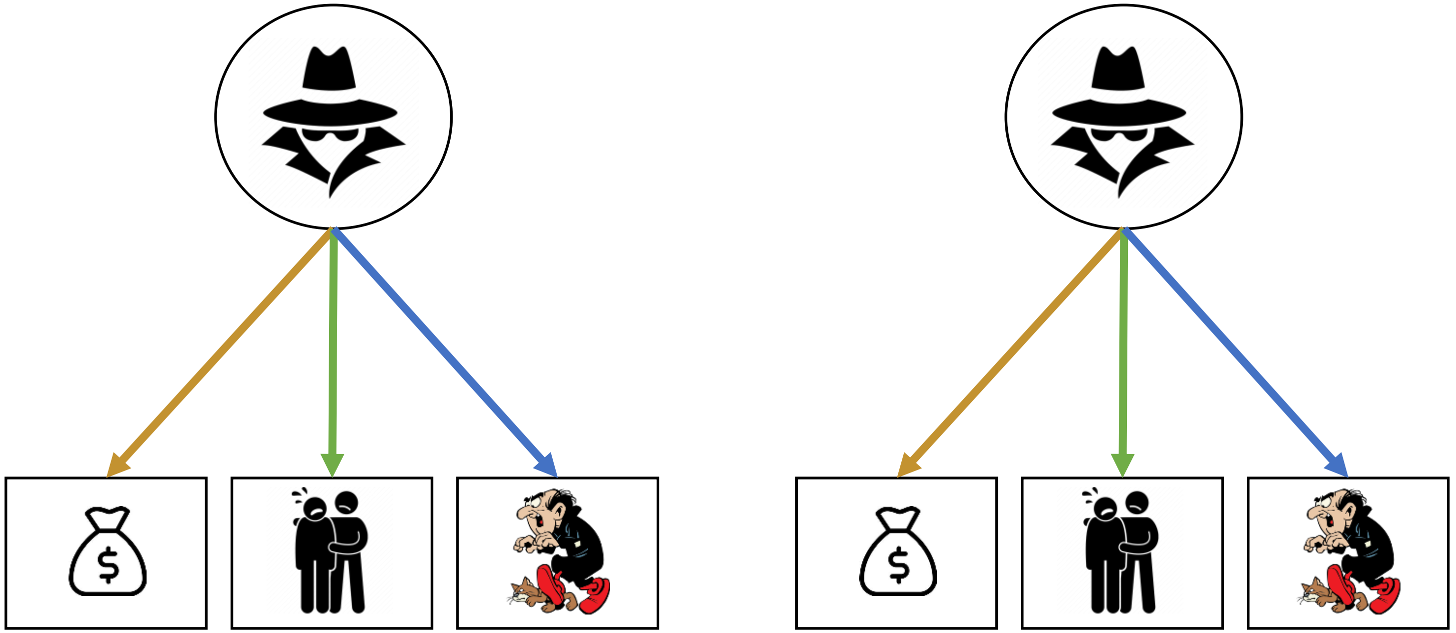


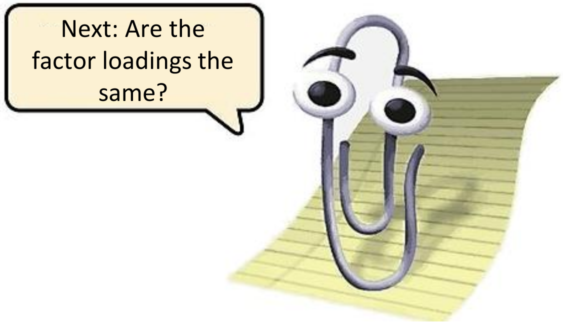
Configural invariance



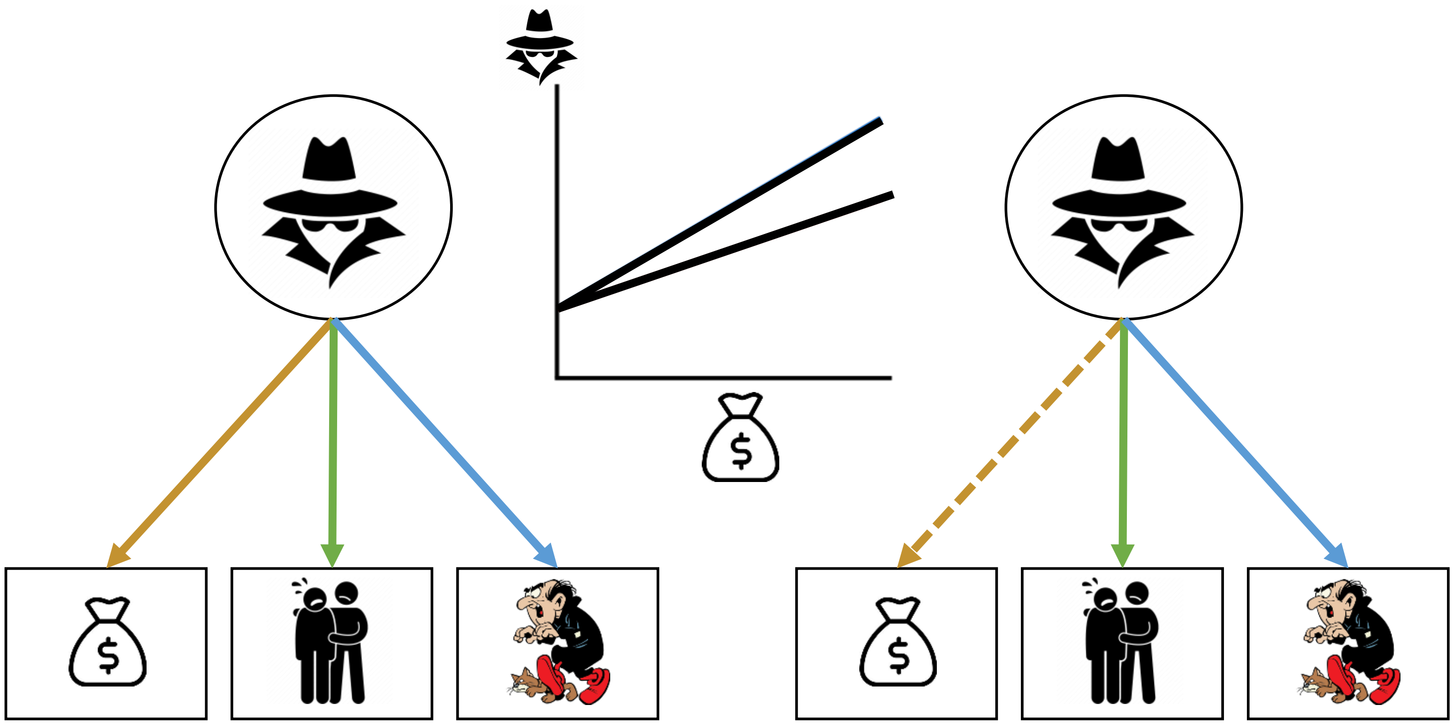
lavaan
latent variable analysis

Metric invariance

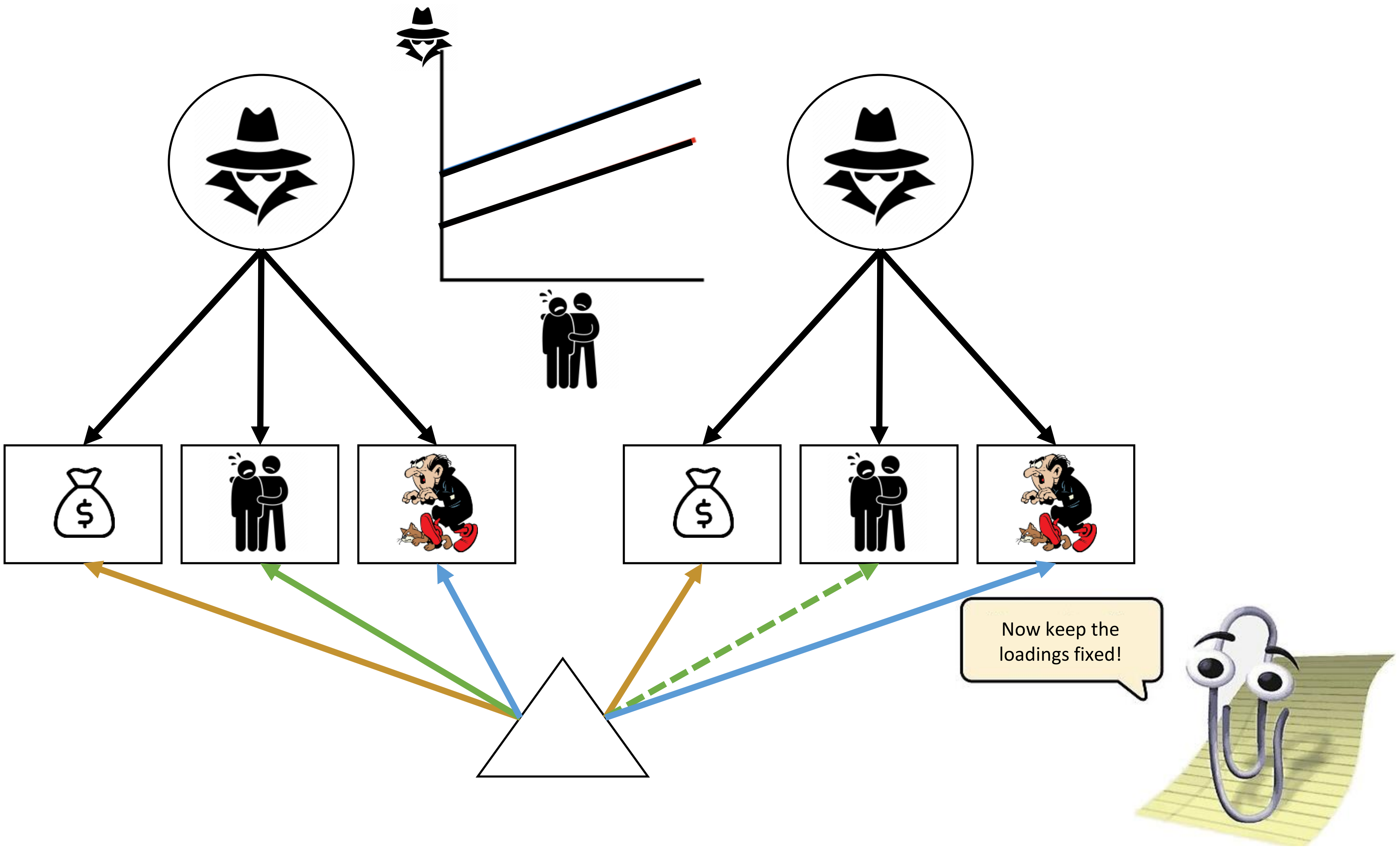




Metric invariance



Scalar invariance



Invariance for Cross-Sectional Data

Regression notation:

$$\boldsymbol{y}_{ig} = \tau_g + \boldsymbol{\Lambda}_g \boldsymbol{\eta}_{ig} + \epsilon_{ig}$$

Reliability

Reliability (assuming invariance)

- Several statistics developed in 1930s-1950s as short cut estimates of reliability, for instance, Cronbach's alpha.
  - Developed before technological advances to find factor structures of scales.
  - Now we can use model-based estimates for reliability that do consider factor structure of scales.
  - This is important in case not all items measure the factors equally well.
  - We will use omega:
  - $\omega = \frac{\left(\sum_{i=1}^{k} \lambda_i\right)^2 Var(\psi)}{\mathbf{1}'\hat{\Sigma}\mathbf{1}}$
    - with $\hat{\Sigma}$ as the observed covariance matrix,
    - k as the number of items
    - $Var(\psi)$ as the variance of the factor scores
    - $\mathbf{1}$ as a k-dimensional row-vector used to sum elements in the matrix.

Lab 1

Invariance & Reliability for Cross-Sectional Data

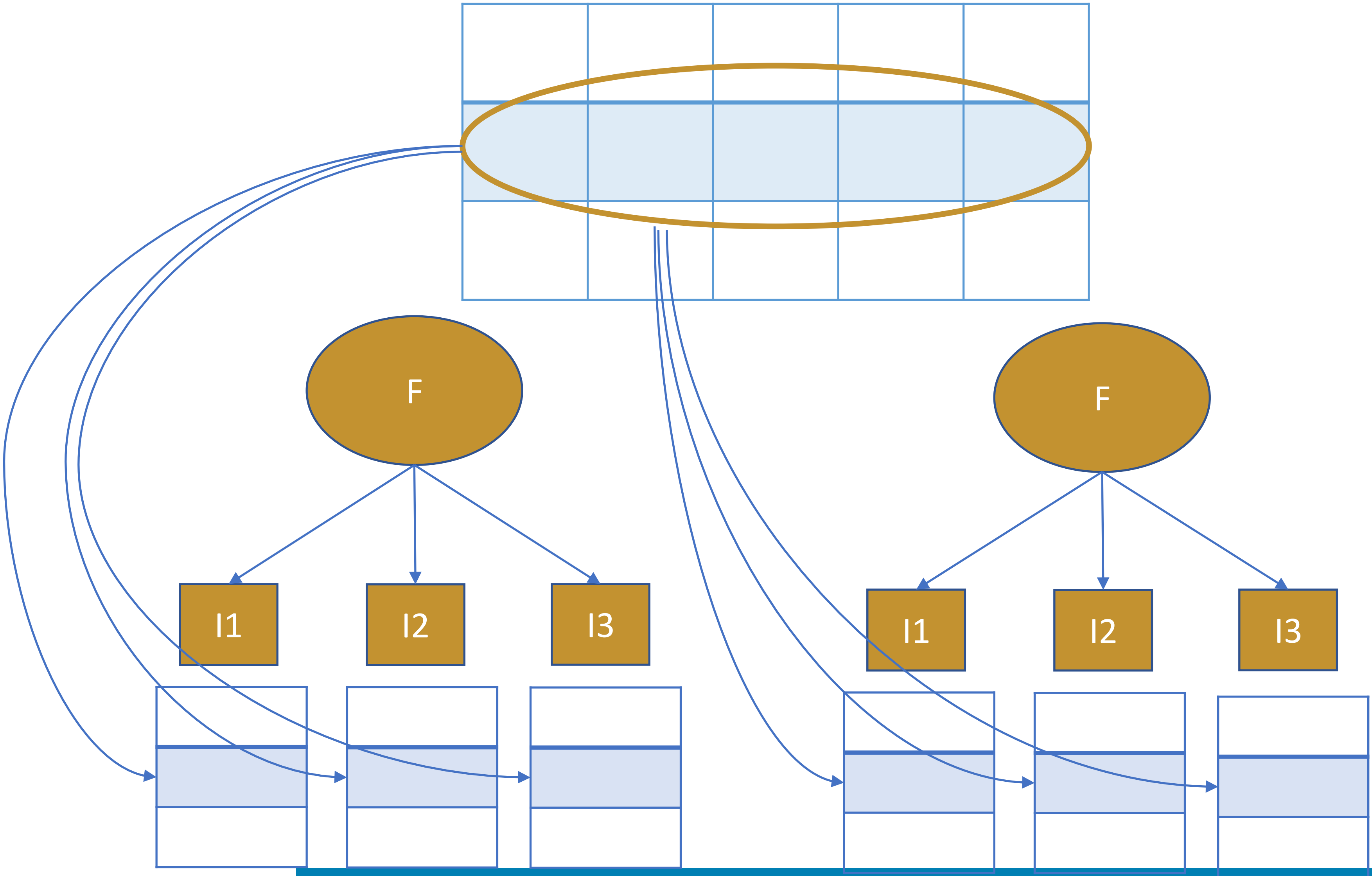
Psychometrics for Panel and Intensive Longitudinal Data

Psychometrics for Panel Data & ILD

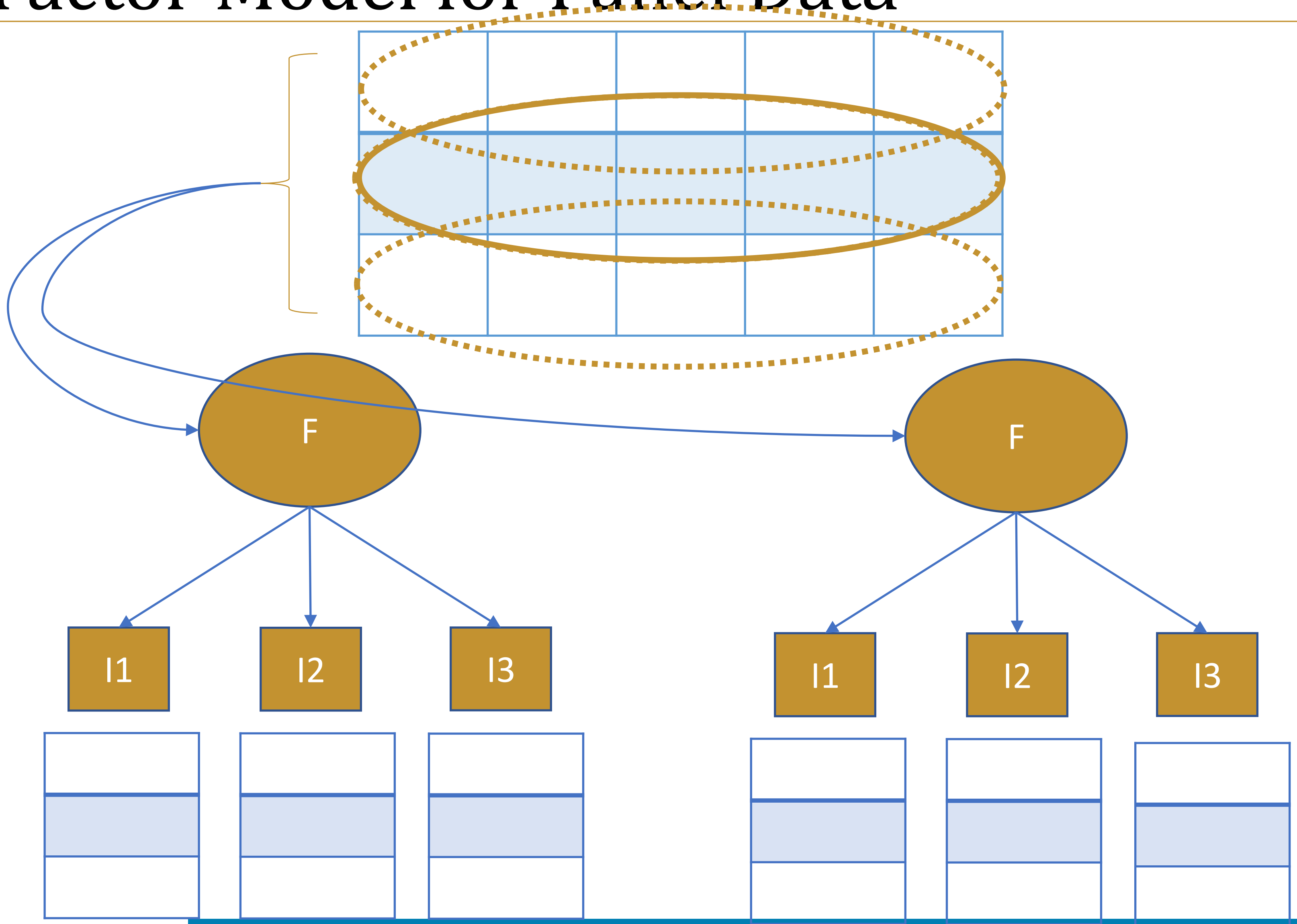
- Factor structure
  - Assess per measurement occasion (“classic” vs. multilevel)
- Invariance
  - Complicated!
- Reliability (assuming invariance)
  - Also complicated!

Factor Model

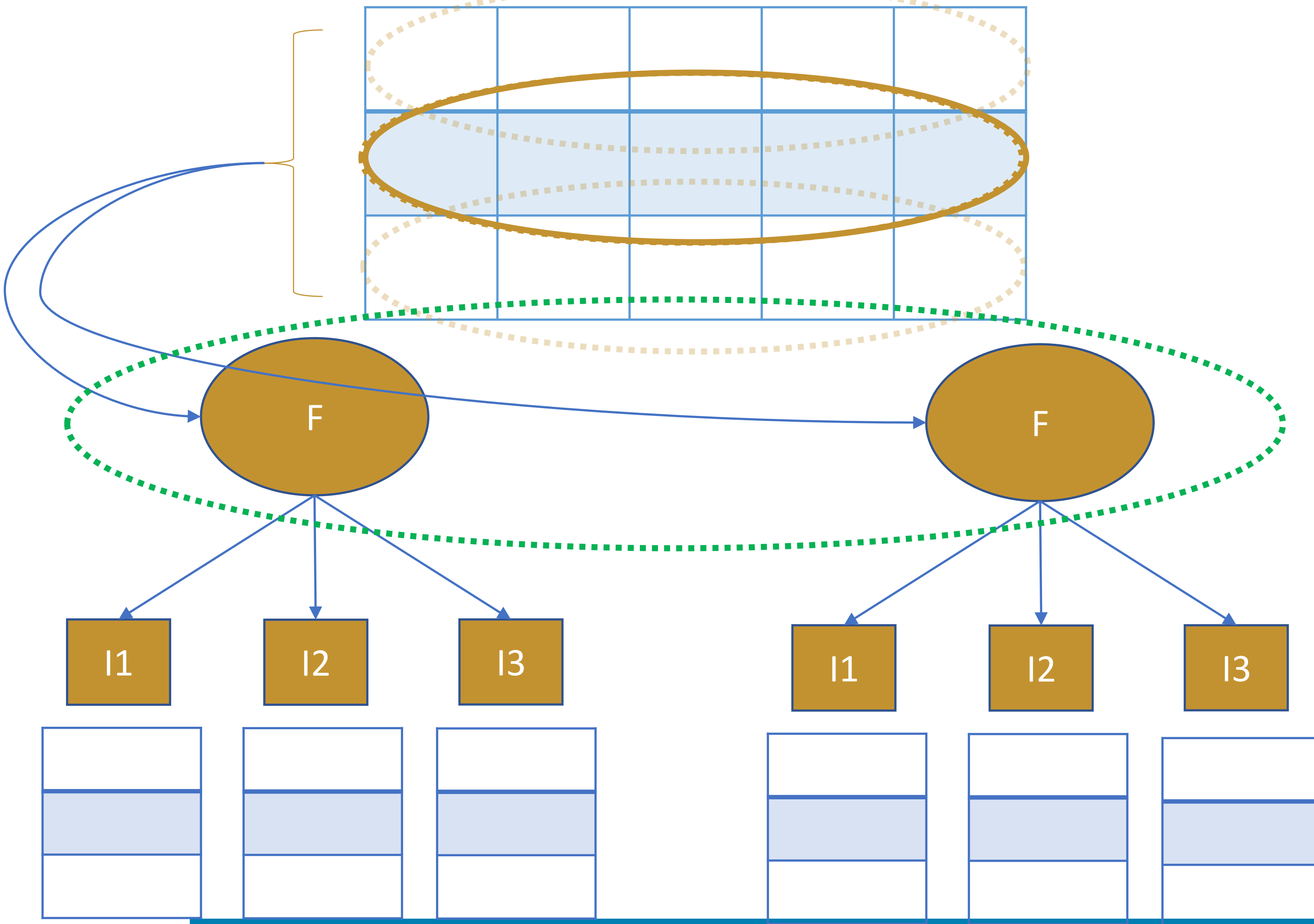
Factor Model for Panel Data



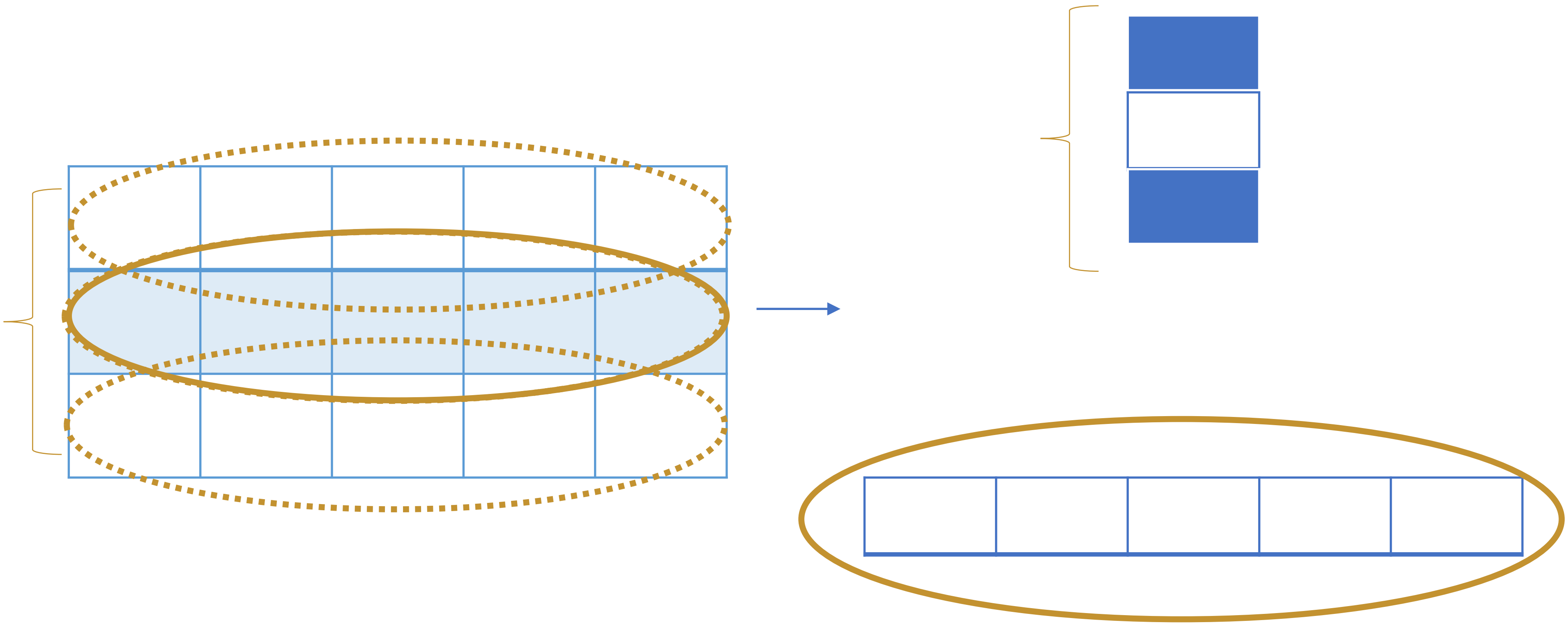
Factor Model for Panel Data



Factor Model for Panel Data



Multilevel



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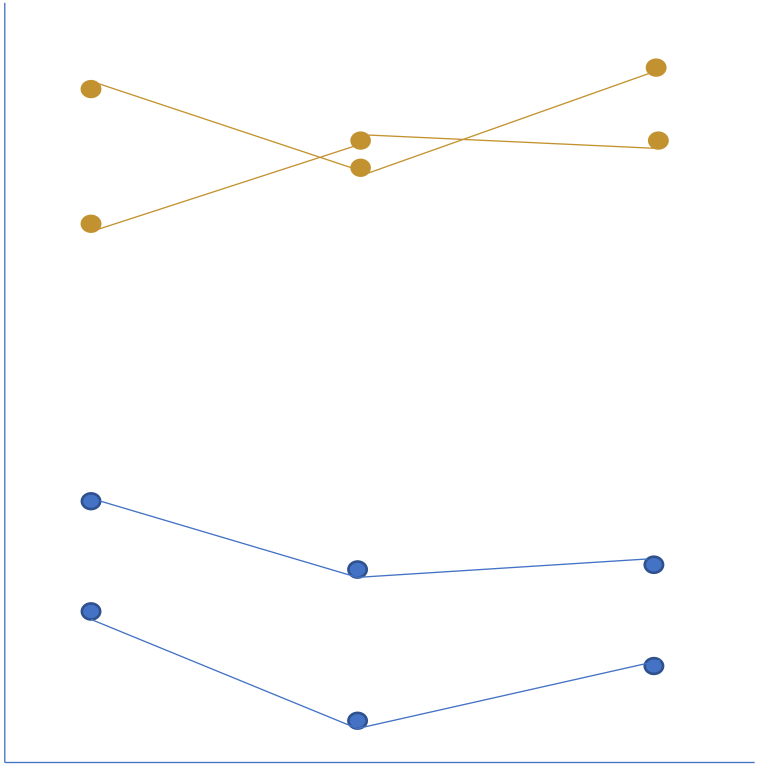
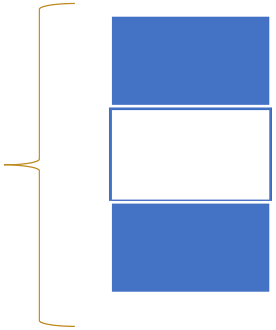
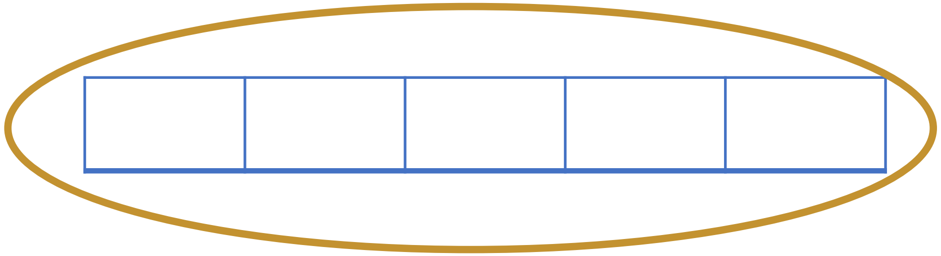
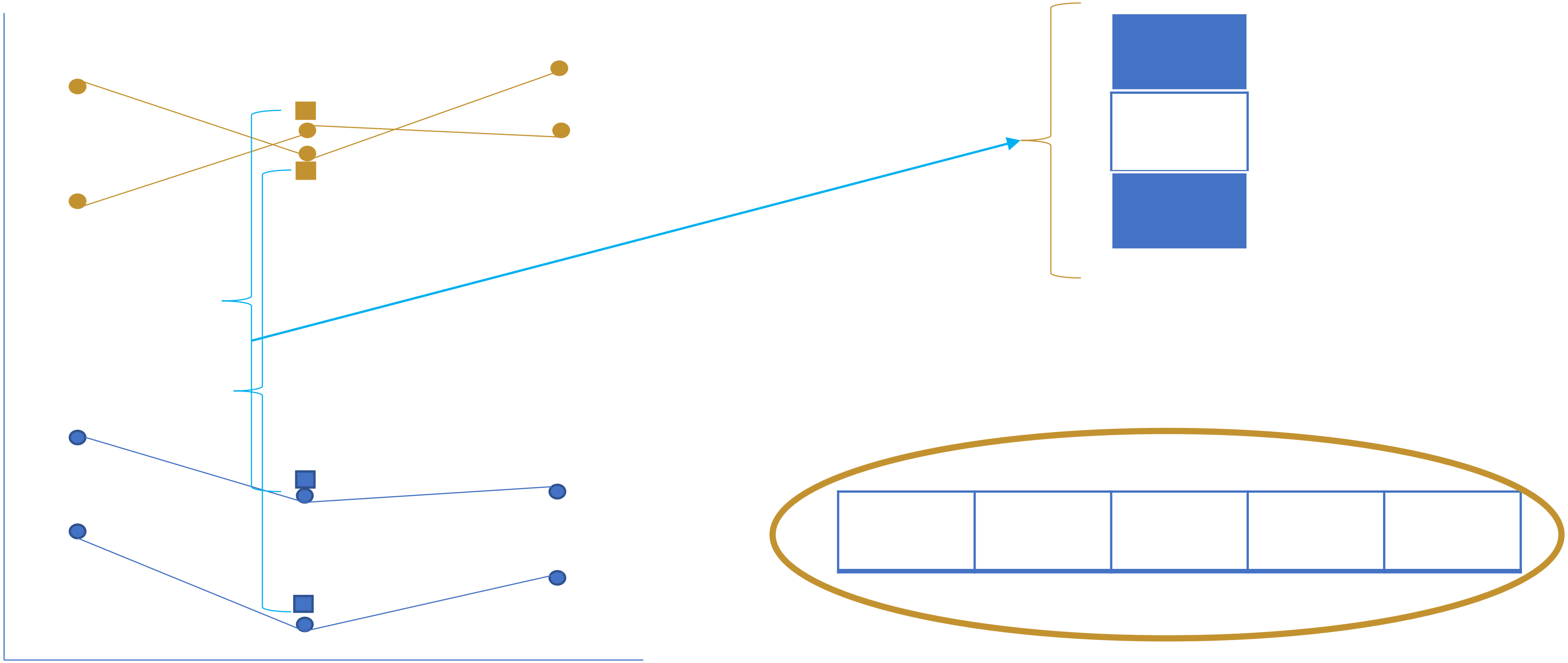


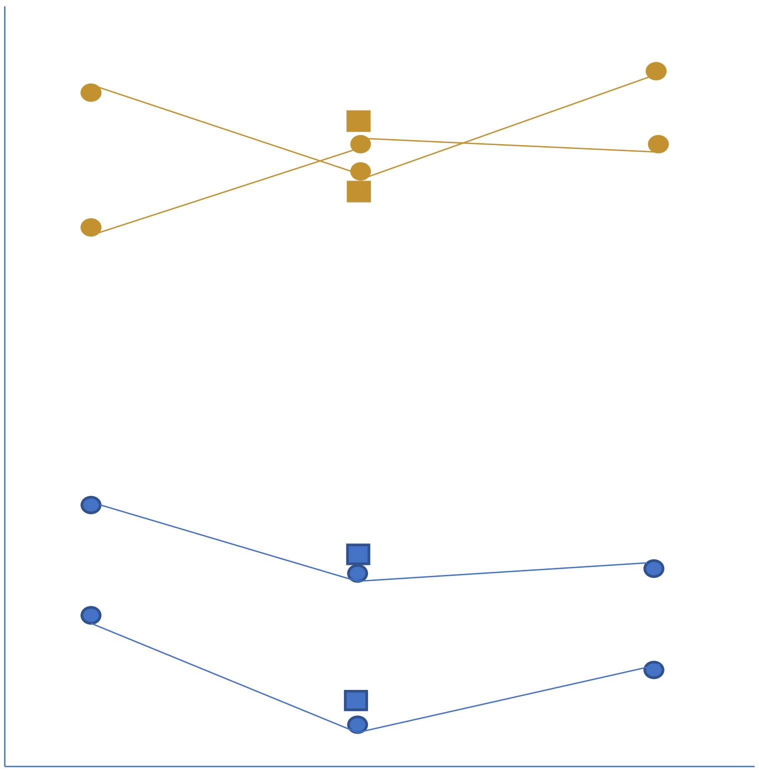


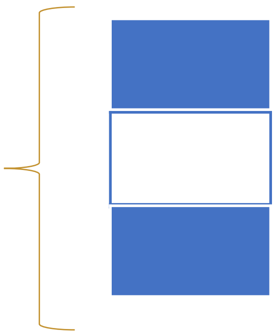


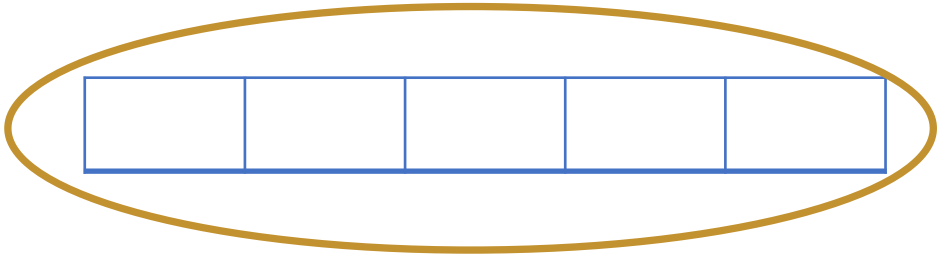
Multilevel



Multilevel







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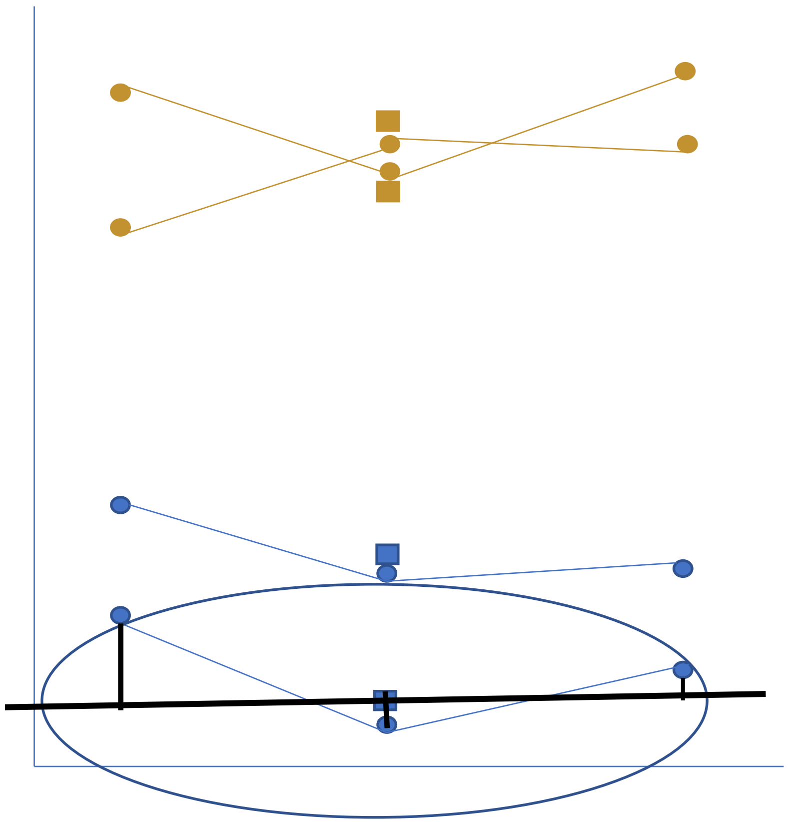
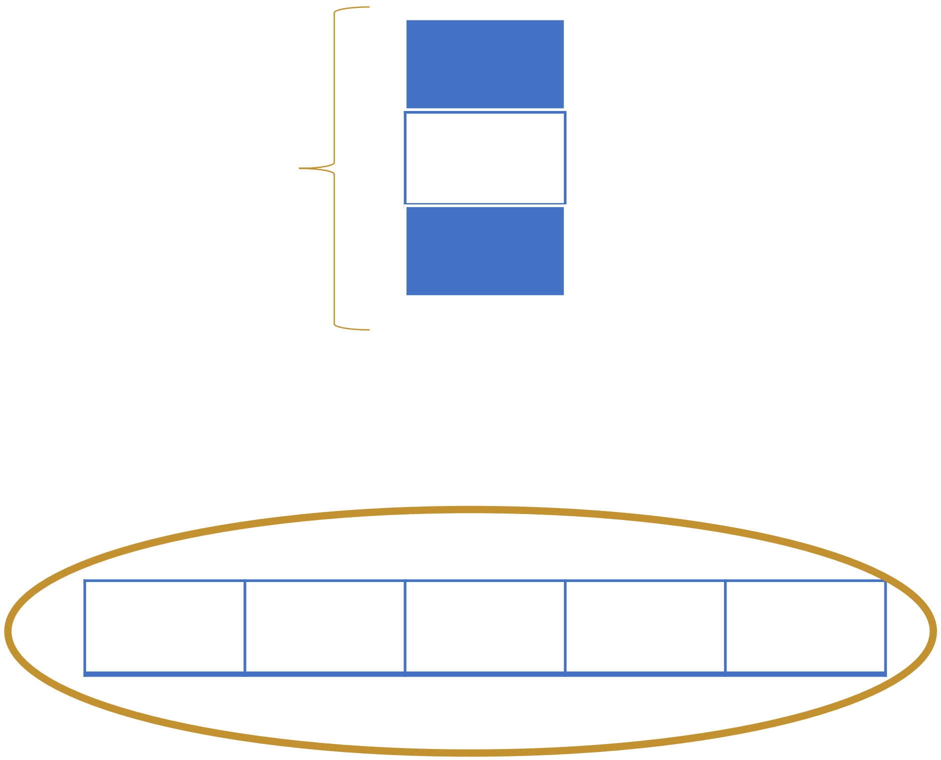
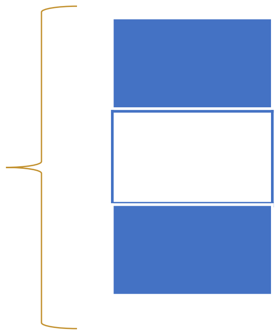
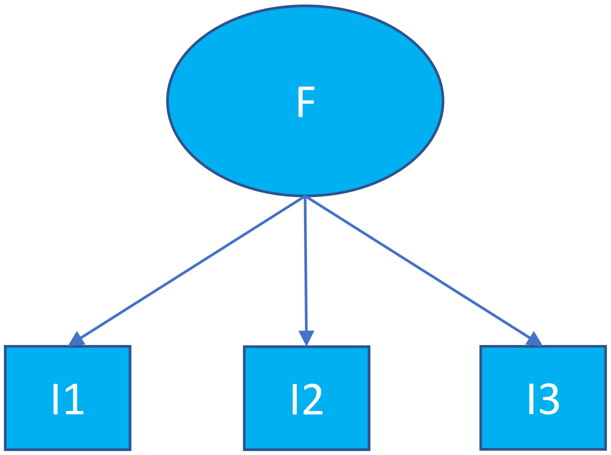


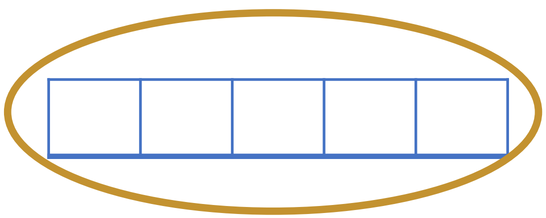


Multilevel

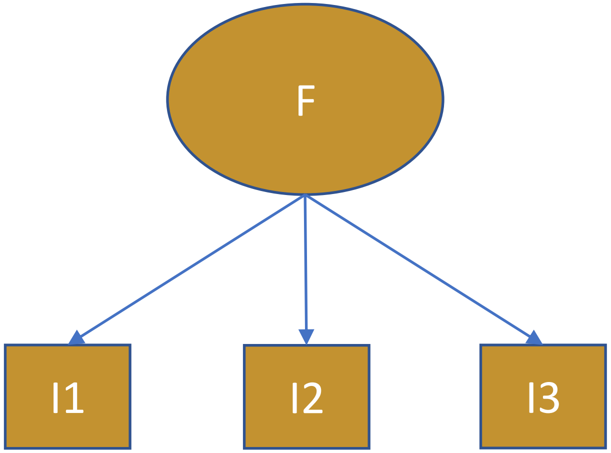


Between

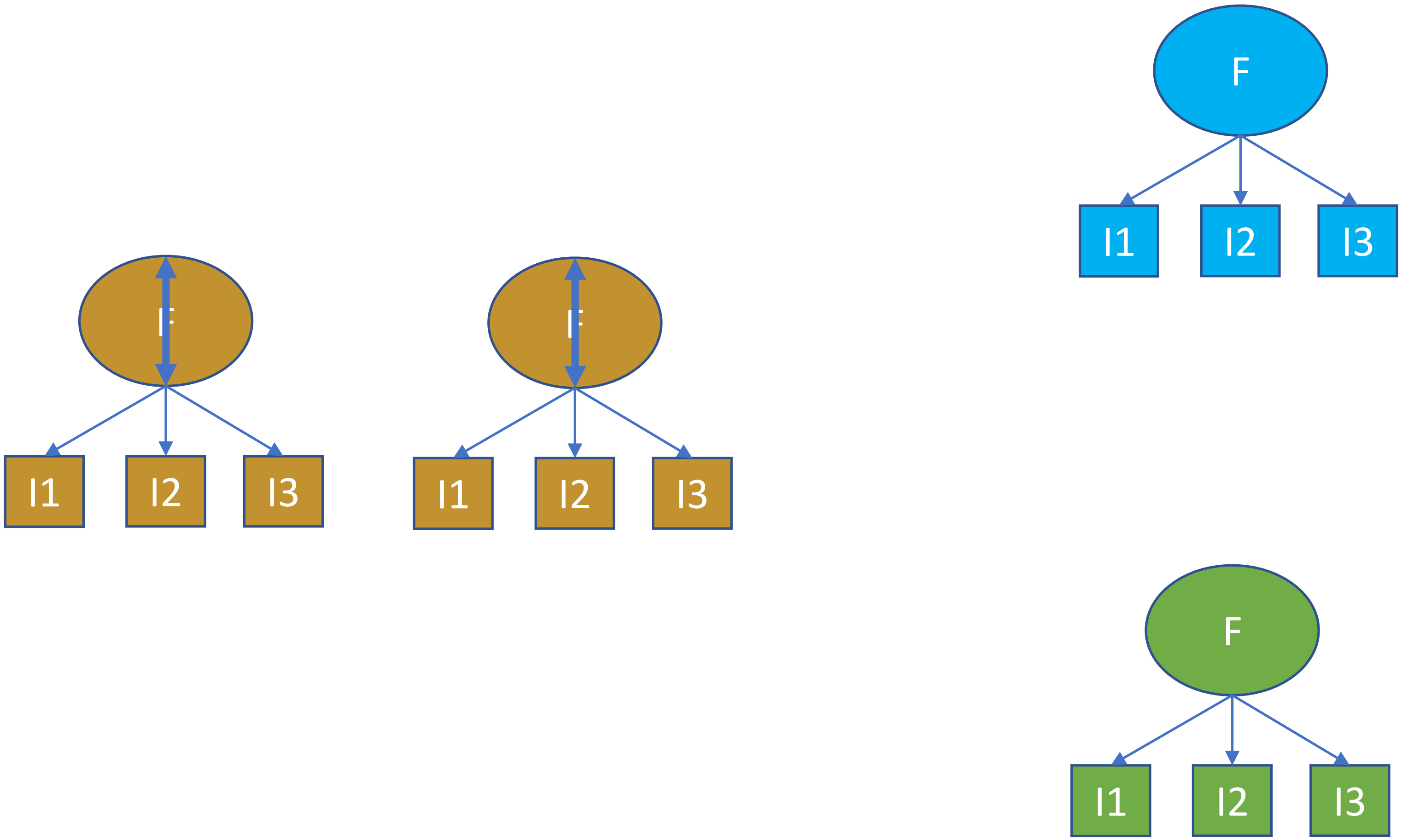




Within



Multilevel



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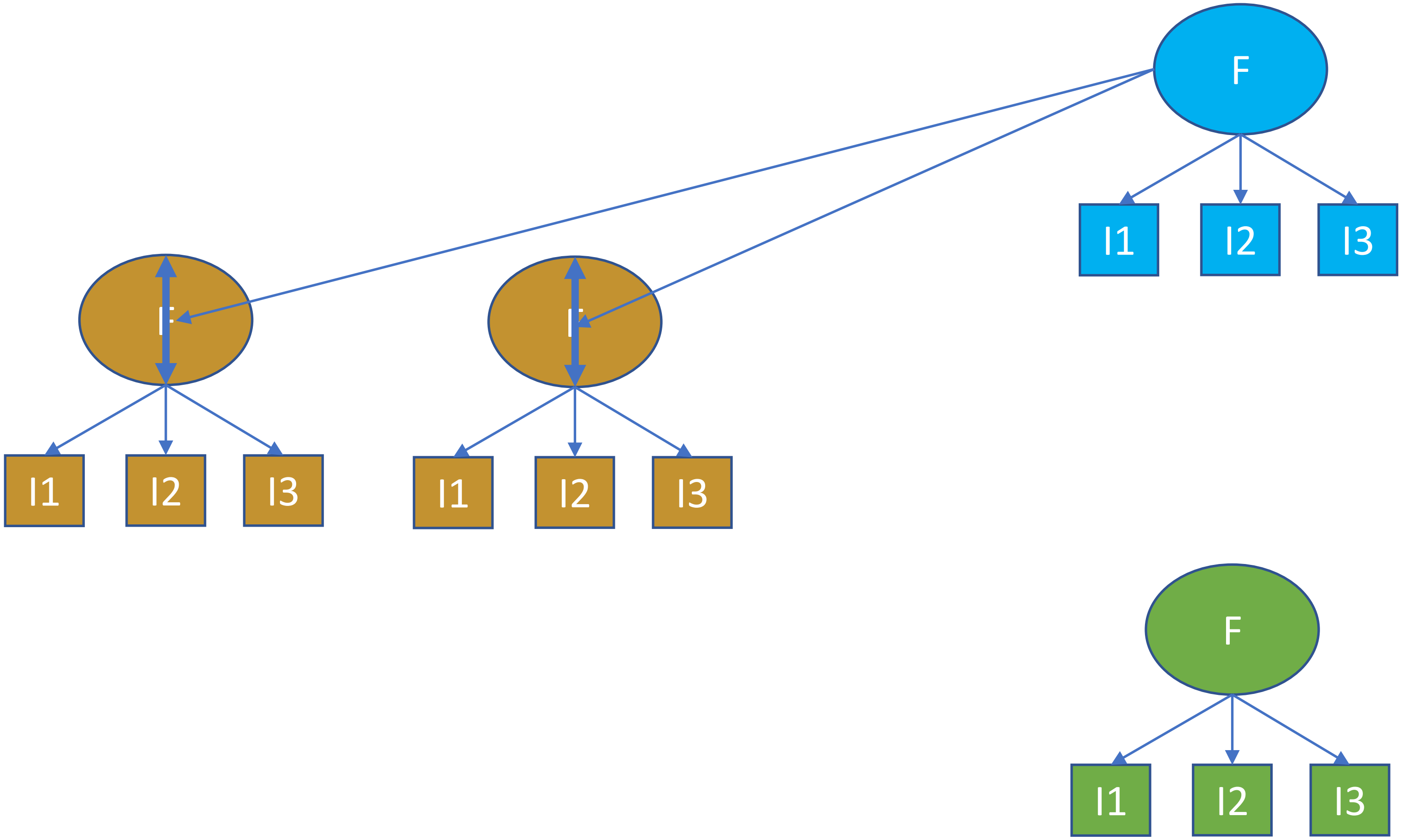

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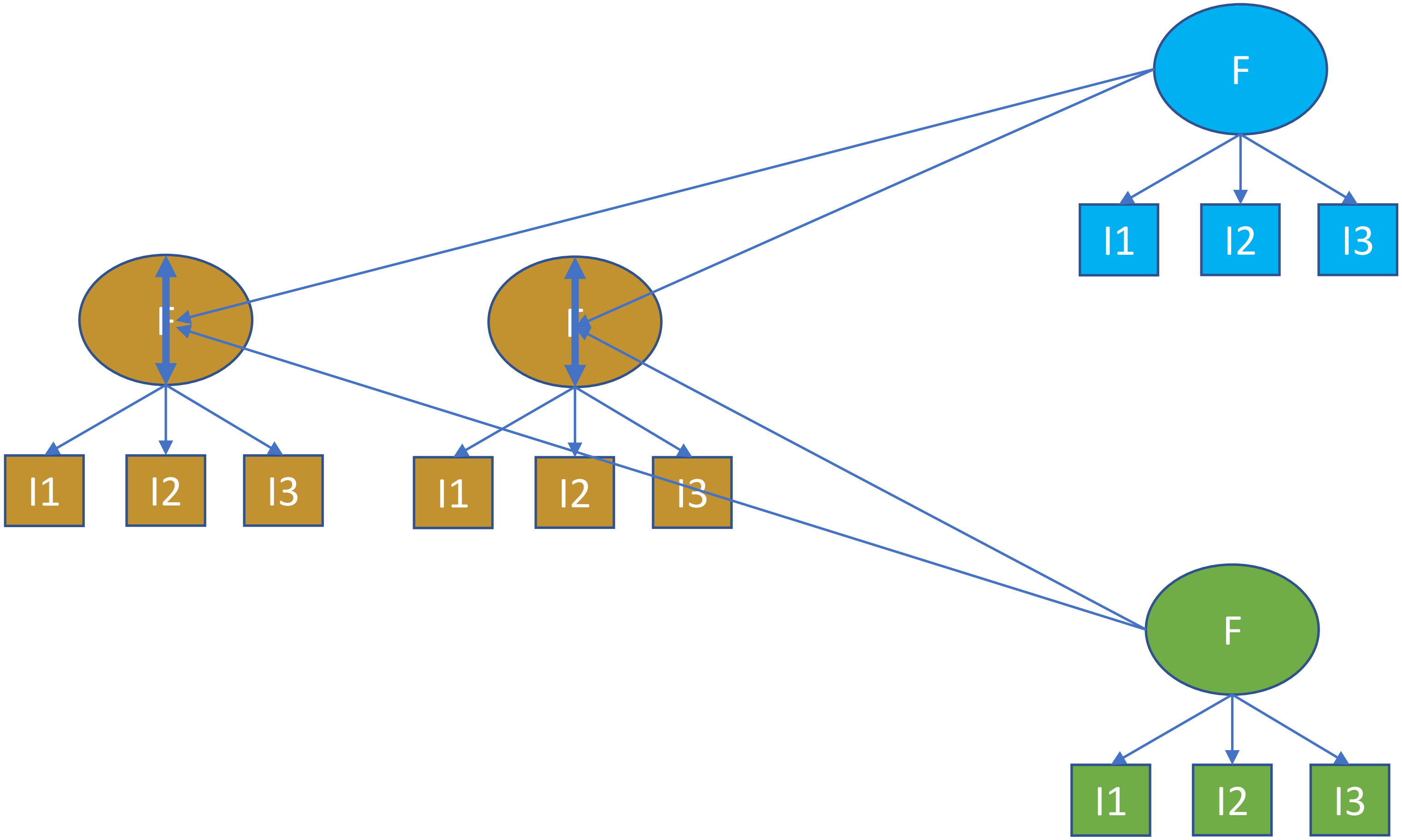

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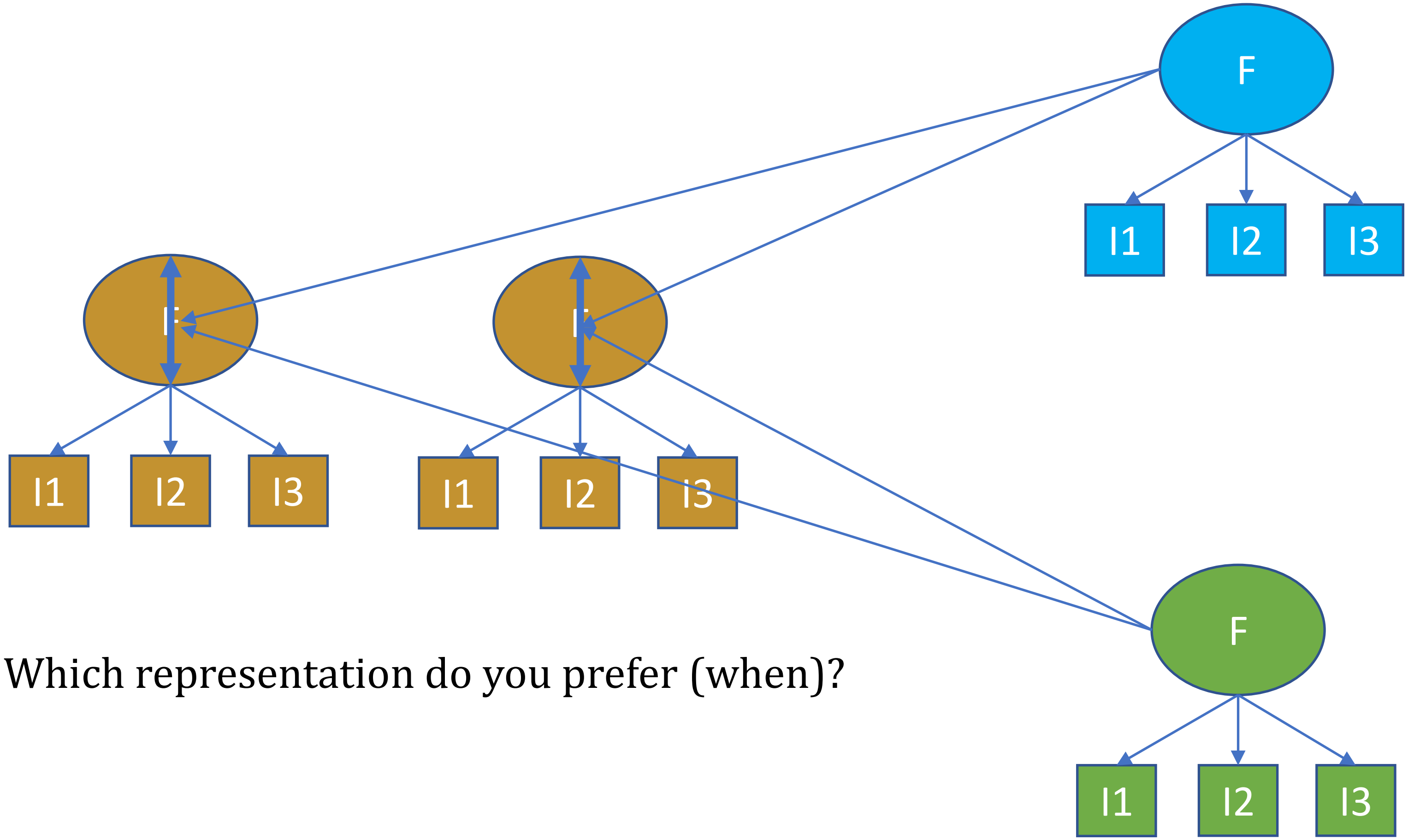


Which representation do you prefer (when)?

Multilevel vs. Panel

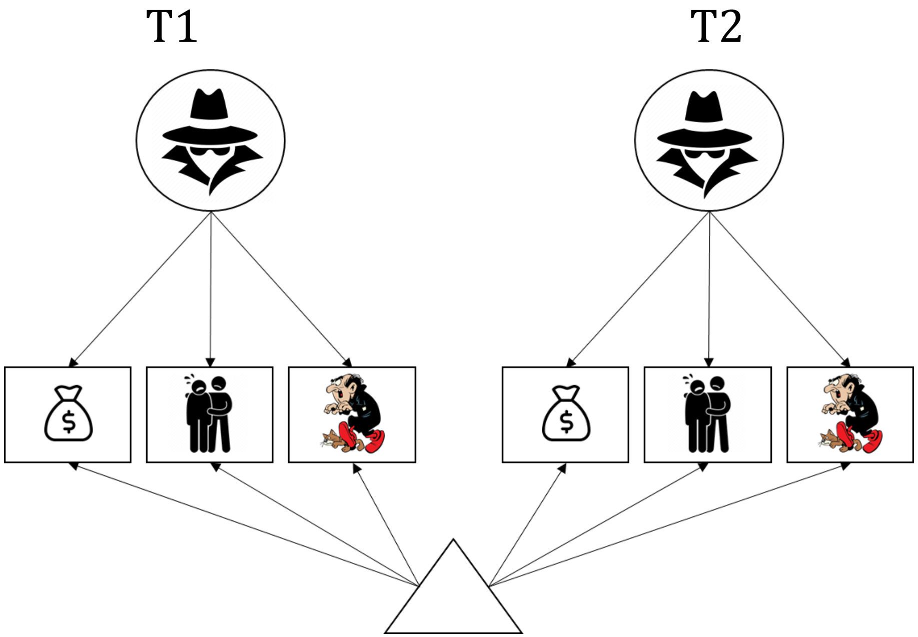
Which representation do you prefer (when)?

- If the focus is more on idiographic analysis (intra-individual change), the multilevel approach provides results “closer to your question”.
- If you want to know if there is change on average (e.g., due to an intervention), the panel model matches your question better.

Measurement Invariance

Invariance for Panel Data

- Now we (typically) don't compare differences across groups but across waves.
- You run pairwise comparisons.
- Multilevel approach is usually not considered here.



Invariance for ILD

Theoretically, we could allow for full heterogeneity (Adolf et al. 2014).

$$\boldsymbol{y}_{i,t} = \tau_{i,t} + \boldsymbol{\Lambda}_{i,t}\boldsymbol{\eta}_{i,t} + \epsilon_{i,t}$$

Where:

- $\boldsymbol{y}_{i,t}$ is the observed score on an indicator for individual *i* on timepoint *t*.
- $\tau_{i,t}$ is the intercept on an indicator for individual *i* on timepoint *t.*
- $\boldsymbol{\eta}_{i,t}$ is the factor-score for individual *i* on timepoint *t*.
- $\boldsymbol{\Lambda}_{i,t}$ is the factor-loading for an indicator for individual *i* on timepoint *t*.
- $\epsilon_{i,t}$ is the residual error on an indicator for individual *i* at timepoint *t*.

Invariance for ILD

This model is not identified.
We can't use this as a baseline.

Time \ Person	No restrictions	Invariance constraints on the measurement model
No restrictions	$\boldsymbol{y}_{i,t} = \boldsymbol{\tau}_{i,t} + \boldsymbol{\Lambda}_{i,t}\,\boldsymbol{\eta}_{i,t} + \boldsymbol{\varepsilon}_{i,t}$ No invariance over time and subjects	$\boldsymbol{y}_{i,t} = \boldsymbol{\tau}_{t} + \boldsymbol{\Lambda}_{t}\,\boldsymbol{\eta}_{i,t} + \boldsymbol{\varepsilon}_{i,t}$ No invariance over time Measurement invariance over subjects
Invariance constraints on the measurement model	$\boldsymbol{y}_{i,t} = \boldsymbol{\tau}_{i} + \boldsymbol{\Lambda}_{i}\,\boldsymbol{\eta}_{i,t} + \boldsymbol{\varepsilon}_{i,t}$ No invariance over subjects Measurement invariance over time	$\boldsymbol{y}_{i,t} = \boldsymbol{\tau} + \boldsymbol{\Lambda}\,\boldsymbol{\eta}_{i,t} + \boldsymbol{\varepsilon}_{i,t}$ Measurement invariance over time and subjects

Invariance for ILD

We could start by assuming invariance over time and test increasingly more restrictive models (i.e., adding constraints to subject-specific parameters) using pairwise comparisons treating subjects as groups (using AIC and BIC).

Time \ Person	No restrictions	Invariance constraints on the measurement model
No restrictions	$\boldsymbol{y}_{i,t} = \boldsymbol{\tau}_{i,t} + \boldsymbol{\Lambda}_{i,t}\, \boldsymbol{\eta}_{i,t} + \boldsymbol{\varepsilon}_{i,t}$ No invariance over time and subjects	$\boldsymbol{y}_{i,t} = \boldsymbol{\tau}_{t} + \boldsymbol{\Lambda}_{t}\, \boldsymbol{\eta}_{i,t} + \boldsymbol{\varepsilon}_{i,t}$ No invariance over time Measurement invariance over subjects
Invariance constraints on the measurement model	$\boldsymbol{y}_{i,t} = \boldsymbol{\tau}_{i} + \boldsymbol{\Lambda}_{i}\, \boldsymbol{\eta}_{i,t} + \boldsymbol{\varepsilon}_{i,t}$ No invariance over subjects Measurement invariance over time	$\boldsymbol{y}_{i,t} = \boldsymbol{\tau} + \boldsymbol{\Lambda}\, \boldsymbol{\eta}_{i,t} + \boldsymbol{\varepsilon}_{i,t}$ Measurement invariance over time and subjects

Invariance for ILD

Person

Time	No restrictions	Invariance constraints on the measurement model
No restrictions	$\boldsymbol{y}_{i,t} = \boldsymbol{\tau}_{i,t} + \boldsymbol{\Lambda}_{i,t}\, \boldsymbol{\eta}_{i,t} + \boldsymbol{\varepsilon}_{i,t}$ No invariance over time and subjects	$\boldsymbol{y}_{i,t} = \boldsymbol{\tau}_{t} + \boldsymbol{\Lambda}_{t}\, \boldsymbol{\eta}_{i,t} + \boldsymbol{\varepsilon}_{i,t}$ No invariance over time Measurement invariance over subjects
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Just like with panel data, you could see if there are differences across specific time-points (assuming invariance over sybjects).

Or you could test if there is an effect of a time-varying covariate on a loading for example.

Invariance for ILD

Person

Time	No restrictions	Invariance constraints on the measurement model
No restrictions	$\mathbf{y}_{i,t} = \boldsymbol{\tau}_{i,t} + \boldsymbol{\Lambda}_{i,t}\, \boldsymbol{\eta}_{i,t} + \boldsymbol{\varepsilon}_{i,t}$ No invariance over time and subjects	$\mathbf{y}_{i,t} = \boldsymbol{\tau}_{t} + \boldsymbol{\Lambda}_{t}\, \boldsymbol{\eta}_{i,t} + \boldsymbol{\varepsilon}_{i,t}$ No invariance over time Measurement invariance over subjects
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Often unrealistic. If even the lowest level of invariance fails, it might make more sense to explore non-invariance and see it as potentially meaningful phenomenon. After all, we are interested in dynamics. We'll get back to that later.

Invariance for ILD

Theoretically, we could allow for full heterogeneity.

$$\boldsymbol{y}_{i,t} = \tau_{i,t} + \boldsymbol{\Lambda}_{i,t}\boldsymbol{\eta}_{i,t} + \epsilon_{i,t}$$

Where:

- $\boldsymbol{y}_{i,t}$ is the observed score on an indicator for individual i on timepoint t.
- $\tau_{i,t}$ is the intercept on an indicator for individual i on timepoint t.
- $\boldsymbol{\eta}_{i,t}$ is the factor-score for individual i on timepoint t.
- $\boldsymbol{\Lambda}_{i,t}$ is the factor-loading for an indicator for individual i on timepoint t.
- $\epsilon_{i,t}$ is the residual error on an indicator for individual i at timepoint t.

- We could constrain all parameter to be equal across time and/or persons.
- Problem: Many pairwise comparisons! Which observations are in the end comparable?

Invariance for ILD

- Consider ILD as cross-classified data (McNeish et al. 2020).
    - Comes with assumptions that are not feasible for typical ILD
        - Equal number of time-points, same starting and end day, same intervals, covariate scores are the same for everyone at the same measurement occasion (e.g., weekend vs weekday).

	ID = 1	ID = 2	ID = 3
T = 1	Y_{11}	Y_{12}	Y_{13}
T = 2	Y_{21}	Y_{22}	Y_{23}
T = 3	Y_{31}	Y_{32}	Y_{33}

	ID = 1	ID = 2	ID = 3
T = 1	Y_{11}	Y_{12}	Y_{13}
T = 2	Y_{21}	Y_{22}	Y_{23}
T = 3	Y_{31}	Y_{32}	Y_{33}

- There is variance across time/subjects. And then?
- Is it even high variance?
- What are the reasons? Could we remove some time-points/observations?

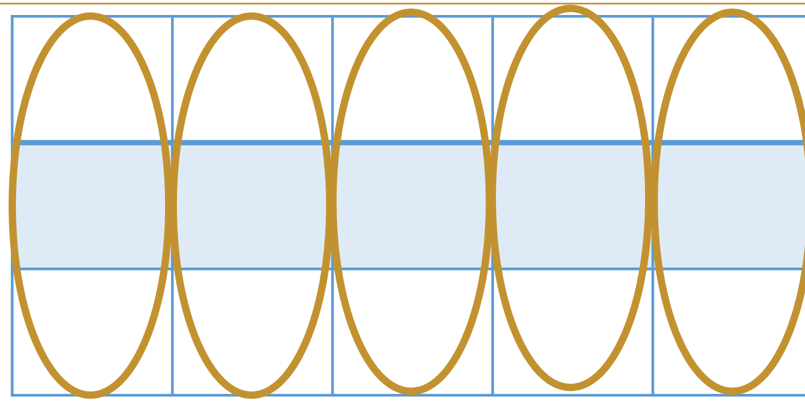
Reliability

Reliability for Panel Data & ILD

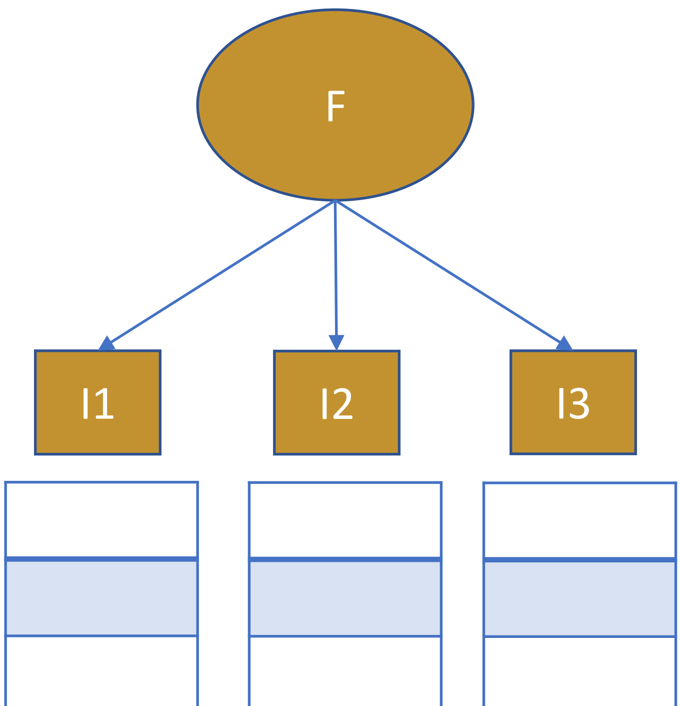
- Reliability (assuming invariance)
- In addition to the cross-sectional/between-person reliability, we can also check:
  - The reliability per person
  - Longitudinal reliability:
    - Factor correlation (like a test-retest reliability)
    - Interrater reliability: whether rank ordering of factor scores is stable across time
- Whether you go “classic” or multilevel, it’s basically doing cross-sectional reliability several times (plus some extras).

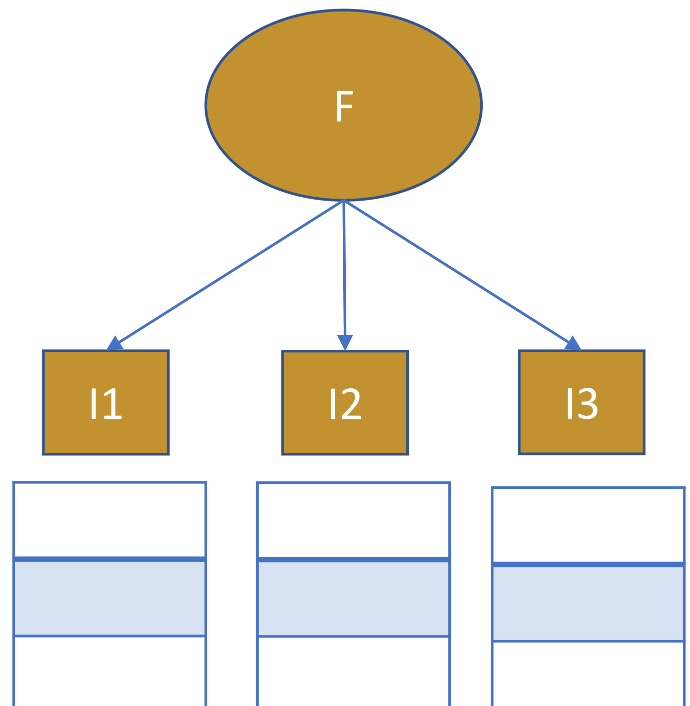
Reliability for Panel Data

$$\omega = \frac{\left(\sum_{i=1}^{k} \lambda_i\right)^2 Var(\psi)}{\mathbf{1}'\hat{\Sigma}\mathbf{1}}$$

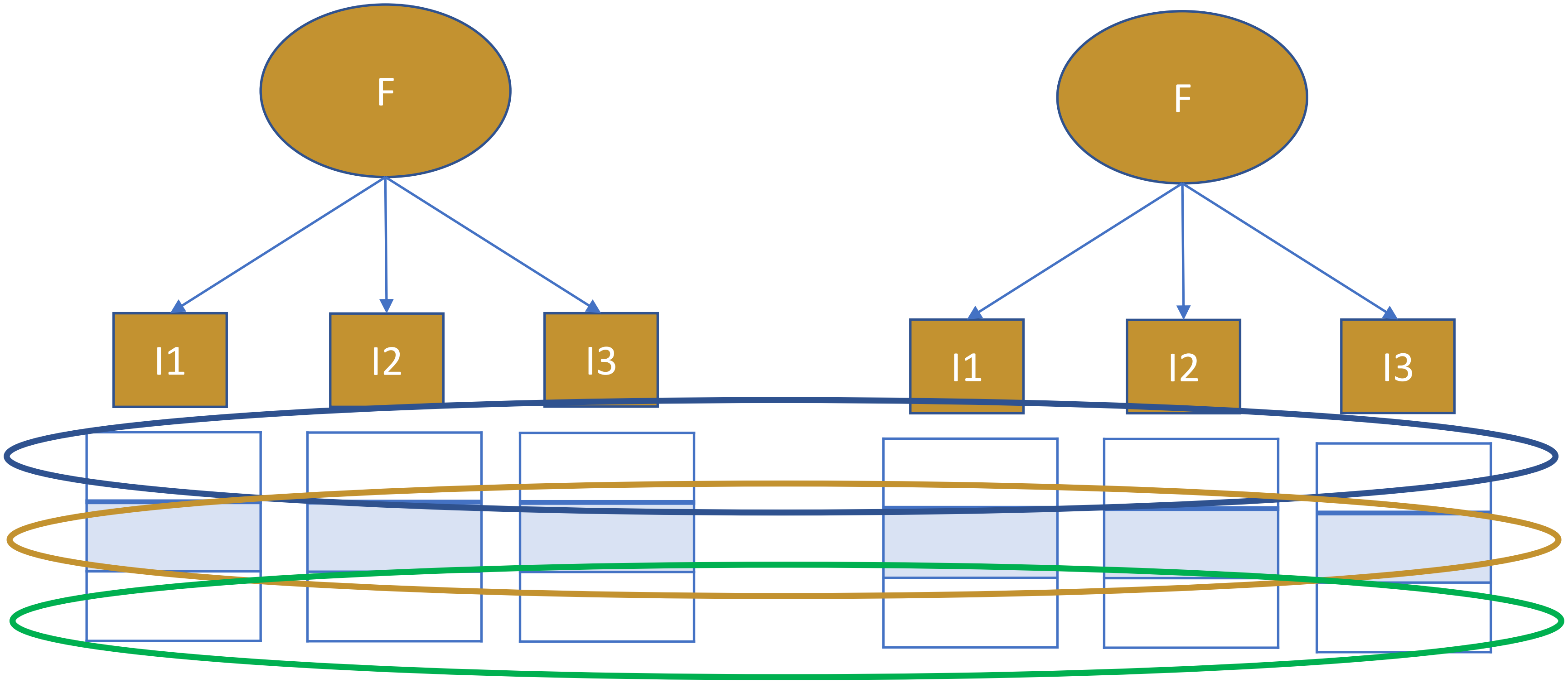


$$\omega = \frac{\left(\sum_{i=1}^{k} \lambda_i\right)^2 Var(\psi)}{\mathbf{1}'\hat{\Sigma}\mathbf{1}}$$

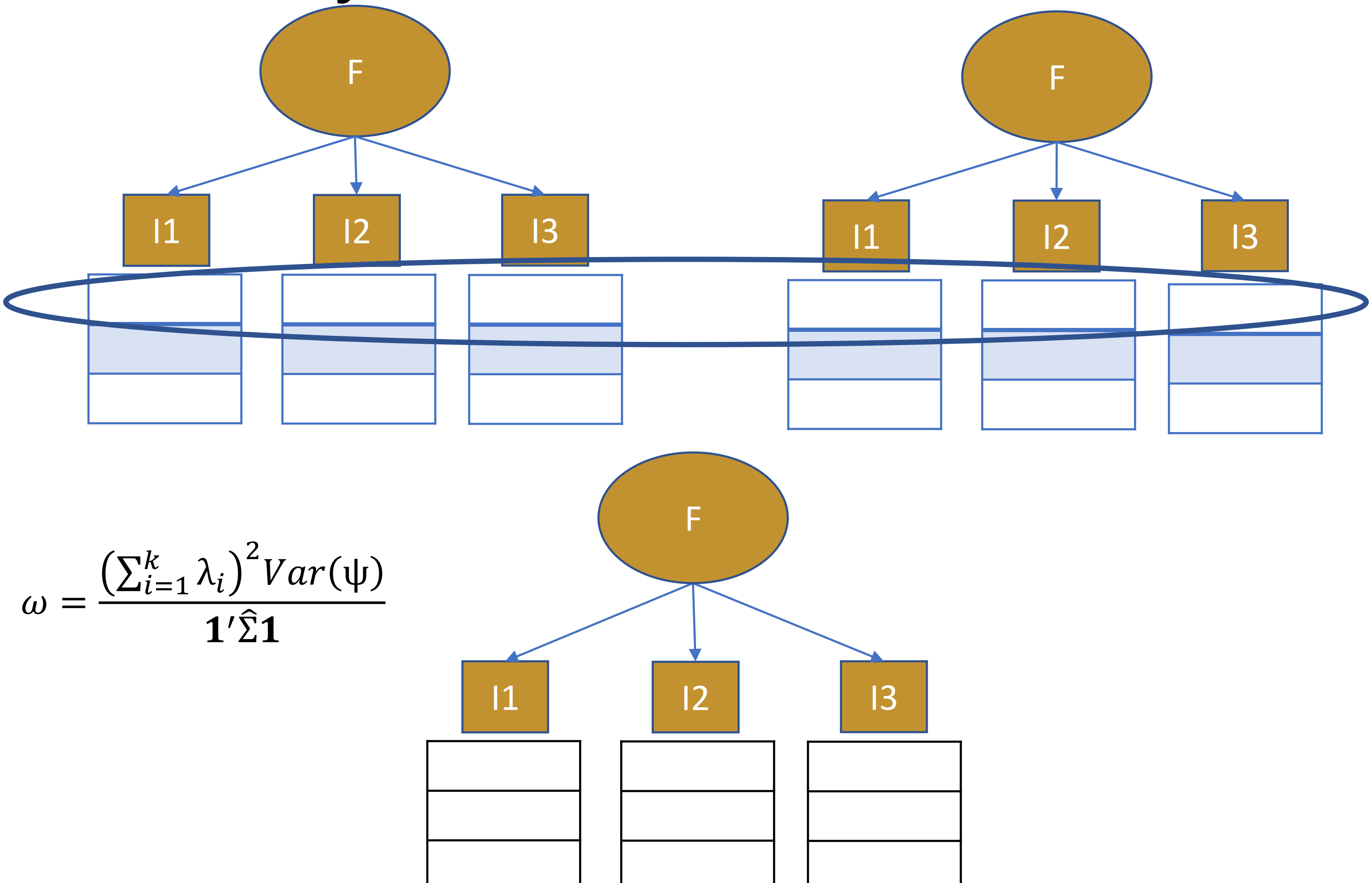




Reliability for Panel Data

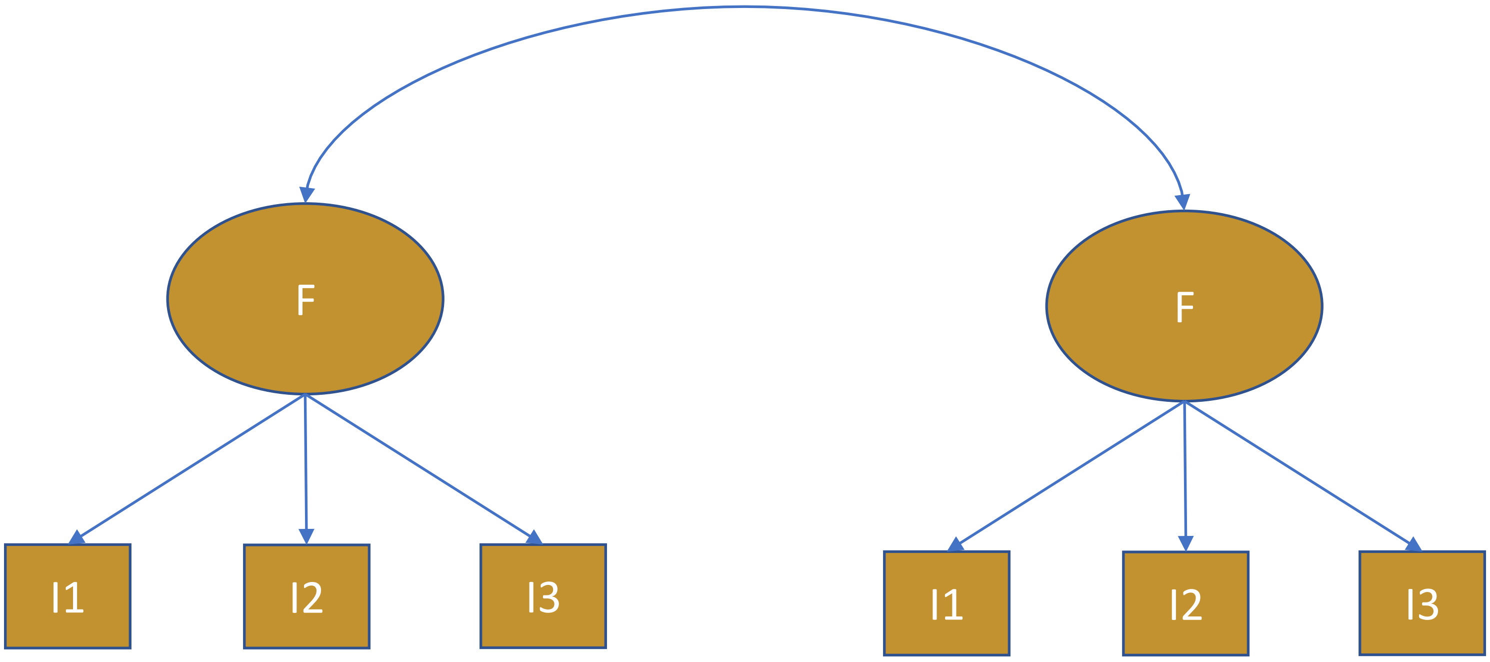


Reliability for Panel Data

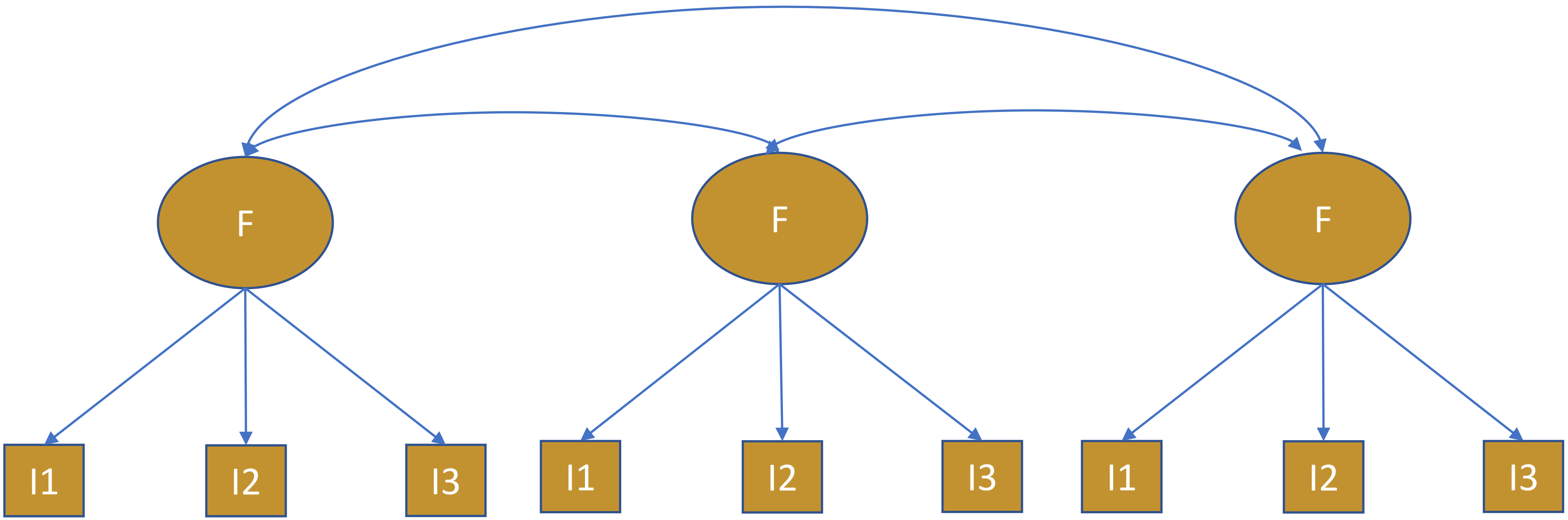


$$\omega = \frac{\left(\sum_{i=1}^{k} \lambda_i\right)^2 Var(\psi)}{\mathbf{1}'\hat{\Sigma}\mathbf{1}}$$

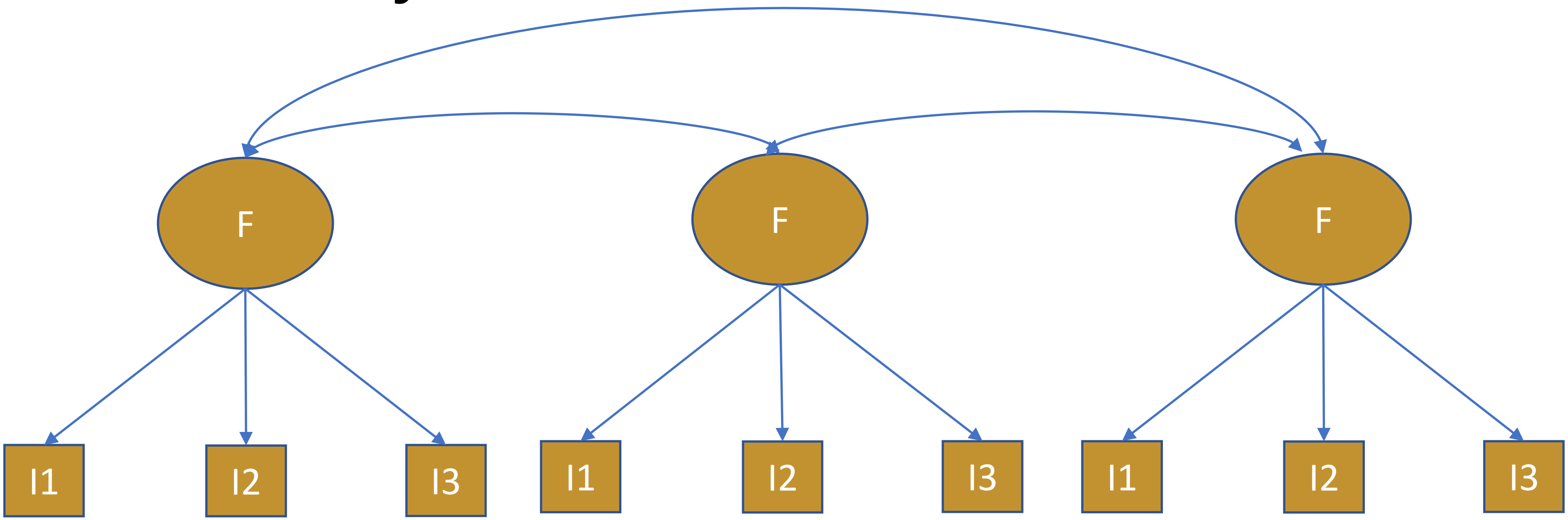
Reliability for Panel Data



Reliability for Panel Data



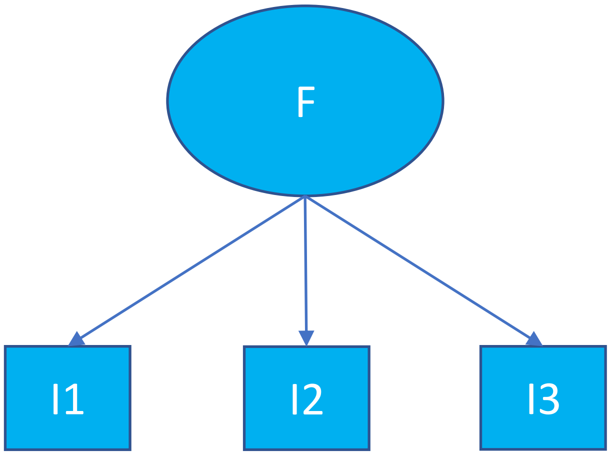
Reliability for Panel Data



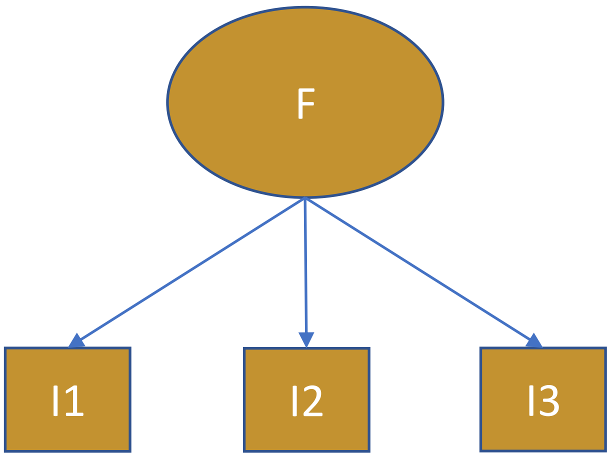
- Could do multiple pairwise comparisons, or
- Inter-rater reliability which checks for the agreement on two or more occasions at the same time
  - Multiple options: Fleiss' Kappa, ICC, etc.

Multilevel Reliability

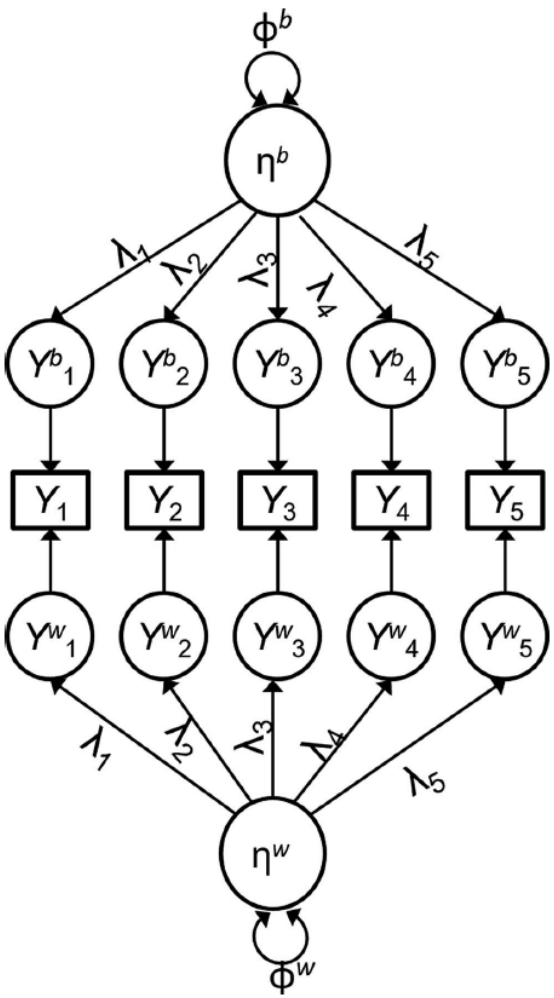
Between



Within



Reliability for Panel Data & ILD

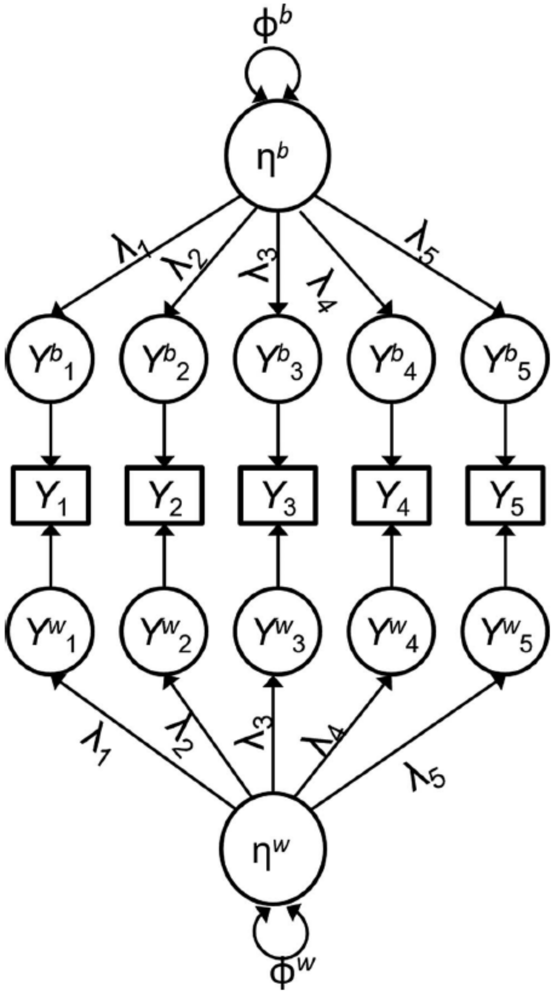


Between Level:

$$\omega = \frac{\left(\sum_{i=1}^{k} \lambda_i\right)^2 Var(\psi)}{\mathbf{1}'\hat{\Sigma}\mathbf{1}} \quad \text{for } \boldsymbol{\eta}^B$$

$$\omega = \frac{\left(\sum_{i=1}^{k} \lambda_i\right)^2 Var(\phi^b)}{\mathbf{1}'\widehat{\Sigma^b}\mathbf{1}}$$

Reliability for Panel Data & ILD

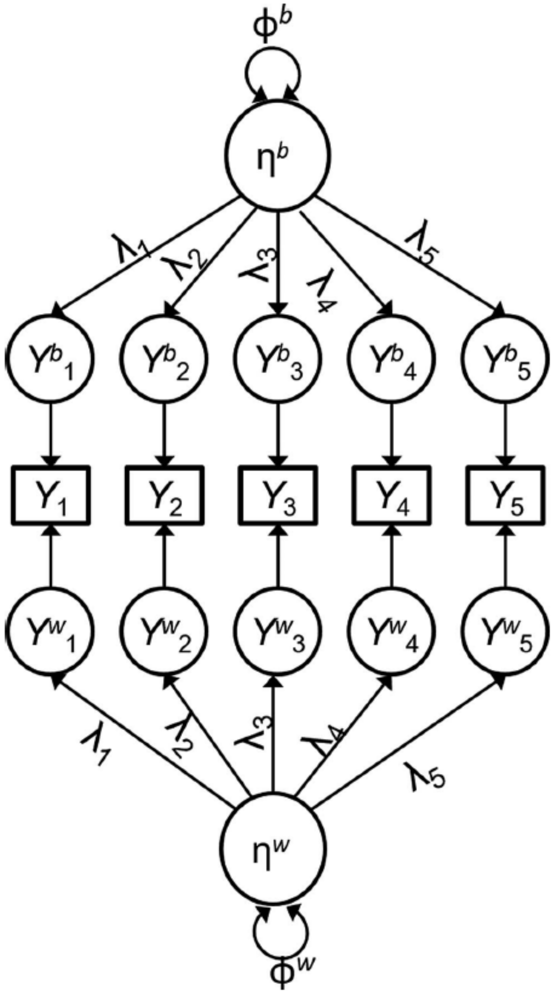


Within Level:

$$\omega = \frac{\left(\sum_{i=1}^{k} \lambda_i\right)^2 Var(\psi)}{\mathbf{1}'\hat{\Sigma}\mathbf{1}} \quad \text{for } \boldsymbol{\eta}^w \text{ or}$$

$$\omega = \frac{\left(\sum_{i=1}^{k} \lambda_i\right)^2 Var(\phi^w)}{\mathbf{1}'\widehat{\Sigma^w}\mathbf{1}}$$

Reliability for Panel Data & ILD



Assume invariance between the within- and between factor model and use

$$\omega = \frac{\left(\sum_{i=1}^{k} \lambda_i\right)^2 Var(\psi)}{\mathbf{1}'\hat{\Sigma}\mathbf{1}}$$

$$ICC = \frac{\boldsymbol{\eta}^b}{(\boldsymbol{\eta}^b + \boldsymbol{\eta}^w)}$$

Reliability for Panel Data & ILD

- With multilevel approach, you sometimes get slidely different Omega's
- Lai (2021): "Reliability is a characteristic of an observed composite"
  - So doesn't like using latent variables, should use actual composites you can calculate from the data (e.g., sum-scores).

Reliability for Panel Data & ILD

- Overall composite of *p* items (Y) for person *i* in cluster *j* is simply the sum of the *p* item scores:

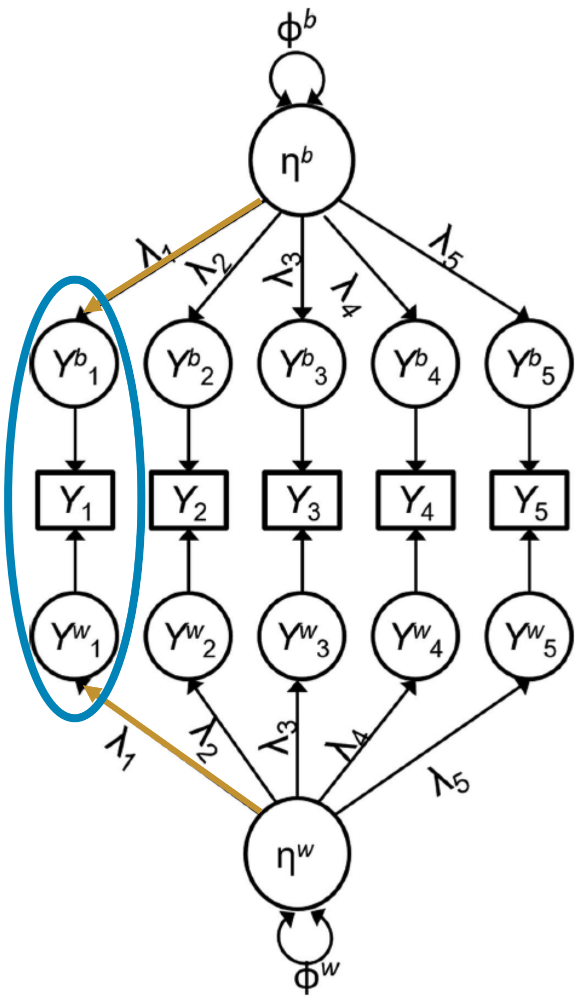
$$Z_{ij} = \sum_{k=1}^{p} Y_{ijk}$$

- The between-level observed composite score is:

$$Z_i^b = \sum_{i=1}^{n_j} \frac{Z_{ij}}{n_j}$$

- Within composite score is Z_{ij} - Z_i^b

Reliability for Panel Data & ILD



True score variance in Y1

For raw-scores:

$$Y_{ip} = T_i + e_i$$

$$Var(\eta) = \phi^b + \phi^w$$

$$\omega^{2L} = \frac{\left(\sum_{i=1}^{k} \lambda_i\right)^2 (\phi^b + \phi^w)}{\left(\sum_{i=1}^{k} \lambda_i\right)^2 (\phi^b + \phi^w) + \mathbf{1}'\mathbf{\Theta}^B\mathbf{1} + \mathbf{1}'\mathbf{\Theta}^W\mathbf{1}}$$

$$\omega = \frac{\left(\sum_{i=1}^{k} \lambda_i\right)^2 Var(\psi)}{\mathbf{1}'\hat{\Sigma}\mathbf{1}}$$

Reliability for Panel Data & ILD

- Always compare true score variance to total variance.

- Different methods only differ in what they consider true- and total variance, and in terms of possible constraints on the multilevel factor-model.
  - In factor models, true-score variance is the variance of the factors
  - For composites, true-score variance can be determined based on the factor-model.

Introduce Example ILD

Introduce ILD

Delusions of Grandeur

Question	0-100
How much does the following apply to you: I can do anything	
How much does the following apply to you: I don’t fail	
How much does the following apply to you: If something bad happens, it's someone else’s fault	
How much does the following apply to you: I'm smarter than other people	
How much does the following apply to you: I avoid making the mistakes other people make	
How much does the following apply to you: I would definitely hire myself	
How much does the following apply to you: I'm better looking than others	

Lab 2

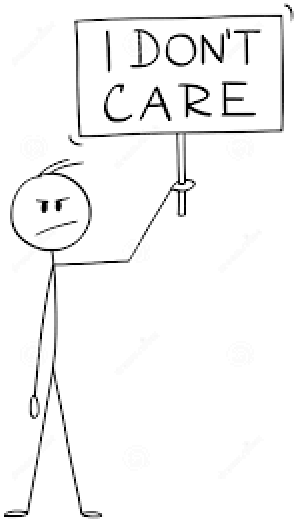
Reliability & Invariance for Longitudinal Data

Exploring Non-Invariance for Intensive Longitudinal Data

Exploring Longitudinal Invariance

- Pairwise comparisons to detect all types of differences and changes are almost impossible.

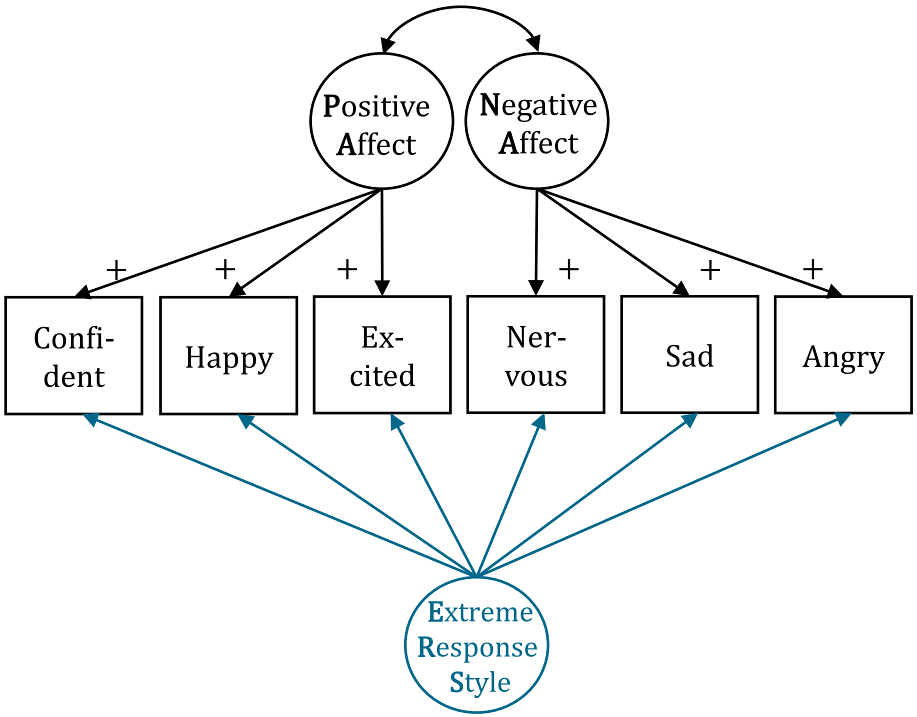
- No method, then…



Response Styles

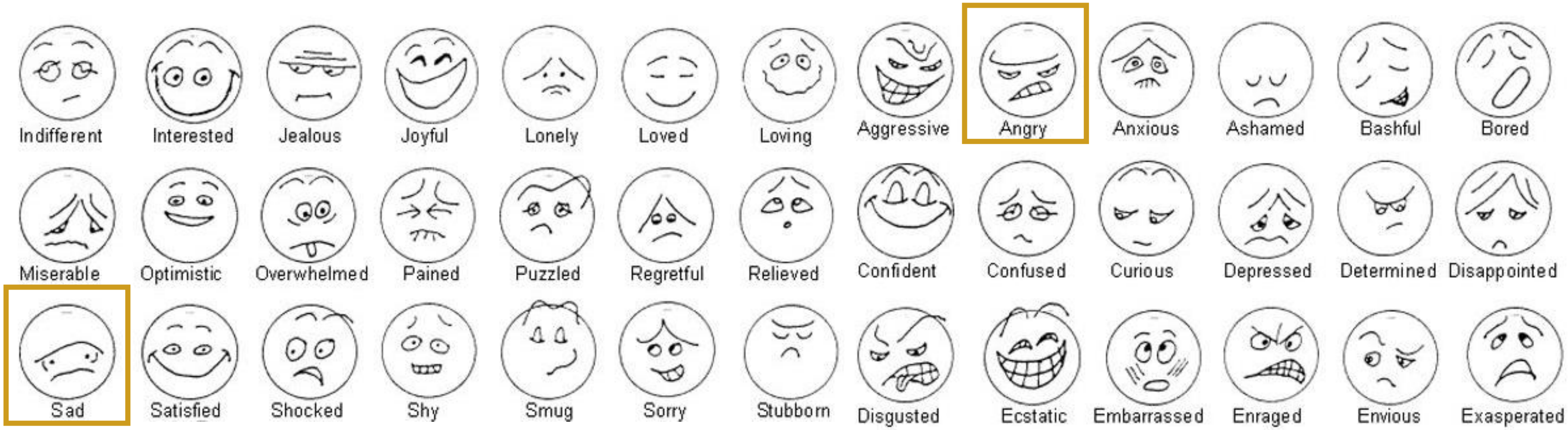
Changes in the factor models (the measurement models) are even more likely than in panel or cross-sectional data.

- Distracting situations
- Getting demotivated



Substantive differences

- Ability to differentiate between emotions
  - High differentiators: Label in a differentiated and context dependent way
  - Low differentiators: Less specific emotional experiences



Substantive differences

- Ability to differentiate between emotions
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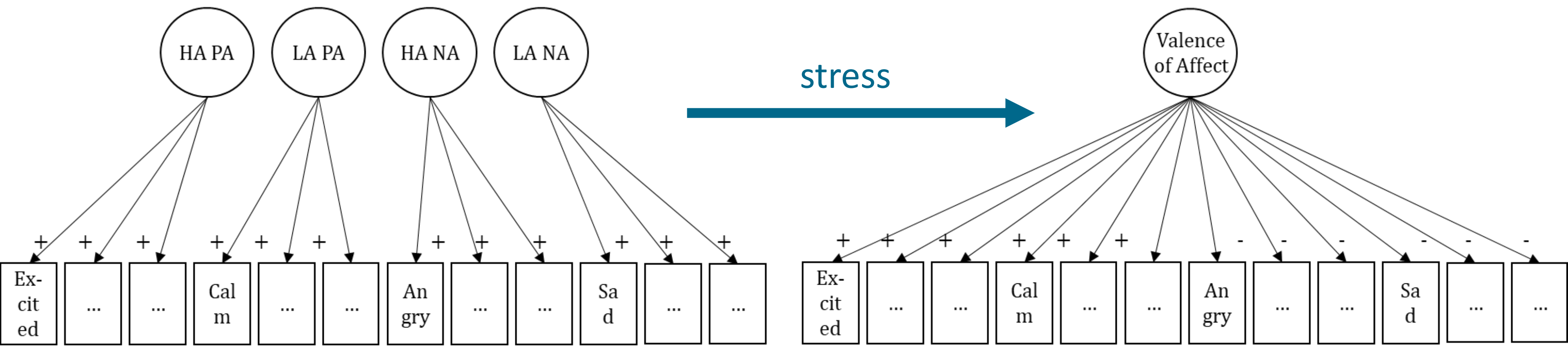
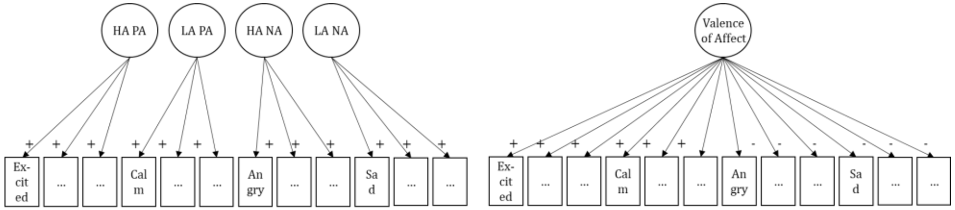


Existing Approaches Were Limited

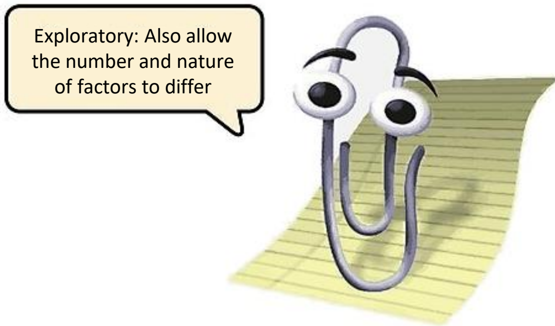
- Only tested if invariance across subjects OR if invariance across time was violated
- Assumed that the number and nature of factors are the same



- No pinpointing for which subjects and time-points the MM differs
- No insights into what the MM differences look like

Latent Markov Factor Analysis

- Vogelsmeier et al. 2019
- Latent Markov model:
  - Latent class model that allows for transitions
  - initial state probabilities
  - Transition probabilities
  - Probabilities may depend on individual- or time-point-specific covariates



- Exploratory factor analysis per state
  - State-specific intercepts, loadings, and unique variances
  - States may differ regarding all levels of invariance, thus, also regarding number of factors
  - For observations within the same state, invariance holds

$$\boldsymbol{y}_{itk} = \tau_k + \boldsymbol{\Lambda}_k \boldsymbol{\eta}_{it} + \epsilon_{itk,}$$

Estimation

- Can be estimated with a FIML or three-step approach

```
install.packages("devtools")
library("devtools")
install_github("LeonieVm/lmfa@0.1.3")
library("lmfa")
```

Step 1: Investigating measurement models

Step 1a: Selecting the number of states and factors

- run `step1()` with the modelselection option and store the results (e.g., into the object "`modelselection`")
- inspect the models using `summary(modelselection)` and rerun non-converged models using `step1()` without the model selection option
- choose best model(s) based on the BIC and CHull using `plot(modelselection)` and `chull_lmfa(modelselection)`
- store the best model(s) (e.g., into the object "`measurementmodel`")

Step 1b: Interpreting the measurement models

- use `summary(measurementmodel)` to obtain the measurement model parameters from the chosen model(s)
- investigate state-specific (obliquely rotated and standardized) loadings
- investigate state-specific intercepts
- investigate state-specific unique variances

Step 1c: Attach factor scores to the dataset

- store the outcome of `factorscores_lmfa(ESM, measurementmodel)` into a new dataset

Step 2: Obtaining state assignments & classification errors

- run `step2()` to calculate classification errors, posterior state-membership probabilities, and modal state assignments and store the results (e.g., into the object "`classification`")
- obtain the results with `summary(classification)`
- if desired, attach posterior state-membership probabilities and the modal state assignments to the dataset by storing `classification$data` into a new dataset

Step 3: Investigating transition model

Step 3a: Selecting the covariates for the transition model

- run `step3()` using the posterior state-membership probabilities of the `classification` object from `step2()` and all covariates of interest and store the results (e.g., into the object "`transitionmodel`")
- inspect Wald test results with `summary(transitionmodel)` and look at the p-values to decide which covariates should be included in the final transition model

Step 3b: Interpreting the transition model

- use `summary(transitionmodel)` to obtain the transition model parameters and probabilities for covariates being equal to their sample means
- use `probabilities(transitionmodel)` to obtain initial state and transition probabilities for any covariate value (and interval length) of interest

Step 3c: Updating state assignments & investigating state memberships

- if desired, attach posterior state-membership probabilities and the modal state assignments to the dataset by storing `transitionmodel$data` into a new dataset
- use `invariance(transitionmodel)` to investigate for which subjects within- and between-person invariance holds
- use `plot(transitionmodel)` to investigate subjects' individual transitions

Data (simulated for the *lmfa* package)

- Long format
- Column indicating time between previous and current observation
- Columns with indicator items
- Explanatory variables

	id	deltaT	negativeEvent	intervention	Interested	Joyful	Determined	Calm
1	1	0.00	53	0	45	16	8	75
2	1	0.56	37	0	52	42	35	50
3	1	1.04	55	0	71	80	70	78
4	1	1.81	59	0	62	77	75	94
5	1	0.80	73	0	27	40	46	17
6	1	2.45	49	0	55	53	18	45

Step 1: Investigating Measurement Models

- The state-specific MMs are estimated …
- …while disregarding the transitions and the covariate effects on these transitions.
- What is the best model in terms of the number of factors and states?
    - Model selection

Step 1: Investigating Measurement Models

```
modelselection <- step1(data = ESM,
                        indicators = c(
                          "Interested","Joyful","Determined","Calm",
                          "Lively","Enthusiastic","Relaxed","Cheerful",
                          "Content","Energetic","Upset","Gloomy",
                          "Sluggish","Anxious","Bored","Irritated",
                          "Nervous","Listless"),
                       modelselection = TRUE,
                       n_state_range = 1:4,
                       n_fact_range = 2:3)
```

Step 1: Investigating Measurement Models

```
summary(modelselection)

##                  LL       BIC convergence n par
## [323]  -353166.8 708485.3           1   254
## [333]  -353149.0 708602.3           1   272
## [3232] -353085.0 708940.1           1   327
## [3233] -353067.8 709058.2           0   345
## [3333] -353047.8 709170.6           0   363
## [3222] -353316.0 709249.7           1   309
## [322]  -353855.3 709709.8           1   236
## [33]   -354421.0 710375.3           1   181
## [2222] -353976.8 710418.8           1   291
## [32]   -355010.3 711401.4           1   163
## [222]  -355095.1 712037.0           1   218
## [22]   -356377.4 713983.1           1   145
## [3]    -361759.6 724281.6           1    90
## [2]    -363744.0 728098.0           1    72
```

Step 1: Investigating Measurement Models

Configural invariance clearly violated!

```
## Obliquely rotated standardized loadings:
##
##                S1F1  S1F2  S1F3   S2F1  S2F2   S3F1  S3F2  S3F3
## Interested     0.66  0.04  0.00   0.68  0.01   0.57 -0.01  0.02
## Joyful         0.60  0.02  0.02   0.65 -0.01   0.88  0.01  0.06
## Determined     0.37  0.03 -0.55   0.61  0.00   0.84  0.02 -0.01
## Calm           0.37 -0.58 -0.01   0.59  0.00   0.18 -0.15  0.82
## Lively         0.63  0.03  0.03   0.65  0.00   0.88 -0.01  0.01
## Enthusiastic   0.65 -0.01  0.02   0.64  0.00   0.89  0.02  0.00
## Relaxed        0.64  0.02  0.00   0.64  0.01   0.16 -0.14  0.85
## Cheerful       0.63  0.07  0.01   0.63 -0.01   0.91  0.01  0.02
## Content        0.61  0.00  0.03   0.67  0.02   0.93  0.02  0.01
## Energetic      0.64 -0.01  0.00   0.63 -0.01   0.90  0.05 -0.01
## Upset          0.09  0.62 -0.01   0.00  0.53   0.03  0.83 -0.03
## Gloomy        -0.24  0.39  0.44  -0.01  0.53   0.02  0.82 -0.01
## Sluggish       0.07 -0.01  0.73  -0.01  0.50  -0.29  0.34  0.77
## Anxious        0.09  0.70 -0.02   0.00  0.52   0.05  0.79 -0.01
## Bored          0.07 -0.01  0.74  -0.01  0.52   0.04  0.47 -0.04
## Irritated      0.06  0.51 -0.05   0.01  0.58   0.04  0.85 -0.02
## Nervous        0.08  0.73 -0.04   0.00  0.51   0.03  0.74  0.01
## Listless       0.06 -0.05  0.73   0.01  0.54   0.02  0.46 -0.03
```

Step 1: Investigating Measurement Models

```
## Intercepts:
##
##                 S1    S2    S3
## Interested   49.24 61.46 51.98
## Joyful       48.92 61.12 49.95
## Determined   46.60 61.20 50.35
## Calm         46.25 61.14 54.76
## Lively       49.29 60.85 50.57
## Enthusiastic 48.99 61.16 50.24
## Relaxed      49.00 61.12 54.90
## Cheerful     49.03 61.02 50.42
## Content      49.39 60.84 49.98
## Energetic    49.35 60.90 50.41
## Upset        44.12 26.54 36.42
## Gloomy       45.88 27.09 35.93
## Sluggish     44.95 26.54 33.26
## Anxious      45.81 26.48 35.83
## Bored        44.98 26.75 29.94
## Irritated    43.48 26.69 35.66
## Nervous      46.39 26.50 35.94
## Listless     45.35 26.84 29.67
```

```
## Unique variances:
##
##                  S1    S2     S3
## Interested   273.26 53.37  96.43
## Joyful       273.82 48.67  92.81
## Determined   261.88 49.93  92.70
## Calm         265.99 51.75  99.17
## Lively       286.09 48.20 104.04
## Enthusiastic 257.06 50.09 107.07
## Relaxed      270.54 49.55  99.75
## Cheerful     284.69 50.47  83.05
## Content      271.52 41.15  92.38
## Energetic    271.12 53.24  95.55
## Upset        278.71 46.13  92.89
## Gloomy       256.03 46.46  73.85
## Sluggish     245.57 51.70  82.24
## Anxious      276.61 45.65  87.14
## Bored        253.52 47.69 103.11
## Irritated    267.30 44.56  84.99
## Nervous      261.57 49.30  86.21
## Listless     269.07 47.29  92.10
```

Step 2: Obtaining State Assignments

- Each observation is assigned to the state with the highest state-membership probability.
- The inherent classification uncertainty is calculated.
- Relevant for obtaining unbiased estimates for the transition model

```
classification <- step2(data = ESM_fs, model = measurementmodel323)
```

Step 3: Investigating Transition Model

- The MMs (i.e., the factor parameters) are kept fixed
- The transitions between the states are estimated (while correcting for step 2's assignment uncertainty)

Step 3: Investigating Transition Model

```
transitionmodel <- step3(data = ESM,
                         identifier = "id",
                         n_state = 3,
                         postprobs =
                           classification$classification_posteriors[,-1],
                         timeintervals  = "deltaT",
                         initialCovariates = NULL,
                         transitionCovariates =
                           c("intervention", "negativeEvent"))
```

```
Wald tests:

                   Wald df p-value
intervention  213.3821  6       0
negativeEvent  55.7629  6       0
```

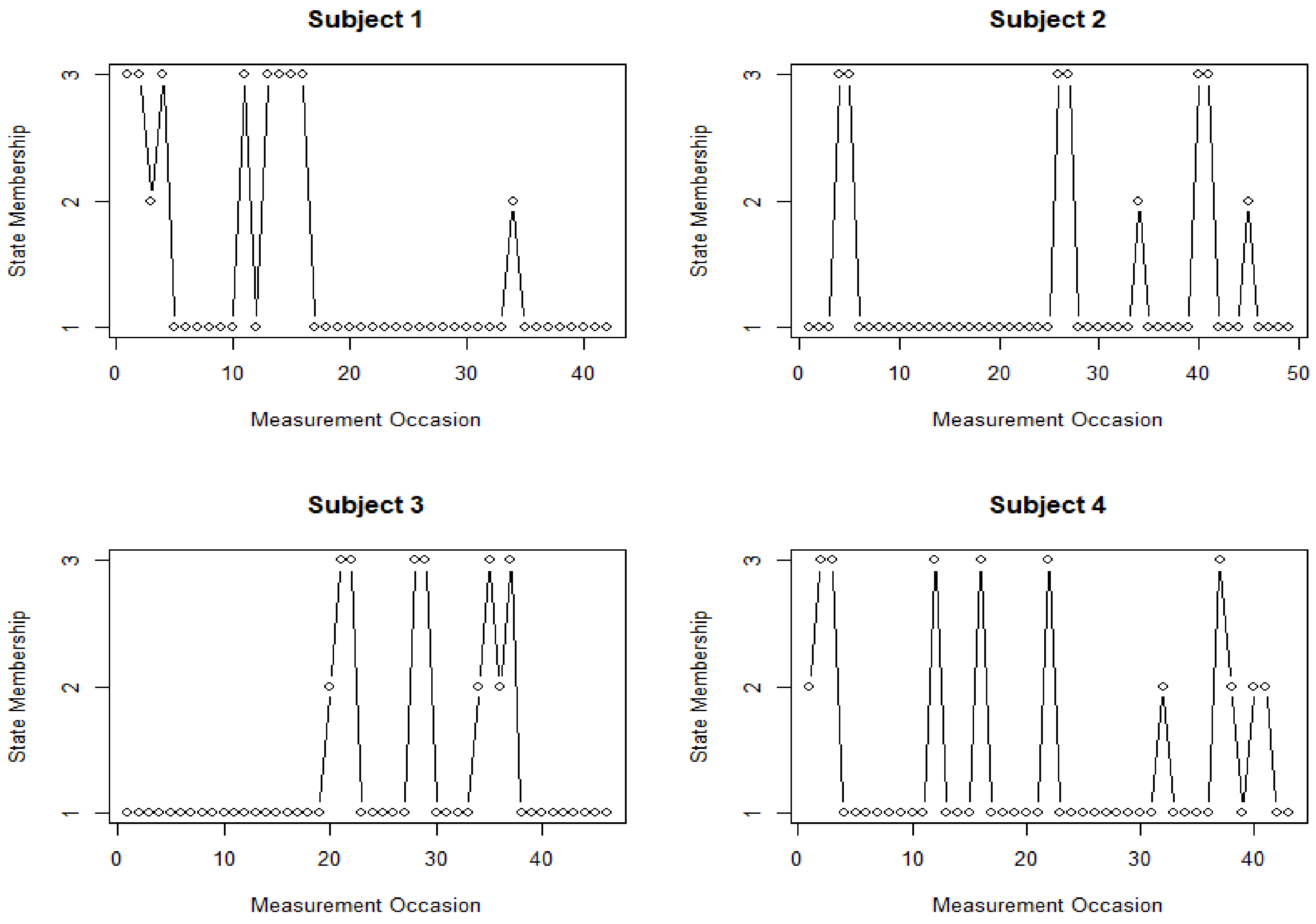
Step 3: Investigating Transition Model

```
probabilities(model = transitionmodel,
              deltaT = 1,
              initialCovariateScores = NULL,
              transitionCovariateScores = c(0, 49.65))
```

```
## 1. Initial state probabilities:
##
## (no covariates defined)
##
##    S1   S2   S3
## 0.42 0.34 0.24
##
## 2. Transition probabilities:
##
## interval length: 1
## intervention score: 0
## negativeEvent score: 49.65
##
##       S1   S2   S3
## S1 0.84 0.07 0.08
## S2 0.37 0.44 0.19
## S3 0.54 0.08 0.39
```

```
## 1. Initial state probabilities:
##
## (no covariates defined)
##
##    S1   S2   S3
## 0.42 0.34 0.24
##
## 2. Transition probabilities:
##
## interval length: 1
## intervention score: 1
## negativeEvent score: 49.65
##
##       S1   S2   S3
## S1 0.71 0.16 0.13
## S2 0.14 0.66 0.20
## S3 0.23 0.15 0.61
```

Step 3: Investigating Transition Model



Accounting for Non-Invariance?

- Obtain dataset with state memberships and state-specific factor scores

```
ESM_fs <- factorscores_lmfa(data = ESM, model = measurementmodel323)
```

```
ESM_fs_cl <- classification$data
```

- Continue with one state only
- Accept that invariance does not hold and focus on substantively interesting results:
  - E.g., learning about situations in which emotion differentiation is reduced

Lab 3

Latent Markov Factor Analysis

References

- Adolf, J., Schuurman, N. K., Borkenau, P., Borsboom, D., & Dolan, C. V. (2014). Measurement invariance within and between individuals: a distinct problem in testing the equivalence of intra- and inter-individual model structures. *Frontiers in Psychology, 5*, 1–14. doi:10.3389/fpsyg.2014.00883
- Lai, M. H. C. (2021). Composite reliability of multilevel data: It's about observed scores and construct meanings. Psychological Methods, 26, 90–102. doi:https://doi.org/10.1037/met0000287
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- Vogelsmeier, L. V. D. E., Vermunt, J. K., van Roekel, E., & De Roover, K. (2019). Latent Markov factor analysis for exploring measurement model changes in time-intensive longitudinal studies. Structural Equation Modeling: A Multidisciplinary Journal, 26, 557–575. doi:10.1080/10705511.2018.1554445