Multilevel

CH-CH-CH-Changes
(David Bowie)

Tilburg University

tesc

Fixed or changing?

There used to be a crude categorization in the types of constructs (social) scientist study.

- Fixed constructs (Traits):
  - Aspects of an individual that do not change over time.
  - E.g., Personality, IQ (although debatable now).

- Changing constructs (States):
  - Aspects that do change.
  - E.g., Mood, Concentration .

What is change?

- Nothing is fixed!
  - “Zoom out” to a longer time-scale, and traits become states.
  - Even mountains (and Betty White) aren’t permanent.

TILBURG UNIVERSITY tesc

What is change?

- Nothing is fixed!
  - “Zoom out” to a longer time-scale, and traits become states.
  - Even mountains aren’t permanent.
- Change also doesn’t exist!

TILBURG UNIVERSITY

tesc

What is change?

- Nothing is fixed!
  - “Zoom out” to a longer time-scale, and traits become states.
  - Even mountains (and Betty White) aren’t permanent.

- Change also doesn’t exist!
  - Change isn’t one thing
  - Very important to realize this

What is change?

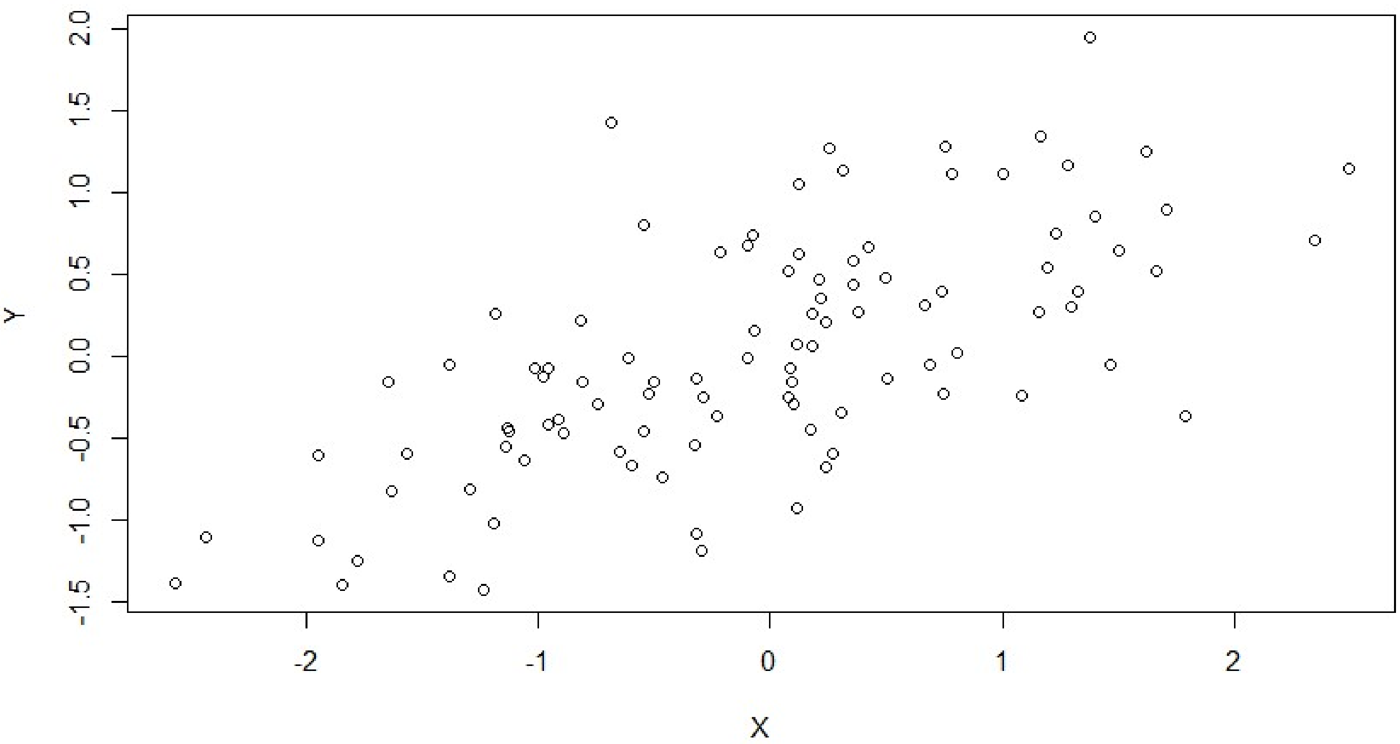
What do you think I mean with “change” isn’t one thing?

Can you think of different types of change?

Tilburg University

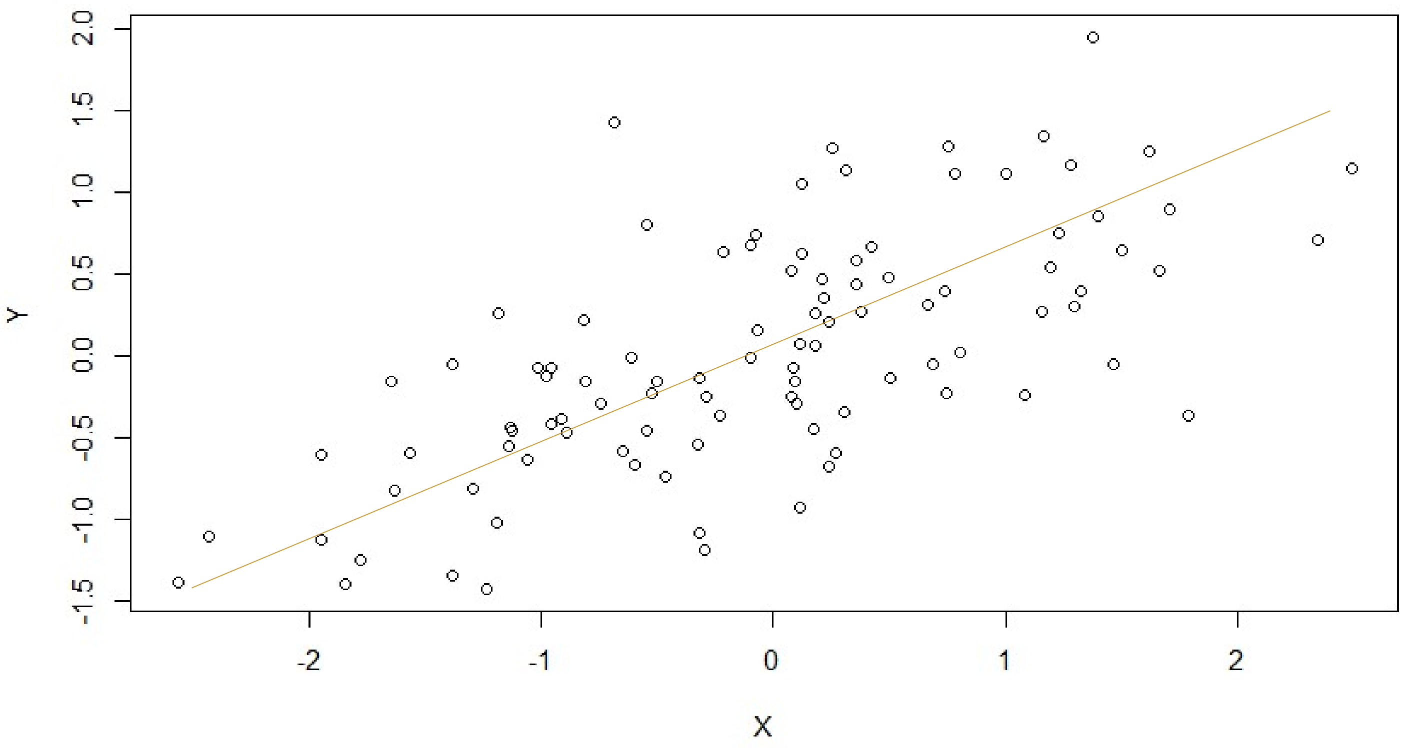
tesc

What is change?



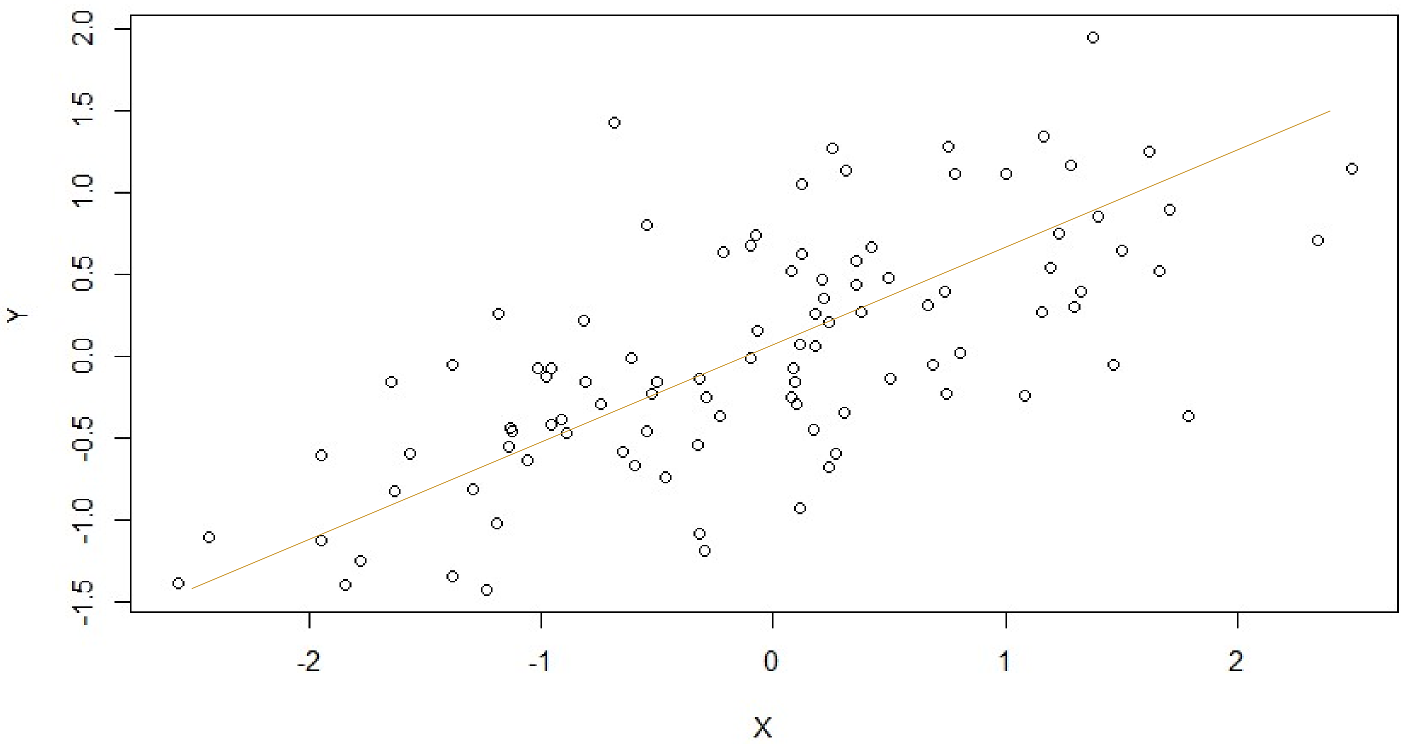
TILBURG

What is change?



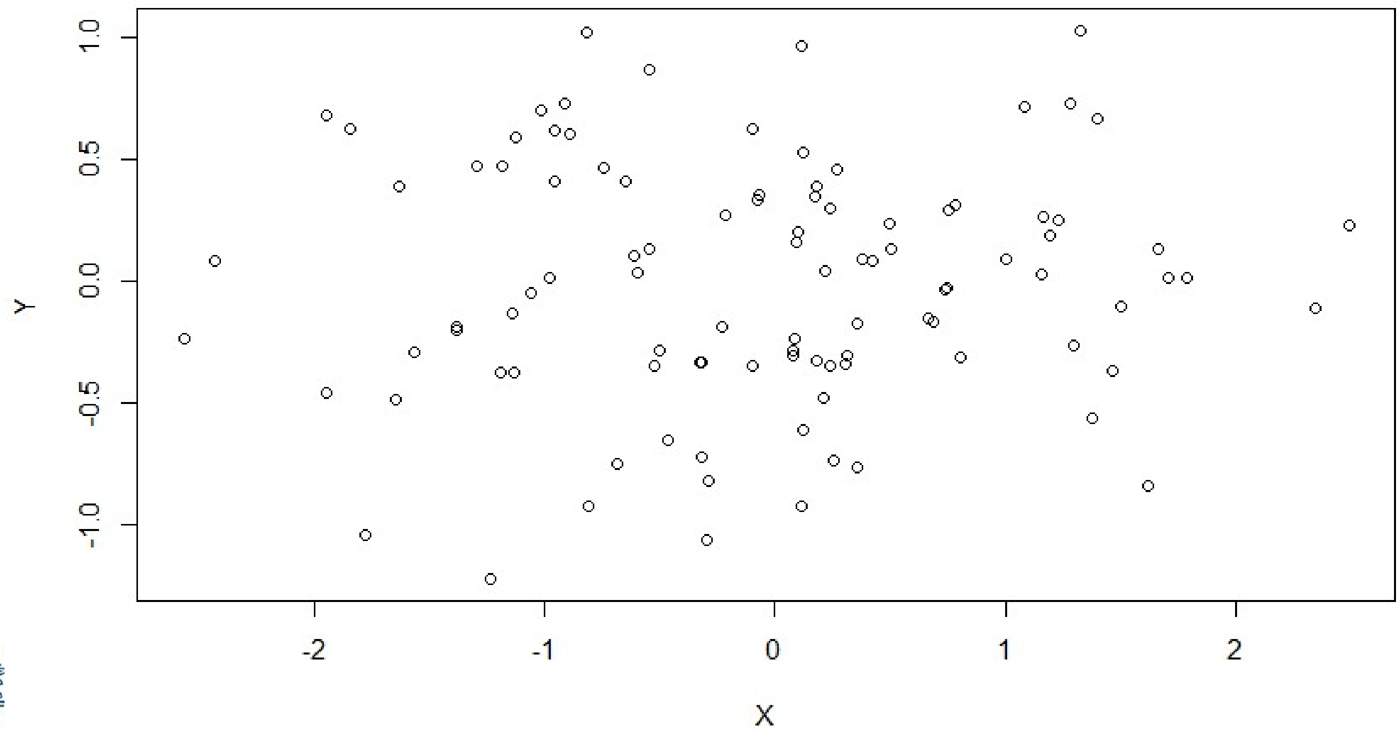
What is change?

What if this is cross-sectional data?

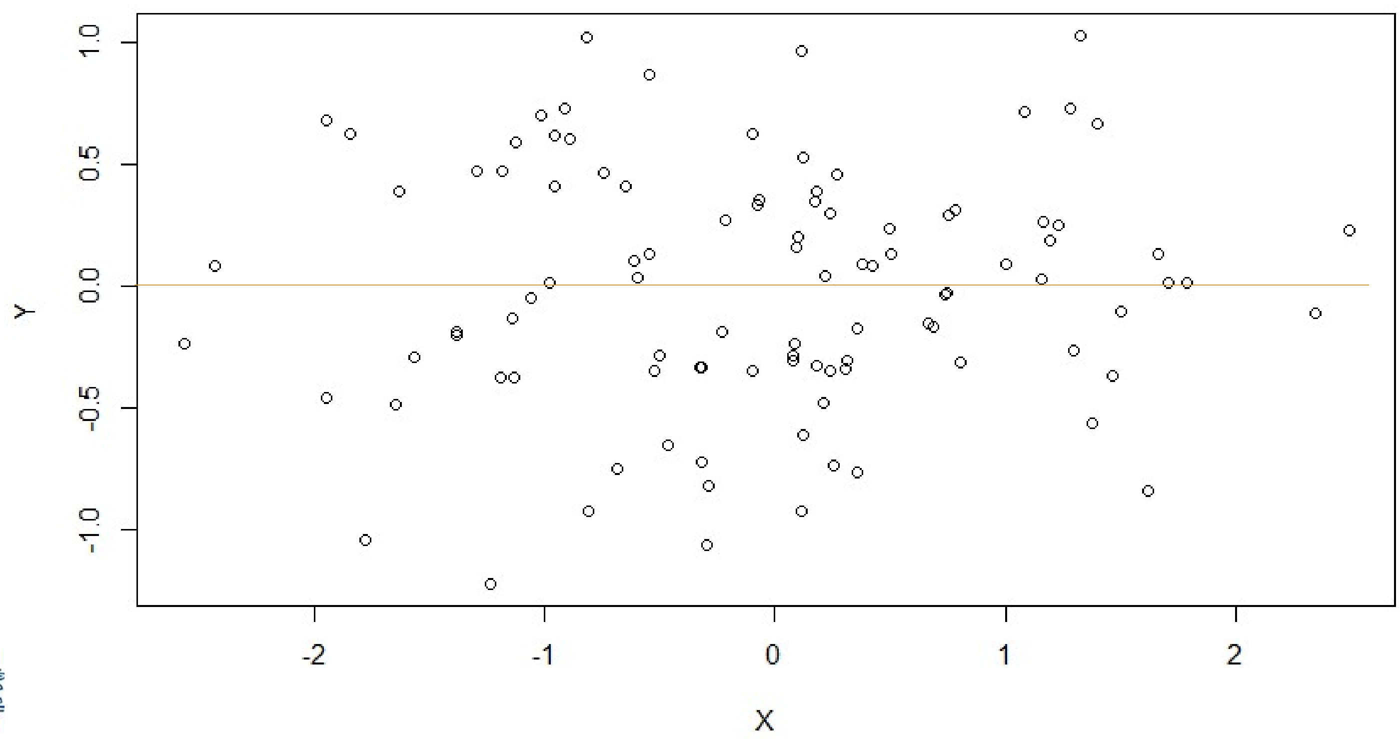


TILBURG

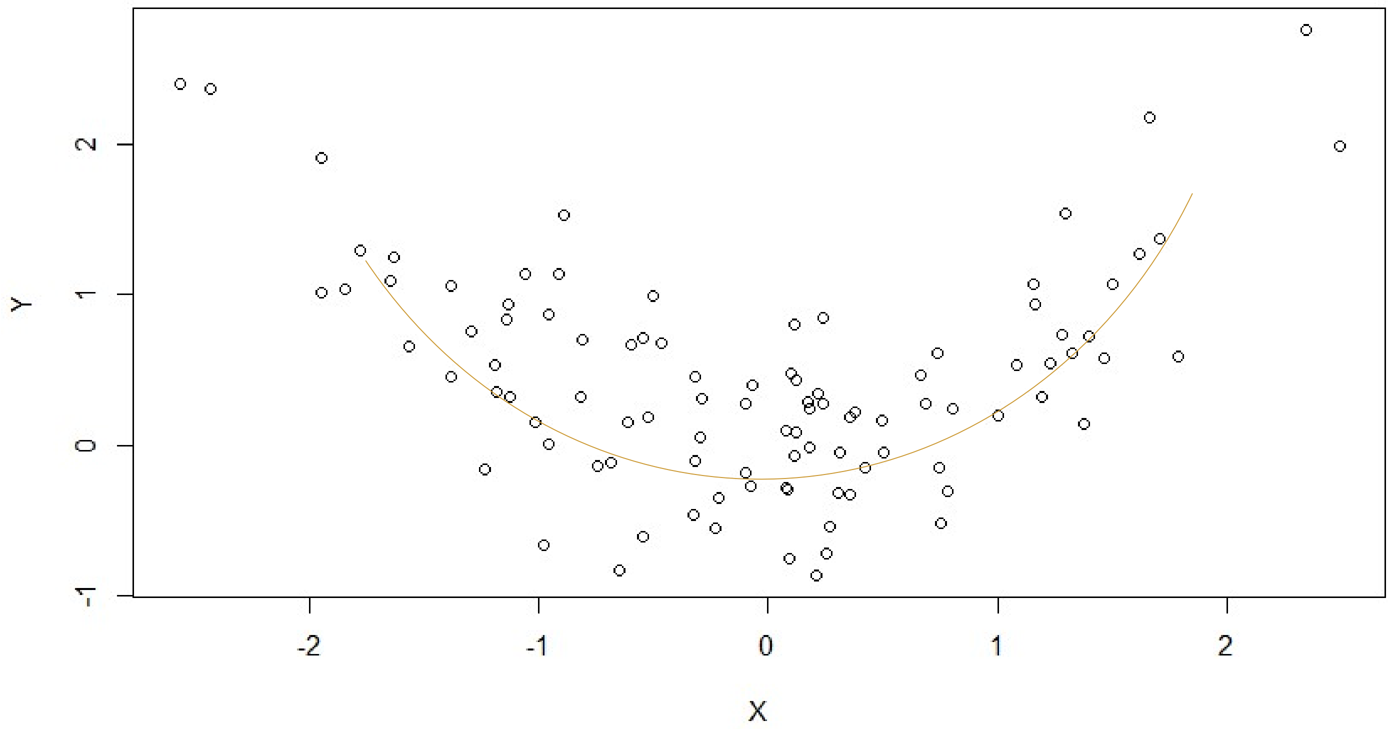
What is change?



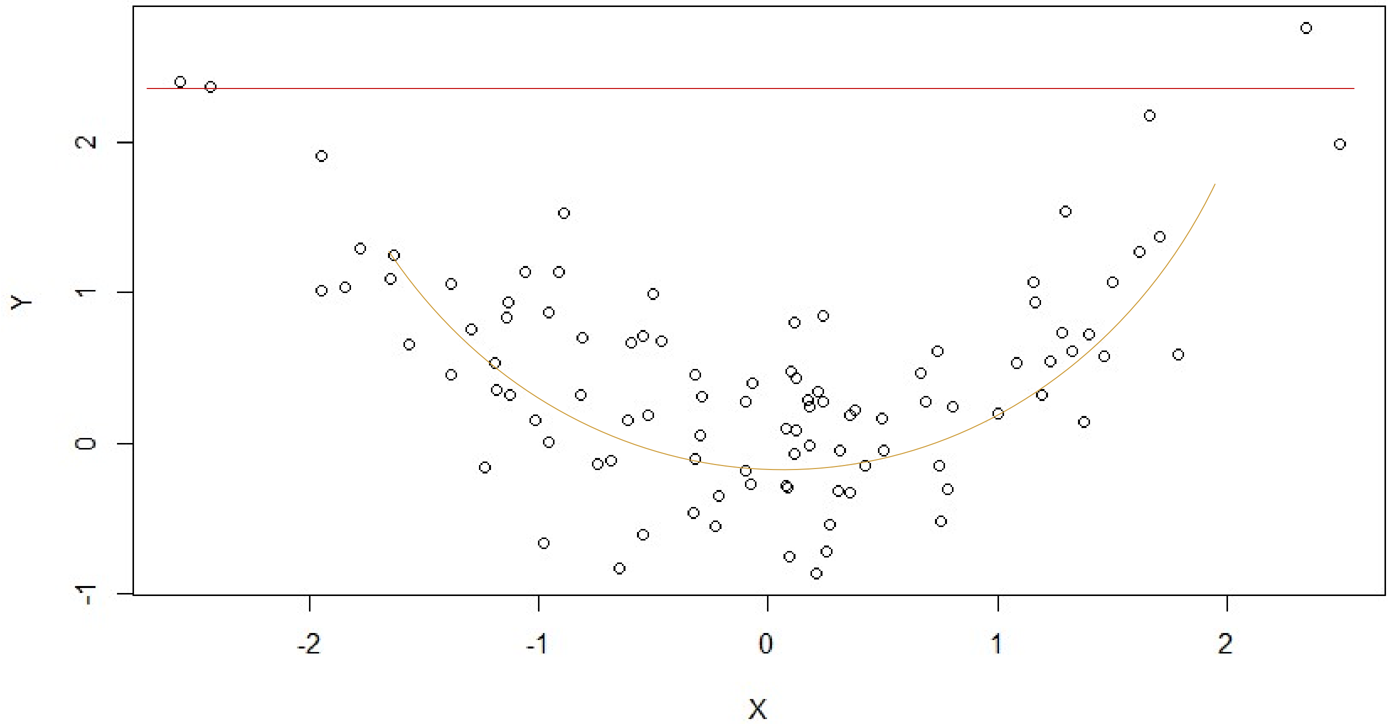
What is change?



What is change?



What is change?



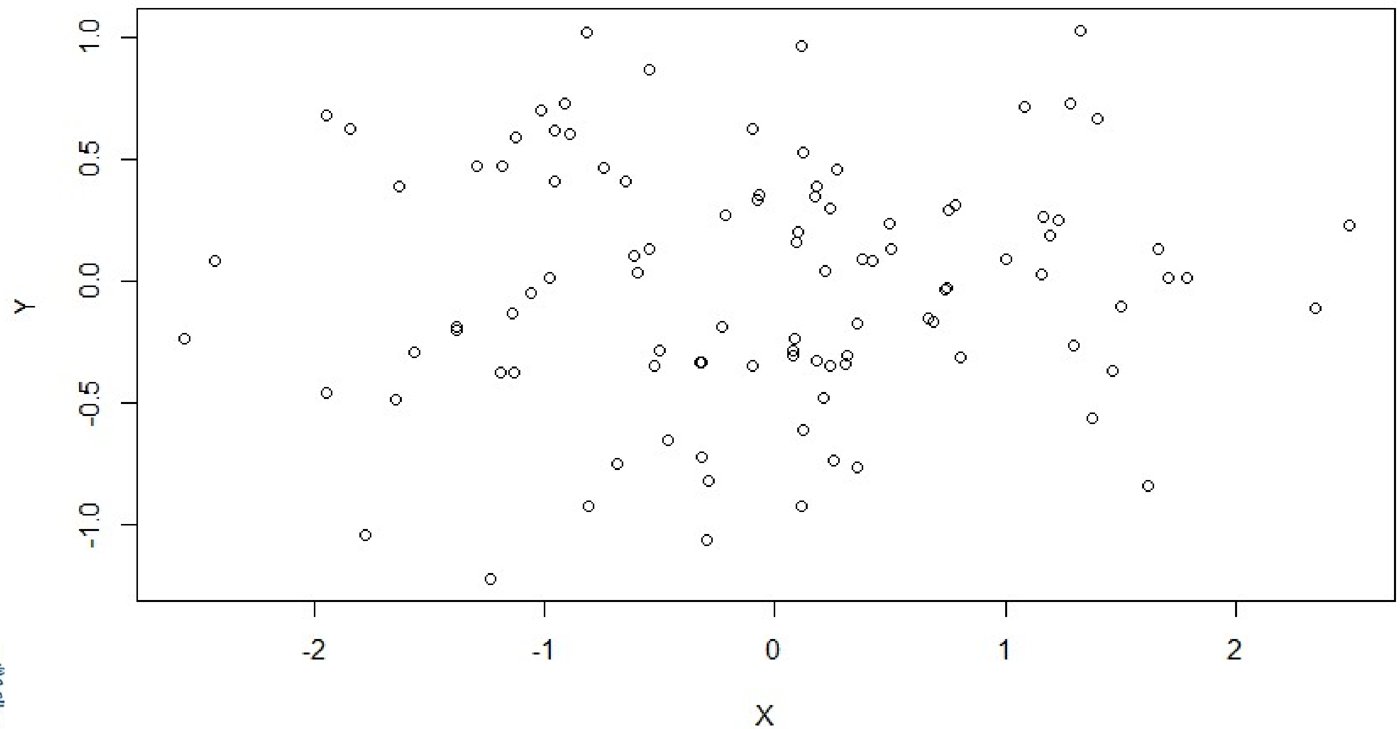
What is change?

- Can think about the following different types of change:
  - Long-term vs Short-term (Note! Long-term not necessarily the sum of short-term)

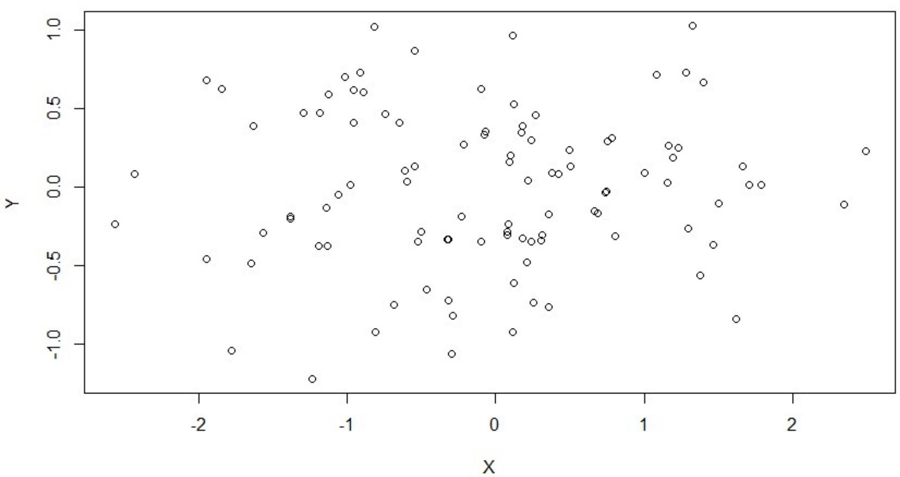
What is change?

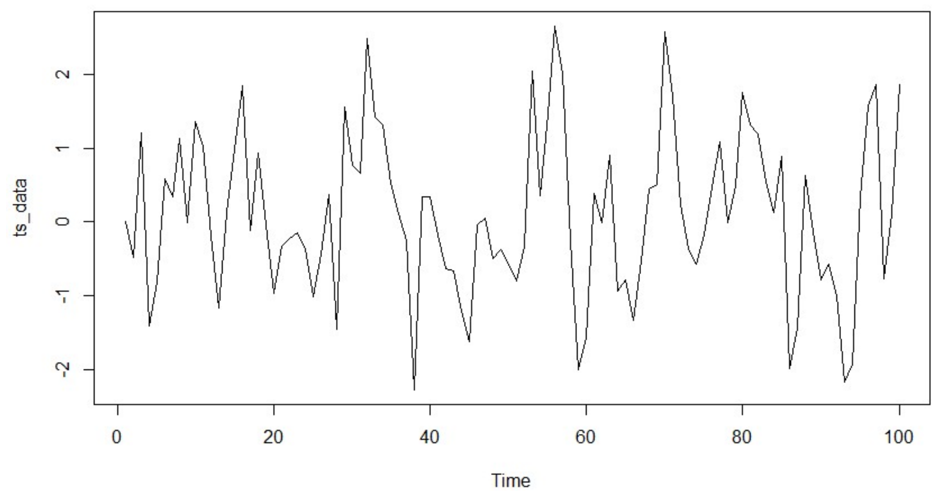
- Can think about the following different types of change:
  - Long-term vs Short-term (Note! Long-term not necessarily the sum of short-term)
  - Reversible vs non-reversible/trend.

What is change?



What is change?





What is change?

- Moreover, "change" can also be a trait(!)…
  - How variable your mood is, could be something that is characteristic of you!
  - Different people might have different amounts of variability in mood.

- …and a state of its own.
  - One person's variability can change over time!

What is change?

- Koval et al.:
  - Inertia (i.e., “lack of change”) in sad/dysphoric affect related to rumination and depression.
  - Amount of inertia differs between individuals.

- Groot:
  - Mood variation different within one person at different times.

What is change?

- Koval et al.:
  - Inertia (i.e., “lack of change”) in sad/dysphoric affect related to rumination and depression.
  - Amount of inertia differs between individuals.

- Groot:
  - Mood variation different within one person at different times.

How could you show this with multilevel analysis?

What is change?

- Usually, focus on long-term, trend change.
  - Mean-level differences.

- Intensive Longitudinal Data/Multilevel can be used for this as well but think about the process your studying!
  - Do you really need to measure someone multiple times a day over several weeks?
  - **Can you give me an example for a construct for which you do? And of a construct for which you don’t?**

What is change?

- Intensive Longitudinal Data really for studying short-term (reversible changes) (and trait-like differences therein).

- What is Intensive Longitudinal Data
  - Many observation per person.
  - Observations close together in time.
  - But…definition not well defined.

What is change?

- Intensive Longitudinal Data really for studying short-term (reversible changes) (and trait-like differences therein).

- What is Intensive Longitudinal Data
  - Many observation per person.
  - Observations close together in time.
  - But…definition not well defined.

- This type of change has its “own” collection of useful models!

What is change?

- This type of change has its “own” collection of useful models!

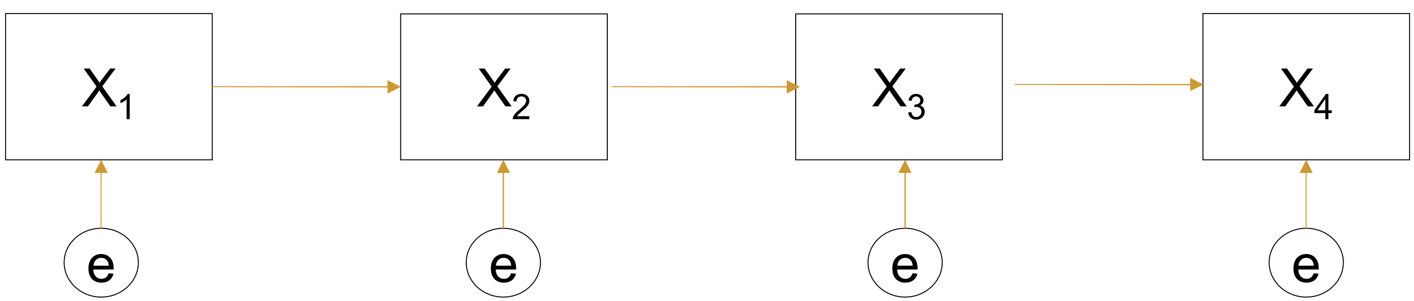
- Would you use multilevel regression models for reversible changes?
  - Why?
  - Why not?

What is change?

- This type of change has its “own” collection of useful models!
- Would you use multilevel regression models for reversible changes?
  - Why?
  - Why not?
- What Intercept and Slope values do you expect to find?
- And what do you expect for the variances in the intercepts and slopes?

Reversible Change

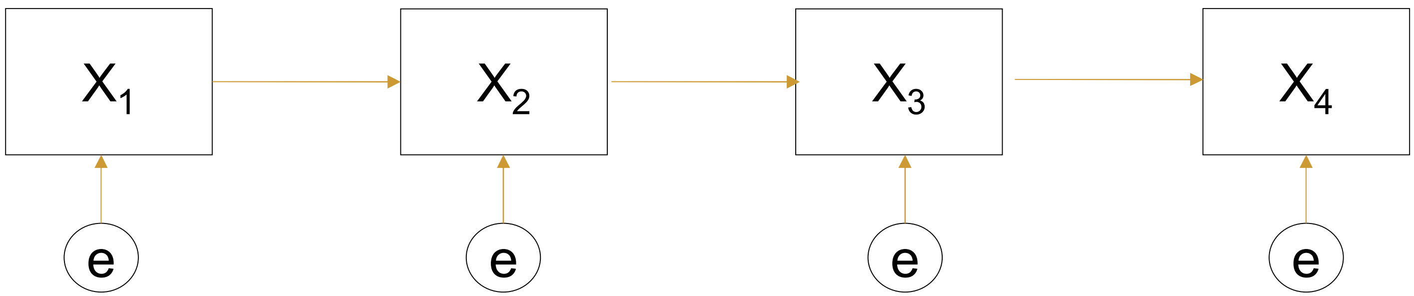
- One type of model used is the AR model



Reversible Change

- One type of model used is the AR model.

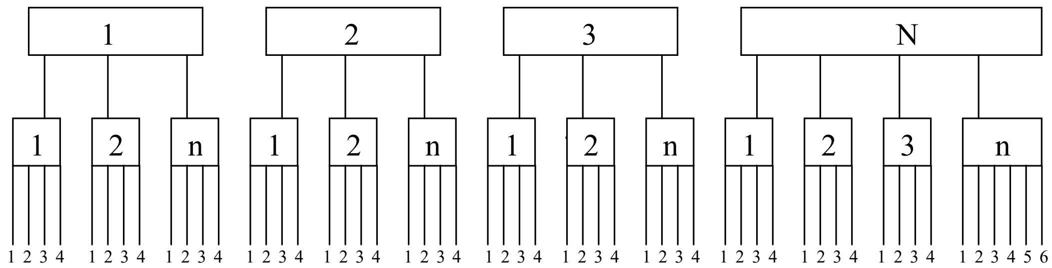
- More on that later 😉…(spoiler: can use multilevel regression for this model too! Just need a small tweak 😉).



Longitudinal Data: Systematic Mean Level Change

Hierarchical analysis:

Figure 1: Example hierarchical data structure



Examples:	**Education**	**Organizational**	**Longitudinal**
Level 3	schools	organizations	classes
Level 2	classes	departments	pupils
Level 1	pupils	individuals	observations

Longitudinal data

- Just another multilevel analysis
- Advantages:
  - Fixed occasions and varying occasions.
  - Change can differ between individuals
  - Balanced data is not a requirement.
  - Can add higher level predictors.

Longitudinal data

- Fixed occasions example:
- 200 students
- Dependent variable:
  - *GPA*: grade point average for six successive semesters
- Independent variables:
  - *job* (number of hours worked, scale from 0-4)
  - *sex* (boy = 0, girl = 1)
  - *HighGPA* (mean high school *GPA*)

Multilevel regression model for longitudinal data

- Regression equation with lower level variables:

$$Y_{ti} = \pi_{0i} + \pi_{1i}Time_{ti} + \pi_{pi}X_{pti} + e_{ti}$$

- coefficients at the lowest level: π

- person level coefficients: β

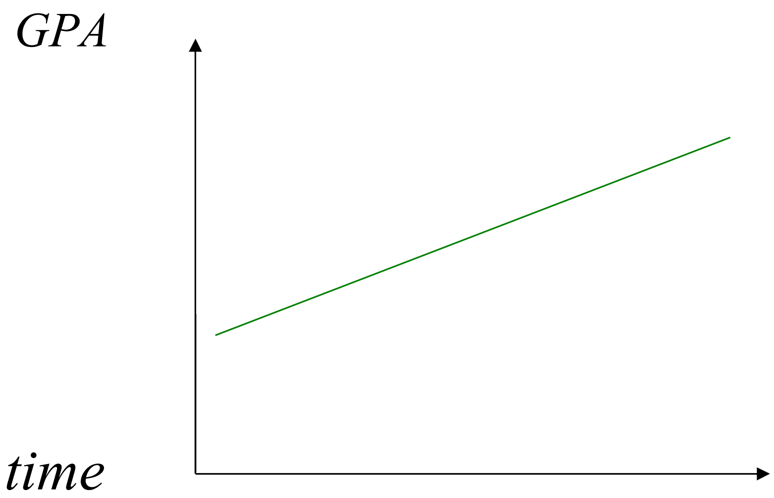
- Y_{ti}: response variable of individual i measured at time point t

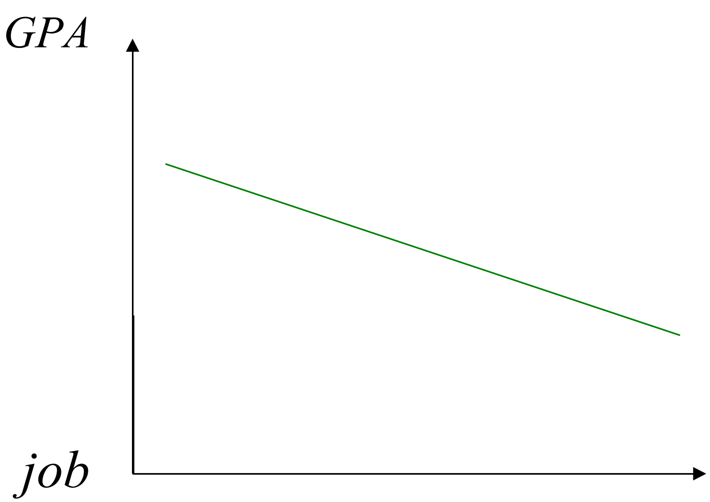
- $Time_{ti}$: time variable, that indicates the time point

- X_{pti}: time varying covariates ($p \neq 0, 1$)

- Example: $$GPA_{ti} = \pi_{0i} + \pi_{1i}Time_{ti} + \pi_{2i}Job_{ti} + e_{ti}$$

First research question





Multilevel regression model for longitudinal data

- Add *time invariant covariates* (second level variables, student characteristics).

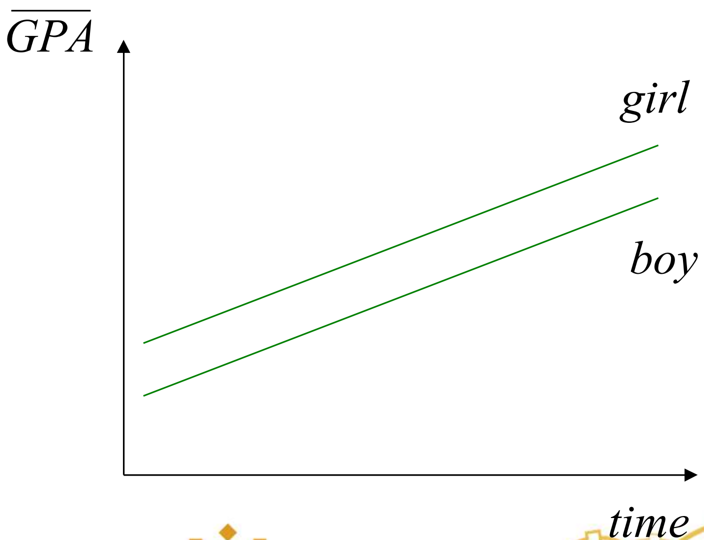
$$Y_{ti} = \pi_{0i} + \pi_{1i} Time_{ti} + \pi_{pi} X_{pti} + e_{ti}$$

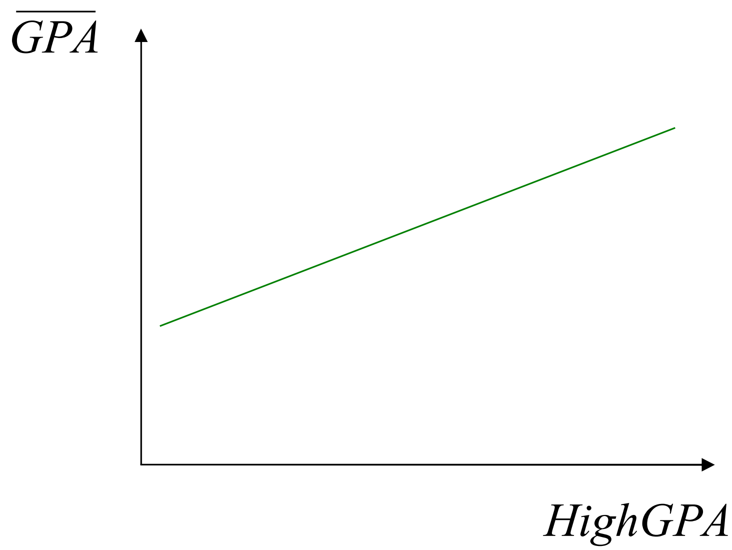
$$\pi_{0i} = \beta_{00} + \beta_{0q} Z_{qi} + u_{0i}$$

Second research question

$$GPA_{ti} = \pi_{0i} + \pi_{1i}Time_{ti} + \pi_{2i}Job_{ti} + e_{ti}$$

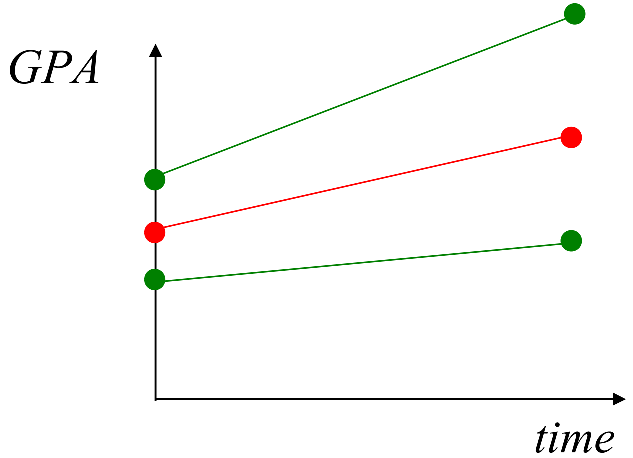
$$\pi_{0i} = \beta_{00} + \beta_{01}sex_i + \beta_{02}HighGPA_i + u_{0i}$$



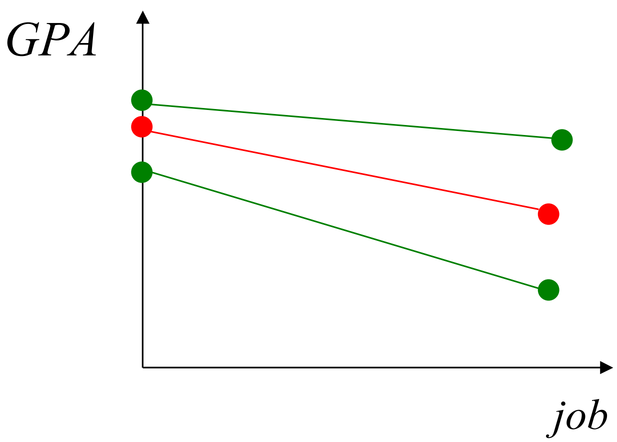


Third research question

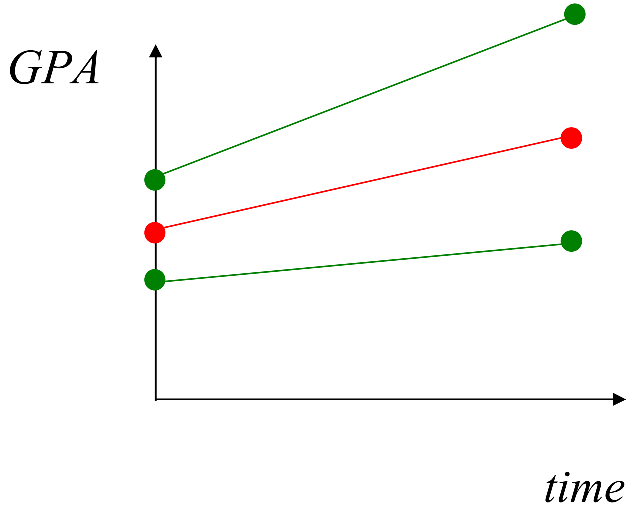
$$\pi_{1i} = \beta_{10} + u_{1i}$$



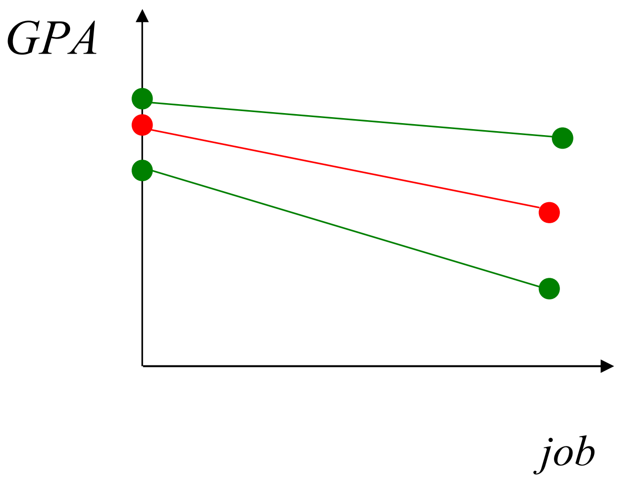
$$\pi_{2i} = \beta_{20} + u_{2i}$$



Fourth research question



$$\pi_{1i} = \beta_{10} + \beta_{1q} Z_{qi} + u_{1i}$$



$$\pi_{2i} = \beta_{20} + \beta_{2q} Z_{qi} + u_{2i}$$

TILBURG UNIVERSITY

tesc

Multilevel regression equation

$$GPA_{ti} = \beta_{00} + \beta_{01} sex_i + \beta_{02} HighGPA_i$$
$$+ \pi_{1i} Time_{ti} + \pi_{2i} Job_{ti} + u_{0i} + e_{ti}$$

$$\pi_{1i} = \beta_{10} + \beta_{11} sex_i + \beta_{12} HighGPA_i + u_{1i}$$

$$\pi_{2i} = \beta_{20} + \beta_{21} sex_i + \beta_{22} HighGPA_i + u_{2i}$$

$$GPA_{ti} = \beta_{00} + \beta_{01} sex_i + \beta_{02} HighGPA_i + \beta_{10} Time_{ti} + \beta_{20} Job_{ti} + \beta_{11} Time_{ti} \times sex_i$$
$$+ \beta_{12} Time_{ti} \times HighGPA_i + \beta_{21} Job_{ti} \times sex_i + \beta_{22} Job_{ti} \times HighGPA_i$$
$$+ u_{1i} Time_{ti} + u_{2i} Job_{ti} + u_{0i} + e_{ti}$$

Steps of a multilevel analysis

1. Check whether multilevel is necessary
2. Add all level 1 main effects and interactions
3. Add all level 2 main effects and interactions
4. Check level 1 effects for random slopes
5. If random slopes are present: add cross-level interactions

TILBURG UNIVERSITY

tesc

Results longitudinal multilevel analysis, fixed effects

- **Step 1: Intercept Only Model (is multilevel necessary?)**

Results multilevel analysis of GPA		
Model:	M0: intercept only model	
Fixed part		
Predictor	coefficient	p-value
Intercept	2.87	0.000
time		
job		
sex		
highGPA		
Random part		
σ^2_e	0.098	
σ^2_{u0}	0.057	0.000

$$GPA_{it} = \pi_{0i} + e_{it}$$

$$\pi_{0i} = \beta_{00} + u_{0t}$$

Results longitudinal multilevel analysis, fixed effects

- **Step 2a: Add Time**

Results multilevel analysis of GPA		
Model:	M1: time model	
Fixed part		
Predictor	coefficient	p-value
Intercept	2.60	0.000
time	0.11	0.000
job		
sex		
highGPA		
Random part		
σ^2_e	0.058	
σ^2_{u0}	0.064	0.000

$$GPA_{it} = \pi_{0i} + \pi_1 Time + e_{it}$$
$$\pi_{0i} = \beta_{00} + u_{0t}$$

Results longitudinal multilevel analysis, fixed effects

- Step 2a: add *time* variable

$$GPA_{ti} = 2.60 + 0.11 time_{ti} + u_{0i} + e_{ti}$$

- Interpretation and relevance of the regression coefficients:
    - intercept:
        - the model predicts a *GPA*-score of 2.60 at the first time point
    - *time*:
        - b = 0.11,
        - relevance: maximum difference: 5 $\times$ 0.11 = 0.55 on the *GPA*-scale 1.7-4

TILBURG UNIVERSITY tesc

Results longitudinal multilevel analysis, fixed effects

- **Step 2b: Add other level 1 predictors**

Results multilevel analysis of GPA						
Model:	M0: intercept only model		M1: time model		M2: M1 + job	
Fixed part						
Predictor	coefficient	p-value	coefficient	p-value	coefficient	p-value
Intercept	2.87	0.000	2.60	0.000	2.61	0.000
time			0.11	0.000	0.10	0.000
job					-0.17	0.000
sex						
highGPA						
Random part						
σ^2_e	0.098		0.058		0.055	
σ^2_{u0}	0.057	0.000	0.064	0.000	0.053	0.000

$$GPA_{it} = \pi_{0i} + \pi_1 Time + \pi_2 Job + e_{it}$$
$$\pi_{0i} = \beta_{00} + u_{0t}$$

Results longitudinal multilevel analysis, fixed effects

- **Step 2b: add *job* variable:**

$$GPA_{ti} = 2.61 + 0.10 time_{ti} - 0.17\, job_{ti} + u_{0i} + e_{ti}$$

- Interpretation and relevance of the regression coefficient:
  - *job*:
    - b = -0.17,
    - relevance: maximum difference: 2 × -0.17 = -0.34 on the *GPA*-scale 1.7-4

Explained variance

- **Compare the random effects of model 0, 1 and 2**

Results multilevel analysis of GPA						
Model:	M0: intercept only model		M1: time model		M2: M1 + job	
Fixed part						
Predictor	coefficient	p-value	coefficient	p-value	coefficient	p-value
Intercept	2.87	0.000	2.60	0.000	2.61	0.000
time			0.11	0.000	0.10	0.000
job					-0.17	0.000
sex						
highGPA						
Random part						
σ^2_e	0.098		0.058		0.055	
σ^2_{u0}	0.057	0.000	0.064	0.000	0.053	0.000

Explained variance

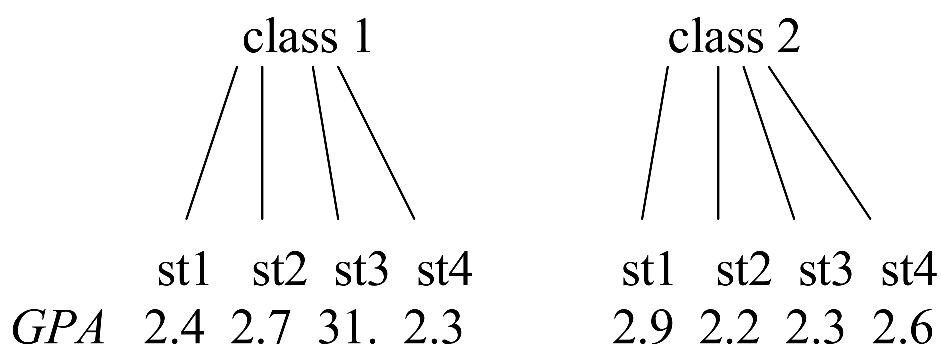
- **Compare the random effects of model 0, 1 and 2**

Results multilevel analysis of GPA						
Model:	M0: intercept only model		M1: time model		M2: M1 + job	
Fixed part						
Predictor	coefficient	p-value	coefficient	p-value	coefficient	p-value
Intercept	2.87	0.000	2.60	0.000	2.61	0.000
time			0.11	0.000	0.10	0.000
job					-0.17	0.000
sex						
highGPA						
Random part						
σ^2_e	0.098		0.058		0.055	
σ^2_{u0}	0.057	0.000	0.064	0.000	0.053	0.000

- Model 1: second level error variance is bigger than in model 0

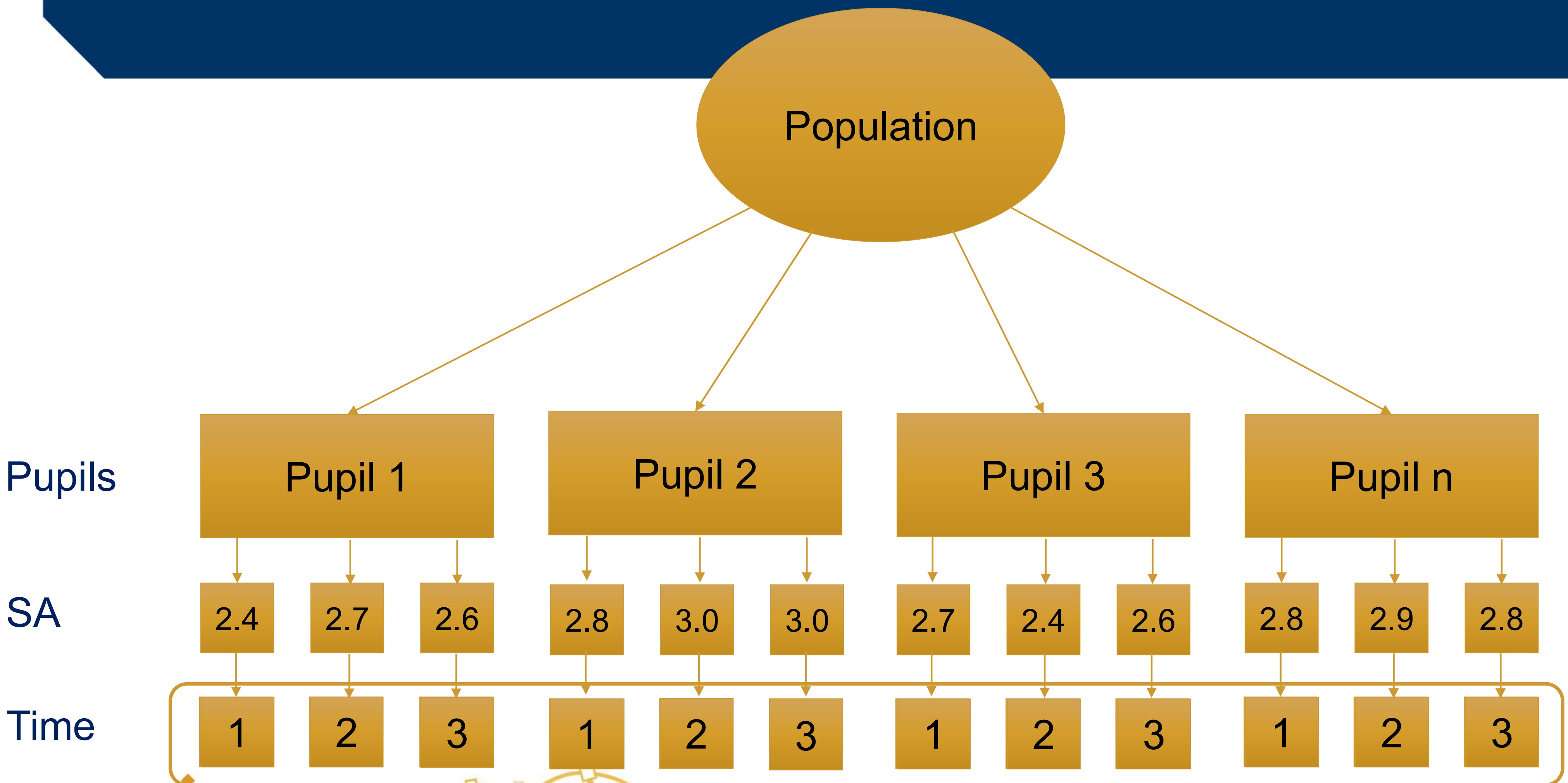
Explained variance

- Multilevel divides total variance assuming 2-stage sampling
  - So first draw a random sample of level 2 units.
  - Then draw a random sample from these level 2 units to get your level 1 observations.
  - Because of this the model assumes at least some variance on all variables (even if it is just sampling error.
- Nested data example:



Explained variance

- With Fixed Timepoints, all scores on the time variable are exactly the same!
  - So, less variance on level 1 then expected!
  - The model notices this when ‘Time’ is added as a predictor and corrects its mistake.



Tilburg University tesc

Explained variance

- With Fixed Timepoints, all scores on the time variable are exactly the same!
  - So, less variance on level 1 then expected!
  - The model notices this when 'Time' is added as a predictor and corrects its mistake.

- Variance on level 1 was too high and is lowered
  - Variance on level 2 goes up since total variance is fixed

Explained variance

• Results multilevel analysis

Results multilevel analysis of GPA

Model:	M0: intercept only model		M1: time model		M2: M1 + job	
Fixed part						
Predictor	coefficient	p-value	coefficient	p-value	coefficient	p-value
Intercept	2.87	0.000	2.60	0.000	2.61	0.000
time			0.11	0.000	0.10	0.000
job					-0.17	0.000
sex						
highGPA						
Random part						
σ^2_e	0.098		0.058		0.055	
σ^2_{u0}	0.057	0.000	0.064	0.000	0.053	0.000

Simple Solution: Don't use an intercept only model!

The model with time is the baseline model.

Explained variance

- *Time* model:
  - σ_e^2 = 0.058
  - $\sigma_{u_0}^2$ = 0.063

$$R_1^2 = \frac{0.058 - 0.055}{0.058} = 0.05$$

- *Job* model:
  - σ_e^2 = 0.055
  - $\sigma_{u_0}^2$ = 0.052

$$R_2^2 = \frac{0.063 - 0.052}{0.063} = 0.17$$

Tilburg University

tesc

Results longitudinal multilevel analysis, fixed effects

- **Step 3: add second level variables**

Results multilevel analysis of GPA				
Model:	M2b: lower level variables		M3: M2b + higher level variables	
Fixed part				
Predictor	coefficient	p-value	coefficient	p-value
Intercept	2.61	0.000	2.53	0.000
time	0.10	0.000	0.10	0.000
job	-0.17	0.000	-0.17	0.000
sex			**0.15**	**0.000**
highGPA			**0.085**	**0.000**
Random part				
σ^2_e	0.055		0.055	
σ^2_{u0}	0.053	0.000	0.046	0.000

$$GPA_{it} = \pi_{0i} + \pi_1 Time + \pi_1 Job + e_{it}$$
$$\pi_{0i} = \beta_{00} + \beta_{01} Sex + \beta_{02} highGPA + u_{0t}$$

Results longitudinal multilevel analysis, fixed effects

- **Step 3: add second level variables**

$$GPA_{ti} = 2.53 + 0.10 time_{ti} - 0.17 job_{ti} + 0.15 sex_i + 0.085 HighGPA + u_{0i} + e_{ti}$$

- Interpretation and relevance of the regression coefficients:
- *sex*:
  - b = 0.15, difference between boys and girls
  - relevance: girls perform 0.15 better than boys on the mean *GPA*-scale 1.7-4

- *HighGPA*:
  - b = 0.085
  - relevance: maximum difference: 2 $\times$ 0.085 = 0.17 on the mean *GPA*-scale 1.7-4

Tilburg University

Explained variance

- *Time* model:
  - $\sigma_e^2 = 0.058$
  - $\sigma_{u_0}^2 = 0.063$

$$R_1^2 = \frac{0.058 - 0.055}{0.058} = 0.05$$

- *Job* model:
  - $\sigma_e^2 = 0.055$
  - $\sigma_{u_0}^2 = 0.052$

$$R_2^2 = \frac{0.063 - 0.052}{0.063} = 0.17$$

- *Job*, *sex* and *HighGPA*:
  - $\sigma_e^2 = 0.055$
  - $\sigma_{u_0}^2 = 0.045$

$$R_2^2 = \frac{0.063 - 0.045}{0.063} = 0.29$$

Results longitudinal multilevel analysis, random slopes

- **Step 4: add random slopes**

- H_0: the relation between *time* and *GPA* is the same within all students
- H_1: the relation between *time* and *GPA* is not the same within all students

- H_0: the relation between *job* and *GPA* is the same within all students
- H_1: the relation between *job* and *GPA* is not the same within all students

Results longitudinal multilevel analysis, random slopes

- **Step 4: add random slope *time***

Results multilevel analysis of GPA				
Model:	M3: lower and higher level variables		M4: M3 + random slope time	
Fixed part				
Predictor	coefficient	p-value	coefficient	p-value
Intercept	2.53	0.000	2.55	0.000
time	0.10	0.000	0.10	0.000
job	-0.17	0.000	-0.13	0.000
sex	0.15	0.000	0.12	0.000
highGPA	0.085	0.000	0.089	0.001
Random part				
σ^2_e	0.055		0.042	
σ^2_{u0}	0.046	0.000	0.039	0.000
σ^2_{u1}			**0.0039**	**0.000**
σ^2_{u0u1}			-0.0025	

$$GPA_{it} = \pi_{0i} + \pi_{1j}Time + \pi_2 Job + e_{it}$$
$$\pi_{0i} = \beta_{00} + \beta_{01}Sex + \beta_{02}highGPA + u_{0t}$$
$$\pi_{1i} = \beta_{10} + u_{1t}$$

TILBURG UNIVERSITY

tesc

Results longitudinal multilevel analysis, random slopes

- **Step 4: add random slope *job***

Results multilevel analysis of GPA				
Model:	M3: lower and higher level variables		M4: M3 + random slope job	
Fixed part				
Predictor	coefficient	p-value	coefficient	p-value
Intercept	2.53	0.000	2.53	0.000
time	0.10	0.000	0.10	0.000
job	-0.17	0.000	-0.18	0.000
sex	0.15	0.000	0.15	0.000
highGPA	0.085	0.000	0.084	0.000
Random part				
σ^2_e	0.055		0.055	
σ^2_{u0}	0.046	0.000	0.045	0.000
σ^2_{u1}			**0.0045**	**>.500**
σ^2_{u0u1}			-0.014	

$$GPA_{it} = \pi_{0i} + \pi_1 Time + \pi_{2j} Job + e_{it}$$
$$\pi_{0i} = \beta_{00} + \beta_{01} Sex + \beta_{02} highGPA + u_{0t}$$
$$\pi_{2i} = \beta_{20} + u_{2t}$$

Tilburg University
tesc

Results longitudinal multilevel analysis, cross-level interactions

- **Step 5: add cross-level interactions**

- H_0: *sex* can't explain the different relations between *GPA* and *time* in different individuals
- H_1: *sex* explains (a part of) the different relations between *GPA* and *time* in different individuals

- H_0: *HighGPA* can't explain the different relations between *GPA* and *time* in different individuals
- H_1: *HighGPA* explains (a part of) the different relations between *GPA* and in different individuals

Results longitudinal multilevel analysis, cross-level interactions

- **Step 5: add cross-level interactions**

Results multilevel analysis of GPA				
Model:	M3: lower and higher level variables		M4: M3 + random slope time	
Fixed part				
Predictor	coefficient	p-value	coefficient	p-value
Intercept	2.55	0.000	2.57	0.000
time	0.10	0.000	0.088	0.000
job	-0.13	0.000	-0.13	0.000
sex	0.12	0.000	0.076	0.029
highGPA	0.089	0.001	0.091	0.002
time×sex			**0.029**	**0.009**
time×highGPA			**-0.0018**	**0.853**
Random part				
σ^2_e	0.042		0.042	
σ^2_{u0}	0.039	0.000	0.039	0.000
σ^2_{u1}	0.0039	0.000	0.0037	0.000
σ^2_{u0u1}	-0.0025		-0.0037	

$$GPA_{it} = \pi_{0i} + \pi_{1j}Time + \pi_2 Job + e_{it}$$
$$\pi_{0i} = \beta_{00} + \beta_{01}Sex + \beta_{02}highGPA + u_{0t}$$
$$\pi_{1i} = \beta_{10} + \beta_{11}Sex + \beta_{12}highGPA + u_{1t}$$

Results longitudinal multilevel analysis, cross-level interactions

- **Step 5: add cross-level interaction**

$$GPA_{ti} = 2.57 + 0.088time_{ti} - 0.13job_{ti} + 0.076sex_{i} + 0.089HighGPA + 0.030time_{ti} \times sex_{j} + u_{1i}time_{ij} + u_{0i} + e_{ti}$$

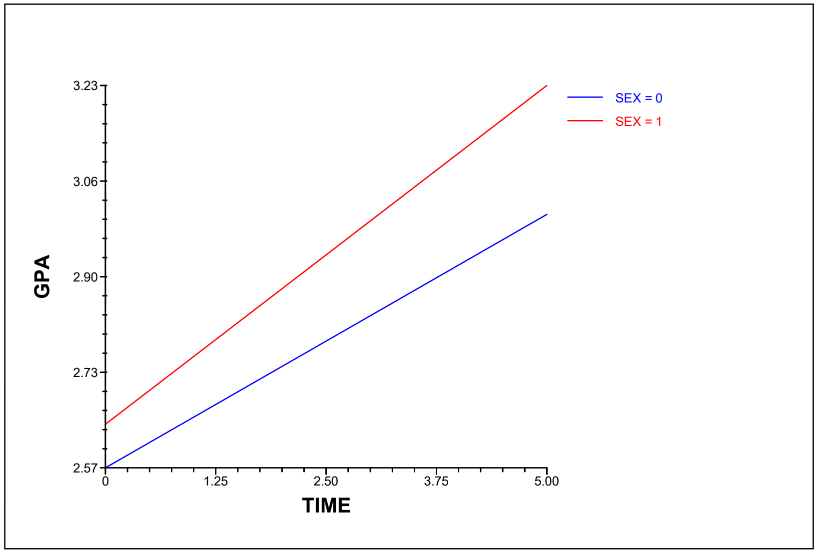
- Interpretation and relevance of regression coefficients:
  - *time.sex*:
    - b = 0.030

- Explained slope variance:

$$R^2_{slope} = \left(\frac{0.0039 - 0.0037}{0.0039} \right) = 0.05$$

Tilburg University

Results longitudinal multilevel analysis, cross-level interactions



- differences between male and female students increases

Practical 6

Residuals

Longitudinal Multilevel Regression

- Remember that you always need to think about what your model says about the data.

- And a multilevel regression (with a fixed effect of time) is saying something that might be a bit weird with longitudinal data.

- Remember:

$$\rho = \frac{\sigma_{u_0}^2}{\sigma_{u_0}^2 + \sigma_e^2}$$

Multilevel Repeated Measures

$$\rho = \frac{\sigma_{u_0}^2}{\sigma_{u_0}^2 + \sigma_e^2}$$

$$\Sigma(\mathrm{Y}) = \begin{pmatrix} \sigma_e^2 + \sigma_{u_0}^2 & \sigma_{u_0}^2 & \sigma_{u_0}^2 & \cdots & \sigma_{u_0}^2 \\ \sigma_{u_0}^2 & \sigma_e^2 + \sigma_{u_0}^2 & \sigma_{u_0}^2 & \cdots & \sigma_{u_0}^2 \\ \sigma_{u_0}^2 & \sigma_{u_0}^2 & \sigma_e^2 + \sigma_{u_0}^2 & \cdots & \sigma_{u_0}^2 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ \sigma_{u_0}^2 & \sigma_{u_0}^2 & \sigma_{u_0}^2 & \cdots & \sigma_e^2 + \sigma_{u_0}^2 \end{pmatrix}.$$

- Makes certain assumptions about variance-covariance matrix
  - Related to Sphericity in RM ANOVA
  - Compound Symmetry
  - All variances and covariances equal (likely?)
  - Inflated Type I errors if violated

- What do we do if violated?

Multilevel Repeated Measures

$$GPA_{it} = \pi_{1i}T1_{it} + \pi_{2i}T2_{it} + \pi_{3i}T3_{it} + \pi_{4i}T4_{it} + \pi_{5i}T5_{it} + \pi_{6i}T6_{it}$$

$$\pi_{1i} = \beta_{10} + u_{1t}$$

$$\pi_{2i} = \beta_{20} + u_{2t}$$

$$\vdots$$

$$\pi_{6i} = \beta_{60} + u_{6t}$$

- Use dummies for all measurement occasions
  - Remove Intercept
  - Each dummie has random slope

- This is basically a MANOVA

SPSS MANOVA analysis

- *Random part*: (co)variance matrix

	GPA1	*GPA2*	*GPA3*	*GPA4*	*GPA5*	*GPA6*
GPA1	0.097					
GPA2	0.036	0.113				
GPA3	0.026	0.051	0.125			
GPA4	0.021	0.056	0.083	0.126		
GPA5	0.024	0.061	0.090	0.107	0.128	
GPA6	0.025	0.055	0.089	0.111	0.119	0.142

SPSS MANOVA analysis

- Disadvantages:
  - no time variable
  - no time varying covariates
  - listwise deletion

- No direct test for “change”, but can use a contrast

SPSS MANOVA analysis

- Categorical

  H_0: all means are equal ($\mu_0 = \mu_1 = \mu_2 = \mu_3 = \mu_4 = \mu_5$)

  H_1: not all means are equal

- Linear

  H_0: there isn't linear relation between *GPA* and *time*

  H_1: there is a linear relation between *GPA* and *time*

MANOVA significance tests on GPA example data			
Effect tested:	F	df	*p*
GPA (categorical)	4.53	5/193	.001
GPA (linear trend)	12.77	1/197	.000
HighGPA	9.16	1/197	.003
sex	7.23	1/197	.000

Tilburg University

SPSS MANOVA analysis

- H_0: there isn't a relation between *GPA* and *sex*

  H_1: there is a relation between *GPA* and *sex*

- H_0: there isn't relation between *GPA* and *HighGPA*

  H_1: there is a positive relation between *GPA* and *HighGPA*

MANOVA significance tests on GPA example data			
Effect tested:	F	df	*p*
GPA (categorical)	4.53	5/193	.001
GPA (linear trend)	12.77	1/197	.000
HighGPA	9.16	1/197	.003
sex	7.23	1/197	.000

Multilevel Repeated Measures

$$GPA_{it} = \pi_{0i} + \pi_{1i}\, Time + \pi_{2i} T1_{it} + \pi_{3i} T2_{it} + \pi_{4i} T3_{it} + \pi_{5i} T4_{it} + \pi_{6i} T5_{it} + \pi_{7i} T6_{it}$$

$$\pi_{0i} = \beta_{00}$$
$$\pi_{1i} = \beta_{10}$$

$$\pi_{2i} = 0 + u_{2t}$$
$$\pi_{3i} = 0 + u_{3t}$$

$$\vdots$$

$$\pi_{7i} = 0 + u_{7t}$$

- This model can be used to model a linear trend with multilevel regression.
  - Not saturated like the MANOVA model

- Build in to certain software packages.

Multilevel Repeated Measures

- Many more possibilities for residual (co)variance structure!

- Observations closer in time more highly correlated?

$$\Sigma(\mathbf{Y}) = \frac{\sigma_\varepsilon^2}{(1-\rho^2)} \begin{pmatrix} 1 & \rho & \rho^2 & \cdots & \rho^{k-1} \\ \rho & 1 & \rho & \cdots & \rho^{k-2} \\ \rho^2 & \rho & 1 & \cdots & \rho^{k-3} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ \rho^{k-1} & \rho^{k-2} & \rho^{k-3} & \cdots & 1 \end{pmatrix}.$$

AR

$$\Sigma(\mathbf{Y}) \sigma_e^2 \begin{pmatrix} 1 & \rho_1 & \rho_2 & \cdots & \rho_{k-1} \\ \rho_1 & 1 & \rho_1 & \cdots & \rho_{k-2} \\ \rho_2 & \rho_1 & 1 & \cdots & \rho_{k-3} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ \rho_{k-1} & \rho_{k-2} & \rho_{k-3} & \cdots & 1 \end{pmatrix}.$$

Toeplitz

TILBURG UNIVERSITY

tesc

Multilevel Repeated Measures

- Or, add a random slope!!! (another reason to not use the step-wise modelbuilding approach!)

$$\mathrm{var}(Y_t) = \sigma^2_{u_0} + \sigma_{u_{01}}(t - t_0) + \sigma^2_{u_1}(t - t_0) + \sigma^2_e,$$

$$\mathrm{cov}(Y_t, Y_s) = \sigma^2_{u_0} + \sigma_{u_{01}}[(t - t_0) + (s - s_0)] + \sigma^2_{u_1}(t - t_0)(s - s_0)$$

TILBURG UNIVERSITY

tesc

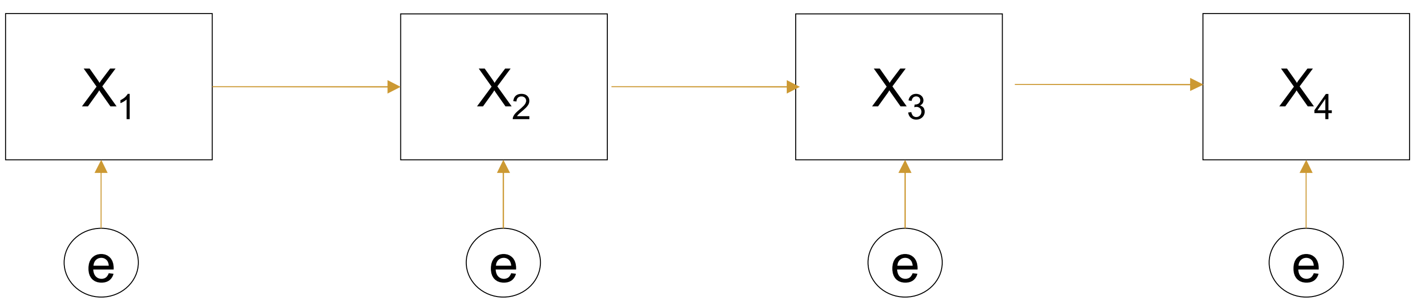
Reversible Change

TILBURG UNIVERSITY

tesc

Reversible Change

- One type of model used is the AR model

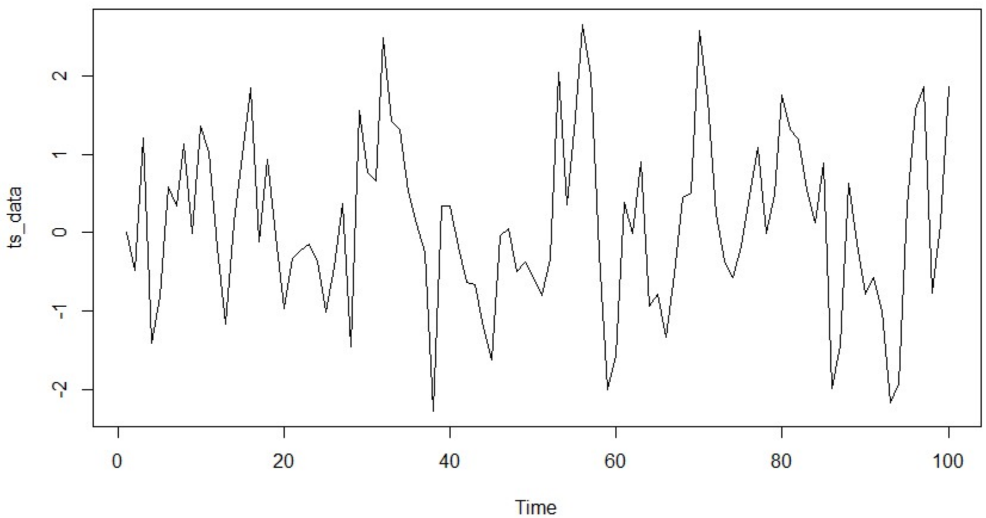


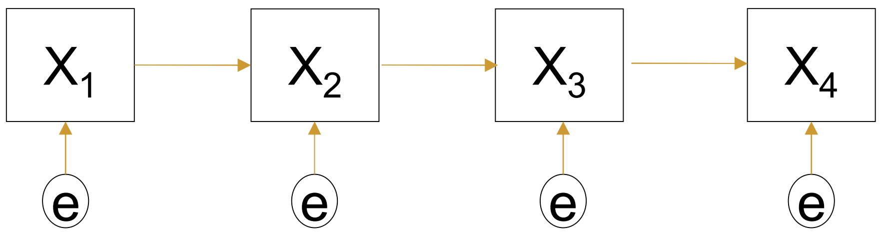
Reversible Change

Can you tell me how different forms of change could be reflected in the AR model?

Reversible Change

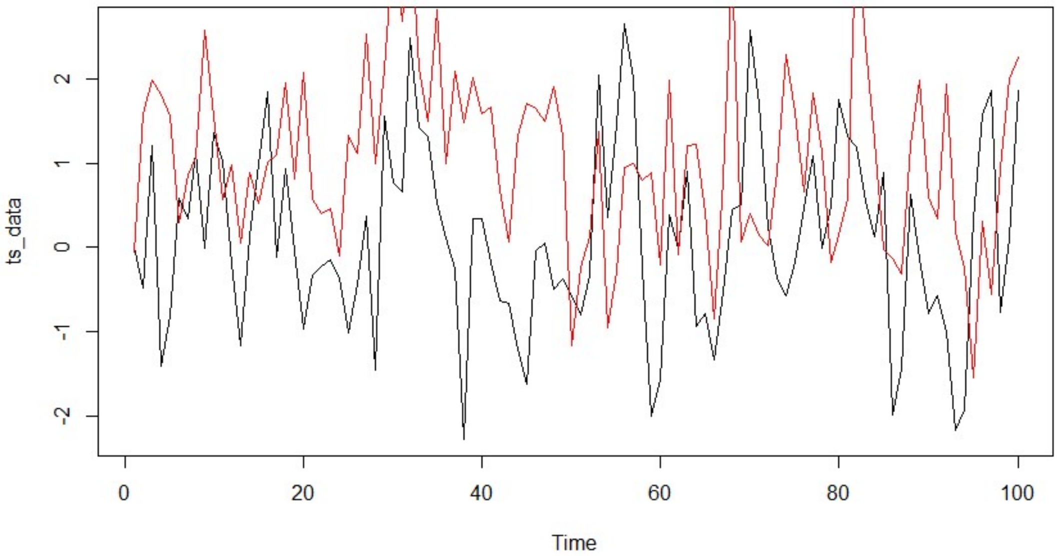
Can you tell me how different forms of change could be reflected in the AR model?

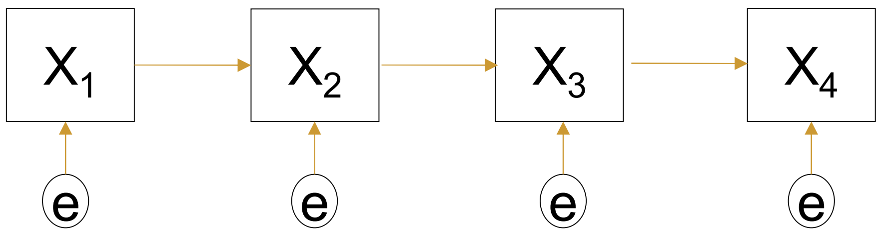




Reversible Change

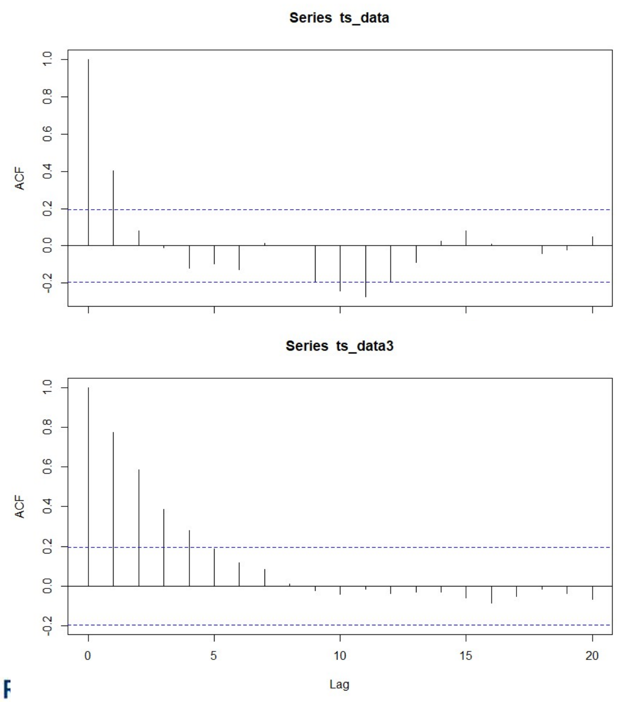
Can you tell me how different forms of change could be reflected in the AR model?

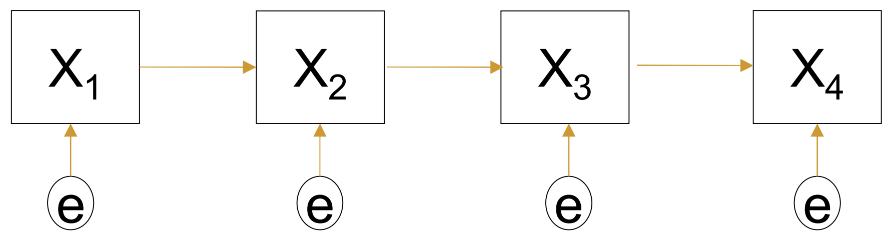




Reversible Change

Can you tell me how different forms of change could be reflected in the AR model?





Reversible Change

How could you run an AR-model using tools you already know?

Reversible Change

- Can run an AR model with multilevel regression too!

$$Y_t = b_0 + b_1 Y_{t-1} + \epsilon$$

Reversible Change

- Can run an AR model with multilevel regression too!

$$Y_t = b_0 + b_1 Y_{t-1} + \epsilon$$

$$Y = b_0 + b_1 X + \epsilon$$

Reversible Change

$$Y_t = b_0 + b_1 Y_{t-1} + \epsilon_t$$

b_0 = Long run tendency → Think "mean".

b_1 = Autoregressive parameter → inertia.

ϵ_t = Residual/Innovation → All variation that can not be predicted by previous measurement.

Reversible Change

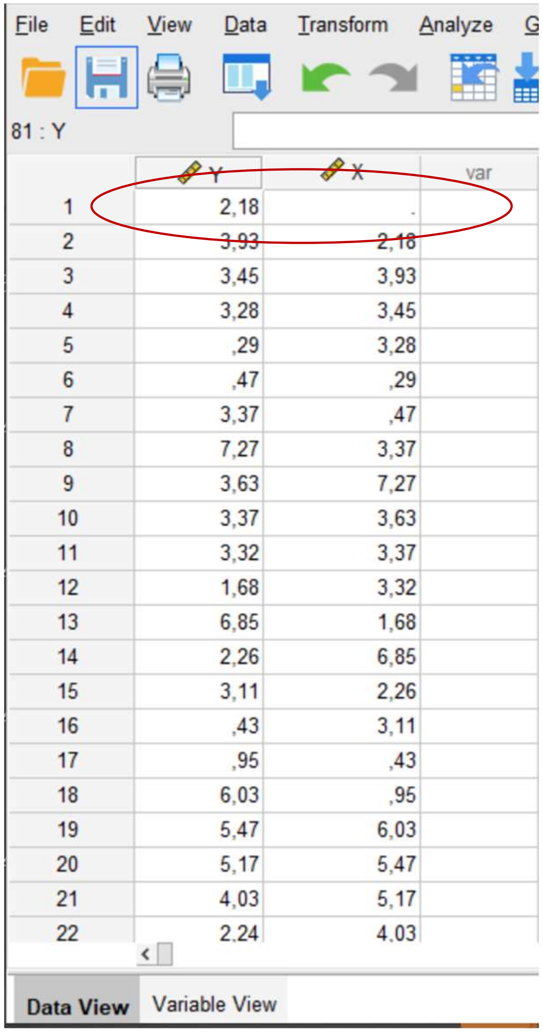
File Edit View Data Transform Analyze G

81 : Y

	Y	X	var
1	2,18	.	
2	3,93	2,18	
3	3,45	3,93	
4	3,28	3,45	
5	,29	3,28	
6	,47	,29	
7	3,37	,47	
8	7,27	3,37	
9	3,63	7,27	
10	3,37	3,63	
11	3,32	3,37	
12	1,68	3,32	
13	6,85	1,68	
14	2,26	6,85	
15	3,11	2,26	
16	,43	3,11	
17	,95	,43	
18	6,03	,95	
19	5,47	6,03	
20	5,17	5,47	
21	4,03	5,17	
22	2,24	4,03	

Data View Variable View

Reversible Change



	Y	X	var
1	2,18	.	
2	3,93	2,18	
3	3,45	3,93	
4	3,28	3,45	
5	,29	3,28	
6	,47	,29	
7	3,37	,47	
8	7,27	3,37	
9	3,63	7,27	
10	3,37	3,63	
11	3,32	3,37	
12	1,68	3,32	
13	6,85	1,68	
14	2,26	6,85	
15	3,11	2,26	
16	,43	3,11	
17	,95	,43	
18	6,03	,95	
19	5,47	6,03	
20	5,17	5,47	
21	4,03	5,17	
22	2,24	4,03	

Reversible Change

$$Y_{it} = b_{0i} + b_{1i}Y_{i,t-1} + \epsilon_{it}$$

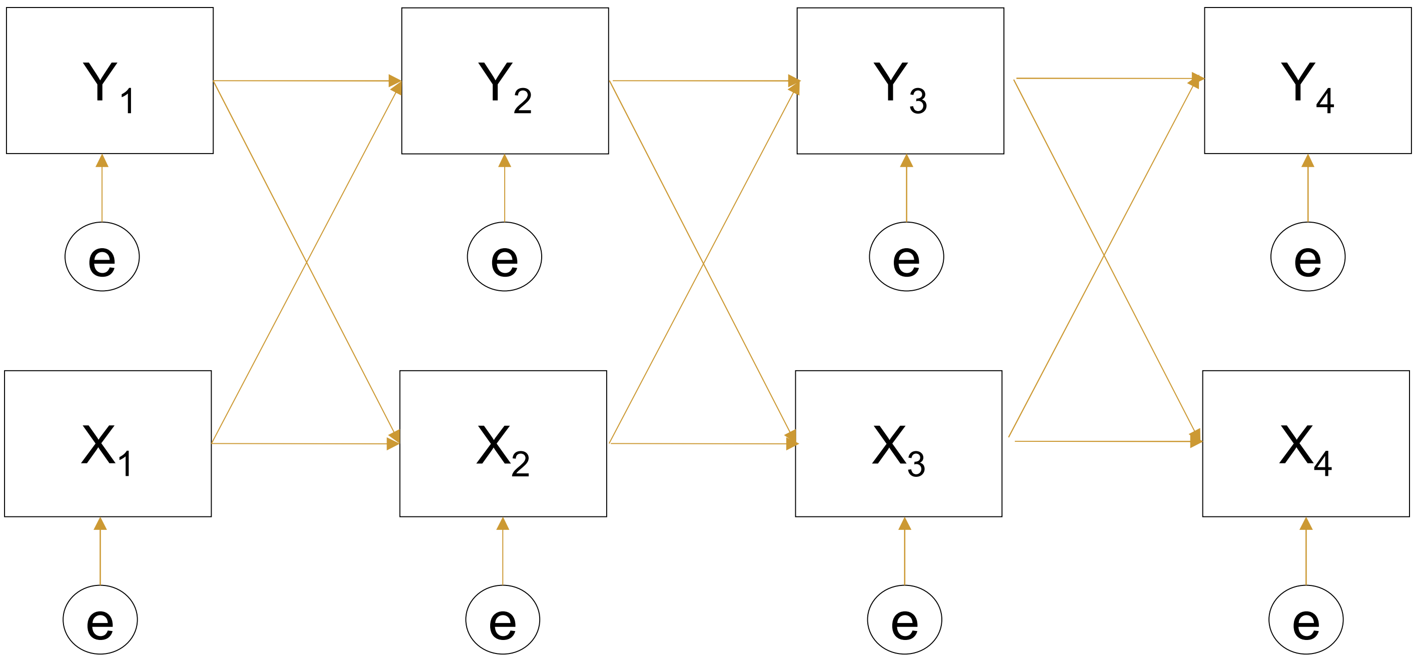
- Couple of things:
  - The intercept is not the mean!

$$\mu = \frac{b_o}{1 - b_1^2}$$

  - To get the mean you need to group-mean center the lagged predictor $Y_{i,t-1}$
  - This model assumes there is no trend! If there is, remove it first!!

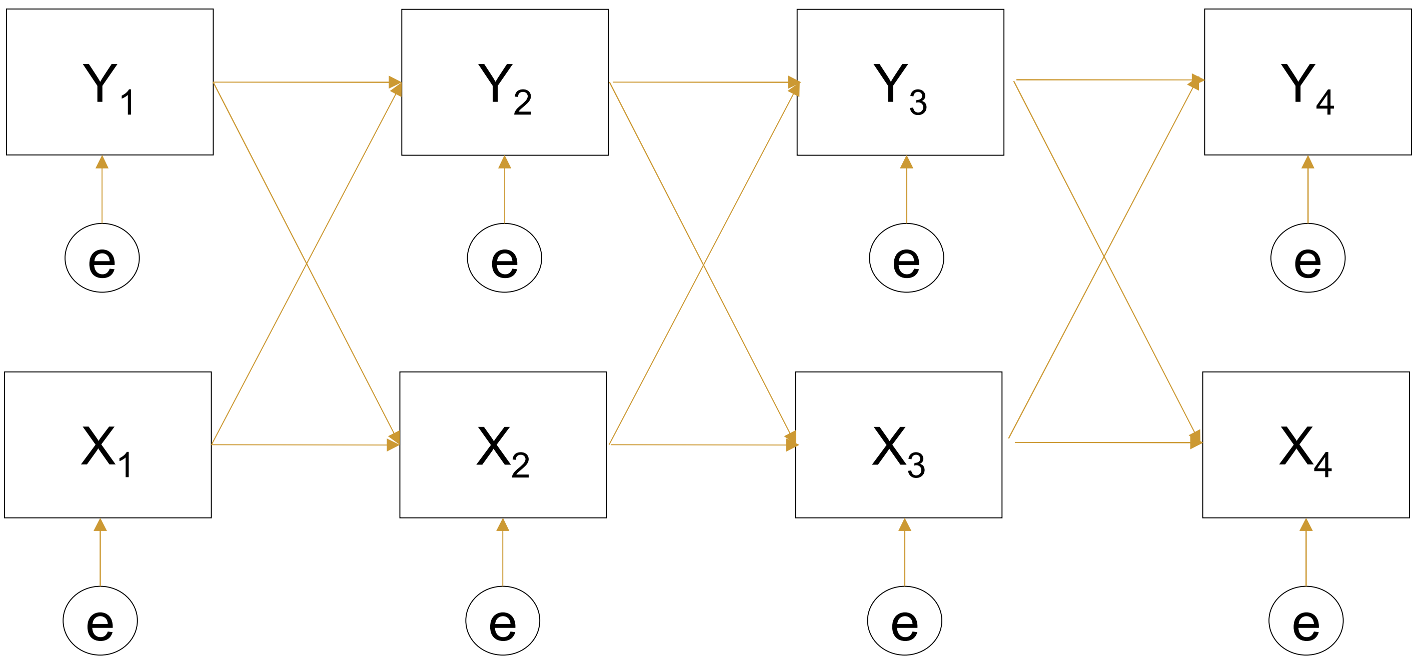
Reversible Change

- Will often be interested in (longitudinal) relation between two or more variables



Reversible Change

- This is called a VAR model



Reversible Change

$$\begin{bmatrix} Y_t \\ X_t \end{bmatrix} = \begin{bmatrix} b_{Y0} \\ b_{X0} \end{bmatrix} + \begin{bmatrix} b_{Y1} & b_{XY} \\ b_{YX} & b_{X1} \end{bmatrix} \begin{bmatrix} Y_{t-1} \\ X_{t-1} \end{bmatrix} + \begin{bmatrix} \epsilon_{Yt} \\ \epsilon_{Xt} \end{bmatrix}$$

b_{Y0}/b_{X0}= Long run tendency → Think “mean”.

b_{Y1}/b_{X1}= Autoregressive parameter → inertia.

b_{YX}/b_{XY}= Cross-lagged effects.

$\epsilon_{Yt}/\epsilon_{Xt}$= Residual/Innovation → All variation that can not be predicted by previous measurement.

TILBURG UNIVERSITY

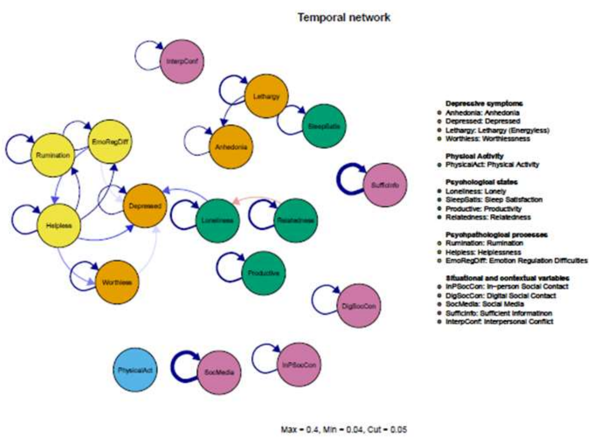
tesc

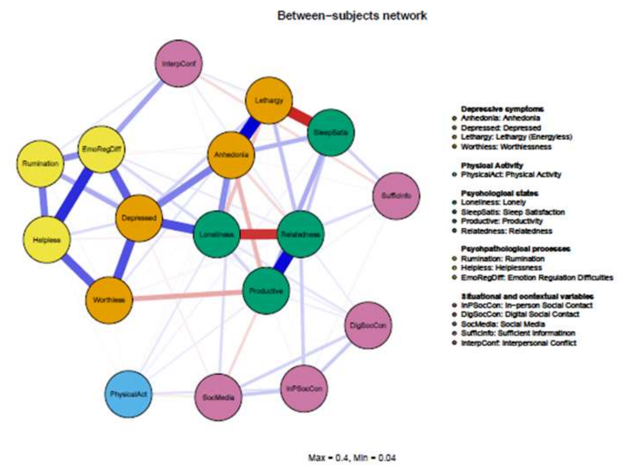
Reversible Change

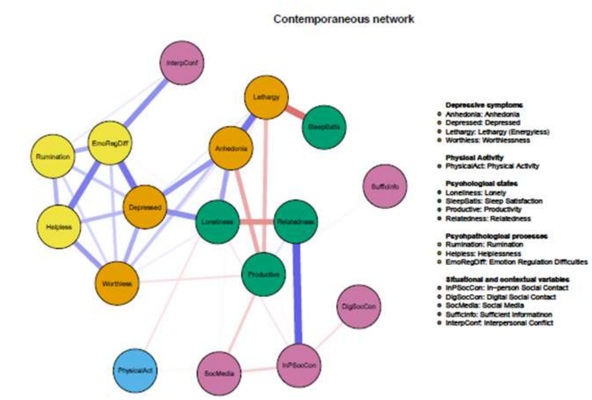
$$\begin{bmatrix} Y_t \\ X_t \end{bmatrix} = \begin{bmatrix} b_{Y0} \\ b_{X0} \end{bmatrix} + \begin{bmatrix} b_{Y1} & b_{XY} \\ b_{YX} & b_{X1} \end{bmatrix} \begin{bmatrix} Y_{t-1} \\ X_{t-1} \end{bmatrix} + \begin{bmatrix} \epsilon_{Yt} \\ \epsilon_{Xt} \end{bmatrix}$$

This model is very close to the longitudinal network models you see in the literature!

Reversible Change







- Only real difference is the estimation method
    - Networks not really multilevel
    - Inference is hard, more descriptive
    - But, scale better to larger data sizes.

What is change? Part II

- Remember change is multifaceted and can appear in many places/ways.

- Intensive Longitudinal Data well suited for certain types of change/differences.
  - **Which ones again?**

What is change? Part II

- Still did not consider all types of change/differences!
  - Residual variances can differ between people and change across time.
  - This implies there is no such thing as reliability!
  - Reliability changes across individuals and time!

What is change? Part II

- Still did not consider all types of change/differences!
  - Residual variances can differ between people and change across time.
  - This implies there is no such thing as reliability!
  - Reliability changes across individuals and time!

- And, to make it really meta:
- If inertia/stability can change, then can that change in stability also change and/or differ between individuals?

Tilburg University

What is change? Part II

- Change is everywhere and people can differ in many, many ways!
- To capture some of these ways, you need to look really “intensively” (i.e., moment to moment).
- For others you don’t!!!

Practical 7

ESM: Current Issues

Experience Sampling

What do you think are some of the difficulties EMA studies pose?

Experience Sampling

- Drop-out.
- Missed measurements.
- Measurement(!!💀…here be monsters).
- Time-scale/lag.
- Timing (fixed interval or randomly timed).

Experience Sampling

- Drop-out.
- Missed measurements.
- Measurement(!!💀…here be monsters).
  - Measurement always difficult.
  - Nothing works for everyone all the time.
  - **NO** measure designed to be administered multiple times a day.
- Time-scale/lag.
- Timing (fixed interval or randomly timed).

Tilburg University

tesc