Multilevel

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Hierarchical Analyses: The Equations

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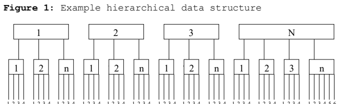
Hierarchical analysis:

Figure 1: Example hierarchical data structure



Examples:	**Education**	**Organizational**	**Longitudinal**
Level 3	schools	organizations	classes
Level 2	classes	departments	pupils
Level 1	pupils	individuals	observations

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Why Multilevel?

- Variables at different levels/Dependency.
  - Level 1: Need to correct for dependency in data.

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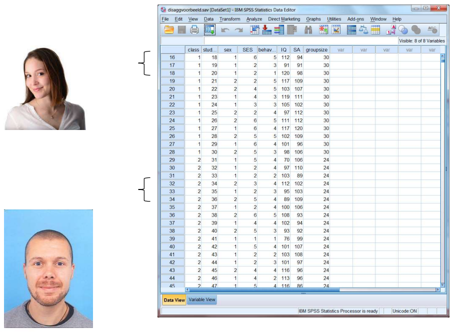

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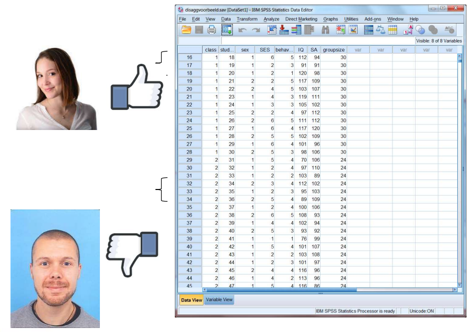

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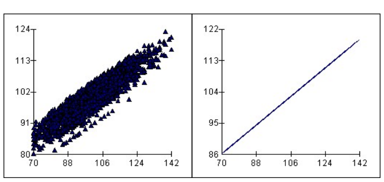
Hierarchical analysis: Research questions



- Use multilevel because observations are likely to de dependent, however:
  - These type of questions could still be answered with normal regression.
  - Merely involves adjusting obtained s.e.

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Hierarchical analysis: Research questions

- Could correct for this with the formula by Kish (1965):
- $v_{eff} = v(1 + (n_{clus} - 1)\rho)$, where
- $\rho = \frac{(MS_b - MS_w)}{MS_b + (n_{clus} - 1)MS_w)}$
- This obviously does not provide the nice missing data handling of multilevel

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Why Multilevel?

- Variables at different levels/Dependency.
  - Level 1: Need to correct for dependency in data.
  - Level 2: Disaggregation

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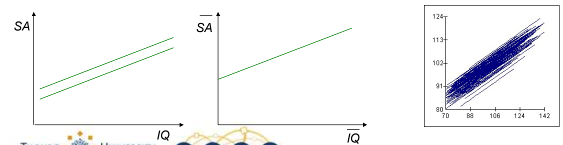
Multilevel research questions

- Questions with respect to the influence of variables at a higher level on the dependent variable at the lowest level:
  - *mean intelligence* of a class (*MIQ*) as predictor of mean *school achievement* (*SA*); (controlling for individual *IQ*)

$$SA_{ij} = b_0 + b_1 IQ_{ij} + b_2 MIQ_j + u_j + e_{ij}$$



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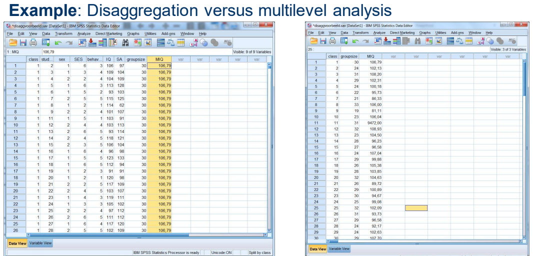
Why multilevel analysis?

Example: Disaggregation versus multilevel analysis



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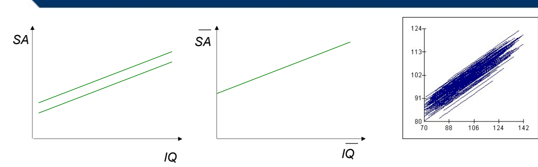
Multilevel research questions



- This type of question could also be answered with normal regression by running a regression on mean scores:
  - Power obviously an issue.
  - Should means from groups of different size be treated the same?

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Why Multilevel?

- Variables at different levels.
  - Level 1: Need to correct for dependency in data.
  - Level 2: Disaggregation
  - Level 1&2: Cross-level interaction
    - We want to MODEL differences between level 2 units in the effects of level 1 variables.

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Hierarchical Analyses: The Steps

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Analysis Strategy

Analysis strategy

- Multilevel model with:
  - *p* explanatory variables at the lowest level
  - *q* explanatory variables at the highest level

$$Y_{ij} = \gamma_{00} + \gamma_{p0} X_{pij} + \gamma_{0q} Z_{qj} + \gamma_{pq} Z_{qj} X_{pij} + u_{pj} X_{pij} + u_{0j} + e_{ij}$$

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Steps of a multilevel analysis

1. Check whether multilevel is necessary
2. Add all level 1 main effects and interactions
3. Add all level 2 main effects and interactions
4. Check level 1 effects for random slopes
5. If random slopes are present: add cross-level interactions

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Step 1: two level intercept-only model

- Analyze a two-level model with no explanatory variables: the *intercept-only* model:

$$Y_{ij} = \beta_{0j} + e_{ij}$$
$$\beta_{0j} = \gamma_{00} + u_{0j}$$
$$Y_{ij} = \gamma_{00} + u_{0j} + e_{ij}$$

- γ_{00} : the regression intercept
- u_{0j} : residual error at the second level
- e_{ij} : residual error at the first level

- Decomposition of the total variance in two terms
- Should we perform a multilevel analysis?

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Step 1: two level intercept-only model

$$Y_{ij} = \gamma_{00} + u_{0j} + e_{ij}$$

- H_0: individuals in the same group aren't more alike than individuals in different groups
  H_0: $\sigma^2_{u_0}$ = 0
- H_1: individuals in the same group are more alike than individuals in different groups
  H_1: $\sigma^2_{u_0}$ > 0

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Intraclass correlation

$$\rho = \frac{\sigma^2_{u_0}}{\sigma^2_{u_0} + \sigma^2_e}$$

- variance of the second level errors u_{0j} $\sigma^2_{u_0}$
- variance of the first level errors e_{ij} σ^2_e

- The 'percentage' variance at the second level
- The expected correlation between the observations on the dependent variable of two randomly chosen units that are in the same group.

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Step 2: first level variables

Add all first level variables (+ their interactions) with fixed effects:

$$Y_{ij} = \beta_{0j} + \beta_{pj} X_{pij} + e_{ij}$$
$$\beta_{0j} = \gamma_{00} + u_{0j}$$
$$\beta_{pj} = \gamma_{p0}$$
$$Y_{ij} = \gamma_{00} + \gamma_{p0} X_{pij} + u_{0j} + e_{ij}$$

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Step 2: first level variables

Add all first level variables (+ their interactions) with fixed effects:

$$Y_{ij} = \gamma_{00} + \gamma_{p0} X_{pij} + u_{0j} + e_{ij}$$

- X_{pij}: *p* explanatory variables at the first level (e.g., gender, extraversion, and gender*extraversion)
- H_0: There is no relation between the dependent variable *Y* and the explanatory variable *X*
- H_1: There is a relation between the dependent variable *Y* and the explanatory variable *X* (+ direction)

- Calculate the explained variance at level 1 and at level 2

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Step 2: first level variables

- Calculate the explained variance at level 1 and level 2.

- Why?

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Step 2: first level variables

- Calculate the explained variance at level 1 and level 2.

- Variables at the lowest level can explain variance at the first level
  - Girls are more popular than boys.

- Variables at the lowest levels can explain variance at the higher levels.
  - Girls are more popular than boys.
  - In a class with a lot of girls the mean popularity is higher.

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Step 3: Second level variables

Add all second level variables (+ their interactions) with fixed effects:

$$Y_{ij} = \beta_{0j} + \beta_{pj} X_{pij} + e_{ij}$$

$$\beta_{0j} = \gamma_{00} + \gamma_{0q} Z_{qj} + u_{0j}$$
$$\beta_{pj} = \gamma_{p0}$$

$$Y_{ij} = \gamma_{00} + \gamma_{p0} X_{pij} + \gamma_{0q} Z_{qj} + u_{0j} + e_{ij}$$

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Step 3: Second level variables

Add all second level variables (+ their interactions) with fixed effects:

$$Y_{ij} = \gamma_{00} + \gamma_{p0} X_{pij} + \gamma_{0q} Z_{qj} + u_{0j} + e_{ij}$$

- Z_{qj} : q explanatory variables at the second level (e.g., teacher experience).
- H_0: there is no relation between the explanatory variable Z and the mean score of the dependent variable *Y*
- H_1: there is a relation between the explanatory variable *Z* and the mean score of the dependent variable *Y* (+ direction?)
- Calculate the explained variance at level 2

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Step 4: random slopes

Assess whether any of the slopes of the explanatory variables of the first level has a significant variance between the second level units:

$$Y_{ij} = \beta_{0j} + \beta_{pj} X_{pij} + e_{ij} \qquad \begin{aligned} \beta_{0j} &= \gamma_{00} + \gamma_{0q} Z_{qj} + u_{0j} \\ \beta_{pj} &= \gamma_{p0} + u_{pj} \end{aligned}$$

$$Y_{ij} = \gamma_{00} + \gamma_{p0} X_{pij} + \gamma_{0q} Z_{qj} + u_{pj} X_{pij} + u_{0j} + e_{ij}$$

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Step 4: random slopes

Assess whether any of the slopes of the explanatory variables of the first level has a significant variance between the second level units:

$$Y_{ij} = \gamma_{00} + \gamma_{p0} X_{pij} + \gamma_{0q} Z_{qj} + u_{pj} X_{pij} + u_{0j} + e_{ij}$$

- u_{pj} : second level residuals of the slopes of the first level explanatory variables X_{pij}
- H_0: the relation between the explanatory variable *X* and the dependent variable *Y* is the same within all level two units (H_0: $\sigma^2_{u_{1j}} = 0$)
- H_1: the relation between the explanatory variable *X* and the dependent variable *Y* is not the same within all level two units (H_1: $\sigma^2_{u_{1j}} > 0$)

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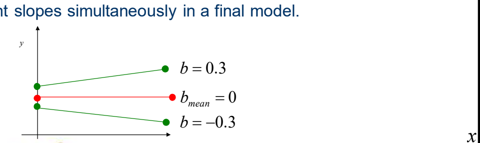
Remarks on step 4: random slopes

- Testing for random slope variation: variable by variable
- Variables that were omitted in step 2 may be analyzed again:
  - it is possible that an explanatory variable has no significant mean regression slope, but that there is slope variance
- Add all significant slopes simultaneously in a final model.



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Step 5: cross-level interactions

Add cross-level interactions between those level one explanatory variables that had significant slope variation and explanatory variables of the second level:

$$Y_{ij} = \beta_{0j} + \beta_{pj} X_{pij} + e_{ij}$$

$$\beta_{0j} = \gamma_{00} + \gamma_{0q} Z_{qj} + u_{0j}$$

$$\beta_{pj} = \gamma_{p0} + \gamma_{pq} Z_{qj} + u_{pj}$$

$$Y_{ij} = \gamma_{00} + \gamma_{p0} X_{pij} + \gamma_{0q} Z_{qj} + \gamma_{pq} Z_{qj} X_{pij} + u_{pj} X_{pij} + u_{0j} + e_{ij}$$

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Step 5: cross-level interactions

Add cross-level interactions between those level one explanatory variables that had significant slope variation and explanatory variables of the second level:

$$Y_{ij} = \gamma_{00} + \gamma_{p0} X_{pij} + \gamma_{0q} Z_{qj} + \gamma_{pq} Z_{qj} X_{pij} + u_{pj} X_{pij} + u_{0j} + e_{ij}$$

- $Z_{qj} X_{pij}$: cross-level interactions
- H_0: the explanatory variable *Z* can't explain the different relations between the explanatory variable *X* and the dependent variable *Y* in different level two units
- H_1: the explanatory variable *Z* explains (a part of) the different relations between the explanatory variable *X* and the dependent variable *Y* in different level two units
- Calculate the explained slope variance

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Multilevel Regression

- Level 1: $SA_{ij} = b_{0j} + b_{1j} IQ_{ij} + e_{ij}$
- Level 2: $b_{0j} = \gamma_{00} + \gamma_{01} Schoolsize_j + u_{0j}$
  $b_{1j} = \gamma_{10} + u_{1j}$

Combined:

$$SA_{ij} = \gamma_{00} + \gamma_{10} IQ_{ij} + \gamma_{01} Schoolsize_j + eij + u_{0j} + u_{1j}\, IQ_{ij}$$

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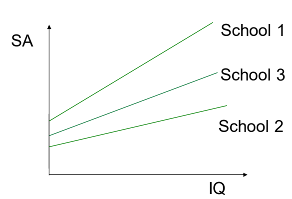
Multilevel Regression



Differences in intercept (partially) explained by school size:

$$b_{0j} = \gamma_{00} + \gamma_{01} Schoolsize_j + u_{0j}$$

So far, no predictor for differences in slope:

$$b_{1j} = \gamma_{10} + u_{1j}$$

Could school size explain differences in slope as well?

$$b_{1j} = \gamma_{10} + \gamma_{11} Schoolsize_j + u_{1j}$$

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Multilevel Regression

Level 1: $SA_{ij} = b_{0j} + b_{1j}IQ_{ij} + e_{ij}$

Level 2: $b_{0j} = \gamma_{00} + \gamma_{01}Schoolsize_j + u_{0j}$

$\boxed{b_{1j} = \gamma_{10} + \gamma_{11}Schoolsize_j + u_{1j}}$

Smaller School → More help → Pupils less reliant on own IQ? → Negative γ_{11}

OR

Smaller School → Less help → Pupils more reliant on own IQ?

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Multilevel Regression

- Level 1: $SA_{ij} = b_{0j} + b_{1j}IQ_{ij} + e_{ij}$
- Level 2: $b_{0j} = \gamma_{00} + \gamma_{01}Schoolsize_j + u_{0j}$
  $b_{1j} = \gamma_{10} + \gamma_{11}Schoolsize_j + u_{1j}$

Combined:

$$SA_{ij} = \gamma_{00} + \gamma_{10}IQ_{ij} + \gamma_{01}Schoolsize_j \boxed{+ \gamma_{11}Schoolsize_j IQ_{ij}} + eij + u_{0j} + u_{1jIQij}$$

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Hierarchical Analyses: The Steps

Update!...Don't do the steps!!

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Maximum Approach

- Going through the steps is a good way to see what is going on, but…
- We always know that things will differ between people/units?
  - So why assume fixed effects at first?
  - Why not go for a full model straight away?

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Maximum Approach

- Going through the steps is a good way to see what is going on, but…
- We always know that things will differ between people/units?
  - So why assume fixed effects at first?
  - Why not go for a full model straight away? (Maximum Approach)
- What are possible advantages of staring with a full model?
- And what are disadvantages?

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Maximum Approach

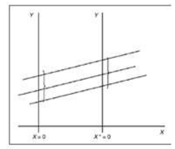

Figure 4.1 Parallel regression lines, with shift on *X*.

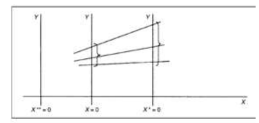

Figure 4.2 Varying regression lines, with shifts on *X*.

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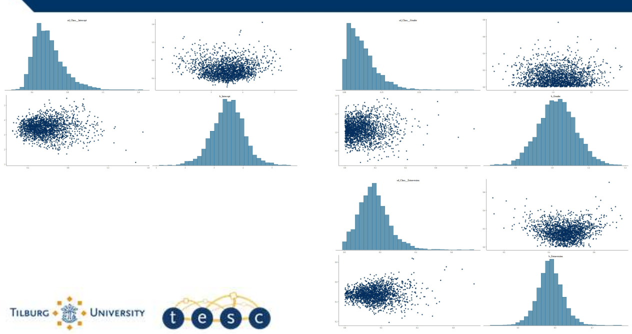
Maximum Approach



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Methodological Considerations: Centering

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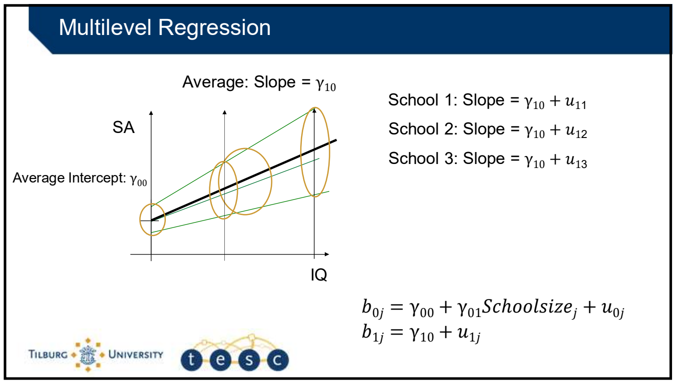
Multilevel Regression
Average: Slope = γ_{10}
SA
Average Intercept: γ_{00}
IQ
School 1: Slope = $\gamma_{10} + u_{11}$
School 2: Slope = $\gamma_{10} + u_{12}$
School 3: Slope = $\gamma_{10} + u_{13}$
$b_{0j} = \gamma_{00} + \gamma_{01}Schoolsize_j + u_{0j}$
$b_{1j} = \gamma_{10} + u_{1j}$
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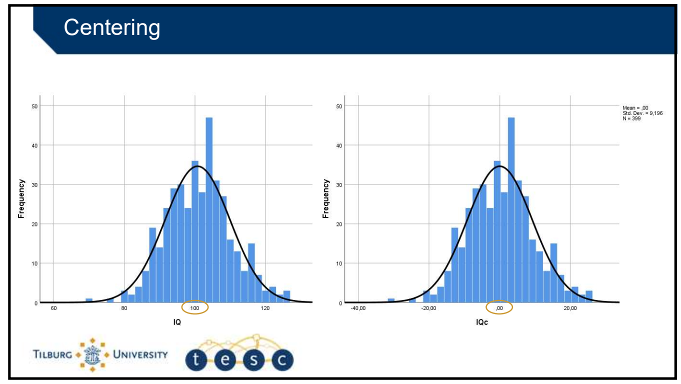
Centering
Frequency
IQ
IQc
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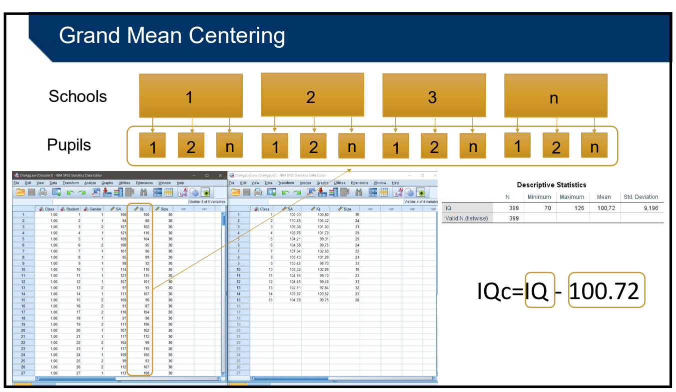
Grand Mean Centering
Schools
1
2
3
n
Pupils
1 2 n 1 2 n 1 2 n 1 2 n
Descriptive Statistics
IQc=IQ - 100.72

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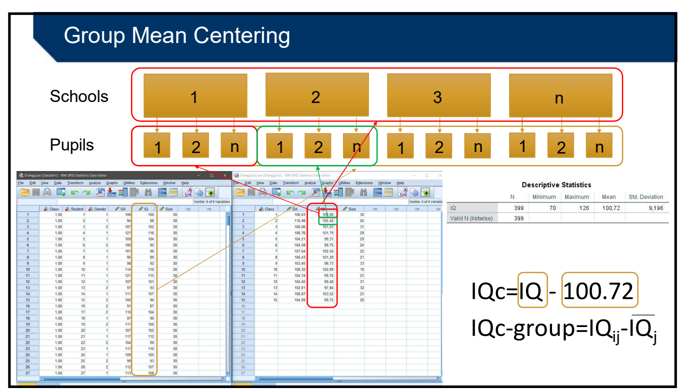
Group Mean Centering
Schools
1
2
3
n
Pupils
1 2 n 1 2 n 1 2 n 1 2 n
Descriptive Statistics
IQc=IQ - 100.72
IQc-group=IQ_{ij}-$\overline{IQ}_j$

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Centering

- Group mean centering changes interpretation of variable:
  - No longer “raw” score.
  - Relative position within higher level unit.
- Used for so-called frog-pond effects:
  - Someone’s actual level of an attribute doesn’t matter, what matters is how one compares to the rest of their group (or pond).

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Conclusion

- With random slopes important to think about the intercept, and therefore the meaning of a 0-score on predictors.
- Centering advised, but…
  - 2 methods: Grand mean centering and group mean centering.
  - Group mean centering complicates interpretation of model.

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Practical 3

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Methodological Considerations: Small Level 2 N

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Fixed Effects Models

- Multilevel is great, but we are modeling on level 2 as well!
- Would you run a regression on N=4?
- Would you calculate a variance on N=2?
- Same with multilevel, if N on level 2 is small (let's say less than 10), modeling distributions there is probably not a great idea.
- Fortunately, there is a solution! ☺.

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Fixed Effects Models

- Hint, the solution is not cluster robust se's as is sometimes suggested
  - Guess what, that correction also need to be estimated ;).
- How have you corrected for things in regression in the past?

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Fixed Effects Models

- Hint, the solution is not cluster robust se's as is sometimes suggested
  - Guess what, that correction also need to be estimated ;).
- How have you corrected for things in regression in the past?

$$SA_{ij} = b_{01}Class1 + b_{02}Class2 + \ \dots + b_{0x}ClassX + b_1IQ_{ij} + e_{ij}$$

- Add dummies! One for each class (and remove the intercept)

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Fixed Effects Models

$$SA_{ij} = b_{01}Class1 + b_{02}Class2 + \ \dots + b_{0x}ClassX + b_1IQ_{ij} + e_{ij}$$

- This is called a Fixed Effects model and is used often in economics.
- It works reaaaaaly well, as the dummies take care of all the level 2 differences.
- Estimates of level 1 predictors unbiased.
- Also deal with “unmodeled” level 2 influences, so is very robust.

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Fixed Effects Models

$$SA_{ij} = b_{01}Class1 + b_{02}Class2 + \ \dots\ + b_{0x}ClassX + b_1 IQ_{ij} + e_{ij}$$

- But! No free lunch.
- You can't model level 2 variables!
  - Since all level 2 variance is in the dummies they are perfectly colinear with level 2 predictors.
  - Not a big problem, if level 2 N is small what do you hope to find there anyway?
- Can model interactions between level 1 and level 2 predictors though.

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Three-level Models

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Multilevel Regression

Level 1: $Pop_{ij} = b_{0j} + b_{1j}Ext_{ij} + e_{ij}$

Level 2: $b_{0j} = \gamma_{00} + \gamma_{01}Texp_j + u_{0j}$

$b_{1j} = \gamma_{10} + u_{1j}$

Combined:

$$\begin{aligned} Pop_{ij} &= b_{0j} + b_{1j}Ext_{ij} + eij \\ &= \gamma_{00} + \gamma_{10}Ext_{ij} + \gamma_{01}Texp_j + e_{ij} + u_{0j} + u_{1j}Ext_{ij} \end{aligned}$$

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Multilevel Regression

Level 1: $Pop_{ijk} = b_{0jk} + b_{1jk}Ext_{ijk} + e_{ijk}$

Level 2: $b_{0jk} = \boxed{\gamma_{00k}} + \gamma_{01k}Texp_{jk} + u_{0jk}$

$b_{1jk} = \boxed{\gamma_{10k}} + u_{1jk}$

Level 3: $\boxed{\gamma_{00k}} = \gamma_{000} + v_{ok}$

$\boxed{\gamma_{10k}} = \gamma_{100} + v_{1k}$

Combined:

$$\begin{aligned} Pop_{ijk} &= b_{0jk} + b_{1jk}Ext_{ijk} + e_{ijk} \\ &= \gamma_{000} + \gamma_{100}Ext_{ijk} + \gamma_{01k}Texp_{jk} + e_{ijk} + u_{0jk} + u_{1jk}Ext_{ijk} + v_{ok} + v_{1k}Ext_{ijk} \end{aligned}$$

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Multilevel Regression

Level 1: $Pop_{ijk} = b_{0jk} + b_{1jk}Ext_{ijk} + e_{ijk}$

Level 2: $b_{0jk} = \boxed{\gamma_{00k}} + \gamma_{01k}Texp_{jk} + u_{0jk}$
$b_{1jk} = \boxed{\gamma_{10k}} + u_{1jk}$

Level 3: $\boxed{\gamma_{00k}} = \boxed{\gamma_{000}} + \boxed{v_{ok}}$
$\boxed{\gamma_{10k}} = \boxed{\gamma_{100}} + \boxed{v_{1k}}$

Combined:

$$Pop_{ijk} = b_{0jk} + b_{1jk}Ext_{ijk} + e_{ijk}$$
$$= \gamma_{000} + \gamma_{100}Ext_{ijk} + \gamma_{01k}Texp_{jk} + e_{ijk} + u_{0jk} + u_{1jk}Ext_{ijk} + v_{ok} + v_{1k}Ext_{ijk}$$

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Multilevel Regression

Level 1: $Pop_{ijk} = b_{0jk} + b_{1jk}Ext_{ijk} + e_{ijk}$

Level 2: $b_{0jk} = \boxed{\gamma_{00k}} + \gamma_{01k}Texp_{jk} + u_{0jk}$
$b_{1jk} = \boxed{\gamma_{10k}} + u_{1jk}$

Level 3: $\boxed{\gamma_{00k}} = \boxed{\gamma_{000}} + \boxed{v_{ok}}$
$\boxed{\gamma_{10k}} = \boxed{\gamma_{100}} + \boxed{v_{1k}}$

Combined:

$$Pop_{ijk} = b_{0jk} + b_{1jk}Ext_{ijk} + e_{ijk}$$
$$= \gamma_{000} + \gamma_{100}Ext_{ijk} + \gamma_{01k}Texp_{jk} + e_{ijk} + u_{0jk} + \boxed{u_{1jk}Ext_{ijk}} + v_{ok} + \boxed{v_{1k}Ext_{ijk}}$$

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Multilevel Regression

Level 1: $Pop_{ijk} = b_{0jk} + b_{1jk}Ext_{ijk} + e_{ijk}$

Level 2: $b_{0jk} = \gamma_{00k} + \gamma_{01k}Texp_{jk} + u_{0jk}$
$b_{1jk} = \gamma_{10k} + u_{1jk}$

Level 3: $\gamma_{00k} = \gamma_{000} + v_{ok}$
$\gamma_{10k} = \gamma_{100} + v_{1k}$

Combined:

$$Pop_{ijk} = b_{0jk} + b_{1jk}Ext_{ijk} + e_{ijk}$$
$$= \gamma_{000} + \gamma_{100}Ext_{ijk} + \gamma_{01k}Texp_{jk} + e_{ijk} + u_{0jk} + u_{1jk}Ext_{ijk} + v_{ok} + v_{1k}Ext_{ijk}$$

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Multilevel Regression

Level 1: $Pop_{ijk} = b_{0jk} + b_{1jk}Ext_{ijk} + e_{ijk}$

Level 2: $b_{0jk} = \gamma_{00k} + \boxed{\gamma_{01k}}Texp_{jk} + u_{0jk}$
$b_{1jk} = \gamma_{10k} + u_{1jk}$

Level 3: $\gamma_{00k} = \gamma_{000} + v_{ok}$
$\gamma_{10k} = \gamma_{100} + v_{1k}$

Combined:

$$Pop_{ijk} = b_{0jk} + b_{1jk}Ext_{ijk} + e_{ijk}$$
$$= \gamma_{000} + \gamma_{100}Ext_{ijk} + \gamma_{01k}Texp_{jk} + e_{ijk} + u_{0jk} + u_{1jk}Ext_{ijk} + v_{ok} + v_{1k}Ext_{ijk}$$

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Multilevel Regression

Level 1: $Pop_{ijk} = b_{0jk} + b_{1jk}Ext_{ijk} + e_{ijk}$

Level 2: $b_{0jk} = \gamma_{00k} + \boxed{\gamma_{01k}}Texp_{jk} + u_{0jk}$

$b_{1jk} = \gamma_{10k} + u_{1jk}$

Level 3: $\gamma_{00k} = \gamma_{000} + v_{ok}$

$\gamma_{10k} = \gamma_{100} + v_{1k}$

$\boxed{\gamma_{01k}} = \gamma_{010} + v_{2k}$

Combined:

$$Pop_{ijk} = b_{0jk} + b_{1jk}Ext_{ijk} + e_{ijk}$$
$$= \gamma_{000} + \gamma_{100}Ext_{ijk} + \boxed{\gamma_{010}}Texp_{jk} + e_{ijk} + u_{0jk} + u_{1jk}Ext_{ijk} + v_{ok} + v_{1k}Ext_{ijk} + \boxed{v_{2k}Texp_{jk}}$$

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Intraclass correlation

- $\sigma^2_{v_0}$ variance of the third level errors v_{0j}
- $\sigma^2_{u_0}$ variance of the second level errors u_{0j}
- σ^2_{e} variance of the first level errors e_{ij}

$$\rho_{class} = \frac{\sigma^2_{u_0}}{\sigma^2_{v_0} + \sigma^2_{u_0} + \sigma^2_{e}} \qquad \rho_{school} = \frac{\sigma^2_{v_0}}{\sigma^2_{v_0} + \sigma^2_{u_0} + \sigma^2_{e}}$$

- The ‘percentage’ variance at the second and third level

$$\rho_{class} = \frac{\sigma^2_{v_0} + \sigma^2_{u_0}}{\sigma^2_{v_0} + \sigma^2_{u_0} + \sigma^2_{e}}$$

- The expected correlation between the observations on the dependent variable of two randomly chosen units that are in the same class (and thus in the same school).

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Extra difficulty with 3 level models

- What do you do when there is a significant amount of variance on level 3, but not level 2?

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Extra difficulty with 3 level models

- What do you do when there is a significant amount of variance on level 3, but not level 2?
- The “higher you go” the smaller your sample size.
  - What do you do if the number of level 3 units is very small?

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Fixed Effect Models

Level 1: $Pop_{ijk} = b_{0j} + b_{1jk}Ext_{ij} + e_{ij}$

Level 2: $b_{0j} = \gamma_{01}School1 + \gamma_{02}School2 + \ldots + \gamma_{0x}SchoolX + \gamma_{0,x+1}Texp_{jk} + u_{0j}$

$b_{1j} = \gamma_{11}School1 + \gamma_{12}School2 + \ldots + \gamma_{1x}SchoolX + u_{1j}$

Combined:

$$\begin{aligned} Pop_{ijk} &= b_{0jk} + b_{1jk}Ext_{ijk} + e_{ij} \\ &= \gamma_{01}School1 + \gamma_{02}School2 + \ldots + \gamma_{0x}SchoolX + \gamma_{0,x+1}Texp_{jk} \\ &\quad \gamma_{11}School1 * Ext_{ij} + \gamma_{12}School2 * Ext_{ij} + \ldots + \gamma_{1x}SchoolX * Ext_{ij} + \\ &\quad e_{ij} + u_{0j} + u_{1j}Ext_{ij} \end{aligned}$$

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Practical 4

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Cross-Nested Data

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Cross-Classified Models

- Sometimes the nesting structure is not constant.
  - You follow students as the transition from primary school to secondary school.
  - Some students that attend the same primary school will attend different secondary schools.
  - So students nested in a cross-classification of primary and secondary schools.

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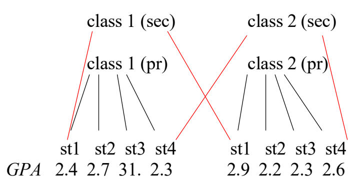
Cross-classified data



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Cross-Classified Models

- $Y_{i(jk)}$ Score of student i within cross-classification of primary school j and secondary school k

$$Y_{i(jk)} = \beta_{0(jk)} + e_{i(jk)}$$

- $\beta_{0(jk)}$ is the intercept/overall mean
- $e_{i(jk)}$ is the residual error term
- the subscript *(jk)* is written in parentheses to indicate that they are conceptually at the same level

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Cross-Classified Models

- The intercept $\beta_{0(jk)}$ varies independently across both primary and secondary schools, so

$$Y_{i(jk)} = \beta_{0(jk)} + e_{i(jk)}$$

$$\beta_{0(jk)} = \gamma_{00} + u_{0j} + v_{ok}$$

- u_{0j} error for primary schools
- v_{ok} error for secondary schools

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Cross-Classified Models

- Individual-level explanatory variables can be added to the equation.
  - Their regression slopes may be allowed to vary across primary and/or secondary schools.

$$Y_{i(jk)} = \beta_{0(jk)} + \beta_{1(jk)} X_{i(jk)} + e_{i(jk)}$$

- School-level variables can also be added,
  - Used to explain variation in the slopes of individual-level variables across schools
  - Similar to ordinary multilevel regression models.

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Cross-Classified Models

- How do you specify these things?
- Do they sort of remind you of a model we saw earlier when looking at the equations?

$$Y_{i(jk)} = \beta_{0(jk)} + e_{i(jk)}$$
$$\beta_{0(jk)} = \gamma_{00} + u_{0j} + v_{ok}$$

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Cross-Classified Models

- How do you specify these things?
- We can start by ignoring the secondary school level, specify the individual and primary school level as usual.
  - individuals at the first level and primary schools at the second level.
- But now what?
  - We need to add another level, without actually adding another level.
  - Have we seen this before?

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Fixed Effect Models

Level 1: $Pop_{ijk} = b_{0j} + b_{1jk}Ext_{ij} + e_{ij}$

Level 2: $b_{0j} = \gamma_{01}School1 + \gamma_{02}School2 + \ \dots + \gamma_{0x}SchoolX + \gamma_{0,x+1}Texp_{jk} + u_{0j}$

$b_{1j} = \gamma_{11}School1 + \gamma_{12}School2 + \ \dots + \gamma_{1x}SchoolX + u_{1j}$

Combined:

$$\begin{aligned} Pop_{ijk} &= b_{0jk} + b_{1jk}Ext_{ijk} + e_{ijk} \\ &= \gamma_{01}School1 + \gamma_{02}School2 + \ \dots + \gamma_{0x}SchoolX + \gamma_{0,x+1}Texp_{jk} \\ &\quad \gamma_{11}School1 * Ext_{ij} + \gamma_{12}School2 * Ext_{ij} + \ \dots + \gamma_{1x}SchoolX * Ext_{ij} + \\ &\quad e_{ij} + u_{0j} + u_{1j}Ext_{ij} \end{aligned}$$

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Cross-Classified Models

- We can start by ignoring the secondary school level, specify the individual and primary school level as usual.
  - individuals at the first level and primary schools at the second level.
- To create a place to specify the crossed effects of the secondary school level, we introduce a third ‘dummy’ level
- At the pupil level: specify a full set of dummy variables to indicate all of the secondary schools.

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Fixed Effect Models/Cross-Classified Models

Level 1: $Pop_{ijk} = b_{0jk} + b_{1jk}Ext_{ij} + b_{2jk}School1 + b_{3jk}School2 + \ldots + b_{xjk}SchoolX + e_{ijk}$

Level 2:
$$b_{0jk} = \gamma_{00k} + \gamma_{01k}Texp_{jk} + u_{0jk}$$
$$b_{1jk} = \gamma_{01k} + u_{1jk}$$
$$b_{2jk} = \gamma_{02k} + u_{2jk}$$
$$\vdots$$
$$b_{xjk} = \gamma_{0xk} + u_{xj}$$

"Level" 3:
$$\gamma_{00k} = \gamma_{000} + v_{000}$$
$$\gamma_{01k} = \gamma_{001} + v_{001}$$
$$\gamma_{02k} = \gamma_{002} + v_{002}$$
$$\vdots$$
$$\gamma_{0xk} = \gamma_{00x} + v_{00x}$$

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Cross-Classified Models

- At the pupil level: specify a full set of dummy variables to indicate all of the secondary schools.
- But this has one problem left! Which one?

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Multilevel Regression

Level 1: $Pop_{ij} = b_{0j} + b_{1j}Ext_{ij} + e_{ij}$

Level 2: $b_{0j} = \gamma_{00} + \gamma_{01}Texp_j + \boxed{u_{0j}}$

$b_{1j} = \gamma_{01} + u_{1j}$

Combined:

$$Pop_{ij} = b_{0j} + b_{1j}Ext_{ij} + eij$$
$$= \gamma_{00} + \gamma_{01}Ext_{ij} + \gamma_{01}Texp_j + e_{ij} + u_{0j} + u_{1j}Ext_{ij}$$

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Fixed Effect Models/Cross-Classified Models

Level 1: $Pop_{ijk} = b_{0jk} + b_{1jk}Ext_{ij} + b_{2jk}School1 + b_{3jk}School2 + \ldots + b_{xjk}SchoolX + e_{ijk}$

Level 2:
$$b_{0jk} = \gamma_{00k} + \gamma_{01k}Texp_{jk} + u_{0jk}$$
$$b_{1jk} = \gamma_{01k} + u_{1jk}$$
$$b_{2jk} = \gamma_{02k} + u_{2jk}$$
$$\vdots$$
$$b_{xjk} = \gamma_{0xk} + u_{xjk}$$

"Level" 3:
$$\gamma_{00k} = \gamma_{000} + v_{000}$$
$$\gamma_{01k} = \gamma_{001} + v_{001}$$
$$\gamma_{02k} = \gamma_{002} + v_{002}$$
$$\vdots$$
$$\gamma_{0xk} = \gamma_{00x} + v_{00x}$$

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Cross-Classified Models

- The fixed regression coefficients of these dummies are excluded from the model,
  - their slopes are allowed to vary at the third, ‘dummy’, level.
  - covariances between dummies all constrained to be zero,
  - variances all constrained to be equal.
- Thus, in the end we estimate one variance component for the secondary schools
- By putting the secondary schools in a separate “level” we assure that there are no covariances between residuals for the primary and the secondary schools.

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Fixed Effect Models/Cross-Classified Models

Level 1: $Pop_{ijk} = b_{0jk} + b_{1jk}Ext_{ij} + b_{2jk}School1 + b_{3jk}School2 + \ldots + b_{xjk}SchoolX + e_{ijk}$

Level 2:

$$b_{0jk} = \gamma_{00k} + \gamma_{01k}Texp_{jk} + u_{0jk}$$
$$b_{1jk} = \gamma_{01k} + u_{1jk}$$
$$b_{2jk} = \gamma_{02k} + u_{2jk}$$
$$\vdots$$
$$b_{xjk} = \gamma_{0xk} + u_{xj}$$

“Level” 3:

$$\gamma_{00k} = \gamma_{000} + v_{000}$$
$$\gamma_{01k} = \gamma_{001} + v_{001}$$
$$\gamma_{02} = \gamma_{002} + v_{002}$$
$$\vdots$$
$$\gamma_{0x} = \gamma_{00x} + v_{00x}$$

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Fixed Effect Models/Cross-Classified Models

Level 1: $Pop_{ijk} = b_{0jk} + b_{1jk}Ext_{ij} + b_{2jk}School1 + b_{3jk}School2 + \ldots + b_{xjk}SchoolX + e_{ijk}$

Level 2:

$$b_{0jk} = \gamma_{00k} + \gamma_{01k}Texp_{jk} + u_{0jk}$$
$$b_{1jk} = \gamma_{01k} + u_{1jk}$$
$$b_{2jk} = \gamma_{02k}$$
$$\vdots$$
$$b_{xjk} = \gamma_{0xk}$$

“Level” 3:

$$\gamma_{00k} = \gamma_{000} + v_{000}$$
$$\gamma_{01k} = \gamma_{001} + v_{001}$$
$$\gamma_{02} = \gamma_{002} + v_{002}$$
$$\vdots$$
$$\gamma_{0x} = \gamma_{00x} + v_{00x}$$

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Fixed Effect Models/Cross-Classified Models

Level 1: $Pop_{ijk} = b_{0jk} + b_{1jk}Ext_{ij} + b_{2jk}School1 + b_{3jk}School2 + \ldots + b_{xjk}SchoolX + e_{ijk}$

Level 2:

$$b_{0jk} = \gamma_{00k} + \gamma_{01k}Texp_{jk} + u_{0jk}$$
$$b_{1jk} = \gamma_{01k} + u_{1jk}$$
$$b_{2jk} = \gamma_{02k}$$
$$\vdots$$
$$b_{xjk} = \gamma_{0xk}$$

“Level” 3:

$$\gamma_{00k} = \gamma_{000} + v_{000}$$
$$\gamma_{01k} = \gamma_{001} + v_{001}$$
$$\gamma_{02k} = 0 + v_{002}$$
$$\vdots$$
$$\gamma_{0xk} = 0 + v_{00x}$$

$$\sigma^2_{v002} = \sigma^2_{v003} = \cdots = \sigma^2_{v0}$$

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Fixed Effect Models/Cross-Classified Models

$$\text{achiev}_{ijk} \sim \text{N}(XB, \Omega)$$

$$\begin{aligned}\text{achiev}_{ijk} = {} & \beta_{30ij}\text{const} + v_{0k}\text{c101}_{ijk} + v_{1k}\text{c102}_{ijk} + v_{2k}\text{c103}_{ijk} + v_{3k}\text{c104}_{ijk} + v_{4k}\text{c105}_{ijk} \\ & + v_{5k}\text{c106}_{ijk} + v_{6k}\text{c107}_{ijk} + v_{7k}\text{c108}_{ijk} + v_{8k}\text{c109}_{ijk} + v_{9k}\text{c110}_{ijk} \\ & + v_{10k}\text{c111}_{ijk} + v_{11k}\text{c112}_{ijk} + v_{12k}\text{c113}_{ijk} + v_{13k}\text{c114}_{ijk} + v_{14k}\text{c115}_{ijk} \\ & + v_{15k}\text{c116}_{ijk} + v_{16k}\text{c117}_{ijk} + v_{17k}\text{c118}_{ijk} + v_{18k}\text{c119}_{ijk} + v_{19k}\text{c120}_{ijk} \\ & + v_{20k}\text{c121}_{ijk} + v_{21k}\text{c122}_{ijk} + v_{22k}\text{c123}_{ijk} + v_{23k}\text{c124}_{ijk} + v_{24k}\text{c125}_{ijk} \\ & + v_{25k}\text{c126}_{ijk} + v_{26k}\text{c127}_{ijk} + v_{27k}\text{c128}_{ijk} + v_{28k}\text{c129}_{ijk} + v_{29k}\text{c130}_{ijk}\end{aligned}$$

$$\beta_{30ij} = 6.349(0.078) + u_{30jk} + e_{30ijk}$$

$$\begin{bmatrix} v_{0k} \\ v_{1k} \\ v_{2k} \\ v_{3k} \\ v_{4k} \end{bmatrix} \quad \begin{bmatrix} 0.065(.022) & & & \\ 0 & 0.065(.022) & & \\ 0 & 0 & 0.065(.022) & \\ 0 & 0 & 0 & 0.065(.022) \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

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Cross-Classified Models

Model	Intercept-only Coeff. (s.e.)	+ pupil vars Coeff. (s.e.)	+ school vars Coeff. (s.e.)	+ *ses* random Coeff. (s.e.)
Fixed part				
Intercept	6.35 (.08)	5.76 (.11)	5.52 (.19)	5.53 (.14)
Pupil gender		0.26 (.05)	0.26 (.05)	0.25 (.05)
Pupil ses		0.11 (.02)	0.11 (.02)	0.11 (.02)
Primary denom			0.20 (.12)	0.20 (.12)
Secondary denom			0.18 (.10)	0.17 (.09)
Random part				
$\sigma^2_{\text{int/pupil}}$	0.51 (.02)	0.47 (.02)	0.47 (.02)	0.46 (.02)
$\sigma^2_{\text{int/primary}}$	0.17 (.04)	0.17 (.04)	0.16 (.04)	0.14 (.08)
$\sigma^2_{\text{int/secondary}}$	0.07 (.02)	0.06 (.02)	0.06 (.02)	0.05 (.02)
$\sigma^2_{\text{ses/primary}}$				0.008 (.004)
Deviance	2317.8	2243.5	2237.5	2224.5
AIC	2325.8	2255.5	2253.5	2244.5

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Cross-Classified Models

- Alternatively you might have network data.
- E.g. you ask all members of a group to rate all other members
  - how much they would like to share some activity with the rated person
  - Sociometric rating

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Sociometric rating

	group	child	age	sex	grsize	rating1	rating2	rating3	rating4	rating5	rating6	rating7	rating8	rating9
1	1	1	8	1	7	.	3	6	4	4	7	6	.	.
2	1	2	10	1	7	5	.	6	4	5	7	5	.	.
3	1	3	11	1	7	4	6	.	4	5	7	6	.	.
4	1	4	9	0	7	4	4	6	.	5	7	5	.	.
5	1	5	11	0	7	5	5	6	5	.	7	6	.	.
6	1	6	10	1	7	4	5	6	3	4	.	6	.	.
7	1	7	10	1	7	3	5	6	5	3	6	.	.	.
8	2	1	9	0	9	.	3	5	3	4	6	6	4	5
9	2	2	9	0	9	2	.	4	5	6	5	4	4	5
10	2	3	9	0	9	5	3	.	4	3	6	5	4	6
11	2	4	8	1	9	3	2	5	.	6	6	5	3	4
12	2	5	9	1	9	4	4	5	5	.	5	7	4	5
13	2	6	9	0	9	3	4	4	4	4	.	5	4	5
14	2	7	9	1	9	4	4	6	5	6	5	.	4	5
15	2	8	11	0	9	3	4	5	4	5	6	6	.	5
16	2	9	8	1	9	3	4	5	5	4	6	7	5	.
17	3	1	11	0	5	.	5	7	5	6	.	.	.	.
18	3	2	11	0	5	5	.	7	6	6	.	.	.	.
19	3	3	13	1	5	5	5	.	6	8	.	.	.	.
20	3	4	12	1	5	4	4	6	.	6	.	.	.	.

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Cross-Classified Models

- DV: Ratings
- Nested:

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Cross-Classified Models

- DV: Ratings
- Nested: Cross-classification of senders and receivers
- Can be further nested within groups

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Sociometric rating

	group	sender	receiver	rating	agesend	sexsend	agerec	sexrec	grsize
1	1	1	2	3	8	1	10	1	7
2	1	1	3	6	8	1	11	1	7
3	1	1	4	4	8	1	9	0	7
4	1	1	5	4	8	1	11	0	7
5	1	1	6	7	8	1	10	1	7
6	1	1	7	6	8	1	10	1	7
7	1	2	1	5	10	1	8	1	7
8	1	2	3	6	10	1	11	1	7
9	1	2	4	4	10	1	9	0	7
10	1	2	5	5	10	1	11	0	7
11	1	2	6	7	10	1	10	1	7
12	1	2	7	5	10	1	10	1	7
13	1	3	1	4	11	1	8	1	7
14	1	3	2	6	11	1	10	1	7
15	1	3	4	4	11	1	9	0	7
16	1	3	5	5	11	1	11	0	7
17	1	3	6	7	11	1	10	1	7
18	1	3	7	6	11	1	10	1	7
19	1	4	1	4	9	0	8	1	7
20	1	4	2	4	9	0	10	1	7

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Cross-Classified Models

- $Y_{i(jk)l}$ Score of student i within cross-classification of sender j and receiver k, and within group l

$$Y_{i(jk)l} = \beta_{0(jk)l} + e_{i(jk)l}$$

- $\beta_{0(jk)l}$ is the intercept/overall mean
- $e_{i(jk)l}$ is the residual error term
- the subscript *(jk)* is written in parentheses to indicate that they are conceptually at the same level

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Cross-Classified Models

- The intercept $\beta_{0(jk)l}$ varies independently across both senders, receivers, and group, so

$$Y_{i(jk)l} = \beta_{0(jk)l} + e_{i(jk)l}$$
$$\beta_{0(jk)l} = \beta_{0l} + u_{0jl} + v_{okl}$$
$$\beta_{0l} = \gamma_{00} + f_{0l}$$

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Cross-Classified Models

$$Y_{i(jk)l} = \beta_{0(jk)l} + e_{i(jk)l} \qquad \beta_{0(jk)l} = \beta_{0l} + u_{0jl} + v_{okl}$$
$$\beta_{0l} = \gamma_{00} + f_{0l}$$

- Score consists of:
  - Overall mean γ_{00},
  - residual error term f_{0l} for group l,
  - individual-level residual error terms u_{jl} for sender j in group l
  - individual-level residual error v_{kl} for receiver k in group l, and
  - the measurement-level error term $e_{i(jk)l}$.

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Cross-Classified Models

- The crossed effects of the receiver level are incorporated using dummies.
- At the lowest level, the ratings, we specify:
  - Dummy variables that indicate the receivers.
  - The fixed coefficients of these dummies are excluded from the model, but their slopes are allowed to vary.
  - Since the cross-classification is nested within the sociometric groups, the slopes of the dummy variables are set to vary at a third group level.
  - In addition, the covariances between the receiver dummies are constrained to be zero, and
  - the variances are constrained to be equal.

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Cross-Classified Models

- The covariances between the receiver dummies are constrained to be zero, and the variances are constrained to be equal.
  - Thus, we estimate one variance component for the receivers.
- By putting the variance term(s) for the receivers on a "separate" level we assure that there are no covariances between the residuals for the sender and the receiver level.

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Cross-Classified Models

- Both sender and receiver characteristics like *age* and *gender* and group characteristics like *group size* can be added to the model as predictors.
- Child characteristics may be allowed to have random slopes at the group level.
- The analysis proceeds along exactly the same lines as outlined for the cross-classification of primary and secondary schools.

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Cross-Classified Models: Hard in some software!

- Since the third 'group' level is already used to specify the random variation for the receiver dummies:
  - we must make sure that the intercept and possible slope variation at the 'real' group level are not correlated with the dummies.
  - This can be accomplished by adding the appropriate constraints to the model.
- When the software supports more than three levels (e.g., R/MLWin/HLM), the same result can be accomplished more conveniently:
  - Add a fourth level to the model; also for the groups
  - Used for the random part at the real group level.
  - Conceptually we still have three levels, with senders and receivers crossed at the second level

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Cross-Classified Models

- Individual-level explanatory variables can be added to the equation.
  - Their regression slopes may be allowed to vary across primary and/or secondary schools.

$$Y_{i(jk)} = \beta_{0(jk)} + \beta_{1(jk)} X_{i(jk)} + e_{i(jk)}$$

- School-level variables can also be added,
  - Used to explain variation in the slopes of individual-level variables across schools
  - Similar to ordinary multilevel regression models.

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Non-Normal Data

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Dichotomous data and proportions

- Standard regression: continuous outcome variable with normal distribution of errors
  - What do we do again if one or more violations are violated?
- Some outcome variables *must* violate this assumption
  - Dichotomous outcome variable
  - Outcome variable is a proportion

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Nonlinear Regression models

- Classic solution: transform the dependent variable
  - Proportions: f(p) = logit(p) = ln((p)/(1-p))
  - Breaks down if p = 0,1
    - Does not work with dichotomous variable
- Modern solution: use generalized linear model
  - i.e., Logistic regression

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Nonlinear Regression models

- Generalized linear model:
- Don’t transform DV and then run a “normal” regression.
- Run a “normal” regression and then transform the prediction of the analysis!

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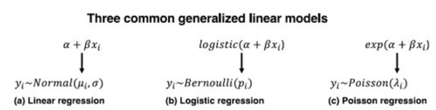
Generalized Linear Model



$y = b_0 + b_1X_1 + b_2X_2 + e$

$\text{pred} = b_0 + b_1X_1 + b_2X_2 + e$

$$\text{logistic(pred)} = \frac{exp^{(pred)}}{1 + exp^{(pred)}}$$

$y \sim Bernoulli(logistic(pred))$

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Generalized Linear Model

Three common generalized linear models

(a) Linear regression	(b) Logistic regression	(c) Poisson regression
$\alpha + \beta x_i$	$logistic(\alpha + \beta x_i)$	$exp(\alpha + \beta x_i)$
$\downarrow$	$\downarrow$	$\downarrow$
$y_i \sim Normal(\mu_i, \sigma)$	$y_i \sim Bernoulli(p_i)$	$y_i \sim Poisson(\lambda_i)$

- The output of $b_0 + b_1X_1 + b_2X_2 + e$ can be any value ranging from negative to positive infinity.
- A 'link' function is introduced to ensure that only sensible parameter values are allowed.
- Logistic regression: the logistic function ensures that values from negative to positive infinity are compressed into the range [0, 1].

$$\text{pred} = b_0 + b_1X_1 + b_2X_2 + e$$

$$\text{logistic(pred)} = \frac{exp^{(pred)}}{1 + exp^{(pred)}}$$

$$y \sim Bernoulli(logistic(pred))$$

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Generalized Linear Model

- Easy to do in R!!
- Just have to change the distribution we use in an analysis!
  - Just like changing from normal to t-distribution
  - But now use Bernoulli or Poisson or…
- Just need to remember that the parameter-estimates are in terms of the non-transformed "pred" part, and so we need to transform them to the "original" scale for interpretation.

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Example: Multilevel Logistic model

- Thailand Education Data
  - 8582 pupils in 411 primary schools
  - Outcome variable dichotomous
  - Repeat a class: yes (1) or no (0)

  - Pupil level predictors
  - *sex* (1 = male, 0 = female)
  - *pre-primary education* (PPED: 1 = yes, 0 = no)
  - School level predictor
  - Mean *SES*

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Example: Multilevel Logistic model

```
Final estimation of fixed effects: (Unit-specific model)
----------------------------------------------------------------------------
                                   Standard             Approx.
   Fixed Effect         Coefficient   Error      T-ratio   d.f.     P-value
----------------------------------------------------------------------------
For       INTRCPT1, B0
   INTRCPT2, G00          -2.038493   0.093685   -21.759       355    0.000
For      SEX slope, B1
   INTRCPT2, G10           0.509270   0.073925     6.889      7513    0.000
For     PPED slope, B2
   INTRCPT2, G20          -0.609289   0.095220    -6.399      7513    0.000
----------------------------------------------------------------------------
```

$$p_{ij} = logistic(-2.04 + 0.51 sex - 0.61 pped)$$

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Example: Multilevel Logistic model

- Interpretation: $p_{ij} = logistic(-2.04 + 0.51 sex - 0.61 pped)$

  - girls, no pped: predict: –2.04 on the underlying continuous scale.
  Transform: $g(x) = \frac{e^x}{1+e^x}$ = 0.115

  - boys, no pped: predict: -2.04 + 0.51 = -1.53
  Transformed: 0.178

  - The estimated repeat rate for the girls is 11.5% and for the boys 17.8% (no pped).

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Example: Multilevel Logistic model

- Interpretation: $p_{ij} = logistic(-2.04 + 0.51 sex - 0.61 pped)$
  - girls, with pped: predict: –2.65 on the underlying continuous scale
  Transformed: 0.066
  - boys, with pped: predict: -2.04 + 0.51 – 0.61 = -2.14
  Transformed: 0.105
- The estimated repeat rate for the girls with pped is 6.6% and for the boys 10.5%.

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Practical 5

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