Automatic Key Word and Key Sentence Extraction

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Agenda

- Background
- Rank-1 Approximation

  saliency score

  mutual reinforcement principle

  weekness
- Rank-k Approximation

  extraction of 1st key sentence

  extraction of 2nd key sentence

  extraction of 2nd key sentence
- Summary

Automatic Key Word and Key Sentence Extraction

Why:

- the explosion of the amount of textual information
- a need to develop automatic procedures for text summarization

How:

- Saliency Score
- Rank-k Approximation

Goal :

- Extract the most important content of a text(key words and key sentences)

Preprocessing

1. Stemming

    eats, eating, ate --> eat

2. Stop-words removing
3. Special symbolic removing

    mathematics or mark-up language tags (HTML, LATEX)

Text Presentation

- consider each sentence as a separate document
- a term is a stemmed word
- term-sentence Matrix A ∈ $\mathbb{R}^{m*n}$

  m: the number of different terms

  n: the number of sentences

	sentence 1	sentence 2	sentence 3	…	sentence n
apple	5.25	3.18	0	…	0
bill	1.21	0	2.54	…	0.88
…	…	…	…	…	…
search	0	1.37	2.28	…	1.59

Term-Sentence Matrix

- Term-Document(term-sentence) Matrix A ∈ $\mathbb{R}^{m*n}$

  m: the number of different terms

  n: the number of sentences
- a_{ij}: the weighted frequency of term i in sentence j
- column vector $(a_{1j}, \ a_{2j}, \cdots\cdots a_{mj})^T$: the terms occurring in sentence j
- row vector $(a_{i1}, \ a_{i2}, \cdots\cdots a_{in})$: sentences containing term i

	sentence 1	…	sentence j	…	sentence n
term 1	5.25	…	$\boldsymbol{a_{1j}}$	…	$\boldsymbol{a_{1n}}$
…	1.21	…	$\boldsymbol{a_{2j}}$	…	$\boldsymbol{a_{2n}}$
term i	$\boldsymbol{a_{i1}}$	…	$\boldsymbol{a_{ij}}$	…	$\boldsymbol{a_{in}}$
…	…	…	…	…	…
term m	0	…	$\boldsymbol{a_{mj}}$	…	$\boldsymbol{a_{mn}}$

Matrix SVD Recap

- A = $\mathbf{U} \sum V^T$

□columns of U and V^T are singular vectors

□diagonal elements of $\sum$ are singular values σ_i

□Av_i=$\sigma_i u_i$

□$A^T u_i$=$\sigma_i v_i$

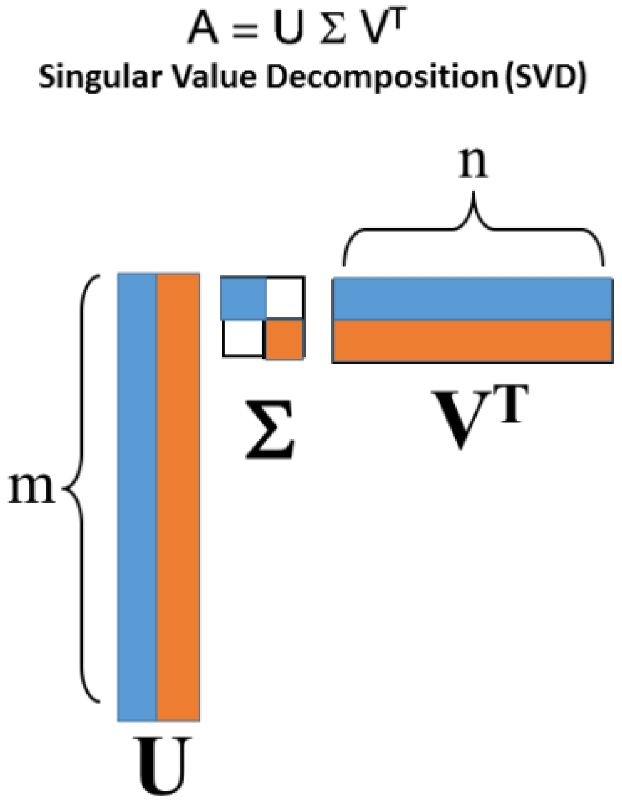
□A $A^T u_i$ = $\sigma_i \sigma_i u_i$

□A^T Av_i = σ_i $\sigma_i v_i$

Eigenvectors of A A^T are the singular vectors u_i

Eigenvectors of A^T A are the singular vectors v_i

AAT and ATA have the same eigenvalues of the form σ_i^2



$\boxed{\mathbf{A}_{[m \times n]} = \mathbf{U}_{[m \times r]}\, \Sigma_{[r \times r]}\, (\mathbf{V}_{[n \times r]})^T}$

Mutual reinforcement principle

☐ Saliency Scores

- u_i : saliency score of term i
- v_j : saliency score of sentence j

☐ mutual reinforcement principle

- A term should have a high saliency score if it appears in many sentences with high saliency scores.

$u_i \propto \sum_{j=1}^{n} a_{ij} v_j,\quad i = 1, 2, 3, \cdots\cdots m.$

- A sentence should have a high saliency score if it contains many words with high saliency scores.

$v_j \propto \sum_{i=1}^{m} a_{ij} u_i,\quad j = 1, 2, 3, \cdots\cdots n.$

reformed as:

$$\sigma_u \, u = Av$$

$$\sigma_v v = A^T u$$

σ_u and σ_v are proportionality constants

Equation Derivation

σ_u u = Av

σ_vv = A^Tu

insert one equation to the other, we get:

σ_u u = $\frac{1}{\sigma_v}$AA^Tu

σ_v v = $\frac{1}{\sigma_u}$ A^TAv

these shows that u and v are eigenvectors of A A^T and A^T A respectively, with the same eigenvalue σ_u σ_v

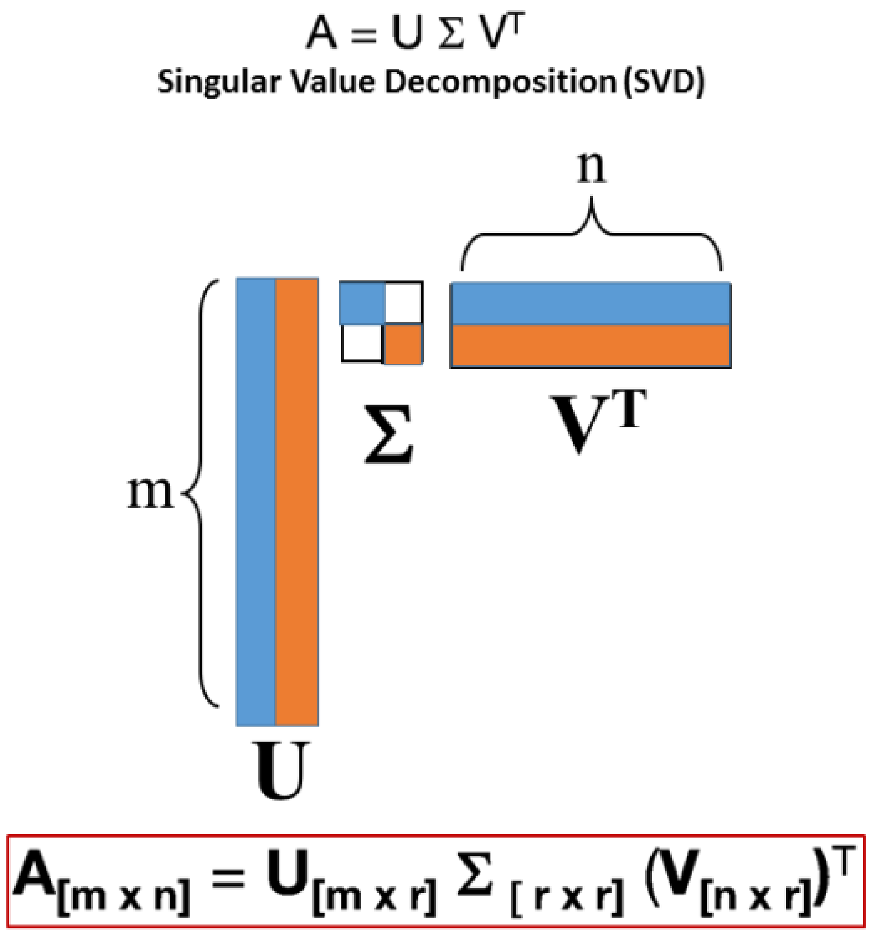
Matrix SVD Recap

- A = $\mathbf{U} \sum V^T$
  - columns of U and V^T are singular vectors
  - diagonal elements of $\sum$ are singular values σ_i
  - Av_i=$\sigma_i u_i$
  - $A^T u_i$=$\sigma_i v_i$
  - A $A^T u_i = \sigma_i \sigma_i u_i$
  - A^T A$\mathrm{v}_i = \sigma_i\ \sigma_i v_i$

Eigenvectors of A A^T are the singular vectors u_i

Eigenvectors of A^T Aare the singular vectors v_i

A A^T and A^T A have the same eigenvalues of the form σ_i^2



Equation Derivation

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σ_v v = $\frac{1}{\sigma_u}$$A^T$Av

these shows that u and v are eigenvectors of A A^T and A^T A respectively, with the same eigenvalue σ_u σ_v

reform as：

σ_v σ_u u = AA^Tu

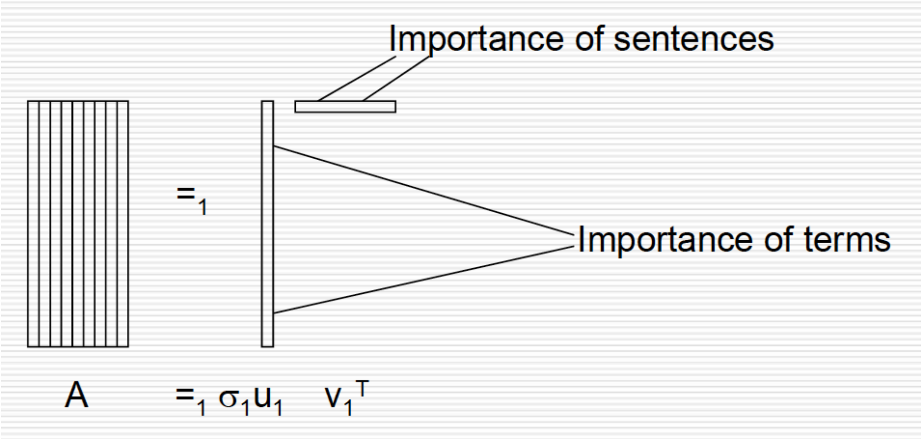
σ_u σ_v v = A^TAv

→ u and v are singular vectors corresponding to the same singular value σ_v σ_u

Saliency Score-Rank 1 Approximation

For some singular value si of SVD of A

- vector u of saliency scores for terms is the singular vector u_i
- vector v of saliency scores for sentences is the singular vector v_i
- $\sigma_\mathbf{v}\ \sigma_\mathbf{u} = \sigma_i\ \sigma_i$
- We select the largest singular value σ_1



The saliency scores for

- terms are the entries of the singular vector u_1
- sentences are the entries of the singular vector v_1
- for the largest singular value σ_1
- $A \approx \sigma_1 u_1 v_1^T$

From Rank-1 Approximation to Rank-k Approximation

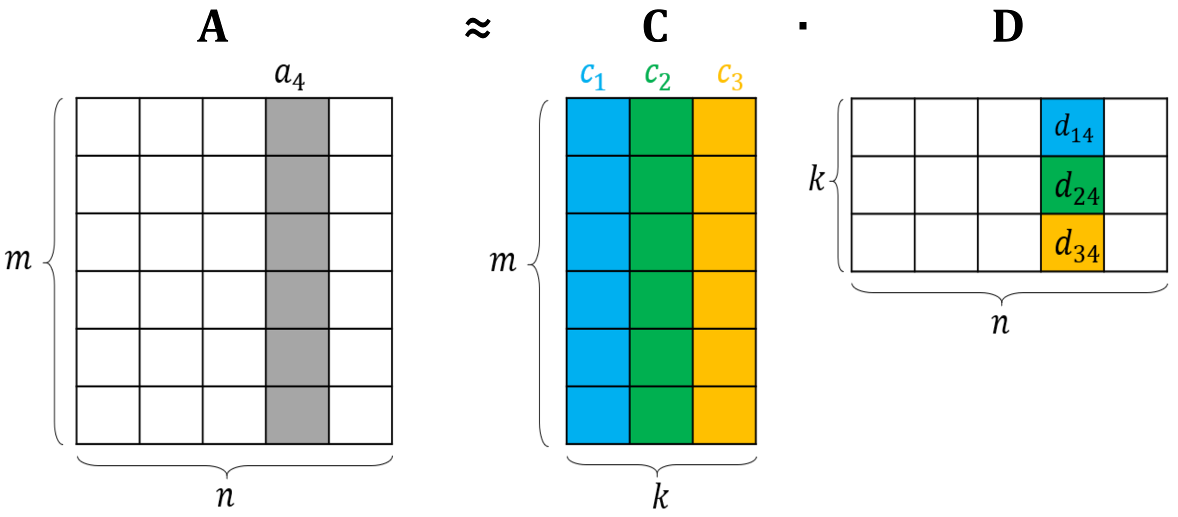
□Problems of rank-1 approximation:

- mutual reinforcement principle prefers long sentences
- consider only the most important (semantic) direction in “sentence space”
- cannot avoid extracting similar sentences

➢Solution: use rank-k approximation to extract key sentences

Key Sentence Extraction from a Rank-k Approximation

- Assume we have computed a rank-k approximation of $\boldsymbol{A}$: $\boldsymbol{A} \approx \boldsymbol{CD}$ (using SVD, clustering, or nonnegative matrix factorization)
- $\boldsymbol{C}$ can be seen as **k basis vectors** representing the most important (semantic) directions in the “sentence space”. (k ≥ number of key sentences)
- $\boldsymbol{D}$ holds **coordinates** of corresponding columns in A with regards to the basis vectors. （weights of semantic directions）
- So each column (sentence) in $\boldsymbol{A}$ is a linear combination of basis vectors in $\boldsymbol{C}$.

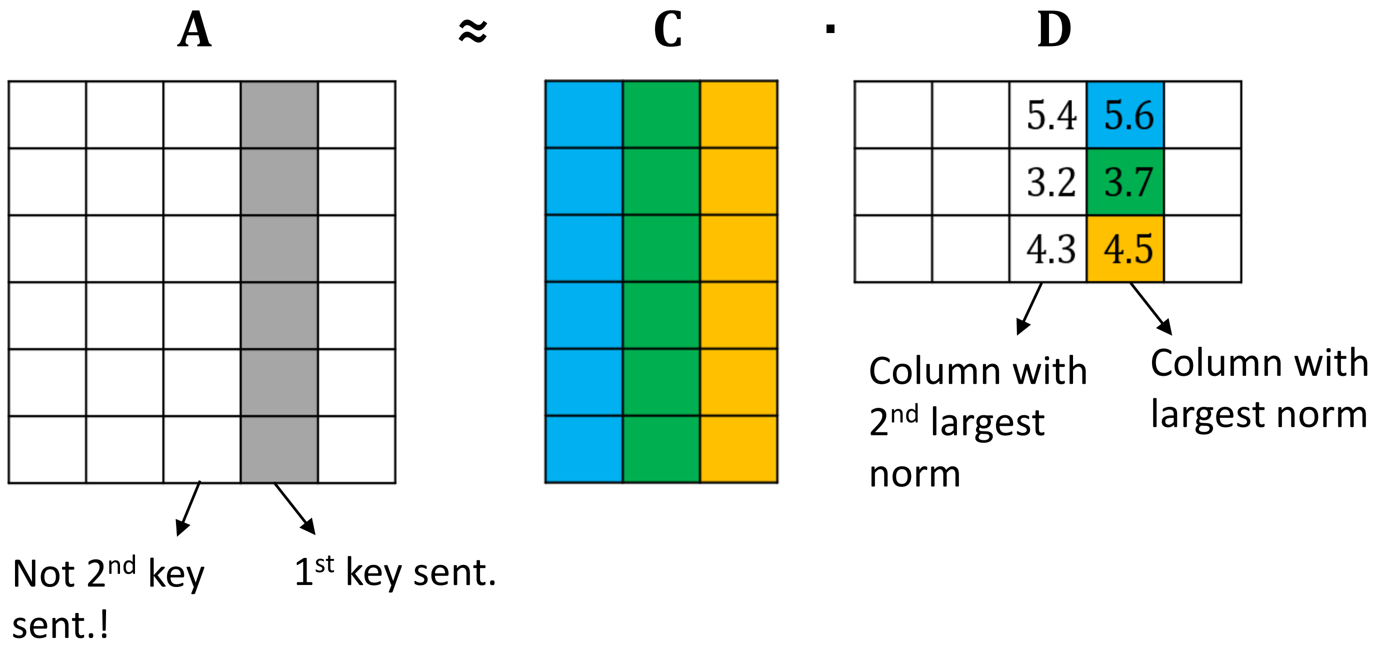


$$m = 6, n = 5, k = 3$$

$$a_4 = d_{14}c_1 + d_{24}c_2 + d_{34}c_3$$

Key Sentence Extraction from a Rank-k Approximation

- Find the most important column in $\boldsymbol{A}$ by looking for the column in $\boldsymbol{D}$ with largest 2-norm. (largest "weights" with regards to basis $\boldsymbol{C}$)
- But! To find the second key sentence, we cannot simply find the second largest column in $\boldsymbol{D}$. Because it may be similar to the first one.
- To avoid similarity, we cannot extract k sentences directly. we must apply some transformation before we determine each key sentence.



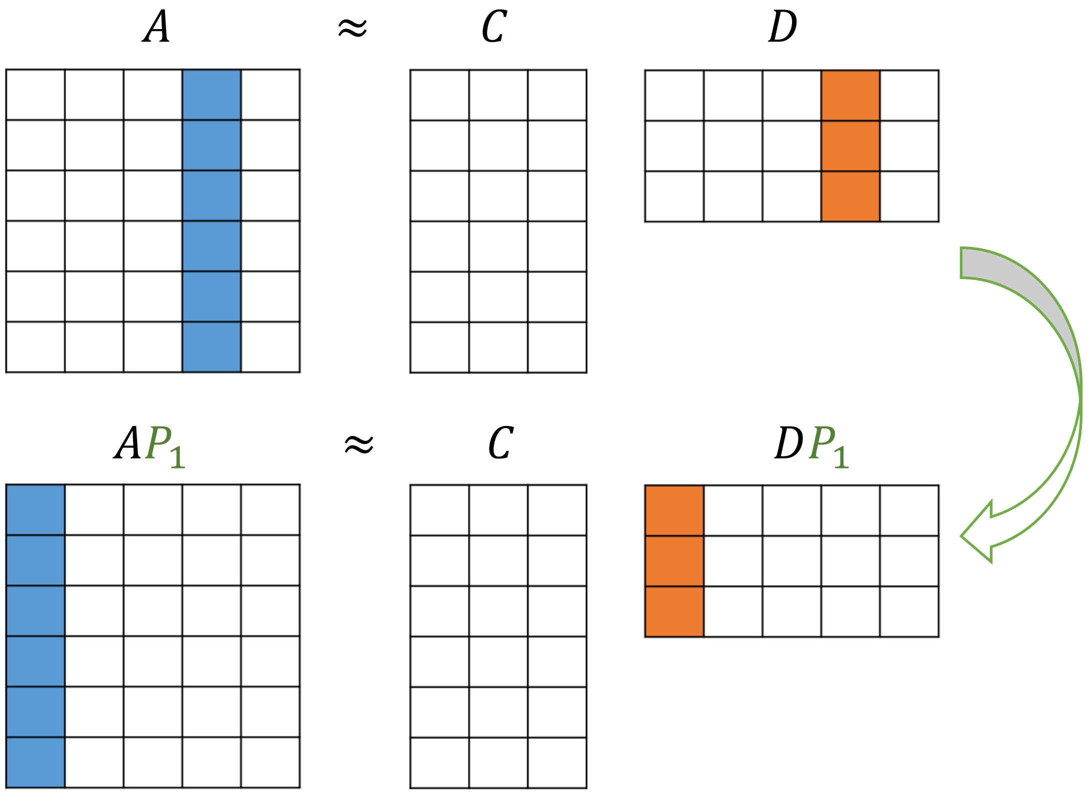
2-norm of a vector:

$$\|d_4\|_2 = \sqrt{d_{14}^2 + d_{24}^2 + d_{34}^2}$$

Extraction of 1st key sentence

Step 1: Permutation

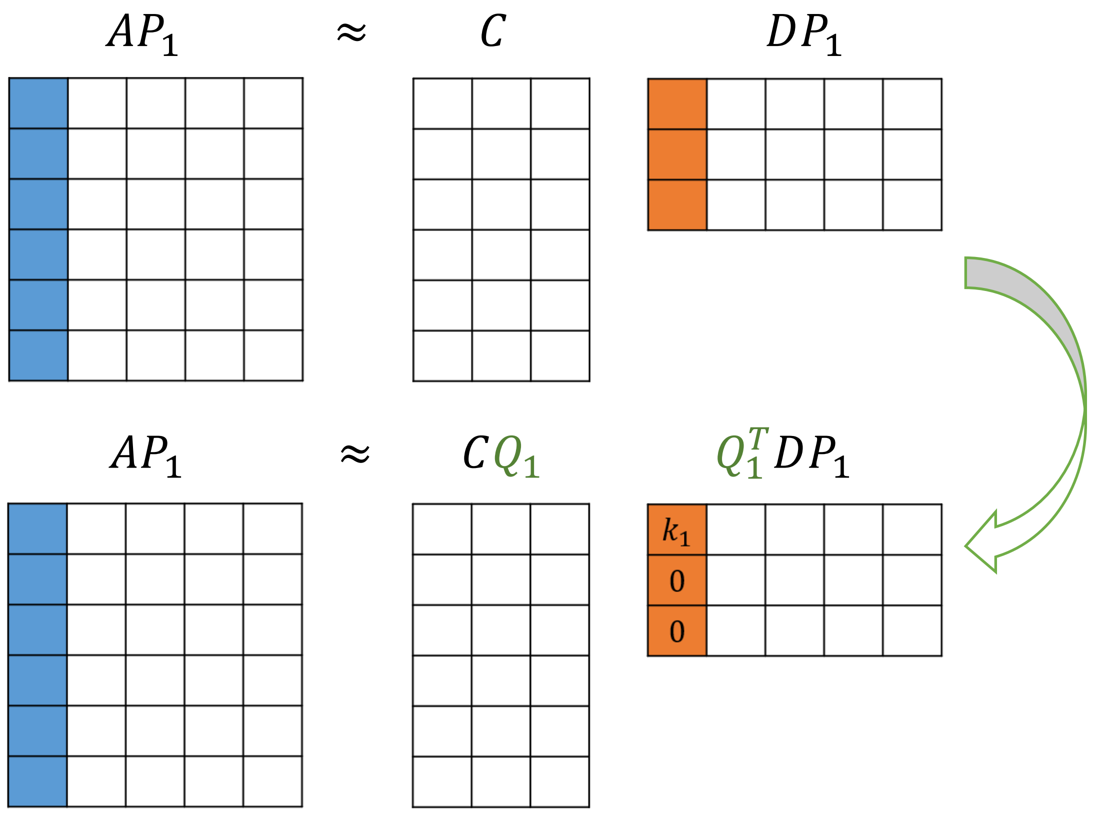
- Permute column in $\boldsymbol{D}$ with largest 2-norm to first position
- By applying a permutation $\boldsymbol{P_1}$ to $\boldsymbol{D}$ (multiply $\boldsymbol{D}$ by $\boldsymbol{P_1}$)
- To keep the equation balanced, $\boldsymbol{P_1}$ is also applied to $\boldsymbol{A}$.
- The corresponding column in $\boldsymbol{A}$ is also moved to first position.



Extraction of 1st key sentence

Step 2: Householder Transformation

- Zeros the elements in the first column below the element in position *(1,1)* in $\boldsymbol{DP_1}$
- By applying a Householder Transformation $\boldsymbol{Q_1^T}$ to $\boldsymbol{DP_1}$
- Then multiply $\boldsymbol{C}$ by $\boldsymbol{Q_1}$, so that the equation $AP_1 \approx CQ_1Q_1^T DP_1$ remains true.
- Because $Q_1Q_1^T = I$ for a householder transformation

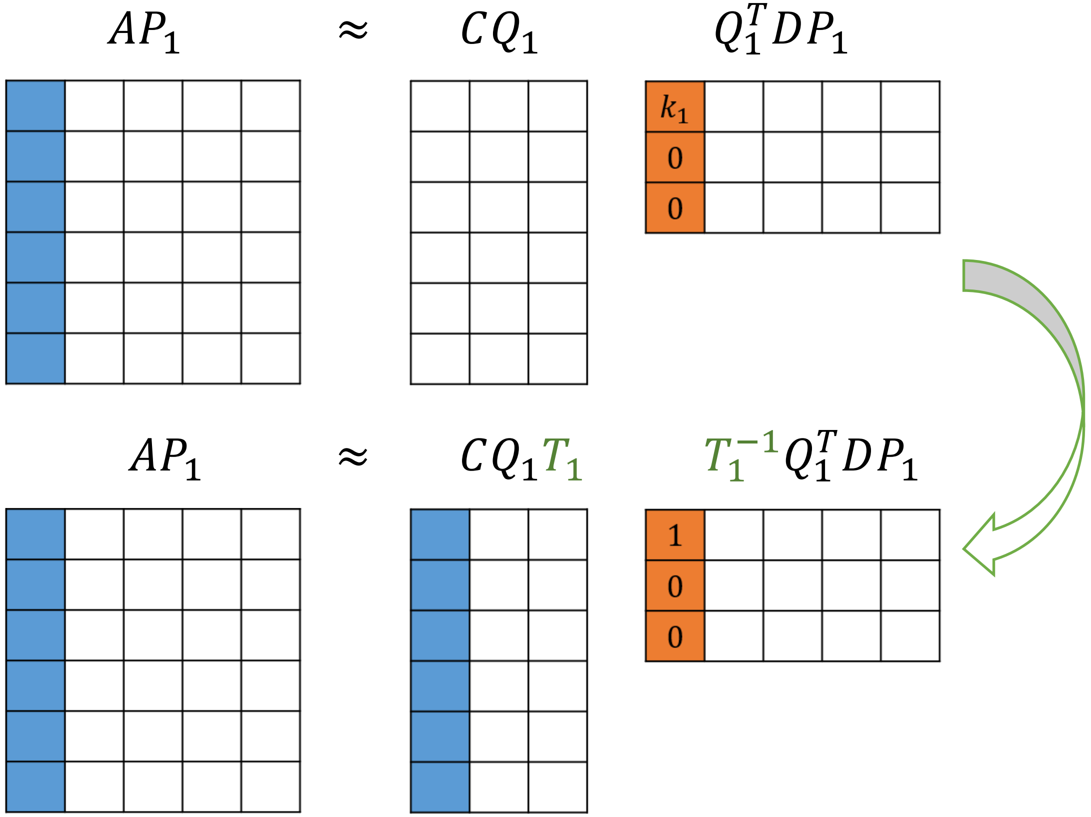


Extraction of 1st key sentence

Step 3: Scaling

- Normalize the first column in $\boldsymbol{Q_1^T D P_1}$ by applying transformation $\boldsymbol{T_1^{-1}}$
- The multiply $\boldsymbol{CQ_1}$ by $\boldsymbol{T_1}$, so that the equation remains true. $(T_1 T_1^{-1} = I)$

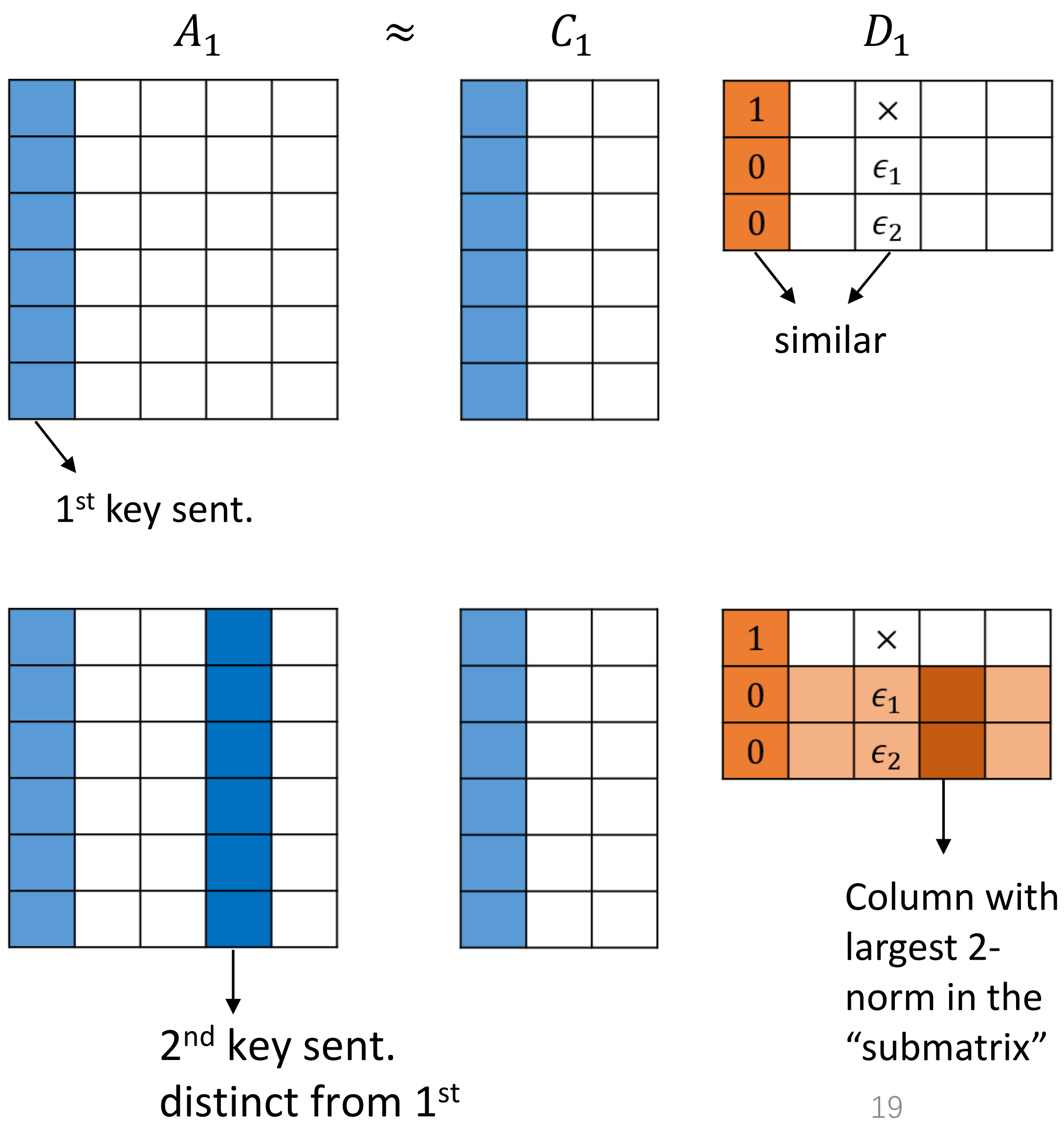
- Now the first column in $\boldsymbol{AP_1}$ is approximately equal to the first column in new basis $\boldsymbol{CQ_1 T_1}$
- We have found the first key sentence



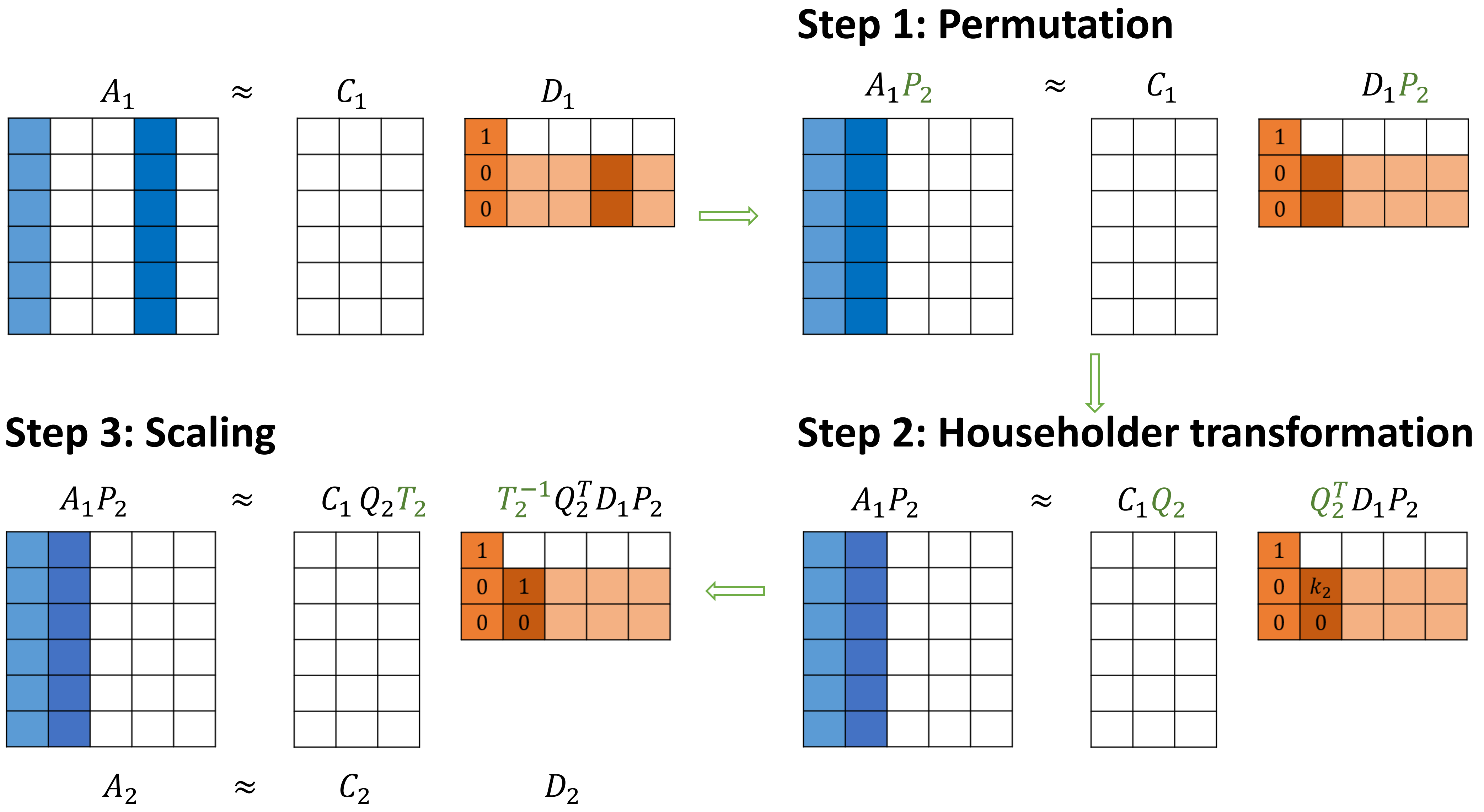
$$T_1 = \begin{bmatrix} k_1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \qquad T_1^{-1} = \begin{bmatrix} \frac{1}{k_1} & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Extraction of 2nd key sentence

- If another column in D was similar to the one that is now in position 1, it has small entries in rows 2 and 3.
- To avoid that column, we ignore the first row in D_1 while computing 2-norm in the next iteration
- Do the transformation again to extract second key sentence

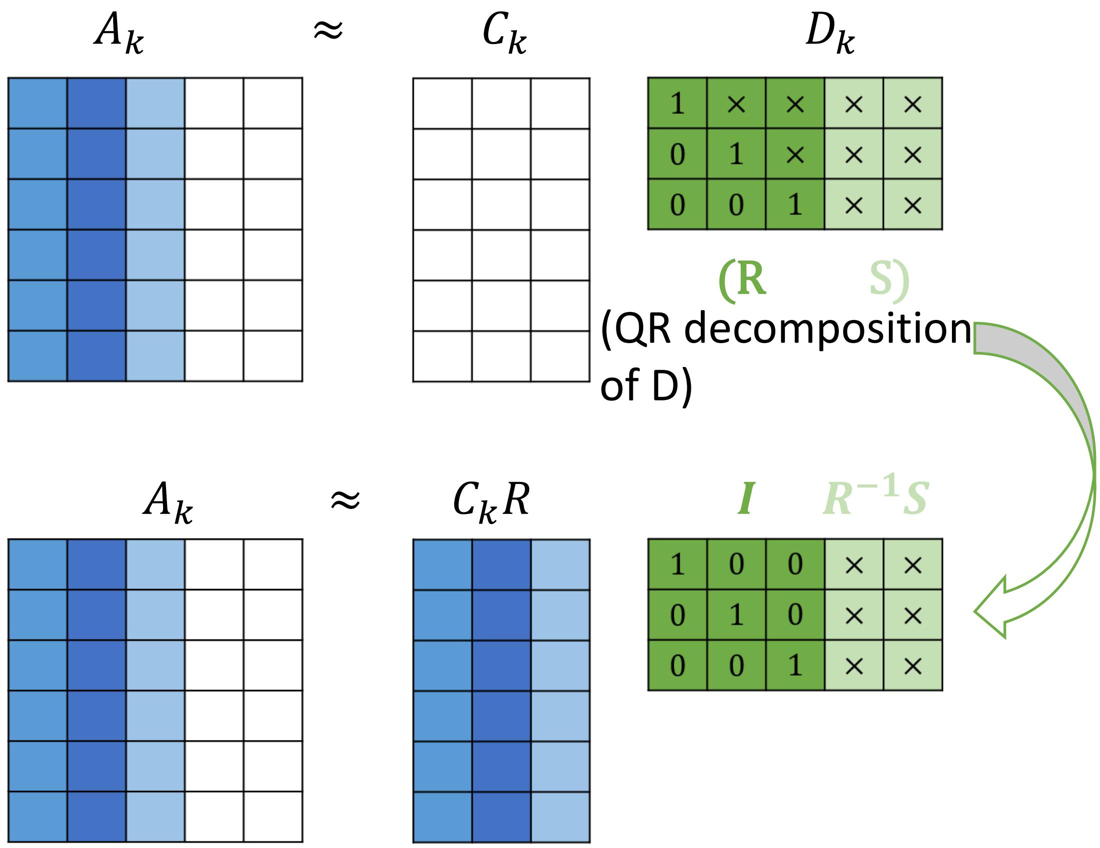


Extraction of 2nd key sentence



Key Sentence Extraction from a Rank-k Approximation

- After k iterations: $\boldsymbol{A_k} \approx \boldsymbol{C_k D_k}$
- The first k columns in $\boldsymbol{A_k}$ are now the k key sentences.
- If we apply $\boldsymbol{R}$ and $\boldsymbol{R^{-1}}$, we get new basis vectors directly corresponding to the key sentences.
- In practice, we just apply QR decomposition to $\boldsymbol{D}$ to get $\boldsymbol{D_k} = (\boldsymbol{R}\ \boldsymbol{S})$, remember the permutation order and find the corresponding columns in $\boldsymbol{A}$.
- If we are only interested in finding the top k sentences, we don't need to apply any transformations to the matrix $\boldsymbol{C}$.

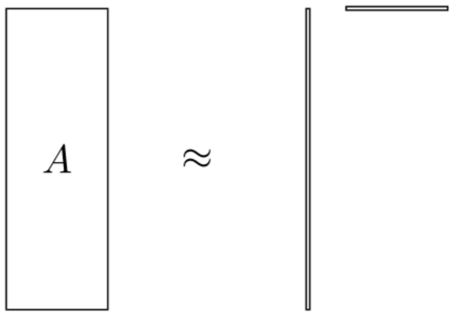


Summary

Automatic key word and key sentence extraction through

- Rank-1 approximation of term-sentence matrix A:

$$A \approx \sigma u v$$



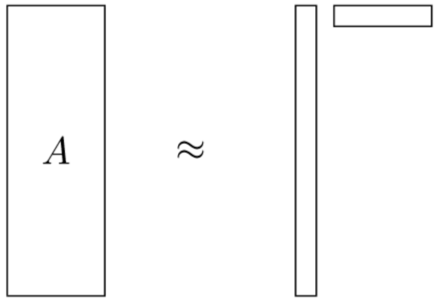
u: saliency scores of terms (1st left singular vector)

v: saliency scores of sentences (1st right singular vector)

Problem: extract similar sentences

- Rank-k approximation of term-sentence matrix A:

$$A \approx CD$$



Transform D and extract k key sentences step by step to avoid similarity

Bibliography

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