CLRS 4-7

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September 24, 2008

a. i. That Monge$(A_{m,n}) \to \forall 1 \leq i < m, 1 \leq j < n(a_{i,j} + a_{i+1,j+1} \leq a_{i,j+1} + a_{i+1,j})$ follows directly from the definition of a monge array.

ii. **Basis**

$$a_{0,0} + a_{1,1} \leq a_{0,1} + a_{1,0} \to \text{Monge}(A_{2,2})$$

Hypothesis

$$\forall 1 \leq i < m, 1 \leq j < n(a_{i,j} + a_{i+1,j+1} \leq a_{i,j+1} + a_{i+1,j}) \to \text{Monge}(A_{m,n})$$

Induction

$$\begin{aligned} &\forall 1 \leq i < m, 1 \leq j < n(a_{i,j} + a_{i+1,j+1} \leq a_{i,j+1} + a_{i+1,j}) \to \\ &\qquad \forall j < n(a_{m,j} + a_{m+1,j+1} \leq a_{m,j+1} + a_{m+1,j}) \wedge \\ &\qquad\qquad \text{Monge}(A_{m,n}) \qquad (1) \\ &\to \text{Monge}(A_{m+1,n}) \qquad (2) \end{aligned}$$

Similarly for $A_{m,n+1}$.

b. Changing either a or b to the given range will work.

$$\begin{bmatrix} 37 & 23 & 22 & 32 \\ 21 & 6 & 7 & 10 \\ 53 & 34 & 30 & 31 \\ 43 & 21 & 15 & 8 \end{bmatrix} \Rightarrow \begin{bmatrix} 37 & 23 & a \geq 23 & 32 \\ 21 & 6 & b \leq 6 & 10 \\ 53 & 34 & 30 & 31 \\ 43 & 21 & 15 & 8 \end{bmatrix}$$

c. $f(k) > f(k+1)$ violates the Monge definition by creating a minimum sum that no other combination of elements can undersum.

Let $f(k) > f(k+1)$; pick $1 \leq q < r < m$ such that:

$$\forall q \leq i < r, q < j \leq r(a_{k,i} > a_{k,r} \wedge a_{k+1,j} > a_{k+1,q})$$

It follows, then, that

$$\forall q \leq i < r, q < j \leq r(a_{k,i} + a_{k+1,j} > a_{k,r} + a_{k+1,q})$$

contradicting the Monge definition.

d. For each odd row i, start at column $j = f(i-1)$, where $f(0) = 0$; let $m_i = \infty$; iterate to $j = f(i+1)$, where $f(n+1) = m$; setting m_i to $\min(m_i, a_{i,j})$.

Running time is:

$$\begin{aligned} O(m) + \sum_{i=1}^{m} f(i-1) - f(i+1) = O(m) + f(0) + f(m) + \\ \sum_{i=1}^{m} f(i) - f(i) \end{aligned} \tag{3}$$

$$= O(m) + O(n) + 0 \tag{4}$$

$$= O(m+n) \tag{5}$$

e.

$$T(m) = T(m/2) + O(m+n) \tag{6}$$

$$= \sum_{i=0}^{\lg m} 2^i + n \tag{7}$$

$$= \Theta(m + n \lg m) \tag{8}$$