

CLRS 15.2-4

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Since in MATRIX-CHAIN-ORDER the i -loop runs $n - l + 1$ times for every l ; and, for every i , the k -loop runs $j - i = i + l - 1 - i = l - 1$ times; and, for every k , two $m[i, j]$ s are looked up; it gelds the following:

$$\sum_{i=1}^n \sum_{j=i}^n R(i, j) = \sum_{l=2}^n 2(n - l + 1)(l - 1) \quad (1)$$

$$= 2 \sum_{l=1}^{n-1} (n - l)l \quad (2)$$

$$= 2 \sum_{l=1}^{n-1} nl - 2 \sum_{l=1}^{n-1} l^2 \quad (3)$$

$$= \frac{2n(n-1)n}{2} - \frac{2(n-1)n(2n-1)}{6} \quad (4)$$

$$= \frac{n^3 - n}{3} \quad (5)$$