CLRS 16.2-1

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Given amounts $a_1 \ldots a_n$ and value densities $\frac{v_1}{w_1} \geq \frac{v_2}{w_2} \geq \cdots \geq \frac{v_n}{w_n}$, $\frac{v_1}{w_1}(a_1 + \cdots + a_n)$ is at least as much as $\frac{v_1}{w_1}a_1 + \cdots + \frac{v_n}{w_n}a_n$:

$$\sum_{i=1}^{n} \frac{v_1}{w_1}a_i \geq \sum_{i=1}^{n} \frac{v_i}{w_i}a_i \tag{1}$$

$$n\frac{v_1}{w_1}\sum_{i=1}^{n}a_i \geq n\max_{1\leq i\leq n}\left(\frac{v_i}{w_i}\right)\sum_{i=1}^{n}a_i \tag{2}$$

$$\frac{v_1}{w_1} \geq \max_{1\leq i\leq n}\left(\frac{v_i}{w_i}\right) \tag{3}$$