CLRS 15.2-5

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Full parenthesization maintains the multiplication tree property (see figures 1 and 2 *infra*); *id est*, that a tree be a full binary tree where:

1. leaves are matrices and only matrices;
2. every parenthesis has two and only two children.

Let $P(e) = n$ state that a fully parenthesized expression of e elements requires n pairs of parentheses.

Basis: $P(1) = 0$

Hypothesis: $P(k) = k - 1$

Praebendum: $P(k + 1) = k$

To insert a matrix into a multiplication tree of k matrices, resulting in a multiplication tree of $k + 1$ matrices and while maintaining the multiplication tree property; the matrix must be added at a leaf by parenthesizing the leaf and adding itself and the new node as children, resulting in k pairs of parentheses.

Therefore:

$$(P(1) = 0 \wedge (P(k) = k - 1 \rightarrow P(k + 1) = k)) \rightarrow \forall n P(n) = n - 1$$

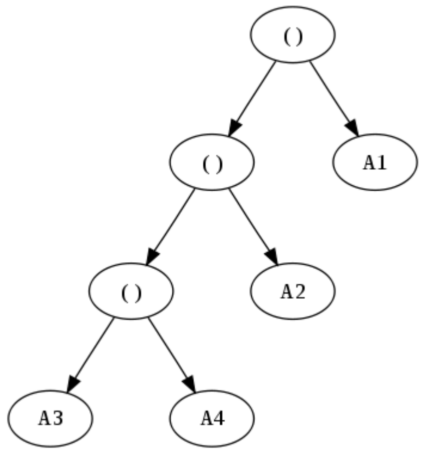

Figure 1: Multiplication tree for $(A1(A2(A3A4)))$

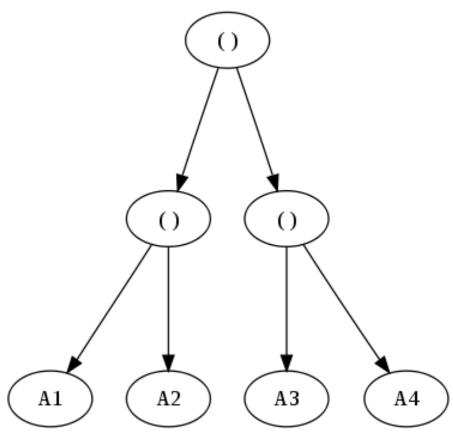

Figure 2: Multiplication tree for $((A1A2)(A3A4))$