CLRS 15.2-4

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Since in Matrix-Chain-Order the i-loop runs $n-l+1$ times for every l; and, for every i, the k-loop runs $j-i=i+l-1-i=j-l$ times; and, for every k, two $m[i,j]$s are looked up; it gelds the following:

$$\begin{aligned}
\sum_{i=1}^{n}\sum_{j=i}^{n} &= \sum_{l=2}^{n} 2(n-l+1)(l-1) && (1)\\
&= 2\sum_{l=1}^{n-1}(n-l)l && (2)\\
&= 2\sum_{l=1}^{n-1} nl - 2\sum_{l=1}^{n-1} l^2 && (3)\\
&= \frac{2n(n-1)n}{2} - \frac{2(n-1)n(2n-1)}{6} && (4)\\
&= \frac{n^3-n}{3} && (5)
\end{aligned}$$