

## CLRS 15.2-5

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Full parenthesization maintains the multiplication tree property (see figures 1 and 2 *infra*); *id est*, that a tree be a full binary tree where:

1. leaves are matrices and only matrices;
2. every parenthesis has two and only two children.

Let  $P(e) = n$  state that a fully parenthesized expression of  $e$  elements requires  $n$  pairs of parentheses.

**Basis:**  $P(1) = 0$

**Hypothesis:**  $P(k) = k - 1$

***Praebendum:***  $P(k + 1) = k$

To insert a matrix into a multiplication tree of  $k$  matrices, resulting in a multiplication tree of  $k + 1$  matrices and while maintaining the multiplication tree property; the matrix must be added at a leaf by parenthesizing the leaf and adding itself and the new node as children, resulting in  $k$  pairs of parentheses.

Therefore:

$$(P(0) \wedge (P(k) \rightarrow P(k + 1))) \rightarrow \forall n P(n)$$

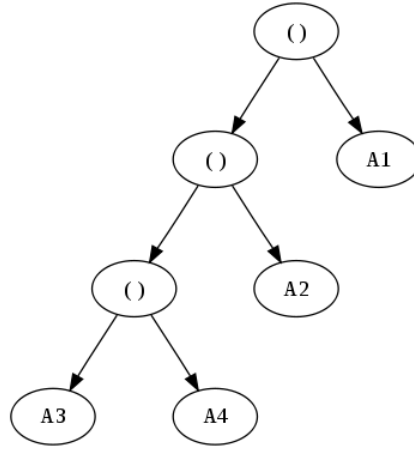


Figure 1: Multiplication tree for  $(A1(A2(A3A4)))$

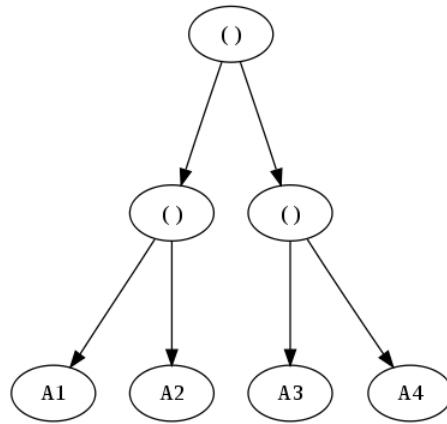


Figure 2: Multiplication tree for  $((A1A2)(A3A4))$