Mohamed Rahouti

Gradient Descent for Linear Regression with One Variable

- Linear Regression with One Variable
    - The Hypothesis Function
    - Cost Function
    - Gradient Descent
- How it works
    - Create demo data
    - Run Gradient Descent
    - Now we can make predictions
    - Predict with **predict.lm**

The Hypothesis Function

Hypothesis function in general form: $h_\theta(x) = \theta_0 + \theta_1 x$

Example:

Given: first row from training set $x_1 = 2, y_1 = 10$

Now we can randomly iterate through θ_0 and θ_1.

So that h_θ for $\theta_0 = 3$ and $\theta_1 = 5$ becomes:

$h_\theta(x) = 3 + 5x,$

and for given $x_1 = 2$ our $h_\theta = 13$. It's greater by 3 from y_1.

Same example with R:

Hypothesis function is represented as: $h_\theta(x) = \theta_0 x_0 + \theta_1 x_1$, where $x_0 = 1$ all times for matrix multiplication.

```
theta <- c(3, 5)
x1 <- c(1, 2) # add 1 as x_0
# find h_theta
h <- x1 %*% theta
print(paste("h =", h[1]))
```

```
## [1] "h = 13"
```

```
# or we can calculate h for all training set at once
x <- matrix(c(rep(1, 3), c(10,20,30)), ncol=2)
x
```

```
##      [,1] [,2]
## [1,]    1   10
## [2,]    1   20
## [3,]    1   30
```

```
x %*% theta
```

```
##      [,1]
## [1,]   53
## [2,]  103
## [3,]  153
```

The Cost Function

This function is also known as “Squared error function”, or “Mean squared error”.

$$J(\theta_0, \theta_1) = \frac{1}{2m} \sum_{i=1}^{m} (h_\theta(x_i) - y_i)^2$$

```
computeCost <- function (X, y, theta){
    # number of training examples
    m <- length(y);
    # need to return
    J <- 0;

    predictions <-  X %*% theta;
    sqerrors = (predictions - y)^2;
    J = 1/(2*m)* sum(sqerrors);

    J
}
```

Example:

Given:

$x_1 = 2, y_1 = 10, m = 1$

$\theta_0 = 3, \theta_1 = 5$:

$h_\theta(x) = 3 + 5x,$

for given $x_1 = 2$, our $h_\theta = 13$.

It’s greater by 3 from y_1.

$$J(3,5) = \frac{1}{2*1} \sum_{i=1}^{1} (13 - 10)^2 = \frac{9}{2} = 4.5$$

```
theta
```

```
## [1] 3 5
```

```
x1
```

```
## [1] 1 2
```

```
print(paste("J =", computeCost(x1, 10, theta)))
```

```
## [1] "J = 4.5"
```

The Gradient Descent Method

Repeat until convergence:

$$\{$$

$$\theta_0 := \theta_0 - \alpha \frac{1}{m} \sum_{i=1}^{m} (h_\theta(x_i) - y_i)$$

$$\theta_1 := \theta_1 - \alpha \frac{1}{m} \sum_{i=1}^{m} ((h_\theta(x_i) - y_i) x_i)$$

$$\}$$

Where:

m – size of training set,

$theta_0, theta_1$ – values to change **simultaneously**,

x_i, y_i – items of training set,

α – step size.

```
gradientDescent <- function(X, y, theta, alpha, num_iters){
    m <- length(y);
    J_history = rep(0, num_iters);
    for (iter in 1:num_iters){
        predictions <-  X %*% theta;
        updates = t(X) %*% (predictions - y);
        theta = theta - alpha * (1/m) * updates;
        J_history[iter] <- computeCost(X, y, theta);
    }
    list("theta" = theta, "J_history" = J_history)
}
```

Example:

```
theta <- c(0, 0)
iterations <- 1500
alpha <- 0.01
X <- matrix(c(1, 1, 3, 3), ncol=2)
y <- matrix(c(10, 10), ncol=1)
# answer must be 1,3
# h(1, 3) = 1 + 3*x = 1 + 3*3 = 10
result <- gradientDescent(X, y, theta, alpha, iterations)
result$theta
```

```
##       [,1]
## [1,]     1
## [2,]     3
```

How Does it Work?

1. Upload Data

For this demo, we are going to use existing data in R, called **mtcars.**

> *> attach (mtcars)*
> *> mtcars # to visualize this data*

Declare two variables X and y and initialize them to mtcars$cyl and mtcars$mpg, respectively. Herein, our X variable will reflect observations of car cylinder in the column 'cyl' of 'mtcars' dataset, while y will represent observations about gas consumption (mile per gallon) specified by the column 'mpg' in the mtcars datset.

```
> x <- mtcars$cyl

> y <- mtcars$mpg

> X <- cbind(1, matrix(x))
```

Let's call our previously implemented function 'gradientDescent' to find the optimal value for our cost function.

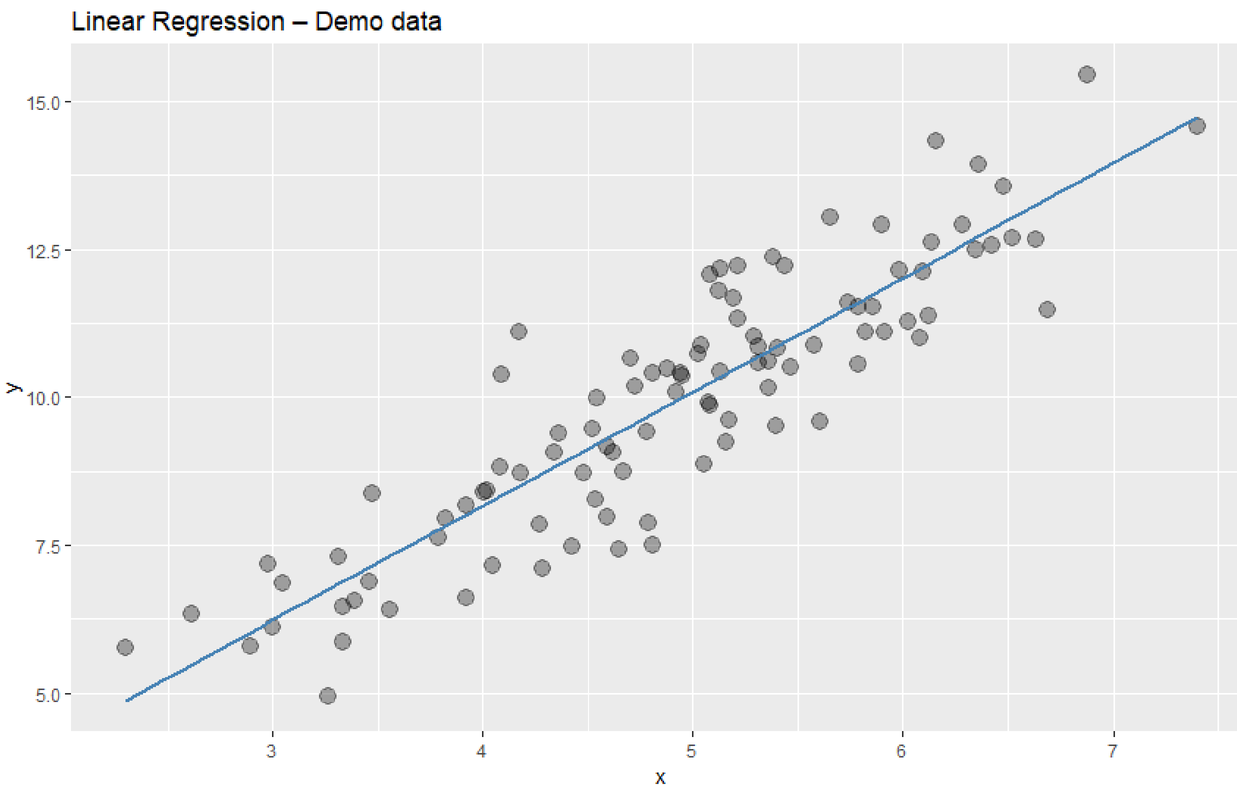
```
> result <- gradientDescent(X, y, theta, alpha, iterations)

> result$theta
```

```
> result <- gradientDescent(X, y, theta, alpha, iterations)
> result$theta
          [,1]
[1,] -102.35505
[2,]   54.47107
>
```

2. Let's Create Demo Data & Compare LR with our Gradient Descent Algorithm

```
library(ggplot2)

set.seed(37)
x <- rnorm(n=100, mean=5, sd=1)
e <- rnorm(n=100, mean=0, sd=1)
y <- e + 2*x

# show data
data = data.frame(x=x, y=y)
g <- ggplot(data, aes(x=x, y=y))  +
    geom_point(alpha=1/3, size=4) +
    geom_smooth(method="lm", se=F, col="steelblue") +
    labs(title = "Linear Regression - Demo data")
```

```
g
```




```
# Add a column of ones to x
X <- matrix(c(rep(1,length(x)),x), ncol=2)
head(X)
```

```
     [,1]     [,2]
[1,]    1 5.124754
[2,]    1 5.382075
[3,]    1 5.579243
[4,]    1 4.706252
[5,]    1 4.171651
[6,]    1 4.667286
> |
```

3. Run the Gradient Descent

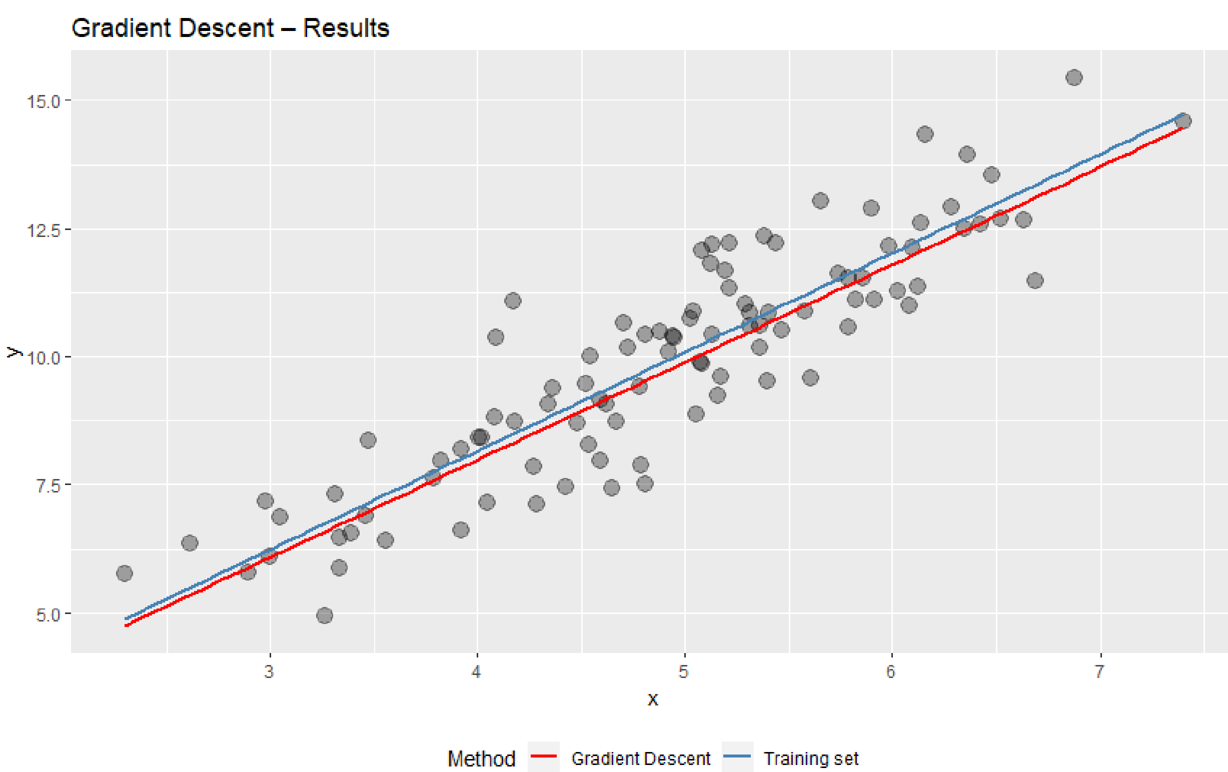
Now let's initialize Gradient Descent parameters and execute function.

```
 # Initialize
theta <- c(0, 0)
iterations <- 1500
# to be precise pick alpha=0.0002
alpha <- 0.0001 # for difference on plot

# run gradient descent
result <- gradientDescent(X, y, theta, alpha, iterations);
theta <- result$theta
print("theta found:");print(theta)
```

```
[1] "theta found:"
          [,1]
[1,] 0.3725434
[2,] 1.9043863
>
```

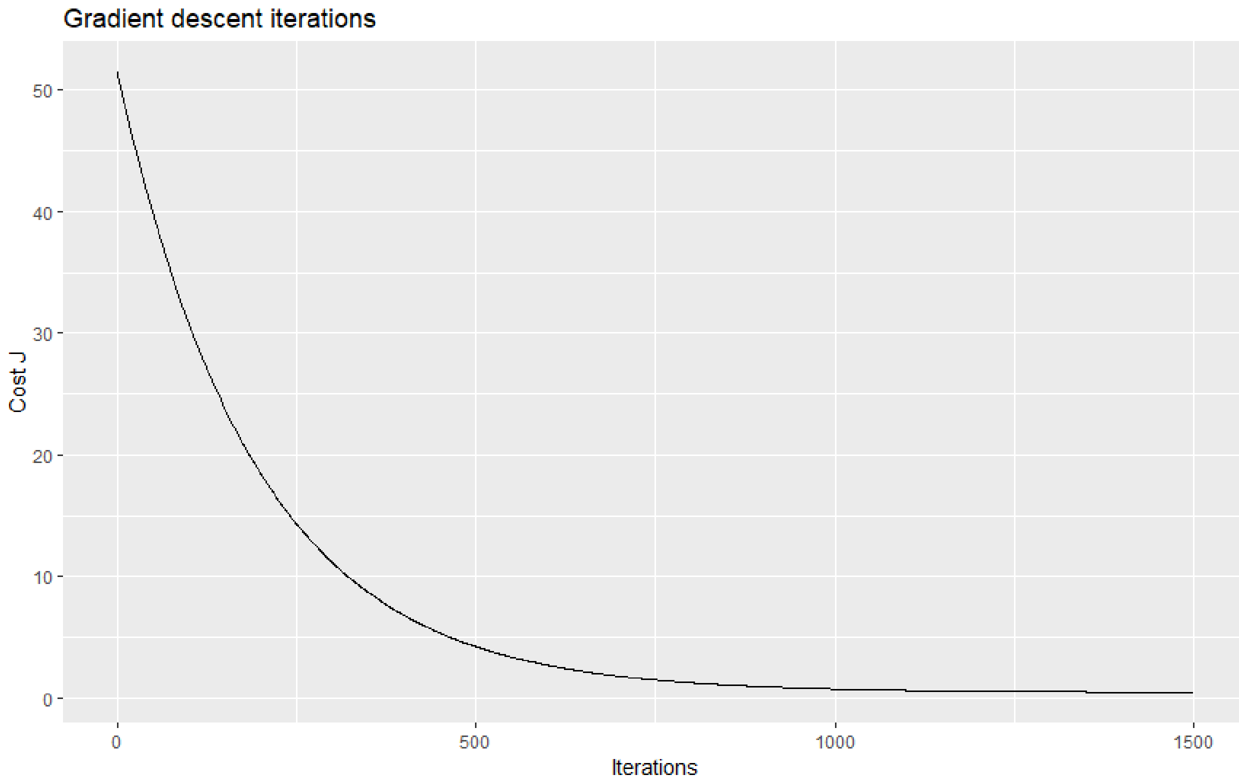
Let's show new line based on found theta.

```
# data with prediction
data = data.frame(x=x, y=y, test = X%*%theta)


ggplot(data, aes(x=x, y=y, test=test))  +
    geom_point(alpha=1/3, size=4) +
    stat_smooth(method = "lm", formula = test ~ x, size = 1, se = FALSE,
                aes(color="Gradient Descent")) +
    geom_smooth(method="lm", se=F, aes(color="Training set")) +
    scale_colour_manual(name="Method", values=c("red", "steelblue")) +
    theme(legend.position = "bottom") +
    labs(title = "Gradient Descent - Results")
```



History of executed cost functions stored in **result$J_history**.

```
data <- data.frame(x=seq(1, length(result$J_history)),
                   y=result$J_history)
ggplot(data, aes(x=x, y=y)) +
    geom_line() +
    labs(title="Gradient descent iterations",
         x="Iterations", y="Cost J")
```



4. Now, we can make predictions

```
predict1 <- c(1, 3.5) %*% theta
predict2 <- c(1, 7) %*% theta
```

```
> predict1
         [,1]
[1,] 7.037895
> predict2
         [,1]
[1,] 13.70325
>
```

Run again Gradient Descent with $\alpha = 0.0002$ (more precise) to compare:

```
theta <- c(0, 0)
iterations <- 1500
alpha <- 0.0002 # set alpha more precisely
result <- gradientDescent(X, y, theta, alpha, iterations);
matrix(c(1, 1, 3.5, 7), ncol=2) %*% result$theta
```

```
          [,1]
[1,]  7.177233
[2,] 13.974120
>
```

Finally, make prediction with **lm**:

```
lm <- lm(y ~ x)
newdata <- data.frame(x=c(3.5, 7))
predict(lm, newdata, interval="none")
```

```
       1         2
7.202689 13.953160
> |
```

Values are amazingly close. Good job!

Lab credit:

- 1
- 2
- 3