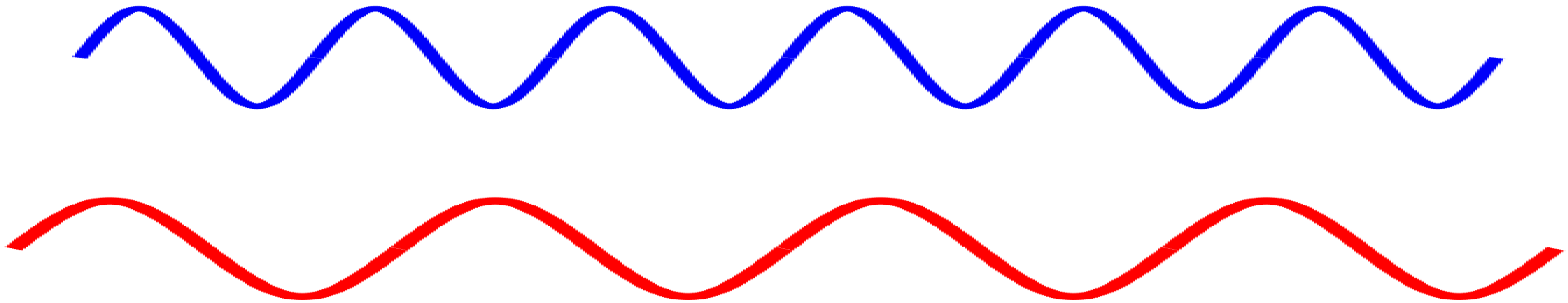
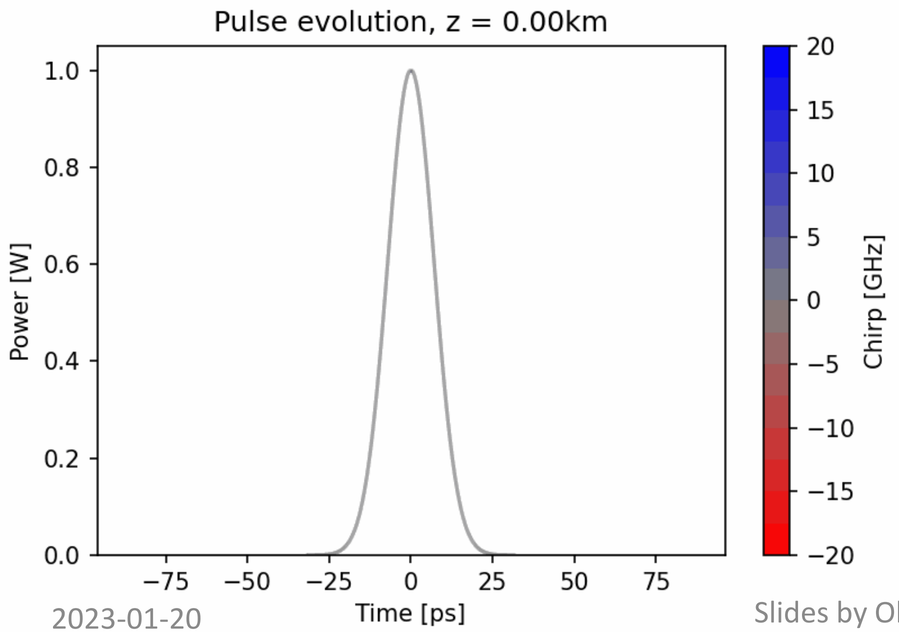
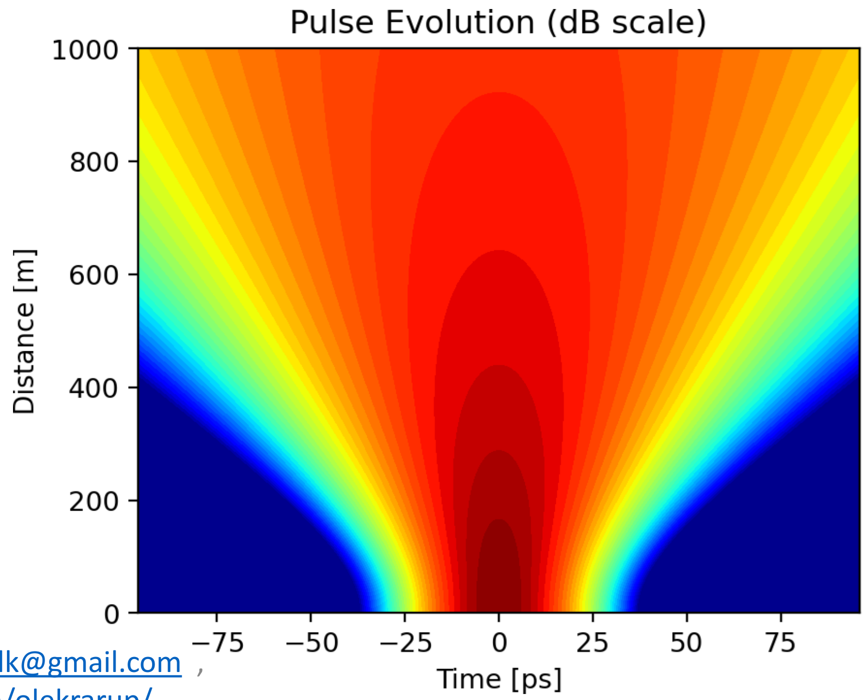


Understanding dispersion





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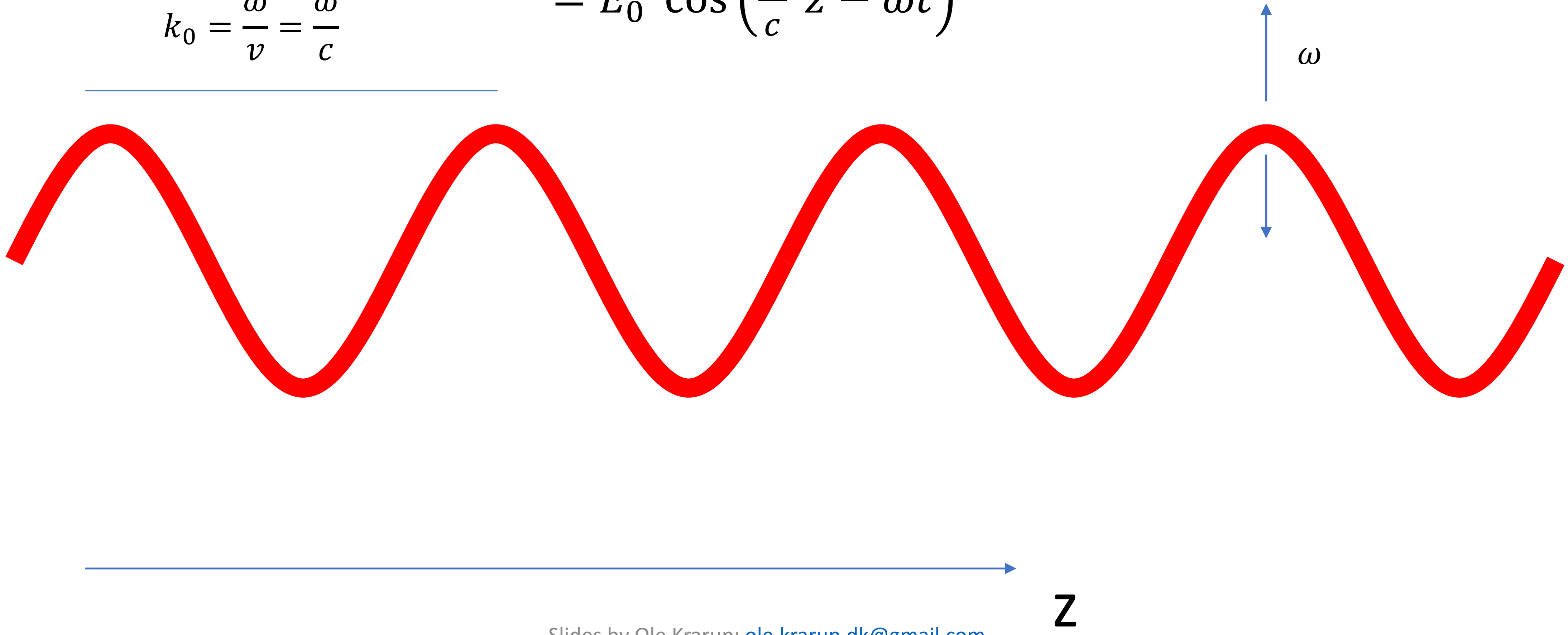
EM wave in vacuum

$$\vec{E} = \overrightarrow{E_0}\cos(k_0 z - \omega t)$$
$$= \overrightarrow{E_0}\cos\left(\frac{\omega}{c}\,z - \omega t\right)$$

$$k_0 = \frac{\omega}{v} = \frac{\omega}{c}$$



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EM wave in bulk material

P. Floyd (1973)

$$\vec{E} = \overrightarrow{E_0}\cos(\beta z - \omega t)$$

$$\beta = \frac{\omega}{v_p} = \frac{\omega}{c/n(\omega)} = \frac{\omega}{c} n(\omega)$$

ω ←unchaged!

EM waves in bulk material

Normal dispersion!!!

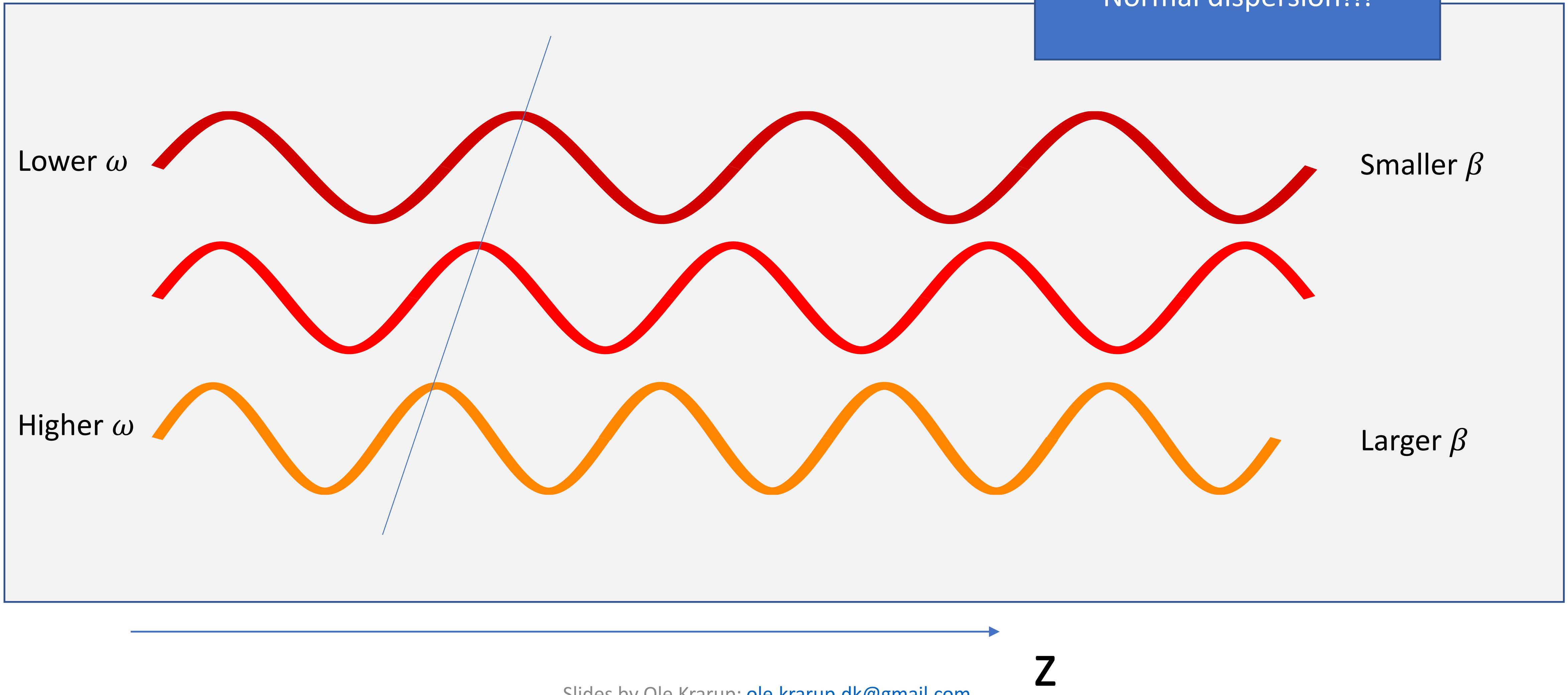

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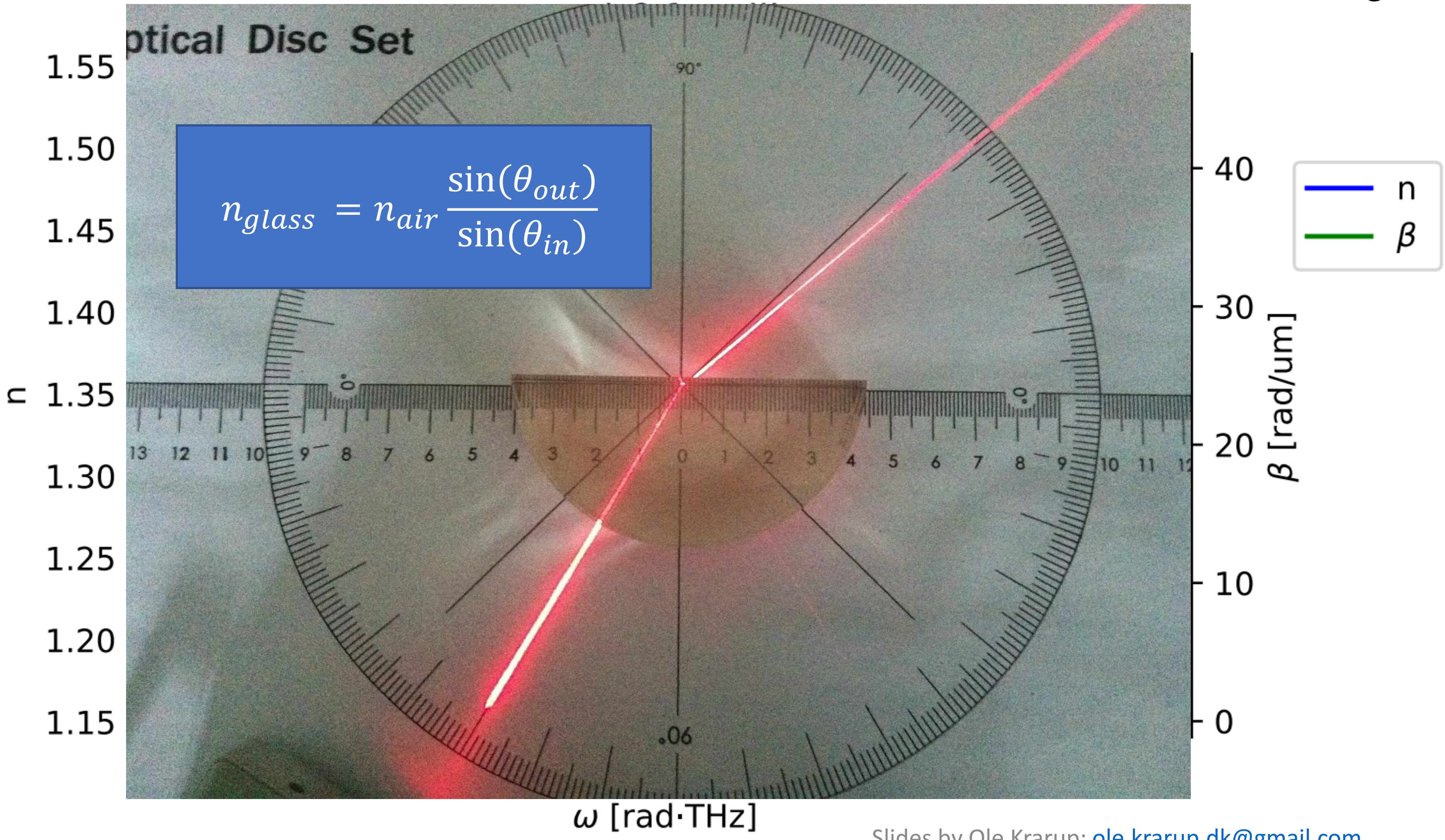
Chart of $n(\omega)$ and $\beta(\omega)$

$$\beta = \frac{\omega}{c} n(\omega)$$



$$n_{glass} = n_{air} \frac{\sin(\theta_{out})}{\sin(\theta_{in})}$$

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Chart of $n(f)$ and $\beta(f)$

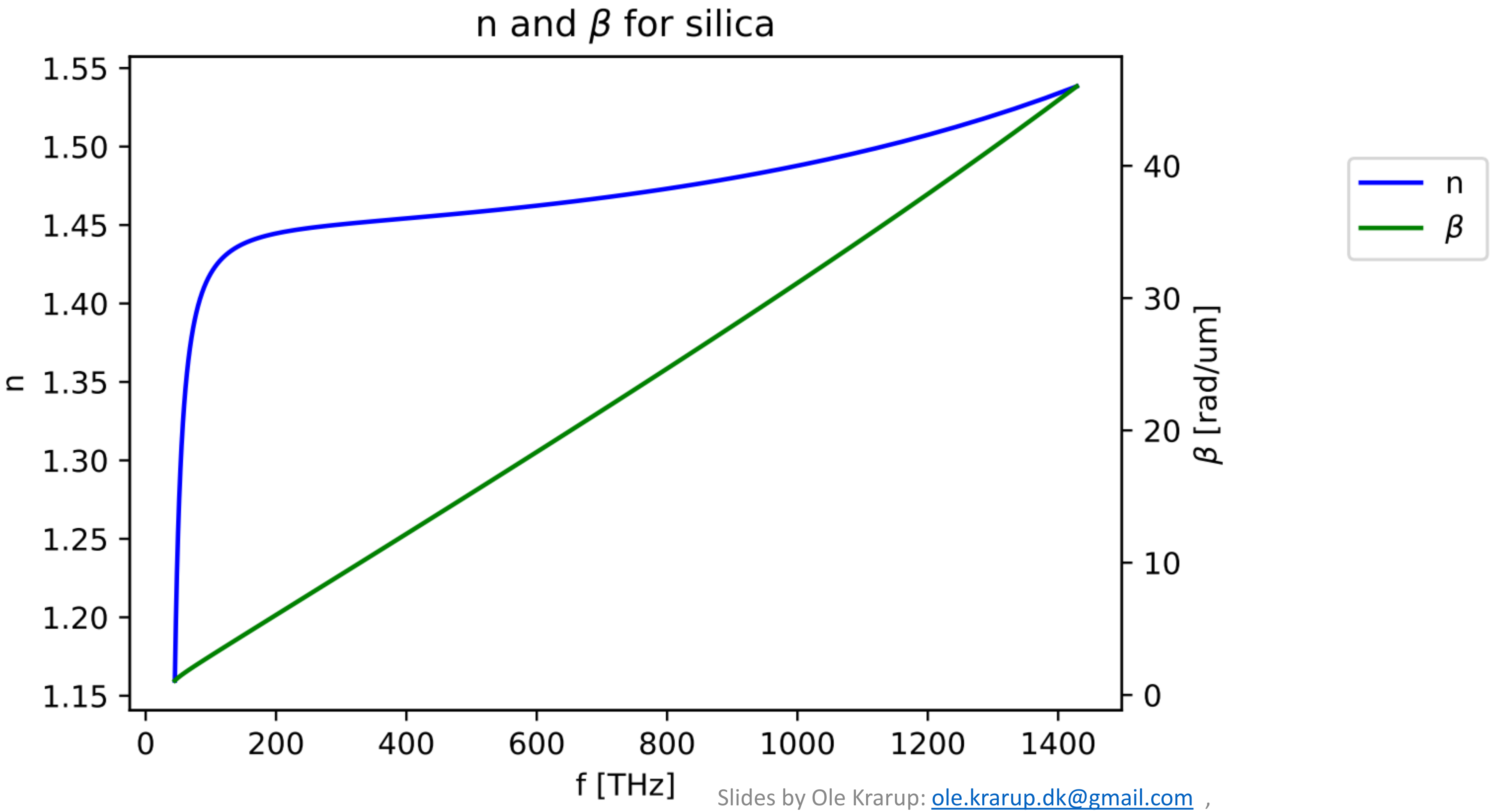

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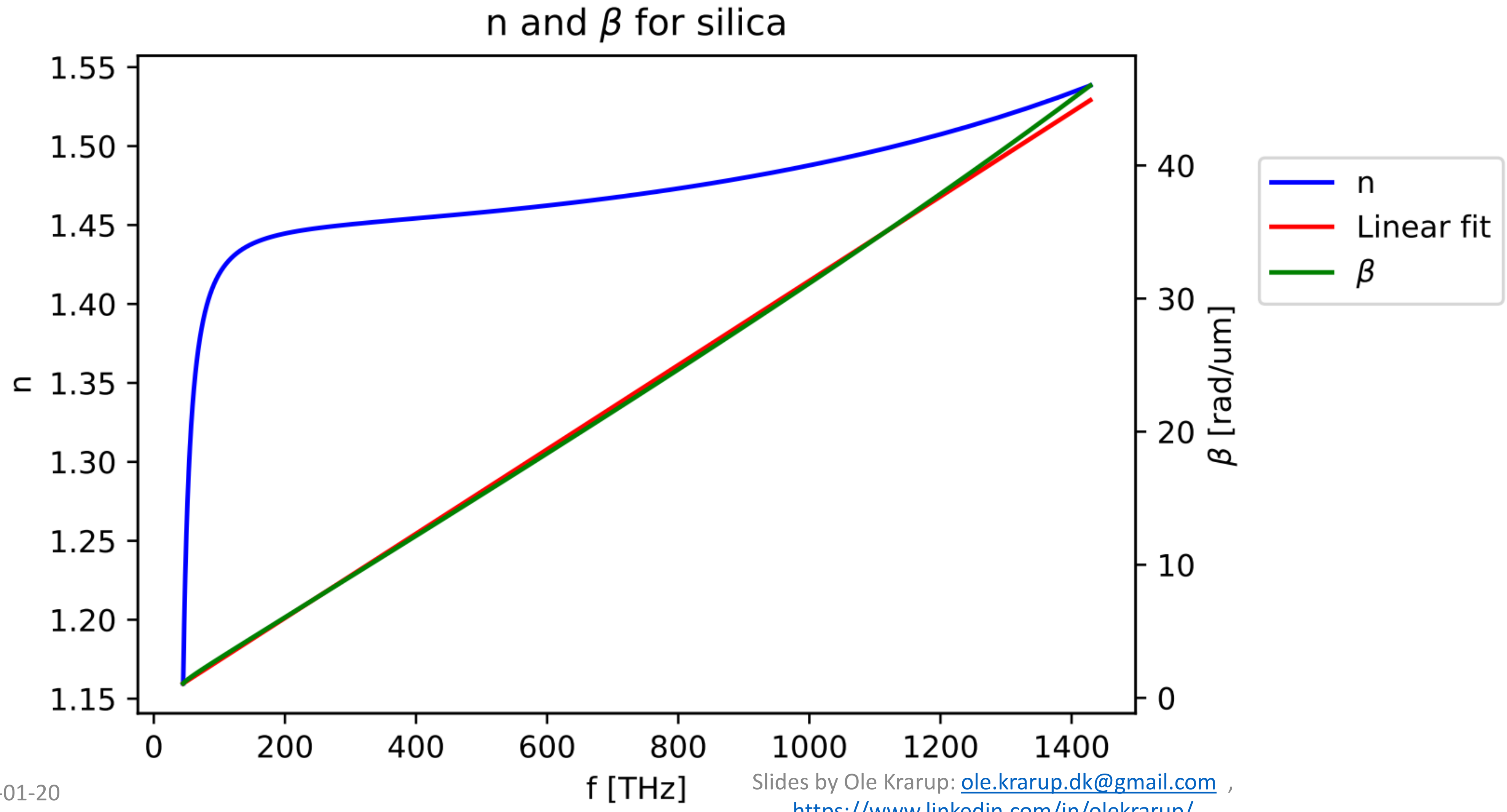
Chart of $n(f)$ and $\beta(f)$



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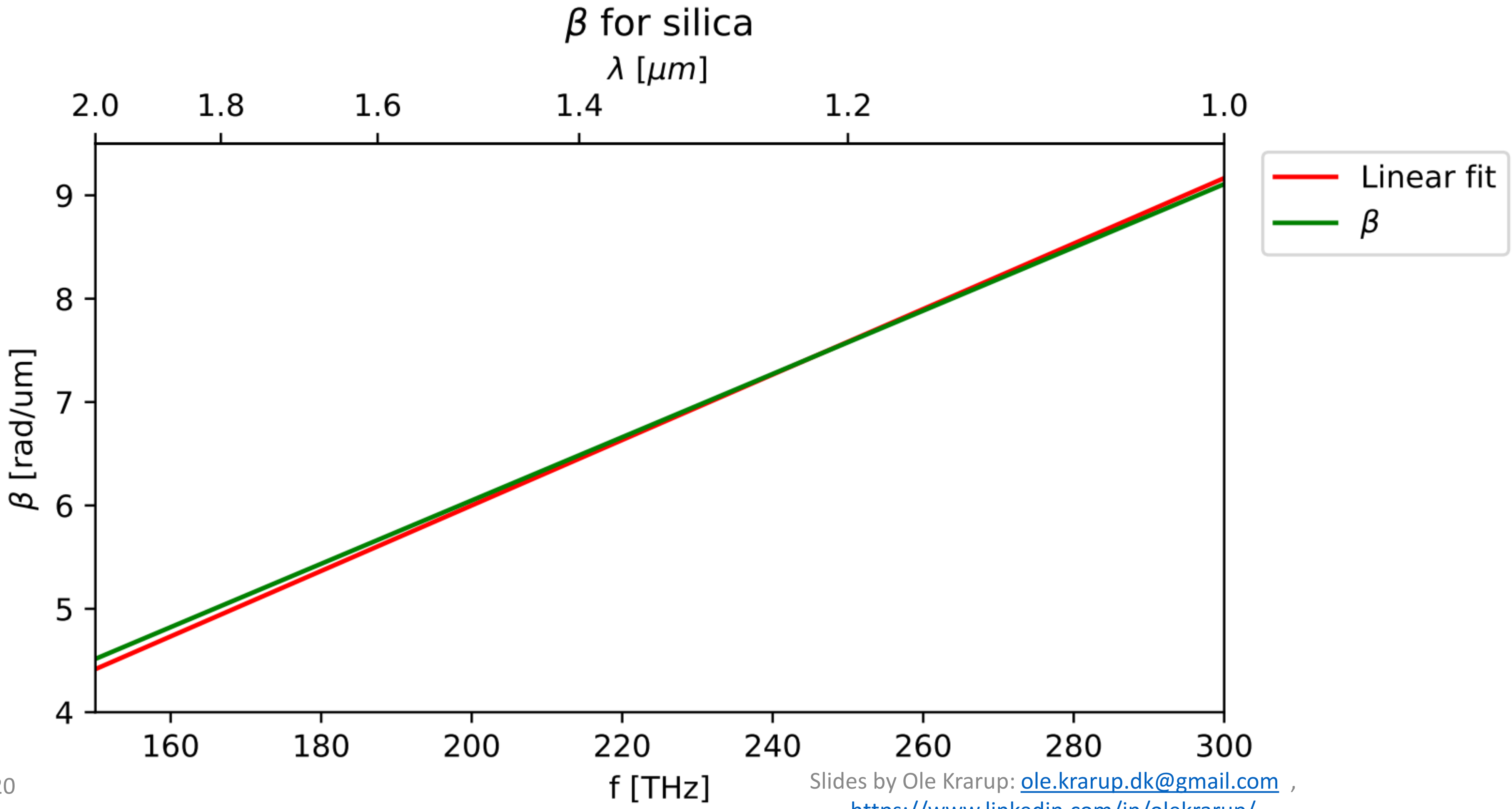
Zoom on NIR range



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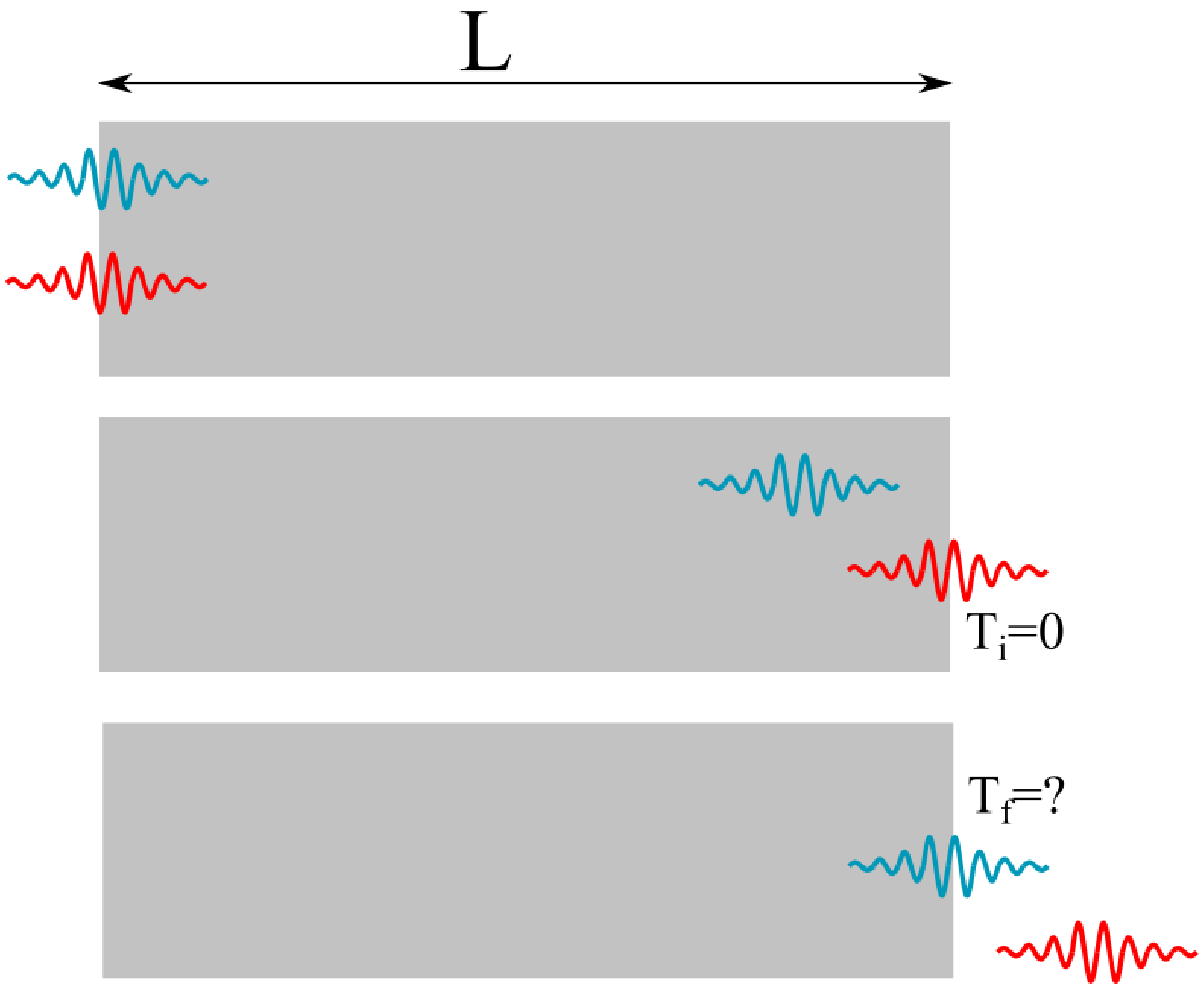
Difference in arrival time?

$$\Delta T = \frac{L}{v} - \frac{L}{v} = L\left(\frac{1}{v} - \frac{1}{v}\right)$$



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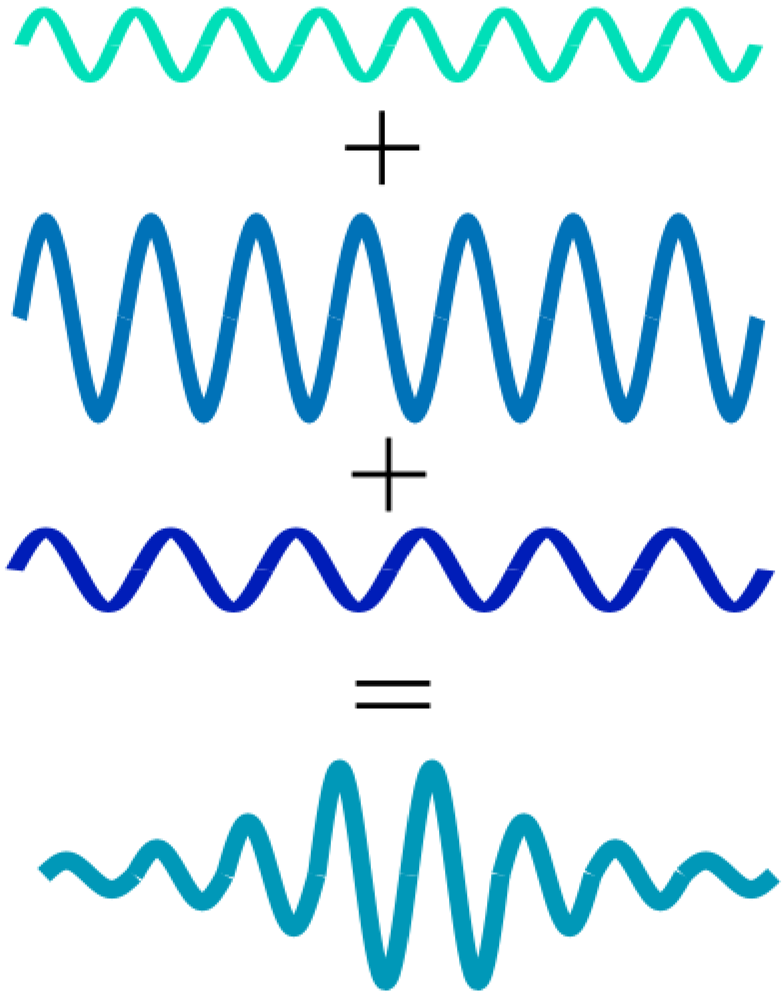
What is a “pulse”?

$$E(z,t) = \int_{-\infty}^{\infty} \hat{E}(\omega) e^{i(\beta(\omega)z-\omega t)} \, d\omega$$



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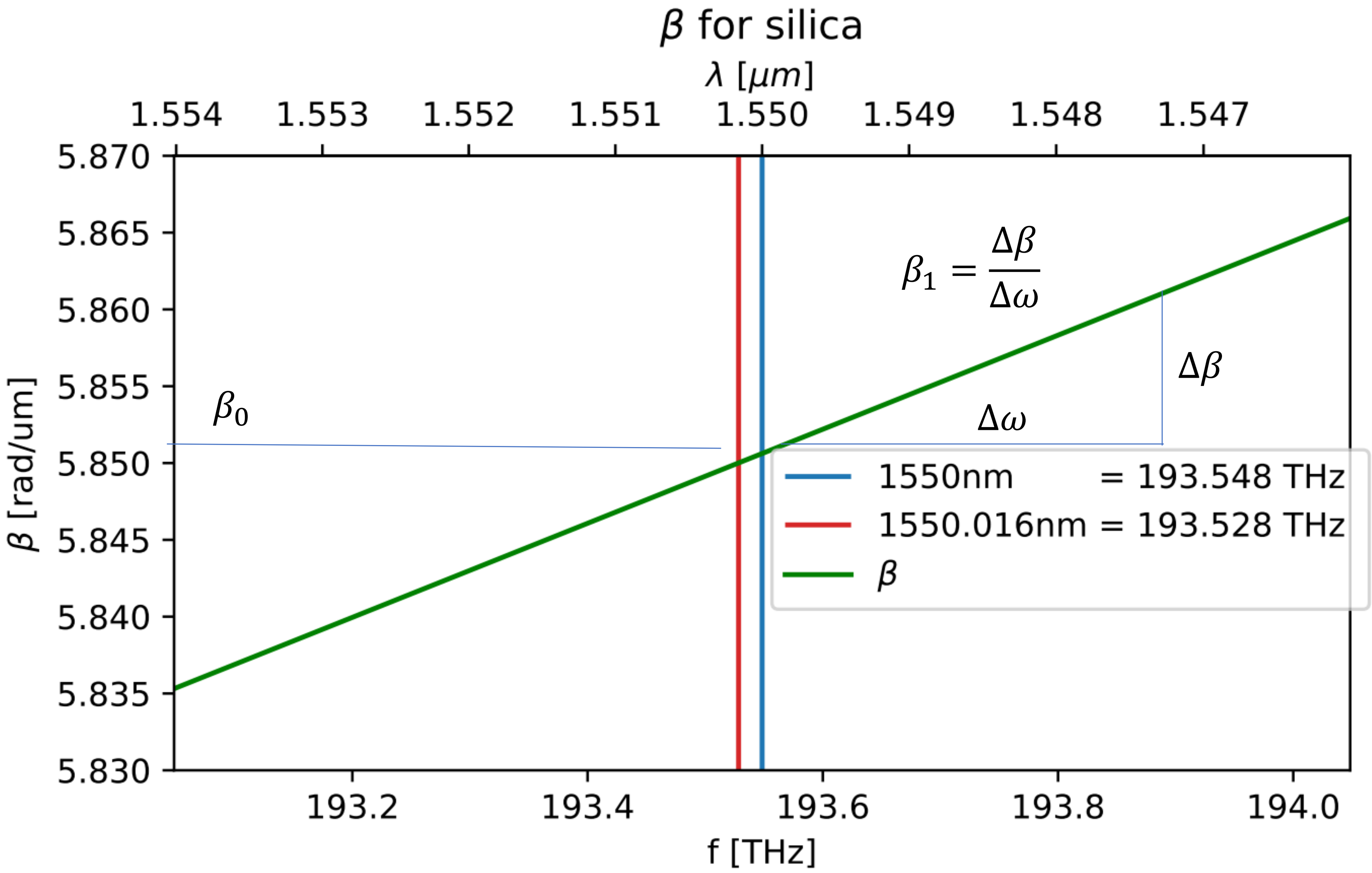

$$\beta(\omega) = \beta_0 + \beta_1(\omega_0)(\omega - \omega_0) + \cdots (higher\ order\ terms)$$

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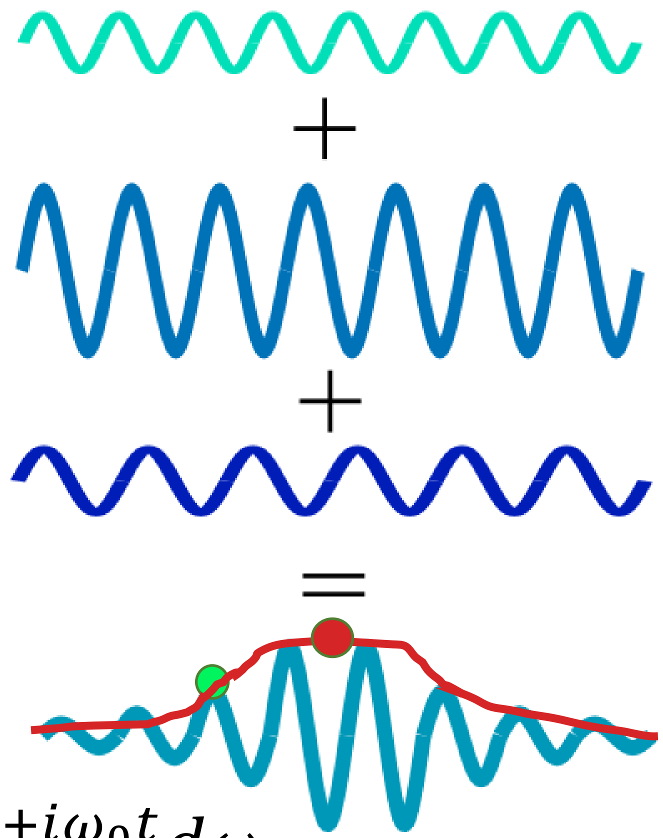
Use approximation of $\beta(\omega)$



$$E(z,t) \approx \int_{-\infty}^{\infty} \hat{E}(\omega) e^{i([\beta_0(\omega_0)+\beta_1(\omega_0)(\omega-\omega_0)]z-\omega t)}\, d\omega$$

$$E(z,t) \approx e^{i\beta_0(\omega_0)z} \int_{-\infty}^{\infty} \hat{E}(\omega) e^{i(\beta_1(\omega_0)[\omega-\omega_0]z-\omega t)}\, d\omega$$

$$E(z,t) \approx e^{i\beta_0(\omega_0)z} e^{-i\omega_0 t} \int_{-\infty}^{\infty} \hat{E}(\omega) e^{i(\beta_1(\omega_0)[\omega-\omega_0]z-\omega t)} e^{+i\omega_0 t}\, d\omega$$

$$E(z,t) \approx e^{i(\underline{\beta_0(\omega_0)z-\omega_0 t})} \int_{-\infty}^{\infty} \hat{E}(\omega) e^{i(\underline{\beta_1(\omega_0)[\omega-\omega_0]z-(\omega-\omega_0)t})}\, d\omega$$

How does a "phase peak" move?

$$v_{phase} = \frac{\omega_0}{\beta_0}$$

How does the "Envelope peak" move?

$$v_{group} = \frac{(\omega-\omega_0)}{\beta_1(\omega_0)[\omega-\omega_0]} = \frac{1}{\beta_1(\omega_0)}$$

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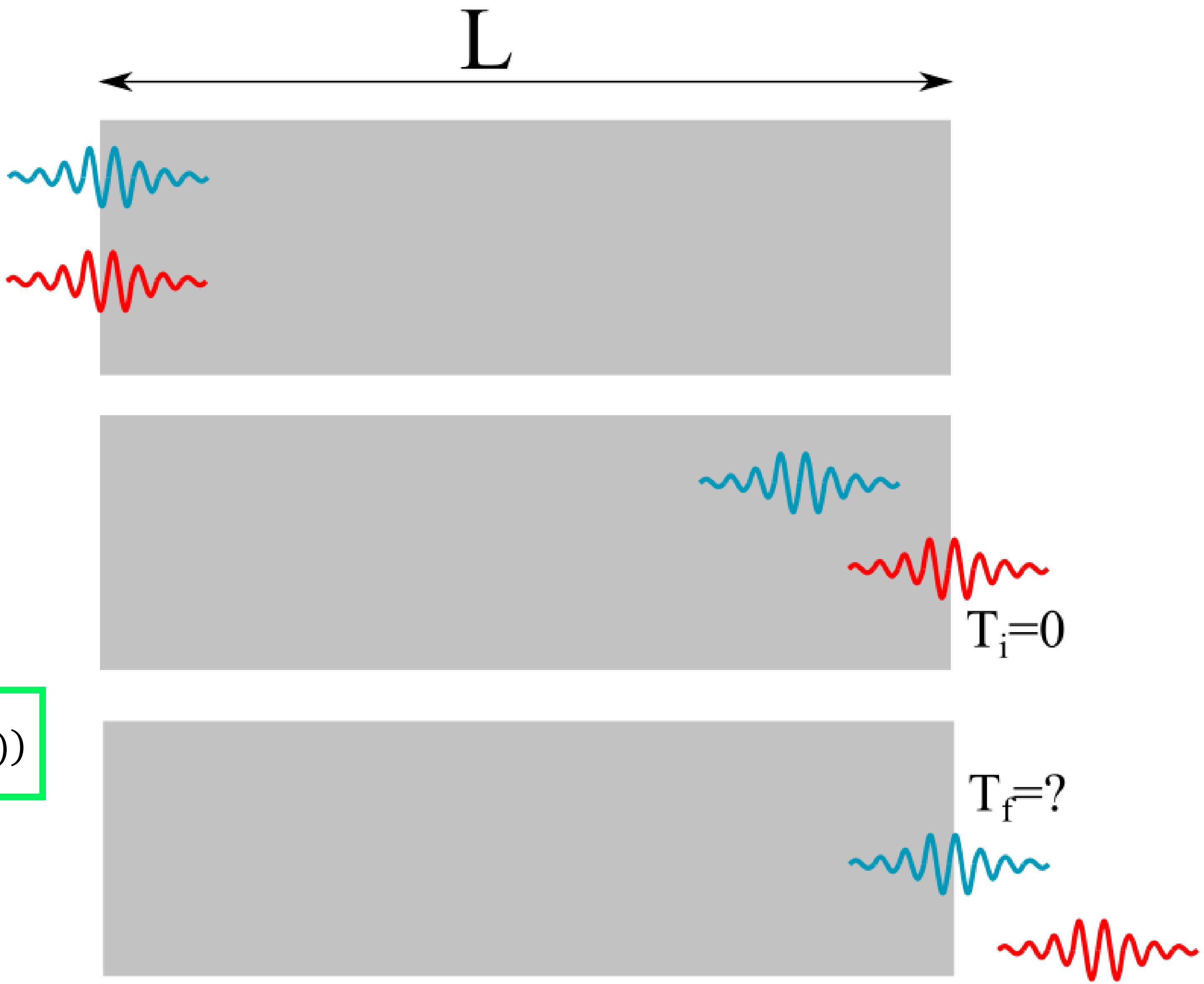
Difference in arrival time?

$$\Delta T = \frac{L}{\color{blue}{v}} - \frac{L}{\color{red}{v}} = \boxed{L(\beta_1(\color{blue}{\omega}) - \beta_1(\color{red}{\omega}))}$$



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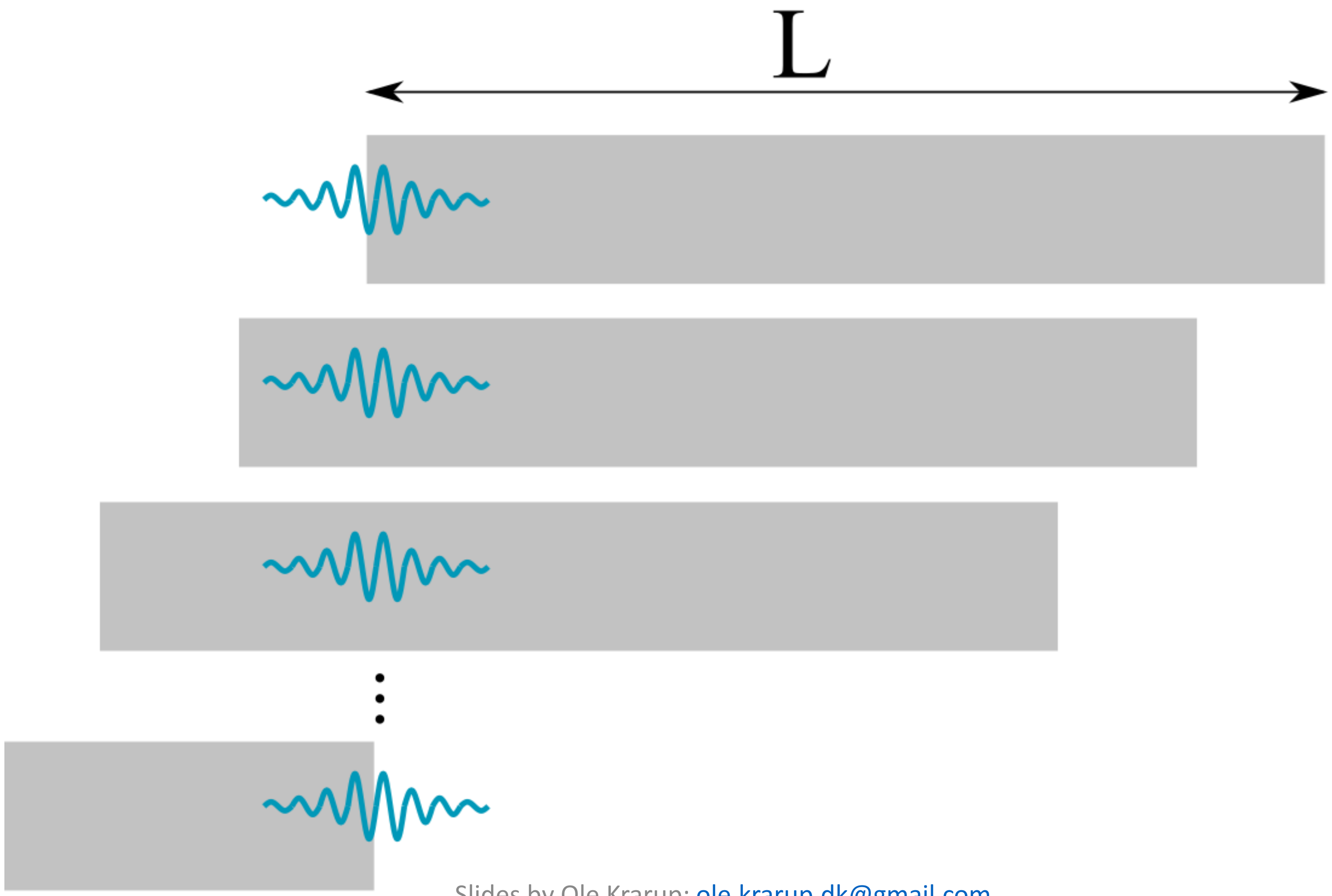
What if we “follow” a pulse?



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What if we “follow” a pulse?

Note: From now on, it’s implicit that the value of every coefficient in the Taylor expansion of $\beta(\omega)$ depends on the chosen carrier frequency, i.e. $\beta_1 = \beta_1(\omega_0)$, $\beta_2 = \beta_2(\omega_0)$ etc.

$$A(z,t) = E(z,t)e^{-i(\beta_0 z-\omega_0 t)}$$

$$A(z,t) = e^{-i(\beta_0 z-\omega_0 t)} \int_{-\infty}^{\infty} \hat{A}(\omega-\omega_0)e^{i(\beta(\omega)z-\omega t)}d\omega$$

$$A(z,t) = e^{-i(\beta_0 z-\omega_0 t)} \int_{-\infty}^{\infty} \hat{A}(\omega-\omega_0)e^{i\left(\left[\beta_0+\beta_1(\omega-\omega_0)+\frac{1}{2}\beta_2(\omega-\omega_0)^2+\cdots\right]z-\omega t\right)}d\omega$$

$$A(z,t) = e^{-i(\beta_0 z-\omega_0 t)}\, e^{-i\omega_0 t} \int_{-\infty}^{\infty} \hat{A}(\omega-\omega_0)\, -\omega_0 e^{i\left(\left[\beta_0+\beta_1(\omega-\omega_0)+\frac{1}{2}\beta_2(\omega-\omega_0)^2+\cdots\right]z-\omega t\right)}\, e^{+i\omega_0 t}d\omega$$

$$A(z,t) = \int_{-\infty}^{\infty} \hat{A}(\omega-\omega_0)e^{i\left(\left[\beta_1(\omega-\omega_0)+\frac{1}{2}\beta_2(\omega-\omega_0)^2+\cdots\right]z-(\omega-\omega_0)t\right)}d\omega$$

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Change time variable

$$T = t - \beta_1 z$$

$$t = T + \beta_1 z$$

$$A(z,T) = \int_{-\infty}^{\infty} \hat{A}(\omega - \omega_0) e^{i\left(\left[\beta_1(\omega-\omega_0)+\frac{1}{2}\beta_2(\omega-\omega_0)^2+\cdots\right]z-(\omega-\omega_0)[T+\beta_1 z]\right)} d\omega$$

$$A(z,T) = \int_{-\infty}^{\infty} \hat{A}(\omega - \omega_0) e^{i\left(\left[\cancel{\beta_1(\omega-\omega_0)}+\frac{1}{2}\beta_2(\omega-\omega_0)^2+\cdots\right]z-(\omega-\omega_0)T-\cancel{(\omega-\omega_0)\beta_1 z}\right)} d\omega$$

$$A(z,T) = \int_{-\infty}^{\infty} \hat{A}(\omega - \omega_0) e^{i\left(\left[\frac{1}{2}\beta_2(\omega-\omega_0)^2+\cdots\right]z-(\omega-\omega_0)T\right)} d\omega$$

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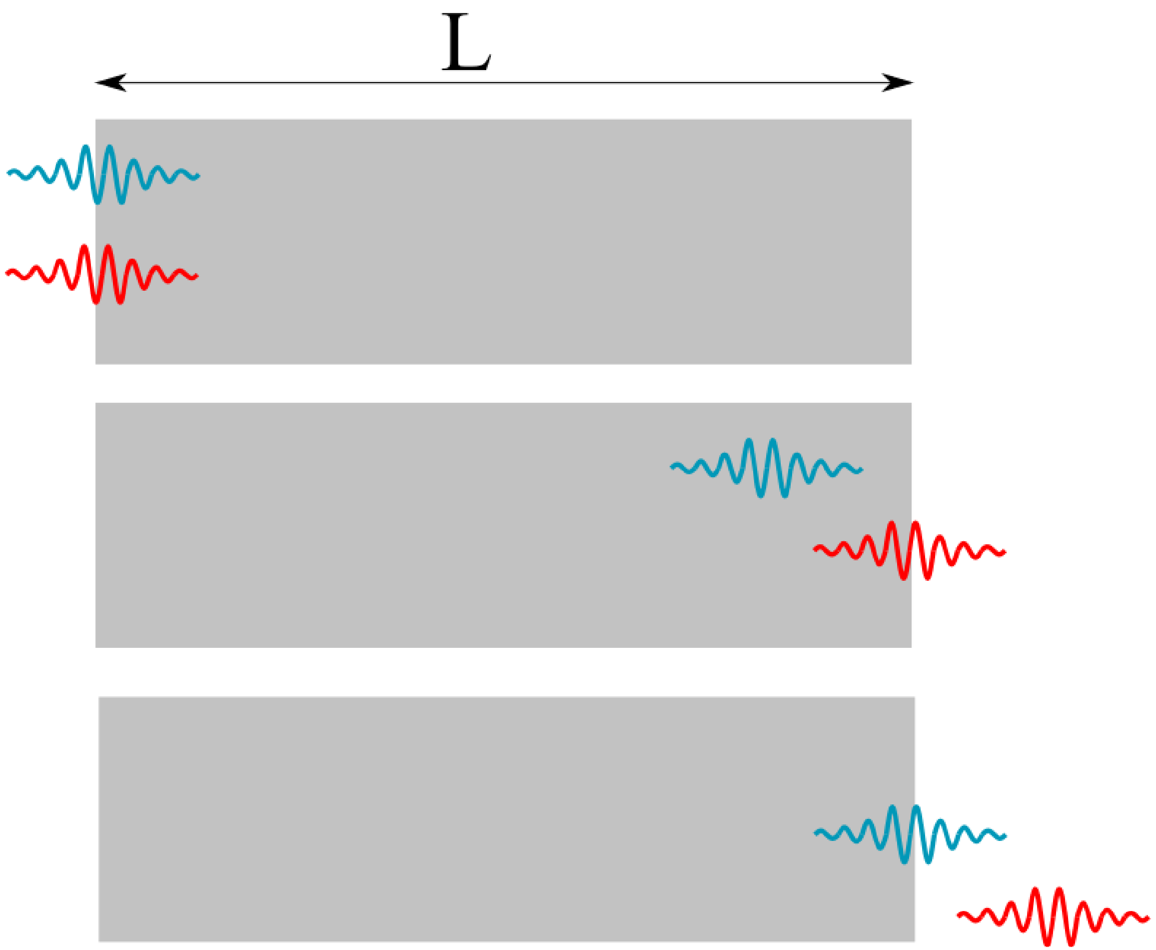
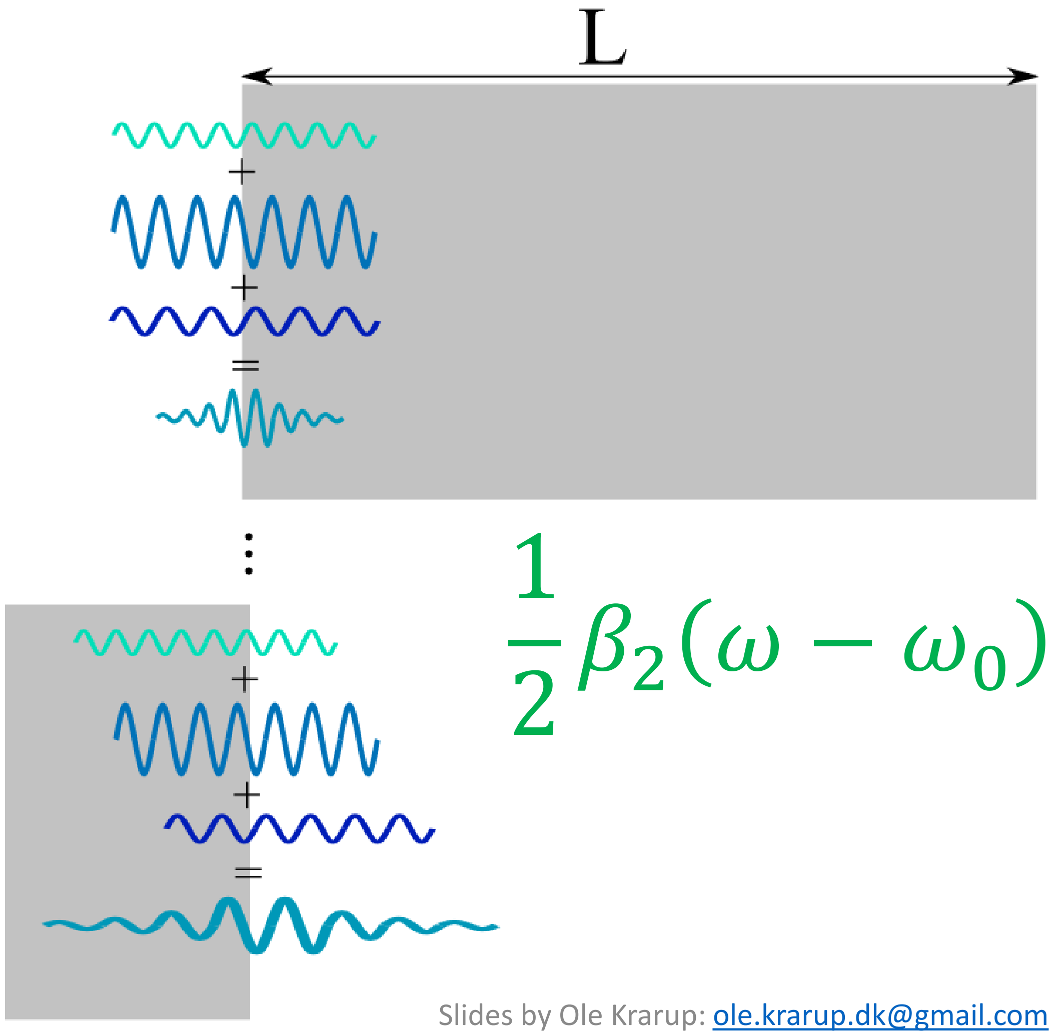
Before: Speed difference between two "frequency packets" centered at different carrier frequencies leads to difference in their arrival times.



Now: Speed difference of different waves within the same "frequency packet" leads to time delay between them. Pulse "spreads out" in time domain!



$$\frac{1}{2}\beta_2(\omega-\omega_0)^2$$

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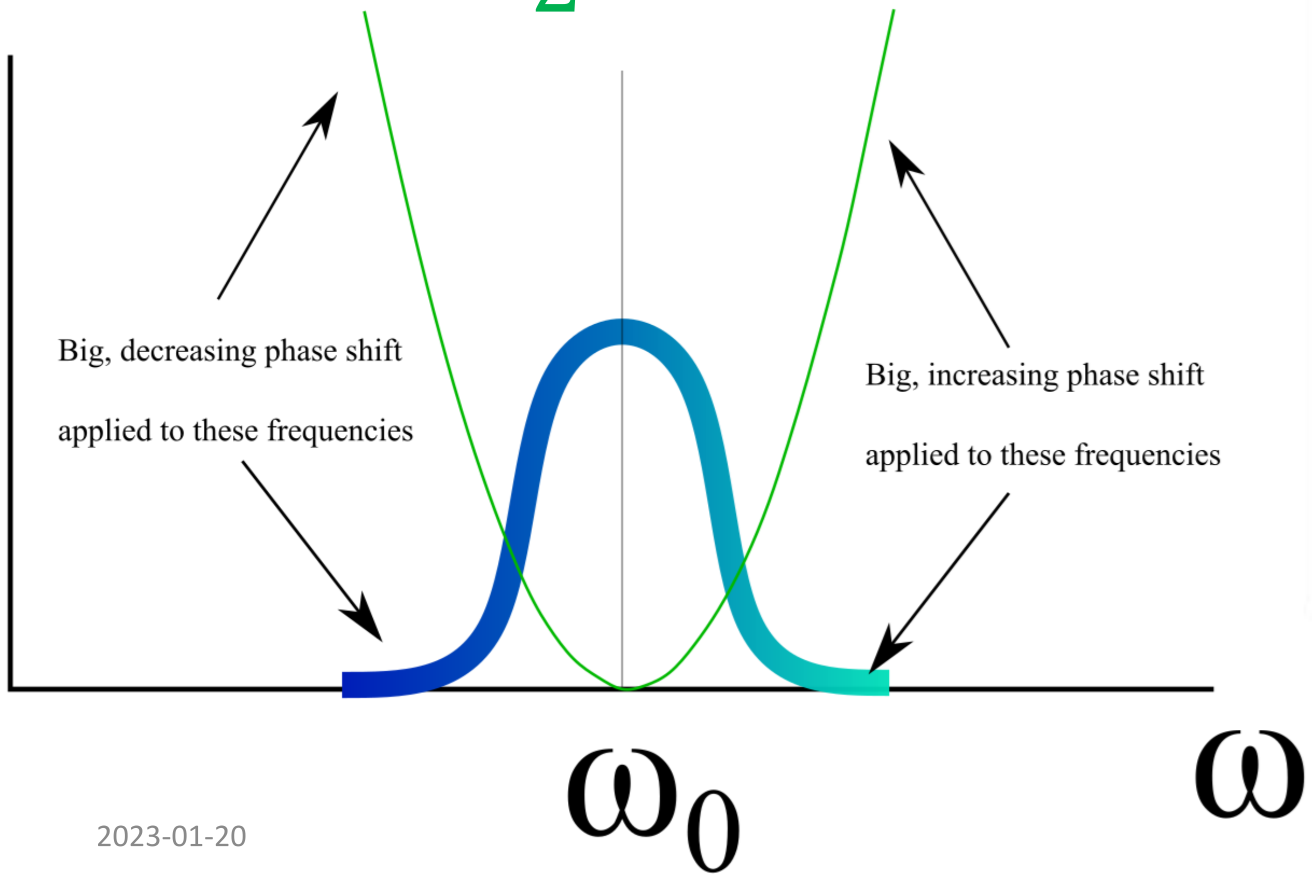
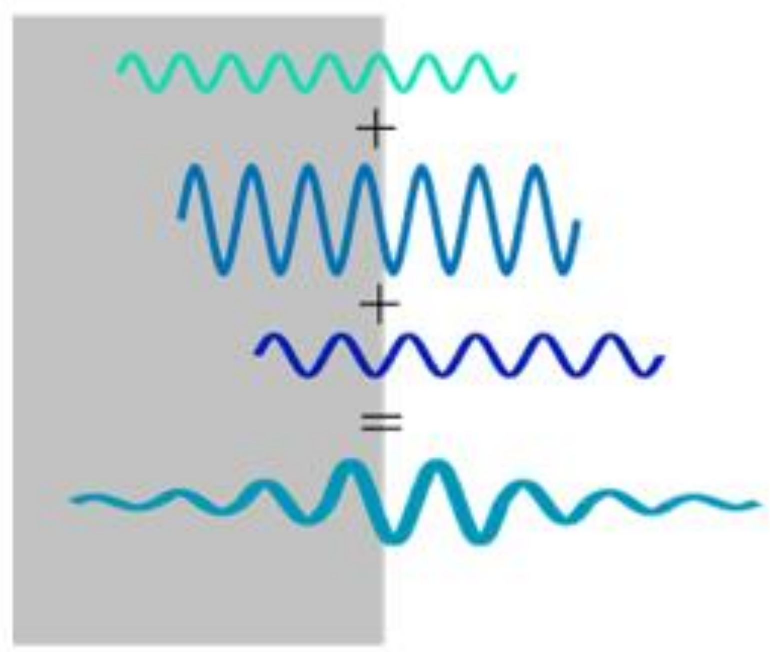
Changing the phase of frequency components

- $\beta_2 > 0$

$$\frac{1}{2}\beta_2(\omega - \omega_0)^2$$





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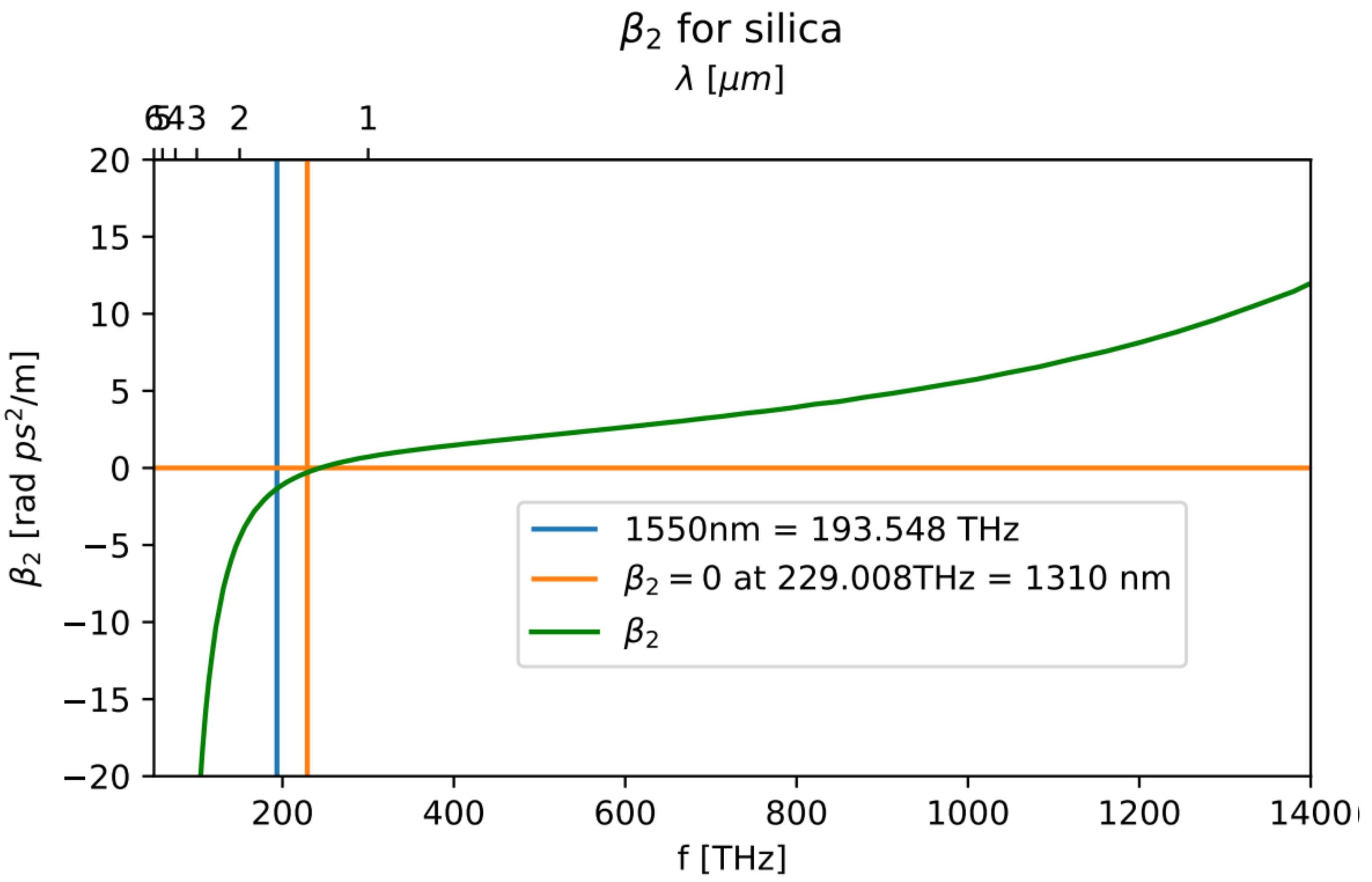
How to find β_2 ?

- Take derivative of $\beta(\omega)$ to find $\beta_1(\omega)$.

- Take derivative of $\beta_1(\omega)$ to find $\beta_2(\omega)$.



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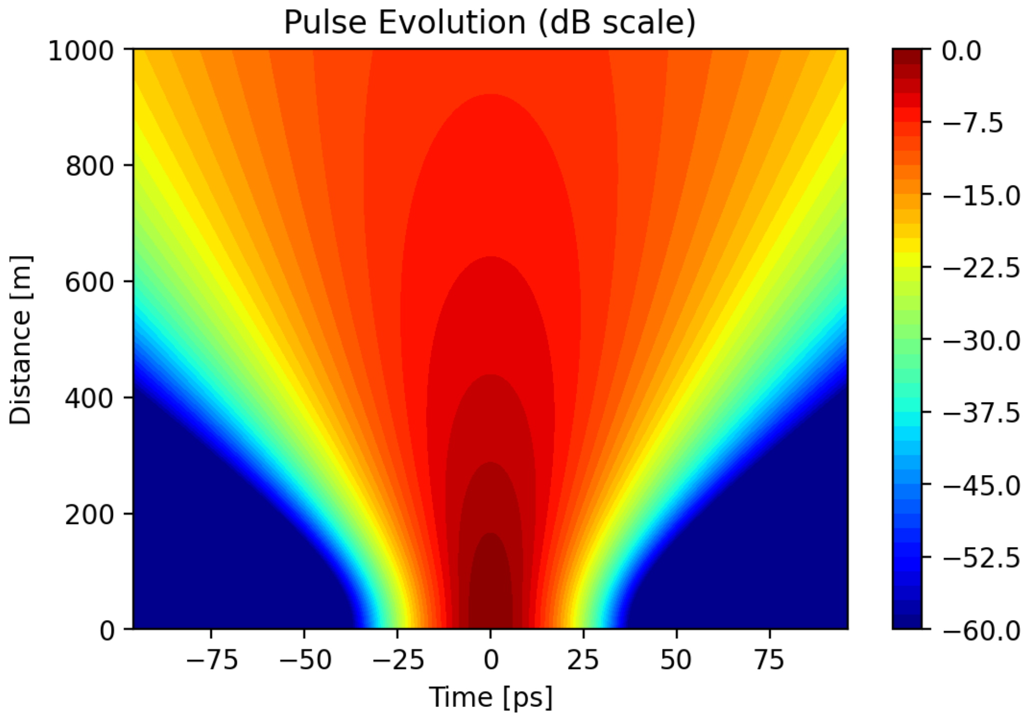
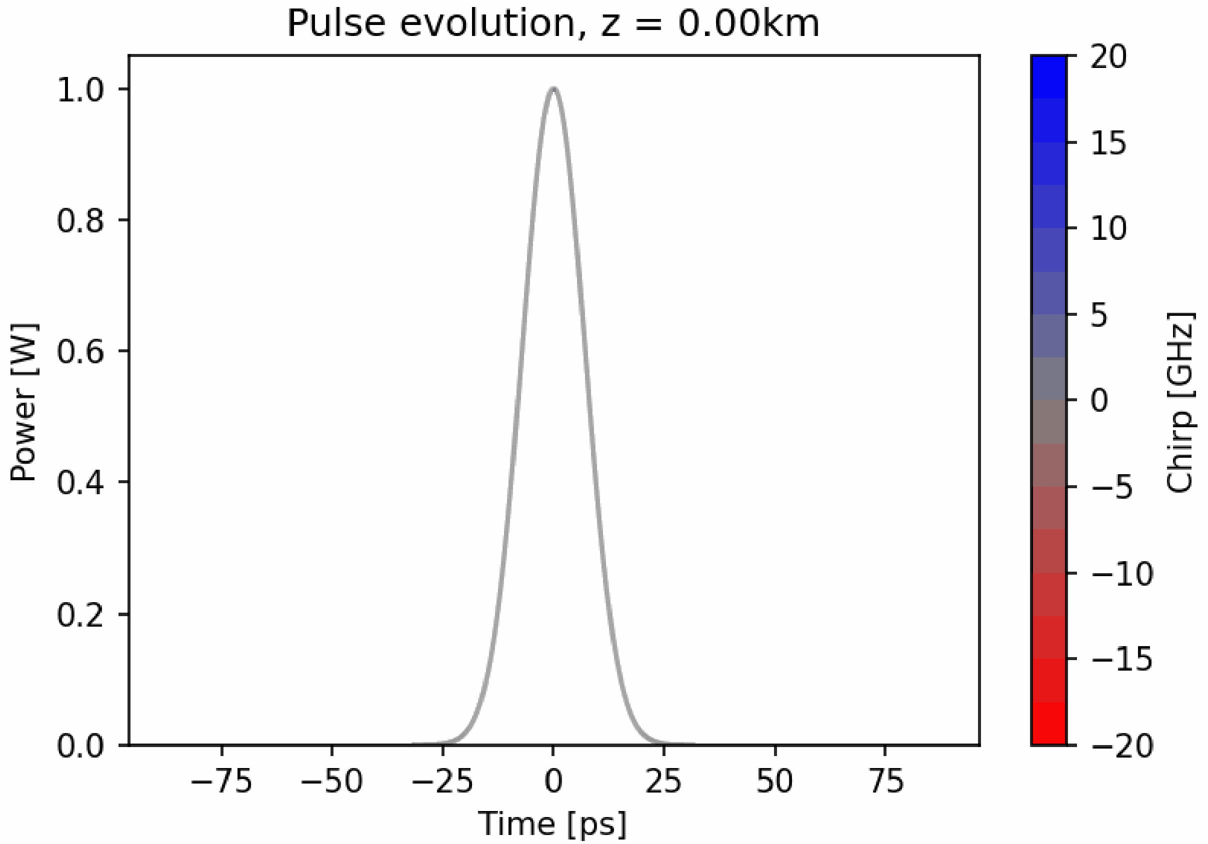
$\beta_2 > 0$ (Normal dispersion)





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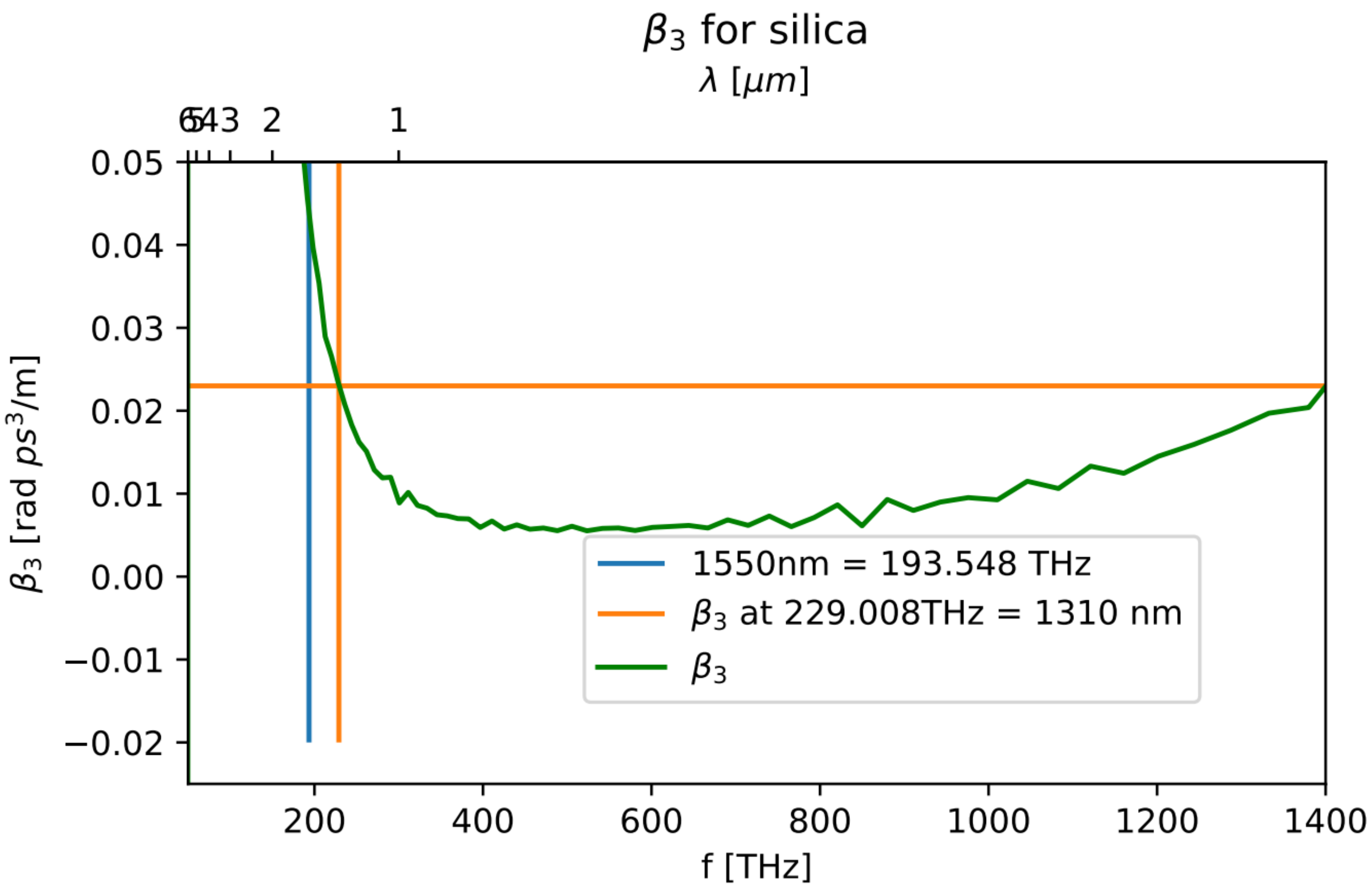
Why stop at β_2?

- Take one more derivative to get β_3!

- What happens for $\beta_2 = 0$ and $\beta_3 > 0$?



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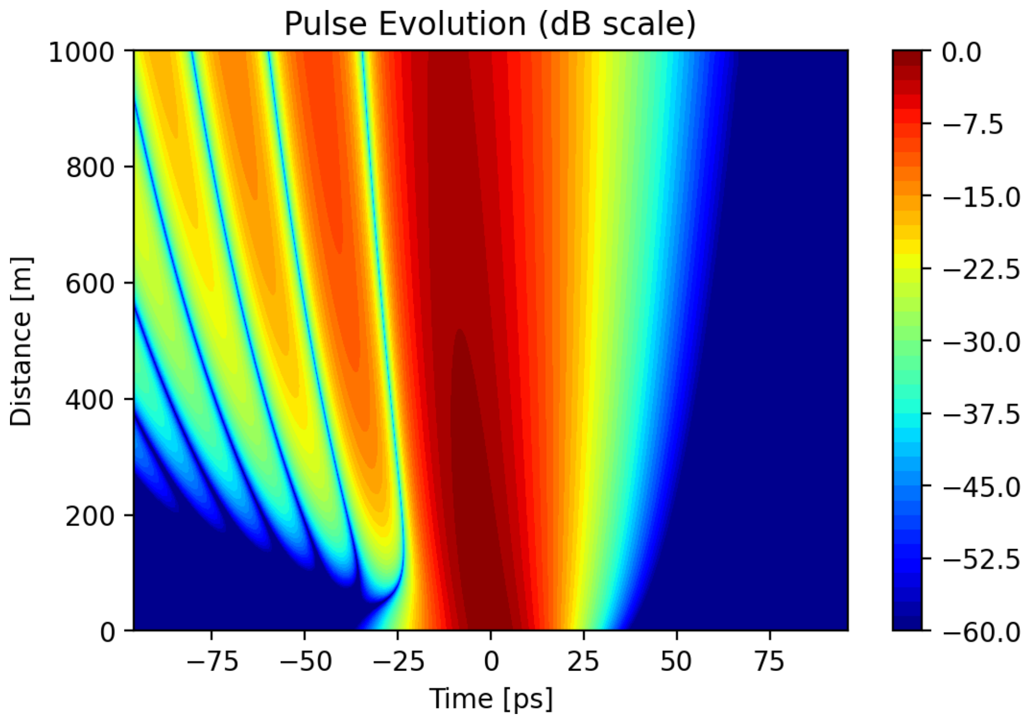
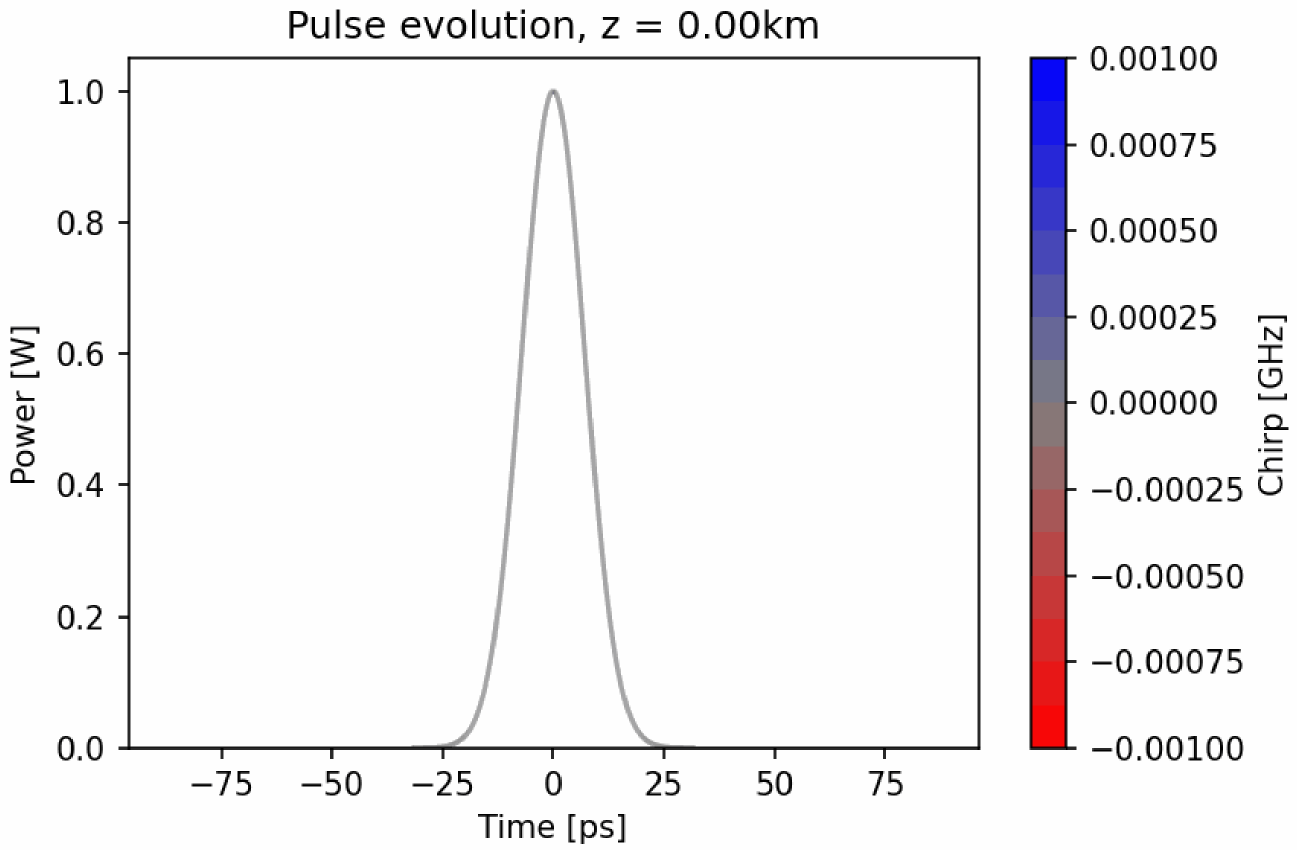
$$\beta_2 = 0 \;\; and \;\; \beta_3 > 0$$





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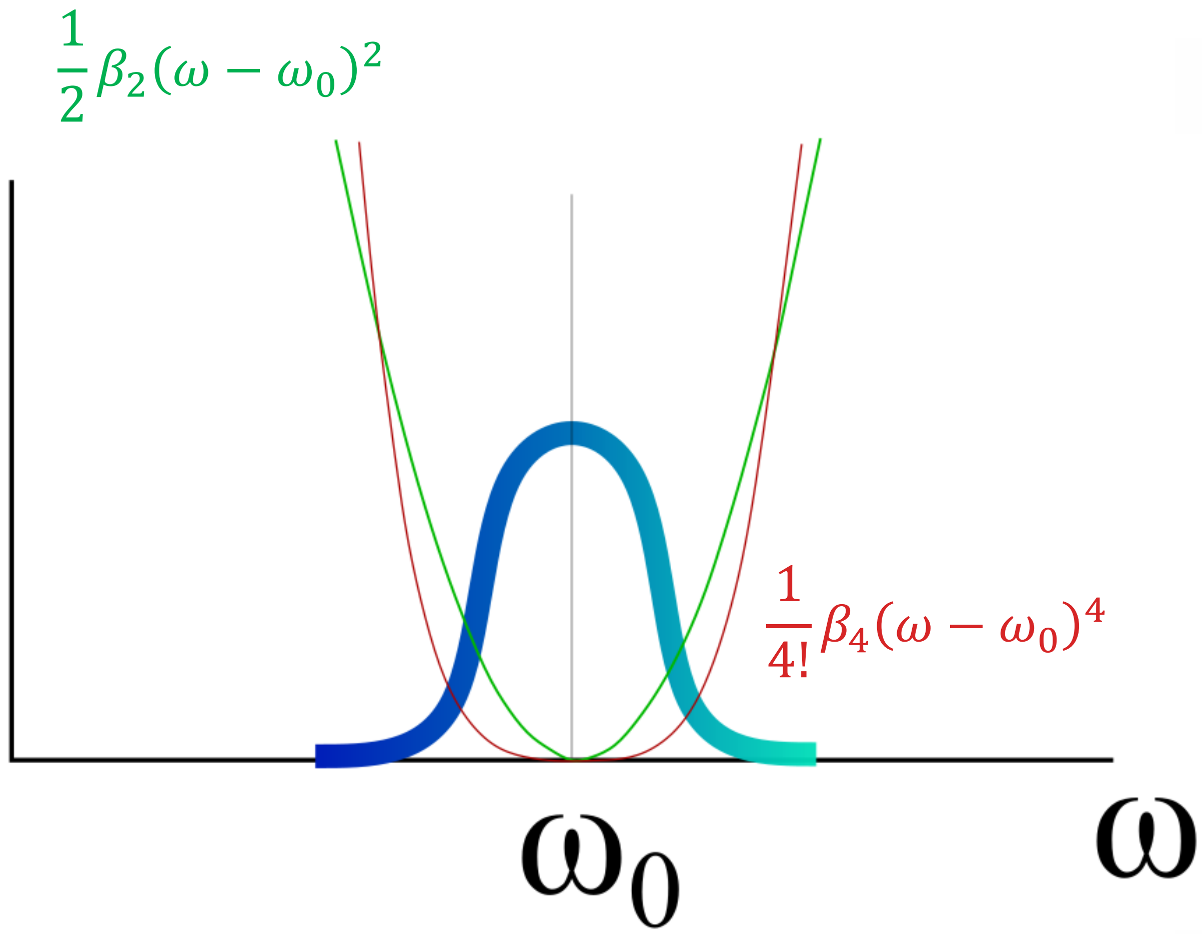
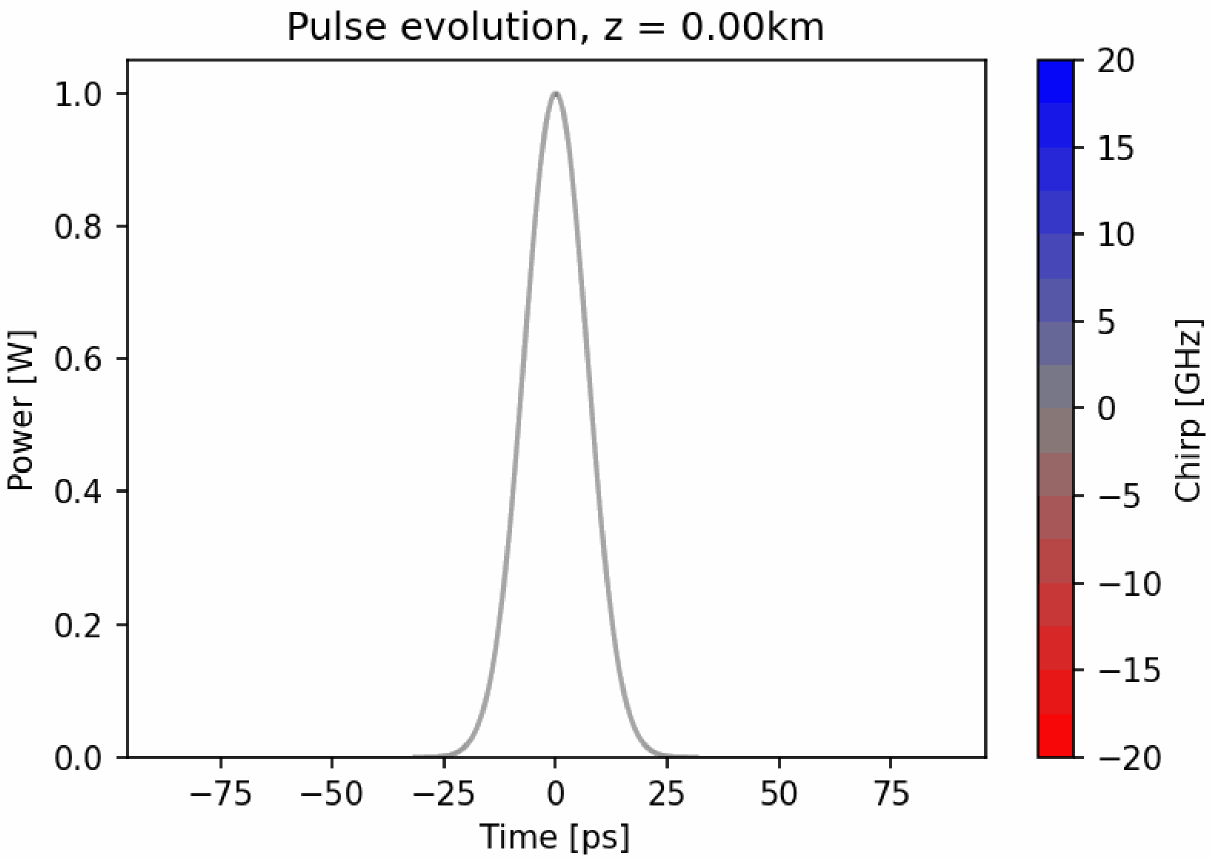
$$\beta_2 = 0 \;\; and \;\; \beta_3 = 0 \;\; and \;\; \beta_4 > 0$$





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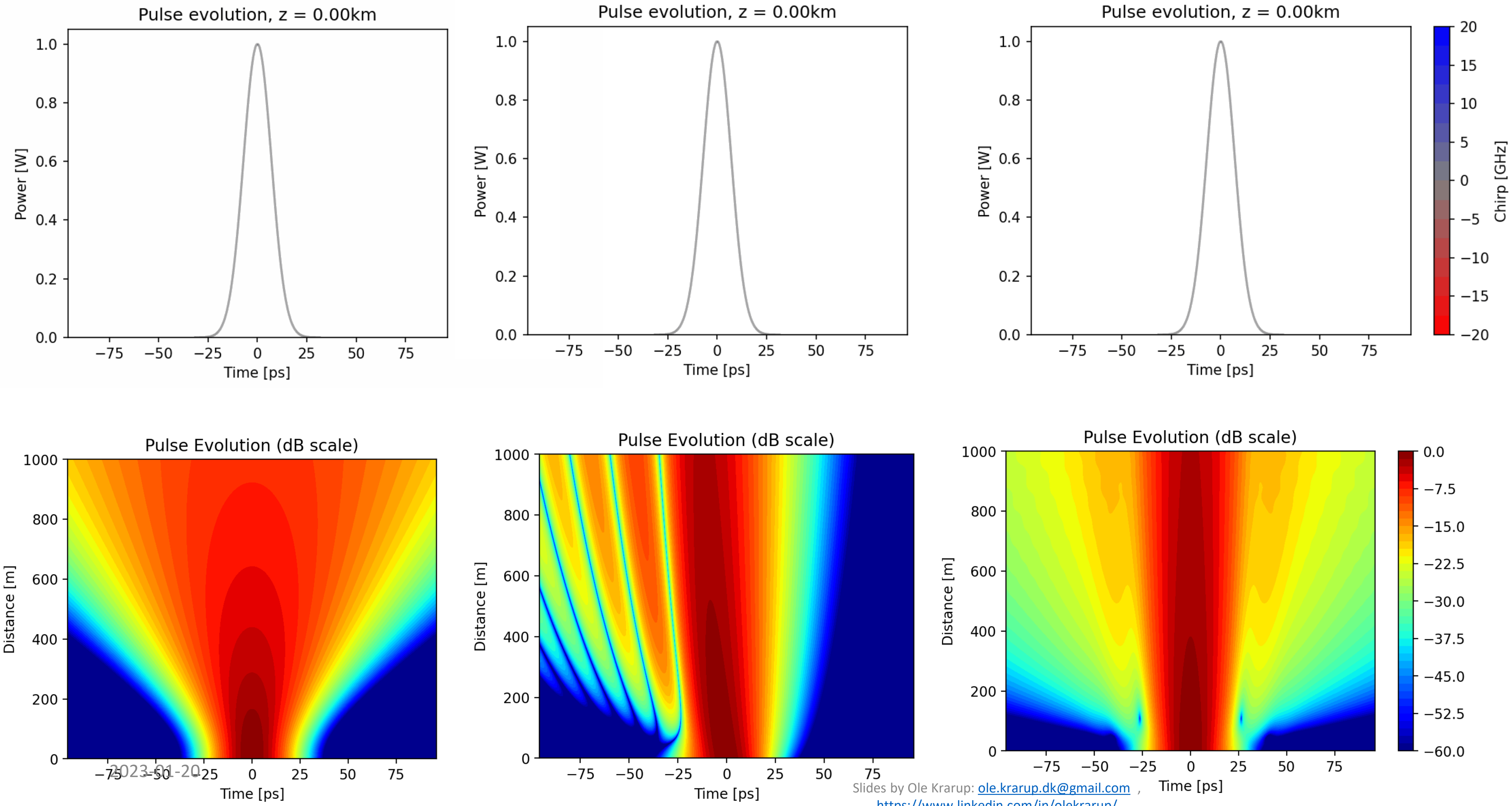
$\beta_2 > 0$ | $\beta_2 = 0, \quad \beta_3 > 0$ | $\beta_2 = 0, \quad \beta_3 = 0, \quad \beta_4 > 0$



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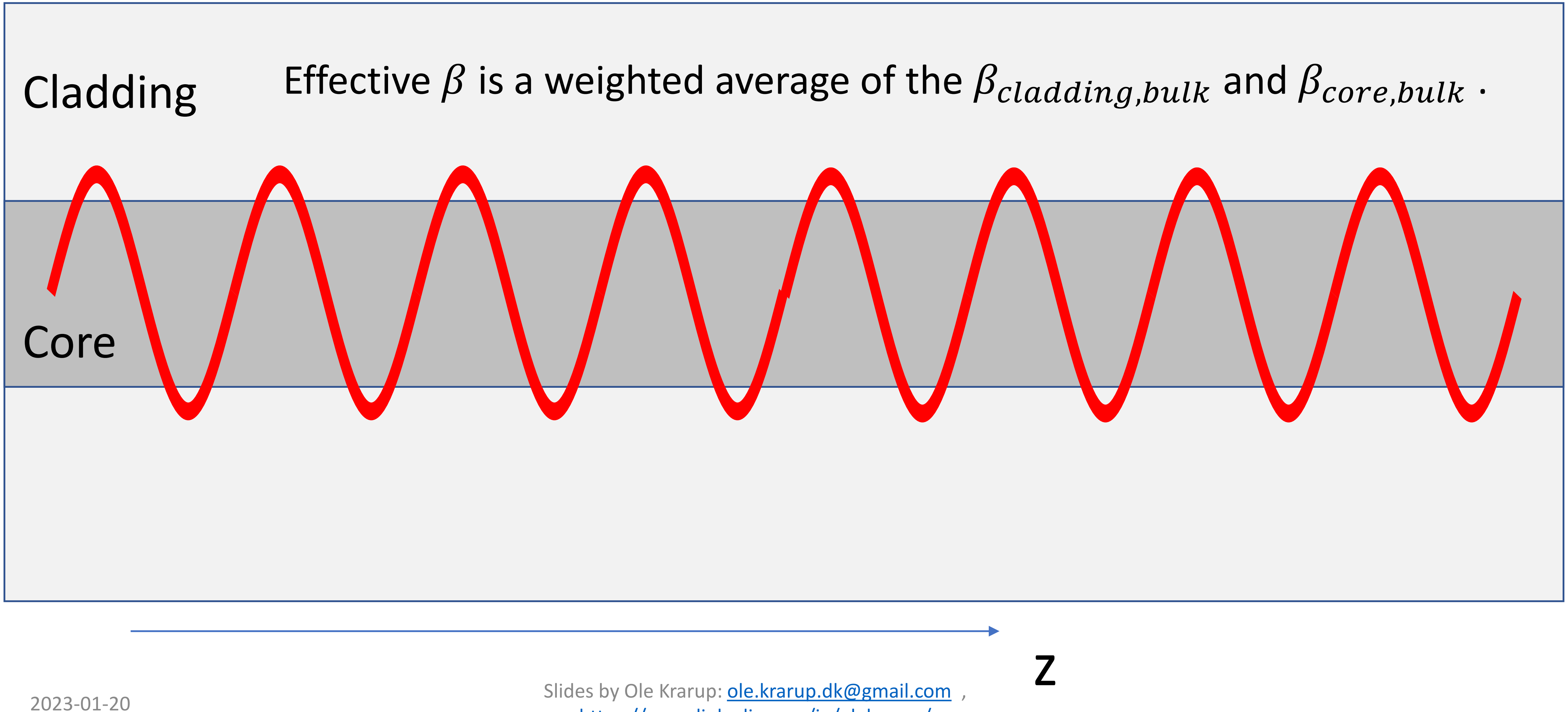
EM waves in waveguides



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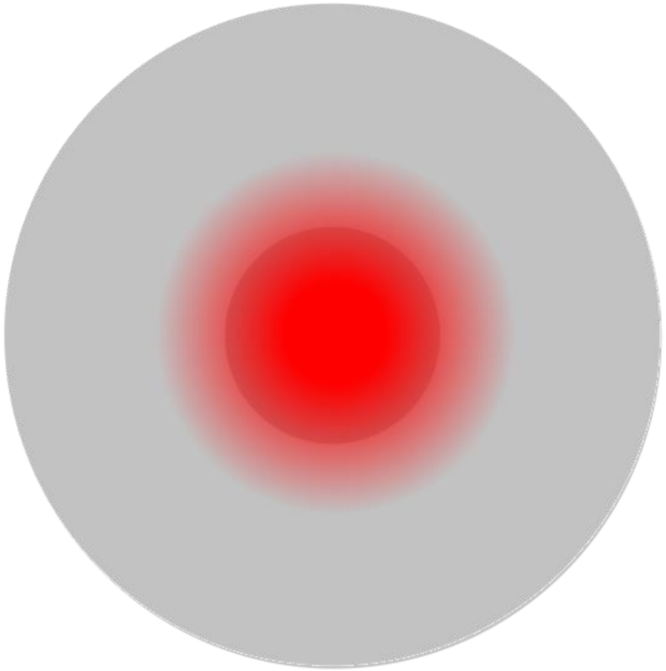
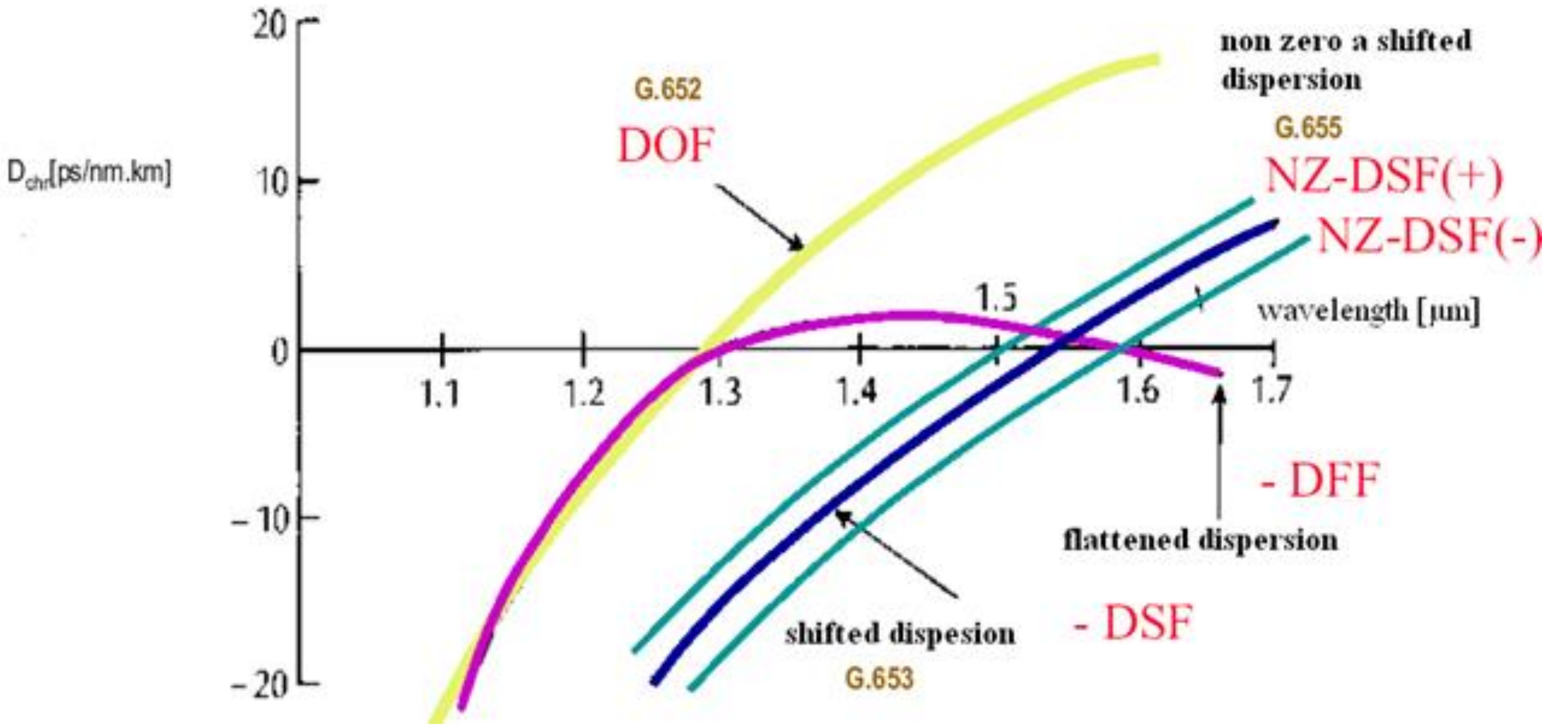
EM in waveguides

$$n(\omega, x, y) \quad {\color{red}A(\omega, x, y)} \qquad n_{eff}(\omega) = \frac{\iint n(\omega, x, y)\,{\color{red}|A(\omega, x, y)|}^2 dxdy}{\iint {\color{red}|A(\omega, x, y)|}^2 dxdy}$$





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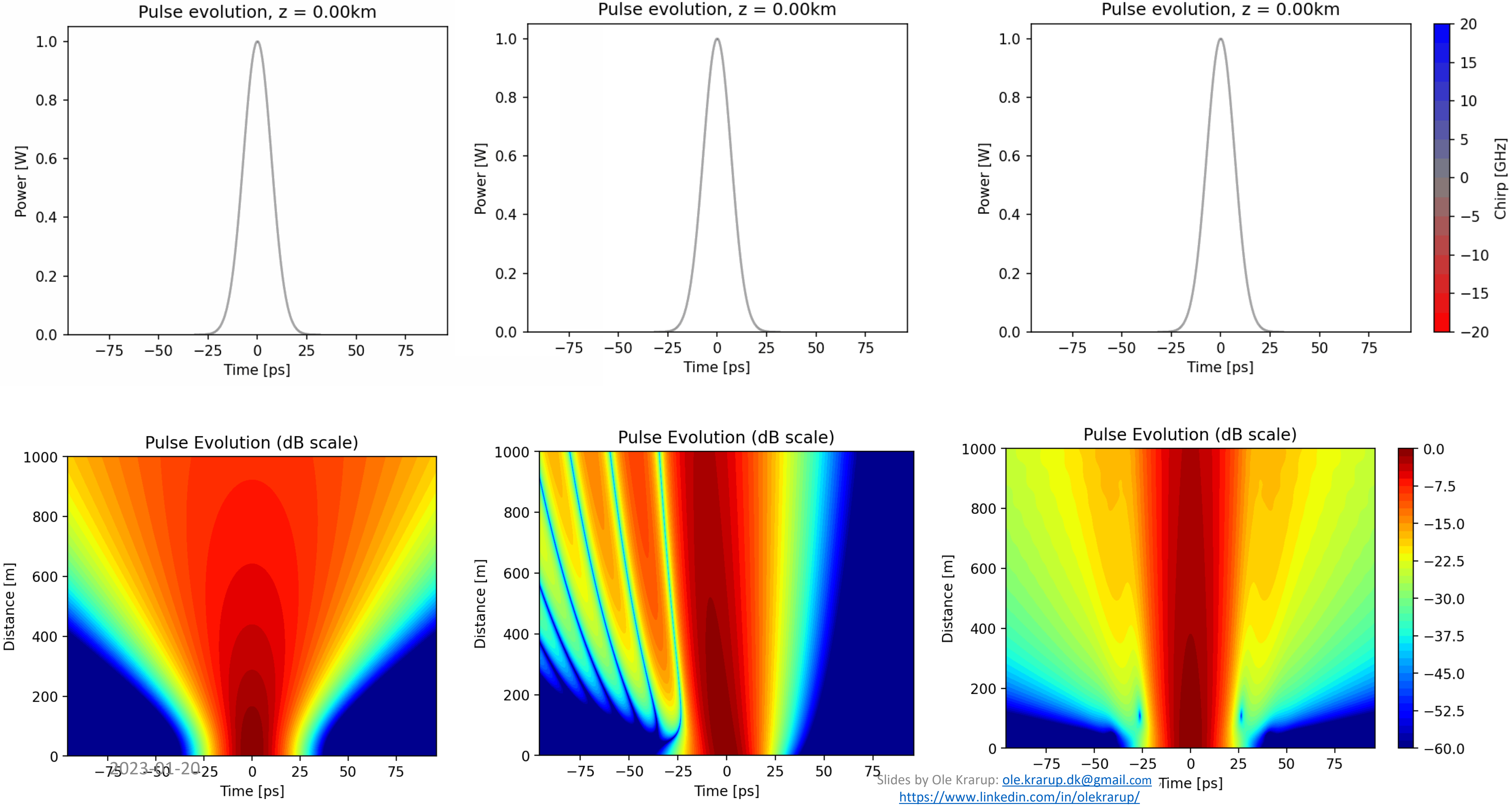
$\beta_2 > 0$ | $\beta_2 = 0, \quad \beta_3 > 0$ | $\beta_2 = 0, \quad \beta_3 = 0, \quad \beta_4 > 0$



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