P8106 Midterm Project

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Load Data

```
load("data/dat1.RData")
train data = dat1 |>
  janitor::clean_names() |>
  mutate(
    gender = as.factor(gender),
    diabetes = as.factor(diabetes),
    hypertension = as.factor(hypertension),
    race = fct_recode(race,
                  White = "1",
                  Asian = "2",
                  Black = "3",
                  Hispanic = "4"),
    gender = fct_recode(gender,
                        Male = "1",
                        Female = "0"),
    smoking = fct_recode(smoking,
                         "Never smoked" = "0",
                         "Former smoker" = "1",
                          "Current smoker" = "2"))
load("data/dat2.RData")
test_data = dat2 |>
  janitor::clean_names() |>
  mutate(
    gender = as.factor(gender),
    diabetes = as.factor(diabetes),
    hypertension = as.factor(hypertension),
    race = fct_recode(race,
                  White = "1",
                  Asian = "2",
                  Black = "3",
                  Hispanic = "4"),
    gender = fct_recode(gender,
                        Male = "1",
                        Female = "0"),
    smoking = fct_recode(smoking,
                         "Never smoked" = "0",
                          "Former smoker" = "1",
```

```
"Current smoker" = "2")
)
```

Modify Data

```
train_data1 =
    train_data %>%
    select(-id, -height, -weight, -hypertension)

x_train = model.matrix(log_antibody ~ ., train_data1)[, -1]
colnames(x_train) = make.names(colnames(x_train), unique = TRUE)
y_train = train_data1[, "log_antibody"]

test_data =
    test_data %>%
    select(-id, -height, -weight, -hypertension)

x_test = model.matrix(log_antibody ~ ., test_data)[, -1]
colnames(x_test) = make.names(colnames(x_test), unique = TRUE)
y_test = test_data[, "log_antibody"]

ctrl1 = trainControl(method = "cv", number = 10)
```

Descriptive Analysis

Numeric Variables

```
train_data |>
  pivot_longer(
    cols = c(sbp, ldl, time),
    names_to = "variable",
    values_to = "value"
) |>
  ggplot(aes(x = value, y = log_antibody, color = variable)) +
  geom_point(alpha = 0.5, size = 0.6) +
```

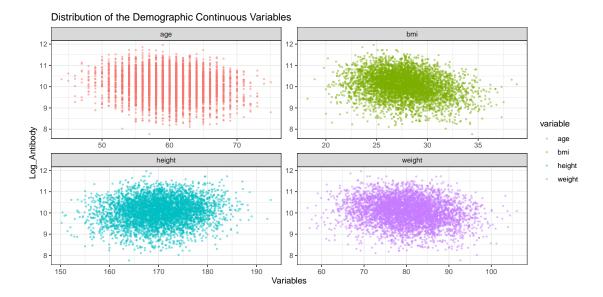


Figure 1: Distribution of the Demographic Continuous Variables

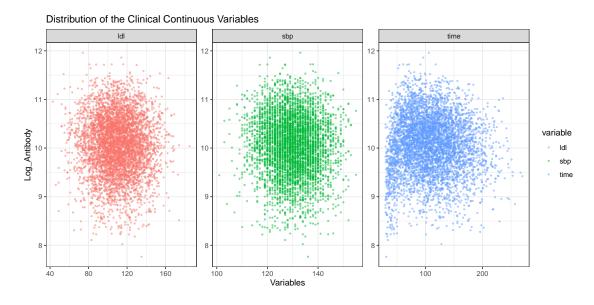


Figure 2: Distribution of the Clinical Continuous Variables

```
train_data %>%
  pivot_longer(
    cols = c(age, height, weight, bmi, sbp, ldl, time, log_antibody),
    names_to = "variable_name",
    values_to = "value"
```

```
group_by(variable_name) %>%
group_by(variable_name) %>%
summarize(
    mean = mean(value),
    median = median(value),
    min = min(value),
    first_quantile = quantile(value, probs = 0.25),
    third_quantile = quantile(value, probs = 0.75),
    max = max(value)
) %>%
ungroup() %>%
arrange(desc(variable_name == "log_antibody"), variable_name) %>%
knitr::kable(digits = 3, caption = "Descriptive Statistics for the Continuous Variables")
```

Table 1: Descriptive Statistics for the Continuous Variables

$variable_name$	mean	median	\min	$first_quantile$	$third_quantile$	max
log_antibody	10.064	10.089	7.765	9.682	10.478	11.961
age	59.968	60.000	44.000	57.000	63.000	75.000
bmi	27.740	27.600	18.200	25.800	29.500	38.800
height	170.126	170.100	150.200	166.100	174.225	192.900
ldl	109.909	110.000	43.000	96.000	124.000	185.000
sbp	129.900	130.000	101.000	124.000	135.000	155.000
time	108.863	106.000	30.000	76.000	138.000	270.000
weight	80.109	80.100	56.700	75.400	84.900	106.000

Categorical Variables

Correlation Plot

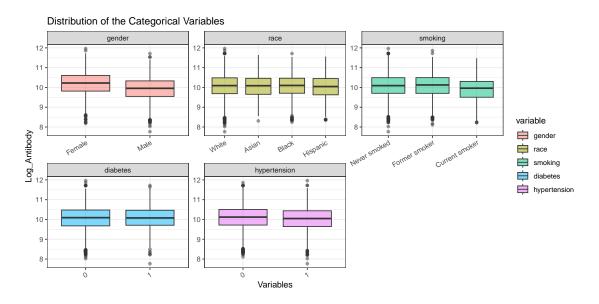


Figure 3: Distribution of the Categorical Variables

```
x_corr = model.matrix(log_antibody ~ ., train_data[, -1])[, -1]
corrplot(cor(x_corr), method = "circle", type = "full")
```

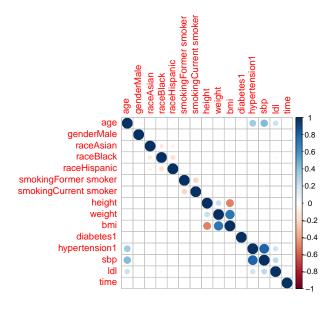


Figure 4: Correlation Plot

Regression

Elastic Net

```
set.seed(37)
enet_fit = train(log_antibody ~ .,
                data = train_data1,
                method = "glmnet",
                 tuneGrid = expand.grid(alpha = seq(0, 1, length = 21),
                                       lambda = exp(seq(-2, -8, length = 100))),
                 trControl = ctrl1)
enet_fit$bestTune
       alpha
                  lambda
## 2044 1 0.004544037
coef(enet_fit$finalModel, enet_fit$bestTune$lambda)
## 13 x 1 sparse Matrix of class "dgCMatrix"
## (Intercept)
                        1.272351e+01
## age
                        -1.914947e-02
## genderMale
                      -2.859704e-01
## raceAsian
## raceBlack
## raceHispanic
                       -2.469963e-02
## smokingFormer smoker 1.592525e-02
## smokingCurrent smoker -1.770936e-01
## bmi
                        -4.820177e-02
## diabetes1
                         4.263422e-05
## sbp
## ldl
## time
                       -1.850422e-04
mycol = rainbow(25)
mypar = list(superpose.symbol = list(col = mycol),
superpose.line = list(col = mycol))
plot(enet_fit, par.settings = mypar, xTrans = log)
```

PCR

Y dimension: 5000 1

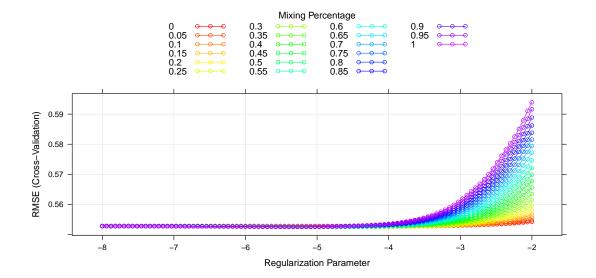


Figure 5: Effect of Tuning Parameters on Train Error (Elastic Net)

```
## Fit method: svdpc
## Number of components considered: 12
## TRAINING: % variance explained
##
             1 comps 2 comps 3 comps
                                         4 comps
                                                  5 comps
                                                            6 comps
                                                                     7 comps
## X
              13.522
                        23.760
                                 33.489
                                          42.534
                                                     51.17
                                                             59.563
                                                                       67.70
               1.296
                                                              2.032
                                                                       13.44
                         1.493
                                  1.512
                                           1.512
                                                      1.54
##
  .outcome
##
             8 comps
                      9 comps
                                10 comps
                                          11 comps
                                                    12 comps
               75.76
                         82.63
                                   89.32
                                             95.36
                                                        100.0
## X
## .outcome
               13.48
                         13.68
                                   13.78
                                             13.84
                                                         14.5
```

```
ggplot(pcr_fit, highlight = TRUE)
```

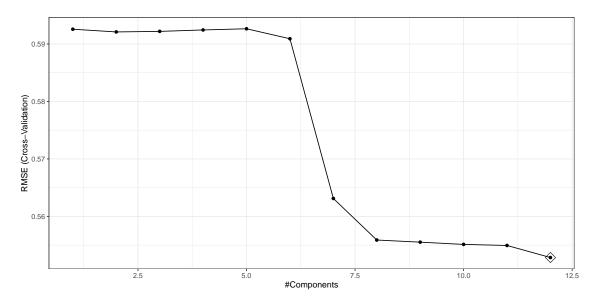


Figure 6: Component Selection (PCR)

PLS

```
set.seed(37)
pls_fit = train(x_train, y_train,
                method = "pls",
                tuneGrid = data.frame(ncomp = 1:12),
                trControl = ctrl1,
                preProcess = c("center", "scale"))
summary(pls_fit)
## Data:
            X dimension: 5000 12
## Y dimension: 5000 1
## Fit method: oscorespls
## Number of components considered: 3
## TRAINING: % variance explained
##
             1 comps 2 comps 3 comps
                        21.21
## X
               9.295
                                 26.88
## .outcome
              13.885
                        14.45
                                 14.50
```

ggplot(pls_fit, highlight = TRUE)

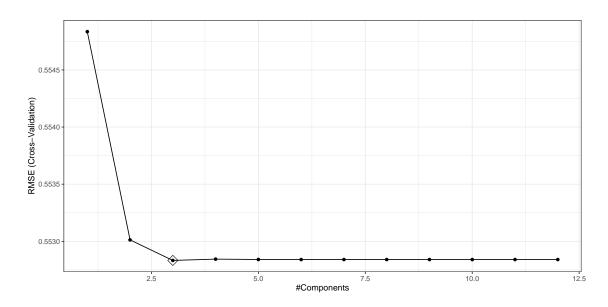


Figure 7: Component Selection (PLS)

GAM

```
gam.fit$bestTune
    select method
## 2 TRUE GCV.Cp
gam.fit$finalModel
##
## Family: gaussian
## Link function: identity
## Formula:
## .outcome ~ genderMale + raceAsian + raceBlack + raceHispanic +
       smokingFormer.smoker + smokingCurrent.smoker + diabetes1 +
##
       s(age) + s(sbp) + s(ldl) + s(bmi) + s(time)
##
## Estimated degrees of freedom:
## 0.992 0.000 0.000 4.179 7.915 total = 21.09
##
## GCV score: 0.2786375
par(mfrow = c(3, 2))
plot(gam.fit$finalModel)
par(mfrow = c(1, 1))
```

MARS

```
mars.fit$bestTune
```

```
## nprune degree
## 8 9 1
```

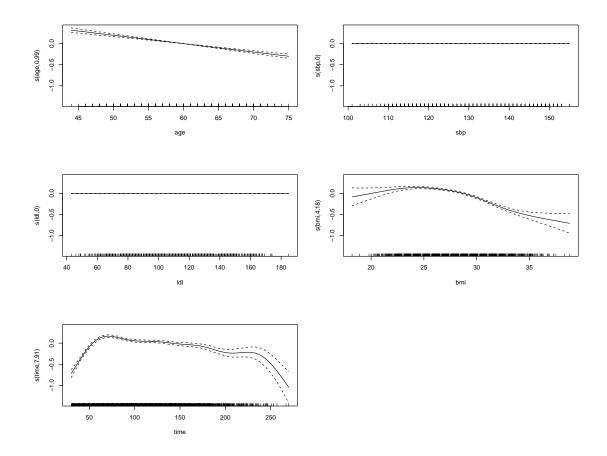


Figure 8: Degree of Predictors (GAM)

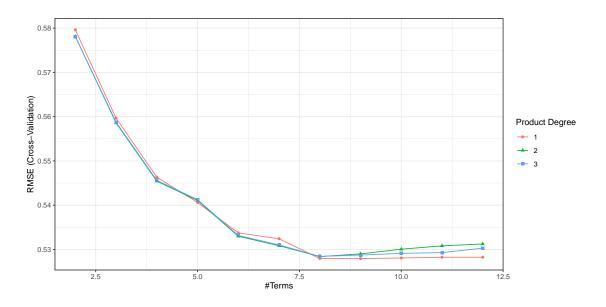


Figure 9: Term and Degree Selection (MARS)

coef(mars.fit\$finalModel)

```
##
             (Intercept)
                                   h(27.8-bmi)
                                                           h(time-57)
            10.847446930
                                   -0.061997354
                                                         -0.002254182
##
              h(57-time)
##
                                     genderMale
                                                            h(age-59)
            -0.033529326
                                  -0.296290451
                                                         -0.022957648
##
                                                          h(bmi-23.7)
##
               h(59-age) smokingCurrent.smoker
             0.016138468
                                  -0.205126851
                                                         -0.084380175
##
```

Regression Trees

size of tree

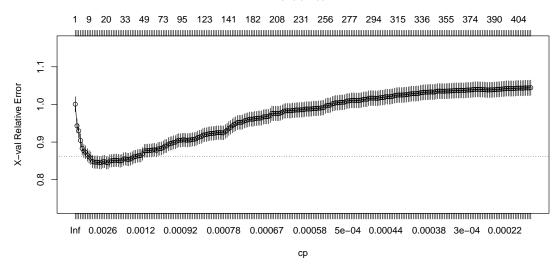


Figure 10: Tuning Parameter Selection (Regression Tree)

```
## Find the cp that yields the minimum cross-validation error
minErr = which.min(cpTable[,4])
tree_final = rpart::prune(tree_full, cp = cpTable[minErr, 1])
rpart.plot(tree_final)
```

```
summary(tree_final)

## Call:
## rpart(formula = log_antibody ~ ., data = train_data1, control = rpart.control(cp = 0))
```

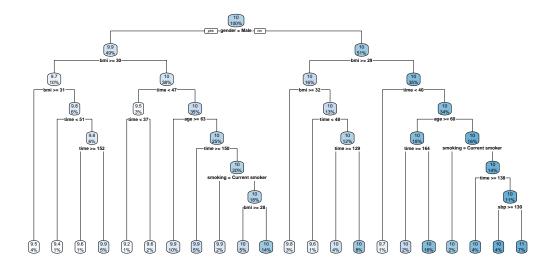


Figure 11: Final Regression Tree Model

```
##
     n = 5000
##
##
               CP nsplit rel error
                                       xerror
##
     0.057918182
                       0 1.0000000 1.0007146 0.02004169
  1
  2
     0.027293800
                       1 0.9420818 0.9432020 0.01870086
##
##
  3
     0.025172140
                       2 0.9147880 0.9295966 0.01846651
                       3 0.8896159 0.9042136 0.01812179
## 4
      0.020759383
## 5
      0.010560350
                        4 0.8688565 0.8829836 0.01767741
## 6
                       5 0.8582961 0.8761783 0.01736189
      0.007928454
      0.007483148
                        6 0.8503677 0.8729469 0.01732773
  7
## 8
     0.006830408
                       7 0.8428845 0.8657230 0.01716499
                       8 0.8360541 0.8604904 0.01704889
## 9
      0.006412912
## 10 0.006407846
                       9 0.8296412 0.8572604 0.01697315
                      10 0.8232334 0.8485364 0.01678947
## 11 0.004852623
## 12 0.003284421
                      11 0.8183808 0.8465873 0.01664462
## 13 0.003033803
                      12 0.8150963 0.8465219 0.01667932
                      13 0.8120625 0.8466921 0.01669623
## 14 0.002914817
## 15 0.002790737
                       14 0.8091477 0.8452142 0.01665712
## 16 0.002730117
                       15 0.8063570 0.8450512 0.01661721
## 17 0.002560197
                       16 0.8036269 0.8476923 0.01669101
## 18 0.002327749
                       17 0.8010667 0.8483089 0.01668839
## 19 0.002223936
                      19 0.7964112 0.8451645 0.01661475
                      21 0.7919633 0.8447269 0.01660173
## 20 0.002164228
##
##
   Variable importance
                                                         ldl
##
              time
       bmi
                    gender
                                age smoking
                                                 sbp
##
                28
                         27
                                                   3
                                                           1
##
                                         complexity param=0.05791818
##
  Node number 1: 5000 observations,
     mean=10.06434, MSE=0.355722
##
     left son=2 (2427 obs) right son=3 (2573 obs)
##
##
     Primary splits:
##
         gender splits as
                            RL,
                                        improve=0.057918180, (0 missing)
```

```
##
                 < 29.65 to the right, improve=0.049512600, (0 missing)
         bmi
##
                 < 46.5 to the left, improve=0.041303690, (0 missing)
         time
##
                 < 60.5 to the right, improve=0.018144830, (0 missing)
                                       improve=0.008813525, (0 missing)
##
         smoking splits as RRL,
##
     Surrogate splits:
##
         sbp < 119.5 to the left, agree=0.518, adj=0.007, (0 split)
##
         time < 37.5 to the left, agree=0.518, adj=0.006, (0 split)
         ldl < 140.5 to the right, agree=0.517, adj=0.005, (0 split)
##
##
         age < 66.5 to the right, agree=0.516, adj=0.003, (0 split)
##
         bmi < 20.75 to the left, agree=0.515, adj=0.002, (0 split)
##
## Node number 2: 2427 observations,
                                         complexity param=0.02517214
     mean=9.91655, MSE=0.3395789
##
     left son=4 (524 obs) right son=5 (1903 obs)
##
##
     Primary splits:
##
         bmi
                 < 29.85 to the right, improve=0.054323820, (0 missing)
##
                 < 46.5 to the left, improve=0.049666810, (0 missing)
         time
##
                 < 60.5 to the right, improve=0.017649280, (0 missing)
         age
##
                                       improve=0.009762305, (0 missing)
         smoking splits as RRL,
                 < 141.5 to the right, improve=0.004210017, (0 missing)
##
##
## Node number 3: 2573 observations,
                                        complexity param=0.0272938
##
     mean=10.20375, MSE=0.3309125
     left son=6 (820 obs) right son=7 (1753 obs)
##
##
     Primary splits:
                 < 28.95 to the right, improve=0.057015340, (0 missing)
##
         bmi
##
                 < 46.5 to the left, improve=0.036007110, (0 missing)
         time
                 < 59.5 to the right, improve=0.023746020, (0 missing)
##
##
                                       improve=0.010252650, (0 missing)
         smoking splits as RRL,
##
                 < 128.5 to the right, improve=0.006713466, (0 missing)
         sbp
##
     Surrogate splits:
##
         ldl < 164.5 to the right, agree=0.683, adj=0.005, (0 split)
         sbp < 108.5 to the left, agree=0.682, adj=0.002, (0 split)
##
##
##
  Node number 4: 524 observations,
                                       complexity param=0.004852623
     mean=9.657717, MSE=0.3166048
##
##
     left son=8 (207 obs) right son=9 (317 obs)
##
     Primary splits:
##
                 < 31.45 to the right, improve=0.05202457, (0 missing)
         bmi
##
                 < 50.5 to the left, improve=0.04364456, (0 missing)
##
                                       improve=0.01948327, (0 missing)
         smoking splits as RRL,
                 < 120.5 to the right, improve=0.01676732, (0 missing)
##
         sbp
                 < 59.5 to the right, improve=0.01274723, (0 missing)
##
         age
##
     Surrogate splits:
                      to the right, agree=0.615, adj=0.024, (0 split)
##
         ldl < 152
         time < 31.5 to the left, agree=0.609, adj=0.010, (0 split)
##
                     to the left, agree=0.607, adj=0.005, (0 split)
##
         sbp < 109
##
##
  Node number 5: 1903 observations,
                                        complexity param=0.02075938
     mean=9.987821, MSE=0.3223782
##
##
     left son=10 (137 obs) right son=11 (1766 obs)
##
     Primary splits:
##
         time
                 < 46.5 to the left, improve=0.060185330, (0 missing)
                 < 62.5 to the right, improve=0.023530880, (0 missing)
##
         age
```

```
##
                 < 141.5 to the right, improve=0.008688514, (0 missing)
         ldl
##
                 < 28.35 to the right, improve=0.008031965, (0 missing)
         bmi
##
         smoking splits as RRL,
                                       improve=0.007860098, (0 missing)
##
## Node number 6: 820 observations,
                                       complexity param=0.006830408
     mean=10.00291, MSE=0.3091452
##
     left son=12 (162 obs) right son=13 (658 obs)
##
##
     Primary splits:
##
         bmi < 32.25 to the right, improve=0.047923770, (0 missing)
##
         time < 47.5 to the left, improve=0.047697360, (0 missing)
##
         age < 60.5 to the right, improve=0.021396790, (0 missing)
         ldl < 73.5 to the right, improve=0.011921340, (0 missing)
##
         sbp < 137.5 to the right, improve=0.008408437, (0 missing)
##
##
## Node number 7: 1753 observations,
                                        complexity param=0.01056035
##
     mean=10.29769, MSE=0.313402
##
     left son=14 (53 obs) right son=15 (1700 obs)
##
     Primary splits:
##
                 < 39.5 to the left, improve=0.034188120, (0 missing)
         time
                 < 59.5 to the right, improve=0.028638110, (0 missing)
##
         age
##
         smoking splits as RRL,
                                       improve=0.017123260, (0 missing)
##
                 < 128.5 to the right, improve=0.009433160, (0 missing)
                 < 26.75 to the right, improve=0.005238462, (0 missing)
##
         bmi
## Node number 8: 207 observations
##
     mean=9.498897, MSE=0.2980599
##
## Node number 9: 317 observations,
                                       complexity param=0.002223936
##
     mean=9.761427, MSE=0.3014877
##
     left son=18 (27 obs) right son=19 (290 obs)
##
     Primary splits:
##
         time
                 < 51
                         to the left,
                                       improve=0.03944451, (0 missing)
##
         smoking splits as RRL,
                                       improve=0.03307206, (0 missing)
##
                 < 58.5 to the right, improve=0.02350311, (0 missing)
##
                 < 121.5 to the right, improve=0.02005357, (0 missing)
         sbp
##
                 < 80.5 to the right, improve=0.01272572, (0 missing)
         ldl
##
## Node number 10: 137 observations,
                                        complexity param=0.003033803
     mean=9.487714, MSE=0.3536406
##
     left son=20 (46 obs) right son=21 (91 obs)
##
##
     Primary splits:
##
         time < 36.5 to the left, improve=0.11137440, (0 missing)
         sbp < 118.5 to the right, improve=0.05219794, (0 missing)
##
         bmi < 26.65 to the right, improve=0.04629031, (0 missing)
##
         age < 65.5 to the right, improve=0.04068284, (0 missing)
##
         ldl < 88.5 to the left, improve=0.02321797, (0 missing)
##
##
## Node number 11: 1766 observations,
                                         complexity param=0.007483148
##
     mean=10.02662, MSE=0.2990454
##
     left son=22 (508 obs) right son=23 (1258 obs)
##
     Primary splits:
##
         age
                 < 62.5 to the right, improve=0.025202130, (0 missing)
##
                 < 97.5 to the right, improve=0.015378130, (0 missing)
         time
                 < 142.5 to the right, improve=0.012513270, (0 missing)
##
         ldl
```

```
##
                                       improve=0.009228690, (0 missing)
         smoking splits as RRL,
##
                 < 28.35 to the right, improve=0.008363651, (0 missing)
         bmi
##
     Surrogate splits:
         sbp < 140.5 to the right, agree=0.737, adj=0.085, (0 split)
##
##
         ldl < 159.5 to the right, agree=0.715, adj=0.008, (0 split)
##
## Node number 12: 162 observations
     mean=9.757604, MSE=0.2946382
##
##
## Node number 13: 658 observations,
                                        complexity param=0.006407846
     mean=10.06331, MSE=0.2942538
     left son=26 (51 obs) right son=27 (607 obs)
##
##
     Primary splits:
         time < 47.5 to the left, improve=0.058863320, (0 missing)
##
##
         age < 61.5 to the right, improve=0.018077840, (0 missing)
##
         bmi < 31.35 to the right, improve=0.011334980, (0 missing)
         ldl < 73.5 to the right, improve=0.009520452, (0 missing)
##
##
         sbp < 137.5 to the right, improve=0.009235878, (0 missing)
##
## Node number 14: 53 observations
##
    mean=9.71145, MSE=0.4536552
##
## Node number 15: 1700 observations,
                                         complexity param=0.007928454
     mean=10.31597, MSE=0.2979808
##
     left son=30 (914 obs) right son=31 (786 obs)
##
##
     Primary splits:
##
                 < 59.5 to the right, improve=0.027837610, (0 missing)
         age
                                       improve=0.016679170, (0 missing)
##
         smoking splits as RRL,
##
                 < 159.5 to the right, improve=0.014116890, (0 missing)
##
                 < 128.5 to the right, improve=0.009671418, (0 missing)
         sbp
                 < 26.75 to the right, improve=0.004398652, (0 missing)
##
         bmi
##
     Surrogate splits:
##
         sbp < 126.5 to the right, agree=0.645, adj=0.232, (0 split)
         ldl < 91.5 to the right, agree=0.569, adj=0.069, (0 split)
##
         time < 176.5 to the left, agree=0.545, adj=0.017, (0 split)
##
##
         bmi < 21.35 to the right, agree=0.540, adj=0.005, (0 split)
##
## Node number 18: 27 observations
     mean=9.404035, MSE=0.2771699
##
##
## Node number 19: 290 observations,
                                        complexity param=0.002223936
##
     mean=9.794701, MSE=0.2907525
     left son=38 (62 obs) right son=39 (228 obs)
##
##
     Primary splits:
##
         time
                 < 151.5 to the right, improve=0.04911456, (0 missing)
                                       improve=0.04214043, (0 missing)
##
         smoking splits as RRL,
##
         sbp
                 < 121.5 to the right, improve=0.02250072, (0 missing)
                 < 58.5 to the right, improve=0.02190715, (0 missing)
##
         age
##
         ldl
                 < 80.5 to the right, improve=0.01055996, (0 missing)
##
## Node number 20: 46 observations
     mean=9.208578, MSE=0.2594872
##
##
## Node number 21: 91 observations
```

```
##
     mean=9.628815, MSE=0.3419384
##
## Node number 22: 508 observations
     mean=9.890003, MSE=0.3020677
##
##
## Node number 23: 1258 observations,
                                          complexity param=0.006412912
     mean=10.08178, MSE=0.287245
     left son=46 (250 obs) right son=47 (1008 obs)
##
##
     Primary splits:
                 < 149.5 to the right, improve=0.031564790, (0 missing)
##
         time
##
                 < 54.5 to the right, improve=0.011762700, (0 missing)
         age
                 < 126.5 to the right, improve=0.011752420, (0 missing)
##
         ldl
                                       improve=0.010707140, (0 missing)
##
         smoking splits as RRL,
##
                 < 109.5 to the left, improve=0.007957896, (0 missing)
##
     Surrogate splits:
##
         ldl < 162.5 to the right, agree=0.802, adj=0.004, (0 split)
##
## Node number 26: 51 observations
##
     mean=9.609269, MSE=0.2289111
##
## Node number 27: 607 observations,
                                        complexity param=0.002730117
     mean=10.10146, MSE=0.2809679
##
     left son=54 (225 obs) right son=55 (382 obs)
##
     Primary splits:
         time < 128.5 to the right, improve=0.02847191, (0 missing)
##
##
         age < 61.5 to the right, improve=0.02293396, (0 missing)
##
         bmi < 31.35 to the right, improve=0.01993356, (0 missing)
         ldl < 111.5 to the right, improve=0.01092987, (0 missing)
##
##
         sbp < 137.5 to the right, improve=0.01007648, (0 missing)
##
     Surrogate splits:
         sbp < 114.5 to the left, agree=0.636, adj=0.018, (0 split)
##
##
         age < 52.5 to the left, agree=0.631, adj=0.004, (0 split)
         bmi < 32.15 to the right, agree=0.631, adj=0.004, (0 split)
##
##
## Node number 30: 914 observations,
                                        complexity param=0.002914817
     mean=10.23151, MSE=0.2886223
##
##
     left son=60 (101 obs) right son=61 (813 obs)
##
     Primary splits:
##
                 < 163.5 to the right, improve=0.019652420, (0 missing)
         time
##
                                       improve=0.012542120, (0 missing)
         smoking splits as RRL,
                 < 24.85 to the right, improve=0.010077130, (0 missing)
##
##
                 < 70.5 to the right, improve=0.004980359, (0 missing)
         age
                 < 79.5 to the right, improve=0.004331232, (0 missing)
##
         ldl
##
                                        complexity param=0.002790737
## Node number 31: 786 observations,
##
     mean=10.41418, MSE=0.2909223
##
     left son=62 (78 obs) right son=63 (708 obs)
##
     Primary splits:
##
         smoking splits as RRL,
                                       improve=0.021707010, (0 missing)
                 < 184.5 to the right, improve=0.020284770, (0 missing)
##
##
                 < 129.5 to the right, improve=0.013958710, (0 missing)
         sbp
##
                 splits as LRLL,
                                       improve=0.008200332, (0 missing)
##
                 < 54.5 to the right, improve=0.007226599, (0 missing)
         age
##
     Surrogate splits:
```

```
##
         sbp < 149
                    to the right, agree=0.903, adj=0.026, (0 split)
##
## Node number 38: 62 observations
     mean=9.565542, MSE=0.2656172
##
## Node number 39: 228 observations
     mean=9.857017, MSE=0.2794241
##
##
## Node number 46: 250 observations
     mean=9.890584, MSE=0.2961252
##
##
## Node number 47: 1008 observations,
                                          complexity param=0.003284421
     mean=10.12921, MSE=0.273727
##
     left son=94 (87 obs) right son=95 (921 obs)
##
##
     Primary splits:
##
         smoking splits as RRL,
                                        improve=0.021171970, (0 missing)
##
         bmi
                 < 28.25 to the right, improve=0.016615490, (0 missing)
##
         ldl
                 < 126.5 to the right, improve=0.013732310, (0 missing)
##
                 < 52.5 to the right, improve=0.009102790, (0 missing)
         age
##
         sbp
                 < 109.5 to the left, improve=0.008120837, (0 missing)
##
## Node number 54: 225 observations
##
     mean=9.984915, MSE=0.2376411
## Node number 55: 382 observations
##
     mean=10.1701, MSE=0.293776
##
## Node number 60: 101 observations
     mean=10.01783, MSE=0.30527
##
##
## Node number 61: 813 observations
##
     mean=10.25805, MSE=0.2801773
##
## Node number 62: 78 observations
##
     mean=10.17476, MSE=0.3236525
##
## Node number 63: 708 observations,
                                         complexity param=0.002327749
##
     mean=10.44056, MSE=0.2803057
     left son=126 (182 obs) right son=127 (526 obs)
##
##
     Primary splits:
         time < 137.5 to the right, improve=0.019352530, (0 missing)
##
##
         sbp < 133.5 to the right, improve=0.014196050, (0 missing)
         age < 55.5 to the right, improve=0.009180245, (0 missing)
##
##
         race splits as LRLL,
                                    improve=0.008380573, (0 missing)
##
         bmi < 28.45 to the right, improve=0.007482769, (0 missing)
     Surrogate splits:
##
         ldl < 154.5 to the right, agree=0.744, adj=0.005, (0 split)
##
##
## Node number 94: 87 observations
     mean=9.881514, MSE=0.2388818
##
##
## Node number 95: 921 observations,
                                         complexity param=0.002560197
##
    mean=10.1526, MSE=0.2706758
     left son=190 (236 obs) right son=191 (685 obs)
```

```
##
     Primary splits:
##
         bmi < 28.25 to the right, improve=0.018266070, (0 missing)
##
         ldl < 127.5 to the right, improve=0.013821740, (0 missing)
##
         age < 55.5 to the right, improve=0.012300610, (0 missing)
##
         sbp < 109.5 to the left, improve=0.011041330, (0 missing)
        time < 95.5 to the right, improve=0.007309987, (0 missing)
##
##
     Surrogate splits:
                    to the right, agree=0.746, adj=0.008, (0 split)
##
         ldl < 164
         age < 46.5 to the left, agree=0.745, adj=0.004, (0 split)
##
##
## Node number 126: 182 observations
    mean=10.31535, MSE=0.2119669
##
##
## Node number 127: 526 observations,
                                         complexity param=0.002327749
##
     mean=10.48388, MSE=0.2966499
##
     left son=254 (191 obs) right son=255 (335 obs)
##
     Primary splits:
##
         sbp < 129.5 to the right, improve=0.028452590, (0 missing)
##
        time < 50.5 to the left, improve=0.013522720, (0 missing)
##
         race splits as LRLL,
                                    improve=0.012365230, (0 missing)
##
        bmi < 28.35 to the right, improve=0.009747016, (0 missing)
##
         age < 54.5 to the right, improve=0.007912211, (0 missing)
##
     Surrogate splits:
        ldl < 145.5 to the right, agree=0.646, adj=0.026, (0 split)
##
         bmi < 22.7 to the left, agree=0.644, adj=0.021, (0 split)
##
##
         time < 42.5 to the left, agree=0.641, adj=0.010, (0 split)
##
## Node number 190: 236 observations
    mean=10.03281, MSE=0.2619733
##
## Node number 191: 685 observations
##
    mean=10.19387, MSE=0.2670264
##
## Node number 254: 191 observations
    mean=10.36221, MSE=0.2972116
##
## Node number 255: 335 observations
     mean=10.55325, MSE=0.2830768
```

Comparison

```
##
## Call:
## summary.resamples(object = resamp)
##
```

```
## Models: elastic_net, pcr, pls, gam, mars
## Number of resamples: 10
##
## MAE
##
                    Min.
                           1st Qu.
                                       Median
                                                   Mean
                                                          3rd Qu.
## elastic net 0.4294666 0.4333146 0.4414904 0.4404188 0.4479709 0.4502491
               0.4283770 0.4331431 0.4418042 0.4406867 0.4484017 0.4516552
## pcr
               0.4283860 0.4331977 0.4418812 0.4407023 0.4483486 0.4517004
## pls
                                                                                0
## gam
               0.4038951 0.4165022 0.4250940 0.4229309 0.4317016 0.4337601
                                                                                0
               0.4068067 0.4144508 0.4248340 0.4221903 0.4299231 0.4327355
                                                                                0
## mars
##
## RMSE
##
                    Min.
                           1st Qu.
                                       Median
                                                   Mean
                                                          3rd Qu.
                                                                        Max. NA's
## elastic_net 0.5346950 0.5401593 0.5540536 0.5525646 0.5599590 0.5733181
               0.5348357 0.5398323 0.5544163 0.5528409 0.5604510 0.5739030
                                                                                0
## pls
               0.5347609 0.5398986 0.5544173 0.5528333 0.5603197 0.5740022
                                                                                0
               0.5101573 0.5227377 0.5286720 0.5286310 0.5327635 0.5511329
                                                                                0
## gam
## mars
               0.5115426 0.5227742 0.5278403 0.5279391 0.5326822 0.5440279
##
## Rsquared
##
                    Min.
                           1st Qu.
                                       Median
                                                   Mean
                                                          3rd Qu.
                                                                        Max. NA's
## elastic net 0.1039358 0.1362602 0.1422293 0.1424292 0.1471805 0.1856091
               0.1036311 0.1369615 0.1396188 0.1414866 0.1456933 0.1846009
                                                                                0
## pcr
               0.1037262 0.1368722 0.1397305 0.1415124 0.1458437 0.1846014
                                                                                0
## pls
## gam
               0.1629446 0.2041048 0.2183934 0.2154693 0.2374062 0.2522148
                                                                                0
## mars
               0.1561416 0.2106884 0.2225692 0.2174307 0.2382161 0.2471323
                                                                                0
```



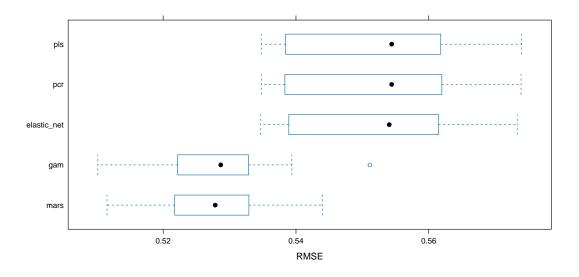


Figure 12: Model Selection

Model Performance

Residuals vs Fitted Values (MARS)

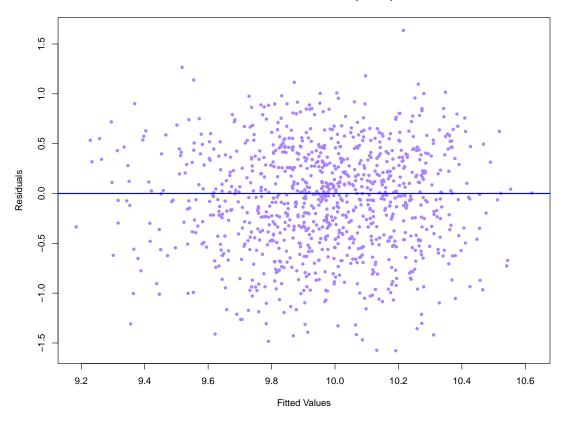


Figure 13: Residuals vs Fitted Values (MARS)

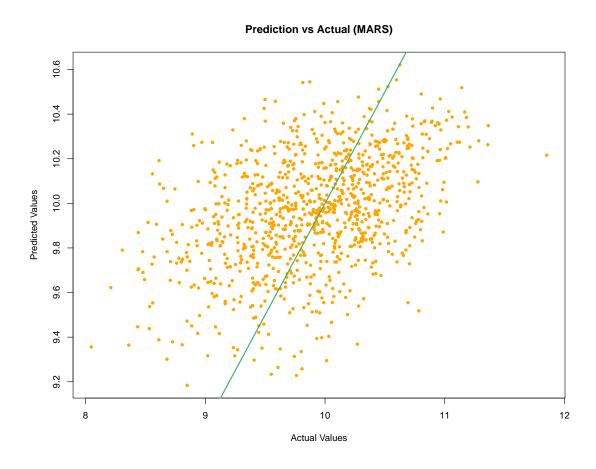


Figure 14: Prediction vs Actual (MARS)

```
rmse = sqrt(mean((y_test - predicted_values)^2))
rmse
```

[1] 0.5327718