

6.1

$$I = \int_0^{+\infty} \frac{\ln x \, dx}{x^2 + 1} = 2 \int_{x=\pm i}^{\infty} \frac{\ln x \, dx}{(x-i)(x+i)}$$

$$x_1 = i$$

$$x_2 = -i$$

$$\oint = \int_{C_1} + \int_{C_2} + \int_{C_3} + \int_{C_4} = 2\pi i \operatorname{Res} f(z)$$

$$\ln i = \ln \left| \frac{i}{x+i0} \right| + \ln |x+i0| + i \Delta \arg y(z) =$$

$$= i \cdot \frac{\pi}{2}$$

$$\operatorname{Res} f(z) = \frac{i \frac{\pi}{2}}{2i} = \frac{\pi}{4}$$

$$I_{1+2+3} = \frac{i \pi^2}{2}$$

$$\cancel{f(-x)} \quad f(-x) = \left| \frac{f(-x)}{f(x+i0)} \right| \cdot f(x+i0) + i \pi$$

$$f(-x) = f(x+i0) + i\pi$$

Res(z)

$$\ln(-x) = \ln(x+i0) + i\pi$$

$$e^{-x} = x e^{i\pi}$$

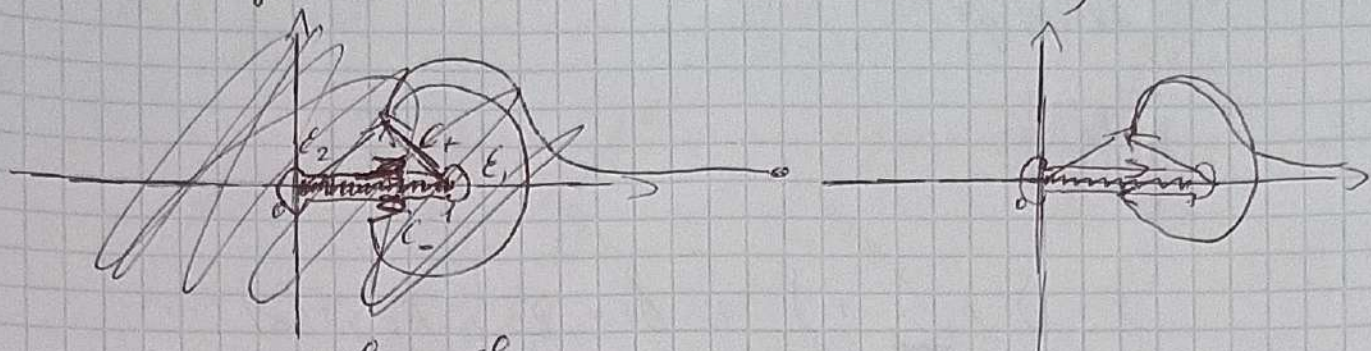
$$\oint = 2I + \int_0^{+\infty} \frac{2\pi i}{x^2+1} dx = \frac{i\pi^2}{2} \Rightarrow I = 0$$

$$I(\alpha) = \int_0^1 \frac{x^\alpha (1-x)^{2-\alpha}}{x+1} dx$$

6.2

Согласно б о мы ~~д~~ $\alpha \geq 0$

Помогательное мнимое сечение $x=1; 0$



$$\oint = \int_{\epsilon_+} + \int_{\epsilon_-} + \int_{C_+} + \int_{C_-} = 2\pi i (\text{Res}_{z=-1} + \text{Res}_{z=\infty})$$

$$f(x-i0) = f(x+i0) \cdot e^{-2\pi i(2-\alpha)} = e^{2\pi i\alpha}$$

$$I - I e^{-2\pi i(2-\alpha)} = 2\pi i (\text{Res}_{z=-1} + \text{Res}_{z=\infty})$$

$$\text{Res}_{z=-1} f(z) = 4 e^{i\pi\alpha} 2^{2-\alpha} e^{i\pi\alpha}$$

$$f(-1) = \frac{|f(-1)|}{|f(x_0+i0)|} f(x_0+i0) e^{i4\arg f} =$$

$$= \frac{1}{2} |f(-1)| \cdot e^{i\pi\alpha} = e^{i\pi\alpha} 2^{2-\alpha}$$

$$|f(-1)| = |-1| \cdot |2^{2-\alpha}| = 2^{2-\alpha}$$

$$\text{Res}_{z=\infty} f(z) = 0$$

$$I (1 - e^{-2\pi i(2-\alpha)}) = 2\pi i \cdot 4 e^{i\pi\alpha}$$

$$I (-2i \sin \pi\alpha) = 2\pi i \cdot 4 e^{i\pi\alpha}$$

$$I = \frac{\pi}{\sin \pi\alpha} \cdot 4$$

$$J(z) = \lim_{z \rightarrow \infty} \frac{f(z)}{z} = \lim_{z \rightarrow \infty} \frac{f(z)}{z} \cdot e^{-i\pi(2-d)} =$$

$$= |f(z)| \cdot e^{i\pi d}$$

$$\text{Res}_{z \rightarrow \infty} \left(\frac{z^d (z-1)^{2-d}}{z+1} e^{i\pi d} \right) = \frac{z \left(1 - \frac{1}{z}\right)^{2-d}}{\left(1 + \frac{1}{z}\right)} =$$

$$= \frac{z \left(1 - \frac{2-d}{z} + \frac{(2-d)(1-d)}{2z^2}\right)}{\left(1 + \frac{1}{z}\right)} =$$

$$z \left(1 - \frac{2-d}{z} + \frac{(2-d)(1-d)}{2z^2}\right) \left(1 - \frac{1}{z} + \frac{1}{z^2}\right) =$$

$$\frac{(2-d)(1-d)}{2} = \frac{(2-d)(1-d)}{2}$$

$$= \frac{(2-d)(1-d)}{2} + \frac{1}{2} + \dots = \frac{(2-d)(1-d) + 1}{2}$$

$$C_{-1} = \frac{d^2 - 2d - d + 2 + 1}{2} = \frac{d^2 - 3d + 3}{2}$$

$$I(1 - e^{2\pi i d}) = 2\pi i \left(\frac{z^{2-d}}{z} e^{i\pi d} + \frac{d^2 - 3d + 4}{2} e^{i\pi d} \right)$$

$$I(-2i \sin \pi d) = -2\pi i \sin \pi d$$

$$I = \frac{\pi}{\sin \pi d} \left(\frac{1}{2} d^2 - \frac{3}{2} d + 2 + \frac{2-d}{2} \right)$$

$$= \frac{8 + d^2 - 5d}{2} + \dots$$

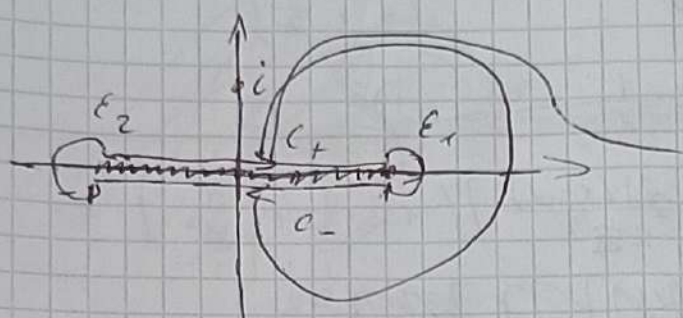
$$C_{-1} = - \frac{8 + d^2 - 5d}{2}$$

$$I(1 - e^{2\pi i d}) = 2\pi i \left(\frac{2-d}{2} - \frac{8 + d^2 - 5d}{2} \right) e^{i\pi d}$$

$$I = \frac{\pi}{\sin \pi d} \left(\frac{8 + d^2 - 5d}{2} - 2^{2-d} \right)$$

6.3

$$\int_{-1}^1 \frac{(1-x)^\alpha (1+x)^{1-\alpha}}{(x^2+1)} dx = \int_{-1}^1 \frac{(1-x)^\alpha (1+x)^{1-\alpha}}{(x+i)(x-i)} dx$$



$$\oint = \int_{C_+} + \int_{C_-} + \int_{\epsilon_2} + \int_{\epsilon_1} = 2\pi i (\text{Res}_{z=i} + \text{Res}_{z=-i} + \text{Res}_{z=\infty})$$

$$f(x-i0) = f(x+i0) \cdot e^{-2\pi i \alpha}$$

$$I(1 - e^{-2\pi i \alpha}) = 2\pi i (\dots)$$

$$\text{Res}_{z=i} = \frac{(1-z)^\alpha (1+z)^{1-\alpha}}{z+i} \Big|_{z=i}$$

$$f(i) = |f(i)| \cdot e^{-\frac{\pi}{4}} \left(i \left(-\frac{\pi}{4} \alpha \right) + \frac{\pi}{4} (1-\alpha) \right) =$$

$$= |f(i)| e^{i\frac{\pi}{4}(1-2\alpha)} = \sqrt{2} |1-i|^\alpha |1+i|^{1-\alpha} e^{i\frac{\pi}{4}(1-2\alpha)} =$$

$$= \frac{\sqrt{2}}{2} e^{i\frac{\pi}{4}(1-2\alpha)} \sqrt{2} e^{i\frac{\pi}{4}(1-2\alpha)}$$

$$\text{Res}_{z=i} = \frac{\sqrt{2} e^{i\frac{\pi}{4}(1-2\alpha)}}{2i}$$

$$\text{Res}_{z=-i} = \frac{\sqrt{2} e^{-i\frac{\pi}{4}(1-2\alpha)}}{-2i}$$

$$f(z) \Big|_{z \rightarrow \infty} = |f(x+i0)| e^{-i\pi \alpha} =$$

$$= (x-1)^\alpha (1+x)^{1-\alpha} e^{-i\pi \alpha}$$

+

$$f(z) = \frac{(z-1)^{\alpha} (z+1)^{1-\alpha}}{z^2+1} = \frac{z^{\alpha} z^{\frac{1-\alpha}{2}} \left(1 - \frac{1}{z}\right)^{\alpha} \left(1 + \frac{1}{z}\right)^{1-\alpha}}{z^2 \left(1 + \frac{1}{z^2}\right)} =$$

$$\frac{1}{z} \left(1 + \dots\right) \quad C_1 = 1$$

$$I(1 - e^{-2\pi i \alpha}) = 2\pi i \left(-1 + \frac{\sqrt{2}}{2i} \left(e^{i\frac{\pi}{4}(1-2\alpha)} - e^{i\frac{\pi}{4}(1-2\alpha)} \right) \right) =$$

$$e^{-i\pi\alpha} I(2i \sin \pi\alpha) = 2\pi i \left(-1 + \sqrt{2} \sin \frac{\pi}{4} (1-2\alpha) \right)$$

$$I e^{-i\pi\alpha} (2i \sin \pi\alpha) = 2\pi i \left(-1 + \frac{\sqrt{2}}{2i} \left(e^{i(\frac{\pi}{4} + \frac{\pi}{2}\alpha)} + e^{-i(\frac{\pi}{4} + \frac{\pi}{2}\alpha)} \right) \right)$$

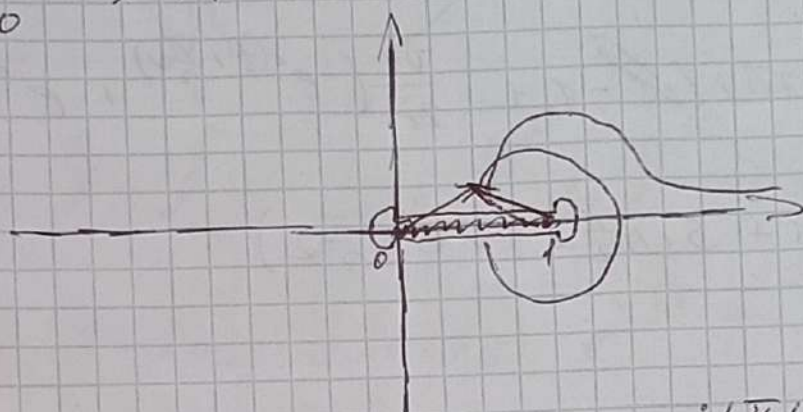
$$I = \frac{\pi}{\sin \pi\alpha} \left(-1 + \sin \frac{\pi}{2}\alpha + \cos \frac{\pi}{2}\alpha \right)$$

6.5

$$I = \int_0^1 \ln \frac{1-x}{x} \frac{dx}{x^2+1}$$

$$\frac{d}{dd} \left(\frac{1-x}{x} \right)^d \Big|_{d=0} = (1-x)^d \cdot x^{-d}$$

$$\int_0^1 \frac{(1-x)^d x^{-d}}{x^2+1} dx$$



$$\begin{aligned} \text{Res}_i &= f(i) = \left| f(i) \right| \cdot e^{i \left(\frac{\pi}{2} (-d) - \frac{\pi}{4} d \right)} e^{-\frac{3\pi}{4} d} \\ &= |(1-i)^d i^{-d}| e^{-i \left(\frac{\pi}{2} + \frac{\pi}{4} d \right)} = \sqrt{2}^d e^{-i \left(\frac{\pi}{2} + \frac{\pi}{4} d \right)} \frac{1}{2i} \end{aligned}$$

$$f(-i) = \sqrt{2}^d \cdot e^{i \left(\frac{\pi}{2} - \frac{\pi}{4} d \right)} \cdot \frac{1}{-2i} \cdot e^{\frac{3\pi}{4} d}$$

$$f(z) \Big|_{z \rightarrow \infty} = (x-1)^d \cdot x^{-d} \cdot e^{-i\pi d} e^{-i\pi d}$$

$$\begin{aligned} \text{Res}_\infty &= \frac{x^d \bar{x}^d \left(1 - \frac{1}{x} \right)^d}{x^2 \left(1 + \frac{1}{x^2} \right)} = \frac{1}{x^2} \left(1 - \frac{d}{x} + \dots \right) \left(1 + \frac{1}{x^2} + \dots \right) \\ &= 0 \end{aligned}$$

$$I (1 - e^{-2\pi i d}) = 2\pi i \left(\frac{\sqrt{2}^d}{2i} \left(\frac{e^{i \left(\frac{\pi}{2} - \frac{\pi}{4} d \right)}}{e^{i \left(\frac{\pi}{2} + \frac{\pi}{4} d \right)}} - e^{\frac{3\pi}{4} d} \right) \right)$$

$$I \sin \pi d = \pi \left(\sqrt{2}^d \frac{\sin \frac{3\pi}{4} d}{\sin \frac{\pi}{4} d} \right)$$

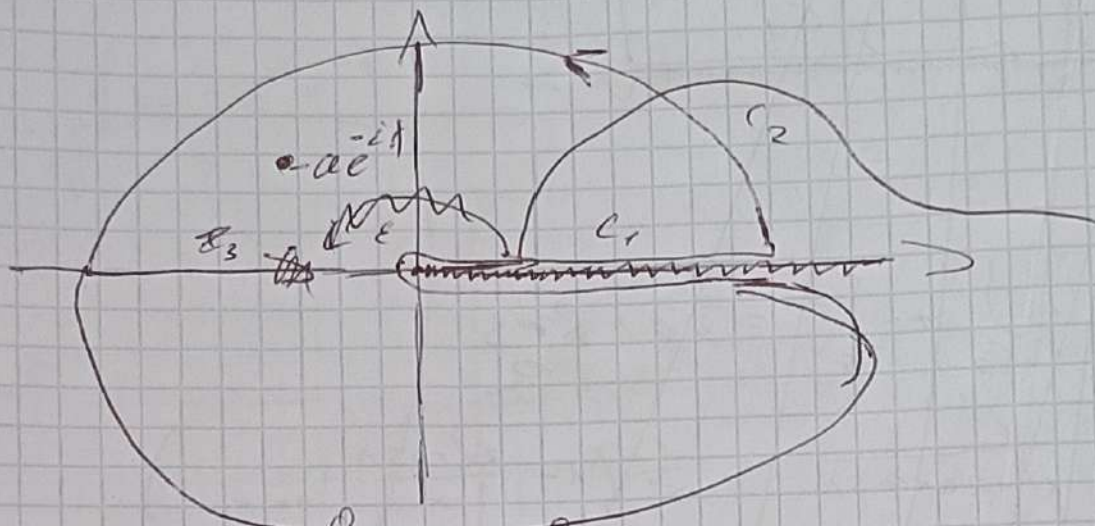
$$I = \frac{\pi}{\sin \pi d} \left(\sqrt{2}^d \frac{\sin \frac{3\pi}{4} d}{\sin \frac{\pi}{4} d} \right)$$

$$\frac{dI}{dd} \Big|_{d=0} = \frac{1}{8} \pi \ln 2$$

6.6

$$I = \int_0^{+\infty} \frac{\ln x dx}{x^2 + 2ax \cos \lambda + a^2}$$

$$\int_0^{+\infty} \frac{x^2 // \varphi(x)}{x^2 + 2ax \cos \lambda + a^2} = \frac{x^2}{(x - (-ae^{i\lambda}))(x + ae^{-i\lambda})}$$



$$\oint = \int_{C_1} + \int_{C_2} + \int_{C_3} = -2\pi i \sum \text{Res}$$

$$\varphi(z(x)) \Big|_{x \rightarrow \infty} = x^2$$

$$\varphi(z(x)) \Big|_{x \rightarrow \infty} = \frac{x^2}{x^2 + 2ax \cos \lambda} = \frac{x^{2-2}}{(1 + \frac{2a \cos \lambda}{x})}$$

$$\text{Res}_{\infty} = 0$$

$$\text{Res}_{-ae^{i\lambda}} =$$

$$\varphi(-ae^{i\lambda}) = |a^2 e^{-i\lambda\lambda}| \cdot e^{i(\pi-\lambda)\lambda} = a^2 e^{i(\pi-\lambda)\lambda}$$

$$\text{Res}_{-ae^{i\lambda}} = \frac{a^2 e^{i(\pi-\lambda)\lambda}}{a(e^{i\lambda} - e^{-i\lambda})} = \frac{a^2 e^{i(\pi-\lambda)\lambda}}{2ia \sin \lambda}$$

$$\cancel{f(x)} = \cancel{f(x+i0)} e^{i\pi\lambda}$$

$$f(x-i0) = f(x+i0) e^{2\pi i \lambda}$$

$$\text{Res}_{z=-ae^{i\lambda}} = \frac{a^2 e^{i(\pi+\lambda)\alpha}}{-2ia \sin \lambda}$$

$$\text{Res}_{-ae^{i\lambda}} + \text{Res}_{-ae^{-i\lambda}} = \frac{a^{2-\alpha}}{2i \sin \lambda} (e^{i(\pi-1)\alpha} - e^{i(\pi+1)\alpha})$$

$$I(1 - e^{2\pi i \alpha}) = 2\pi i \frac{\pi a^{2-\alpha}}{\sin \lambda} (\dots)$$

$$e^{i\pi\alpha} I(2i \sin \lambda) = \frac{\pi a^{2-\alpha}}{\sin \lambda} (2i \sin \lambda)$$

$$I(1 - e^{2\pi i \alpha}) = \frac{\pi a^{2-\alpha}}{\sin \lambda} 2i \sin \lambda$$

$$I = \frac{\pi a^{2-\alpha}}{\sin \lambda}$$

$$I = \frac{\pi a^{2-\alpha}}{\sin \lambda} \cdot \frac{\sin \lambda \alpha}{\sin 2\pi}$$

$$\lim_{\alpha \rightarrow 0} \frac{dI}{d\alpha} = \frac{\lambda \ln a}{a \sin \lambda} = I$$

6. 4

$$I = \int_0^{\infty} \frac{\ln x}{x^{1/3} (x+1)^2} dx$$

more

$$\int_0^{\infty} \ln x \sin x dx$$

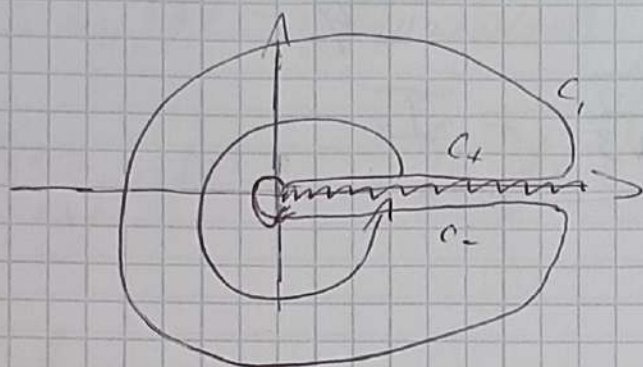
5. 10.

$$\begin{aligned} \int_1^2 \left(\frac{1}{w} + \frac{2}{w^3} \right) \cos w dw &= \int_1^2 \frac{\cos w}{w} + \int_1^2 \frac{2 \cos w}{w^3} dw \\ &= \int_1^2 \frac{\cos w}{w} + 2 \left(\dots + \int_1^2 \frac{\sin w}{2w^2} \right) = \\ &= \int_1^2 \frac{\cos w}{w} + 2 \left(\dots + \int_1^2 \frac{\cos w}{2w} dw \right) = \\ &= \int_1^2 \left(\frac{\cos w}{w} - \frac{2 \cos w}{2w} \right) dw \Rightarrow 2 = 2 \end{aligned}$$

6.4

$$I = \int_0^{\infty} \frac{\ln x}{x^{1/3}(x+1)^2} dx$$

$$I' = \int_0^{\infty} \frac{x^{\alpha - \frac{1}{3}}}{(x+1)^2} dx$$



$$f(-1) = |f(-1)| \cdot e^{+i\pi(\alpha - \frac{1}{3})} = e^{2\pi i(\alpha - \frac{1}{3})}$$

$$\oint = \int_{C_+} + \int_{C_-} + \int_{C_1} + \int_{C_2} = 2\pi i \operatorname{Res}_{z=-1}$$

$$\operatorname{Res}_{z=-1} = \frac{d}{dz} \frac{z^{\alpha - \frac{1}{3}}}{(z+1)^2} = \left(\alpha - \frac{1}{3}\right) z^{\alpha - \frac{4}{3}} \Big|_{z=-1} = -\left(\alpha - \frac{1}{3}\right) e^{i\pi(\alpha - \frac{1}{3})}$$

$$f(x+io) = f(x-io) e^{i2\pi(\alpha - \frac{1}{3})} \Rightarrow -\left(\alpha - \frac{1}{3}\right) e^{i\pi(\alpha - \frac{1}{3})} = -\left(\alpha - \frac{1}{3}\right) e^{i\pi(\alpha - \frac{1}{3})} e^{i2\pi(\alpha - \frac{1}{3})}$$

$$I' (1 - e^{i2\pi(\alpha - \frac{1}{3})}) = -2\pi i \left(\alpha - \frac{1}{3}\right) e^{i\pi(\alpha - \frac{1}{3})}$$

$$e^{i\pi(\alpha - \frac{1}{3})} I' (-2i \sin \pi(\alpha - \frac{1}{3})) = -2\pi i \left(\alpha - \frac{1}{3}\right) e^{i\pi(\alpha - \frac{1}{3})}$$

$$I' = \frac{\pi}{\sin \pi(\alpha - \frac{1}{3})} \left(\alpha - \frac{1}{3}\right)$$

$$\lim_{\alpha \rightarrow 0} \frac{dI'}{d\alpha} = \left(\alpha - \frac{1}{3}\right) - \frac{\frac{1}{3} \sin \pi(\alpha - \frac{1}{3}) + (\alpha - \frac{1}{3}) \cos \pi(\alpha - \frac{1}{3})}{\sin^2 \pi(\alpha - \frac{1}{3})} \Big|_{\alpha=0} =$$

$$= \frac{\frac{1}{3} \pi(-\frac{1}{3}) - \frac{1}{3}}{\pi^2 \frac{1}{9}} =$$

$$\frac{dI}{d\alpha} = -\pi\left(\alpha - \frac{1}{3}\right) \cdot \frac{1}{\sin^2 \pi\left(\alpha - \frac{1}{3}\right)} \cdot \pi \cos \pi\left(\alpha - \frac{1}{3}\right) + \frac{\pi}{\sin \pi\left(\alpha - \frac{1}{3}\right)}$$

$$\left. \frac{dI}{d\alpha} \right|_{\alpha \rightarrow 0} = \frac{\frac{1}{3}\pi}{\frac{1}{2} \cdot \frac{1}{9}} + \frac{\frac{1}{3}\pi^2}{\frac{3}{2}} \cdot \frac{5}{2} +$$

$$- \frac{\pi}{\frac{\sqrt{3}}{2}} = \frac{2\pi^2}{9} - \frac{2\pi}{\sqrt{3}} = I$$

5.1

$$I_0(z) = \int_0^{\infty} t^z e^{-t} dt$$

$$I_1(z) = \frac{1}{z+1} \int_0^{\infty} t^{z+1} e^{-t} dt$$

$$I_2(z) = \frac{1}{(z+2)(z+1)} \int_0^{\infty} t^{z+2} e^{-t} dt$$

$$I_3(z) = \frac{1}{(z+3)(z+2)(z+1)} \int_0^{\infty} t^{z+3} e^{-t} dt$$

$$\text{Res}_{z=-3} I_3(z) = \frac{1}{2} \int_0^{\infty} e^{-t} dt = \frac{1}{2}$$

5.2

$$\begin{aligned} f(z) &= \sum_{n=0}^{\infty} \frac{z^n}{(n+1)(n+2)} = \sum_{n=1}^{\infty} \frac{z^n}{n} - \sum_{n=2}^{\infty} \frac{z^n}{n} = \frac{1}{z} \sum_{n=1}^{\infty} \frac{z^n}{n} - \\ &= \frac{1}{z^2} \sum_{n=2}^{\infty} \frac{z^n}{n} = -\frac{1}{z} \ln(1-z) + \frac{1}{z^2} (\ln(1-z) + z) = \\ &= \frac{\ln(1-z) + z - z \ln(1-z)}{z^2} \end{aligned}$$

$$\gamma_1: \ln z = \ln|-1| - i\pi = -i\pi$$

$$f_{\gamma_1}(z) = \frac{1}{z} + \frac{i\pi}{4}$$

$$\gamma_2: \ln z = i\pi$$

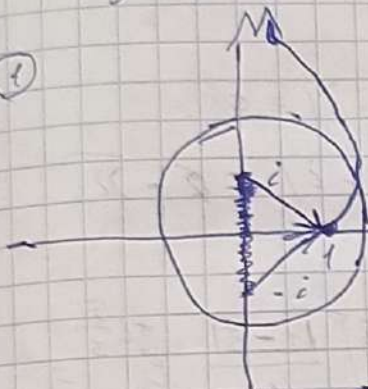
$$f_{\gamma_2}(z) = \frac{1}{z} - \frac{i\pi}{4}$$

$$I_{1/2} = 0$$

4.3.

$$f(z) = \sqrt{1+z^2} = \sqrt{(z+i)(z-i)}$$

①



$$f(1) = \sqrt{2}$$

$$\int f(z) dz = -\text{Res}_{z=i\infty} f(z) \cdot 2\pi i$$

$$z = iy$$

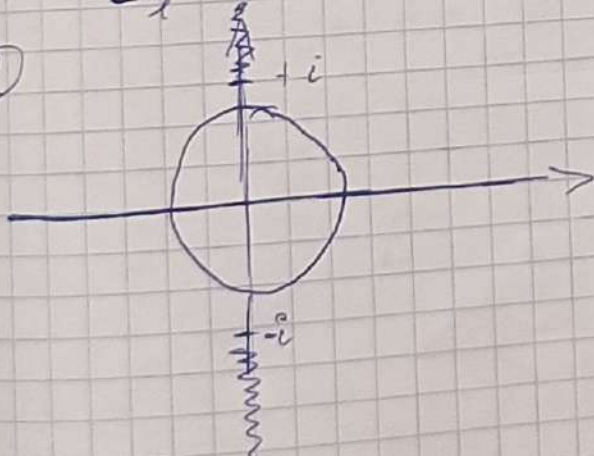
$$f(iy) = \sqrt{\frac{1-y^2}{2}} \sqrt{2} e^{\frac{1}{2}i\pi \left(\frac{\pi}{4} + 2\pi n - \frac{\pi}{4} \right)} =$$

$$= (1-y^2)^{1/2} \cdot e^{i\frac{\pi}{2}} = iy \left(1 - \frac{1}{y^2} \right)^{1/2} = iy \left(1 - \frac{1}{2y^2} + \dots \right) =$$

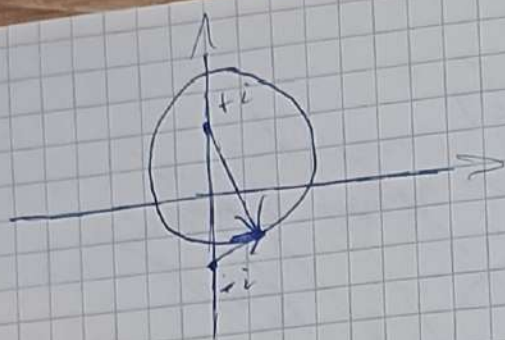
$$C_{-1} = -\frac{1}{2}i$$

$$I_1 = \pi i$$

②



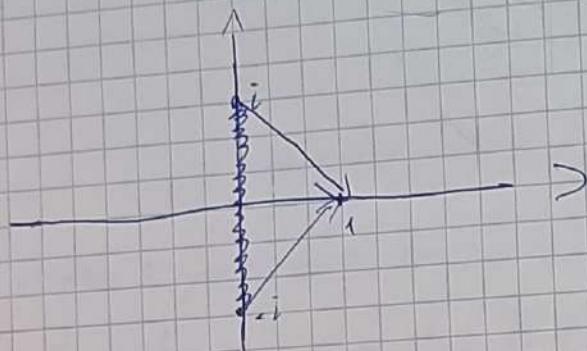
$I_2 = 0$ (выпуск не
особых точек и
ф-ия аналитична и огранич.)



$$\Delta \arg(z+i) = 0$$

$$\Delta \arg(z-i) = 2\pi$$

$$f = f_0 e^{i\pi} = -f_0$$



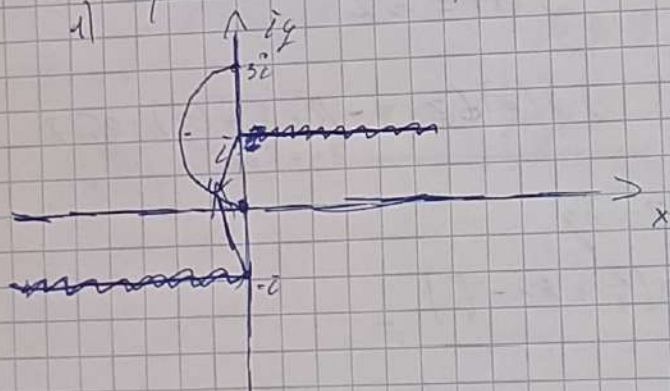
$$f(1) = \sqrt{2}$$

$$f(-1) = \sqrt{2} \cdot e^{i(\frac{\pi}{2} + \frac{3\pi}{2})}$$

$$= \sqrt{2} e^{i\pi} = -\sqrt{2}$$

4.6

$$1) \quad \varphi(z) = \sqrt[3]{1+z^2}; \quad \varphi(0) = 1$$

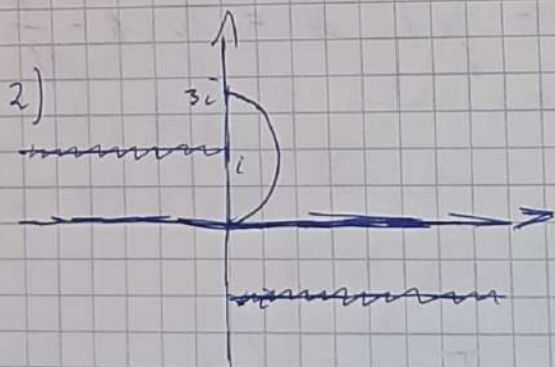


$$\Delta \arg(z+i) = 0$$

$$\Delta \arg(z-i) = 2\pi$$

$$\varphi(3i) = \sqrt[3]{|1-9|} e^{i\frac{\pi}{3}} =$$

$$= 2 e^{-i\frac{\pi}{3}} = 2 \left(\frac{1}{2} - i\frac{\sqrt{3}}{2} \right)$$

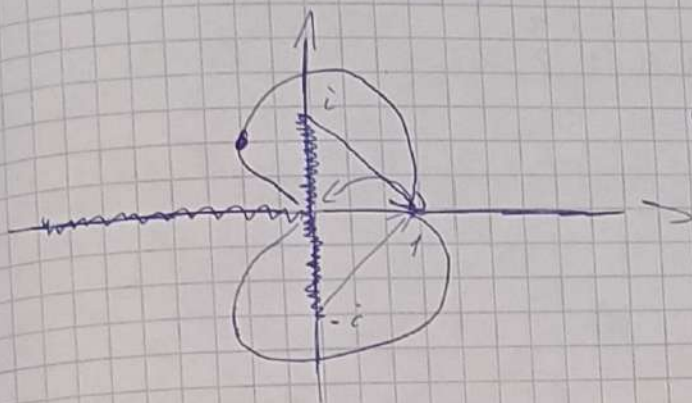


$$\varphi(3i) = 2 \cdot e^{i\frac{\pi}{3}}$$

4.8

$$f(x) = \ln(1+x^2)^{1/2} \quad \text{---} \quad \frac{1}{2} \ln(1+x^2)$$

$$F(z) = \frac{1}{2} \ln(z+i)(z-i)$$



$$F(1) = \frac{1}{2} \ln(2)$$

$$\begin{aligned} \textcircled{1} \lim_{\epsilon \rightarrow 0} F(\epsilon) &= \frac{1}{2} \left(\ln \left| \frac{g(\epsilon)}{g_0} \right| + i \Delta \arg + \right. \\ &\quad \left. + F_0 \right) = \frac{1}{2} \ln g(\epsilon) = \\ &= \frac{1}{2} \ln 1 = 0. \end{aligned}$$

$$\textcircled{2} \lim_{\epsilon \rightarrow 0} F(\epsilon e^{3\pi i/4}) +$$

$$\Delta \arg g = \frac{\pi}{4} + 2\pi - \frac{\pi}{4} = 2\pi$$

$$F(\epsilon e^{3\pi i/4}) = \ln(g(\epsilon)) + i \frac{3\pi}{4} = \ln(g(\epsilon)) + i \frac{3\pi}{4}$$

$$= \frac{1}{2} (\ln(g(\epsilon)) + i 2\pi) = i\pi$$

$\textcircled{3}$ Answer no

$$F(\epsilon e^{3\pi i/4}) = \frac{1}{2} (\ln(g(\epsilon)) - 2\pi i) = -\pi i$$

4.9

$$\cancel{f(z)} = z^2(z-1)^3 \quad f = z^2(z-1)^3$$

$$N(1, 1) = 0 \quad (f = z^2(z-1)^3)$$

$$N(1, \frac{1}{2}) = 2 \quad (z-1; z=\infty)$$

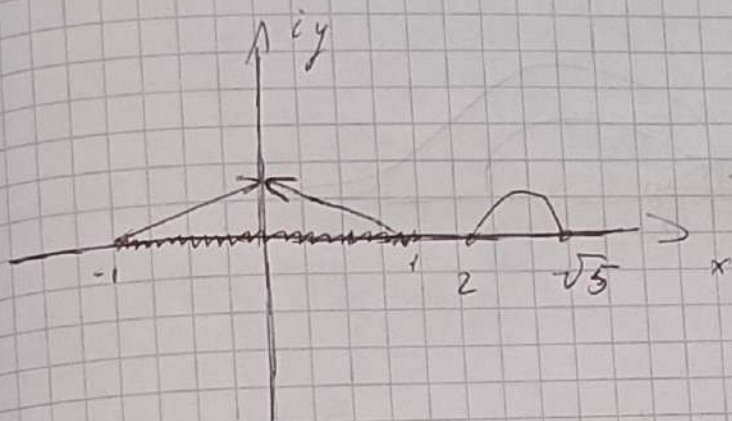
$$N(\frac{1}{2}, \frac{1}{3}) = 3 \quad \left(\cancel{f} = f = z^{1/2}(z-1)^{1/3} \quad \begin{matrix} z=0, z=1, \\ z=\infty \end{matrix} \right)$$

$$N(\frac{2}{3}, \frac{1}{3}) = 2 \quad (f = z^{2/3}(z-1)^{1/3} \quad z=0, z=1)$$

4/10

$$f(z)^2 - 2f(z) + z^2 = 0$$

$$f(z) = 1 \pm \sqrt{1-z^2} = 1 \pm \sqrt{(1-z)(1+z)} = 1 \pm i\sqrt{(z-1)(z+1)}$$



$$y(z) = \ln(1 - f(z))$$

$$S(\theta(z)) = \sqrt{3}$$

$$S(z) = (z-1)(z+1)$$

$$\theta(z) = \sqrt{(z-1)(z+1)}$$

$$\theta(z) = \sqrt{1}$$

$$\theta(\sqrt{5}) = \sqrt{\frac{|S(\sqrt{5})|}{|S(2)|}} \cdot |\theta(2)| \cdot e^{\frac{1}{2}i \cdot 0} = 2$$

$$g(z) = \ln(z \pm i\sqrt{3})$$

$$g(\sqrt{5}) = \ln \left| \frac{2 \pm i2}{2 \pm i\sqrt{3}} \right| + \ln(z \pm i\sqrt{3}) + i \cdot 0 =$$

$$= \ln(2 \pm i \cdot 2) = \ln(\sqrt{2} e^{\pm i \frac{\pi}{4}}) = \ln \sqrt{2} \pm i \frac{\pi}{4}$$

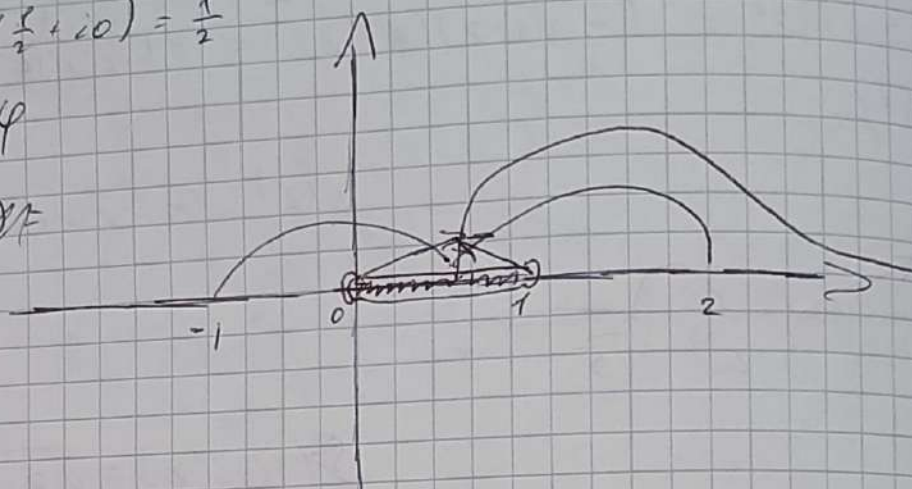
4.4.

$$\varphi(z) = z^\mu (1-z)^{1-\mu}$$

$$\varphi\left(\frac{1}{2} + i0\right) = \frac{1}{2}$$

$$\varphi(z) = \varphi$$

$$\varphi(z) \neq \varphi$$



$$1) \varphi(z) = \frac{1}{2} \left| \frac{z}{\frac{1}{2} + i0} \right|^\mu \cdot \left| \frac{1-z}{-\frac{1}{2} - i0} \right|^{1-\mu} e^{-i\pi(1-\mu)} = 2^{-\mu} e^{i\pi(\mu-1)}$$

$$2) \varphi(-1) = \frac{1}{2} \left| \frac{-1}{\frac{1}{2} + i0} \right|^\mu \cdot \left| \frac{2}{-\frac{1}{2} - i0} \right|^{1-\mu} e^{i\pi\mu} = 2^{1-\mu} e^{i\pi\mu}$$

$$3) \lim_{z \rightarrow \infty} \frac{\varphi(z)}{z} = \lim_{z \rightarrow \infty} (z^{\mu-1} (1-z)^{1-\mu})$$

$$\varphi(z) \neq \varphi$$

$$\frac{\varphi\left(\frac{1}{2} + i0\right)}{\frac{1}{2} + i0} = 1$$

$$\left| \frac{\varphi(z)}{z} \right|_{z \rightarrow \infty} = \left| \frac{z^{\mu-1}}{\frac{1}{2} + i0} \cdot \frac{1-z}{-\frac{1}{2} - i0} \right|^{1-\mu} e^{i\pi(\mu-1)} =$$

$$e^{i\pi(\mu-1)} \cdot \left| \frac{1-z}{z} \right|^{1-\mu} = e^{i\pi(\mu-1)}$$