

18.09.22  
1)  $e^{-\frac{1}{z^2}} \rightarrow e^{-\frac{1}{z^2}}$

$z^2 = \frac{1}{t^2}$

$e^{-\frac{1}{z^2}} = 1 - \frac{1}{z^2} + \frac{1}{2!z^4} - \frac{1}{3!z^6} + \dots$

$t=0$ ;  $z=\infty$  - угу өсөөдөг ~~нэгдсэн~~ нөхцөл.

3)  $\sin \frac{\pi}{z^2} (a=0)$

$\lim_{z \rightarrow 0} (\sin \frac{\pi}{z^2}) \neq$  угу. өсөөдөг нөхцөл.

19.15

1)  $\frac{(1+z^2)^2}{1-z^2} = \frac{(1+z^2)^2}{(1-z)(1+z)} = \left\{ \begin{array}{l} 1-z=\varepsilon \\ z=1-\varepsilon \end{array} \right\} =$   
 $= \frac{(1+1-\varepsilon)^2}{\varepsilon(2-\varepsilon)} = \frac{1}{\varepsilon} \frac{(\varepsilon^2 - 2\varepsilon + 2)^2}{(2-\varepsilon)}$   
 (а)  $\frac{1}{\varepsilon}$  - үргэлж нэгдсэн, өсөөдөг  
 (б)  $\frac{(\varepsilon^2 - 2\varepsilon + 2)^2}{(2-\varepsilon)}$  - үргэлж нэгдсэн, өсөөдөг

2)  $\cot z = \frac{\cos z}{\sin z} = \frac{(1 - \frac{z^2}{2!} + \frac{z^4}{4!} - \dots)}{z(1 - \frac{z^2}{3!} + \frac{z^4}{5!} - \dots)}$

$= \frac{1}{z} (1 - \frac{z^2}{2!} + \frac{z^4}{4!} - \dots) (1 + (\frac{z^2}{3!} - \frac{z^4}{5!} + \dots) + \dots)^2 + \dots$   
 $\Rightarrow \frac{1}{z} + \pi k$  - нэгдсэн нөхцөл,  $\frac{\pi}{2} + \pi k$  - нэгдсэн

3)  $z \cot^2 z = \frac{z^3 (1 - \frac{z^2}{2!} + \frac{z^4}{4!} - \dots)^2}{z (\frac{1}{z} - \frac{z}{2!} + \frac{z^3}{4!} - \dots)} = z$   
 $z=0$ ;  
 $z=\pi k$  - нэгдсэн.

$z = z - \frac{\pi}{2}$   
 $z = z - \frac{\pi}{2}$

$(\frac{\pi}{2} + \frac{\pi}{2}) \cot^2 z = (\frac{\pi}{2} + \frac{\pi}{2}) \cdot \frac{1}{z^2} (1 - \frac{z^2}{2!} + \frac{z^4}{4!} - \dots) (1 + (\frac{z^2}{3!} - \frac{z^4}{5!} + \dots) + \dots)^2 + \dots$   
 $= \frac{\pi}{2} \frac{1}{z^2} (1 - \frac{z^2}{2!} + \frac{z^4}{4!} - \dots) + \dots$   
 $+ \frac{1}{z} (1 - \frac{z^2}{2!} + \frac{z^4}{4!} - \dots) + \dots$   
 (а)  $\frac{\pi}{2}$  - нэгдсэн, өсөөдөг  
 $z = \frac{\pi}{2} + \pi k$  - нэгдсэн нөхцөл

$$3) 4) \sum_{n=-\infty}^{+\infty} 2^{-n^2} (z+1)^n =$$

$$= \sum_{n=0}^{\infty} \frac{2^{-n^2}}{(z+1)^n} + \sum_{n=1}^{\infty} \frac{(z+1)^n}{2^{n^2}} =$$

$$= \sum_{n=0}^{\infty} \frac{1}{(z+1)^n} + \sum_{n=1}^{\infty} \frac{(z+1)^n}{2^{n^2}}$$

$$\lim_{n \rightarrow \infty} \sqrt[n]{\frac{(z+1)^n}{2^{n^2}}} = \frac{z+1}{2^n} = 0 < 1$$

$$\lim_{n \rightarrow \infty} \sqrt[n]{\frac{1}{2^{n^2} (z+1)^n}} = 0 < 1$$

$$0 < |z+1| < \infty$$



$$-\frac{1}{t(t-3)} - \frac{1}{3t(1-\frac{t}{3})} = \sum_{n=0}^{\infty} \frac{t^{n+1}}{3^{n+1}}$$

$$\begin{aligned} \frac{(t-1)^3}{t(t-3)} &= \frac{1}{3t} + \frac{1}{9} + \frac{2}{27} + \sum_{n=0}^{\infty} \frac{t^{n+1}}{3^{n+1}} - \sum_{n=0}^{\infty} \frac{t^{n+2}}{3^{n+1}} - 1 \\ &+ \sum_{n=1}^{\infty} \frac{t^{n+1}}{3^n} + t - \sum_{n=2}^{\infty} \frac{t^n}{3^n} - 1 - \frac{t}{3} + = \\ &= \frac{1}{3t} - \frac{8}{9} + \frac{19}{27}t + \sum_{n=2}^{\infty} \left( \frac{t^n}{3^{n+2}} - \left(\frac{t}{3}\right)^n + \frac{t^n}{3^{n-1}} - \frac{t^n}{3^{n-1}} \right) \quad (t = z+1) \end{aligned}$$

$$8) \quad \frac{1}{(z^2-1)(z^2+4)} = \frac{1}{5} \left( \frac{1}{z^2-1} - \frac{1}{z^2+4} \right)$$

$$\frac{1}{z^2-1} = \frac{1}{z^2} \frac{1}{1-\frac{1}{z^2}} = \sum_{n=0}^{\infty} \frac{1}{z^{2n+2}}$$

$$\frac{1}{z^2+4} = \frac{1}{z^2(1+\frac{4}{z^2})} = \sum_{n=0}^{\infty} (-1)^n \frac{4^n}{z^{2n+2}}$$

$$\frac{1}{(z^2-1)(z^2+4)} = \frac{1}{5} \sum_{n=0}^{\infty} \frac{(-1)^n 4^n + 1}{z^{2n+2}}$$

20.16

$$1) \quad z^3 e^{\frac{1}{z}} = z^3 + z^2 + \frac{1}{2}z + \frac{1}{6} + \sum_{n=1}^{\infty} \frac{z^{-n}}{(n+3)!}$$

$$\begin{aligned} 2) \quad z^2 \sin\left(\pi \frac{z+1}{z}\right) &= z^2 \sin\left(\pi + \frac{\pi}{z}\right) = -z^2 \sin\left(\frac{\pi}{z}\right) = \\ &= -z^2 \left( \frac{\pi}{z} - \frac{\left(\frac{\pi}{z}\right)^3}{3!} + \frac{\left(\frac{\pi}{z}\right)^5}{5!} - \dots \right) \end{aligned}$$

$$6) \frac{ze^{iz}}{(z^2+b^2)^2} = \frac{ze^{iz}}{(z-ib)^2(z+ib)^2} = \frac{(z-ib)e^{iz} + ibe^{iz}}{(z-ib)^2(z+ib)^2}$$

$$= \frac{ze^{iz}}{(z-ib)^2(z+ib)^2} + \frac{e^{iz}}{(z-ib)(z+ib)} \approx$$

$$z = -\frac{ib}{4b(z-ib)^2} - \frac{e^{-6}}{4b^2} \frac{1}{z-ib} = -\frac{(ib+z-ib)e^{-6}}{4b^2(z-ib)^2} =$$

$$= -\frac{ie^{-6}}{4b} \frac{1}{(z-ib)^2}$$

$$7) \frac{z}{(z^2+b^2)^2} = \frac{1}{(z-ib)(z+ib)^2} + \frac{ib}{(z+ib)^2(z+ib)^2} \approx$$

$$\approx -\frac{1}{4b^2(z-ib)} - \frac{i}{4b(z-ib)^2} = -\frac{i}{4b} \cdot \frac{1}{(z-ib)^2}$$



20.01

$$1) \sum_{n=-\infty}^{\infty} 2^{-|n|} \cdot z^n = \sum_{n=-\infty}^{\infty} \frac{1}{2^{|n|}} \cdot z^n =$$

$$= \sum_{n=0}^{\infty} \frac{z^n}{2^n} + \sum_{n=1}^{\infty} \left(\frac{z}{2}\right)^n =$$

$$\neq \text{~~scribble~~} \quad n = -m$$

$$= \sum_{m=0}^{\infty} \frac{1}{(2z)^m} + \sum_{n=1}^{\infty} \left(\frac{z}{2}\right)^n$$

$$\begin{cases} \left|\frac{1}{2z}\right| < 1 \\ \left|\frac{z}{2}\right| < 1 \end{cases} \Rightarrow \frac{1}{2} < |z| < 2$$

$$2) \sum_{n=-\infty}^{\infty} \frac{z^n}{3^n+1} = \text{~~scribble~~} \Rightarrow \text{~~scribble~~}$$

$$\begin{cases} \left|\frac{z}{3}\right| < 1 \\ \left|\frac{1}{z}\right| < 1 \end{cases}$$

$$\begin{aligned} &= \sum_{n=0}^{\infty} \frac{z^n}{3^n+1} + \sum_{n=1}^{\infty} \frac{z^n}{3^n+1} = \sum_{n=0}^{\infty} \frac{1}{\left(\frac{1}{3^n}+1\right)z^n} + \\ &+ \sum_{n=1}^{\infty} \frac{z^n}{3^n+1} = \sum_{m=0}^{\infty} \frac{3^m}{3^m+1} \cdot \frac{1}{z^m} + \sum_{n=1}^{\infty} \left(\frac{z}{3}\right)^n = \\ &= \sum_{m=0}^{\infty} \frac{1}{z^m} + \sum_{n=1}^{\infty} \left(\frac{z}{3}\right)^n \end{aligned}$$

$$\begin{cases} \left|\frac{z}{3}\right| < 1 \\ \left|\frac{1}{z}\right| < 1 \end{cases} \Rightarrow 1 < |z| < 3$$



20.09 (1, 5, 9, 8)

1)  $\frac{1}{z(z-3)^2}$  ( $a=1$ ;  $D: 1 < |z-1| < 2$ )

$\xi = z - 1$

$\frac{1}{(\xi+1)(\xi-2)^2} = \frac{A}{\xi+1} + \frac{B}{\xi-2} + \frac{C}{(\xi-2)^2}$

$= \frac{1}{9} \cdot \frac{1}{(\xi+1)} + \frac{1}{9(\xi-2)} + \frac{1}{3(\xi-2)^2}$

$= \frac{1}{9\xi(1+\frac{\xi}{2})} + \frac{1}{18(\frac{\xi}{2}+1-\frac{\xi}{2})} + \frac{1}{12(1-\frac{\xi}{2})^2}$

$= \frac{1}{9\xi} \sum_{n=0}^{\infty} \left(\frac{1}{2}\right)^n \xi^n + \frac{1}{18} \sum_{n=0}^{\infty} \left(\frac{\xi}{2}\right)^n + \frac{1}{12} \left(\sum_{n=0}^{\infty} \left(\frac{\xi}{2}\right)^n\right)^2 \quad (\xi = z-1)$   
 $(-\frac{3}{2} \in D)$

2) 5)  $\frac{1}{z(z-1)(z-2)} = \frac{1}{2z} + \frac{1}{2(z-2)} - \frac{1}{(z-1)}$

$= \frac{1}{2z} + \frac{1}{2} \sum_{n=0}^{\infty} \left(\frac{1}{2}\right)^n - \frac{1}{z(1-\frac{1}{2})} =$

$= \frac{1}{2z} + \frac{1}{2} \sum_{n=0}^{\infty} \left(\frac{1}{2}\right)^n - \sum_{n=0}^{\infty} \left(\frac{1}{2}\right)^{n+1}$

7)  $z^3 \left( \frac{1}{3(z-2)} - \frac{1}{3(z+1)} \right) = \left\{ \begin{array}{l} z \neq z+1 \\ 0 < |z| < 3 \end{array} \right\} = \frac{(z-1)^3}{z(z+1)}$

$= \frac{z^3 - 3z^2 + 3z - 1}{z(z+1)} = \frac{z^2}{(z+1)} - \frac{3z}{z+1} + \frac{3}{z+1} - \frac{1}{z(z+1)}$

$\frac{z^2}{z+1} = - \frac{z^2}{3(1+\frac{z}{3})} = - \frac{1}{3} \sum_{n=0}^{\infty} \frac{z^{n+2}}{3^n} = - \sum_{n=0}^{\infty} \frac{z^{n+2}}{3^{n+1}}$

$-\frac{3z}{z+1} = \frac{z}{1+\frac{z}{3}} = \sum_{n=0}^{\infty} \frac{z^{n+1}}{3^n}$

$\frac{3}{z+1} = - \frac{1}{1-\frac{z}{3}} = - \sum_{n=0}^{\infty} \left(\frac{z}{3}\right)^n$



20.21

$$1) \frac{z}{(z+2)^2} = \frac{z+2-2}{(z+2)^2} = \frac{1}{z+2} - \frac{2}{(z+2)^2}$$

$$2) \frac{e^z + 1}{e^z - 1} = \frac{e^{z+2\pi i k} + 1}{e^{z+2\pi i k} - 1} = \frac{e^{\frac{z+2\pi i k}{2}} (e^{\frac{z+2\pi i k}{2}} + e^{-\frac{z+2\pi i k}{2}})}{e^{\frac{z+2\pi i k}{2}} (e^{\frac{z+2\pi i k}{2}} - e^{-\frac{z+2\pi i k}{2}})} =$$

$$= \frac{2 \cos(\frac{z+2\pi i k}{2})}{-2i \sinh(\frac{z+2\pi i k}{2})} = \frac{1}{-i(\frac{z}{2} + \pi i k)}$$

(c.m.m.g. no q.m.m.) =  $\frac{2}{z - 2\pi i k}$

$$3) \frac{z-1}{\sin^2 z} \approx \frac{z-1}{z^2} \approx -\frac{1}{z^2} + \frac{1}{z}$$

$$4) \frac{e^{iz}}{z^2 + b^2} = \frac{e^{iz}}{(z+ib)(z-ib)} \quad \left. \begin{array}{l} z-ib = \xi \\ z+ib = \xi \end{array} \right\}$$

$$= \frac{e^{i\xi} \cdot e^{-b}}{(\xi+2ib)\xi} = \frac{1}{\xi} \frac{e^{-b}}{\xi+2ib} \approx \frac{1}{\xi} \frac{e^{-b}}{2ib} = \frac{1}{z-ib} \frac{e^{-b}}{2ib}$$

$$5) \frac{(z^2+1)^2}{z^2+b^2} = \frac{(z+i)^2(z-i)^2}{(z+ib)(z-ib)} = \left\{ \xi = \frac{1}{z} \right\} = \frac{(\frac{1}{\xi}+i)^2(\frac{1}{\xi}-i)^2}{(\frac{1}{\xi}+ib)(\frac{1}{\xi}-ib)} =$$

$$= \frac{(1+i\xi)^2(1-i\xi)^2}{(1+ib\xi)(1-ib\xi)} = \frac{1(1+i\xi)^2(1-i\xi)^2}{\xi^2(1+ib\xi)(1-ib\xi)} \approx$$

$$\approx \frac{1}{\xi^2} \approx \frac{1}{z^2}$$