

2.1

$$\frac{1+2z^2}{z^3+z^5} = \frac{1+2z^2}{z^3(1+z^2)} = \frac{1}{z^3(1+z^2)} + \frac{2z}{z(1+z^2)} =$$

$$= \frac{1}{z^3(z+i)(z-i)} + \frac{2}{z(z+i)(z-i)} = \frac{2}{z} \cancel{\frac{1}{z}} + \frac{1}{z-i} +$$

$$+ \frac{1}{z+i} + \frac{1}{z^3} + \frac{1}{z(z-i)} + \frac{1}{z(z+i)} = \frac{1}{z} =$$

$$= \frac{1}{z} + \frac{1}{z^3} + 0 + 0$$

2.2.

$$f(z) = \frac{1}{z(e^z-1)} \neq \frac{1}{z} \sum_{n=0}^{\infty} \frac{1}{n!} =$$

$$\approx \frac{1}{z} (1 + z + \frac{z^2}{2} + \dots)$$

$$\frac{1}{z} \sum_{n=0}^{\infty} \frac{z^n}{n!} = \frac{1}{z^2} \underbrace{\left(1 + \frac{z}{1!} + \frac{z^2}{2!} + \dots\right)}_{p(z)} =$$

$$= \frac{1}{z^2} \sum_{n=0}^{\infty} (p(z)) \frac{z^n}{n!} = \frac{1}{z^2} - \frac{1}{2z} + \dots$$

2.6

$$\textcircled{1} \int_C \frac{z e^z}{\tan z^2} dz = 2\pi i \sum \text{Res} = 2\pi i$$



~~$$f(z) = \frac{z e^z}{\tan z^2} = \frac{z(1+z+\dots)}{z^2+\dots} = \frac{1}{z} + 1 + \dots$$~~

$$\int_C f(z) dz = 1$$

$$\textcircled{2} \int_C e^{-1/2} \sin\left(\frac{1}{z}\right) dz = \frac{1}{2} \int_C \sin\left(\frac{1}{z}\right) dz = \frac{1}{2} \int_C \left(-\frac{1}{z^2} + \dots\right) dz = \frac{1}{2} \cdot 2\pi i = \pi i$$

~~$$\int_C e^{-1/2} \sin\left(\frac{1}{z}\right) dz = \pi i$$~~

$$f(z) = \left(1 - \frac{1}{z} + \frac{1}{2!z^2} + \dots\right) \left(\frac{1}{z} - \frac{1}{3!z^3} + \dots\right) = \frac{1}{z} - \frac{1}{z^2} + \dots$$

$$\textcircled{3} 2\pi i$$

$$\textcircled{3} \int_C \frac{e^z}{z^n} dz = 2\pi i \frac{1}{(n-1)!}$$

$$f(z) = \frac{1+z+z^2+\dots}{z^n} = \frac{1}{z^n} + \frac{1}{z^{n-1}} + \frac{1}{z^{n-2}} + \dots + \frac{1}{(n-1)!z} + \dots$$

$$1) f(z) = \frac{\sin z}{1 - \frac{\sin z}{\cos z}} = \sin z \cdot \frac{\cos z}{\cos z - \sin z} \quad E = z - \frac{\pi}{4}$$

$$= \frac{\sin z \cos z}{\cos z - \sin z} = \frac{\frac{1}{2} (2z - \frac{1}{2!} 2z^3 + \dots)}{1 - \frac{1}{2!} z^2 + \frac{1}{4!} 2^4 z^4 - \dots} =$$

$$= \frac{\sin z}{1 - z - \frac{1}{6} z^3 + \dots} = \frac{z - \frac{z^3}{6} + \dots}{1 - z - \frac{1}{6} z^3 + \dots}$$

$$\#1 = \frac{\sin(E + \frac{\pi}{4})}{1 - \tan(\frac{\pi}{4} + E)} = \frac{\frac{1}{\sqrt{2}} \sin E + \frac{1}{\sqrt{2}} \cos E}{1 - \frac{1 + \tan E}{1 - \tan E}} =$$

$$= \frac{1}{\sqrt{2}} \left(\frac{\sin E + \cos E}{1 - \tan E - 1 - \tan E} \right) \cdot (1 - \tan E) =$$

$$= \frac{(\sin E + \cos E)(1 - \tan E)}{-2\sqrt{2} \tan E} = \frac{(E + 1)(1 - E)}{-2\sqrt{2} E} =$$

$$\frac{1 - E^2}{-2\sqrt{2} E} = -\frac{1}{2\sqrt{2} E} + \dots$$

$$2) f(z) = \frac{e^{\frac{c}{z-a}}}{e^{\frac{c}{2a}} - 1} = \frac{e^{c(E + 2\pi i a)}}{\exp\left\{\frac{E + 2\pi i a}{a}\right\} - 1}$$

$$\frac{z}{a} = 2\pi i \Rightarrow z = 2\pi i a n \quad E = \frac{c}{2\pi i a} - z - 2\pi i a$$

$$= \frac{\exp\left\{\frac{c}{E + 2\pi i a}\right\}}{\exp\left\{\frac{E}{a}\right\} - 1} = -\frac{\exp\left\{\frac{c}{E + 2\pi i a}\right\}}{(1 - e^{\frac{E}{a}})} \approx$$

$$= -\exp\left\{\frac{c}{E + 2\pi i a}\right\} \cdot \left(1 + \frac{E}{a}\right) = -\left(1 + \frac{c}{2\pi i a \left(1 + \frac{E}{2\pi i a}\right)}\right) \cdot \left(1 + \frac{E}{a}\right)$$

$$\left(1 + \frac{E}{a}\right) = -\left(1 + \frac{c}{2\pi i a} \left(1 - \frac{E}{2\pi i a} + \dots\right)\right) \left(1 + \frac{E}{a}\right)$$

$$\approx \exp\left\{\frac{c}{E + 2\pi i a}\right\} \cdot \left(\frac{a}{E}\right) + \dots \quad (\text{approxim. nashch})$$

$$\text{f m. } z=a \Rightarrow f(z) = \left(1 + \frac{c}{E} + \frac{c^2}{2E^2} + \dots\right) \left(\frac{a}{E} + \dots\right) \quad \text{чл. раз.}$$

2.2.

$$f(z) = z e^{\frac{1}{z}} e^{-\frac{1}{2z}} \quad z \neq 0$$

$$= z \left(1 + \frac{1}{z} + \frac{1}{2z^2} + \dots \right) \left(1 - \frac{1}{2z} + \frac{1}{2z^2} + \dots \right)$$

лог. ряд. метода.

2.3

$$\frac{1}{z(z-1)}$$

$$\textcircled{1} |z| \in (0; 1)$$

$$= -\frac{1}{z} \sum_{n=0}^{\infty} z^n = -\sum_{n=0}^{\infty} z^{n-1}$$

$$\textcircled{2} |z| \in (1; +\infty)$$

$$\frac{1}{2z} \sum_{n=0}^{\infty} \left(\frac{1}{z}\right)^n = \sum_{n=0}^{\infty} \left(\frac{1}{z}\right)^{n+2}$$

2.4

$$\frac{z}{z^2+1} = \frac{z}{(z+i)(z-i)} = \frac{1}{2} \left(\frac{1}{z+i} - \frac{1}{z-i} \right)$$

$$z = i; \quad \xi = z - i$$

$$\textcircled{=} \frac{1}{2} \left(\frac{1}{\xi+2i} - \frac{1}{\xi} \right) = \frac{1}{2} \left(\frac{1}{2i} \left(\frac{1}{1+\frac{\xi}{2i}} \right) - \frac{1}{\xi} \right)$$

$$= \frac{1}{2} \left(\frac{1}{2i} \left[\frac{1}{2i} \sum_{n=0}^{\infty} (-1)^n \left(\frac{\xi}{2i}\right)^n - \frac{1}{\xi} \right] \right) =$$

$$= -\frac{1}{2} \frac{(\xi+i)i}{\xi} + \frac{1}{4} i \sum_{n=0}^{\infty} (-1)^n \left(\frac{\xi}{2i}\right)^n = -\frac{1}{2} \frac{2i}{z-i}$$

$$= -\frac{1}{z-i} + \frac{1}{4} i \sum_{n=0}^{\infty} \left(\frac{\xi}{2} (z-i)\right)^n$$