1.04

1. $\frac{1}{1-i} \cdot \frac{1+i}{1+i} = \underline{\frac{1}{2} + \frac{1}{2} i}$

2. $\left(\frac{1-i}{1+i}\right)^3 = \frac{1}{8}\left(1-i\right)^6 = e^{-\frac{\pi}{4} \cdot 6 \cdot i} = e^{-\frac{\pi}{2} 3 \cdot i} = \underline{i}$

3. $\left(\frac{1}{2} - i \frac{\sqrt{3}}{2}\right)^3 = e^{-\frac{\pi}{3} i \cdot 3} = \underline{-1}$

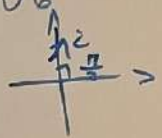
4. $\left(\frac{i^5 + 2}{i^{19} + 1}\right)^2 = \left(\frac{i + 2}{-i + 1}\right)^2 = \frac{((i+2)(1+i))^2}{4} =$

$= \frac{(1+3i)^2}{4} = \underline{-2 + \frac{3}{2} i}$

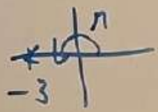
5. $\frac{(1+i)^5}{(1-i)^3} = \frac{\sqrt{2}^5}{\sqrt{2}^3} \cdot \frac{e^{i\frac{3\pi}{4}}}{e^{i(-\frac{3\pi}{4})}} = 2e^{i2\pi} =$

$= \underline{2}$

1.06

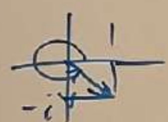
1.  $\arg i = \frac{\pi}{2}$

$|i| = 1$

2.  $\arg(-3) = \pi$

$|-3| = 3$

$i^{123} = -i$

3.  $\arg(1 + i^{123}) = \frac{7\pi}{4}$

$|z| = \sqrt{2}$

4. $-\frac{1}{2} + i\frac{\sqrt{3}}{2}$ $\quad \arg z = \frac{2\pi}{3}$

$|z| = 1$

5. $\frac{1-i}{1+i} = \frac{1}{2}(1-i)^2 = -i$ $\quad \arg z = \frac{3\pi}{2}$

$|z| = 1$

6. $-\cos\frac{\pi}{7} + i\sin\frac{\pi}{7}$ $\quad \arg z = \frac{6\pi}{7}$

$|z| = 1$

7. $(-4+3i)^3 = 125e^{3(\pi - \operatorname{arctg}\frac{3}{4})i}$ $\quad |z| = 125$

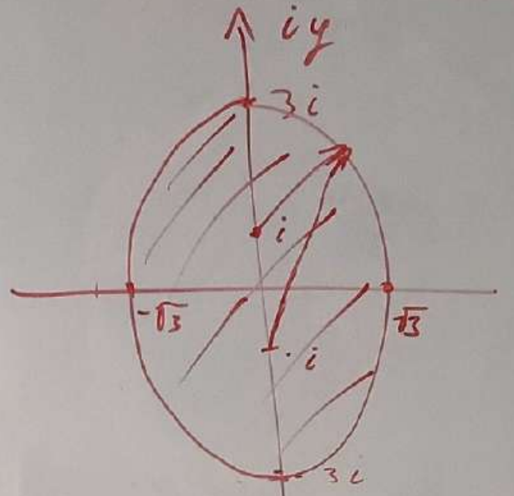
8. $(1+i)^8(1-i\sqrt{3})^{-6} =$ $\quad \arg z = 3$

$= \sqrt{2}^{8} e^{i2\pi} \cdot 2^{-6} \cdot e^{i2\pi} = \frac{1}{4}e^{i0\pi}$

$\arg z = 0$

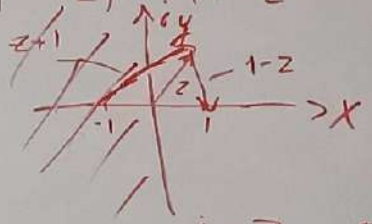
$|z| = \frac{1}{4}$

(3) $|z-i| + |z+i| < 4$ 1.21



$$\frac{x^2}{3} + \frac{y^2}{4} = 1$$

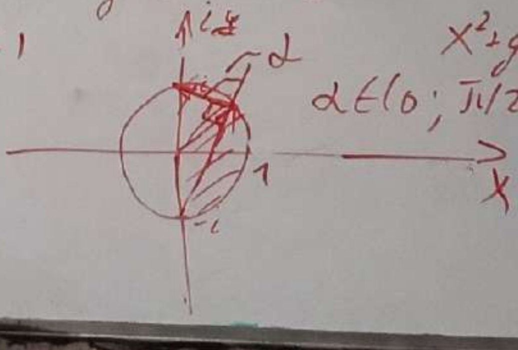
(4) $|1+z| < |1-z|$



$x < 0$

(5) $0 < \arg \frac{i-z}{z+i} < \frac{\pi}{2}$

$$\begin{cases} x^2+y^2<1 \\ x>0 \end{cases}$$



$x^2+y^2 \geq 1$

$\alpha \in (0; \pi/2)$

№ 51

1. $\sin\theta + \sin 2\theta + \ldots + \sin n\theta = \frac{\sin\frac{n+1}{2}\theta}{\sin\frac{\theta}{2}} \sin\frac{n\theta}{2}$

$\operatorname{Im}\left(e^{i\theta} + e^{2i\theta} + e^{3i\theta} + \ldots e^{in\theta}\right) =$

$\operatorname{Im}\left(e^{i\theta} \cdot \left(\frac{e^{in\theta} - 1}{e^{i\theta} - 1}\right)\right) = \cancel{\frac{e^{(n+1)i\theta} - e^{i\theta}}{e^{i\theta} - 1}} =$

$\cancel{e^{\frac{i\theta}{2}}\left(e\right.}$

$\operatorname{Im}\left(e^{i\theta}\left(\frac{e^{\frac{in\theta}{2}}\left(e^{\frac{in\theta}{2}} - e^{-\frac{in\theta}{2}}\right) \cdot 2i}{e^{\frac{i\theta}{2}}\left(e^{\frac{i\theta}{2}} - e^{-\frac{i\theta}{2}}\right) \cdot 2i}\right) = \right)$

$\operatorname{Im}\left(e^{\frac{i\theta}{2}} \cdot e^{\frac{in\theta}{2}}\right) \frac{\sin\frac{n\theta}{2}}{\sin\frac{\theta}{2}} = e^{\frac{i\theta}{2}(n+1)} \cdot \frac{\sin\frac{n\theta}{2}}{\sin\frac{\theta}{2}}$

$= \frac{\sin\frac{(n+1)\theta}{2}}{\sin\frac{\theta}{2}} \sin\frac{n\theta}{2}$

$\cancel{\varepsilon = \cos\frac{\theta + 2k\pi}{n} + i\sin\frac{\theta + 2k\pi}{n}}, \ k = 0, 1, \ldots, n-1$

1.52

1. $\cos\theta + \cos 3\theta + \ldots \cos(2n-1)\theta = \frac{\sin 2n\theta}{2\sin\theta}$

$Re(e^{i\theta} + e^{3i\theta} + e^{i5\theta} + \ldots + e^{i(2n-1)\theta}) =$

$= Re(e^{i\theta}(1 + e^{2i\theta} + e^{4i\theta} + \ldots e^{2n\theta i})) =$

$= Re\left(e^{i\theta}\left(\frac{e^{i2n\theta} - 1}{e^{i2\theta} - 1}\right)\right) = Re\left(e^{i\theta}\frac{e^{in\theta}(e^{in\theta} - e^{-in\theta})}{e^{i\theta}(e^{i\theta} - e^{-i\theta})}\right) =$

$= \frac{\sin n\theta \cdot \cos n\theta}{\sin\theta} = \frac{\sin 2n\theta}{2\sin\theta}$

2. $\sin\theta - \sin 3\theta + \ldots + (-1)^{n+1}\sin(2n-1)\theta = (-1)^{n+1}\frac{\sin 2n\theta}{2\cos\theta}$

$Im(e^{i\theta}(1 - e^{i2\theta} + e^{i4\theta} + \ldots + (-1)^{n+1}e^{i(2n-1)\theta})) =$

$= Im\left(e^{i\theta}\frac{(-1)^{n}e^{i2n\theta} - 1}{-e^{i2\theta} - 1}\right) =$

$= Im\left(e^{i\theta}\frac{(-1)^{n+1}e^{i2\theta n} + 1}{2\cos\theta\, e^{i\theta}}\right) =$

$= \frac{(-1)^{n+1}}{2\cos\theta} Im(e^{i2\theta n} + 1) = (-1)^{n+1}\frac{\sin 2n\theta}{2\cos\theta}$

1.60

$$1 + 2\varepsilon + 3\varepsilon^2 + \dots n\varepsilon^{n-1} = \frac{n}{\varepsilon - 1}$$

$$\frac{d}{d\varepsilon}\left(\varepsilon + \varepsilon^2 + \varepsilon^3 + \dots + \varepsilon^n\right) =$$

$$= \frac{d}{d\varepsilon}\left(\frac{\varepsilon^{n+1} - \varepsilon}{\varepsilon - 1}\right) =$$

$$\frac{(\varepsilon - 1)\left((n+1)\varepsilon^n - 1\right) - \varepsilon^{n+1} + \varepsilon}{(\varepsilon - 1)^2} = \frac{(n+1)\varepsilon^{n+1} - \varepsilon -}{(\varepsilon - 1)^2}$$

$$\frac{- \varepsilon^{n+1} + \varepsilon - (n+1)\varepsilon^n + 1}{} = \frac{n\varepsilon^{n+1} - n\varepsilon^n - \varepsilon^n + 1}{(\varepsilon - 1)^2} =$$

$$\# \quad \varepsilon = e^{i\frac{2\pi k}{n}} \qquad \left(e^{i2\pi k} = 1\right)$$

$$= \frac{n\varepsilon - n}{\varepsilon - 1} = \frac{n}{\varepsilon - 1}$$

1.53

$$z^n = 1$$

6.51

1\. $u = \operatorname{Re} f = x^3 + 6x^2y - 3xy^2 - 2y^3$, $f(0) = 0$

$\frac{\partial u}{\partial x} = 3x^2 + 12xy - 3y^2 = \cancel{6y^2}$

$\cancel{\frac{\partial u}{\partial y} = 12xy = 6xy = 6y^2}$

$\frac{\partial u}{\partial x} = \frac{\partial v}{\partial y} \Rightarrow v = 3x^2y + 6xy^2 - y^3 + C(x)$

$\frac{\partial u}{\partial y} = \cancel{}\, 6x^2 - 6xy - 6y^2 \cancel{}$

$\frac{\partial v}{\partial x} = -\frac{\partial u}{\partial y} \Rightarrow 6xy + 6y^2 + C'(x) = -6x^2 + 6yx + 6y^2$

$C'(x) = -6x^2 \Rightarrow C(x) = -2x^3$

$v = 3x^2y + 6xy^2 - y^3 - 2x^3 \cancel{} + C \quad (f(0) = 0 \Rightarrow C = 0)$

$f = x^3 + 6x^2y - 3xy^2 - 2y^3 + i(3x^2y + 6xy^2 - 2x^3 - y^3) =$

$= \underbrace{x^3 + 3ix^2y - 3xy^2 - iy^3}_{(x+iy)^3} - 2i\underbrace{(x^3 + i3x^2y - 3xy^2 - iy^3)}_{(x+iy)^3} =$

$= z^3 - 2i\,z^3 = z^3(1 - 2i)$

8.51

2. $u = \operatorname{Re} f = e^x(x\cos y - y\sin y)$

$$\frac{\partial u}{\partial x} = (x\cos y - y\sin y)e^x + e^x\cos y$$

$$\frac{\partial u}{\partial x} = \frac{\partial v}{\partial y} \Rightarrow v = e^x x\sin y - e^x\int y\sin y\,dy +$$

$$+ e^x\sin y + C(x) = e^x x\sin y - e^x(\sin y - y\cos y) + e^x\sin y + C(x) = e^x(x\sin y + y\cos y) + C(x)$$

$$\frac{\partial v}{\partial x} = (x\sin y + y\cos y)e^x + e^x\sin y + C'(x)$$

$$\frac{\partial u}{\partial y} = -e^x(x\sin y + \sin y + y\cos y)$$

$$C'(x) = 0 \Rightarrow C(x) = C \qquad (C = 0, \text{ т.к. } f(0) = 0)$$

$$f = e^x((x\cos y - y\sin y) + i(x\sin y + y\cos y)) =$$

$$= e^x(xe^{iy} - y\sin y + iy\cos y) = e^x(xe^{iy} + iy(\cos y + i\sin y)) =$$

$$= e^x(xe^{iy} + iye^{iy}) = e^{x+iy}(x+iy) = \underline{e^z \cdot z}$$

8. 2.51

5. $|f| = (x^2+y^2)e^x$ $\qquad f = e^{\ln f}$

$f = |f| e^{i \arg f}$

$\ln f = \ln|f| + i\arg f$

$g = u + iv$

$u = \ln|f| = \ln(x^2+y^2) + x$

$\frac{\partial u}{\partial x} = \frac{2x}{x^2+y^2} + 1$

$v = 2\,\mathrm{arctg}\,\frac{y}{x} + y + C(x)$

$\frac{\partial v}{\partial x} = 2\,\frac{1}{1+\left(\frac{y}{x}\right)^2}\left(-\frac{y}{x^2}\right) + C'(x) = -\frac{2y}{x^2+y^2} + C'(x)$

$\frac{\partial u}{\partial y} = \frac{2y}{x^2+y^2} \Rightarrow C'(x) = 0 \Rightarrow C(x) = C$

$f = e^{\ln(x^2+y^2) + x + i(2\,\mathrm{arctg}\left(\frac{y}{x}\right) + y + C)} =$

$= e^{x+iy} \cdot \left(\sqrt{x^2+y^2}\, e^{i\,\mathrm{arctg}\,\frac{y}{x}}\right)^2 \cdot e^{iC} =$

$= e^{z} \cdot z^2 \cdot e^{iC}$

9.16

1. $C: z = it + 1 \qquad 0 \le t \le 1; \quad w = z^2$

$$w = (it+1)^2 = (1 - t^2) + i\,2t$$

$$l = \int_0^1 \sqrt{(x_t')^2 + (y_t')^2}\,dt = 2\int_0^1 \sqrt{1+t^2}\,dt$$

$t = tg\theta$

$dt = \left(\frac{1}{\cos\theta}\right)^2 d\theta$

$$\int \frac{1}{\cos^3\theta}\,d\theta = \int sec^3\theta\,d\theta =$$

D	I
+ $sec\theta$	$sec^2\theta$
− $sec\theta\, tg\theta$	$tg\theta$

$$= sec\theta\, tg\theta - \int tg^2\theta\, sec\theta\, d\theta =$$

$$= sec\theta\, tg\theta - \int sec^3\theta\, d\theta + \int sec\theta\, d\theta) =$$

$$\int sec^3\theta = \frac{1}{2}(sec\theta\, tg\theta + \int sec\theta\, d\theta) =$$

$$= \frac{1}{2}\left(sec\theta\, tg\theta + \int \frac{d(tg\theta + sec\theta)}{tg\theta + sec\theta}\right) =$$

$$= \frac{1}{2}(sec\theta\, tg\theta + \ln|tg\theta + sec\theta| =$$

$$= \frac{1}{2}\left(t\sqrt{1+t^2} + \ln|\sqrt{1+t^2} + t|\right)$$

$$l = 2\int_0^1 \sqrt{1+t^2}\,dt = (t\sqrt{1+t^2}\Big|_0^1 + \ln|\sqrt{1+t^2} + t|\Big|_0^1) =$$

$$= \sqrt{2} + \ln(\sqrt{2} + 1)$$

9_51

2. $C: z = it \,,\quad 0 \le t \le 2\pi \,;\quad w = e^z$

$w = \cos t + i \sin t$

$$\ell = \int_0^{2\pi} \sqrt{\sin^2 t + \cos^2 t}\, dt = 2\pi$$