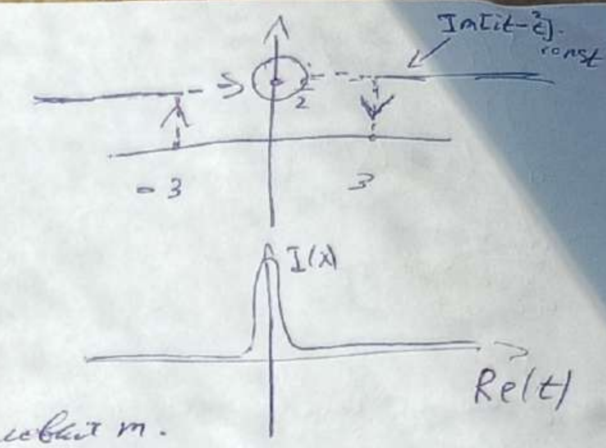
N1. $I(\lambda)=\int\limits_{-3}^{3}\frac{e^{\lambda(it-t^2)}}{1+t^2}\,dt$



$f(t)=it-t^2$

$\frac{df}{dt}=i-2t$

$f'=0 \Rightarrow t=\frac{i}{2}$ - седловая т.

Доминирующий вклад имеет окрестность точки $\frac{i}{2}$

$$I(\lambda)=\sqrt{\frac{2\pi}{\lambda f''(\frac{i}{2})}}\cdot g(\tfrac{i}{2})\cdot e^{-\lambda f(\frac{i}{2})}=\underline{\sqrt{\frac{\pi}{\lambda}}\cdot\frac{3}{4}\cdot e^{-\frac{\lambda}{4}}}$$

N2. $Ai(x)=\frac{1}{2\pi}\int\limits_{-\infty}^{+\infty}e^{i(xt+\frac{t^3}{3})}\,dt$

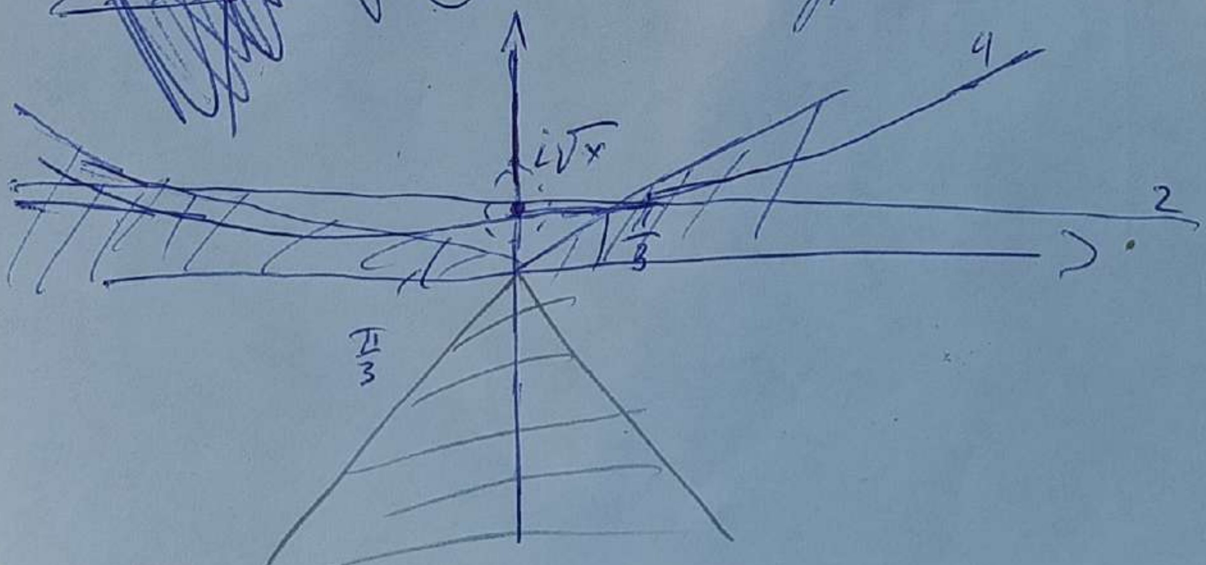
$Re\left[i(xt+\frac{t^3}{3})\right]<0$

$-Im(xt+\frac{t^3}{3})<0 \xRightarrow{t\to\infty} Im(\frac{t^3}{3})>0 \Rightarrow \sin 3\varphi>0$

$\cancel{\pi+2\pi n<3\varphi<2\pi+2\pi n}$ $\quad 2\pi n<3\varphi<\pi+2\pi n$

$\cancel{\frac{\pi}{3}+\frac{2\pi n}{3}<\varphi<\frac{2\pi}{3}+\frac{2\pi n}{3}}$ $\quad \frac{2\pi n}{3}<\varphi<\frac{\pi}{3}+\frac{2\pi n}{3}$

В разрешенн. попадают контура (2) и (4)



$f(t) = i\left(xt + \frac{t^3}{3}\right)$

$\frac{df}{dt} = i(x + t^2)$ $\qquad t = \pm i\sqrt{x}$ - седл. точки.

$\frac{d^2 f}{dt^2} = 2it$ $\qquad f(t_0) = i\left(i x^{3/2} + \frac{i^3 x^{3/2}}{3}\right) =$

$= -\frac{2}{3} x^{3/2}$

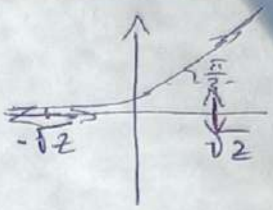
Разложим в ряд до 2го члена:

$\cancel{Ai(t)} = Ai(x) \approx \frac{1}{2\pi} \int_{-\infty}^{+\infty} e^{f(t_0) + \frac{f''(t_0)}{2}(t - t_0)^2} dt =$

$= \frac{1}{2\pi} e^{-\frac{2}{3}x^{3/2}} \int_{-\infty}^{+\infty} e^{-x^{3/2} \cdot (ix^{1/2} - t)^2} dt$

№ 4.

$$f(z) = \int_{-\infty}^{e^{\frac{i\pi}{3}}\infty} e^{\frac{t^3}{3} - zt}\, dt$$



Θ

$f' = t^2 - z$; $t = \pm\sqrt{z}$ - седл. точки. $(z \to +\infty)$

$f'' = 2t$ $\quad t = \pm i\sqrt{|z|}$ $(z \to -\infty)$

Напр. наиск. спуска:

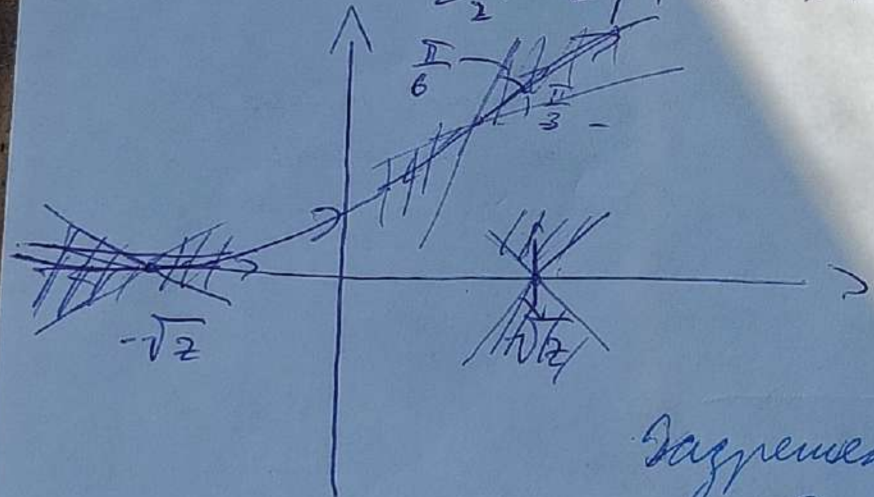
$$f'' = \begin{cases} -2\sqrt{z}, & \arg f'' = \pi \\ 2\sqrt{z}, & \arg f'' = 0 \end{cases}$$

$t_1 = \sqrt{z}$; $t_2 = -\sqrt{z}$

$2\varphi + \arg f'' = \pi + 2\pi n$

t_1: $2\varphi + 0 = \pi + 2\pi n \Rightarrow \varphi = \frac{\pi}{2} + \pi n$

t_2: $2\varphi + \pi = \pi + 2\pi n \Rightarrow \varphi = \pi n$



при $z \to \infty$
первый перевал
достигнем, а
второй нет.

Запрещенные области:

$Re\left(\frac{t^3}{3}\right) < 0$ $\left(t = e^{\frac{i\pi}{3}} \cdot p \Big|_{p \to \infty} \right)$

$-Re\left(\frac{p^3}{3}\right) < 0$

$\cos 3\alpha > 0$

$-\frac{\pi}{6} + \frac{2\pi n}{3} < \alpha < \frac{\pi}{6} + \frac{2\pi n}{3}$

(т.е можно деформировать контур на угол $\frac{\pi}{6}$ или $-\frac{\pi}{6}$ от напр $\frac{\pi}{3}$)

$$f(t) = f(-\sqrt{z}) + \frac{f''(-\sqrt{z})}{2}(t+\sqrt{z})^2$$

Подставляем в интеграл и получаем аналогичное выр., как и в пред. задаче

$$I = e^{+\frac{2}{3}z^{3/2}} \int_{-\infty}^{e^{\frac{i\pi}{3}}\infty} e^{-\sqrt{z}(t+\sqrt{z})^2} dt =$$

$$= \frac{e^{+\frac{2}{3}z^{3/2}}}{z^{1/4}} \int_{-\infty}^{e^{\frac{i\pi}{3}}\infty} e^{-\rho^2} d\rho = \frac{\sqrt{\pi}}{z^{1/4}} e^{+\frac{2}{3}z^{3/2}}$$

$z \to -\infty$; $z = -|z|$

Перевалы: $t_{1,2} = \pm i\sqrt{z}$

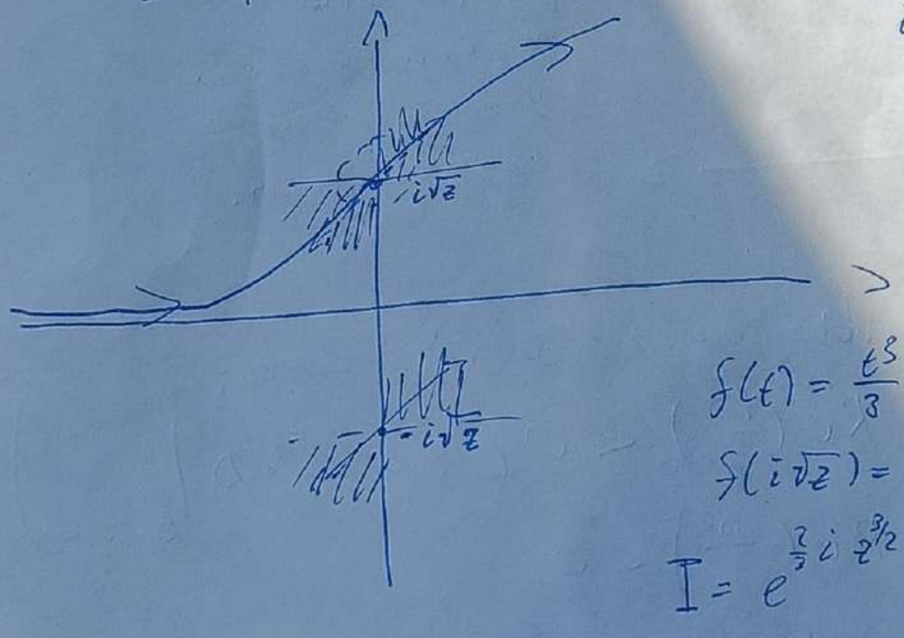
$$f'' = \begin{cases} 2i\sqrt{z}, & \arg f_1 = \frac{\pi}{2} \\ -2i\sqrt{z} & \arg f_2 = -\frac{\pi}{2} \end{cases}$$

t_1: $2\alpha_1 + \arg f_1 = \pi + 2\pi n$

$\alpha_1 = \frac{\pi}{4} + \pi n$

t_2: $\alpha_2 = \frac{3\pi}{4} + \pi n$

Достигним только $i\sqrt{z}$ перевал.



$$f(t) = \frac{t^3}{3} + |z|t$$

$$f(i\sqrt{z}) = \frac{2}{3}iz^{3/2}$$

$$I = e^{\frac{2}{3}iz^{3/2}} \int_{-\infty}^{+\infty} e^{i\sqrt{z}(t-i\sqrt{z})^2} dt =$$

$$= e^{\frac{i\pi}{4}} \frac{\sqrt{\pi}}{z^{1/4}} e^{2/3\, i z^{3/2}}$$

№5.

1) $I(\lambda) = \int\limits_{-\infty}^{+\infty} e^{-\frac{x^4}{4} + i\lambda x}\, dx$ $\quad \{\lambda \to +\infty\}$

$$e^{-\frac{\lambda^{4/3}}{4}\cdot\frac{2i\pi}{3} + \frac{2i\pi}{3}\lambda^{4/3}} = e^{\frac{2i\pi}{3}\cdot\frac{3}{4}\lambda^{4/3}} = e^{\frac{i\pi}{2}\lambda^{4/3}}$$

$$e^{\frac{i\pi}{2}\lambda^{4/3} - i\frac{\pi}{6}}$$

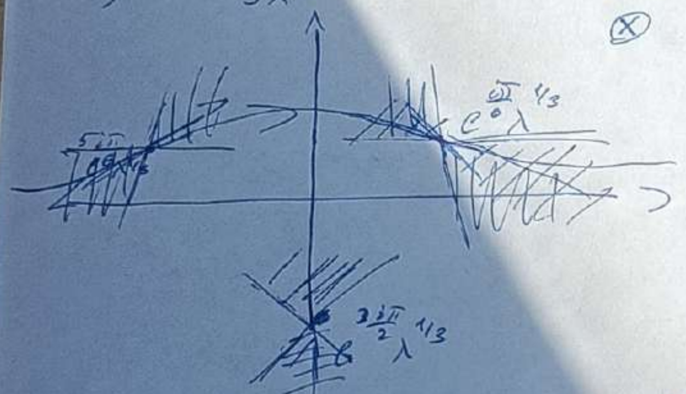
$f(x) = i\lambda x - \frac{x^4}{4}$

$f' = i\lambda - x^3 \Rightarrow x_{1,2,3} = \begin{cases} e^{\frac{i\pi}{6}}\lambda^{1/3} \\ e^{\frac{5i\pi}{6}}\lambda^{1/3} \\ e^{\frac{3i\pi}{2}}\lambda^{1/3} \end{cases}$

$f'' = -3x^2$

$\otimes$

3й перевал недостижим.



$$\frac{e^{-\frac{i\pi}{6}}\cdot e^{\frac{i\pi}{6}\frac{2i\pi}{2}\lambda^{2/3}}\cdot\sqrt{\frac{2\pi}{3\lambda^{2/3}}}}{e^{\frac{3}{4}i\frac{3\pi}{2}\lambda^{4/3}}\sqrt{\frac{2\pi}{3e^{\frac{5i\pi}{3}}\lambda^{2/3}}}} =$$

$f'' = \begin{cases} -3e^{\frac{i\pi}{3}}\lambda^{2/3} & \arg f_1'' = \frac{4\pi}{3} \\ -3e^{\frac{5i\pi}{3}}\lambda^{2/3} & \arg f_2'' = \frac{2\pi}{3} \\ -3e^{3i\pi}\lambda^{2/3} & \arg f_3'' = 0 \end{cases}$

$2\varphi_1 + \frac{4\pi}{3} = \pi + 2\pi n$

$\varphi_1 = -\frac{\pi}{6} + \pi n$

$2\varphi_2 + \frac{2\pi}{3} = \pi + 2\pi n$

$\varphi_2 = \frac{\pi}{6} + \pi n$

$2\varphi_3 + 0 = \pi + 2\pi n$

$\varphi_3 = \frac{\pi}{2} + \pi n.$

$$I(\lambda) \sim \sqrt{\frac{2\pi}{3|\lambda|^{2/3}}}\, e^{\frac{3}{4}e^{\frac{2i\pi}{3}}\lambda^{4/3} - i\frac{\pi}{6}} +$$

$$+ \sqrt{\frac{2\pi}{3|\lambda|^{2/3}}}\, e^{\frac{3}{4}e^{\frac{4i\pi}{3}} + i\frac{\pi}{6}} =$$

$$= \sqrt{\frac{2\pi}{3}}\,|\lambda|^{-1/3}\, e^{-\frac{3}{8}|\lambda|^{4/3}} \cos\left(\frac{3}{8}\sqrt{3}|\lambda|^{4/3} - \frac{\pi}{6}\right)$$

2) $\lambda \to +i\infty$

$$\int_{-\infty}^{+\infty} e^{-\frac{x^4}{4}+i\lambda x}\,dx \to \int_{-\infty}^{+\infty} e^{-\frac{x^4}{4}-|\lambda|x}\,dx$$

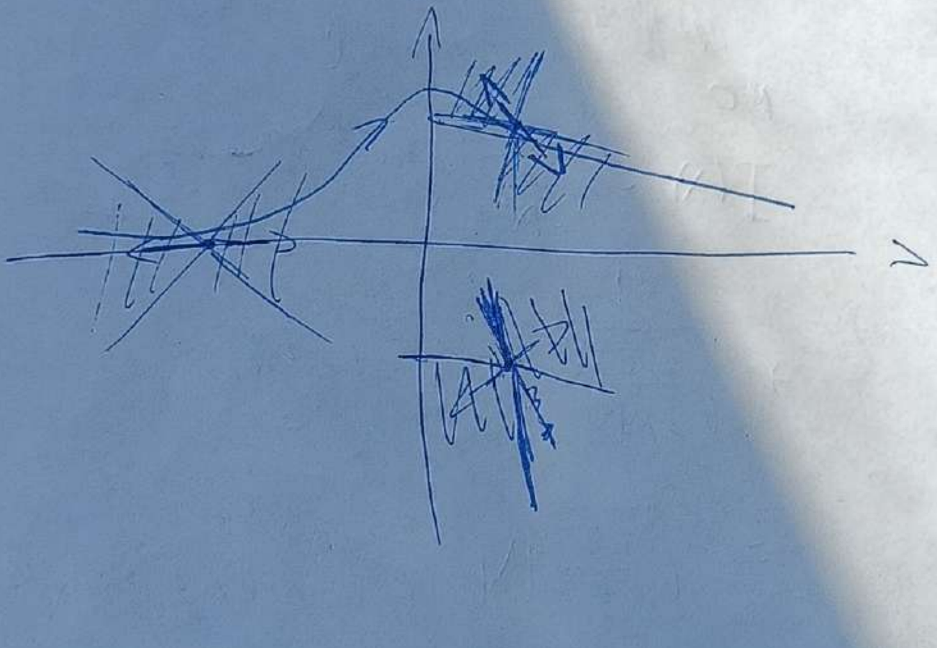
Перевальные точки:

$$x_{1,2,3} = \begin{cases} e^{\frac{i\pi}{3}}|\lambda|^{1/3} \\ e^{i\pi}|\lambda|^{1/3} \\ e^{\frac{i5\pi}{3}}|\lambda|^{1/3} \end{cases}$$

Аналогично находим напр. наиб. спуска:

$$\arg f''_{1,2,3} = \begin{cases} \frac{5\pi}{3} \\ \pi \\ \frac{\pi}{3} \end{cases} ; \quad \varphi_{1,2,3} = \begin{cases} -\frac{\pi}{3} \\ 0 \\ \frac{\pi}{3} \end{cases}$$

$$I(\lambda) \sim \sqrt{\frac{2\pi}{3}}\,|\lambda|^{-1/3} e^{3/4|\lambda|^{4/3}}$$



№6

$$I(\lambda) = \int_{-\infty}^{+\infty} e^{-\lambda(x^2-3ix)} F(x)\,dx = \int_{-\infty}^{+\infty} e^{\lambda(3ix-x^2)} F(x)\,dx$$

$$F(x) = \int_0^{\infty} \frac{(1+ix)y^{ix}}{(1+y)^{2+ix}} e^{-y}\,dy \qquad \lambda \to +\infty$$

$$f' = \cancel{2x-3i}\ \ 3i-2x \Rightarrow x_0 = +\frac{3}{2}i \text{ - седл. точка.}$$

$$f'' = -2.$$

$$F\left(\frac{3}{2}i\right) = \int_0^{\infty} \frac{\left(1+\frac{3}{2}i^2\right)y^{\frac{3}{2}i^2}}{(1+y)^{2+\frac{3}{2}i^2}}\,dy = -\frac{1}{2}\int_0^{\infty} \frac{y^{-\frac{3}{2}}e^{-y}}{(1+y)^{-1}}\,dy =$$

$$= -\frac{1}{2}\left(\int_0^{+\infty} y^{-1/2-1}e^{-y}\,dy + \int_0^{+\infty} y^{1/2-1}e^{-y}\,dy\right) =$$

$$= -\frac{\Gamma(1/2)+\Gamma(-1/2)}{2} = \frac{\sqrt{\pi}}{2}$$

Восп. формулой для метода Лапласа и получим:

$$I(\lambda) \sim \frac{\pi}{2\sqrt{\lambda}} e^{-\frac{9}{4}\lambda}$$

№7.

$$I(\lambda)=\int_{-\infty}^{+\infty}\cos(\lambda\cos x)\,\frac{\sin x}{x}\,dx = Re\left\{\int_{-\infty}^{+\infty} e^{i\lambda\cos x}\cdot\frac{\sin x}{x}\,dx\right\}$$

$$I'(\lambda)=\int_{-\infty}^{+\infty}e^{i\lambda\cos x}\,\frac{\sin x}{x}\,dx.$$

$f(x) = i\cos x$

$f' = -i\sin x$

$x_0 = \pi n$ - седл. точки.

$f'' = -i\cos x$

$f''(x_0) = (-1)^{n+1}\cdot i \quad \arg f''(x_0) = \frac{\pi}{2}$

$2\varphi + \frac{\pi}{2} = \pi + 2\pi n$

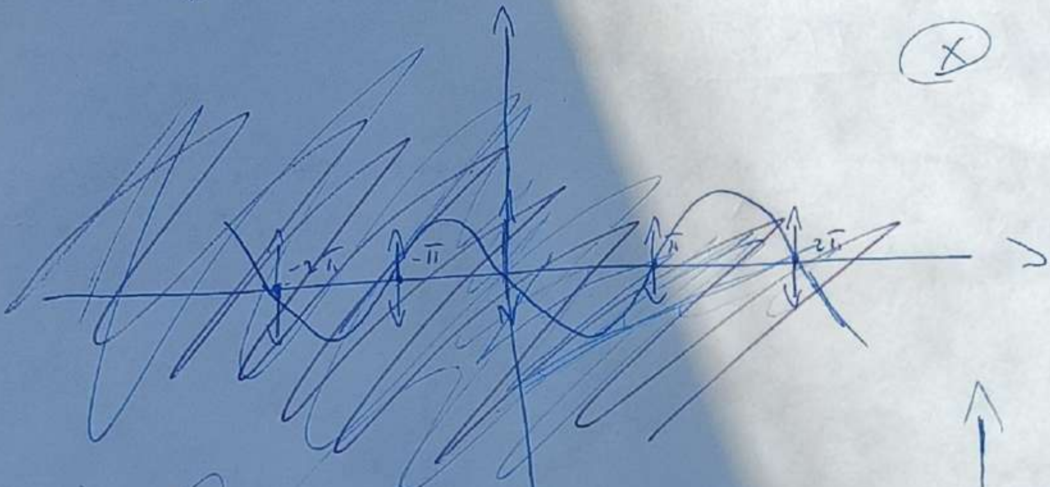
$\varphi = \frac{\pi}{4} + \pi n \quad [x_0 \in (\pi, 3\pi \ldots)]$

$\varphi = -\frac{\pi}{4} + \pi n \quad [x_0 \in (0, 2\pi, 4\pi \ldots)]$

$I'(\lambda) \sim \sqrt{\frac{2\pi}{i\lambda}}\cdot e^{i\lambda}$

$$I(\lambda) = Re\{I'(\lambda)\} = \sqrt{\frac{2\pi}{\lambda}}\cos\left(\lambda - \frac{\pi}{4}\right)$$

X



~~$I'(\lambda) = \sum_{n=0}^{\infty}\left(\sqrt{\frac{2\pi}{(-1)^n\lambda i}}\right)e$~~