6.1

$$I = \int_0^{+\infty} \frac{\ln x \, dx}{x^2+1} = \int_{0}^{\infty} \frac{\ln x \, dx}{(x+i)(x-i)}$$

$x = \pm i$

$$\oint = \int_{C_1} + \int_{C_2}^{\to 0} + \int_{C_3} + \int_{\varepsilon}^{\to 0} = 2\pi i \operatorname{Res}_{z=i} f(z)$$

$$\ln i = \ln\left|\frac{i}{x+i\cdot 0}\right| + \ln|x+i0| + i\Delta \arg g(z) =$$

$$= i \cdot \frac{\pi}{2}$$

$$\operatorname{Res} f(z) = \frac{i\frac{\pi}{2}}{2i} = \frac{\pi}{4}$$

$$I_{1+2+3} = \frac{i\pi^2}{2}$$

$$f(-x) = \left|\frac{f(-x)}{f(x+i0)}\right| \cdot f(x+i0) + \Delta \arg g(z)\, i$$

$$f(-x) = f(x+i0) + i\pi$$

$\operatorname{Res}(z)$

$$\ln(-x) = \ln(x+i0) + i\pi$$

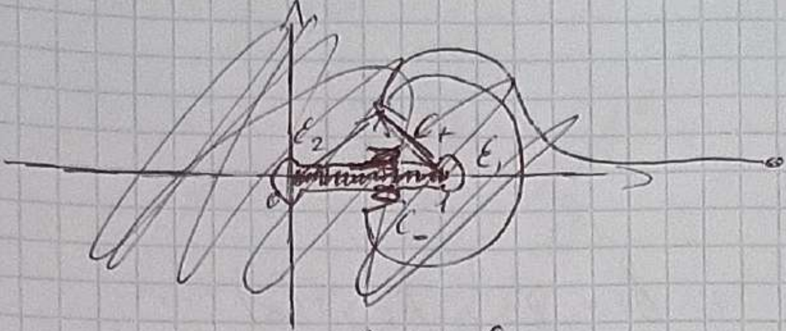
$$e^{-x} = x e^{i\pi}$$

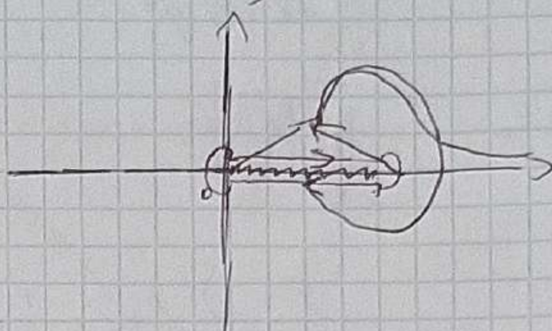
$$\oint = 2I + \int_0^{+\infty} \frac{\pi i}{x^2+1} dx = \frac{i\pi^2}{2} \Rightarrow I = 0$$

$$I(\alpha) = \int_0^1 \frac{x^\alpha (1-x)^{2-\alpha}}{x+1}\, dx \qquad 6.2$$

Сходимость в 0 при $\alpha \geq 0$

Потенциальные точки ветвления: $x = 1;\ 0$





$$\oint = \int_{\varepsilon_1}^{\to 0} + \int_{\varepsilon_2}^{\to 0} + \int_{c_+} + \int_{c_-} = 2\pi i \left(\operatorname*{Res}_{z=-1} + \operatorname*{Res}_{z=\infty}\right)$$

$$f(x - i0) = f(x+i0) \cdot e^{-2\pi i (2-\alpha)} = e^{2\pi i \alpha}$$

$$I - I e^{-2\pi i (2-\alpha)} = 2\pi i \left(\operatorname*{Res}_{z=-1} + \operatorname*{Res}_{z=\infty}\right)$$

$$\operatorname*{Res}_{z=-1} f(z) = \cancel{4 e^{i\pi\alpha}}\ 2^{2-\alpha} e^{i\pi\alpha}$$

$$f(-1) = \frac{|f(-1)|}{|f(x_0+i0)|} f(x_0+i0)\, e^{i\Delta \arg f} =$$

$$= |f(-1)| \cdot e^{i\pi\alpha} = 2^{2-\alpha} e^{i\pi\alpha}$$

$$|f(-1)| = |-1^\alpha \cdot 2^{2-\alpha}| = 2^{2-\alpha}$$

$$\operatorname*{Res}_{z=\infty} f(z) = 0$$

$$I\left(1 - e^{2\pi i \alpha}\right) = 2\pi i \cdot 4 e^{i\pi\alpha}$$

$$e^{\pi i \alpha} I\left(-2i \sin \pi\alpha\right) = 2\pi i \cdot 4 e^{i\pi\alpha}$$

$$I = -\frac{\pi}{\sin \pi \alpha} \cdot 4$$

$$f(z)\Big|_{z} \underset{z\to\infty}{=} \cancel{f(x_0+i0)}\;|f(z)|\cdot e^{-i\pi(2-\alpha)} =$$

$$= |f(z)|\cdot e^{i\pi\alpha}$$

$$\operatorname*{Res}_{z\to\infty}\left(\frac{z^{\alpha}(z-1)^{2-\alpha}}{z+1}e^{i\pi\alpha}\right) = \frac{z\left(1-\frac{1}{z}\right)^{2-\alpha}}{\left(1+\frac{1}{z}\right)} =$$

$$= \cancel{z\left(1-\frac{2-\alpha}{z}\right)\left(1-\frac{1}{z}+\frac{1}{z^2}\right)}$$

$$z\left(1-\frac{2-\alpha}{z}+\frac{(2-\alpha)(1-\alpha)}{2z^2}\right)\left(1-\frac{1}{z}+\frac{1}{z^2}\right) =$$

$$\cancel{= \frac{(\alpha-1)(\alpha-2)}{2z}+\frac{1}{z}+\dots = \frac{(\alpha-1)(\alpha-2)+2}{2z}}$$

$$\cancel{C_{-1} = \frac{\alpha^2-2\alpha-\alpha+2+2}{2} = \frac{\alpha^2-3\alpha+4}{2}}$$

$$\cancel{I\left(1-e^{2\pi i\alpha}\right) = +2\pi i\left(2^{2-\alpha}e^{i\pi\alpha}+\frac{\alpha^2-3\alpha+4}{2}e^{i\pi\alpha}\right)}$$

$$\cancel{e^{i\pi\alpha}\, I\left(-2i\sin\pi\alpha\right) = -2\pi i(\dots)}$$

$$\cancel{I = \frac{-\pi}{\sin\pi\alpha}\left(\frac{1}{2}\alpha^2-\frac{3}{2}\alpha+2+2^{2-\alpha}\right)}$$

$$= \frac{8+\alpha^2-5\alpha}{2z}+\dots$$

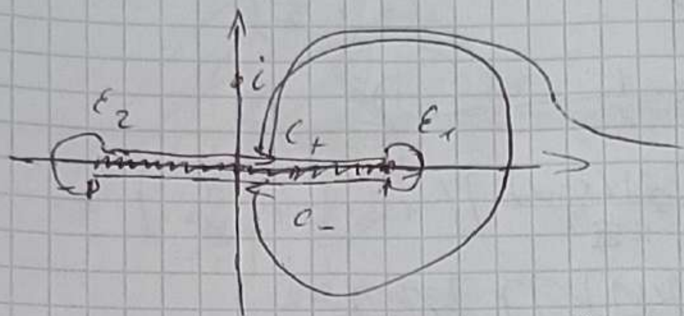
$$-C_{-1} = -\frac{8+\alpha^2-5\alpha}{2z}$$

$$e^{i\pi\alpha}\, I\left(1-e^{2\pi i\alpha}\right) = 2\pi i\left(2^{2-\alpha}-\frac{8+\alpha^2-5\alpha}{2}\right)e^{i\pi\alpha}$$

$$I = \frac{\pi}{\sin\alpha\pi}\left(\frac{8+\alpha^2-5\alpha}{2}-2^{2-\alpha}\right)$$

6.3

$$\int_{-1}^{1} \frac{(1-x)^{\alpha}(1+x)^{1-\alpha}}{(x^2+1)}\,dx = \int_{-1}^{1} \frac{(1-x)^{\alpha}(1+x)^{1-\alpha}}{(x+i)(x-i)}$$



$$\oint = \int_{C_+} + \int_{C_-} + \oint_{\varepsilon_2}^{0} + \oint_{\varepsilon_1}^{0} = 2\pi i\left(\operatorname*{Res}_{z=i} + \operatorname*{Res}_{z=-i} + \operatorname*{Res}_{z=\infty}\right)$$

$$f(x-i0) = f(x+i0)\cdot e^{-2\pi i\alpha}$$

$$I\left(1-e^{-2\pi i\alpha}\right) = 2\pi i(\dots)$$

$$\operatorname*{Res}_{z=i} = \frac{(1-z)^{\alpha}(1+z)^{1-\alpha}}{z+i}\Bigg|_{z=i}$$

$$f(i) = |f(i)|\cdot e^{\cancel{\frac{i\pi}{4}}\left(i\left(-\frac{\pi}{4}(\alpha)+\frac{\pi}{4}(1-\alpha)\right)\right)} =$$

$$= |f(i)|\, e^{i\frac{\pi}{4}(1-2\alpha)} = \cancel{\sqrt{2}}\, |1-i|^{\alpha}|1+i|^{1-\alpha} e^{i\frac{\pi}{4}(1-2\alpha)} =$$

$$= \cancel{\frac{\sqrt{2}}{2} e^{i\frac{\pi}{4}(1-2\alpha)}}\ \sqrt{2}\, e^{i\frac{\pi}{4}(1-2\alpha)}$$

$$\operatorname*{Res}_{z=i} = \frac{\sqrt{2}\, e^{i\frac{\pi}{4}(1-2\alpha)}}{2i}$$

$$\operatorname*{Res}_{z=-i} \frac{\sqrt{2}\, e^{-i\frac{\pi}{4}(1-2\alpha)}}{-2i}$$

$$f(z)\big|_{z\to\infty} = |f(x+i0)|\, e^{-i\pi\alpha} =$$

$$= (x-1)^{\alpha}(1+x)^{1-\alpha} e^{-i\pi\alpha}$$

$$f(z)=\frac{(z-1)^{\alpha}(z+1)^{1-\alpha}}{z^2+1}=\frac{z^{\alpha}\cdot z^{1-\alpha}\left(1-\frac{1}{z}\right)^{\alpha}\left(1+\frac{1}{z}\right)^{1-\alpha}}{z^2\left(1+\frac{1}{z^2}\right)}=$$

$$\frac{1}{z}\left(1+\ldots\right) \qquad C_{-1}=1$$

$$I\left(1-e^{-2\pi i\alpha}\right)=2\pi i\left(-e^{i\pi\alpha}+\frac{\sqrt{2}}{2i}\left(e^{i\frac{\pi}{4}(1-2\alpha)}-e^{i\frac{\pi}{4}(1-2\alpha)}\right)\right)=$$

$$\cancel{e^{-i\pi\alpha}I(2i\sin\pi\alpha)=2\pi i\left(-1+\sqrt{2}\sin\left(\frac{\pi}{4}(1-2\alpha)\right)\right)}$$

$$\cancel{I}$$

$$I\,\cancel{e^{-i\pi\alpha}}(2i\sin\pi\alpha)=2\pi i\left(\cancel{e}-1+\frac{\sqrt{2}}{2i}\left(e^{i\left(\frac{\pi}{4}+\frac{\pi}{2}\alpha\right)}+e^{-i\left(\frac{\pi}{4}+\frac{\pi}{2}\alpha\right)}\right)\right)$$

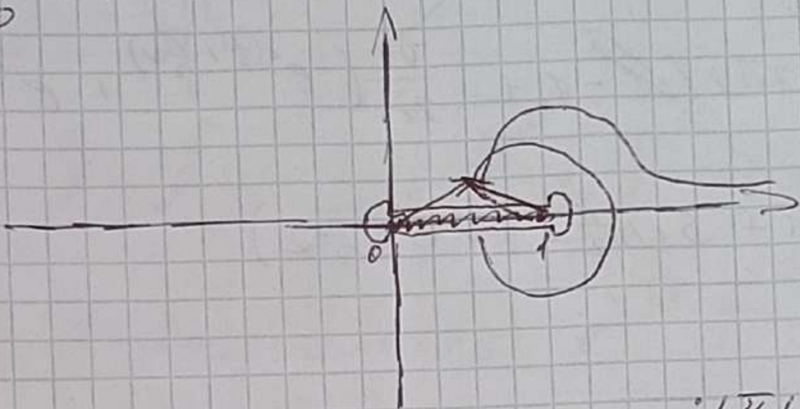
$$I=\frac{\pi}{\sin\pi\alpha}\left(-1+\sin\frac{\pi}{2}\alpha+\cos\frac{\pi}{2}\alpha\right)$$

6.5

$$I = \int_0^1 \ln\frac{1-x}{x}\,\frac{dx}{x^2+1}$$

$$\frac{d}{d\alpha}\left(\frac{1-x}{x}\right)^{\alpha}\Big|_{\alpha=0} = (1-x)^{\alpha}\cdot x^{-\alpha}$$

$$\int_0^1 \frac{(1-x)^{\alpha}x^{-\alpha}}{x^2+1}\,dx$$



$$\underset{i}{\mathrm{Res}} = f(i) = |f(i)|\cdot e^{i\left(\frac{\pi}{2}(-\alpha)-\frac{\pi}{4}\alpha\right)} = e^{-\frac{3\pi}{4}\alpha}$$

$$= |(1-i)^{\alpha} i^{-\alpha}|\, e^{-i\left(\frac{\pi}{2}+\frac{\pi}{4}\alpha\right)} = \sqrt{2}^{\alpha}\, \cancel{e^{-i\left(\frac{\pi}{2}+\frac{\pi}{4}\alpha\right)}}\,\frac{1}{2i}$$

$$f(-i) = \sqrt{2}^{\alpha}\cdot \cancel{e^{i\left(\frac{\pi}{2}+\frac{\pi}{4}\alpha\right)}}\cdot\frac{1}{-2i}\cdot e^{\frac{3\pi}{4}\alpha}$$

$$f(z)\Big|_{z\to\infty} = (x-1)^{\alpha}\cdot x^{-\alpha}\cdot \cancel{e^{-i\pi\alpha}}\; e^{-i\pi\alpha}$$

$$\underset{\infty}{\mathrm{Res}} = \frac{x^{\alpha}x^{-\alpha}\left(1-\frac{1}{x}\right)^{\alpha}}{x^2\left(1+\frac{1}{x^2}\right)} = \frac{1}{x^2}\left(1-\frac{\alpha}{x}+\dots\right)\left(1+\frac{1}{x^2}+\dots\right) =$$

$$=0$$

$$I\left(1-e^{-2\pi i\alpha}\right) = 2\pi i\left(\sqrt{2}^{\alpha}\left(\frac{\overset{e^{\frac{3\pi}{4}\alpha}}{\cancel{e^{i\left(\frac{\pi}{2}+\frac{\pi}{4}\alpha\right)}}} - \overset{e^{-\frac{3\pi}{4}\alpha}}{\cancel{e^{i\left(\frac{\pi}{2}+\frac{\pi}{4}\alpha\right)}}}}{2i}\right)\right)$$

$$I\sin\pi\alpha = \pi\left(\sqrt{2}^{\alpha}\,\overset{\sin\frac{3\pi}{4}\alpha}{\cancel{\cos\frac{\pi}{4}\alpha}}\right)$$

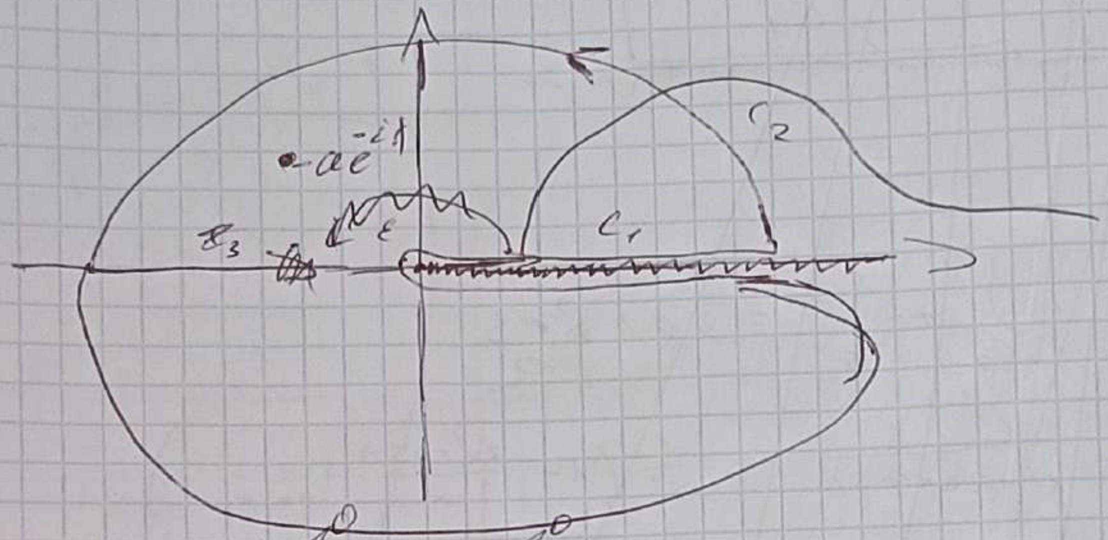
$$I = \frac{\pi}{\sin\pi\alpha}\left(\sqrt{2}^{\alpha}\,\overset{\sin\frac{3\pi}{4}\alpha}{\cancel{\cos\frac{\pi}{4}\alpha}}\right)$$

$$\frac{dI}{d\alpha}\Big|_{\alpha=0} = \frac{1}{8}\pi\ln 2$$

6.6

$$I = \int_0^{+\infty} \frac{\ln x \, dx}{x^2 + 2ax\cos\lambda + a^2}$$

$$\int_0^{+\infty} \frac{x^\alpha}{x^2+2ax\cos\lambda+a^2} \quad // \ \varphi(x) \qquad = \frac{x^\alpha}{(x-(-ae^{i\lambda}))(x+ae^{-i\lambda})}$$



$$\oint = \int_{\varepsilon} + \int_{C_1} + \int_{C_2} + \int_{C_3} = -2\pi i \, \cancel{Res} \sum Res$$

$$\cancel{\varphi(x)\Big|_{x\to\infty} = x^\alpha}$$

$$\cancel{f(x)\Big|_{x\to\infty} = \frac{x^\alpha}{x^2+2ax\cos\alpha} = \frac{x^{\alpha-2}}{(1+\frac{2a\cos\lambda}{x})}}$$

$$\cancel{\underset{\infty}{Res} = 0}$$

$$\cancel{I}$$

$$\underset{-ae^{i\lambda}}{\cancel{Res}} = \varphi(-ae^{-i\lambda}) = |a^\alpha e^{-i\lambda\alpha}| \cdot e^{i(\pi-\lambda)\alpha} =$$
$$= a^\alpha e^{i(\pi-\lambda)\alpha}$$

$$\underset{z=-ae^{-i\lambda}}{Res} = \frac{a^\alpha e^{i(\pi-\lambda)\alpha}}{a(e^{i\lambda}-e^{-i\lambda})} = \frac{a^\alpha e^{i(\pi-\lambda)\alpha}}{2ia\sin\lambda}$$

$$\cancel{f(x) = f(x+i0)\, e^{i\pi\alpha}}$$

$$f(x-i0) = f(x+i0)\, e^{2\pi i\alpha}$$

$$\operatorname*{Res}_{z=-ae^{i\lambda}} = \frac{a^{\alpha} \cancel{\alpha} e^{i(\pi+\lambda)\alpha}}{-2ia\sin\lambda}$$

$$\operatorname*{Res}_{-ae^{i\lambda}} + \operatorname*{Res}_{-ae^{-i\lambda}} = \cancel{a^{\alpha-1}} \; \frac{a^{\alpha-1}}{2i\sin\lambda}\left(e^{i(\pi-\lambda)\alpha} - e^{i(\pi+\lambda)\alpha}\right)$$

$$I(1-e^{2\pi i\alpha}) = \cancel{2\pi} i \frac{\pi a^{\alpha-1}}{\sin\lambda}(\ \ldots\ )$$

$$\cancel{e^{\pi i\alpha} I(-2i\sin\alpha) = \frac{\pi a^{\alpha-1}}{\sin\alpha}(2i\sin\lambda)}$$

$$I(-2i\sin\alpha\pi) = \frac{\pi a^{\alpha-1}}{\sin\lambda} 2i\sin\lambda\alpha$$

$$\cancel{I = -\frac{\pi a^{\alpha-1}}{\sin\alpha}}$$

$$I = \frac{\pi a^{\alpha-1}}{\sin\lambda}\cdot\frac{\sin\lambda\alpha}{\sin\alpha\pi}$$

$$\lim_{\alpha\to 0}\frac{dI}{d\alpha} = \frac{\lambda \ln a}{a\sin\lambda} = I$$

~~6.4 $I = \int_0^\infty \frac{\ln x}{x^{1/3}(x+1)^2}dx$~~

pomocn.

$\int \ln \omega \sin \omega d\omega$

5.10.

$$\int_1^2 \left(\frac{1}{\omega} + \frac{\alpha}{\omega^3}\right)\cos\omega \, d\omega = \int_1^2 \frac{\cos\omega}{\omega} + \int_1^2 \frac{\alpha\cos\omega}{\omega^3} d\omega \; \mathrm{I}$$

$$= \int_1^2 \frac{\cos\omega}{\omega} + \alpha\left(\ldots + \int_1^2 \frac{\sin\omega}{2\omega^2}\right) =$$

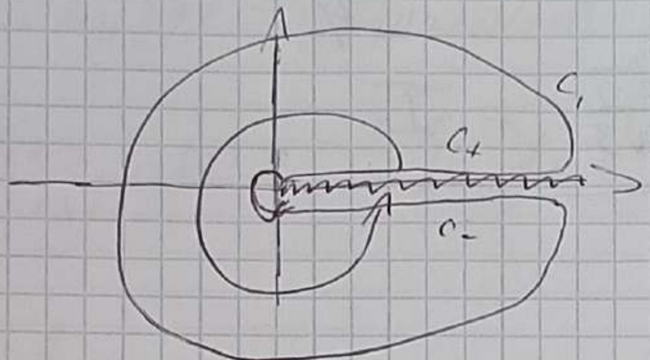
$$= \int_1^2 \frac{\cos\omega}{\omega} + \alpha\left(\ldots + \int_1^2 \frac{\cos\omega}{2\omega} d\omega\right) =$$

$$\int\left(\frac{\cos\omega}{\omega} - \frac{\alpha\cos\omega}{2\omega}\right) d\omega \Rightarrow \alpha = 2$$

6.4

$$I = \int_0^{\infty} \frac{\ln x}{x^{1/3}(x+1)^2}$$

$$I' = \int_0^{+\infty} \frac{x^{\alpha-\frac{1}{3}}}{(x+1)^2}$$



$$f(-1) = |f(-1)| \cdot e^{+i\pi(\alpha-\frac{1}{3})} = e^{i\pi(\alpha-\frac{1}{3})}$$

$$\oint = \int_{C_+} + \int_{C_-} + \int_{C_1} = 2\pi i \operatorname*{Res}_{z=-1}$$

$$\operatorname*{Res}_{z=-1} = \cancel{\frac{d}{dz}\frac{z^{\alpha-\frac{1}{3}}}{}} \quad \frac{d}{dz}\frac{z^{\alpha-\frac{1}{3}}}{(z+1)^2}(x+1)^2 = \left(\alpha-\frac{1}{3}\right) z^{\alpha-\frac{4}{3}} = -\left(\alpha-\frac{1}{3}\right) e^{i\pi(\alpha-\frac{1}{3})}$$

$$f(x+i0) = f(x-i0)\, e^{i2\pi(\alpha-\frac{1}{3})}$$

$$I'\left(1 - e^{2\pi i(\alpha-\frac{1}{3})}\right) = -2\pi i\left(\alpha-\frac{1}{3}\right) e^{i\pi(\alpha-\frac{1}{3})}$$

$$e^{i\pi(\alpha-\frac{1}{3})} I'\left(-2i \sin\pi\left(\alpha-\frac{1}{3}\right)\right) = -2\pi i\left(\alpha-\frac{1}{3}\right) e^{i\pi(\alpha-\frac{1}{3})}$$

$$I' = \frac{\pi}{\sin\pi\left(\alpha-\frac{1}{3}\right)}\left(\alpha-\frac{1}{3}\right)$$

$$\lim_{\alpha\to 0} \frac{dI'}{d\alpha} = \cancel{\left(\alpha-\frac{1}{3}\right)} \quad \cancel{\frac{-\frac{1}{3}\sin\pi\left(\alpha-\frac{1}{3}\right) + \left(\alpha-\frac{1}{3}\right)\cos\ldots}{\sin^2\pi\left(\alpha-\frac{1}{3}\right)}}\Bigg|_{\alpha\to 0} =$$

$$\cancel{\frac{\frac{1}{3}\pi\left(-\frac{1}{3}\right) - \frac{1}{3}}{\pi^2\frac{1}{9}}} =$$

$$\frac{dI}{d\alpha} = -\pi\left(\alpha - \frac{1}{3}\right) \cdot \frac{1}{\sin^2 \pi\left(\alpha - \frac{1}{3}\right)} \cdot \pi \cdot \cos \pi\left(\alpha - \frac{1}{3}\right) +$$

$$+ \frac{\pi}{\sin \pi\left(\alpha - \frac{1}{3}\right)}$$

$$\left.\frac{dI}{d\alpha}\right|_{\alpha \to 0} = + \frac{\frac{1}{3}\pi^2}{\frac{3}{4}} \cdot \frac{1}{2} -$$

$$- \frac{\pi}{\frac{\sqrt{3}}{2}} = \frac{2\pi^2}{9} - \frac{2\pi}{\sqrt{3}} = I$$

5.1

$$I_0(z) = \int_0^\infty t^z e^{-t}\,dt$$

$$I_1(z) = \frac{1}{z+1}\int_0^\infty t^{z+1} e^{-t}\,dt$$

$$I_2(z) = \frac{1}{(z+2)(z+1)}\int_0^\infty t^{z+2} e^{-t}\,dt$$

$$I_3(z) = \frac{1}{(z+3)(z+2)(z+1)}\int_0^\infty t^{z+3} e^{-t}\,dt$$

$$\operatorname*{Res}_{z=-3} I_3(z) = \frac{1}{2}\int_0^\infty e^{-t}\,dt = \frac{1}{2}$$

5.2

$$f(z) = \sum_0^\infty \frac{z^n}{(n+1)(n+2)} = \sum \frac{z^n}{(n+1)} - \sum \frac{z^n}{n+2} = \frac{1}{z}\sum_1^\infty \frac{z^n}{n} -$$

$$- \frac{1}{z^2}\underbrace{\sum_2^\infty \frac{z^n}{n}}_{=\sum_1^\infty \frac{z^n}{n} - z} = -\frac{1}{z}\ln(1-z) + \frac{1}{z^2}\left(\ln(1-z) + z\right) =$$

$$= \frac{\ln(1-z) + z - z\ln(1-z)}{z^2}$$

γ_1: $\ln 2 = \ln|-1| - i\pi = -i\pi$

$$f_{\gamma_1}(2) = \frac{1}{2} + \frac{i\pi}{4}$$

γ_2: $\ln 2 = i\pi$

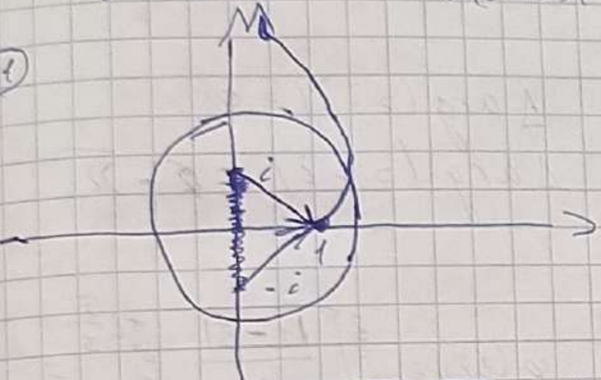
$$f_{\gamma_2}(2) = \frac{1}{2} - \frac{i\pi}{4}$$

$I_{1,2} = 0$

4.3.

$f(z) = \sqrt{1+z^2} = \sqrt{(z+i)(z-i)}$

①



$f(1) = \sqrt{2}$

$\oint f(z)dz = -\underset{z=+i\infty}{Res} f(z) \cdot 2\pi i$

$z = iy$

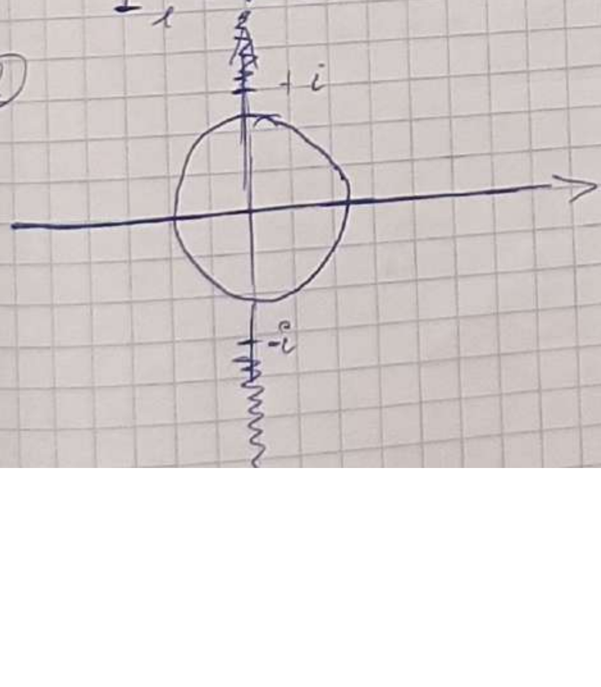
$f(iy) = \sqrt{\frac{|1-y^2|}{2}} \sqrt{2} \; e^{\frac{1}{2} i (\frac{\pi}{4} + \pi - \frac{\pi}{4})} =$

$= (1 - y^2)^{1/2} \cdot e^{i\frac{\pi}{2}} = iy\left(1 - \frac{1}{y^2}\right)^{1/2} = iy\left(1 - \frac{1}{2y^2} + \ldots\right) =$

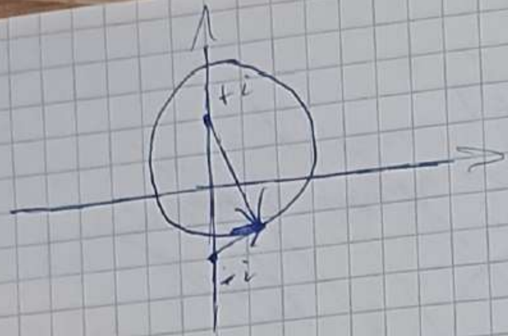
$C_{-1} = -\frac{1}{2} i$

$I_1 = \pi i$

②



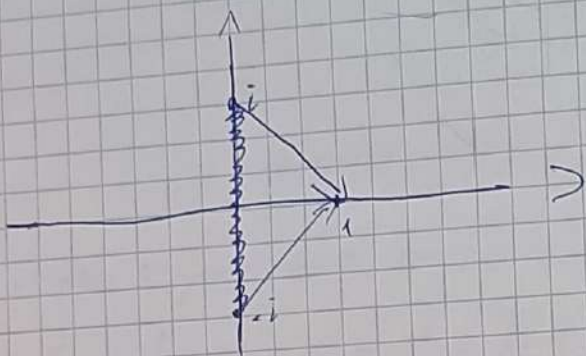
$I_2 = 0$ (внутри нет особых точек и ф-ия аналитична и однозн.)



$\Delta \arg(z+i) = 0$

$\Delta \arg(z-i) = 2\pi$

$f = f_0 e^{i\pi} = -f_0$



$f(1) = \sqrt{2}$

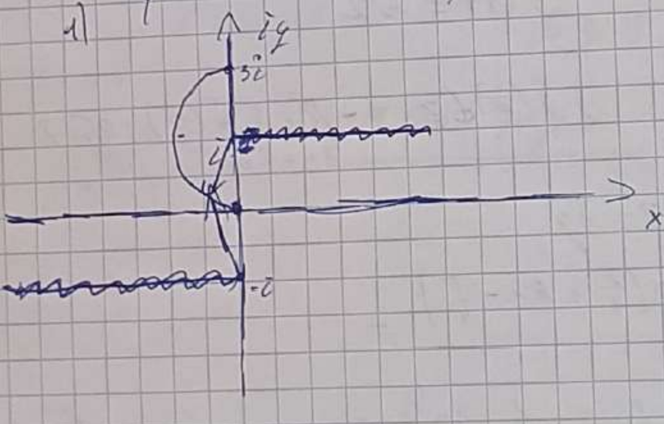
$f(-1) = \sqrt{2}\cdot e^{i\left(\frac{\pi}{2}+\frac{3\pi}{2}\right)} =$

$= \sqrt{2}\, e^{i\pi} = -\sqrt{2}$

4.6

$\varphi(z) = \sqrt[3]{1+z^2}$; $\varphi(0) = 1$

1)



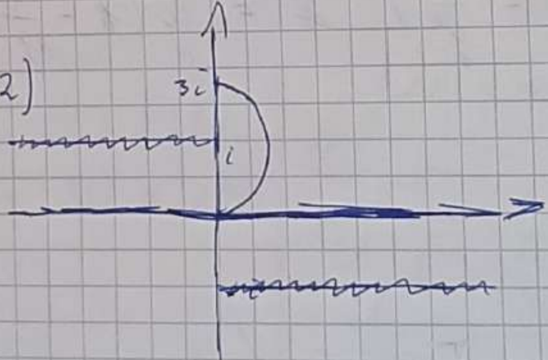
$\Delta \arg(z+i) = 0$

$\Delta \arg(z-i) = -\pi$

$\varphi(3i) = \sqrt[3]{\frac{|1-3^2|}{1}}\, e^{-i\frac{\pi}{3}} =$

$= 2\cdot e^{-i\frac{\pi}{3}} = 2\left(\frac{1}{2} - i\frac{\sqrt{3}}{2}\right.$

2)

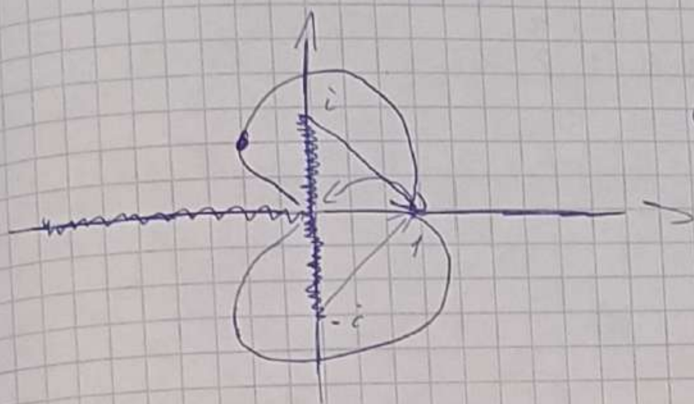


$\varphi(3i) = 2\cdot e^{i\frac{\pi}{3}}$

4.7

$f(x) = \ln(1+x^2)^{1/2} = \frac{1}{2}\ln(1+x^2)$

$F(z) = \frac{1}{2}\ln(z+i)(z-i)$



$F(1) = \frac{1}{2}\ln(2)$

$\lim_{\varepsilon\to 0} F(\varepsilon)$

① $F(\varepsilon) = \frac{1}{2}\left(\ln\left|\frac{g(\varepsilon)}{g_0}\right| + i\Delta \overset{0}{\arg} + F_0\right) = \frac{1}{2}\ln g(\varepsilon) = \frac{1}{2}\ln 1 = 0.$

② $\lim_{\varepsilon\to 0} F(\varepsilon e^{3\pi i/4})$

$\Delta \arg g = \frac{\pi}{4} + 2\pi - \frac{\pi}{4} = 2\pi$

$F(\varepsilon e^{3\pi i/4}) = \ln(g(\varepsilon))$

$= \frac{1}{2}(\ln(g(\varepsilon)) + i2\pi) = i\pi$

③ Аналогично

$F(\varepsilon e^{\frac{3\pi i}{4}}) = \frac{1}{2}(\ln(g(\varepsilon)) - 2\pi i) = -\pi i$

4.9 $f = z^a(z-1)^b$

$N(1,1) = 0$ $(f = z(z-1))$

$N(1, \frac{1}{2}) = 2$ $(z=1;\ z=\infty)$

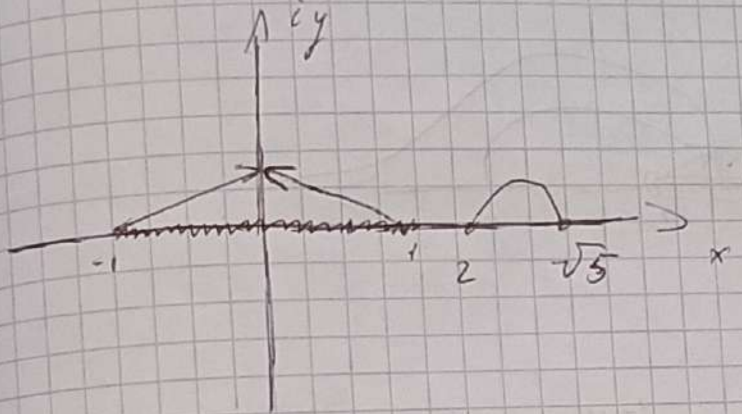
$N(\frac{1}{2}; \frac{1}{3}) = 3$ $(f = z^{1/2}(z-1)^{1/3}$ $z=0;\ z=1;\ z=\infty)$

$N(\frac{2}{3}; \frac{1}{3}) = 2$ $(f = z^{2/3}(z-1)^{1/3}$ $z=0;\ z=1)$

4.10

$$f(z)^2 - 2f(z) + z^2 = 0$$

$$f(z) = 1 \pm \sqrt{1-z^2} = 1 \pm \sqrt{(1-z)(1+z)} = 1 \pm i\sqrt{(z-1)(z+1)}$$



$$g(z) = \ln(1+f(z))$$

$$\Theta(2) = \sqrt{3}$$

$$S(z) = (z-1)(z+1)$$

$$\Theta(z) = \sqrt{(z-1)(z+1)}$$

$$\cancel{\Theta(z) = \sqrt{1}}$$

$$\Theta(\sqrt{5}) = \sqrt{\frac{|S(\sqrt{5})|}{|S(2)|}} \cdot |\Theta(2)| \cdot e^{\frac{1}{2} i \cdot 0} = 2$$

$$g(2) = \ln(2 \pm i\sqrt{3})$$

$$g(\sqrt{5}) = \ln\left|\frac{2 \pm i2}{2 \pm i\sqrt{3}}\right| + \ln(2 \pm i\sqrt{3}) + i \cdot 0 =$$

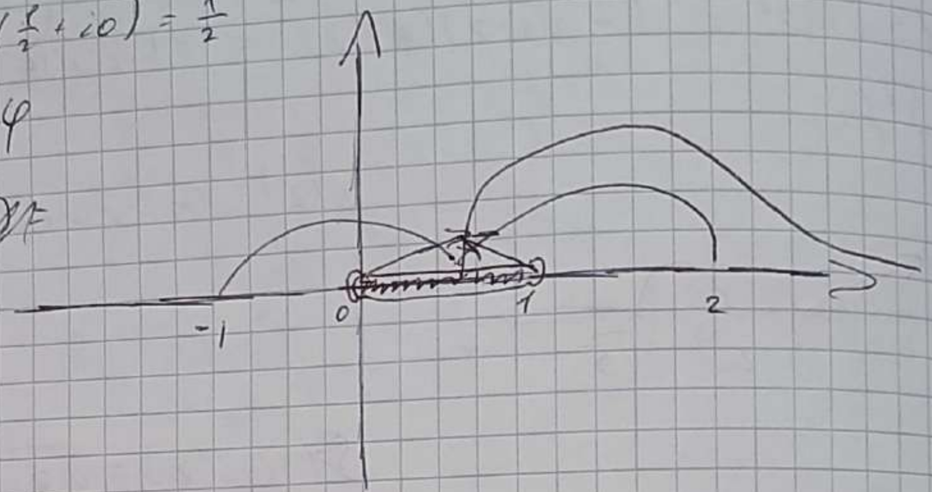
$$= \ln(2 \pm i \cdot 2) - \ln(\sqrt{2}\, e^{\pm i\frac{\pi}{4} + 2\pi n}) = \ln\sqrt{2} \pm i\frac{\pi}{4} + 2\pi n$$

4.4.

$$\varphi(z) = z^{\mu}(1-z)^{1-\mu}$$

$$\varphi\left(\tfrac{1}{2}+i0\right) = \tfrac{1}{2}$$

$\cancel{\varphi(z)} = \varphi$



1) $\varphi(2) = \frac{1}{2}\left|\frac{2}{\frac{1}{2}+i0}\right|^{\mu}\cdot\left|\frac{1-2}{-\frac{1}{2}-i0}\right|^{1-\mu} e^{-i\pi(1-\mu)} = 2^{\mu} e^{(\mu-1)\ldots}$

2) $\varphi(-1) = \frac{1}{2}\left|\frac{-1}{\frac{1}{2}+i0}\right|^{\mu}\cdot\left|\frac{2}{-\frac{1}{2}-i0}\right|^{1-\mu} e^{i\pi\mu} = 2^{1-\mu} e^{i\pi\mu}$

3) $\lim\limits_{z\to\infty} \frac{\varphi(z)}{z} = \lim\limits_{z\to\infty}\left(z^{\mu-1}(1-z)^{1-\mu}\right)$

$$\frac{\varphi\left(\frac{1}{2}+i0\right)}{\frac{1}{2}+i0} = 1$$

$$\left.\frac{\varphi(z)}{z}\right|_{z\to\infty} = \left|\frac{z}{\frac{1}{2}+i0}\right|^{\mu-1}\cdot\left|\frac{1-z}{-\frac{1}{2}-i0}\right|^{1-\mu} e^{i\pi(\mu-1)} =$$

$$e^{i\pi(\mu-1)}\cdot\left|\frac{1-z}{z}\right|^{1-\mu} = e^{i\pi(\mu-1)}$$