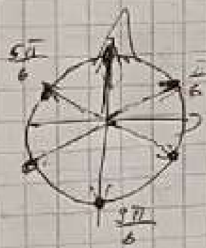
1) $\int\limits_{-\infty}^{+\infty} \frac{x^4}{1+x^6}\,dx = \int + \int\limits_{C_R} \to \frac{1}{R^5} \to 0 \quad = 2\pi i \sum \operatorname{Res}_{z_0} f(z) =$

$$f(z) = \frac{z^4}{1+z^6} = \frac{z^4}{(z^3-i)(z^3+i)}$$

$$z^3 = \pm i = \pm e^{i\left(\frac{\pi}{2}+2\pi k\right)}$$

$$z = \pm e^{i\left(\frac{\pi}{6}+\frac{2\pi k}{3}\right)}$$



3 простых полюсов (все [illegible] [illegible] 3)

$$\operatorname{Res}_{z_0} f(z) = \frac{z^4}{\left((z^3+i)(z^3-i)\right)'} =$$

$$= \frac{z^4}{3z^2(z^3+i) + 3z^2(z^3-i)} = \frac{z^4}{3z^2 \cdot 2z^3} = \frac{1}{6z} = \begin{cases} \frac{1}{6} e^{-i\frac{\pi}{6}} \\ \frac{1}{6} e^{-i\frac{5\pi}{6}} \\ \frac{1}{6} e^{-i\frac{9\pi}{6}} \end{cases}$$

$$= \frac{1}{6} 2\pi i \left(\frac{\sqrt{3}}{2} + \frac{1}{2} i - \frac{\sqrt{3}}{2} + \frac{1}{2} i + i\right) = \frac{2}{3}\pi$$

7) $\displaystyle\int_0^{2\pi} \frac{\cos 2\theta}{2+\cos\theta}\, d\theta$ $\qquad \cos 2\theta = \frac{1}{2}\left(z^2+\frac{1}{z^2}\right)$

$\qquad\qquad\qquad\qquad\qquad\qquad\qquad \cos\theta = \frac{1}{2}\left(z+\frac{1}{z}\right)$

$z = e^{i\theta} \qquad dz = i e^{i\theta}\, d\theta \Rightarrow d\theta = \frac{dz}{iz}$

$I = \oint_\Gamma f(z)\,dz \quad \oint \frac{\frac{1}{2}\left(z^2+\frac{1}{z^2}\right)}{iz\left(2+\frac{1}{2}\left(z+\frac{1}{z}\right)\right)}\,dz = \frac{z^4+1}{iz^2(z^2+4z+1)} =$

$= \oint \frac{z^4+1}{iz^2(z+2+\sqrt{3})(z+2-\sqrt{3})}\,dz =$

в $z=0$: $f(z) \sim \frac{1}{iz^2(1-(-4z-z^2))} =$

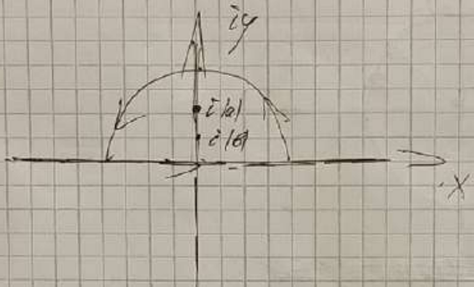
$= -i\frac{1}{z^2}(1-4z-z^2) \Rightarrow c_{-1} = 4i$

$\operatorname{Res}_{z=\sqrt{3}-2} f(z) = \frac{(\sqrt{3}-2)^4+1}{i(\sqrt{3}-2)^2(2\sqrt{3})} = -7\frac{1}{\sqrt{3}}i$

$= 2\pi i\left(-\frac{7}{\sqrt{3}}i+4i\right) = 2\pi\left(\frac{7}{\sqrt{3}}-4\right)$

3) $$\int_{-\infty}^{+\infty} \frac{dx}{(x^2+a^2)(x^2+b^2)^2} = \oint + \int_{C_R} \to \frac{1}{R^5} \xrightarrow[R\to\infty]{} 0 \;=$$

$$f(z) = \frac{1}{(z^2+a^2)(z^2+b^2)^2} = \frac{1}{(z+i|a|)(z-i|a|)(z+i|b|)^2(z-i|b|)^2}$$



$$\operatorname*{Res}_{z=i|a|} f(z) = \frac{1}{(z+i|a|)(z+i|b|)^2(z-i|b|)^2} = \frac{1}{2i|a|(a^2-b^2)^2}$$

$$\operatorname*{Res}_{z=i|b|} f(z) = \frac{d}{dz}\left(\frac{1}{(z+i|a|)(z-i|a|)(z+i|b|)^2}\right) =$$

$$= \frac{1}{(z+i|a|)^2(z-i|a|)^2(z+i|b|)^4}\Big((z-i|a|)(z+i|b|)^2 + 2(z+ib)(z+i|a|)(z-i|a|) +$$

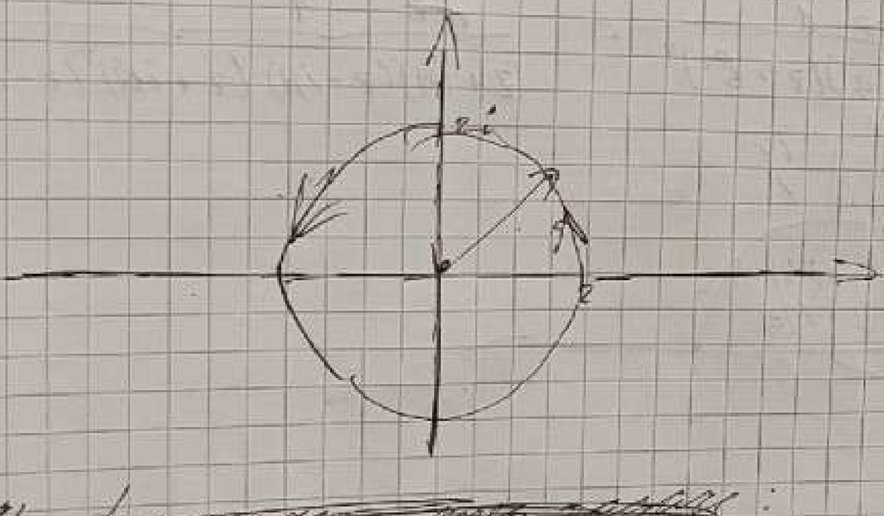
$$-2\cdot\frac{i2b+2z^2+a^2}{(z^2+a^2)(z+ib)^3}$$

$$+ (z+i|a|)(z+i|b|)^2\Big) = \;\ldots\; = -i\,\frac{a^2-3b^2}{4b^3(a^2-b^2)^2}$$

$$= -2\pi i\left(-i\,\frac{a^2-3b^2}{4b^3(a^2-b^2)^2} + \frac{1}{2|a|(b^2-a^2)^2}\,i\right) = \pi\left(\frac{2b^3+a^3-3ab^2}{2ab^3(b-a)^2(b+a)^2}\right) =$$

$$= \frac{2b+a}{2ab^3(b+a)^2}\,\pi$$

$$\int_c \frac{z^5 dz}{1+z^6} = \int_c \frac{z^5 dz}{(z^3+i)(z^3-i)}$$



~~[illegible]~~ :

$\oint f(z)\,dz =$ ~~[illegible]~~

$$f(z) = \frac{z^5}{z^6\left(1+\frac{1}{z^6}\right)} = \frac{1}{z\left(1+\frac{1}{z^6}\right)} = \frac{1}{z}\left(1-\frac{1}{z^6}+\dots\right)$$

$$\oint f(z)\,dz = 2\pi i$$

$f(z) = z^3 \cos \frac{1}{z-2} \qquad z = \infty$

$\underset{z=2}{\mathrm{Res}}\ \cancel{z^3}$ $\qquad z^3 \cos \frac{1}{z-2} \cdot (z-2) \Big|_{z=2}$

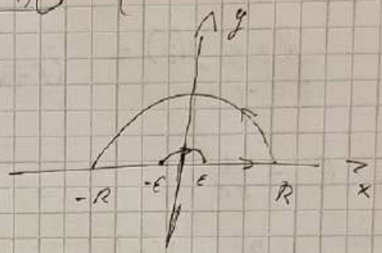
$\lim\limits_{z \to 2} z^3 (z-2) \cdot \cos \frac{1}{z-2} =$

$(\varepsilon + 2)^3 \cos \frac{1}{\varepsilon} =$

$= (\varepsilon^3 + 3\varepsilon^2 \cdot 2 + 3\varepsilon \cdot 4 + 8)\left(1 - \frac{1}{2\varepsilon^2} + \frac{1}{24\varepsilon^4} + \dots\right)$

$-6 + \frac{1}{24} = -\frac{143}{24}.$

$$\int_{-\infty}^{+\infty} \frac{\sin^2 x\,dx}{x^2(x^2+1)} = \cancel{\int_{-\infty}^{+\infty} \frac{1-\cos 2x}{2x^2(x^2+1)}\,dx} =$$



$$= \frac{1}{2} Re\left[\int_{-\infty}^{+\infty} \frac{1-e^{2ix}}{x^2(x^2+1)}\,dx \right]$$

$$\oint_C = \int_{-\infty}^{-\varepsilon} + \int_{\varepsilon}^{+\infty} + \int_{\varepsilon} + \oint_{C_R} \qquad O\sim\frac{1}{R^5} \quad =$$

$$\oint f(z)\,dz = 2\pi i \operatorname*{Res}_{z=i} f(z) = 2\pi i \frac{1-e^{2iz}}{z^2(z+i)} =$$

$$= 2\pi i \frac{1-e^{-2}}{(-1)\cdot 2i} = \pi\left(\frac{1}{e^2}-1\right)$$

$$\int_{\varepsilon} = \int_{\pi}^{0} \frac{1-e^{2i\varepsilon e^{i\varphi}}}{(\varepsilon e^{i\varphi})^2((\varepsilon e^{i\varphi})^2+1)}\, i\varepsilon e^{i\varphi}\,d\varphi =$$

$$= \int_{\pi}^{0} \frac{1-1+2i\varepsilon e^{i\varphi}}{\varepsilon^2(\varepsilon^2+1)}\, i\cdot\varepsilon e^{i\varphi}\,d\varphi =$$

$$2\int_{\pi}^{0} d\varphi = -2\pi$$

$$\int_{-\infty}^{+\infty} = \oint - \int_{\varepsilon} = \pi\left(\frac{1}{e^2}-1\right) + 2\pi = \pi\left(\frac{1}{e^2}+1\right)$$

$$f(z) = \frac{1}{z^3 - z^5} = \frac{1}{z^3(1-z^2)}$$

$$\operatorname*{Res}_{z=-1} f(z) = \frac{1}{(t-1)^3(1-t^2+2t-1)} \approx \frac{1}{(t^3-3t^2+3t-1)(2t-t^2)}$$

$$\approx -\frac{1}{2t}$$

$\operatorname*{Res}_{z=1} f(z)$ аналогично.

$$\operatorname*{Res}_{z=0} f(z) = \frac{1}{z^3(1-z^2)} \quad \frac{1}{z^3}\left(1+z^2+\dots\right) + \dots$$

$$\operatorname*{Res}_{z=0} f(z) = 1$$

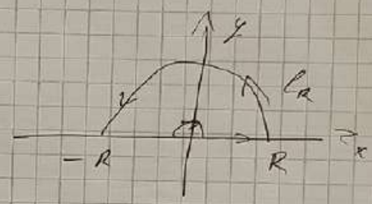
$$\operatorname*{Res}_{z=\infty} f(z) = -\frac{1}{z^5\left(1-\frac{1}{z^2}\right)} = -\frac{1}{z^5}\left(1+\frac{1}{z^2}+\frac{1}{z^4}\right)+\dots$$

$$\operatorname*{Res}_{z=\infty} f(z) = 0.$$

1. $\int_{0}^{+\infty} \frac{x-\sin x}{x^3}\,dx$

$$I = \frac{1}{2}\int_{-\infty}^{+\infty} \frac{x-\sin x}{x^3}\,dx = \frac{1}{2}\int_{-\infty}^{+\infty} \mathrm{Im}\left(\frac{1+ix-e^{ix}}{x^3}\right)dx =$$

$$= \frac{1}{2}\,\mathrm{Im}\int_{-\infty}^{+\infty} \frac{1+ix-e^{ix}}{x^3}$$



$$f(z) = \frac{1+iz-e^{iz}}{z^3}$$

$$\oint f(z)\,dz = 0 = \int_{-\infty}^{+\infty} + \int_{\varepsilon} + \int_{C_R}$$

$$\int_{\varepsilon} = \frac{1+\varepsilon e^{i\varphi} - 1 - i\varepsilon e^{i\varphi} - \frac{(i\varepsilon e^{i\varphi})^2}{2!}}{\varepsilon^2 e^{i\varphi}}\,d\varphi$$

$$i\int_{\pi}^{0} \frac{i^2 \cdot e^{2i\varphi}}{2!\,\varepsilon^2 e^{i\varphi}} = -\frac{\pi}{2}i$$

$$0 = \int_{-\infty}^{+\infty} - \frac{\pi}{2}i \Rightarrow$$

$$I = \frac{1}{2}\,\mathrm{Im}\int_{-\infty}^{+\infty} = \frac{1}{2}\cdot\frac{\pi}{2} = \frac{\pi}{4}$$