1

$$\Delta \bar{A} = -\frac{4\pi}{c}\bar{j}$$

$$\Delta \bar{A} = -\frac{4\pi}{c}\rho\omega r \hat{e}_\varphi \quad (0 \le r \le a) \; (1)$$

$$\Delta \bar{A} = 0 \quad (r \ge a) \; (2)$$

r: $$\frac{A_r}{r^2} - \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial A_r}{\partial r}\right) = 0$$

φ: $$\frac{A_\varphi}{r^2} - \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial A_\varphi}{\partial r}\right) = \frac{4\pi}{c}\rho\omega r$$

z: $$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial A_z}{\partial r}\right) = 0$$

(1): $\Delta \bar{A} = -\frac{4\pi}{c}\rho\omega r \hat{e}_\varphi$:

$$\begin{cases} A_r = C_2 r + \frac{C_1}{r} \\ A_\varphi = \frac{C_3}{r} + C_4 r + \frac{1}{2}\frac{\pi}{c}\rho\omega r^3 \\ A_z = C_5 \ln r + C_6 \end{cases} \Rightarrow \begin{cases} A_r = \frac{C_1}{r} \\ A_\varphi = -\frac{\pi}{2c}\rho\omega r^3 + \frac{\pi}{c}\rho\omega a^2 r + \frac{C_3}{r} \\ A_z = C_6 \end{cases}$$

ум. спивки.

$\Delta \bar{A} = 0$ т.к. $r \to \infty$ (поле конечн.)

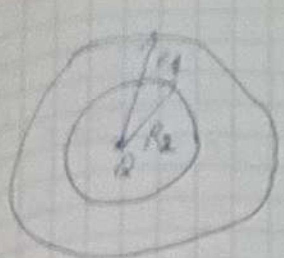
$$\begin{cases} A_r = \tilde{C}_1 r + \frac{\tilde{C}_2}{r} \\ A_\varphi = \tilde{C}_3 r + \frac{\tilde{C}_4}{r} \\ A_z = \tilde{C}_5 \ln r + \tilde{C}_6 \end{cases} \Rightarrow \begin{cases} A_r = \frac{\tilde{C}_2}{r} \\ A_\varphi = \frac{\tilde{C}_3}{r} + \frac{\pi}{2cr}\rho\omega a^4 \\ A_z = \tilde{C}_6 \end{cases}$$

$$\bar{H} = \nabla \times \bar{A} = \left(\frac{\partial A_z}{\partial \varphi}\frac{1}{r} - \frac{\partial A_\varphi}{\partial z}\right)\hat{e}_r + \left(\frac{\partial A_r}{\partial z} - \frac{\partial A_z}{\partial r}\right)\hat{e}_\varphi +$$

$$+ \frac{1}{r}\left(\frac{\partial (rA_\varphi)}{\partial r} - \frac{\partial A_r}{\partial \varphi}\right)\hat{e}_z = \frac{1}{r}\frac{\partial (rA_\varphi)}{\partial r} = \frac{2\pi}{c}\rho\omega(a^2 - r^2)\hat{e}_z$$

$0 \le r \le a$

$H = \nabla \times A$; при $r \ge a$ $\bar{H} = \bar{0}$



$\uparrow E_z$

$$\Delta\varphi = \frac{1}{r^2}\left(\frac{\partial}{\partial r} r^2 \frac{\partial\varphi}{\partial r}\right) + \frac{1}{r^2 \sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta \frac{\partial\varphi}{\partial\theta}\right) = 0$$

при ~~R~~ $r < R_1$

$$E = \frac{Q}{r^2}$$

при $R_1 < r < R_2$

$$E = 0$$

при $r > R_2$:

$$\varphi = \sum_0^\infty \left(A_l r^l + \frac{B_l}{r^{l+1}}\right) P_l(\cos\theta)$$ получено на семинаре.

Поле на бесконечн. $E = (-E_z \cos\theta + E_Q;\ -E_z \sin\theta;\ 0)$

$$E_r = -\frac{\partial\varphi}{\partial r} = -\sum_0^\infty \left(A_l \cdot l \cdot r^{l-1} - \frac{B_l(l+1)}{r^{l+2}}\right) P_l(\cos\theta) = E_z \cos\theta + E_Q$$

Очевидно, что $l = 0;\ 1$

$$E_r = \frac{B_0}{r^2} + \left(\frac{2B_1}{r^3} - A_1\right)\cos\theta = \underbrace{E_Q}_{=\frac{Q}{r^2}} + E_z \cos\theta$$

$$B_0 = Q;\quad A_1 = -E_z$$

$$\frac{\partial\varphi}{\partial\theta} = \cancel{\sum \left(A_l r^l + \frac{B_l}{r^{l+1}}\right)}\ \ \underbrace{A_0 + \frac{B_0}{r}}_{\to 0} + \left(A_1 r + \frac{B_1}{r^2}\right)(-\sin\theta)$$

$$A_1 R_1 + \frac{B_1}{R_1^2} = 0 \Rightarrow B_1 = -A_1 R_1^3 = E_z R_1^3$$

Поле ~~вне шара~~:

$$\varphi = \frac{Q}{r} - E_z r \cos\theta + \frac{E_z R_1^3}{r^2}\cos\theta$$

$$\begin{cases} E_r = +\dfrac{Q}{r^2} + E_z \cos\theta + 2\dfrac{E_z R_1^3}{r^3}\cos\theta \\ E_\theta = -E_z r \sin\theta + \dfrac{E_z R_1^2}{r^2}\sin\theta \\ E_\varphi = 0 \end{cases}$$

3

$$\frac{1}{2^{l+1}\pi i}(1-x^2)(l+2)(l+1)\oint\frac{(t^2-1)^l}{(t-x)^{l+3}}dt-\frac{x}{2^{l}\pi i}(l+1)\oint\frac{(t^2-1)^l}{(t-x)^{l+2}}dt+$$

$$+\,l(l+1)\oint\frac{(t^2-1)^l}{(t-x)^{l+1}}\frac{1}{2^{l+2}\pi i}=0$$

$$\frac{1}{2^{l+1}\pi i}(l+1)\left(\oint\frac{(t^2-1)\left((1-x^2)(l+2)-2x(t-x)+l(t-x)^2\right.}{(t-x)^{l+3}}\right.=$$

$$=\frac{l+1}{2^{l+1}\pi i}\oint\frac{(t^2-1)^l}{(t-x)^{l+3}}\left(t^2 l-2(l+1)tx+(l+2)\right)=0$$

$$\frac{l+1}{2^{l+1}\pi i}\oint\frac{d}{dt}\left(\frac{(t^2-1)^{l+1}}{(t-x)^{l+2}}\right)dt=\frac{l+1}{2^{l+1}\pi i}\oint d(\ldots)=0$$

Ч.Т.Д.