2.1

$$\frac{1+2z^2}{z^3+z^5} = \frac{1+2z^2}{z^3(1+z^2)} = \frac{1}{z^3(1+z^2)} + \frac{2}{z(1+z^2)} =$$

$$= \frac{1}{z^3(z+i)(z-i)} + \frac{2}{z(z+i)(z-i)} = \frac{2}{z} + \frac{1}{z-i} +$$

$$+ \frac{1}{z+i} + \frac{1}{z^3} + \frac{1}{2(z-i)} + \frac{1}{2(z+i)} - \frac{1}{z} =$$

$$= \underline{\frac{1}{z} + \frac{1}{z^3}} + \dots$$

2.2.

$$f(z) = \frac{1}{z(e^z-1)} =$$

$$= \frac{1}{z^2(1+\underbrace{\frac{z}{2!} + \frac{z^2}{3!} + \dots}_{p(z)})} =$$

$$= \frac{1}{z^2}\sum_{0}^{\infty}(p(z))^n(-1)^n = \underline{\frac{1}{z^2} - \frac{1}{2z}} + \dots$$

2.6

① $\int_C \frac{z e^{z^2}}{\tan z^2}\, dz = 2\pi i \sum \mathrm{Res} = 2\pi i$

$f(z) = \frac{z e^{z^2}}{\tan z^2} \approx \frac{z(1+z^2+\dots)}{z^2+\dots} = \frac{1}{z} + 1 + \dots$

$\int_C f(z)\, dz = 1$

② $\int_C e^{-1/z} \sin\left(\frac{1}{z}\right) dz \; \ominus$

$f(z) = \left(1 - \frac{1}{z} + \frac{1}{2!\, z^2} + \dots\right)\left(\frac{1}{z} - \frac{1}{3!\, z^3} + \dots\right) =$

$= \frac{1}{z} - \frac{1}{z^2} + \dots$

$\ominus \; 2\pi i$

③ $\int_C \frac{e^z}{z^n}\, dz = 2\pi i \frac{1}{(n-1)!}$

$f(z) = \frac{\cancel{1+z^2+z}\,\left(1 + \frac{z^2}{2} + \frac{z^3}{3!} + \dots\right)}{z^n} =$

$= \frac{1}{z^n} + \frac{1}{2\, z^{n-2}} + \frac{1}{3!\, z^{n-3}} + \dots + \underline{\frac{1}{(n-1)!\, z}}$

1) $f(z) = \frac{\sin z}{1 - \frac{\sin z}{\cos z}}$ $\qquad \varepsilon = z - \frac{\pi}{4}, \quad z = \varepsilon + \frac{\pi}{4}$

$$= \frac{\sin(\varepsilon + \frac{\pi}{4})}{1 - \tan(\frac{\pi}{4} + \varepsilon)} = \frac{\frac{1}{\sqrt{2}}\sin\varepsilon + \frac{1}{\sqrt{2}}\cos\varepsilon}{1 - \frac{1 + tg\,\varepsilon}{1 - tg\,\varepsilon}} =$$

$$= \frac{1}{\sqrt{2}}\left(\frac{\sin\varepsilon + \cos\varepsilon}{1 - tg\,\varepsilon - 1 - tg\,\varepsilon}\right)\cdot(1 - tg\,\varepsilon) =$$

$$= \frac{(\sin\varepsilon + \cos\varepsilon)(1 - tg\,\varepsilon)}{-2\sqrt{2}\, tg\,\varepsilon} = \frac{(\varepsilon + 1)(1 - \varepsilon)}{-2\sqrt{2}\,\varepsilon} =$$

$$\frac{1 - \varepsilon^2}{-2\sqrt{2}\,\varepsilon} = -\frac{1}{2\sqrt{2}\,\varepsilon} + \dots$$

2) $$f(z) = \frac{e^{c/(z-a)}}{e^{z/a} - 1} = \frac{e^{c/(\varepsilon + 2\pi i a)}}{\exp\left\{\frac{\varepsilon + 2\pi i a}{a}\right\} - 1} =$$

$\frac{z}{a} = 2\pi i \Rightarrow z = 2\pi i a n \qquad \varepsilon = z - 2\pi i a$

$$= \frac{\exp\left\{\frac{c}{\varepsilon + 2\pi i a}\right\}}{\exp\left\{\frac{\varepsilon}{a}\right\} - 1} = -\frac{\exp\left\{\frac{c}{\varepsilon + 2\pi i a}\right\}}{(1 - e^{\varepsilon/a})} \approx$$

$\approx \exp\left\{\frac{c}{\varepsilon + 2\pi i a}\right\}\cdot\left(\frac{a}{\varepsilon} + \dots\right)$ (простой полюс)

в т. $z = a \Rightarrow f(z) = \left(1 + \frac{c}{\varepsilon} + \frac{c^2}{2\varepsilon^2} + \dots\right)\left(\frac{a+1}{\varepsilon} + \dots\right)$ сущ. точка

2.9.

$$f(z) = z e^{\frac{1}{z}} e^{-\frac{1}{z^2}} \qquad z=0$$

$$= z\left(1 + \frac{1}{z} + \frac{1}{2z^2} + \dots\right)\left(1 - \frac{1}{z^2} + \frac{1}{2z^4} + \dots\right)$$

сущ. особ. точка.

2.3

$$\frac{1}{z(z-1)} \qquad ① \ |z| \in (0;1)$$

$$-\frac{1}{z}\sum_0^\infty \cancel{(-1)} z^n = -\sum_0^\infty z^{(n-1)}$$

$$② \ |z| \in (1;+\infty)$$

$$\frac{1}{z^2}\sum_0^\infty \left(\frac{1}{z}\right)^n = \sum_0^\infty \left(\frac{1}{z}\right)^{n+2}$$

2.4

$$\frac{z}{z^2+1} = \frac{z}{(z+i)(z-i)} = \cancel{\varepsilon} \ \frac{1}{2}\cancel{z}\left(\frac{1}{z+i} - \frac{1}{z-i}\right) =$$

$$z = i \ ; \quad \varepsilon = z - i$$

$$= \frac{1}{2}\cancel{(\varepsilon+i)}\left(\frac{1}{\varepsilon+2i} \cancel{+} - \frac{1}{\varepsilon}\right) = \frac{1}{2}\cancel{(\varepsilon+i)}\left(\frac{1}{2i}\left(\frac{1}{1+\left(\frac{\varepsilon}{2i}\right)}\right) - \frac{1}{\varepsilon}\right) =$$

$$= \frac{1}{2}\cancel{(\varepsilon+i)}\left[\frac{1}{2i}\sum_0^\infty (-1)^n\left(\frac{\varepsilon}{2i}\right)^n - \frac{1}{\varepsilon}\right] =$$

$$\cancel{= -\frac{1}{2}\frac{(\varepsilon+i)i}{\varepsilon} + \frac{1}{4}i\sum(-1)^n\left(\frac{\varepsilon}{2}\right)^n = -\frac{1}{2}\frac{zi}{z-i}}$$

$$\cancel{\frac{1}{4}i\sum_0^\infty\left(\frac{(z-i)i}{2}\right)^n}$$

$$= -\frac{1}{2(z-i)} + \frac{1}{4}i\sum_0^\infty\left(\frac{i}{2}(z-i)\right)^n$$