Geometric-Computational Duality: A Cohomological Approach to P vs NP

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Abstract

We propose a novel geometric framework—*Geometric-Computational Duality* (GCD)—to investigate the P vs NP problem via tools from algebraic geometry and sheaf cohomology. Boolean satisfiability (SAT) instances are encoded as algebraic varieties in projective space by homogenizing polynomial representations of logical clauses. We conjecture that problems in class P yield projective varieties with trivial first cohomology group $H^1(\mathcal{O}_V) = 0$, whereas NP-complete problems correspond to geometries with nontrivial topological and cohomological structure.

1 Introduction

The P vs NP problem is a cornerstone of computational complexity theory. We introduce a new perspective that leverages tools from algebraic geometry to formulate a geometric analog of computational difficulty.

2 Background

2.1 Complexity Theory

Overview of P, NP, and SAT. Discussion of natural proofs and relativization barriers.

2.2 Algebraic Geometry

Review of affine and projective varieties, sheaf cohomology, and $H^1(\mathcal{O}_V)$.

3 Geometric Encoding of SAT

3.1 Polynomial Encoding

Boolean variables $x_i \in 0, 1$ with encoding via polynomials over $\mathbb{F}_2$.

3.2 Homogenization to Projective Space

Clauses are homogenized into $\mathbb{P}^n$ for cohomological interpretation.

$$
\begin{array}{c}
\boxed{\text{SAT Formula}} \\
\downarrow \\
\text{Clause } \rightarrow \text{Polynomial} \\
\downarrow \\
\text{Ideal } I \\
\downarrow \\
\text{Variety } V = \mathrm{Proj}(R/I) \\
\downarrow \\
\text{Cohomology } H^1(\mathcal{O}_V)
\end{array}
$$

Figure 1: SAT-to-geometry pipeline in the GCD framework

4 The GCD Framework

We present two conjectures: one linking H^1 to complexity, and one linking symmetry groups to class separation.

5 Case Study: 4-Variable SAT Instance

5.1 Macaulay2 Encoding

We encode three clauses as homogenized polynomials and compute $H^1(\mathcal{O}_V)$, yielding $\dim H^1 = 24$.

6 Implications and Future Work

Outline automation, larger-scale testing, and connections to Geometric Complexity Theory (GCT).

7 Conclusion

Nontrivial cohomology acts as a geometric witness to computational hardness. GCD offers a promising path toward understanding P vs NP.