Differential Equations Notes

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Contents

Remarks

This notes thing was started on 10/19/2025.
I am to work through the WHOLE of the textbook (Elementary Differential Equations and Boundary Value Problems (11th ed)) by the end of this quarter (12/13/2025).
Hopefully, I'll also get around 20-40% of the problems in the textbook done.

1 Introduction

aka chapter 1

1.1 Introduction for the Introduction

Definition (Differential Equations)**.** Equations containing derivatives.

Definition (Slope Field/Direction Field)**.** A buncha line segments on the plane that represent the "motion" of a diff-eq.

Direction Fields are good for studying differential equations of the form

$$\frac{dy}{dt} = f(t, y).$$

(Page 6 – How to construct a diff-eq mathematical model from a real-world situation.)
(7) Newton: Differential equations come in one of these 3 forms:

1. $\dfrac{dy}{dx} = f(x)$,
2. $\dfrac{dy}{dx} = f(y)$,
3. $\dfrac{dy}{dx} = f(x, y)$.

Exercise 11-16.
1.1.5 corresponds with **j**.
1.1.6 corresponds with **c**.
1.1.7 corresponds with **g**.
1.1.8 corresponds with **b**.
1.1.9 corresponds with **h**.
1.1.10 corresponds with **e**. □

Exercise 17.
(a)

$$\frac{dC}{dt} = [\text{chemicals/hour going in}] - [\text{out}] = 0.01 \cdot 300 - 300 \cdot \frac{C}{1000000}$$

where C is the number of gallons of said chemical in the pond and t is time measured in hours.

(b) After a very long time, 10000 gallons will be in the pond; this limiting amount is independent of starting conditions.

(c) Since concentration $= \dfrac{\text{Amount}}{\text{Volume}}$, $C = \text{volume} \cdot c = c \cdot 10^6$ where c stands for concentration. As such,

$$\frac{dc}{dt} = \frac{1}{10^6}\frac{dC}{dt} = \frac{3}{10^6} - \frac{3(c \cdot 10^6)}{10^4 \cdot 10^6}$$

So in final, $\boxed{\dfrac{dc}{dt} = \dfrac{3}{10^6} - \dfrac{3c}{10^4}}$. □

Exercise 18.

$$\frac{dV}{dt} = -k \cdot 4\pi r^2 = -k \cdot 4\pi \left(\frac{3}{4\pi} V\right)^{\frac{2}{3}}$$

$\square$

Exercise 19.

$$\frac{dT}{dt} = -0.05 * (T - 70)$$

where T is the temperature of the object in Farenheit and t is time in minutes. $\square$

1.2 Introduction to Solutions

(11) - Finding the general solutions to diff-eqs of the form $\frac{dy}{dt} = ay - b$;

$$\frac{dy}{dt} = ay - b \Longrightarrow y(t) = \frac{b}{a} + \left(y_0 - \frac{b}{a}\right) e^{at}$$