Designing Markets for Reliability with Incomplete Information

Job Market Paper - Preliminary Draft

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Abstract.

This paper examines the challenges of allocating a good subject to capacity constraints, when considering consumer preferences and investment decisions. A theoretical framework is developed where a market designer sequentially chooses a level of investment and proposes an allocation mechanism to consumers followed by a consumption stage. The market designer uses the allocation to maximize consumer surplus and finance the investment cost. He faces heterogenous consumers who have private information about their demand level and belong to a publicly observed category. We show that the lack of complete information about consumer utility and constraints on the implementable mechanism leads to specific relations between the optimal allocation mechanism and the level of investment. Namely, we find that the optimal allocation implies discriminating consumers based on their types and that discimination depends on the level of investment considered. This implies that the most efficient mechanism is not always Pareto-improving for every consumer. This has significant welfare and distributive implications. We first study the benchmark case with complete information. We then analyze the current second-best situation, in which the market designer cannot obtain information about consumers and must choose fixed prices ex-ante. In the third step, we describe the optimal theoretical second-best allocation mechanism that considers the incentive and individual rationality constraints and the investment decisions.

1 Introduction

Recent events, such as the European energy crisis, have highlighted that investments in infrastructure are crucial for ensuring a reliable and adequate supply of essential goods. These goods are characterized by demand and supply that fluctuate significantly and unpredictably. Moreover, any demand that exceeds the available capacity and cannot be efficiently rationed generates significant welfare losses. For instance, without sufficient investment, the reliability of the electricity supply can be compromised, leading to frequent outages and power interruptions. It affects individuals' daily lives and significantly impacts industries and the overall economy. The COVID crisis has also shown that the lack of production capacity for medical goods can lead to significant systemic effects due to economic and health externalities. The transport sector must also be mentioned due to the growing interest in modal shifts from individual carbon-based means of transport to carbon-free public means. Without proper management of the capacity level and the demand for these goods, we face the risk of sustaining significant congestion costs, preventing a much-needed modal shift.

The central contribution of this paper is to discuss the implications of considering the demand side when it comes to ensuring an efficient level of investment. Indeed, most of the previous discussions in the economic literature have recognized the importance of public intervention when providing investment and a reliable supply of essential goods. However, it is mainly centered around the supply side of the provisions while taking the demand for the good as given. Namely how to procure sufficient investment at the least cost and how to consider the private incentives producers face, compared to the social value investments. Leaving aside any supply-side considerations, we focus on implementing the best mechanism to ensure efficient consumption in the face of capacity constraints and investment decisions. Therefore, the key motivation is that by overlooking the role of demand management, we risk missing opportunities to lower the cost of providing essential goods.

We study a regulated entity, called the market designer, that (i) determines the allocation in prices and quantities of a homogeneous good and (ii) chooses the level of investments that maximizes consumer surplus. After making an investment decision, the market designer faces a set of consumers who wish to consume the essential good. The consumers are characterized by a linear utility function, which is uncertain when the market designer makes investment decisions and proposes an allocation mechanism. This uncertainty has two additive components: (i) a common shock that is identical across all consumers, and (ii) a private shock only observed by consumers before the consumption stage and the realization of the common shock. We also embedded each consumer with a category for which the market designer is publicly informed. Therefore, our environment highlights the role of information constraints the market designer faces when designing the demand side of essential goods. Indeed, the existence of private information with respect to their consumption implies that consumers' private incentives might also differ from the market designer's objective. In a nutshell, the paper provides a conceptual framework for the demand for reliability when the market designer procures the investment from producers and wishes to implement a market for the demand side when he faces heterogeneous consumers with private information. We also describe some results concerning the distributive issues a market designer may face when choosing the efficient allocation, especially regarding the possibility of discriminating between consumer categories.

Our framework uses the canonical investment decision model used in the electricity market that describes how capacity constraints interact with the provisions of a homogeneous good with time-varying uncertain stochastic demand. Therefore, when the market designer chooses the investment level and the efficient allocation of the essential good across its consumers, he needs to consider the potentially different asymmetrical effects that a given allocation can have when the capacity is binding. We will show that depending on the ability to propose an allocation based on the different states of the world, the consideration of the capacity constraint can significantly impact the design of the efficient mechanism. Hence, we also show that the (potential) incompleteness of the mechanism proposed by the market designer due to implementation constraints can have significant impacts.

We also link the investment decision and the choice of allocation mechanism by implementing a budget constraint for the market designer. That is, the prices and quantity that dictate the consumption stage are entirely used by the market designer to cover the fixed cost associated with the capacity. Therefore, our model endogenously determines both the investment decisions and the allocation mechanism, which allows us to fully characterize the initial motivations: an increase in investment allows more consumption via a change in capacity constraint but at the expense of a decrease in consumer surplus due to the need for more revenue due to the budget constraint.

To summarize, our framework encompasses the decisions made by a market designer facing a series of interrelated constraints. (i) A set of feasibility constraints: the capacity and the budget constraint; (ii) to which we add the incentive compatibility and individual rationality constraints due to the incomplete information framework. Finally, to derive some policy implications, we implement market design constraints and, in further work, some distributive preferences.

In the first step, we derive the optimal schedule for the consumption allocation that the market designer offers consumers. Under perfect information, the *spot market* allocation coincides with the optimal transfers that allow individualized prices and quantities for consumers. In other words, when investment decisions are considered, the optimal schedule remains identical to a single price given by a spot market when demand equals supply. This result opposes a short-sighted market designer whose allocation is only chosen to maximize consumer surplus, with investment decisions made only with the residual revenue. In that case, the optimal allocation mechanism implies discriminating between consumers, with lower types being rationed first.

Then, we analyze a framework that mimics the actual design of most electricity markets. The market designer has imperfect information and cannot implement any revelation mechanism. We assume that he faces market design constraints because he can only implement a fixed price for every state of the world concerning the common shock. We first study the single-price case when the market designer cannot discriminate between categories of consumers. This framework allows us to focus only on the revenue effect of our framework without distributive effects between categories. In that case, the price schedule is increasing with the level of investment. We then implement the possibility of discriminating between consumer categories. We find that this generates a specific relation between the level of investment and the price offered for each category. Namely, we show that for the category of consumers with the smaller average private shock, the optimal price first decreases and then increases with the investment level. On the other hand, the price for the category of higher consumers is always increasing with the capacity. We also find that the price for the former category is above the latter for relatively low capacity values, and then the ranking reverses for higher values. These non-monotonicities can be explained by the opposite effect the market designer faces in terms of consumer surplus and revenue effect when choosing prices. We find that the net effect between the two depends on the level of investment.

In the last section, we study a mechanism design setup where the market designer is no longer constrained in the prices and quantities schedule he can offer consumers. He now faces incentive compatibility and individual rationality constraints. We start by extending the canonical setup of mechanism design to consider the specificity of our set-up, that is, budget and capacity constraints. We then derive the optimal allocation. We first describe for which level of investment the market designer is constrained by the revenue used to cover the fixed costs and the information rent that he needs to provide to consumers so they behave truthfully. We find that the market designer can provide an unconstrained first-best allocation only for low values of the investment level due to the concavity of the virtual consumer utility with respect to the capacity. For higher values, the additional expected utility gains from increased capacity cannot compensate the investment costs. Then, we show that the behavior of the optimal allocation depends on the state of the world considered and the type of consumers. Namely, as the investment level increases, consumers always face a decrease in the optimal quantity allocated during off-peak. For on-peak periods, the quantity change depends on the consumer's type. Even if the category exhibits some ranking in average private consumption, decreasing the allocation of smaller types within each category may be optimal.

The remainder of this section discusses the related literature. Section 2 presents the environment. Section 3 characterizes the optimal allocation in the case of perfect information. Section 4 extends the analysis to the case where the consumers' type is private information, and the market designer cannot extract any information. Section 5 implements the mechanism design. Section 6 concludes.

Related Literature

The initial environment of the paper originates from the canonical literature on investment decisions in the electricity sector Boiteux (1949). It has mainly been used in recent work to study the role of market power, as in Léautier (2016), where producers can increase the price on the spot market beyond marginal cost even though they are not capacity-constrained. The paper also introduces some long-term agreements with producers offering in capacity remuneration mechanisms and short-term markets. The effect of price regulation is also analyzed in Leautier (2018), where the author demonstrates that short-term inefficiencies can sometimes have long-term and counterintuitive effects. In this paper, the price cap changes the private incentives producers face, hence the final investment decisions. In line with the current paper, Holmberg and Ritz (2020) study the effect of having inefficient rationing due to inelastic demand. When demand exceeds capacity, there exists an additional welfare loss. Consequently, electricity prices do not internalize this additional cost, and the market designer needs to implement an additional stream of revenue for the producers. Our work introduces two features in the model : (i) heterogeneous consumers with private information and (ii) inefficiencies due to the schedule commitment by the market designer before the uncertainty is resolved.

A second stream of papers is also related to the electricity markets and is based on the seminal paper by Chao and Wilson (1987) on priority service. The central idea is to provide a mechanism design solution in the form of a contractual arrangement where consumers choose at the same time the allocation during the wholesale market, which is in the same vein as the allocation schedule of this paper, and the probability of being disconnected when demand exceed the level of capacity. This framework has been refined by a series of papers by the same authors, including the comparison with other market arrangements Chao et al. (2022) and the role of risk aversion Chao (2012). We also relate to a series of papers focusing on implementing the second-best pricing method for consumers with incomplete information in Spulber (1992a,b, 1993). The work in Spulber (1992b) focuses on an incomplete information framework without investment decisions. The optimal allocation schedule is non-linear because consumers' type is private information. Therefore, the market designer faces some challenges concerning implementing such schedules. In Spulber (1992a), a regulated firm is introduced to consider its budget constraint. However, the focus of this paper remains circumscribed to the design of consumers' second-best rates. Finally, Spulber (1993) studies the case of a monopoly designing the rates under incomplete information. We depart from this literature by deepening the private incentives consumers might have by behaving strategically from the truthful reporting and by tightening the link with the investment decisions framework developed in the previous paragraph.

With those initial results and ongoing work, this paper fits the new literature on industrial organization using an incomplete information framework. Triple-IO (for Incomplete Information Industrial Organization) papers aim at underlining traditional issues of industrial organization and how it can be renewed when imperfect information exists. See, for instance, the literature review by Loertscher and Marx (2021). This paper deals with the effect of capacity-constrained systems where (inefficient) rationing must be implemented. It fits with some works by Loertscher and Muir (2020, 2021) and Gilbert and Klemperer (2000), which studies pricing and rationing decisions within imperfect information. We add to the existing literature by providing a similar framework but by including investment decisions and a different type of rationing mechanism.

Finally, our paper is also related to the literature on optimal mechanism design, especially with distributive concerns. Our framework is particularly related to the papers from Akbarpour et al. (2023a) and Akbarpour et al. (2023b), which study the trade-off between allocating certain vaccines on a free but random basis or using prices to discriminate and extract information from consumers. In the two papers, the authors assumed that the market designer has distributive and exogenous revenue preferences. Therefore, the model exhibits a tension of allocating the good via prices, which generates some revenue, or via a random free allocation that minimizes distributive issues. Our paper allows us to endogenize the revenue preference by implementing investment decisions with a budget constraint. We also provide results when the policymaker can imperfectly implement prices.

2 Environment

We describe in the section the idiosyncratic characteristics of an electricity system. Note that while the terminology is specific, the results can be applied to other essential goods as described in the introduction. (i) The demand side, which can be interpreted as households, industrial consumers, or retailers participating in the electricity market; (ii) The allocation mechanism that defines how the market designer allocates (in terms of quantity and financial transfer) electricity to the demand side. (iii) The supply side concerns how investment and production decisions are made. This current version of the paper focuses solely on a market designer configuration where investment decisions are made to maximize consumer surplus. From an outcome perspective, this is similar to having either a monopolist subject to budget constraint or a set of perfect competitive producers without market failure or public interventions. (iv) The decisions' timing.

2.1 Consumers Preferences

There exists a unit mass of consumers for electricity. Each consumer is characterized by a type vector (i, θ, s). The first characteristic refers to the consumer category, such as, for instance, a consumer being a household or an industry. There is a finite set of categories such that $i \in \{1, 2\}$. It is publicly observed, and the size of each category, i.e., the number of agents, is denoted by $\mu_i > 0$ for each group. Each consumer is characterized by a demand level θ, which, under an incomplete information framework, is assumed to be privately observed by the consumer. Conditional on belonging to a category i, this value is drawn from a common-knowledge cumulative distribution function distribution G_i whose continuous density is $g_i > 0$ has full support and is strictly positive on $[\underline{\theta}_i; \bar{\theta}_i]$. With households, θ could represent the revenue shocks, the lowest type of consumer being the poor household and the highest type of consumer being the more prosperous household. Industrial consumers could also be modeled with this framework, where θ represents their buyers' orders (see Chao (2012) for a micro foundation). When we define a consumer category i as being of a higher type concerning a category j as follows:

Definition 1. *If the consumer category i is of a higher type than consumer category j, then $G_i(\theta)$ first-order stochastically dominates $G_j(\theta)$*

For instance, if we assume the same distribution for both categories, then it implies that we have $\bar{\theta}_i \geq \bar{\theta}_j$ and $\underline{\theta}_i \geq \underline{\theta}_j$. On the other hand, we suppose that consumers are also subject to an individual but identical shock represented by s. Every agent in the game knows this value. It can mean, for instance, weather shock or specific economic conditions (recession) observable by everyone. This shock follows a common-knowledge continuous distribution $F > 0$ whose density $f > 0$ has full support on $s \in [0, \bar{s}]$. In this framework, the demand shock is the same for all consumers, and the aggregate shock equals $2s$. For now, we also assume uniform distribution for the common shock and the private information.

We assume each consumer type is known before the demand shocks are realized in this initial environment. Therefore, this framework encompasses two interpretations of the demand shocks: (i) a static model, where a single shock is realized, and there is uncertainty concerning its realization. (ii) a repetition of multiple shocks over a given period (for example, one year), which are drawn from the distribution $F(.)$ Léautier (2016). In the last interpretation, we assume that the type of consumer does not change between different shocks and is determined before this given period. Expectation operator $\mathbb{E}_s$ will denote the expectations over every state of the world. All agents in the game are assumed risk-neutral.

We define a consumer's utility belonging to a category i of a type θ_i. The value for electricity consumption for each consumer is denoted: $U(q, \theta, s) = \int_0^q u(\tilde{q}, \theta, s) d\tilde{q}$, with q the quantity of electricity allocated to the consumer. u can be interpreted as the marginal willingness to pay for a given quantity of electricity. If a consumer receives a quantity q in exchange for a monetary transfer t, we define the indirect utility function, also referred to as the consumer surplus, as $V(q, \theta, s) = U(q, \theta, s) - tq$. If a consumer does not receive electricity, we assume its value is null. Finally, we assume that u is linear of the form : $u(q, \theta, s) = \theta + s - q$

2.2 Allocation design

Given a total quantity $Q(s)$ of electricity in state of the world s, a general allocation mechanism $\mathcal{M}$ can be described via a collection of functions $q_i : [\underline{\theta}_i, \bar{\theta}_i] \rightarrow \Delta(Q(s))$ where q_i is a function describing the quantity q of electricity allocation to a consumer with type θ in category i at a state

s. The aggregate quantity allocated to a group i of consumers is $Q_i(s) = \mu_i \int_{\theta_i} q_i(\theta, s) dG_i(\theta)$. The total allocation is recover with $Q(s) = \sum_i Q_i(s) = \sum_i \mu_i \int_{\theta_i} q_i(\theta, s) dG_i(\theta)$.

We also define the function $t_i(\theta, s)$ as the monetary transfer assigned to a consumer with type $\theta \in [\underline{\theta}_i, \bar{\theta}_i]$ in category i at state s. To study the optimal second-best mechanism with incomplete information in section 5, we rely on the Revelation Principle. Given a direct mechanism $(q_i, t_i)_{i=\{1,2\}}$, for each category, consumers report their type θ, receive an allocation $q_i(\theta)$, and pays $t_i(\theta)$ to the market designer. This framework can also encompass other mechanisms. We study, in particular, spot market allocations, which are defined by a relationship between the monetary transfer (understood as a price) and quantity such that it respects a demand function. In the case the consumer chooses the level of demand concerning the transfer, then we have the following relationship $q_i(\theta, s) = d(t_i(\theta), \theta, s)$. For a category i of consumers, the aggregate electricity demand given a category is $Q_i = d(t_i, s) = \mu_i \int_{\theta_i} d(t_i(\theta), \theta, s) dG_i(\theta))$. Finally, The inverse demand functions for each category as $p_i(Q_i, s) = D_i^{-1}(Q_i, s)$, and the aggregate function for all consumers is given by $p(Q(s), s) = \sum_i D_i^{-1}(Q_i, s)$.

2.3 Supply side

We assume the most straightforward form for the supply side. A direct interpretation is that the market designer collects total revenues $\sum_i \mu_i \int_{\theta_i} t_i(\theta) q_i(\theta_i) dG_i(\theta)$ and makes investment decisions in productive capacity. It encompasses the literature on the management of a public firm or the direct regulation of a private monopolist. The model could describe a market designer acting as an intermediary between consumers and producers, marking production and investment decisions. In that case, the mechanism between consumers and the market designer could be understood as a theoretical retail market, and the mechanism between producers and the market designer would be a wholesale market. The main idea is that we remain agnostic about the proper form of the allocation mechanism between producers and the market designer. However, we assume it is fully efficient in that the production and investment decisions are made in the same manner under optimal regulation (for instance, the market designer acts as a single buyer under a spot market with perfectly competitive producers). In the rest of the paper, we abstract from those details. We assume a market designer making both production and investment decisions.

We denote the level of investment k. The investment cost is linear with $I(k) = rk$. The production cost is unitary and normalized to 0. The capacity level k implies a capacity constraint such that for any total quantity allocation Q and any realization of s, we must have at $Q(s) \leq k$.

Following the description of the supply side, the market designer chooses the allocation and the investment decisions to maximize a long-term expected and aggregate welfare function :

$$\int_s \underbrace{\sum_i \mu_i \int_{\theta_i} \lambda_i(\theta) V_i(\theta, s) dG_i(\theta)}_{\text{Consumers surplus}} + \underbrace{\sum_i \mu_i \int_{\theta_i} t_i(\theta) q_i(\theta) dG_i(\theta)}_{\text{Revenue}} \; dF(s)$$

Where $\lambda_i(\theta)$ represents welfare weights on a given consumer. We will specify their role later in further research. For now, we assume that they are equal to $G_i(\theta)$. They are neutral for maximizing the welfare function, which boils down to the expected sum of the consumer's surplus function.

2.4 Timing

We assume a multi-period game where :

1. **Information stage.** The consumers (and the market designer under complete information) learn about consumer types.
2. **Investment Decision**. The market designer chooses the level of investment k
3. **Allocation Proposal**.
   (a) The market designer chooses an allocation schedule (which can be market or mechanism-based) offered to the consumers. The allocation can be fully complete if it depends on all the realization of s, or incomplete if there exist some constraints that limit the allocation to some realization of s.[1]

[1]In current practice, political and technological constraints imply that the market designer (or any retail agent) can only propose a finite number of schedules. See, for instance, Astier (2021) for theoretical and empirical implications for consumer surplus of allocation incompleteness.

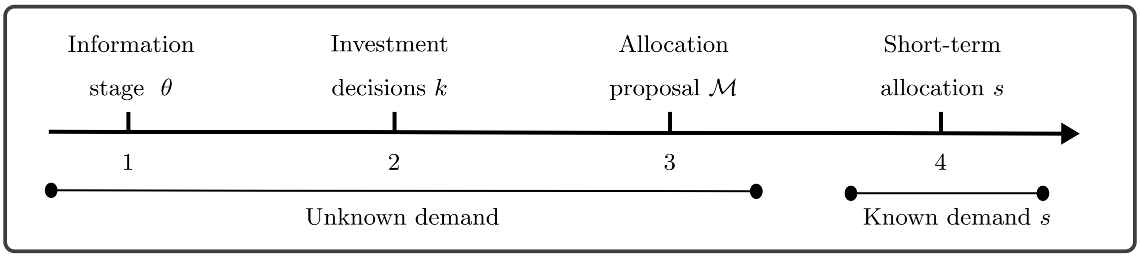
(b) Consumers accept or reject the offer (in this case, the consumer does not participate in the third stage and receives no electricity.

4. **Short-term allocation**.

   (a) The realization of the common shock is known to every agent or the given period that occurs.

   (b) The allocations are realized following what has been proposed in the third stage.

We summarize the timeline of the game below :



3 Complete Information

3.1 Optimal allocation proposal

The first regime we study is the complete information case concerning consumer type. It can be understood as a nonstrategic regime with complete information in the sense that consumers reveal their type honestly. For each realization of the shock s, we define the allocation rule under complete information with $q_i^s(\theta, s)$ that maps the observed type of each consumer for each category to the quantity allocated. The payment rule $t_i^s(\theta, s)$ maps the observed type of each consumer to the per-unit payment made by the consumer to the market designer. This framework can be understood as the market designer offering a price/quantity allocation schedule that varies depending on the demand shock. We derive its problem as follows :

$$\max_{\substack{t_i^s(\theta,s)\to\mathbb{R}^+,\\ q_i^s(\theta,s)\to\mathbb{R}^+,\\ k\geq 0}} \quad CS(k) = \sum_i \ \mu_i \int_s \int_{\theta_i} U(q_i^s(\theta,s),\theta,s) \ - \ t_i^s(\theta,s) q_i^s(\theta,s) dG_i(\theta) dF(s)$$

$$\text{s.t.} \quad I(k) \ \leq \ \sum_i \mu_i \int_s \int_{\theta_i} t_i^s(\theta,s) q_i^s(\theta,s) \ dF(s), \tag{R}$$

$$\sum_i \mu_i \int_{\theta_i} q_i^s(\theta,s) dG_i(\theta) \leq k, \tag{K}$$

The first constraint follows the principle that the market designer should avoid any negative revenue at the optimum level of investment. It is straightforward to show that the constraint is binding under the problem's solution. It allows the rewrite of the objective function by replacing the payment part directly with the investment cost. For consistency with the rest of the analysis, we keep separated this constraint. In other words, under our supply-side assumption and given the absence of production cost, the entire income is allocated to financing the investment costs. The second constraint is the capacity constraint.

Solving for the Lagrangean shows that when the capacity is not biding, the optimal allocation is characterized by an expected marginal utility null: $\int_{\theta_i} u_i(q_i^s(\theta,s),s) dG_i(\theta) = 0$. On the other hand, when the capacity is binding, the optimal allocation should be equal to the marginal investment cost: $\int_{\theta_i} u_i(q_i^s(\theta,s),s) dG_i(\theta) = r$. It also implies that the optimal allocation is such that the marginal utility should be equal in every state of the world. We show in Proposition 1 that a spot market allocation implements the first best allocation given the framework. The spot market allocation is defined by a price schedule t_i^s linked to the optimal allocation schedule q_i^s such that $q_i^s(\theta,s) = d(t_i^s(\theta,s),s)$ with d the aggregate function of individual function d defined as $d = u^{-1}$. It is similar to implementing a price schedule where consumers adjust their quantity to maximize their utility function.

Proposition 1. *(i) A spot market allocation implements the optimal allocation.*

(ii) The price and quantity schedule is defined for each realization of s as follows:

$$t^s(k,s) = \begin{cases} 0 \\ p(k,s) \end{cases} \quad \textit{and} \quad q_i^s(k,\theta,s) = \begin{cases} d(0,\theta,s) & \textit{if} \quad s \in [0, s_1(k)) \\ d(p(k,s),\theta,s) & \textit{if} \quad s \in [s_1(k), \bar{s}] \end{cases}$$

With $s_1(k)$ *defined as the first state of the world when the capacity is binding such that* $s_1 = \left\{ \quad s \quad | \quad \sum_i \mu_i \int_{\theta_i} d(0,\theta,s) dG_i(\theta) = k \right\}$. *Moreover, the optimal level of investment is given by the equality between the marginal investment cost and the expected marginal revenue when the capacity is binding:*

$$k^s = \left\{ \quad k \quad | \quad r = \int_{s_1(k)}^{\bar{s}} p(k,s) dF(s) \right\}$$

The results of this proposition are at the core of how the spot market in the electricity system should work. Whenever the capacity is not constraining, prices equal the short-term marginal cost, i.e. the marginal production cost, which is null in our framework. When the capacity is binding, prices should be raised above the long-term marginal cost such that the expected prices during those periods equal the marginal investment cost. Given the maximization objective, the optimal transfer between consumers and the market designer for each s is identical to implementing the single price given by the aggregate inverse demand function at the capacity level.

With the linear model, we can express the expected consumer surplus in three intermediate cases depending on the level of investment k and the realization of the demand shock s : (i) the capacity always binds for any s, that is for any low $k \in [0, k^-]$ with $s_1(k^-) = 0$, (ii) the capacity never binds for any s, that is for any high $k \in [k^+, +\inf)$ with $s_1(k^+) = \bar{s}$ (iii) the capacity binds for some s, that is for any $k \in [k^-, k^+]]$. For the last case, we can express, for instance, the expected consumer utility under the optimal single-price allocation as follows:

$$\sum_i \mu_i \int_0^{s_1(k)} \underbrace{\int_{\theta_i} U(d(0,\theta,s),\theta,s) dG_i(\theta)}_{\text{off-peak utility}} dF(s) + \sum_i \mu_i \int_{s_1(k)}^{\bar{s}} \underbrace{\int_{\theta_i} U(d(p^k,\theta,s),\theta,s) dG_i(\theta)}_{\text{on-peak utility}} dF(s)$$

$p^k = p(k,s)$ is defined for notation clarity as the aggregate demand function at the investment level $p(k,s)$.

3.2 Long-term vs. Short-term consumer surplus

The previous section showed that the optimal mechanism allocation is identical to a spot market under complete information when the market designer seeks to optimize the expected consumer surplus. In practice, the market designer might also pay attention to consumer surplus on a short-term horizon. In this section, we analyze the consequence of choosing first the investment level and second the consumer surplus. [2] We define the new objective for the market designer as follows. For clarity, we note 0, the period under which the investment decision is made, and period 1, the period under which the choice of mechanism is made.

$$\max_{k\geq 0} \max_{\substack{t_i^{st}(\theta,s)\to\mathbb{R}^+,\\ q_i^{st}(\theta,s)\to\mathbb{R}^+}} CS^{st}(k) = \sum_i \mu_i \int_s \int_{\theta_i} U(q_i^{st}(\theta,s),\theta,s) \; - \; t_i^{st}(\theta,s)q_i^{st}(\theta,s)dG_i(\theta) \quad dF(s)$$

s.t.

$$\text{period} \quad 0: \qquad I(k) \; \leq \; \sum_i \mu_i \int_s \int_{\theta_i} t_i^{st}(\theta,s)q_i^{st}(\theta,s)\; dF(s), \tag{R}$$

$$\text{period} \quad 1: \qquad \sum_i \mu_i \int_{\theta_i} q_i^{st}(\theta,s)dG_i(\theta) \leq k, \tag{K}$$

Compared to the long-term case, we dissociate the maximization problem into two sub-problems, which is solved using backward induction. First, the market designer maximizes the consumer surplus and then chooses the investment level. We find that a spot market allocation with individualized prices implements the optimal allocation under this objective. We describe the mechanism in the following proposition assuming w.l.o.g. that consumers of category 1 are of a higher type than category 2. For clarity we assume that $\bar{\theta}_1 \geq \bar{\theta}_2$ and $\underline{\theta}_1 \leq \underline{\theta}_2$

Proposition 2. *Under short-term consumer surplus maximization, the best allocation schedule is an individualized price system, where prices are set to ration the consumers from the lowest first.*

[2] Recent events in the European electricity markets showed that a market designer might adopt this short-term-oriented policy. Some interventions focused on reducing short-term prices via diverse interventions without considering long-term investment decisions. Therefore, this modeling approach could mirror those interventions.

Assuming w.l.o.g. that consumer category 1 *is of a higher type than category* 2*, then the price and quantity schedule is defined for each realization of* s *as follows.*

- *If* $s \in [0, s_1(k))$ *then* $t_1^{st}(\theta, s) = t_2^{st}(\theta, s) = 0$
- *If* $s \in [s_1(k), s_2(k))$ *then* $t_1^{st}(\theta, s) = 0$ *for all consumers* 1 *and* $t_2^{st}(\theta, s) = 0$ *for consumers* 2 *with* $\theta \in [\underline{\theta}_2, \underline{\theta}_1]$*. Define* $Q_0^1(s)$ *the total quantity for consumers having* $t_i^{st}(\theta, s) = 0$*. Then, for consumers 2 with* $\theta \in [\underline{\theta}_1, \bar{\theta}_2]$*,* $t_2^{st}(\theta, s)$ *is defined such that* $\int_{\underline{\theta}_1}^{\bar{\theta}_2} d(t_2^{st}(\theta, s), \theta, s) dG_2(\theta) + Q_0^1(s) = k$
- *If* $s \in [s_2(k), s_3(k))$*, then* $t_1^{st}(\theta, s) = 0$ *for consumer* 1 *with* $\theta \in [\bar{\theta}_2, \bar{\theta}_1]$*. Define* $Q_0^2(s)$ *the total quantity for consumers having* $t^{st}(\theta, s) = 0$*. Then, for all consumer with* $\theta \in [\underline{\theta}_1, \bar{\theta}_2]$*, a unique price for same types of both category* $t^{st}(\theta, s)$ *is defined such that* $\sum \mu_i \int_{\underline{\theta}_1}^{\bar{\theta}_2} d(t^{st}(\theta, s), \theta, s) dG_i(\theta) + Q_0^2(s) = k$
- *For* $s \in [s_3(k), \bar{s}]$ *then define* $t_1^{st}(\theta, s)$ *for consumers* 1 *such that* $\int_{\bar{\theta}_2}^{\bar{\theta}_1} d(t_1^{st}(\theta, s), \theta, s) dG_1(\theta) = k$

The prices are set to $\theta + s$ *for all other consumers, so their demand is null. Moreover, the demand schedule is determined by the demand function* $d(t, \theta, s)$ *at the defined price schedule.* $s_j(k)$ *with* $j \in \{1, 2, 3\}$ *are defined such that the total quantity of consumers at a null price equals the investment level:* $\sum \mu_i \int_{\theta_i} d(0, \theta, s_1(k)) dG_i(\theta) = k \quad , \quad \mu_1 \int_{\underline{\theta}_1}^{\bar{\theta}_1} d(0, \theta, s_2(k)) dG_1(\theta) + \mu_2 \int_{\underline{\theta}_1}^{\bar{\theta}_2} d(0, \theta, s_2(k)) dG_2(\theta) = k \quad , \quad \mu_1 \int_{\underline{\theta}_2}^{\bar{\theta}_1} d(0, \theta, s_3(k)) dG_1(\theta) = k$

Given the capacity constraint, the individualized price system can be understood as a rationing mechanism. Hence, the price schedule is constructed to ration consumers from the lowest type to the highest one. Given the ordering between the categories, the second price (between s_1 and s_2) consists in reducing the consumers belonging to category 2 whose type is comprised between $\underline{\theta}_2$ and $\underline{\theta}_1$. When $\underline{\theta}_2 = \underline{\theta}_1$ this schedule is not needed. For any states between s_2 and s_3, the market designer is indifferent between rationing consumers from both categories (as soon as it does in increasing order). Finally, the last prices are defined to exclude category 2 from the market while continuing rationing the lowest consumers type from category 1 whose type is between $\bar{\theta}_2$ and $\bar{\theta}_1$. For other consumers whose prices are not defined, we assume that the market designer excludes them such that the price implies a null consumption.

Illustrative Example The short-term mechanism can be understood in a setting with a discrete set of consumers. Assuming that only two consumers with type θ_1 and θ_2 drown from the corresponding distribution, such as $\theta_1 > \theta_2$. In that case, the allocation under a short-term mechanism can be described as follows:

$$t_1^{cs}(k, \theta_1, s) = \{0, 0, u(k, \theta_1, s)\} \quad | \quad t_2^{cs}(k, \theta_1, s) = \{0, u(k - d(0, \theta_1, s), \theta_2, s), \theta_2 + s\}$$
$$q_1^{cs}(k, \theta_1, s) = \{\theta_1 + s, \theta_1 + s, k\} \quad | \quad q_2^{cs}(k, \theta_1, s) = \{\theta_2 + s, k - (\theta_1 + s), 0\}$$

As consumer surplus always decreases with prices, the optimal situation when there is no capacity constraint ($s \in [0, s_1(k)]$) is when $t_1^{st} = t_2^{st} = 0$. It implies a quantity equal to $d(0, \theta_i, s) = \theta_i + s$ and corresponds to the first terms in the set of prices and quantities. When the capacity starts to bind, ($s \in [s_1(k), s_3(k))$), prices must rise to ration consumers. Again, as consumer surplus always decreases with prices, this implies that the constraint will always bind and that it is never optimal to set prices such that quantity is below capacity. Note that the capacity constraint implies : $\sum_i d(t_i^{cs}, \theta_i, s) = k$, under linear marginal utility this is similar as having $t_2^{cs}(t_1^{cs}) = 2s + k - \sum_i \theta_i - t_1^{cs}$. It allows us to express consumer surplus at the capacity constraint only concerning t_1^{cs}. We find that the consumer surplus exhibits a U shape, given $t_2^{cs}(t_1^{cs})$, the first derivative of the consumer surplus under our model assumptions is

$$\begin{aligned}\frac{\partial CS^{cs}(t_1^{cs})}{\partial t_1^{cs}} =& - d(t_1^{cs}, \theta_1, s) - d(t_2^{cs}(t_1^{cs}), \theta_2, s)\frac{\partial t_2^{cs}(t_1^{cs})}{\partial t_1^{cs}} + t_2^{cs}(t_1^{cs}) d_p \frac{\partial t_2^{cs}(t_1^{cs})}{\partial t_1^{cs}} \left(1 - d_p \frac{\partial t_2^{cs}(t_1^{cs})}{\partial t_1^{cs}}\right) \\ =& - 2d(t_1^{cs}, \theta_1, s)\end{aligned}$$

The second-order derivative is equal to 2, confirming the consumer surplus's convexity at the capacity constraint. Note also that at $t_1^{cs} = 0$, $\frac{\partial CS^{cs}(t_1^{cs})}{\partial t_1^{cs}} = -2\theta_1 - 2s < 0$, and at $\tilde{t}_1^{cs} = 0$ such that $t_2^{cs}(\tilde{t}_1^{cs}) = 0$, $\frac{\partial CS^{cs}(t_1^{cs})}{\partial t_1^{cs}} = 2\theta_2 + 2s > 0$. Finally, at the boundaries, the value for the consumer surplus is always higher: $CS^{cs}(0, t_2^{cs}(0)) = (\theta_1 + s)^2 > CS^{cs}(\tilde{t}_1^{cs}, t_2^{cs}(\tilde{t}_1^{cs})) = (\theta_2 + s)^2$, whenever $\theta_1 > \theta_2$. This proves that it is always optimal to ration first the consumer 2 over the residual supply (i.e., $d(t_2^{st}, \theta_2, s) = k - d(0, \theta_1, s)$). The third value for prices and quantities, ($s \in [s_3(k), \bar{s}]$), is determined due to the impossibility of having negative quantities: when $k = d(0, \theta_1, s)$, it implies that $q_2^{cs}(k, \theta_1, s) = 0$. Hence, to maintain this level, the price for consumer 2 must stay at $\theta_2 + s$, which implies raising prices for consumer 1 such that $k = d(t_1^{st}, \theta_1, s)$.

The key difference between the two allocation mechanisms can be understood as follows. The market designer chooses prices and quantities given the investment level in the short-term consumer mechanism allocation. So prices, when understood under a spot market perspective, are set only from a rationing perspective to reduce quantity. In that case, maximizing the consumer surplus always implies a form of discrimination against the lowest consumers as soon as consumers have heterogeneity and a capacity constraint. The revenue from prices acts only as (residual) transfers to cover fixed costs and choose the investment level. Given the investment level, the best allocation follows this personalized price system. On the other hand, under the long-term consumer allocation, the market designer cares about the long-term decisions of choosing the optimal level of investment. In that perspective, prices are chosen to generate revenues and efficiently ration consumers. When increasing the (marginal) level of investment, the market designer internalizes the opposite effect of sustaining a marginal investment cost and increasing the available quantity for consumers. This increase in quantity allows both an increase in consumer surplus and prices to cover the fixed costs. We compare the quantity and price schedule in Figure 1. The red curves represent the higher consumer category, 1, and the blue curves represent the lower consumer category, 2. For the (expected) quantity schedule, the black line represents the total quantity, which is by definition equal between the two mechanisms (the plateau is equal to k). They only differ with respect to the allocation between consumer categories. The long-term mechanism is shown in the first plot with dashed curves. Whenever the capacity is constraining, the central idea of the short-term mechanism is to exclude the lowest type of consumers gradually.[3]. The second plot shows This via individualized (expected) prices. We have excluded prices used to exclude consumers for clarity. Each increasing price is assigned to a specific group of consumers. The first blue line is assigned to consumers of category two between with $\theta \in [\underline{\theta}_2, \underline{\theta}_1]$. The second mixed-colored curve is applied to consumers of both categories with $\theta \in [\underline{\theta}_1, \bar{\theta}_2]$. Finally, when the demand is too high, it is always

[3]Note that even if the market designer does not differentiate between the category for consumers of type $\theta \in [\underline{\theta}_1, \bar{\theta}_2]$, the existence of consumers for category 1 with higher type and priced at 0 implies the increasing red curve between s_2 and s_3.

optimal to exclude all consumers with types lower than $\bar{\theta}_1$ and set a price, given by the red curve, for consumers above such that capacity is binding.

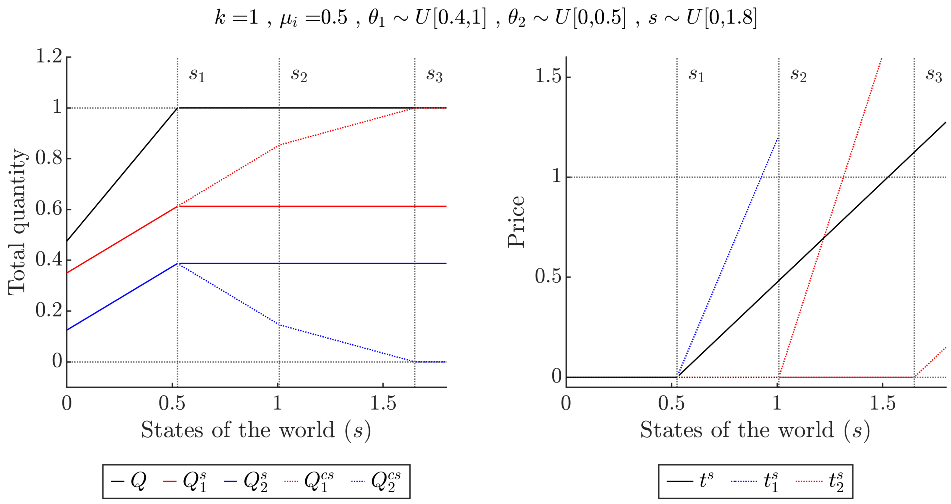


Figure 1: Quantity and price schedules under long-term and short-term consumer surplus mechanism. (The price schedules on the plot represent only prices associated with a positive quantity.)

Proposition 1 and Proposition 2 directly imply the following corollary :

Corollary 1. *The individualized price system that maximizes short-term consumer surplus always leads to a lower expected surplus when considering investment decisions.*

To compare this allocation schedule with the first best solution, we provide in Corollary 2 a description of the optimal quantity (rationing) that maximizes long-term surplus. Instead of setting prices and letting the quantities adjust, the market designer could select quantities and impose a unique transfer on each consumer. Such a policy can be implemented when prices do not emerge due to price regulation, such as in a price cap case (see, for example, Leautier (2018); Zöttl (2011)). When the capacity is sufficiently binding such that the price cap constrains the price on the spot market, the market designer needs to set a quantity rationing policy. The corollary describes this policy under the linear model for ease of exposition.

Corollary 2. *Under linear marginal utility assumption and uniform distribution.*

(i) When the capacity starts to bind, then the optimal rationing is independent of s :

$$\alpha_i^s(s,k) = \mu_i + \frac{\mu_i\mu_j(\theta_i^{av} - \theta_j^{av})}{k} \qquad \forall s \in (s_1(k), \bar{s}]$$

(ii) Assuming that consumer 1 is the higher type ($\theta_1 > \theta_2$) Under the individualized price system, the rationing strategy consists of rationing first the lowest type and then the highest type. If α_i^{st} is the rationing policy under the individualized price system, then we have :

$$\alpha_1^{st}(s,k) = \mu_1 \frac{s + \theta_1^{av}}{k} \quad \text{and} \quad \alpha_2^{st}(s,k) = 1 - \alpha_1^{st}(s,k) \quad \forall s \in (s_1(k), s_2(k)]$$
$$\alpha_0^{st}(s,k) = \frac{1}{k}\left(\frac{\bar{\theta}_1 - \bar{\theta}_2}{\bar{\theta}_1 - \underline{\theta}_1}\right)\left(s + \frac{\bar{\theta}_1 + \bar{\theta}_2}{2}\right) \quad \text{and} \quad \alpha_{-0}^{st}(s,k) = 1 - \alpha_0^{st}(s,k) \quad \forall s \in (s_2(k), s_3(k)]$$

With α_0^{st} the ratio for consumers from category 1 with $\theta > \bar{\theta}_2$ receiving a null price, and $\alpha_{-0}^{st}(s,k)$ the ratio for consumer belonging to both category receiving positive price.

Finally, Lemma 1 describes the level of investment that maximizes the expected short-term mechanism given the previous allocation schedule.

Lemma 1. *The expected consumer surplus under a short-term maximizing regime can exhibit non-concavity. There is at least one local maximum and at most two local maxima.*

The value of each investment derives from the first-order condition of the expected consumer surplus under the price and quantity schedule described in Proposition 2. It significantly differs from the first-order condition of Proposition 1. We express the condition as follows:

$$r = \int_{s_1(k)}^{s_2(k)} \overbrace{\mu_2 \int_{\underline{\theta}_2}^{\underline{\theta}_1} t_2^{st}(\theta,s)\, \frac{\partial t_2^{st}(\theta,s)}{\partial k} dG_2(\theta)}^{\text{marg. utility from category 2}}\, dF(s) + \int_{s_2(k)}^{s_3(k)} \overbrace{\sum \mu_i \int_{\underline{\theta}_1}^{\bar{\theta}_2} t_i^{st}(\theta,s)\, \frac{\partial t_i^{st}(\theta,s)}{\partial k} dG_i(\theta)\, dF(s)}^{\text{marg. utility from cat. 1 and 2}}$$
$$\int_{s_3(k)}^{\bar{s}} \underbrace{\mu_1 \int_{\bar{\theta}_2}^{\bar{\theta}_1} t_1^{st}(\theta,s)\, \frac{\partial t_1^{st}(\theta,s)}{\partial k} dG_1(\theta)\, dF(s)}_{\text{marg. utility from category 1}}$$

When choosing the investment level, the market designer needs to weigh the effect of a change of k on the positive prices that generate positive quantities for consumers (i.e., It excludes consumers that are not rationed (null prices) or fully rationed (null quantity). The first-order conditions capture those effects. The existence of two possible maxima stems from the boundary conditions due to the capacity constraints. Indeed, there must be a coherence between a maximum and the values of all the $s_j(k)$. For instance, if k solve the previous equation, it must be that $s_1(k) > 0$ and $s_3(k) < \bar{s}$. We therefore study the (six) possible maxima for the different values of the s_j. We find that there are only four possible values.

The first one corresponds to the case where the (maximizing) capacity level is very high, such as it never leads to rationing of the highest type of consumers of both categories (above $\underline{\theta}_1$), that is $s_2(k) = \bar{s}$, implying the second and third term in the first order cancels out. The second one corresponds to the case where the capacity level is moderately high, such as it never leads to rationing of the highest type of consumers from category 1 (above $\bar{\theta}_2$), that is $s_3(k) = \bar{s}$, implying the third term in the first order cancels out. Finally, the two last equilibria are mutually exclusive. They can coexist with the other maxima but never with each other. They respectively imply a lower level of capacity such that (i) the lowest type consumers of category 1 are always excluded ($s_2(k) = 0$, first term cancels out), or (ii) the lowest consumers of both categories are always excluded ($s_3(k) = 0$, first and second term cancel out)). On the contrary, the value is always unique for the long-term maximizing investment level, such that the capacity is either always binding or binding with positive probability.

We illustrate the results in Figure 2. The first panel shows the expected consumer utility, and the second shows the surplus for the respective allocations. The solid curves represent the long-term allocation, while the dashed curves represent the short-term allocation. As we can see, the associated consumer surplus is always lower under short-term allocation. Note that the consumer surplus under the single-price system is concave, while under the multi-price system, it exhibits some non-concavity.

4 Incomplete Information - Spot Market Allocation

In this section, we study the second regime under which the market designer has to choose the best allocation, given three assumptions : (i) the consumer's type is unknown to the market designer,

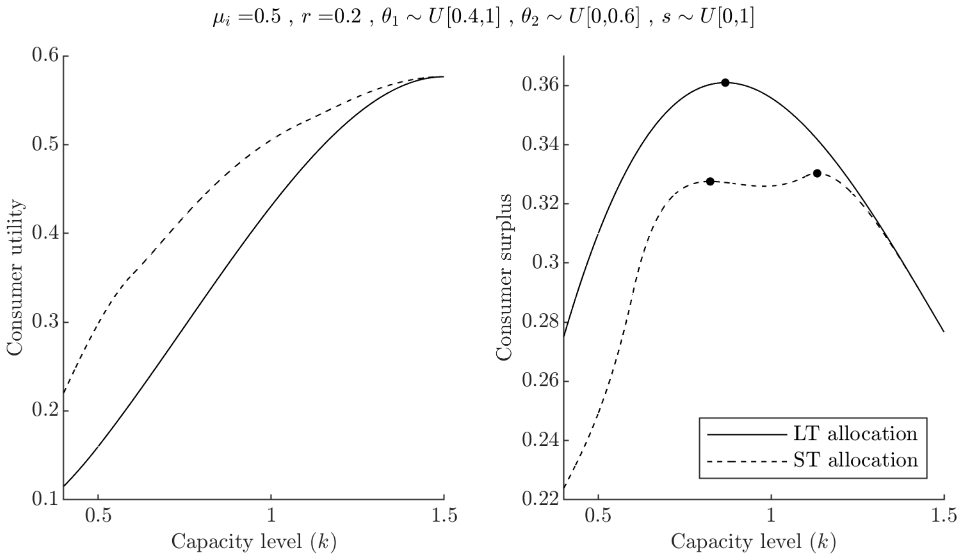
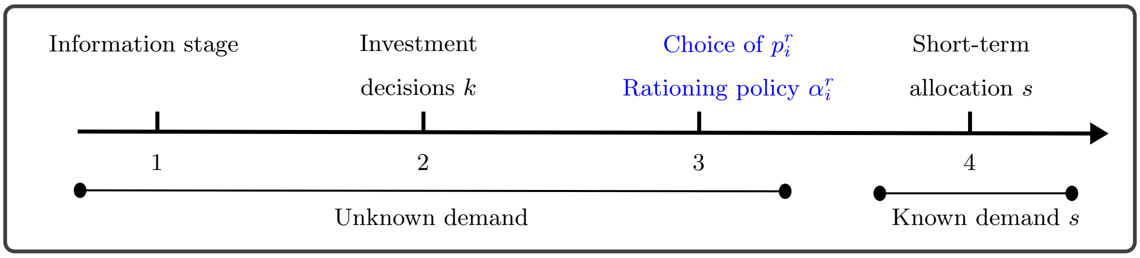


Figure 2: Expected consumer utility and surplus under the short-term (ST) and long-term (LT) allocation.

(ii) the market designer cannot extract any information from consumers, (iii) the schedule offered by the market designer is based on a spot market allocation and constrained to a unique price. The last assumption can be understood as follows. Under the unit mass assumption and if the market designer can implement a spot market, the first-best allocation that maximizes consumer surplus is based only on the expected type of consumers from both categories. In fact, if we assume consumers adjust to prices under a spot market mechanism, the market designer does not need additional information beyond the distribution and the support of consumer type.

To capture the effect of incomplete information, the market designer must be constrained when implementing the mechanism. It can come (i) from the quantity allocation - that is, consumers do not maximize their utility - (ii) or from the proposed monetary transfer. In the last case, the price schedule is incomplete because it is not optimal for every value of s, Which distorts the quantity demanded by consumers, even though they maximize their utility. In this paper, we take the second interpretation: We assume that the market designer can only choose a single price for every state s. From a policy perspective, this is similar to a market designer offering a fixed-price contract to consumers. [4] In this section, we study two cases: (i) the market designer does not discriminate between different categories, and the offered price is unique for every consumer; (ii) the market designer can discriminate between different categories, and he offers a price for each category. The first case allows us to focus our analysis on highlighting the trade-off the market designer faces when collecting revenue for the investment cost. In contrast, the second case highlights the distributive effect between consumers of different groups, even without social welfare weight.

This modeling approach underlines a market designer's trade-off concerning the uncertainty of the consumers' types, even without strategic inefficiencies. The core idea of the model is that, without any information, a market designer has to choose a price p_i^r independent of the world's states s. Then, during the consumption stage, consumer adapts their quantity given the price p_i^r, the realization of s, and their type θ. Following the framework description, we modify the actions taken by the market designer. The allocation proposal comprises (i) choosing p_i^r and (ii) defining the rationing policy α_i^r described below (that is, the share of capacity each consumer is receiving when capacity is binding). We provide below an updated figure considering the decisions the market designer has to make within this framework.



4.1 Single price policy

We start by assuming that the market designer is constrained by setting a unique price for each category, so we drop the index and assume that p^r is the price chosen by the market designer.

[4]This regime offers a more realistic approach between the complete and incomplete information case with a mechanism design setup described in the next section. Indeed, it can approximate the actual management of electricity markets, where a market designer is constrained in its short-term allocation while having imperfect information on its consumers' type.

The incomplete information set-up in this section has an important implication regarding quantity allocation. Indeed, combining a single-price policy and imperfect knowledge implies that some inefficient rationing should be expected in the market. To see this, recall that $d(p^r, \theta, s)$ is the quantity a consumer asks given the price p^r. Let's define $s_1^r(k)$ the first states of the world when the capacity is binding when the price is p^r that is :

$$s_1^r(k, p^r) = \left\{ s : \sum \mu_i \int_{\theta_i} d(p^r, \theta, s) dG_i(\theta) \right\}$$

For any $\leq s_1^r(k, p^r)$, the price is such that capacity is not binding. That is, the quantity asked by each consumer is feasible. In that case, there is no need for rationing. Note, however, that when $p^r > 0$, the model does imply an inefficiency similar to the effect of market power. Due to the price being higher to marginal, it prevents some Pareto-improving trade from happening. For any $s \geq s_1^r(k)$, capacity is binding, and the total quantity each consumer asked is above the available capacity. To avoid market failure, the market designer needs to reallocate quantity between consumers. However, we assumed that he does not observe consumer type. Without any possibility of extracting information, the only option for the market designer is to allocate equally across consumers a quantity equal to the investment level. Therefore, the individual quantity k and the expected quantity for each category is $\mu_i k$. We illustrate the implications by defining the expected utility under the single-price policy with incomplete information.

$$\sum \mu_i \int_0^{s_1^r(k)} \overbrace{\int_{\theta_i} U(d(p^r, \theta, s), \theta, s) dG_i(\theta)}^{\text{off-peak utility}} dF(s) + \sum \mu_i \int_{s_1^r(k)}^{\bar{s}} \overbrace{\int_{\theta_i} U(k, \theta, s) dG_i(\theta)}^{\text{on-peak utility}} dF(s)$$

We turn now to determining the best single-price policy given the framework. Compared to the previous analysis, the optimal price p^r depends not only on the first-best condition but on the budget constraint. The optimal price p^r is given by the following lemma.

Lemma 2. *If it exists, the optimal value $p^r(k, p^r)$ satisfies the net revenue condition $R^k(k, p^r) = 0$ with:*

$$R^k(k,p^r) := \underbrace{p^r\left(\sum \mu_i \int_0^{s_1^r(k,p^r)} \int_{\theta_i} d(p^r,\theta,s)dG_i(\theta)dF(s) + \int_{s_1^r(k,p^r)}^{\bar{s}} kdF(s)\right)}_{Expected\ revenue} - I(k)$$

This result is close to what can be found in the literature on peak pricing with price-inelastic consumers. In that case, the optimal price is simply the average cost. Under our framework, the optimal single price is different due to the price response of the consumers during off-peak periods and to the inefficient rationing occurring in the on-peak periods. Next, we provide in Proposition 3 the relation between the investment level and the optimal single-price

Proposition 3. *If an optimal single-price $p^r(k)$ exists, it increases in k.*

The intuitions of the lemma and proposition are closely related. When choosing the price p^r, the market designer must trade off opposite effects. Indeed, increasing p^r lowers quantity during off-peak. Hence, the revenue effect during off-peak is ambiguous. For on-peak periods, the revenue effect is always positive as the expected quantity is k and is not affected by a change of p^r. Note that the revenue is concave in p^r, meaning that the second-order effects are negative, limiting the market designer's ability to extract revenue from consumers.[5] Those effects can be shown by expressing the first derivative of the expected net revenue :

$$\frac{\partial R^r(k)}{\partial p^r} = \underbrace{\int_0^{s_1^r(k)} \overbrace{d_p p^r}^{-} + \sum_i \mu_i \int_\theta \overbrace{d(p^r,\theta,s)}^{+} dG_i(\theta)dF(s)}_{\text{off-peak marg. revenue}} + \underbrace{\int_{s_1^r(k)}^{\bar{s}} \overbrace{k}^{+} \ dF(s)}_{\text{on-peak marg. revenue}}$$

With $d_p = \frac{\partial d}{\partial t}$ the derivative of the demand function with respect to prices. Calculation shows that $\frac{\partial s_1^r(k)}{\partial p^r} > 0$, as a higher price, means consumers decrease their consumption, and the capacity is binding less often. Next, we show how k modifies the marginal effect of p^r. Again, the market designer faces trade-offs. Increasing k increases the revenue that can be collected from on-peak

[5]Increasing p^r lowers the occurrence of on-peak periods, and the revenue during off-peak is concave due to the linearity assumption of the marginal utility.

periods but also decreases the occurrence of a binding capacity. Therefore, at the margin, it reinforces the negative effect of the price on the quantity during off-periods and increases the marginal revenue during on-peak periods. Those direct effects are represented in the following expression of the cross-derivative of the expected revenue.

$$\frac{\partial^2 R^r(k)}{\partial p^r \partial k} = \overbrace{d_p p^r f(s_1^r(k)) \frac{\partial s_1^r(k)}{\partial k}}^{-} + \overbrace{\int_{s_1^r(k)}^{\bar{s}} 1 dF(s)}^{+}$$

So if $\frac{\partial R^r}{\partial k} > 0$, then at least one quantity is negative, which cannot happen.

Then we use the fact that $\frac{\partial^2 R}{\partial p^r 0^2} < 0$ is concave. At $p_0^r = r \mid k = \tilde{k} : s_1 = 0$. We have $\frac{\partial R}{\partial p^r 0} > 0$, implying that the revenue is increasing at the limit in p_0^r. If the function is concave, there could be at least two potential values for the optimal value of p_0^r. However, note that consumer surplus is always decreasing in prices; therefore, a lower price is always optimal compared to a higher price. So, the optimal value corresponds to the first increasing part.

Proposition 3 shows that expanding the capacity level always leads to the positive (revenue) effect dominating the adverse (price) effects. That is, the effect of the increase in the revenue collected during on-peak periods offsets the compound negative impact of a price increase that (may) lower the revenue during off-peaks and reduces the occurrence of on-peak periods.[6]

In Figure 3, we represent the optimal value of $p^r(k)$. The plot shows two main zones for the optimal value of p^r[7]: (i) The first one on the left of the first vertical line represents optimal values such that the capacity never binds in expectation. Therefore the optimal value solves $p^r \int_0^{\bar{s}} kdF(s) = I(k)$, implying with linear investment costs rk that $p^r(k) = r$. (ii) The second zone corresponds to the case for which p^r is such that k binds for some states.

[6] From a policy perspective, the market designer never wants to lower price so as the increase the consumption during off-peak.

[7] Indeed when solving for p^r, one should ensure that it finds a value in line with the model fundamentals, particularly the values of $s_1^r(k, p^r)$. Namely, there exist three possible values of p^r, such that the capacity never binds $s_1^r(k, p^r) = \bar{s}$, always binds $s_1^r(k, p^r) = 0$, or binds for some world's states. When the investment value is low, the coherent value of p^r corresponds to the case where the capacity level always binds.

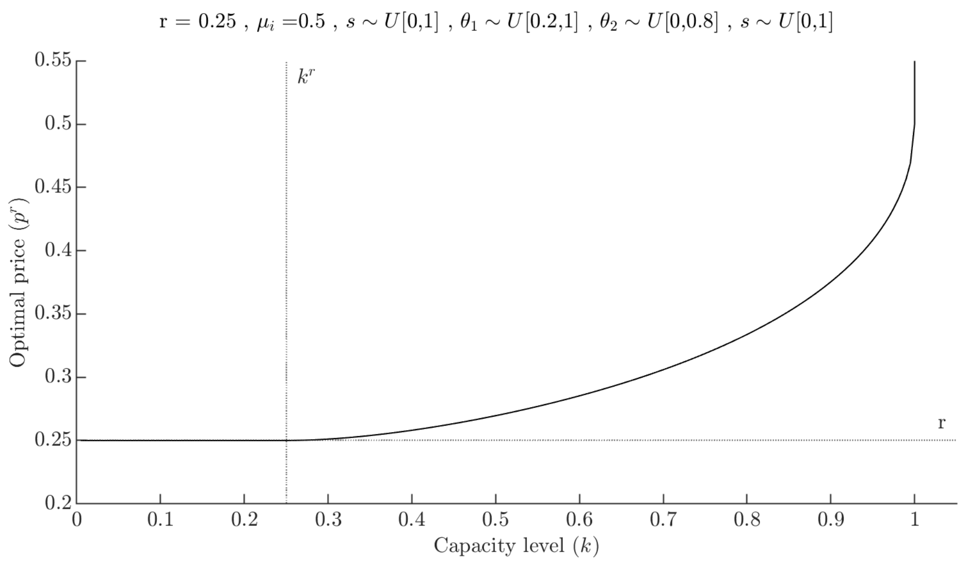


Figure 3: Optimal single-price policy for different investment levels.

4.2 Category-price policy

We extend the previous findings by assuming now that the market designer knows the category each consumer belongs to (but his type remains private information). Therefore, the market designer can imperfectly discriminate between consumers and implement a category-based price to finance its investment cost. We start by defining the new rationing policy under this framework. This stage boils down to allocating the capacity k in the first step between the two categories and the second step, randomly for each consumer within each category. This procedure implies that the market designer allocates the same expected quantity to each category under the first best allocation (even though the within-category allocation remains inefficient). The problem is solved as follows. Let the quantity for category i be q_i; then, when the capacity is constraining, we must have for every state of the world: $\sum_i \mu_i q_i = k$, implying that the relation between the quantity is equal to $q_i(q_j) = \frac{k-\mu_j q_j}{\mu_i}$. Then, we maximize the short-term expected utility : $\sum_i \mu_i \int_{\theta_i} U(q_i, \theta, s) dG_i(\theta) dF(s)$, given the previous relation. Solving using the first-order condition leads to a capacity share for each consumer belonging to a group i of $1 + \frac{\mu_j(\theta_i^{av} - \theta_j^{av})}{k}$, which in for all consumers implies the same allocation as in Corollary 2.

Given this optimal rationing policy, we now define the new problem the market designer faces :

$$\max_{\substack{k,\\ p_i^r \to \mathbb{R}^+}} \quad CS^r(k, p_1^r, p_2^r) = \sum_i \mu_i \int_s \int_{\theta_i} U(q_i^r(\theta, s), \theta, s) \; - \; p_i^r q_i^r(\theta, s) dG_i(\theta) \quad dF(s)$$

$$\text{s.t.} \qquad I(k) \;\leq\; \sum_i \mu_i \int_s \int_{\theta_i} p_i^r q_i^r(\theta, s) \; dF(s), \tag{R}$$

We drop the capacity constraint as the rationing policy implies. To see this, we can redefine the state of the world when the capacity starts binding :

$$s_1^r(k, p_1^r, p_2^r) = \left\{ s : \sum \mu_i \int_{\theta_i} d(p_i^r, \theta, s) dG_i(\theta) \right\}$$

Then, the quantity $q_i^r(\theta, s)$ allocated in the market for each consumer of catagegory i is equal to $d(p_i^r, \theta, s)$ when $s < s_1^r(k, p_1^r, p_2^r)$ and $\alpha_i^r k = k + \mu_j(\theta_i^{av} - \theta_j^{av})$ when $s \geq s_1^r(k, p_1^r, p_2^r)$. Note that while the total quantity for each category is identical under the first-best and this framework, the total utility does differ. It implies a very similar delta in terms of utility as the equation in the previous section with a single price policy.

First, let's note $\mathcal{L}^r$, the Lagrangian associated with the market designer program such that

$$\mathcal{L}^r(k, p_1^r, p_2^r, \gamma^r) = CS^r(k, p_1^r, p_2^r) + \gamma^r R^r(k, p_1^r, p_2^r)$$

With CS^r the aggregate expected consumer surplus defined as the sum of consumers' utility net of monetary transfers, γ^r the lagrangian multiplier associated with the budget constraint, and $R^r(k)$ the budget constraint (expected revenue net of investment costs). Then, using the Envelop Theorem, we can express the derivative of an optimal price with respect to k as follows:

$$\frac{\partial p_i^r(k)}{\partial k} = (\overbrace{st_{ik} + \rho_c CS_{jk}}^{\text{consumer surplus effect}\leq 0} \quad + \quad \overbrace{(R_i - \rho R_j)\gamma_k^r + (R_{ik} - \rho R_{jk})\gamma^r - \rho_r CS_{jk}}^{\text{revenue effect } \geq 0} \quad) \underbrace{\frac{-\mathcal{L}_{jj}}{H^r}}_{\leq 0}$$

$$\frac{\partial p_i^r(k)}{\partial k} = \big(\overbrace{st_{ik} + R_i\gamma_k^r + R_{ik}\gamma^r}^{\text{effect of } p_i^r} - \rho\overbrace{(st_{jk} + R_j\gamma_k^r + R_{ik}\gamma^r)}^{\text{effect of } p_j^r}\big)\underbrace{\frac{-\mathcal{L}_{jj}}{H^r}}_{\leq 0}$$

$$\gamma_k^r = \frac{\sum_i R_i(L_{jj}L_{ik} - L_{ij}L_{jk}) - R_k H^r}{bH^r}$$

$$\gamma_k^r = \frac{\overbrace{\sum_i R_i(L_{jj}CS_{ik} - L_{ij}CS_{jk})}^{\text{CS effect}} + \overbrace{\sum_i \gamma^r R_i(L_{jj}R_{ik} - L_{ij}R_{jk}) - R_k H^r}^{\text{Revenue effect}}}{bH^r}$$

$$\begin{aligned}\frac{\partial p_i^r(k)}{\partial k} = \overbrace{\frac{-\mathcal{L}_{jj}}{H^r}}^{\leq 0} &\overbrace{\left(st_{ik} + \rho CS_{jk} + (R_i - \rho R_j)\sum_i (CS_{ik} - \rho CS_{jk})\frac{R_i}{L_{jj}bH^r}\right.}^{\text{effect of } k \text{ on CS with holding R fixed}} \\ &\underbrace{\left. + \gamma^r\left(R_{ik} - \rho R_{jk} + (R_i - \rho R_j)\sum_i (R_{ik} - \rho R_{jk})\frac{R_i}{L_{jj}bH^r}\right) - R_k\frac{H^r}{bH^r}\right)}_{\text{effect of } k \text{ on } R}\end{aligned}$$

With $\rho_c = \frac{CS_{ij}}{L_{jj}}$, $\rho_r = \gamma^r\frac{R_{ij}}{L_{jj}}$ and $\rho = \rho_r + \rho_c$. $H^r = \mathcal{L}_{11}\mathcal{L}_{22} - \mathcal{L}_{12}\mathcal{L}_{21}$ being the determinant of the Hessian matrix of the lagrangian. $bH^r = \mathcal{L}_{ij}R_iR_j - \mathcal{L}_{jj}R_i^2 - \mathcal{L}_{ii}R_j^2 + \mathcal{L}_{ji}R_iR_j$ being the determinant of the bordered Hessian matrix of the lagrangian. Each variable's index is associated with the corresponding derivative. For instance, CS_{ik} reads as the cross derivative between the price of category i with respect to the investment level. It measures the (marginal) change of the marginal effect of price p_i^r on the consumer surplus. We summarize the findings in the following proposition.

Proposition 4. *Suppose that category 1 consumers are of higher types than category 2 consumers and that the average private demand for each category is sufficiently different, then:*

- $p_1^r(k)$ *is increasing with* k
- $p_2^r(k)$ *is first decreasing, then increasing with* k*.*

Morevover, given a threshold value of k^r, any value below k^r implies : $p_2^r(k) > p_1^r(k)$, and any value above k^r implies : $p_1^r(k) > p_2^r(k)$. At k^r, optimal prices equal the optimal single price, such that $p^r(k) = p_i^r(k)$.

Figure 4 illustrates the results. The red curve shows $p_1^r(k)$, the blue curve shows $p_2^r(k)$, and the black dashed curve shows the optimal single price $p^r(k)$ found in the previous section. Following the proposition, we observe that the blue curves corresponding to the group with a lower expected demand exhibit a non-monotonic relationship with the level of investment such that it decreases for low values of k and then increases again following a similar behavior to the optimal price for the higher category of consumers represented in the red curve.

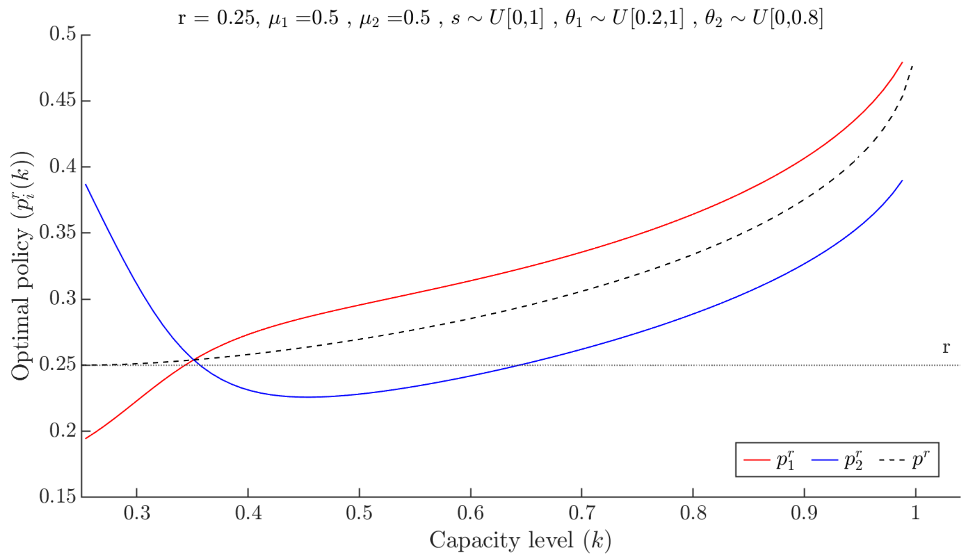


Figure 4: Optimal prices under the category-price policy with respect to the investment

The proof of such behavior of the optimal prices can be understood by distinguishing the first-order and the second-order effects of prices and level of investment on (a) the aggregate consumer surplus and (b) the budget constraint. From a consumer surplus perspective, the market designer prefers (i) discrimination and (ii) favoring the consumers from the category of the highest type. Preferring discrimination implies that consumer surplus exhibits a U shape form. Due to the asymmetry between the consumers, the function is skewed towards lower types, implying that to bring a relatively same amount of surplus, lower types should be paying a higher price. This is because higher types bring relatively more surplus than the smaller types On the other hand, when studying the variation of the expected revenue with respect to prices, the market designer faces the opposite effect. Increasing p_1^r generates more revenue as they consume, on average, more. . To see the fundamental tension between revenue and consumer surplus, we study the difference between the derivative of the consumer surplus with respect to p_1^r and p_2^r. Under the linear and uniform framework, it is negative and the opposite of the difference between the derivative of the expected revenue. In absolute terms, they are both equal to $\frac{1}{2}(\theta_1^{av} - \theta_2^{av}) > 0$. Note that we also have the opposite effect between consumer surplus and revenue from the level of prices. Namely, the consumer surplus is higher when prices are low, and revenue is increasing (although not everywhere) with prices. Therefore, the net effect on the optimal values of p_1^r and p_2^r ultimately depends on which effect dominates.

The next step for understanding the results lies in how those opposite effects change with the level of investment. As shown in the previous section, our framework implies that a change of k does affect both revenue and the consumer surplus, which is captured via the direct effect on prices needed for financing this investment and the change of occurrences between off-peak and on-peak periods. First, the level of investment induces a positive first derivative of the consumer surplus and a negative second derivative. That is, increasing k always increases the surplus, but for a higher level of investment, the positive impact is relatively smaller. On the other hand, an analysis of how the budget constraint behaves shows a convex effect with respect to k. It implies that an increase of k leads to the iso-revenue associated with the constraint shifting at an increasing rate.[8]. The switch between the decreasing and increasing parts is therefore associated with the consumer surplus effect dominating first the budget effect. As k increases, the respective concavity and convexity of the functions lead to the budget effect dominating the surplus effect: this explains the increasing parts and the ranking between the two prices on the right part of Figure 4.

Now, let us turn towards the left part of Figure 4. For low values of k, the budget effect is small. Now, for the sake of clarity, let's assume the budget is fixed. Note that the iso-revenue curve is convex and skewed towards prices of consumers 1 due to the preference for symmetry of the

[8]The pure revenue effect of increasing prices is found in the previous section.

budget constraint. That is, when choosing between two pairs of prices, the market designer needs to balance two opposite effects. The market designer faces a high degree of discrimination for relatively different prices. On the other hand, for prices with closer value, the market designer meets, on average, lower prices. As consumers' surplus benefits from a higher degree of discrimination and lower prices, choosing the optimal pair of prices depends on the net opposite effects. As k increases, the discrimination effect is lower, which induces the market designer to choose relatively less different prices. For the final increasing parts of the optimal prices, the budget constraint and the preference for lower prices imply that both prices are increasing, and the price for the higher type is above the price for smaller types. We illustrate this tension in Figure 5. We represent the contour map of the aggregate consumer surplus with respect to p_1^r and p_2^r for two values of k. The convex curve represents a fixed revenue constraint that we assume is independent of k. As k increases, we show that the indifference curves tend to be more convex. It is this shift in the shape of the consumer surplus that implies a decrease of the optimal price, implying that the gain in lower prices is higher than the gain from discrimination.

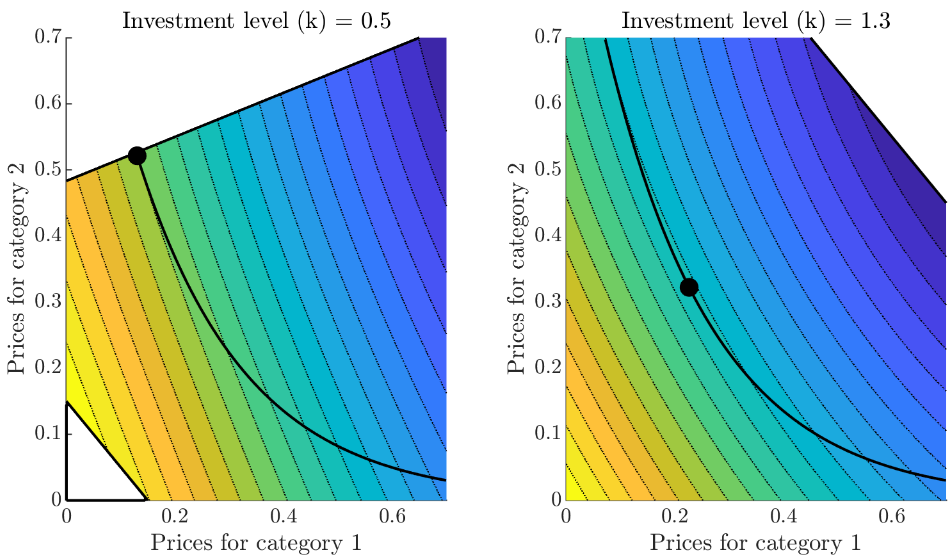


Figure 5: Illustration of the decreasing part of p_2^r with respect to k

We conclude this section by analyzing the implication of the optimal policy when choosing the level of investment to maximize the consumer surplus. We define the first-order conditions using the Envelop Theorem for constrained optimization. That is, it is sufficient to derive the derivative of the Lagrangian with respect to k : $\frac{\partial \mathcal{L}^{\nabla}}{\partial k} = \frac{\partial CS^r}{\partial k} + \gamma^r \frac{\partial R^r}{\partial k}$. We start with the consumer surplus :

$$\frac{\partial s_1^r(k)}{\partial k} \overbrace{\sum \mu_i \int_{\theta_i} \int_{\alpha_i^r k}^{d(p_i^r,\theta,s)} u(q,\theta,s) dq dG_i(\theta)}^{\Delta \text{ in quantity btw. off-peak and on-peak}} |_{s=s_1^r(k)} + \int_{s_1^r(k)}^{\bar{s}} \overbrace{\sum \mu_i \int_{\theta_i} u(\alpha_i^r k,\theta,s) - p_i^r \; dG_i(\theta)}^{+ \text{ in on-peak cons. surplus}} dF(s)$$

The second term is similar to the complete information benchmark. It represents the gain in consumer surplus during on-peak as capacity expands. Note that the gain in surplus does not depend on the price as the quantity is randomly assigned to each consumer in each category due to imperfect information. The first term stands for the change at the margin of quantities for each consumer. Under complete information, the quantities allocation is continuous in s. However, due to incomplete knowledge, the market designer creates a discontinuity in the allocation when capacity starts binding, which implies that the value at $s_1^r(k)$ does not cancel out. For the revenue, the derivative can be expressed as follow :

$$\frac{\partial s_1^r(k)}{\partial k} \overbrace{\sum \mu_i \int_{\theta_i} p_i^r \left(d(p_i^r,\theta,s) - \alpha_i^r k \right) dG_i(\theta)}^{\Delta \text{ in quantity btw. off-peak and on-peak}} |_{s=s_1^r(k)} + \int_{s_1^r(k)}^{\bar{s}} \overbrace{\sum \mu_i p_i^r}^{+ \text{ in on-peak rev.}} \quad dF(s) - r$$

The first term is similar and originates from discontinuity. The second term comes from the increase in available quantity during on-peak. As expected, and similarly to the first-best investment level, the sign of the first-order condition is ambiguous as it has positive and negative effects of an increase in k. For instance, it raises investment costs, but it also raises available revenue. Calculations shows that there exists a level of investment that maximizes the expected aggregate consumer surplus, as the consumer surplus and the revenue are concave in k. We now compare the outcomes in terms of welfare given the optimal policy for the single-price and category-based prices at the aggregate surplus level and also from the surplus for each category. Figure **??** illustrates our findings when we vary either the investment costs or the degree of heterogeneity between the consumers. Namely, we consider that category 1 is of higher types than category 2, and we increase the lower type from category 1: $\underline{\theta}_1$. In the first panel, we show the relation between the optimal investment level given the (i) first-best complete information case (ii) the second-best with single price policy (iii) the second-best with category-based policy for different levels of r. The second panel shows the aggregate consumer surplus at the corresponding investment level. The third panel is the consumer surplus for category 1, and the fourth is the consumer surplus for category 2. The last three figures are shown with different values of $\underline{\theta}_1$

The figure suggests multiple implications for our comparison. First, we find that the imperfect information and the price constraints could imply a lower or higher level of investment at the second best, depending on the model parameters. This result can be compared to other analyses for the cause of underinvestment, such that a price cap implies a lower investment level, and the public good nature of the investment leads to a higher second-best. Then, we find that, on an aggregate basis, allowing the market designer to discriminate between categories of consumers increases the aggregate consumer surplus. Indeed, as we have shown earlier, maximizing consumer surplus or revenue implies a form of preference for discrimination. Therefore, our model provides a basis for the positive effect of discriminating consumers based on their category due to the absence of information and the constraints on prices. This has to be compared to the first-best mechanism under which the market designer does not discriminate.

5 Incomplete Information - Mechanism Design

We extend our framework by allowing the market designer to choose an allocation mechanism such that (i) consumers behave truthfully and (ii) the market designer is not constrained in its choice of prices given the realization of s. The two assumptions combined allow him to bypass the spot market allocation because truthful behavior implies that he can also set quantities for each consumer. In other words, the market designer can now offer a complete set of prices and quantities such that the schedule depends on each consumer, for every state of the world s, and every type θ. The following figure summarizes the new action set for the market designer. As

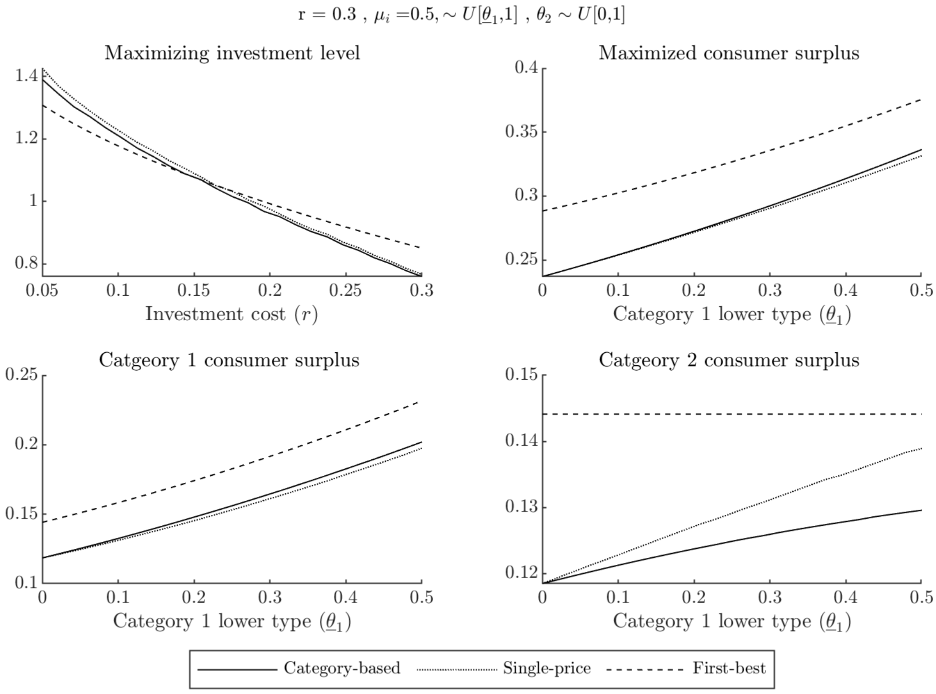
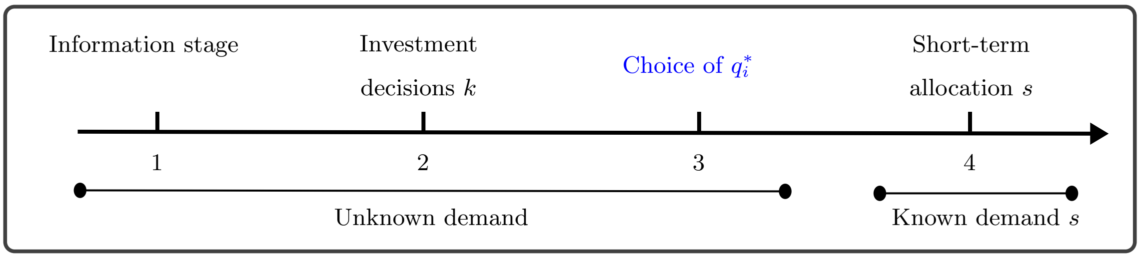


Figure 6: First panel: Optimal investment level. Second panel: Optimal aggregate consumer surplus. Third panel: Optimal category 1 consumer surplus. Fourth panel: Optimal category 2 consumer surplus.

we will show, the incentive compatibility constraint pins down the optimal monetary transfer t_i^*, leaving the market designer only with the quantity choice.



To induce true reporting from consumers, the market designer needs to require the following:

$$\theta_i = \arg\max_{\hat{\theta}} \int_s U(q_i(\hat{\theta}, s), \theta, s) - t_i(\hat{\theta}, s) q_i(\hat{\theta}, s)\; dF(s) \tag{IR}$$

While the participation of every consumer implies that:

$$0 \leq \int_s U(q_i(\theta, s), \theta, s) \;-\; t_i(\theta, s) q_i(\theta, s)\; dF(s) \tag{IC}$$

Putting all of this together, the mechanism design problem faced by the market designer is given by:

$$\max_{k \geq 0} \max_{\substack{t_i(\theta,s)\to\mathbb{R}^+,\\ q_i(\theta,s)\to\mathbb{R}^+}} \quad CS(k) = \sum_i \; \mu_i \int_s \int_{\theta_i} U(q_i^s(\theta, s), \theta, s) \;-\; t_i^s(\theta, s) q_i^s(\theta, s) dG_i(\theta) dF(s)$$

$$\text{s.t.} \quad I(k) \leq \sum_i \mu_i \int_s \int_{\theta_i} t_i(\theta, s) q_i(\theta, s) dG_i(\theta) dF(s), \tag{R}$$

$$\sum_i \mu_i \int_s \int_{\theta_i} q_i(\theta, s) dG_i(\theta) dF(s) \leq k, \tag{K}$$

$$0 \leq \int_s U(q_i(\theta, s), \theta, s) \;-\; t_i(\theta, s) q_i(\theta, s)\; dF(s), \tag{IR}$$

$$\theta_i = \arg\max_{\hat{\theta}} \int_s U(q_i(\hat{\theta}, s), \theta, s) - t_i(\hat{\theta}, s) q_i(\hat{\theta}, s)\; dF(s), \tag{IC}$$

We start describing the optimal allocation schedule given the new constraints and for a given level of investment k. We analyze the implications regarding investment decisions in the Appendix available upon request.

Using the method developed similarly to Guesnerie and Laffont (1984) and the incentive compatibility constraint, we characterize the monetary transfer $t_i(\theta, s)$ in terms of quantity $q_i(\theta, s)$. As our problem is well-defined, the incentive compatibility constraint is satisfied as soon as the optimal allocation $q_i(\theta, s)$ increases with respect to the type θ. Lemma 3 describes the revenue from a consumer given its type, and a realization of s is

Lemma 3. *The payoff equivalence implies the following relation between optimal transfer and quantity allocated to a consumer of type θ, from category i and given a realization of s :*

$$t_i(\theta,s)q_i(\theta,s) = U(q_i(\theta,s),\theta,s) - \int_{\bar{\theta}}^{\theta}\int_s q_i(\tilde{\theta},s)dF(s)d\tilde{\theta} + Cst$$

Where Cst is an arbitrary constant.

The proof uses the canonical approach of the Envelope Theorem (see Milgrom and Segal (2002)) that we modify to consider the capacity constraints. Next, we use the approach from Spulber (1992a) to characterize a feasibility constraint that associates the budget and individual rationality constraints. The core idea is that if one of the constraints is satisfied but not the other, a feasible lump-sum transfer from the non-binding constraint could exist that allows for relaxing the binding constraints. To say it differently, when there is, for instance, some excess revenue but the individual rationality is constraining, it is possible to transfer a lump-sum positive amount of money to the lowest types of consumers, which allows for less constraint optimal allocation. We describe in the following equation the corresponding new constraint, noted $R-IR$:

$$\sum_i \mu_i \int_s\int_\theta U(q_i(\theta,s),\theta,s) - \Gamma_i(\theta)\int_s q_i(\theta,s)dF(s)dG_i(\theta)dF(s) - I(k) \geq 0$$

With $\Gamma_i(\theta) = \frac{1-G_i(\theta)}{g_i(\theta)}$ the inverse hazard rate. Under our uniform distribution assumption considering the distribution of θ, the inverse hazard rate is decreasing with θ. We then solve for the Lagrangian. The following lemma shows the first-order condition to find the optimal allocation $q_i^*(\theta,s)$.

Lemma 4. *Given IC and IR constraints, the optimal allocation for a consumer of type θ from category i and for a given realization of s $q_{i,k}^*$ satisfies the following condition.*

$$u(q_{i,k}^*,\theta,s)(1+\zeta) - \zeta\Gamma_i(\theta) - \varepsilon = 0$$

With ζ and ε, the Lagrangian multipliers for, respectively, the $R-IR$ condition and the capacity constraint. We denote $l = \{1,2,3,4\}$ the index variable such that $3-4$ implies that $R-IR$ is binding while $1-2$ means it does not, and $2-4$ implies that the capacity is binding while $1-3$ means it does not.

Given the lemma, we can prove that the optimal allocation increases with the type. For instance, the following equation shows the derivative of the optimal allocation when both capacity and the $R - IR$ constraint are binding.

$$\frac{\partial q^*_{i,4}}{\partial \theta} = \left(\frac{1}{2} - \frac{\partial u(q^*_{i,4}, \theta_i, s)}{\partial \theta}\right)\left[1 - \frac{\zeta}{1+\zeta}\Gamma'_i(\theta)\right] / \frac{\partial u(q^*_{i,4}, \theta, s)}{\partial q}$$

Following our model specification, $\frac{\partial u}{\partial \theta} = 1$ and $\frac{\partial u}{\partial q} < 0$, the assumption concerning $\Gamma'_i(\theta) < 0$ implies that $\frac{\partial q^*_{i,4}}{\partial \theta} > 0$. Lemma4 provides four solutions to the problem faced by the market designer depending on which constraints are binding or not. We can regroup them in pairs such that $\{q^*_1(\theta, s), q^*_2(\theta, s)\}$ is the set of quantities when the optimal allocation is not constraint by $R - IR$. That is, the revenue generated by the mechanism is sufficient to cover the fixed costs and provide enough incentive for every consumer to consume electricity. On the other hand, $\{q^*_3(\theta, s), q^*_4(\theta, s)\}$ is the set of quantities such that the constraint is binding, implying that the optimal allocation needs to be distorted to covers both fixed costs and participation. In the following corollary, we provide the characterization of the first set of quantities.

Corollary 3. *The optimal allocation under the mechanism design approach when the $R - IR$ constraint is not binding is identical to the spot market allocation.*

When $\zeta = 0$, the condition in Lemma 4 is identical to the conditions described in the complete information section. Moreover, it can also be shown that the spot market schedule in prices and quantities is also incentive-compatible. We next analyze the threshold between the two sets of quantities. That is, we described under which value of k the market designer faces a binding $R - IR$. We summarize our findings in the following proposition.

Proposition 5. *There exists a unique value of k such that the $R - IR$ is null. Moreover, for any value of k below this threshold, the constraint $R - IR$ is not binding, while any value above the constraint is binding.*

Proposition 5 that it is possible to cover both fixed costs and participation constraints without distorting the allocation only when the level of investment is low. The intuition for this result can be understood as follows. First, we denote the marginal virtual utility: $J_{i,k} = u(q_i^*(\theta, s), \theta, s) - \Gamma_i(\theta)$, which is the marginal utility derived from the optimal allocation net of the information rent. Under our framework, it can be interpreted as the feasible gain in utility from the allocation after having remunerated the consumers to behave truthfully. Then, we can express the derivative of the $R_I R$ constraint for the first set of optimal quantities.

$$\overbrace{\sum_i \mu_i \int_{s_1(k)}^{\bar{s}} \int_{\theta_i} \quad J_{i,2} \quad \frac{\partial q_{i,2}^*(\theta, s)}{\partial k} \quad dG_i(\theta) dF(s)}^{\text{aggregate expected marginal virtual revenue}} \quad - \quad r$$

Essentially, the constraint starts binding when the aggregated marginal virtual revenue from the mechanism during the on-peak period equals the marginal investment. Note that both the marginal virtual utility and the derivative of the quantity are, in that case, positive. Under our framework, $\frac{\partial q_{1,2}^*}{\partial k}$ is equal to 1, so to ensure that $\sum_i \mu_i \frac{\partial q_{1,2}^*}{\partial k} = 1$. Therefore, an increase of k generates an ambiguous effect on the constraint : (i) it increases the virtual surplus during on-peak periods, and (ii) it increases the investment costs. However, the expected surplus from consumers is concave. Indeed, note that the derivative of the marginal virtual utility with respect to k is equal to $\frac{\partial J_{i,k}}{\partial k} = -\frac{\partial q_{i,k}^*}{\partial k}$, meaning that if an increase of the investment increases the optimal quantity, then it decreases the possible marginal utility net of information rent. This effect also accumulates with the change in occurrence between off-peak and on-peak. As k increases, the capacity binds less in expectation, implying a decrease in the positive first part of the expression above. This second-order effect, combined with the increase in investment costs, implies that the constraint crosses binds only once.

We illustrate our findings in Figure 7. We plot the $R - IR$ constraint under the optimal allocation set $\{q_{i,1}^*, q_{i,2}^*\}$ for different values of k. The black curve shows the constraint. We decompose it with the blue curve only representing the aggregate expected consumer virtual utility and the red curve representing the investment costs. As previously described, the blue curve is concave in k with an increasing value and then a decreasing part. Note that for sufficiently high values of k, the utility function is even independent of k because, in expectation, the capacity is never binding. When adding the increasing investment costs, the difference between the two has

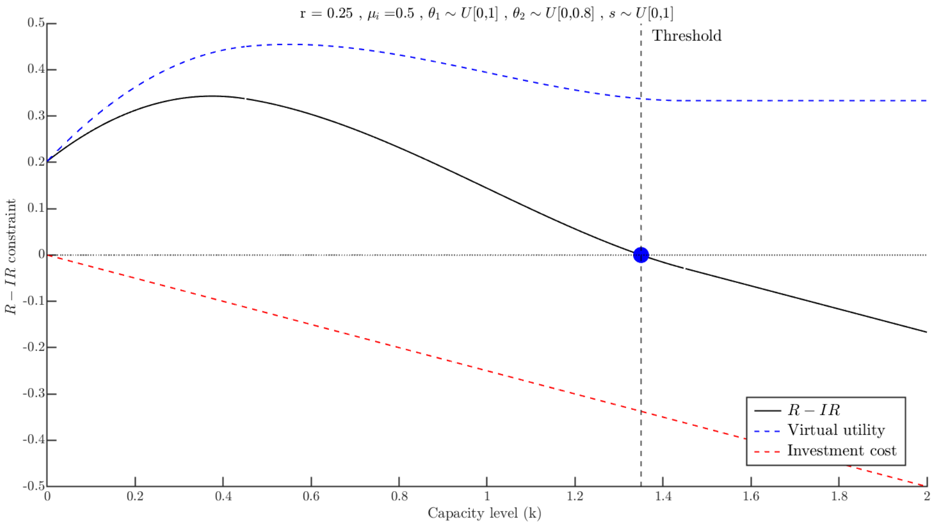


Figure 7: $R - IR$ constraints and its component with respect to the investment level.

to exhibit, at one point, a decreasing behavior. Finally, we have represented the threshold value with the vertical dashed line. For lower values of k, the $R - IR$ constraint is positive, meaning that fixed costs and the information constraint are not binding. Above this value, the value is negative, so the market designer needs to distort the allocation so that $R - IR = 0$. We now describe how the optimal allocation depends on the investment level when the $R - IR$ binds. We summarize our main findings in the following proposition.

Proposition 6. *The investment level directly affects the optimal allocation.*

- *For every consumer, the optimal quantity during off-peak $q^*_{i,3}$ is always decreasing with k.*
- *For the optimal allocation $q^*_{i,4}$, there exists for each category a unique threshold $\theta^*_i(k)$. For consumers of a category i, if his type is below $\theta^*_i(k)$, his allocation $q^*_{i,4}$ decreases with k. If his type is above $\theta^*_i(k)$, his allocation $q^*_{i,4}$ increases with k. Moreover, $\theta^*_i(k)$ is increasing with k.*

The proposition states that for a higher level of investment, every consumer should receive less electricity during off-peak periods. During on-peak, the change of quantity depends on the types. For lower types of both categories, consumers should also receive less electricity. On the other hand, higher types always receive more electricity as capacity expands. When the capacity is not binding, the effect on the quantity is captured in the equation below :

$$\frac{\partial q^*_{i,3}}{\partial k} = \frac{J_{i,3}}{1+\zeta}\frac{\partial \zeta}{\partial k}$$

The derivative is derived from the first-order condition from Lemma 4. Namely, it can be rewritten such that: $u(q^*_{i,k}, \theta, s)(1+\zeta) - \zeta\Gamma_i(\theta) - \varepsilon = u(q^*_{i,k}, \theta, s) + \zeta J_{i,3} - \varepsilon$. When capacity is not binding, we have $\varepsilon = 0$. Hence, the marginal virtual utility at the optimal allocation during off-peak is always negative. Under our framework, and similarly to the previous sections, we know that the budget constraint behaves convexly with respect to k: a higher capacity level implies a higher need for revenue. Thus it implies that $\frac{\partial \zeta}{\partial k} > 0$. The two observations lead to a negative derivative. The economic intuitions can be understood as follows: as k expands, this does not directly generate any additional quantity for consumers during off-peak, as, by definition, the capacity is not binding. On the other hand, the need for revenue is increasing. Combining the absent surplus effect and the negative budget effect implies that the optimal quantities for all consumers are decreasing. For the on-peak allocation, the initial derivative is expressed as follows:

$$\frac{\partial q^*_{i,4}}{\partial k} = \left[J_{i,4}\frac{\partial \zeta}{\partial k} - \frac{\partial \varepsilon}{\partial k}\right]\frac{1}{1+\zeta}$$

As quantity expands, the willingness to pay for less binding constraint decreases, implying that $\frac{\partial \varepsilon}{\partial k} < 0$. Therefore, the sign of the derivative is ambiguous and notably depends on the sign of $J_{i,4}$. Contrary to the off-peak allocation, the initial first-order condition when $\varepsilon > 0$ does not allow a clear-cut answer for the sign of the virtual marginal utility. Using the constraint from the market design problem, we can express the derivative of the Lagrange multiplier ε associated with the capacity constraint as a function of the derivative of ζ with respect to k. Namely, after simplification, we find that the derivative of the optimal quantity can be expressed as follows:

$$\frac{\partial q^*_{i,4}}{\partial k} = \left[J_{i,4} - \mathbb{E}J_4\right]\frac{\partial \zeta}{\partial k}\frac{1}{1+\zeta} + 1$$

With $\mathbb{E}J_4 = \sum_i \mu_i \int_{\theta_i} J_{i,4} dG_i(\theta)$ the aggregate marginal virtual utility over every consumer and across all groups. The equation states a sufficient condition for having a positive derivative for a given consumer: If his virtual marginal utility is (sufficiently) higher than the aggregate marginal virtual utility, then its allocation is increasing with k. This condition captures the fundamental trade-off that the market designer faces when there is an information constraint. First, note the value 1 on the right part of the equation. It describes the positive effect of increasing k for consumers when the capacity is binding, which always implies higher utility. Then, let's assume a consumer that, even with on-peak periods, has a negative marginal virtual utility, either because the marginal utility u is also negative or because the information rent $\Gamma_i(\theta)$ is too high. Similarly to the off-peak case and the previous section, allocating a given quantity of electricity to the smaller consumer is always negative (at the margin). Depending on the model parameters, the potential adverse effect of having a negative marginal virtual utility has to be compared to the positive effect of 1 associated with a less binding capacity. Finally, even when a consumer has $J_{i,4} > 0$, the market designer, due to the capacity constraint and the need to cover the fixed costs, has to favor the consumers for which it is less costly to induce an optimal allocation, that is, for consumers of the highest type. This tension is highlighted by the delta $J_{i,4} - \mathbb{E}J_4$. We illustrate our findings in Figure 8.

The left panel shows the optimal on-peak allocation for each consumer depending on their type and for a given realization of s. The solid lines represent a set of quantities given a value of k, and the dashed lines represent the allocation for a higher value of k. As described in the proposition, we observe a rotation of the allocation, with higher types receiving more goods while lower types endure a decrease in their quantity allocated. Interestingly, we do not observe a strict ranking between consumers of different categories. Namely, optimally reducing the quantities given to each consumer concerns the lowest type in each category but not across categories. The rationale behind those results lies in how incentive compatibility and individual rationality contain the market designer. As he can discriminate the consumers based on their category, which is publicly observed, the cost associated with the information rent (partly) depends on the category

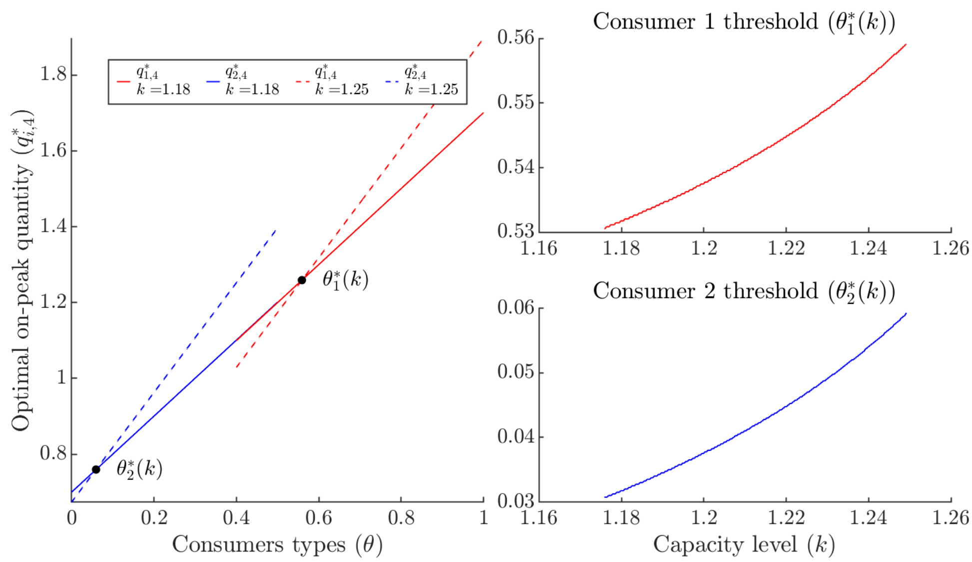


Figure 8: Optimal on-peak allocation with respect to the consumers' type θ, and threshold θ_i^* with respect to k

the consumer belongs to. Therefore, it is less costly to discriminate the consumers of the lowest type negatively.

6 Conclusion

This paper built a tractable framework to analyze the role of market designers in finding the most efficient way of consuming an essential good when faced with investment decisions. Most of the literature has focused either on providing additional remuneration streams for producers to increase the level of investment or on designing the second-best pricing schedule for consumers, given informational and technical constraints. This paper provides a unifying framework linking investment decisions and consumer participation. We show an inherent tension when implementing an allocation mechanism to maximize consumer surplus and generate revenue to cover fixed costs. The paper provides policy and technical results by adding to the initial framework additional constraints. We assume that consumers possess private information with respect to their utility level and that the market designer may be constrained in the allocation mechanism he can propose to consumers. The central result of the paper is that, depending on a set of assumptions, some specific and non-intuitive relations exist between the level of investment and the optimal allocation proposed to consumers, which has significant welfare and distributive implications. Namely, when the market designer faces heterogeneous consumers, an increase in the level of investment may affect different consumers depending on their type. For instance, under a mechanism design approach, low types can experience a decrease in their quantity allocated despite an increase in the capacity level.

Finally, we plan to extend our result with two main extensions : (i) study market design constraints with the mechanism design framework. While market designers may wish to implement some information revelation mechanism, as theoretically studied in the third result, practical contractual arrangements between the market designer and consumers may constrain him in the implementable mechanism. It would lead to specific effects, as highlighted in the second set of results. (ii) Implement specific distribution preferences associated with consumer types and categories. Our current framework does not consider welfare weights, which may distort the optimal allocation. Including such parameters would highlight the tension between generating sufficient revenue and maximizing consumer surplus. From a more extreme view, as our paper shows that the allocation can exhibit some non-monotonicities of the optimal quantities and prices, a market designer may want to avoid any decreasing quantities when the level of investment rises. Including such constraints in our framework could highlight a new trade-off.

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