P1. 12 sided fair die. with side 1 ~ 12.

(a) see following code & result

(b). X ~ Uniform [1, 12]

$p(X=1) = \frac{1}{12}$

$p(X=2) = \frac{1}{12}$

$\vdots$

$p(X=11) = \frac{1}{12}$

$p(X=12) = \frac{1}{12}$

$\therefore p(X=odd) = p(X=1) + p(X=3) + p(X=5) + p(X=7) + p(X=9) + p(X=11) = \frac{1}{2}$

(c) As we can see from the outcome that the probability X is odd for different number of tosses are different but it does closer to 0.5 as n were from 50 to 100000 generally.

(d) prime numbers $p = 2a$ (2. 3. 5. 7. 11)

non-prime numbers $p = a$ (1. 4. 6. 8. 9. 10. 12)

$p(total) = p(X=1) + p(X=2) + \dots + p(X=12) = 1$

$7a + 5 \cdot 2a = 1$

$17a = 1$

$a = \frac{1}{17}$

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```
clear
clc
close all

n = [50, 100, 1000, 2000, 3000, 10000, 100000];
pmf = [1/17, 2/17, 2/17, 1/17, 2/17, 1/17, 2/17, 1/17, 1/17, 1/17, 2/17, 1/17];
population = 1:12;
podd = zeros(1,7);
podd_uneven = zeros(1,7);
for i = 1:length(n)
    sample_size = n(i);
    podd(i) = dicetoss(sample_size);
    random_num_vec = randsample(population,sample_size,true,pmf);
    podd_uneven(i) = findOddProb(random_num_vec);
    fprintf('The probability that X has an odd value (even dice) is %5.4f with number of tosses %d \n', podd(i),sample_size)
    fprintf('The probability that X has an odd value (uneven dice) is %5.4f with number of tosses %d \n \n', podd_uneven(i),sample_size)
end
% display the podd for different nnumber of tossess and observe the trend.


% Another functon declared
function podd = findOddProb(vec)
    count = 0;
    for i = 1:length(vec)
        if mod(vec(i),2) == 1
            count = count+1;

        end
    end
    podd = count/length(vec);
end

function pOdd = dicetoss(n)
count = 0;
for i = 1:n
    m = randi([1,12]);
    if mod(m,2) == 1
        count = count + 1;
    end

end
pOdd = count/n;
end
```

Workspace

Name ▲	Value
i	6
j	10000
m	6.2400
N	[1,2,3,10,30,100]
pmf	1x12 double
population	1x12 double
random_num_...	1x100 double
rnorm	1x10000 double
s	1x1 struct
std	[3.4132,2.4135,1...
t	10000
ZiSample	1x10000 double

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Command Window

```
The probability that X has an odd value (even dice) is 0.5200 with number of tosses 50
The probability that X has an odd value (uneven dice) is 0.7600 with number of tosses 50

The probability that X has an odd value (even dice) is 0.5500 with number of tosses 100
The probability that X has an odd value (uneven dice) is 0.6200 with number of tosses 100

The probability that X has an odd value (even dice) is 0.5090 with number of tosses 1000
The probability that X has an odd value (uneven dice) is 0.6210 with number of tosses 1000

The probability that X has an odd value (even dice) is 0.5050 with number of tosses 2000
The probability that X has an odd value (uneven dice) is 0.6020 with number of tosses 2000

The probability that X has an odd value (even dice) is 0.5060 with number of tosses 3000
The probability that X has an odd value (uneven dice) is 0.5987 with number of tosses 3000

The probability that X has an odd value (even dice) is 0.4984 with number of tosses 10000
The probability that X has an odd value (uneven dice) is 0.5897 with number of tosses 10000

The probability that X has an odd value (even dice) is 0.4993 with number of tosses 100000
The probability that X has an odd value (uneven dice) is 0.5906 with number of tosses 100000

fx >>
```

clarification :

Workspace

Name ▲	Value
i	7
n	[50,100,1000,20...
pmf	1x12 double
podd	[0.5200,0.5500,0...
podd_uneven	[0.7600,0.6200,0...
population	1x12 double
random_num_...	1x100000 double
sample_size	100000

even: fair dice (each side equal prob)

uneven: unfair dice (prime number more prob)

$\therefore$ pmf = $\left[\frac{1}{17}, \frac{2}{17}, \frac{2}{17}, \frac{1}{17}, \frac{2}{17}, \frac{1}{17}, \frac{2}{17}, \frac{1}{17}, \frac{1}{17}, \frac{1}{17}, \frac{2}{17}, \frac{1}{17}\right]$

(vector) $\quad x=1 \quad x=2 \quad 3 \quad 4 \quad 5 \quad 6 \quad 7 \quad 8 \quad 9 \quad 10 \quad 11 \quad 12$

$$p(\text{odd}) = p(x=1) + p(x=3) + p(x=5) + p(x=7) + p(x=9) + p(x=11)$$

$$= \frac{1}{17} + \frac{2}{17} + \frac{2}{17} + \frac{2}{17} + \frac{1}{17} + \frac{2}{17} = \frac{10}{17} \approx 0.588$$

Here, I obtained similar result that as n went from 50 ~ 100 000, the probability (X = odd) goes closer to the value of 0.588 (as can be seen in the output)

0.5600, 0.6400, $\longrightarrow$ 0.5863. 0.5873

Q2.

(a) $X_i \sim N(\mu, \sigma^2)$

$$f_{X_i}(x_i \mid \mu, \sigma) = \frac{1}{\sqrt{2\pi}\sigma} \cdot e^{-\frac{(x_i-\mu)^2}{2\sigma^2}}$$

$$f_{X_1, X_2, \ldots, X_n}(x_1, x_2, x_3, \ldots, x_n \mid \mu, \sigma)$$

$$= f_{X_1}(x_1 \mid \mu, \sigma) \cdot f_{X_2}(x_2 \mid \mu, \sigma) \cdot f_{X_3}(x_3 \mid \mu, \sigma) \cdots f_{X_n}(x_n \mid \mu, \sigma)$$

when $X_1, X_2, \ldots X_n$ are i.i.d.

$$= \left(\frac{1}{\sqrt{2\pi}\sigma}\right)^n \cdot e^{-\left[\frac{(x_1-\mu)^2 + (x_2-\mu)^2 + \cdots + (x_n-\mu)^2}{2\sigma^2}\right]}$$

$$\therefore \log(f_{X_1, X_2, \ldots, X_n}(x_1, x_2, \ldots x_n \mid \mu, \sigma)) = \log(\sqrt{2\pi}\sigma)^{-n} + \sum_{i=1}^{n} \log e^{-\frac{(x_i-\mu)^2}{2\sigma^2}}$$

$$= \log(2\pi\sigma^2)^{-\frac{n}{2}} - \sum_{i=1}^{n} \frac{(x_i-\mu)^2}{2\sigma^2}$$

$$= \boxed{-\frac{n}{2}\log(2\pi\sigma^2) - \sum_{i=1}^{n} \frac{(x_i-\mu)^2}{2\sigma^2}}$$

(b) when $\log(f_{X_1 \ldots X_n}(x_1, \ldots, x_n \mid \mu, \sigma))$ is maximum,

$g(x_1, \ldots, x_n) =$

$g'(x_1, \ldots, x_n) = 0$ (μ is the variable)

$$\Rightarrow -\sum_{i=1}^{n} \frac{2(x_i-\mu)}{2\sigma^2} \cdot (-1) = 0 \Rightarrow \sum_{i=1}^{n} \frac{(x_i-\mu)}{\sigma^2} = 0 \Rightarrow n\mu = X_1 + \ldots X_n$$

$$\boxed{\Rightarrow \mu_{MLE} = \frac{X_1 + \ldots X_n}{n}}$$

$\sigma_{MLE_2} = \arg\max_{\sigma} \underbrace{\log f_{x_1, x_2, \dots, x_n}(x_1, x_2, x_3, \dots, x_n \mid \mu_{MLE_2}, \sigma)}_{①}$

σ^{-2}

$-2\sigma^{-3}$

when σ max in ①, $\frac{d①}{d\sigma} = 0$

$-\frac{n}{\cancel{2}} \cdot \frac{1}{\cancel{2\pi}\sigma^2} \cdot \cancel{4\pi\sigma}^{2} - \sum_{i=1}^{n} \frac{(x_i-\mu)^2}{2} \left(\frac{1}{\sigma^2}\right)'$

$$= -\frac{n}{\sigma} - \sum_{i=1}^{n} \frac{(x_i-\mu)^2}{2} \cdot (-2) \cdot \sigma^{-3} = 0$$

$$\Rightarrow -\frac{n}{\sigma} + \sum_{i=1}^{n} \frac{(x_i-\mu)^2}{\sigma^3} = 0 \qquad \left(\mu = \frac{x_1 + \dots + x_n}{n}\right)$$

$$\Rightarrow \sum_{i=1}^{n} (x_i-\mu)^2 = \sigma^2 \cdot n$$

$$\sqrt{\frac{\sum_{i=1}^{n} (x_i-\mu_{MLE})^2}{n}} = \sigma$$

(c) see codes & plots below

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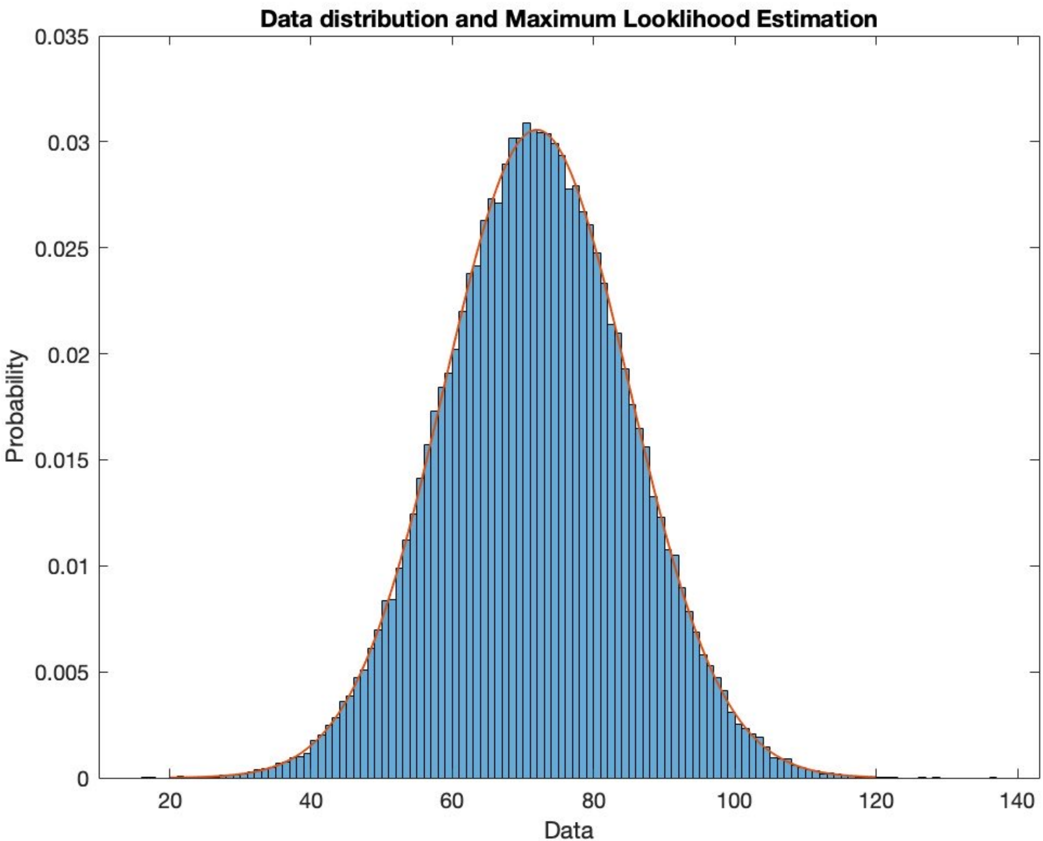
```matlab
clear
clc
close all

% Load the data and initialize mean and variance
data = load('data.txt');
l = length(data);
miuMLE = sum(data)/l;
sigmaMLE = sqrt(sum((data-miuMLE).^2)/l);

% Plot the data with the Maximum likelihood Estimation of Guassian generated
% with mean miuMLE and sigmaMLE
figure;
histogram(data,'Normalization','pdf')
hold on
pd = makedist('Normal','mu',miuMLE,'sigma',sigmaMLE);
x = 20:0.1:120;
y = normpdf(x,miuMLE,sigmaMLE);
plot(x,y,lineWidth = 1)
xlabel('Data')
ylabel('Probability')
title('Data distribution and Maximum Looklihood Estimation')
```

Workspace

Name ▲	Value
data	100000x1 double
l	100000
miuMLE	71.9438
pd	1x1 NormalDistri..
sigmaMLE	13.0547
x	1x1001 double
y	1x1001 double



Data distribution and Maximum Looklihood Estimation
Probability
Data
0.035
0.03
0.025
0.02
0.015
0.01
0.005
0
20
40
60
80
100
120
140

3.

(a) see code attached & plot

(b)

Separate the table into 2 parts where the first part only has rows $B=0$, second part only has rows $B=1$.

$$P(T \mid B=0) = \begin{cases} 0.3229 & T=1 \\ 0.3229 & T=2 \\ 0.3541 & T=3 \end{cases}$$

$$P(T \mid B=1) = \begin{cases} 0.3421 & T=1 \\ 0.3158 & T=2 \\ 0.3421 & T=3 \end{cases}$$

$$P(S \mid B=0) = \begin{cases} 0.8477 & S=0 \\ 0.1523 & S=1 \end{cases}$$

$$P(S \mid B=1) = \begin{cases} 0.6754 & S=0 \\ 0.~~32~~46 & S=1 \end{cases}$$

$P(A \mid B=0)$ & $P(A \mid B=1)$ see plots & data below.

$$\begin{cases} P(A \le 67 \mid B=0) = 0.6587 \\ P(A \le 67 \mid B=1) = 0.6287 \end{cases}$$

$P(B=0) = 0.6144$

$P(B=1) = 0.3856$

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projectP3a.m | projectP3b.m | p3functions.m | projectP4a.m | projectP4d.m | +

Code for 3a

```
l = 887;
Data = csvread('user_data.csv',1);
B = Data(:,1);
T = Data(:,2);
S = Data(:,3);
A = Data(:,4);

[x1,p1] = pmfPlotHelper(B);
figure
subplot(4,1,1)
stem(x1, p1, "LineStyle", '-.', 'MarkerFaceColor', 'red', 'MarkerEdgeColor','green','LineWidth',2);
ylim([0,1])
title('pmf of B');

[x2,p2] = pmfPlotHelper(T);
subplot(4,1,2)
stem(x2, p2, "LineStyle", '-.', 'MarkerFaceColor', 'red', 'MarkerEdgeColor','green','LineWidth',2);
ylim([0,1])
title('pmf of T')

[x3,p3] = pmfPlotHelper(S);
subplot(4,1,3)
stem(x3, p3, "LineStyle", '-.', 'MarkerFaceColor', 'red', 'MarkerEdgeColor','green','LineWidth',2);
ylim([0,1])
title('pmf of S')

[x4,p4] = pmfPlotHelper(A);
subplot(4,1,4)
stem(x4, p4, "LineStyle", '-.', 'MarkerFaceColor', 'red', 'MarkerEdgeColor','green','LineWidth',2);
ylim([0,1])
title('pmf of A')
function nRepeat = repetition(Vec, element)
    count = 0;
    for i = 1:length(Vec)
        if Vec(i) == element
            count = count +1;
        end
    end
    nRepeat = count;
end
 function [fNumberVec, Probability] = pmfPlotHelper(inputVec)
    numberVec = [];
    for i = 1:length(inputVec)
        if ismember(inputVec(i),numberVec) == 0
            numberVec = [numberVec, inputVec(i)];
        end
    end
    fNumberVec = numberVec;
    for j = 1:length(numberVec)
        Probability(j) = repetition(inputVec, numberVec(j))/length(inputVec);
    end
end
```

Command Window

Workspace

Name ▲	Value
data	100000x1 double
l	100000
miuMLE	71.9438
pd	1x1 NormalDistri...
sigmaMLE	13.0547
x	1x1001 double
y	1x1001 double

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projectP3a.m | projectP3b.m | p3functions.m | projectP4a.m | projectP4d.m | +

3b

```
l = 887;
Data = csvread('user_data.csv',1);
% Divide the whole User data list into two lists where the first one only
% has B = 0, and the second only has B = 1.
DataB0 = [];
DataB1 = [];
for i = 1:l
    if Data(i,1) == 0
        DataB0 = [DataB0; Data(i,:)];

    else
        DataB1 = [DataB1; Data(i,:)];
    end

end

% Find out each T,S, and A vector given the conditions of B = 0 or 1. Use
% the function declared before to help us to solve each conditional pmf.
T0 = DataB0(:,2);
T1 = DataB1(:,2);
S0 = DataB0(:,3);
S1 = DataB1(:,3);
A0 = DataB0(:,4);
A1 = DataB0(:,4);

[x10,p10] = pmfPlotHelper(T0);
figure
subplot(3,2,1)
stem(x10, p10, "LineStyle", '-.', 'MarkerFaceColor', 'red', 'MarkerEdgeColor','green','LineWidth',2);
ylim([0,1])
title('pmf of T given B = 0');

[x11,p11] = pmfPlotHelper(T1);
subplot(3,2,2)
stem(x11, p11, "LineStyle", '-.', 'MarkerFaceColor', 'red', 'MarkerEdgeColor','green','LineWidth',2);
ylim([0,1])
title('pmf of T given B = 1');

[x20,p20] = pmfPlotHelper(S0);
subplot(3,2,3)
stem(x20, p20, "LineStyle", '-.', 'MarkerFaceColor', 'red', 'MarkerEdgeColor','green','LineWidth',2);
ylim([0,1])
title('pmf of S given B = 0');

[x21,p21] = pmfPlotHelper(S1);
subplot(3,2,4)
stem(x21, p21, "LineStyle", '-.', 'MarkerFaceColor', 'red', 'MarkerEdgeColor','green','LineWidth',2);
ylim([0,1])
title('pmf of S given B = 1');

[x30,p30] = pmfPlotHelper(A0);
subplot(3,2,5)
```

Workspace

Name ▲	Value
A0	545x1 double
A1	545x1 double
count1	359
count2	215
Data	887x4 double
DataB0	545x4 double
DataB1	342x4 double
i	545
j	342
l	887
p10	[0.3541,0.3229,0..
p11	[0.3158,0.3421,0..
p20	[0.8477,0.1523]
p21	[0.6754,0.3246]
p30	1x81 double
p31	1x81 double
prob1	0.6587
prob2	0.6287
S0	545x1 double
S1	342x1 double
T0	545x1 double
T1	342x1 double
x10	[3,1,2]
x11	[2,3,1]
x20	[0,1]
x21	[1,0]
x30	1x81 double
x31	1x81 double

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projectP3a.m | projectP3b.m * | p3functions.m | projectP4a.m | projectP4d.m | +

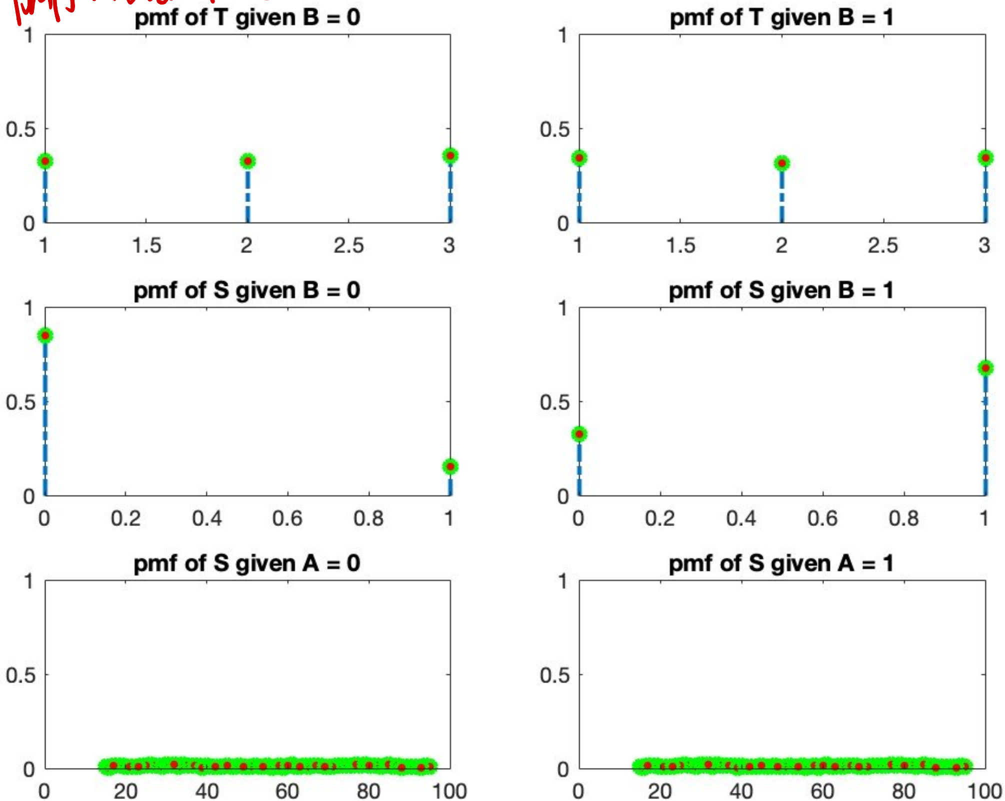
3b continued,

```matlab
subplot(3,2,5)
stem(x30, p30, "LineStyle", '-.', 'MarkerFaceColor', 'red', 'MarkerEdgeColor','green','LineWidth',2);
ylim([0,1])
title('pmf of S given A = 0');

[x31,p31] = pmfPlotHelper(A1);
subplot(3,2,6)
stem(x31, p31, "LineStyle", '-.', 'MarkerFaceColor', 'red', 'MarkerEdgeColor','green','LineWidth',2);
ylim([0,1])
title('pmf of S given A = 1');

% Further compute the probabilty that A <= 67 given B=0 and B=1:

count1 = 0;
for i = 1:length(DataB0)
    if DataB0(i,4) <= 67
        count1 = count1 +1;
    end
end
prob1 = count1/length(DataB0);
fprintf('The probability that A <= 67 given B = 0 is %5.4f. \n',prob1)

count2 = 0;
for j = 1:length(DataB1)
    if DataB1(j,4) <= 67
        count2 = count2 +1;
    end
end
prob2 = count2/length(DataB1);
fprintf('The probability that A <= 67 given B = 1 is %5.4f. \n',prob2)

function nRepeat = repetition(Vec, element)
    count = 0;
    for i = 1:length(Vec)
        if Vec(i) == element
            count = count +1;
        end
    end
    nRepeat = count;
end
 function [fNumberVec, Probability] = pmfPlotHelper(inputVec)
    numberVec = [];
    for i = 1:length(inputVec)
        if ismember(inputVec(i),numberVec) == 0
                numberVec = [numberVec, inputVec(i)];
        end
    end
    fNumberVec = numberVec;
    for j = 1:length(numberVec)
        Probability(j) = repetition(inputVec, numberVec(j))/length(inputVec);
    end
end
```

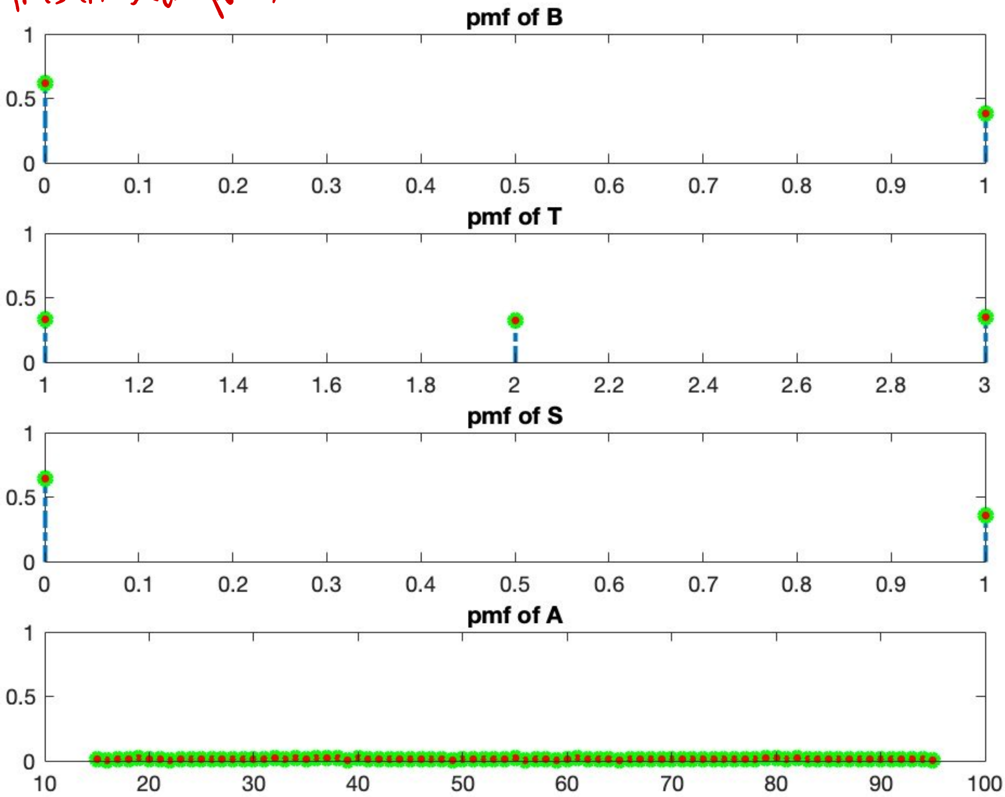
Workspace

Name ▲	Value
A0	545x1 double
A1	545x1 double
count1	359
count2	215
Data	887x4 double
DataB0	545x4 double
DataB1	342x4 double
i	545
j	342
l	887
p10	[0.3541,0.3229,0.
p11	[0.3158,0.3421,0.
p20	[0.8477,0.1523]
p21	[0.6754,0.3246]
p30	1x81 double
p31	1x81 double
prob1	0.6587
prob2	0.6287
S0	545x1 double
S1	342x1 double
T0	545x1 double
T1	342x1 double
x10	[3,1,2]
x11	[2,3,1]
x20	[0,1]
x21	[1,0]
x30	1x81 double
x31	1x81 double

(b) 6 pmfs in stem plots



(a) 4 pmfs in stem plots



(c) ① $P(B=0, T=1, S=0, A\le 67) = P(T=1, S=0, A\le 67 \mid B=0) \cdot P(B=0)$

② $P(B=1, T=1, S=0, A\le 67) = P(T=1, S=0, A\le 67 \mid B=1) \cdot P(B=1)$

① $= P(T=1 \mid B=0) \cdot P(S=0 \mid B=0) \cdot P(A\le 67 \mid B=0) \cdot P(B=0)$

$= 0.3229 \cdot 0.8477 \cdot 0.6587 \cdot 0.6144$

≈ 0.1108

② $= P(T=1 \mid B=1) \cdot P(S=0 \mid B=1) \cdot P(A\le 67 \mid B=1) \cdot P(B=1)$

$= 0.3471 \cdot 0.6754 \cdot 0.6287 \cdot 0.3856$

≈ 0.0560

(d) $P(B=0 \mid T=1, S=0, A\le 67)$

$$= \frac{P(B=0, T=1, S=0, A\le 67)}{P(B=0, T=1, S=0, A\le 67) + P(B=1, T=1, S=0, A\le 67)}$$

$$= \frac{0.1108}{0.1108 + 0.056} \approx \boxed{0.6643}$$

$$P(B=1 \mid T=1, S=0, A\le 67)$$

$$= \frac{P(B=1, T=1, S=0, A\le 67)}{P(B=0, T=1, S=0, A\le 67) + P(B=1, T=1, S=0, A\le 67)}$$

$$= \frac{0.056}{0.1108 + 0.056} \approx \boxed{0.3357}$$

Since $P(B=0 \mid T=1, S=0, A\le 67) > P(B=1 \mid T=1, S=0, A\le 67)$

$\therefore$ She's not likely to buy this product

P4.

(a) $X_i \sim U(10, 16)$

$N = [1, 2, 3, 10, 30, 100]$

(b). X_i mean: $m = \frac{10+16}{2} = \boxed{13}$

Variance: $\sigma^2 = \frac{1}{3}(10^2 + 10 \cdot 16 + 16^2) - \left(\frac{10+16}{2}\right)^2$

$= 172 - 169 = \boxed{3}$

$Z_n = \frac{X_1 + X_2 + \dots + X_n}{n}$.

Z_n's mean: $m_n \overset{(*)}{=} \frac{n \cdot m}{n} = m = \boxed{13}$ * by $X_1 \dots X_n$ are i.i.d.

variance: $VAR(Z_n) = VAR\left(\frac{X_1}{n} + \frac{X_2}{n} + \dots \frac{X_n}{n}\right) = \frac{VAR(X_1) + \dots VAR(X_n)}{n^2} = \boxed{\frac{3}{n}}$

(c) See code.

(d) 12-sided die. Recall from (c)

pmf = $[\frac{1}{17}, \frac{2}{17}, \frac{2}{17}, \frac{1}{17}, \frac{2}{17}, \frac{1}{17}, \frac{2}{17}, \frac{1}{17}, \frac{1}{17}, \frac{1}{17}, \frac{2}{17}, \frac{1}{17}]$

(vector) $x=1$, $x=2$, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12

$E[X_i] = 1 \cdot \frac{1}{17} + 2 \cdot \frac{2}{17} + 3 \cdot \frac{2}{17} + 4 \cdot \frac{1}{17} + 5 \cdot \frac{2}{17} + 6 \cdot \frac{1}{17} + 7 \cdot \frac{2}{17} + 8 \cdot \frac{1}{17} +$

$9 \cdot \frac{1}{17} + 10 \cdot \frac{1}{17} + 11 \cdot \frac{2}{17} + 12 \cdot \frac{1}{17} = \boxed{\frac{106}{17}}$

$VAR(X_i) = E[X_i^2] - E[X_i]^2$

$E[X_i^2] = 1 \cdot \frac{1}{17} + 4 \cdot \frac{2}{17} + 9 \cdot \frac{2}{17} + 16 \frac{1}{17} + 25 \cdot \frac{2}{17} + 36 \cdot \frac{1}{17} + 49 \cdot \frac{2}{17} + 64 \cdot \frac{1}{17} +$

$81 \cdot \frac{1}{17} + 100 \frac{1}{17} + 121 \frac{2}{17} + 144 \frac{1}{17}$

$= \frac{859}{17}$ ~~≈50.53~~

$\therefore VAR(X_i) = \frac{859}{17} - \left(\frac{106}{17}\right)^2 \approx \boxed{11.65}$

$E[Z_n] = \frac{n \cdot E[X_i]}{n} = \boxed{\frac{106}{17} \approx 6.24}$

$VAR(Z_n) = \frac{\sum_{i=1}^{n} VAR(X_i)}{n^2} = \boxed{\frac{11.65}{n}}$ $\quad \sigma = \sqrt{\frac{11.65}{n}}$

(e) $P(Z_n \le z) = P\left(\left(\frac{X_1 + X_2 + \dots + X_n}{n}\right) \le z\right) = P((X_1, X_2, \dots, X_n) \in A_z)$

where $A_z = \{(x_1, \dots, x_n) : x_1 + x_2 + \dots + x_n \le z\}$

$$f_{Z_n}(z_n) = \frac{d}{dz_n} \int_{x_n} \cdots \int_{x_1} f_{X_1 \dots X_n}\left(\frac{x_1 + \dots + x_n}{n} \le z\right) dx_1 dx_2 \dots dx_n$$

Since $X_1, \dots X_n$ are independent RVs.

$f_{Z_n}(z_n) = f_{X_1}(x_1) * f_{X_2}(x_2) * \dots * f_{X_n}(x_n)$

(we have seen the case of $z = x + y$ where x, y in Lecture 13)

=

$f(x_i) = \begin{cases} \frac{1}{6} & 10 < x < 16 \\ 0 & \text{o.w.} \end{cases}$

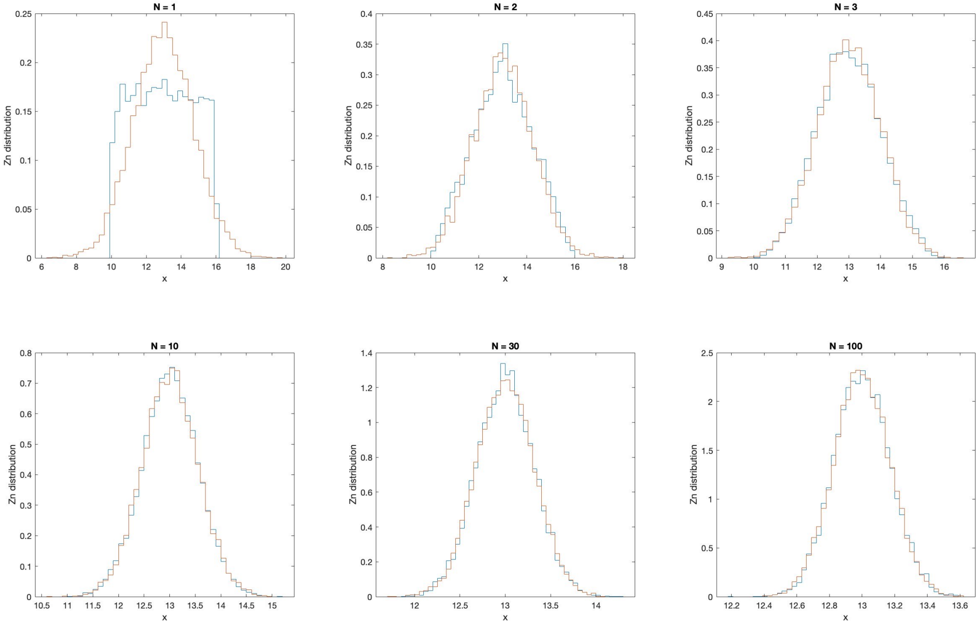
$$f(x_i) * f(x_i) = \left(\frac{1}{6}\right)^2 \int_{-\infty}^{+\infty} [u(\tau - 10) - u(\tau - 16)][u(t - \tau - 10) - u(t - \tau - 16)] d\tau$$

use Matlab to compute $\underbrace{f(x_i) * f(x_i) \dots * f(x_i)}_{n}$ and plot it with the exact Gaussian.

Code At the end & plots

In the following plots, legends are omitted for it will block part of the histograms.

Lia's plot



Editor – projectP4a.m   Variables – mem_blue{1, 3}

+1 | untitled7 * | ComputeNewAngle.m | MovementValidationExecution.m | projectP4a.m | +

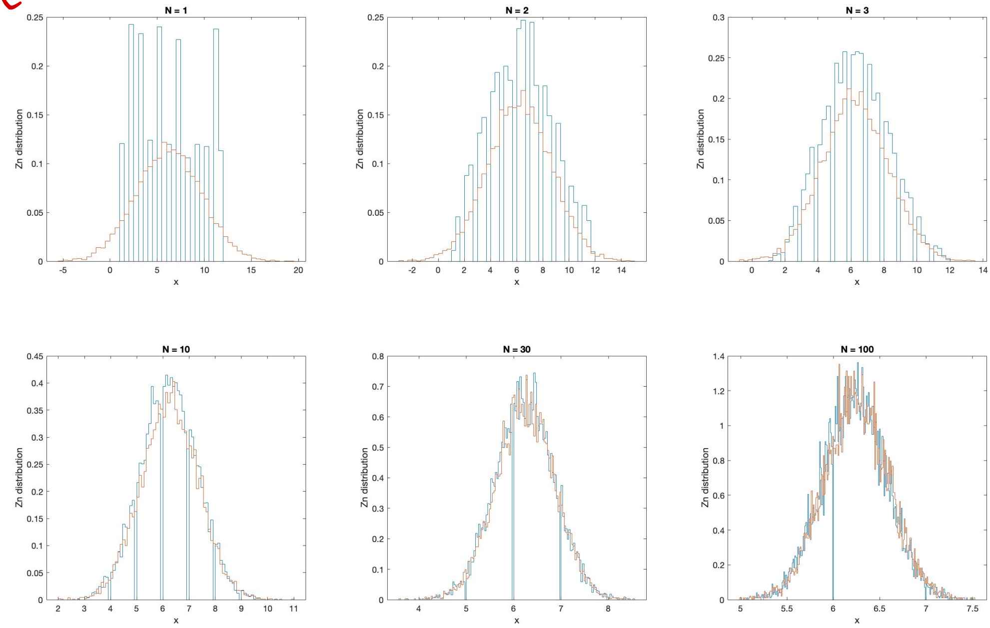
(4a)

```
clear
clc
close all

% Constant declaration
m = 13;
N = [1,2,3,10,30,100];  % n's number
s = 3*ones(1,length(N));
t = 1e4;   % Sample's size
std = sqrt(s./N);   % Standard deviation vector result from different n
nbins = 10;
figure

for i = 1:length(N)
    ZiSample = zeros(1,t); % Generate that zi vector to store generated sample of different n.
    for j = 1:t            % Generate zi t times
        zi = 0;
        for k = 1:N(i)
        zi = zi + unifrnd(10,16);
        end
        ZiSample(j) = zi/N(i); % Store that zi in each position
    end
    subplot(2,3,i)
    histogram(ZiSample,'Normalization','pdf',"DisplayStyle" ,"stairs")
    hold on
    % Plot the Gaussian with analytical parameters
    s = rng;
    rnorm = normrnd(m,std(i),[1,t]);
    histogram(rnorm,'Normalization','pdf',"DisplayStyle" ,"stairs")
    title(['N = ' num2str(N(i))])
    xlabel('x')
    ylabel('Zn distribution')
    rng(s);
    %Commented out because it blocks the histogram
    %lgd = legend('pdf(Zn)','pdf(Gaussian)');
    %fontsize(lgd,3,'points')
end
```

Workspace

Name ▲	Value
i	6
j	10000
k	100
m	13
N	[1,2,3,10,30,100]
nbins	10
rnorm	*1x10000 double*
s	*1x1 struct*
std	[1.7321,1.2247,1...
t	10000
zi	1.2999e+03
ZiSample	*1x10000 double*

Let's plot



Editor – projectP4d.m | Variables – mem_blue{1, 3}

+2 | ComputeNewAngle.m | MovementValidationExecution.m | projectP4a.m | projectP4d.m | +

4d)

```
clear
clc
close all

% Constant declaration

pmf = [1/17, 2/17, 2/17, 1/17, 2/17, 1/17, 2/17, 1/17, 1/17, 1/17, 2/17, 1/17];
population = 1:12;

m = 6.24;
N = [1,2,3,10,30,100];  % n's number
s = 11.65*ones(1,length(N));
% Standard deviation vector result from different n
std = sqrt(s./N);
t = 1e4;    % Sample's size
figure

for i = 1:length(N)
    ZiSample = zeros(1,t); % Generate that zi vector to store generated sample of different n.
    for j = 1:t            % Generate zi t times
        random_num_vec = randsample(population,N(i),true,pmf);
        ZiSample(j) = sum(random_num_vec)/N(i); % Store that zi in each position
    end

    subplot(2,3,i)
    histogram(ZiSample,'Normalization','pdf','BinWidth',1/(1+N(i)) ,"DisplayStyle" ,"stairs")
    hold on
    % Plot the Gaussian with analytical parameters
    s = rng;
    rnorm = normrnd(m,std(i),[1,t]);
    histogram(rnorm,'Normalization','pdf', 'BinWidth',1/(1+N(i)) ,"DisplayStyle" ,"stairs")
    title(['N = ' num2str(N(i))])
    xlabel('x')
    ylabel('Zn distribution')
    rng(s);
    %Commented out because it blocks the histogram
    %lgd = legend('pdf(Zn)','pdf(Gaussian)');
    %fontsize(lgd,3,'points')
end
```

Workspace

Name	Value
i	6
j	10000
k	100
m	13
N	[1,2,3,10,30,100]
nbins	10
rnorm	1x10000 double
s	1x1 struct
std	[1.7321,1.2247,1...
t	10000
zi	1.2999e+03
ZiSample	1x10000 double

Editor – projectP4e.m

Variables – mem_blue{1, 3}

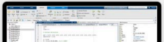
+3 | MovementValidationExecution.m | projectP4a.m | projectP4d.m | projectP4e.m | +

(e)

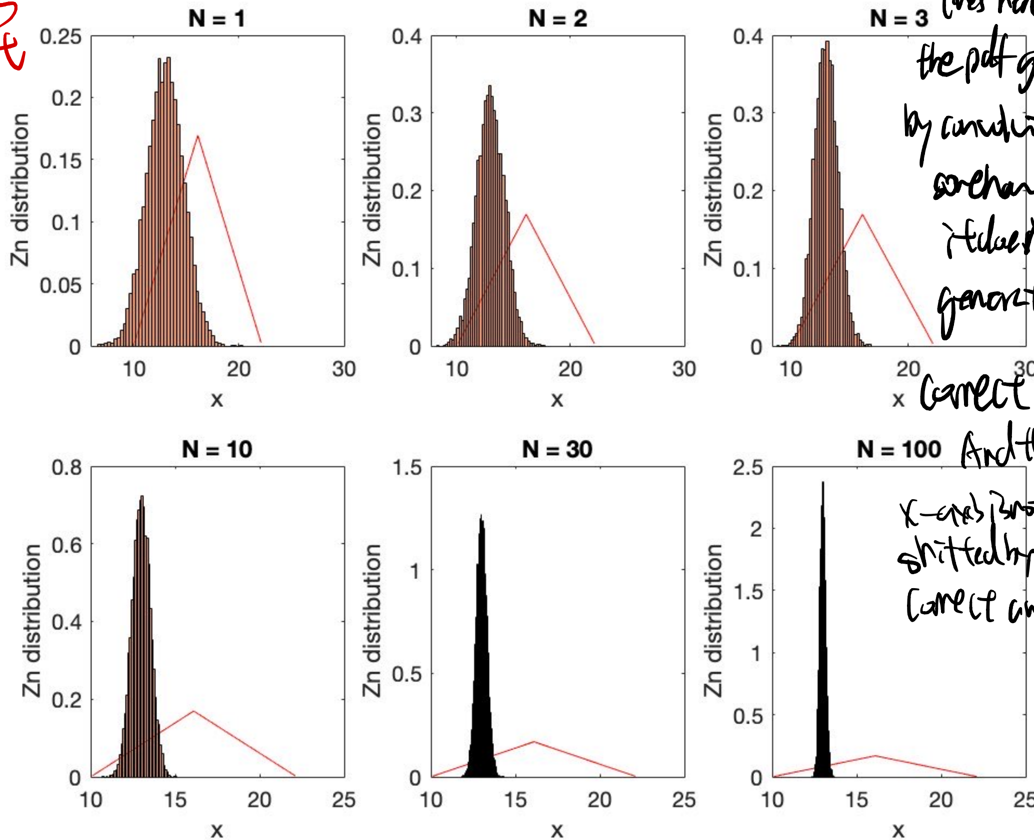
```
clear
clc
close all

% Constant declaration
dx = 0.1;
x = 10:dx:16;
y = 1/6*ones(1,length(x));

m = 13;
N = [1,2,3,10,30,100];  % n's number
s = 3*ones(1,length(N));
t = 1e4;   % Sample's size
std = sqrt(s./N);   % Standard deviation vector result from different n

figure

for i = 1:length(N)
    yi = y;
    for j = 1:length(N(i))
        q = conv(yi, y);
        yi = q;
        %plot((0:(numel(q)-1))*dx,q*dx)

    end
    q_x = (1:length(q)).* dx + x(1);

    subplot(2,3,i)
    plot(q_x, dx*q,'r');
    hold on
    % Plot the Gaussian with analytical parameters
    %s = rng;
    rnorm = normrnd(m,std(i),[1,t]);
    histogram(rnorm,'Normalization','pdf','FaceAlpha',0.6)
    title(['N = ' num2str(N(i))])
    xlabel('x')
    ylabel('Zn distribution')
    %rng(s);
    %Commented out because it blocks the histogram
    %lgd = legend('pdf(Zn)','pdf(Gaussian)');
    %fontsize(lgd,3,'points')
end
```

Workspace

Name ▲	Value
i	6
j	10000
k	100
m	13
N	[1,2,3,10,30,100]
nbins	10
rnorm	1x10000 double
s	1x1 struct
std	[1.7321,1.2247,1...
t	10000
zi	1.2999e+03
ZiSample	1x10000 double



4c's plot



lines here are the pdf generated by convolutions somehow, it doesn't generate the correct bell shape. And the x-axis is not shifted by the correct amount.