10.2 Two vehicles are moving along a straight line. For the first vehicle we use the same model as in exercise 10.1:

$$\begin{bmatrix} s_1(t+1) \\ s_2(t+1) \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ 0 & 0.95 \end{bmatrix} \begin{bmatrix} s_1(t) \\ s_2(t) \end{bmatrix} + \begin{bmatrix} 0 \\ 0.1 \end{bmatrix} u(t), \qquad t = 0, 1, 2, \ldots,$$

$s_1(t)$ is the position at time t, $s_2(t)$ is the velocity at time t, and $u(t)$ is the actuator input. We assume that the vehicle is initially at rest at position 0: $s_1(0) = s_2(0) = 0$. The model for the second vehicle is

$$\begin{bmatrix} p_1(t+1) \\ p_2(t+1) \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ 0 & 0.8 \end{bmatrix} \begin{bmatrix} p_1(t) \\ p_2(t) \end{bmatrix} + \begin{bmatrix} 0 \\ 0.2 \end{bmatrix} v(t), \qquad t = 0, 1, 2, \ldots,$$

$p_1(t)$ is the position at time t, $p_2(t)$ is the velocity at time t, and $v(t)$ is the actuator input. We assume that the second vehicle is initially at rest at position 1: $p_1(0) = 1$, $p_2(0) = 0$.

Formulate the following problem as a least norm problem, and solve it in MATLAB (see the remark at the end of exercise 10.1). Find the control inputs $u(0)$, $u(1)$, ..., $u(19)$ and $v(0)$, $v(1)$, ..., $v(19)$ that minimize the total energy

$$\sum_{t=0}^{19} u(t)^2 + \sum_{t=0}^{19} v(t)^2$$

and satisfy the following three conditions:

$$s_1(20) = p_1(20), \qquad s_2(20) = 0, \qquad p_2(20) = 0. \tag{37}$$

In other words, at time $t = 20$ the two vehicles must have velocity zero, and be at the same position. (The final position itself is not specified, *i.e.*, you are free to choose any value as long as $s_1(20) = p_1(20)$.)

Plot the positions $s_1(t)$ and $p_1(t)$ of the two vehicles, for $t = 1, 2, \ldots, 20$.

10.3 Explain how you would solve the following problems using the QR factorization.

(a) Find the solution of $Cx = d$ with the smallest value of $\sum_{i=1}^n w_i x_i^2$:

$$\begin{array}{ll} \text{minimize} & \sum_{i=1}^n w_i x_i^2 \\ \text{subject to} & Cx = d. \end{array}$$

The problem data are the $p \times n$ matrix C, the p-vector d, and the n vector w. We assume that A has linearly independent rows, and $w_i > 0$ for all i.

(b) Find the solution of $Cx = d$ with the smallest value of $\|x\|^2 - c^T x$:

$$\begin{array}{ll} \text{minimize} & \|x\|^2 - c^T x \\ \text{subject to} & Cx = d. \end{array}$$

The problem data are the n-vector c, the $p \times n$ matrix C, and the p-vector d. We assume that C has linearly independent rows.

10.4 Show how to solve the following problems using the QR factorization of A. In each problem A is an $m \times n$ matrix with linearly independent columns. Clearly state the different steps in your method. Also give the complexity, including all terms that are quadratic (order m^2, mn, or n^2), or cubic (order m^3, m^2n, mn^2, n^3). If you know several methods, give the most efficient one.

(a) Solve the set of linear equations

$$\begin{bmatrix} 0 & A^T \\ A & I \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} b \\ c \end{bmatrix}.$$

The variables are the n-vector x and the m-vector y.