ECE 141 - Project
Spring 2023

Please carefully justify all your answers.
Submit your MATLAB script separately.
You can refer to the additional material posted under Week 8 for useful MATLAB commands.
If you think that there is something wrong with the questions,
just use your best interpretation and state your justification.
You are not allowed to collaborate with others.

Due: June 9, 2023 11:59 pm, 90 points in total

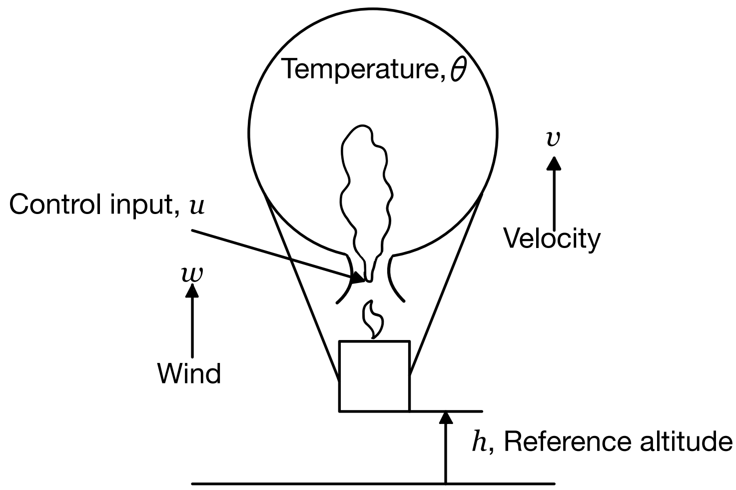

Figure 1: Hot air balloon system where heat is injected as a control input

In this project we will design a controller for a hot air balloon system based on an approximate differential equation describing its motion. The visual representation of the system is given in Figure 1. The equation of motion for the system is as follows

$$\begin{aligned} \dot{\theta} &= -\frac{1}{\tau_1}\theta + u, \\ \dot{v} &= -\frac{1}{\tau_2}v + \sigma\theta + \frac{1}{\tau_2}w, \\ \dot{h} &= v, \end{aligned} \tag{1}$$

where $\theta \in \mathbb{R}$ is a temperature of air inside the balloon [K]; $u \in \mathbb{R}$ is a control input [K/s]; $h \in \mathbb{R}$ is an altitude change from some reference point [m]; $w \in \mathbb{R}$ is a wind disturbance [m/s]; and $v \in \mathbb{R}$ is balloon's vertical velocity [m/s]; $\sigma \in \mathbb{R}$ [m/s^{2}K], $\tau_1 \in \mathbb{R}$ [s], $\tau_2 \in \mathbb{R}$ [s] are all system parameters. You can assume $\tau_1 \neq \tau_2$ for all the questions below. Output of the system is h.

Part I, Laplace Domain

1. (15 points) Assume that $\sigma, \tau_1, \tau_2 > 0$. Find the expression for the open-loop output response of the system in Laplace domain when both the control input and the disturbance are present. Now, suppose you place the complete system in a closed-loop with a unity negative feedback. Draw a block-diagram of the complete system. Can you make the system stable using a proportional controller for the closed-loop system with a unity negative feedback?

2. (30 points) Assume that w is equal to some constant value for all $t \geq 0$. Find a controller such that (1) the system tracks constant reference signals with zero steady-state error; and (2) the error signal is no less than -0.2 for all $t \geq 0$ with convergence to zero for $\sigma = 5$, $\tau_1 = 0.1$, $\tau_2 = 0.1$. Take both constants for the reference and disturbance signals as 1 for (2). Provide a plot of the error signal.

Part II, State-Space

1. (30 points) Assume the disturbance, w, is zero for all t in (1) for the remainder of the project. Write the system in the state-space matrix form. Compute the controllability matrix of the system using the appropriate matrices found in part 1. Find the determinant of the controllability matrix found in part 2. What condition should be satisfied for the system to be controllable? Suppose now that the condition you found in part 3 is violated, what part of the state can you still control?

2. (15 points) Take the parameters as $\sigma = 5$, $\tau_1 = 0.1$, $\tau_2 = 0.1$. Suppose your controller in state-space is given by $u = Kx$ where $K \in \mathbb{R}^{1\times 3}$ is the controller gain and $x \in \mathbb{R}^3$ is the state of the system. Take $K = \begin{bmatrix} 14 & -10 & -1 \end{bmatrix}$. Verify that your closed-loop system is stable.