EECE 562 Assignment 3

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Q1. (a)

$$y[k] = [c_1\ c_2]\,x[k]$$
$$= [c_1\ c_2]\left\{\begin{bmatrix}-1 & -2\\ 1 & 0\end{bmatrix}x[k-1] + \begin{bmatrix}1\\0\end{bmatrix}u[k-1]\right\}$$

where $x[k-1] = [c_1\ c_2]^{-1}\,y[k-1]$

$$= [c_1\ c_2]\begin{bmatrix}-1 & -2\\ 1 & 0\end{bmatrix}[c_1, c_2]^{-1}\,y[k-1] + c_1\,u[k-1]$$

(b) No, $x[0]$ cannot be reconstruted with known $u[0] \sim u[k]$

(c) Let $[c_1\ c_2]\begin{bmatrix}-1 & -2\\ 1 & 0\end{bmatrix}[c_1, c_2]^{-1} = a$, $c_1 = b$, $\theta = \begin{bmatrix}a\\b\end{bmatrix}$

$$y[k] = a\,y[k-1] + b\,u[k-1] = \left[y[k-1],\ u[k-1]\right]\cdot\theta$$

$$\therefore\ \Psi = [\psi_1, \psi_2 \cdots \psi_n]' = \begin{bmatrix} y_1 & u_1\\ y_2 & u_2\\ \vdots & \\ y_n & u_n\end{bmatrix}$$

$$\therefore\ \theta_{LS} = (\Psi'\Psi)^{-1}\cdot\Psi'\cdot Y$$

Q2. Let $x_k = \begin{bmatrix} y_t \\ y_{k-1} \end{bmatrix}$ and $W_t = \begin{bmatrix} u_k \\ 0 \end{bmatrix}$

Then $y_k = \begin{bmatrix} 1 & 0 \end{bmatrix} x_k$

$x_k = \begin{bmatrix} a_1 & a_2 \\ 1 & 0 \end{bmatrix} x_{k-1} - W_t$

All eigenvalues of $\begin{bmatrix} a_1 & a_2 \\ 1 & 0 \end{bmatrix}$ should be stable

$$\begin{vmatrix} a_1 - \lambda & a_2 \\ 1 & -\lambda \end{vmatrix} = \lambda^2 - a_1\lambda - a_2 = 0$$

$$\lambda_{1,2} = \frac{a_1 \pm \sqrt{a_1^2 + 4a_2}}{2} < 0$$

$\therefore\ a_1 < 0$

and $|a_1| = -a_1 > \sqrt{a_1^2 + 4a_2} \Rightarrow a_2 < 0$

Q3. $y_k = a_1 y_{k-1} + a_2 y_{k-2} + u_k$

$$Y = \begin{bmatrix} y_3 \\ y_4 \\ \vdots \\ y_n \end{bmatrix} = \begin{bmatrix} y_2 \\ y_3 \\ \vdots \\ y_{n-1} \end{bmatrix} a_1 + \begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_{n-2} \end{bmatrix} a_2 + u_k$$

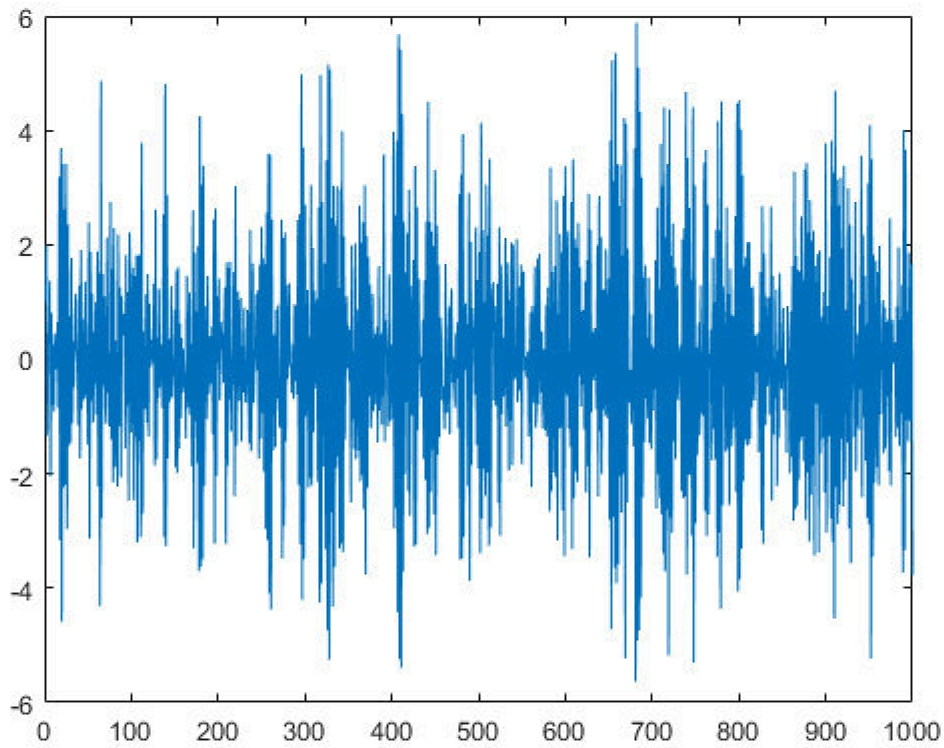
$$\varphi = \begin{bmatrix} y_2 & y_1 \\ y_3 & y_2 \\ \vdots & \vdots \\ y_{n-1} & y_{n-2} \end{bmatrix} \qquad \theta = \begin{bmatrix} a_1 \\ a_2 \end{bmatrix}$$

$$\theta = \begin{bmatrix} a_1 \\ a_2 \end{bmatrix} = (\varphi'\varphi)^{-1}\varphi' Y$$

Table of Contents

q2

```
N=1000;
y=zeros(N,1);
y(1)=1;
y(2)=2;
a1=-1;
a2=-0.1;
for k=3:N
    wk=normrnd(0,1);
    y(k)=a1*y(k-1)+a2*y(k-2)+wk;
end
plot(1:N,y)
```

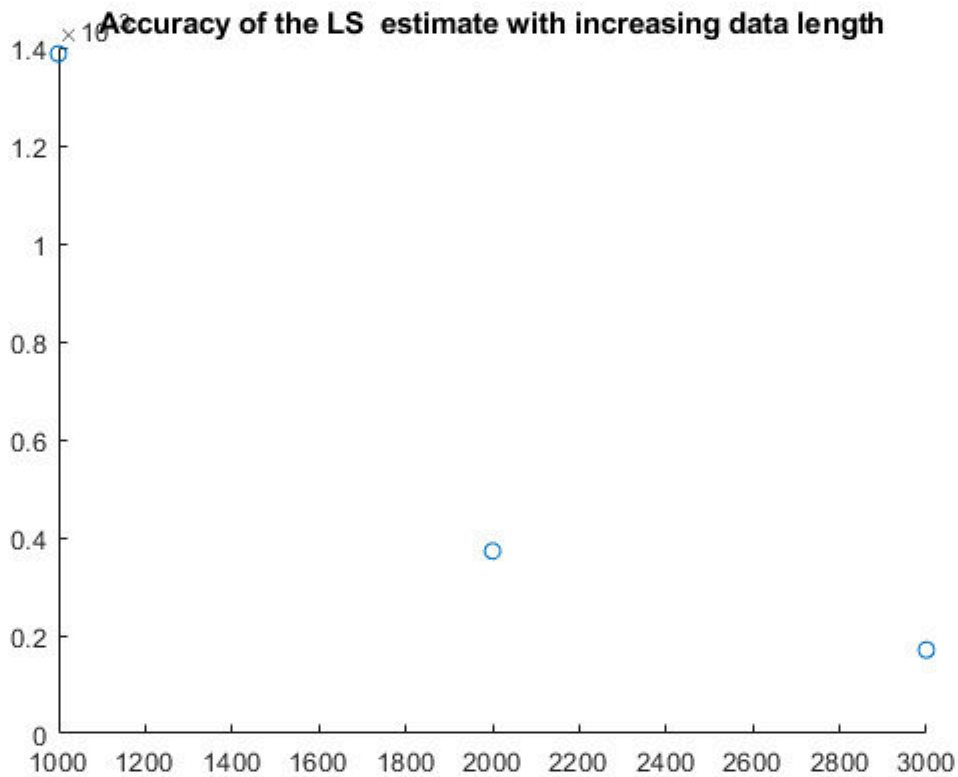


q3

Asymptotically stationary

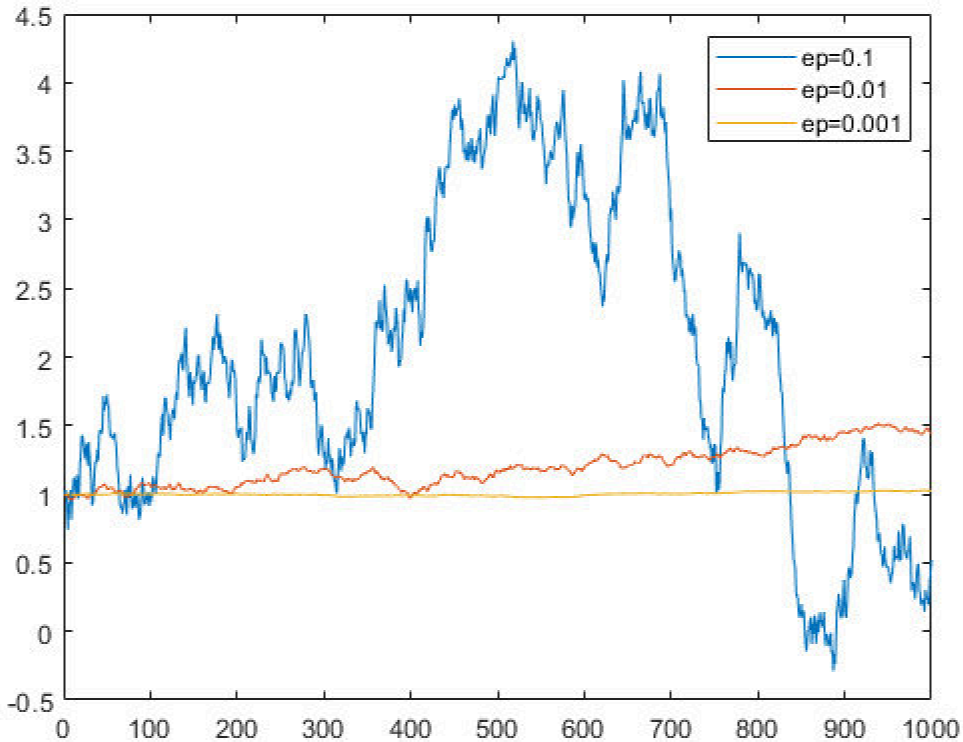
```
for N=1000:1000:3000
    %simulate AR model as q2
    y=zeros(N,1);
    y(1)=1;
    y(2)=2;
    a1=-1;
    a2=-0.1;
    for k=3:N
        wk=normrnd(0,1);
        y(k)=a1*y(k-1)+a2*y(k-2)+wk;
    end
    % LS estimator
    pha=zeros(N-2,2);
    Yout=zeros(N-2,1);
    Yout=y(3:N);
    pha(:,1)=y(2:N-1);
    pha(:,2)=y(1:N-2);
    theta=inv(pha'*pha)*pha'*Yout;
    %mean square error
    accuracy(N/1000)=immse(theta,[a1;a2]);
end
figure
scatter(1000:1000:3000,accuracy)
title("Accuracy of the LS  estimate with increasing data length");

% Non-asymptotically stationary
%When the model is not asymptotically stationary, y(N) will go to
%infinite.Infinite values cannot be calculated here.
```



q4

```
%(a)
N=1000;
theta(1)=1;
eplist=[0.1,0.01,0.001];
for ep=eplist
    for k=1:N
        pha=normrnd(0,1);
        v=normrnd(0,1);
        wk=normrnd(0,1);
        theta(k+1)=theta(k)+ep*wk;
    end
    plot(1:N,theta(1:N));
    hold on
end
legend('ep=0.1','ep=0.01','ep=0.001')
```



(b)

```
N=1000;
pha=zeros(N,1);
theta=zeros(N,1);
v=zeros(N,1);
y=zeros(N,1);
theta(1)=0.6;
ep=0.0001;
for k=1:N
        pha(k)=normrnd(0,1);
        v(k)=normrnd(0,1);
        wk(k)=normrnd(0,1);
        theta(k+1)=theta(k)+ep*wk(k);
        y(k)=pha(k)*theta(k)+v(k);
end
% RLS
theta_est(1)=1;
rou=0.999;
p=zeros(N,1);
p(1)=0.1;
for k=1:N-1
    alpha=rou^(N-k-1);
    theta_est(k+1)=theta_est(k)+(p(k)*pha(k+1))/(1/alpha+pha(k
+1)'*p(k)*pha(k+1))*(y(k+1)-pha(k+1)'*theta_est(k));
```

```
    p(k+1)=p(k)-(p(k)*pha(k+1)*pha(k+1)'*p(k))/(1/alpha+pha(k
+1)'*p(k)*pha(k+1));
end
theta(N)
theta_est(N)


ans =

    0.5945


ans =

    0.6049
```

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