物理学A　2章演習　解説

1.1

$m\ddot{\boldsymbol{r}}=\boldsymbol{F}$ より

$$\begin{cases} m\ddot{x}=0 \cdots\cdots ① \\ m\ddot{y}=-A\sin\omega t \cdots\cdots ② \end{cases}$$

①より　$\dot{x}=C$

$x=Ct+D$

$t=0$ で $x=D=0$

$\dot{x}=C=v_0$

$\therefore x=v_0 t$

②より　$\dot{y}=-\dfrac{A}{m}\int \sin\omega t dt=\dfrac{A}{m\omega}\cos\omega t+C'$

$$y=\int\left(\frac{A}{m\omega}\cos\omega t+C'\right)dt=\frac{A}{m\omega}\sin\omega t+C't+D'$$

$t=0$ で $y=D'=0$

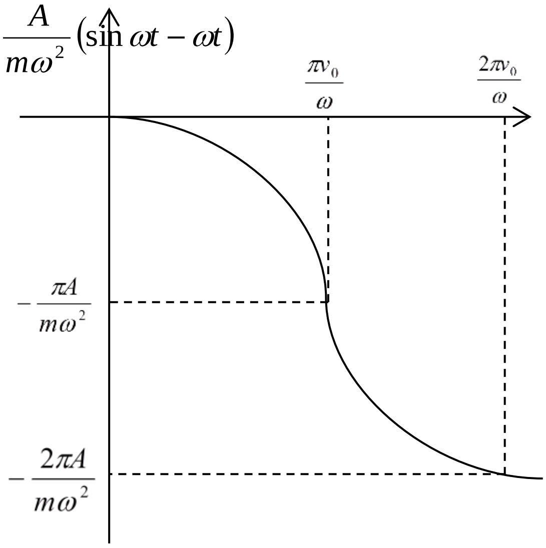
$\dot{y}=\dfrac{A}{m\omega}+C'=0$　　　$\therefore C'=-\dfrac{A}{m\omega}$

$\therefore y=\dfrac{A}{m\omega^2}(\sin\omega t-\omega t)$

よって,

$$\boldsymbol{r}=\left(v_0 t, \frac{A}{m\omega^2}(\sin\omega t-\omega t)\right)$$

$$\boldsymbol{v}=\left(v_0, \frac{A}{m\omega}(\cos\omega t-1)\right)$$



奇跡は t を消去して

$$y=\frac{A}{m\omega^2}\left(\sin\frac{\omega x}{v_0}-\frac{\omega x}{v_0}\right)$$

$$y'=\frac{A}{m\omega v_0}\left(\cos\frac{\omega x}{v_0}-1\right)\leq 0$$

単調減少　$\dfrac{\omega x}{v_0}=2\pi n$ で y'＝0

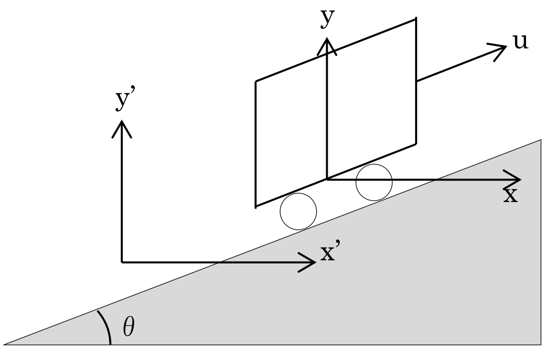
$$y''=-\frac{A}{m v_0{}^2}\sin\frac{\omega x}{v_0}$$

x	・・・	0	・・・	$\frac{\pi v_0}{\omega}$	・・・	$\frac{2\pi v_0}{\omega}$	・・・
y'	−	0	−		−	0	−
y''	＋	0	−	0	＋	0	−
y	↓→	0	→↓	$-\frac{\pi A}{m\omega^2}$	↓→	$-\frac{2\pi A}{m\omega^2}$	→↓

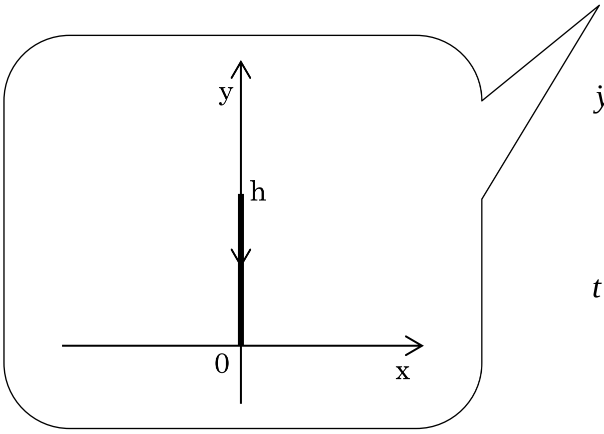
（別解）

$Y=\dfrac{\omega x^2}{A}y, X=\dfrac{\omega x}{v_0}$ とおくと，　$Y=\sin X-X$

2.2



（静止座標系）



S 座標

(i) $\begin{cases} m\ddot{x} = 0 \\ m\ddot{y} = -mg \end{cases}$

(ii) $t = 0$ で，$\dot{x} = 0$

$\dot{y} = 0$ →静かに落としたから.

(iii) $\ddot{x} = 0$ より，$\dot{x} = Cx$

$x = C_x t + D_x$

$t = 0$ で，$x = D_x = 0$

$\dot{x} = Cx = 0 \qquad \therefore x = 0$

$\ddot{y} = -g$ より，$\dot{y} = -gt + C_y$

$$y = -\frac{1}{2}gt^2 + C_y t + D_y$$

$t = 0$ で，$y = D_y = h$

$\dot{y} = C_y = 0 \qquad \therefore y = -\frac{1}{2}gt^2 + h$

S'座標系

(i) $\begin{cases} m\ddot{x} = 0 \\ m\ddot{y} = -mg \end{cases}$

(ii) $t = 0$ で，$\dot{x}' = u\cos\theta$

$\dot{y}' = u\sin\theta$

(iii) $\ddot{x} = 0$ より，$\dot{x}' = C'x$

$x' = C'_x t + D'_x$

$t = 0$ で，$x' = D'_x = 0$

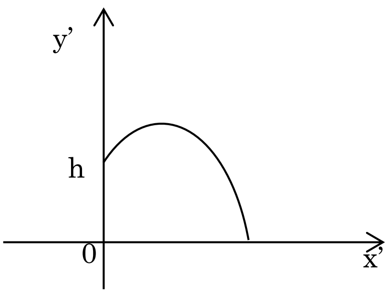
$\dot{x}' = C'x = u\cos\theta \qquad \therefore x' = (u\cos\theta)t$

$\ddot{y}' = -g$ より，$\dot{y}' = -gt + C'_y$

$$y' = -\frac{1}{2}gt^2 + C_y{}' t + D'_y$$

$t = 0$ で，$y' = D_y{}' = h$

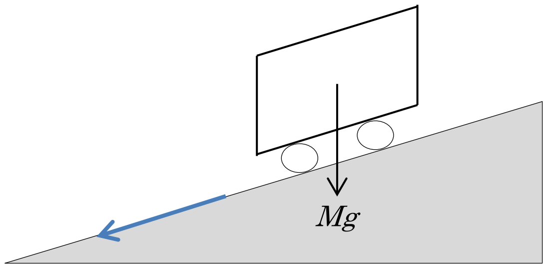
$\dot{y}' = C'_y = u\sin\theta \qquad \therefore y' = -\frac{1}{2}gt^2 + (u\sin\theta)t + h$



軌跡は

$$y' = \frac{1}{2}g\left(\frac{x'}{u\cos\theta}\right)^2 + (\tan\theta)x' + h$$

2.3



ケーブルカーの加速度は斜面に沿って

$$M\ddot{X} = Mg\sin\theta$$

$$\ddot{X} = g\sin\theta$$

$$\therefore \boldsymbol{a} = \left(-g\sin\theta\cos\theta, -g\sin^2\theta\right)$$

(i)慣性力は

$$-m\boldsymbol{a} = mg\left(\sin\theta\cos\theta, \sin^2\theta\right)$$

(ii) $\begin{cases} m\ddot{x} = mg\sin\theta\cos\theta\cdots\cdots① \\ m\ddot{y} = -mg + mg\sin^2\theta\cdots\cdots② \end{cases}$

$$= -mg\left(1-\sin^2\theta\right) = -mg\cos^2\theta$$

(iii)①より

$$\dot{x} = \left(g\sin\theta\cos\theta\right)t + C_x$$

$$x = \frac{1}{2}\left(g\sin\theta\cos\theta\right)t^2 + C_x t + D_x$$

$t = 0$ で　　$x = D_x = 0$

$\dot{x} = C_x = 0$　　　$\therefore x = \frac{1}{2}\left(g\sin\theta\cos\theta\right)t^2$

②より

$$\dot{y} = -\left(g\cos^2\theta\right)t + C_y$$

$$y = -\frac{1}{2}\left(g\cos^2\theta\right)t + C_y t + D_y$$

$t = 0$ で　　$y = D_y = h$

$\dot{y} = C_y = 0$　　　$\therefore y = -\frac{1}{2}\left(g\cos^2\theta\right)t^2 + h$

軌跡は

$$y = -\frac{\cos\theta}{\sin\theta}x + h = -\frac{x}{\tan\theta} + h$$