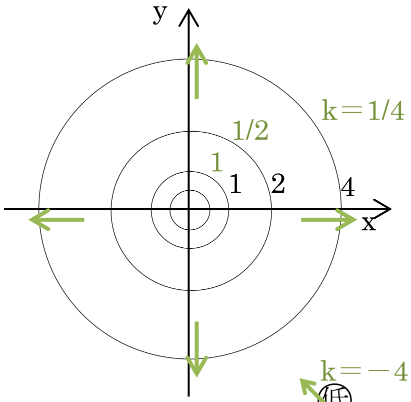
物理学 A　4 章演習　解説

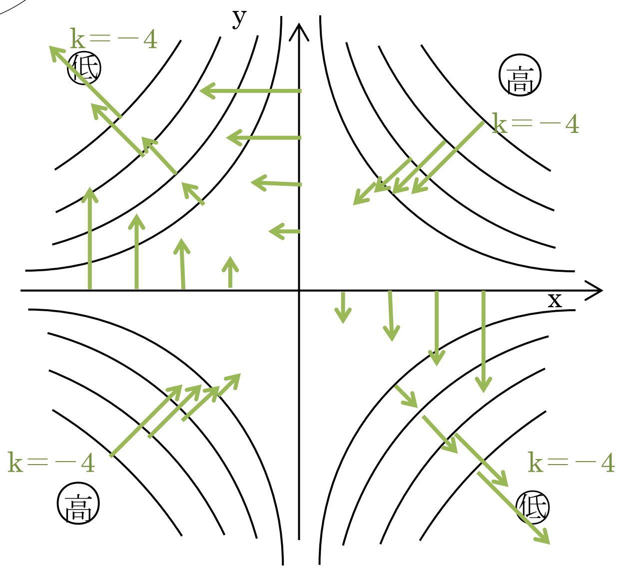
4.1

1)　等高線



$$U_1=\frac{1}{r}=k \text{ より}$$

$$x^2+y^2=\frac{1}{k^2}$$



$$U_2=xy=k \text{ より}$$

$$y=\frac{k}{x}$$

(2)　$\mathbf{F}_1=-\nabla U_1$

$$F_1x=-\frac{\partial}{\partial x}\frac{1}{r}=\frac{1}{r^2}\frac{\partial r}{\partial x}=\frac{1}{r^2}\frac{x}{r}$$

$$F_2y=-\frac{\partial}{\partial y}\frac{1}{r}=\frac{1}{r^2}\frac{\partial r}{\partial y}=\frac{1}{r^2}\frac{y}{r}$$

$$\therefore \mathbf{F}=\frac{1}{r^2}\left(\frac{x}{r},\frac{y}{r}\right)=\frac{1}{r^2}\cdot\frac{r}{r}$$

$$\mathbf{F}_2=-\nabla U_2=-\left(\frac{\partial(xy)}{\partial x},\frac{\partial(xy)}{\partial y}\right)=(-y,-x)$$

(3)　力はポテンシャルの等高線に垂直

U が高い所から低い所へ向かう.

4.2

(1)　(A)のとき　　　　$\boxed{W=\int \mathbf{F}\cdot d\mathbf{r}}$

$$W_{(I)}=\int_0^1 F_x(x,0)dx+\int_1^2 F_y(1,y)dy$$

←直線 y=0 上　←x=1 上

$$=\int_0^1 x\cdot 0dx+\int_0^2 \frac{1}{2}dy=0+1=1$$

$$W_{(II)}=\int_{(0,0)}^{(1,2)}\left(F_xdx+F_ydy\right)=\int_0^1 F_x(x,2x)dx+\int_0^2 F_y\left(\frac{y}{2},y\right)dy$$

$$=\int_0^1 2x^2dx+\int_0^2 \frac{1}{2}\left(\frac{y}{2}\right)^2 dy=\frac{2}{3}+\frac{1}{3}=1$$

$$W_{(III)}=\int_0^2 F_y(0,y)dy+\int_0^2 F_x(x,2)dx$$

$$=\int_0^2 \frac{0}{2}dy+\int_0^1 2xdx=0+1=1$$

(B)のとき

$$W_{(I)}=\frac{13}{6},\qquad W_{(II)}=\frac{3}{2},\qquad W_{(III)}=\frac{1}{6}$$

(2)　(B) $W_{(I)}\neq W_{(II)}$ より保存力である.

(A) $W_{(I)}=W_{(II)}=W_{(III)}$ より

保存力の可能性がある.

$\rightarrow \dfrac{\partial F_y}{\partial x}-\dfrac{\partial F_x}{\partial y}=0$ を調べる.

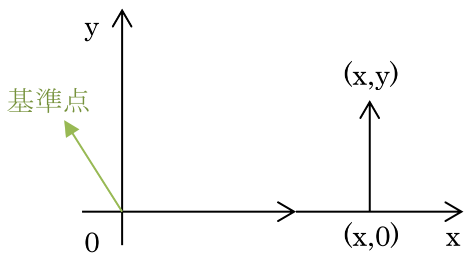
$$\frac{\partial F_y}{\partial x}-\frac{\partial F_x}{\partial y}=\frac{\partial}{\partial x}\left(\frac{x^2}{2}\right)-\frac{\partial}{\partial y}(xy)=x-x=0$$

よって(A)は保存力.

(3)　$U(x,y)=-\int_{(0,0)}^{(x,y)}$ **Fdr**

(A)に対して $U(x,y)=-\int_0^x F_x(x',0)dx'-\int_0^y F_y(x,y')dy'$

$$=-\int_0^x x'\cdot 0dx'-\int_0^y \frac{x^2}{2}dy=-\frac{x^2y}{2}$$



解きやすい経路で！

※U を出したら検算をすること

$$\rightarrow -\frac{\partial U}{\partial x}=\frac{\partial}{\partial x}\left(\frac{x^2y}{2}\right)=xy=F_x$$

$$-\frac{\partial U}{\partial y}=\frac{\partial}{\partial y}\left(\frac{x^2y}{2}\right)=\frac{x^2}{2}=F_y$$

補足

(A) $F_x = xy$,　$F_y = \frac{1}{2}x^2$

保存力$\left(\frac{\partial F_y}{\partial x} - \frac{\partial F_x}{\partial y} = 0\right)$のとき,

$$U_{(x,y)} = -\int_0^r F(r')dr'$$

$$= -\left(\int_0^2 F_x(x',0)dx' + \int_0^y F_y(x,y')dy'\right)$$

$$= 0 - \int_0^y \frac{x^2}{2}dy = -\frac{1}{2}x^2 y$$

別解

$$\begin{cases} -\frac{\partial U}{\partial x} = F_x = xy \cdots ① \\ -\frac{\partial U}{\partial y} = F_y = \frac{1}{2}x^2 \cdots ② \end{cases}$$

①を x で積分

$$U = -\int xydx = -\frac{1}{2}x^2 y + F(y) \cdots ①'$$

↑y の任意の関数

①'を②に代入

$$\frac{1}{2}x^2 - F'(y) = \frac{1}{2}x^2$$

$$\therefore F'(x) = 0$$

$$F(x) = C$$

$$\therefore U = -\frac{1}{2}x^2 y + C$$

$U_{(0,0)} = C = 0$ より,　$\underline{U = -\frac{1}{2}x^2 y}$