EXERCISE 2.2

Prove the following:

1. $3\sin^{-1}x = \sin^{-1}(3x - 4x^3), \quad x \in \left[\frac{-1}{2}, \frac{1}{2}\right]$
2. $3\cos^{-1}x = \cos^{-1}(4x^3 - 3x), \quad x \in \left[\frac{1}{2}, 1\right]$
3. $\tan^{-1}\frac{2}{11} + \tan^{-1}\frac{7}{24} = \tan^{-1}\frac{1}{2}$
4. $2\tan^{-1}\frac{1}{2} + \tan^{-1}\frac{1}{7} = \tan^{-1}\frac{31}{17}$

Write the following functions in the simplest form:

5 $\tan^{-1}\left(\frac{\sqrt{1+x^2}-1}{x}\right), \quad x \neq 0$

6 $\tan^{-1}\left(\frac{1}{\sqrt{x^2-1}}\right), \quad |x| > 1$

7 $\tan^{-1}\left(\sqrt{\frac{1-\cos(x)}{1+\cos(x)}}\right), \quad 0 < x < \pi$

8 $\tan^{-1}\left(\frac{\cos(x)-\sin(x)}{\cos(x)+\sin(x)}\right), \quad -\frac{\pi}{4} < x < \frac{3\pi}{4}$

9 $\tan^{-1}\left(\frac{x}{\sqrt{a^2-x^2}}\right), \quad |x| < a$

10 $\tan^{-1}\left(\frac{3a^2x-x^3}{a^3-3ax^2}\right), \quad a > 0, \quad -\frac{a}{\sqrt{3}} < x < \frac{a}{\sqrt{3}}$

Find the values of each of the following:

11 $\tan^{-1}\left[2\cos\left(2\sin^{-1}\left(\frac{1}{2}\right)\right)\right]$

12 $\sin\left(\sin^{-1}\left(\frac{1}{5}\right) + \cos^{-1}(x)\right) = 1$, find x

13 $\tan\left(\frac{1}{2}\right)\left[\sin^{-1}\left(\frac{2x}{1+x^2}\right) + \cos^{-1}\left(\frac{1-y^2}{1+y^2}\right)\right], \ |x| < 1, \ y > 0, \ xy < 1$

14 $\sin\left(\sin^{-1}\left(\frac{1}{5}\right) + \cos^{-1}(x)\right) = 1$, find x

15 $\tan^{-1}\left(\frac{x-1}{x-2}\right) + \tan^{-1}\left(\frac{x+1}{x-2}\right) = \frac{\pi}{4}$, find x

Find the values of each of the expressions in Exercises 16 to 18.

16 $\sin^{-1}\left(\sin\frac{2\pi}{3}\right)$

17 $\tan^{-1}\tan\frac{3\pi}{4}$

18 $\tan\left(\sin^{-1}\frac{3}{5} + \cos^{-1}\frac{7}{25}\right)$

19 $\cos^{-1}\left(\cos\frac{7\pi}{6}\right)$ is equal to:

(A) $\frac{7\pi}{6}$ (B) $\frac{5\pi}{6}$ (C) $\frac{\pi}{3}$ (D) $\frac{\pi}{6}$

20 $\sin\left(\frac{3\pi}{2} - \sin^{-1}\frac{1}{2}\right)$ is equal to:

(A) $\frac{7\pi}{6}$ (B) $\frac{5\pi}{6}$ (C) $\frac{\pi}{3}$ (D) π

21 $\tan^{-1}\sqrt{3} - \cos^{-1}(\sqrt{3})$ is equal to:

(A) π (B) $\frac{\pi}{2}$ (C) 0 (D) $2\sqrt{3}$