Trading Strategy Back-Tester and Bot Trader Creation

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1 Introduction

This **Warwick Coding Society** report outlines the theory and the creation of a **trading strategy back-testing program and a bot trader** using the **Python** programming language. This document's approach to presenting the theory involves giving a simple explanation of the relevant topics before introducing the mathematics for the topics. The reader can also follow the code shown throughout the report to practice building their own trading toolkit, which is designed to be flexible.

The topics covered in this report include:

- What is **trading** and **arbitrage**?
- What is a **trading strategy** and how use it?
- What is **portfolio optimisation** and how it's done?

The report will also cover existing back testing and portfolio optimisation Python packages like **Backtrader** and **PyPortfolioOpt**.

The motivation for this report is to introduce programmers of any skill level to algorithmic trading and back testing in a clear, concise manner, in line with Warwick Coding Society's goal of making programming accessible. This report is also part of a collaboration project with the WFS Quant Finance Team. This WCS report was made to perfectly complement the WFS report on 'trading strategy comparisons', as one can replicate the results in that report with the tools they learn in this one.

2 Trading Basics

2.1 What is Trading?

Trading is the exchange of value between two entities. A **market** is a field where entities trade. The most active markets in the world today comprise of financial asset markets, including the **New York Stock Exchange (NYSE)** and the **National Association of Securities Dealers Automated Quotations (NASDAQ)**.

Financial asset markets are ever-trending due to their increasing accessibility to the general public and the ease with which one can trade financial assets. For this reason, this report will be focusing on financial asset markets only, though the methodology presented here is applicable to any market.

With the progress of computer technology and financial mathematics in the late 20th century, companies, like **Renaissance Technologies**, investigated the use of algorithmic trading strategies to automate and optimise their trading activity in financial asset markets. Currently, anyone can access financial markets through the many internet brokers that exist, opening the opportunity for anyone to use algorithmic trading methods to automate their trading activity.

There are two common ways to earn profit from trading in financial asset markets.

- **Buying (Long Position)**: When one buys an asset on the market at an agreed price and time. Then, they sell the asset on the market when the it increases in price, thus profiting from the increase in the price.
- **Selling (Short Position)**: When one loans assets from the broker and sells them on the market at an agreed price and time. Then, they buy back the asset when it decreases in price and return it to the broker, thus profiting from the decrease in the price.

For more details on how brokers, financial markets and assets work, the reader can resort to the resources mentioned in this report's bibliography.

2.2 What is Arbitrage?

Arbitrage involves exploiting inefficiencies across markets to trade more profitably.

An **arbitrage opportunity** is characterised by the potential of investing a net zero worth of capital into assets and profiting from them without risk. Simply put, an arbitrage opportunity can give something from nothing without loss.

Arbitrage opportunities are found using information that efficiently reflect the current demand for different assets, allowing one to essentially predict what asset will be profitable. Again, arbitrage is a general concept that is applicable to any set of markets.

The common methods of arbitrage opportunity detection include:

- Multivariate trend analysis
- Internet social trend analysis

This is by no means an exhaustive list of arbitrage opportunity detection methods.

3 Trading Strategy Utilisation Theory

A **trading strategy** is a rule for when and how to trade an asset over time given a set of information. We start by outlining what we expect from a trading strategy.

3.1 The Characteristics of a Good Trading Strategy

3.1.1 General Description

The aim of trade is to achieve **profit**. One can trade in the financial market by either opening long or short positions on financial assets.

Long positions are profitable when assets are bought at low prices and sold at higher prices. **Short positions** are profitable when assets are sold at high prices and bought at lower prices.

Therefore, a good trading strategy:

- Opens and holds long positions while the asset price increases
- Opens and holds short positions while the asset price decreases
- Closes long positions when the asset price starts decreasing
- Closes short positions when the asset price starts increasing

3.1.2 Mathematical Preliminaries

Define the set of times for which the price data of a given asset is observed as the countable set $T \subset \mathbb{R}$.

Brokers and financial websites provide data on the historical prices of assets, including the close, open, low and high prices of assets for different time intervals. Define the close, open, low and high prices as follows:

- Close price series: $P_c : T \mapsto \mathbb{R}^+$
- Open price series: $P_o : T \mapsto \mathbb{R}^+$
- Low price series: $P_l : T \mapsto \mathbb{R}^+$
- High price series: $P_h : T \mapsto \mathbb{R}^+$

These series can be used for the construction of **indicator series**.

3.2 Indicator Series

3.2.1 General Description

A trading strategy involves using information about the asset price trend. This information is usually in the form of a time series, known as an **indicator series**, that shows when the price asset is increasing or decreasing.

An indicator series is:

- Positive when the price is increasing
- Negative when the price is decreasing
- Zero when the price is stable

3.2.2 Mathematical Description

Define an indicator time series $I : T \mapsto \mathbb{R}$ as a function of time.

An example of a trivial indicator series is the difference between the close and open prices of the asset traded:

$$I(t) = P_c(t) - P_o(t)$$

Another example of a trivial indicator series involves using the first order difference of the closed prices:

$$I(t) = P_c(t) - P_c(t - \Delta_t)$$

Where Δ_t is the difference between time t and the latest time before t in set T, or, more precisely:

$$\Delta_t = t - \sup\left(T \cap (\infty, t)\right)$$

Deriving reliable indicators is a separate science that quant traders specialise in. The WFS report on trading strategy comparisons gives some insight into the most commonly used indicators to date.

3.3 Trade Signal Series

3.3.1 General Description

A **trade signal series** gives information on when a given trade type should be open according to an underlying indicator series.

The long trade signal series is:

- Equal to one when the indicator is strictly positive
- Equal to zero otherwise

The short trade signal series is:

- Equal to one when the indicator is strictly negative
- Equal to zero otherwise

3.3.2 Mathematical Description

The **long trade signal series** $V_l : T \mapsto \{0, 1\}$ is defined as:

$$V_l(t) = \text{sign}(\max\{I(t), 0\})$$

The **short trade signal series** $V_s(t)$ is trivially derived by changing the sign of the indicator.

$$V_s(t) = \text{sign}(\max\{-I(t), 0\})$$

3.4 Action Series

3.4.1 General Description

An **action series** informs exactly when to open or close trades of a given type based on its underlying trade signal series.

The action series is:

- Equal to one when the trade must be opened
- Equal to zero when the position must be held (inaction)
- Equal to minus one when the trade must be closed

3.4.2 Mathematical Description

The **action series** $A : T \mapsto \{-1, 0, 1\}$, for a given trade signal series $V(t)$, is defined as:

$$A(t) = \begin{cases} V(t) & \text{if } t = \inf T \\ V(t) - V(t - \Delta_t) & \text{if } t > \inf T \end{cases}$$

3.5 Alpha Series

Finally, the **alpha series** shows the ratio of the value of the investment with its previous value at all times under the use of a trading strategy.

When the trade is inactive, the alpha series will take the value one, since no capital is invested in the asset. When the trade is active, the alpha series will output the ratio change in the investment according to the price series.

The alpha series of a given asset and trading strategy is what's used to evaluate the performance of the trading strategy for the asset.

3.5.1 Mathematical Description

The alpha series $\alpha : T \mapsto \mathbb{R}^+$, for a given trade signal series $V(t)$ and action series $A(t)$, is generated using the following algorithm:

Require: $T \wedge V(t) \wedge A(t) \wedge P_c(t) \wedge r$
Ensure: $\alpha(t)$

for $t = \inf T$ **to** $\sup T$ **do**
  if $A(t) = 1$ **then**
    $\alpha(t) \leftarrow 1$
    $P_0 \leftarrow P_c(t)$
  else if $A(t) = 0$ **then**
    if $V(t) = 0$ **then**
      $\alpha(t) \leftarrow 1$
    else if $V(t) = 1$ **then**
      $\alpha(t) \leftarrow (P_0 + r \cdot (P_c(t) - P_0))/(P_0 + r \cdot (P_c(t - \Delta_t) - P_0))$
    end if
  else if $A(t) = -1$ **then**
    $\alpha(t) \leftarrow (P_0 + r \cdot (P_c(t) - P_0))/(P_0 + r \cdot (P_c(t - \Delta_t) - P_0))$
  end if
end for

The r term is the proportion of invested capital dedicated to the trade. A positive r generates an alpha series for a long position strategy and a negative r generates an alpha series for a short position strategy.

4 Portfolio Implementation Theory

4.1 Why are Portfolios Important?

A **trading portfolio** is an allocation of capital dedicated to a set of assets for trade. Portfolios can significantly affect the efficiency of one's trading activity. Dedicating a more capital into assets with volatile prices may be irrational if there exist more stable assets with higher average returns.

The **volatility** of a random variable is its degree of variation. An asset price that deviate more from its trend that anther asset price is more volatile than the other. Usually, portfolios are optimised according to the volatility and average returns of the asset prices.

4.2 Portfolio Distribution Series

Since a portfolio is an allocation of capital, it is expressed as a vector function called a **portfolio distribution series** $\pi : T \mapsto [0,1]^n$ with entries summing to one. The ith entry of $\pi(t)$, is the proportion $\pi_i(t)$ of capital dedicated towards asset $i \in I$ for trade at time $t \in T$, where I is the index set for the assets.

Setting $\alpha_i(t)$ to be the alpha series of the ith asset, define the portfolio alpha series as follows:

$$\alpha_\pi(t) = \textstyle\sum_{i\in I} \alpha_i(t)\pi_i(t)$$

The **portfolio alpha series** is shows the ratio of the value of the total invested capital with its previous value at all times.

4.3 The Equal Weight Portfolio

The **equal weight portfolio**, or the **trivial portfolio**, is one that uniformly allocates capital across all assets that are being traded. Letting $V_i(t)$ be the trade signal series of asset $i \in I$, the trivial portfolio is such that:

$$\pi_i(t) = \begin{cases} \dfrac{V_i(t)}{\sum_{i\in I} V_i(t)} & \text{if } \sum_{i\in I} V_i(t) \neq 0 \\ 0 & \text{if } \sum_{i\in I} V_i(t) = 0 \end{cases}$$

4.4 Markowitz Modern Portfolio Theory

Markowitz MPT (Modern Portfolio Theory) one of the most successful and useful portfolio optimisation methods created and used to date. MPT optimises a portfolio by maximising its average returns and minimising its variance, used as a measure of volatility in this case. It utilises information on the returns of individual assets, including their average returns and return covariances.

Let $\mu(t)$ be the **vector of average returns** of all assets in I at time $t \in T$. So, $\mu_i(t)$ is the average returns of asset i at time t.

Let $\Sigma(t)$ be the **matrix of covariances between returns** of all assets in I at time $t \in T$. So, $\Sigma_{ij}(t)$ is the covariance between the returns of asset i and j at time t.

Let $d(t)$ be the **wealth distribution**, showing the proportion of wealth to be invested in each asset. The wealth distribution is different to the portfolio distribution $\pi(t)$, since the entries of $d(t)$ can be any real value all summing to one. $d_i(t)$ is the wealth distributed to asset i at time t.

The entries of the portfolio distribution are derived from the wealth distribution as follows:

$$\pi_i(t) = \frac{|d_i(t)|}{\sum_{i\in I} |d_i(t)|}$$

There exist two special wealth distributions.

- The **minimal variance risky wealth distribution**:

$$d_M(t) = \frac{\Sigma^{-1}(t) \cdot \mathbf{1}}{\mathbf{1}^T \cdot \Sigma^{-1}(t) \cdot \mathbf{1}}$$

  With its corresponding average return $d_M(t) \cdot \mu(t) = \mu_M(t)$.

- The **zero rate tangency wealth distribution**:

$$d_T(t) = \frac{\Sigma^{-1}(t) \cdot \mu(t)}{\mathbf{1}^T \cdot \Sigma^{-1}(t) \cdot \mu(t)}$$

  With its corresponding average return $d_T(t) \cdot \mu(t) = \mu_T(t)$.

The **optimal wealth distribution** is derived by solving the following optimisation problem for some desired average return $\mu_0(t)$:

$$\begin{array}{cc} \text{minimise} & d(t)^T \cdot \Sigma(t) \cdot d(t), \quad d(t) \in \mathbb{R}^n \\ \text{subject to} & d(t) \cdot \mathbf{1} = 1, \\ & d(t) \cdot \mu(t) \geq \mu_0(t) \end{array}$$

The problem involves deriving the wealth distribution that minimises the variance of the total returns with a set of necessary constraints. The unique solution to the problem is given by:

$$d_0(t) = \frac{\mu_T(t) - \mu_0(t)}{\mu_T(t) - \mu_M(t)} d_M(t) + \frac{\mu_0(t) - \mu_M(t)}{\mu_T(t) - \mu_M(t)} d_T(t)$$

Therefore, it is best to use $d_0(t)$ for a desired average returns of $\mu_0(t)$.

5 Back Testing and Trading Bot Implementation