Folha 3A - Primitivas por partes e de potências de funções trigonométricas.

1. Partes

a) $P(x\underset{\uparrow}{e^{-5x}}) = -\frac{1}{5}e^{-5x}x + P\left(\frac{1}{5}e^{-5x}\right)$

$= -\frac{x}{5}e^{-5x} - \frac{1}{25}e^{-5x} + c$

b) $P(x^3 e^{3x^2}) = P(x^2 \underbrace{x e^{3x^2}}_{\uparrow})$

$= \frac{1}{6}e^{3x^2}x^2 - P\left(\frac{1}{6}e^{3x^2}2x\right)$

$= \frac{x^2}{6}e^{3x^2} - P\left(\frac{x}{3}e^{3x^2}\right)$

$= \frac{x^2}{6}e^{3x^2} - \frac{1}{18}e^{3x^2} + C$

c) $P\left(\ln\frac{1}{x}\right) = P\left(\underset{\uparrow}{1}\ln\frac{1}{x}\right)$

$= x\ln\frac{1}{x} - P\left(x\,\frac{-\frac{1}{x^2}}{\frac{1}{x}}\right)$

$= x\ln\frac{1}{x} + P\left(x\,\frac{x}{x^2}\right)$

$= x\ln\frac{1}{x} + x + C$

d) $P\left(\ln(5+x)\right) = P\left(\underset{\uparrow}{1}\ln(5+x)\right)$

$= x\ln(5+x) - P\left(x\,\frac{1}{5+x}\right)$

$= x\ln(5+x) - P\left(\frac{x+5-5}{5+x}\right)$

$= x\ln(5+x) - P\left(1 - \frac{5}{5+x}\right)$

$= x\ln(5+x) - x + 5\ln(5+x) + C$

e) $P(\text{arcsen}\,x) = P\left(\underset{\uparrow}{1}\,\text{arcsen}\,x\right)$

$= x\,\text{arcsen}\,x - P\left(x\,\frac{1}{\sqrt{1-x^2}}\right)$

$= x\,\text{arcsen}\,x - P\left(x\,(1-x^2)^{-1/2}\right)$

$= x\,\text{arcsen}\,x + \frac{1}{2}\,(1-x^2)^{1/2}\cdot 2 + C$

$= x\,\text{arcsen}\,x + \sqrt{1-x^2} + C$

f) $P(x\,\underset{\uparrow}{\sec^2}x) = \text{tg}\,x\;x - P(\text{tg}\,x)$

$= x\,\text{tg}\,x + \ln|\cos x| + C$

g) $P(\text{arctg}\,x) = P\left(\underset{\uparrow}{1}\,\text{arctg}\,x\right)$

$= x\,\text{arctg}\,x - P\left(x\,\frac{1}{1+x^2}\right)$

$= x\,\text{arctg}\,x - \frac{1}{2}\ln(1+x^2) + C$

h) exercício extra

$$P(\operatorname{ch}t \operatorname{sen}3t) = \operatorname{sh}t \operatorname{sen}3t - P(\operatorname{sh}t \, 3\cos 3t)$$

$$= \operatorname{sh}t \operatorname{sen}3t - 3P(\operatorname{sh}t \cos 3t)$$

$$= \operatorname{sh}t \operatorname{sen}3t - 3\left[\operatorname{ch}t \cos 3t + P(\operatorname{ch}t \cdot 3\operatorname{sen}3t)\right]$$

$$= \operatorname{sh}t \operatorname{sen}3t - 3\operatorname{ch}t \cos 3t - 9P(\operatorname{ch}t \operatorname{sen}3t)$$

primitiva inicial

Então

$$10P(\operatorname{ch}t \operatorname{sen}3t) = \operatorname{sh}t \operatorname{sen}3t - 3\operatorname{ch}t \cos 3t + C$$

$$\Rightarrow P(\operatorname{ch}t \operatorname{sen}3t) = \frac{1}{10}\operatorname{sh}t \operatorname{sen}3t - \frac{3}{10}\operatorname{ch}t \cos 3t + C$$

2. Potências de funções trigonométricas

a) $$P(\operatorname{sen}^2 x) = P\left(\frac{1-\cos 2x}{2}\right)$$

$$= \frac{1}{2} P(1-\cos 2x)$$

$$= \frac{1}{2}\left(x - \frac{1}{2}\operatorname{sen} 2x\right)$$

$$= \frac{x}{2} - \frac{1}{4}\operatorname{sen} 2x + C$$

$$= \frac{x}{2} - \frac{1}{2}\operatorname{sen} x \cos x + C$$

b) $$P(\cos^3 x) = P\left(\cos x\,(1-\operatorname{sen}^2 x)\right)$$

$$= P(\cos x) - P(\cos x \operatorname{sen}^2 x)$$

$$= \operatorname{sen} x - \frac{1}{3}\operatorname{sen}^3 x + C$$

c) $$P(\operatorname{sen}^4 x) = P\left((\operatorname{sen}^2 x)^2\right)$$

$$= P\left(\left(\frac{1-\cos 2x}{2}\right)^2\right)$$

$$= \frac{1}{4} P\left(1 - 2\cos 2x + \cos^2 2x\right)$$

$$= \frac{1}{4}\left(x - \operatorname{sen} 2x\right) + \frac{1}{4} P\left(\frac{1+\cos 4x}{2}\right)$$

$$= \frac{x}{4} - \frac{1}{4} \operatorname{sen} 2x + \frac{1}{8}\left(x + \frac{1}{4} \operatorname{sen} 4x\right) + C$$

$$= \frac{x}{4} - \frac{1}{4} \operatorname{sen} 2x + \frac{x}{8} + \frac{1}{32} \operatorname{sen} 4x + C$$