



INSTITUTO SUPERIOR DE ENGENHARIA DE COIMBRA

DEPARTAMENTO DE FÍSICA E MATEMÁTICA

ENGENHARIA BIOMÉDICA – 1º ano /1º Semestre

19-Out-2010

Duração:30'

TESTE 1

1- Considere a função $f(x) = 1 + 2\text{sen}(3x + \frac{\pi}{6})$:

- a) Determine o domínio e o contradomínio de f .
- b) Resolva a equação $f(x) > 2$.
- c) Caracterize f^{-1} .
- d) Determine $f^{-1}(0)$.

2- Considere a equação $x^3 + y^3 = 4xy + 1$ que define implicitamente y como função de x .

- a) Determine y' .
- b) Obtenha a equação da recta tangente à curva $y = f(x)$ no ponto de coordenadas $(2,1)$.

Resolução do TESTE 1

$$1. f(x) = 1 + 2 \sin\left(3x + \frac{\pi}{6}\right)$$

$$a) Df = \left\{ x \in \mathbb{R} : 3x + \frac{\pi}{6} \in \mathbb{R} \right\} \\ = \mathbb{R}$$

$$CDf = [-1, 3]$$

$$y = 1 + 2 \sin\left(3x + \frac{\pi}{6}\right)$$

$$-1 \leq \frac{y-1}{2} = \sin\left(3x + \frac{\pi}{6}\right) \leq 1$$

$$-2 \leq y-1 \leq 2$$

$$-1 \leq y \leq 3$$

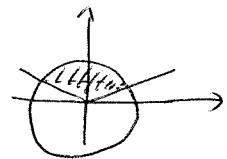
$$b) f(x) > 2$$

$$1 + 2 \sin\left(3x + \frac{\pi}{6}\right) > 2$$

$$2 \sin\left(3x + \frac{\pi}{6}\right) > 1$$

$$\sin\left(3x + \frac{\pi}{6}\right) > \frac{1}{2}$$

$$\sin\left(3x + \frac{\pi}{6}\right) > \sin \frac{\pi}{6}$$



$$2k\pi + \frac{\pi}{6} < 3x + \frac{\pi}{6} < \pi - \frac{\pi}{6} + 2k\pi$$

$$2k\pi + 0 < 3x < \frac{5\pi}{6} - \frac{\pi}{6} + 2k\pi$$

$$2k\pi + 0 < 3x < \frac{4\pi}{6} + 2k\pi$$

$$\frac{2k\pi}{3} + 0 < x < \frac{4\pi}{18} + \frac{2k\pi}{3}, \quad k \in \mathbb{Z}$$

$$c) f^{-1}:$$

$$Df^{-1} = \text{CD}f =$$

$$= \left\{ y \in \mathbb{R} : \frac{y-1}{2} \in [-1, 1] \right\}$$

$$= [-1, 3]$$

$$\text{CD}f^{-1} = Df \text{ (restrição principal)}$$

$$= \left\{ x \in \mathbb{R} : 3x + \frac{\pi}{6} \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right] \right\}$$

$$\text{C.A. } -\frac{\pi}{2} \leq 3x + \frac{\pi}{6} \leq \frac{\pi}{2}$$

$$-\frac{\pi}{2} - \frac{\pi}{6} \leq 3x \leq \frac{\pi}{2} - \frac{\pi}{6}$$

$$-\frac{4\pi}{6} \leq 3x \leq \frac{2\pi}{6}$$

$$-\frac{4\pi}{18} \leq x \leq \frac{2\pi}{18}$$

$$\text{CD}f^{-1} = \left[-\frac{4\pi}{18}, \frac{2\pi}{18}\right]$$

$$d) f^{-1}(0) = -\frac{\pi}{18} + \frac{1}{3} \arcsen\left(\frac{0-1}{2}\right)$$

$$= -\frac{\pi}{18} + \frac{1}{3} \left(-\frac{\pi}{6}\right)$$

$$= -\frac{\pi}{18} - \frac{\pi}{18} = -\frac{2\pi}{18} = -\frac{\pi}{9}$$

$$y = 1 + 2 \sen\left(3x + \frac{\pi}{6}\right)$$

$$y-1 = 2 \sen\left(3x + \frac{\pi}{6}\right)$$

$$\frac{y-1}{2} = \sen\left(3x + \frac{\pi}{6}\right)$$

$$3x + \frac{\pi}{6} = \arcsen\left(\frac{y-1}{2}\right)$$

$$3x = -\frac{\pi}{6} + \arcsen\left(\frac{y-1}{2}\right)$$

$$x = -\frac{\pi}{18} + \frac{1}{3} \arcsen\left(\frac{y-1}{2}\right)$$

$$a) \quad 3x^2 + 3y^2 y' = 4xy' + 4y$$

$$3y^2 y' - 4xy' = 4y - 3x^2$$

$$y' = \frac{4y - 3x^2}{3y^2 - 4x}$$

$$b) \quad y'(2) = \frac{4 \times 1 - 3 \times 4}{3 \times 1 - 4 \times 2} = \frac{4 - 12}{3 - 8} = \frac{-8}{-5} = \frac{8}{5}$$

Eq. da reta tangente à curva no ponto (2, 1)

$$y - 1 = \frac{8}{5} (x - 2)$$

$$y - 1 = \frac{8}{5} x - \frac{16}{5}$$

$$y = \frac{8}{5} x - \frac{11}{5}$$