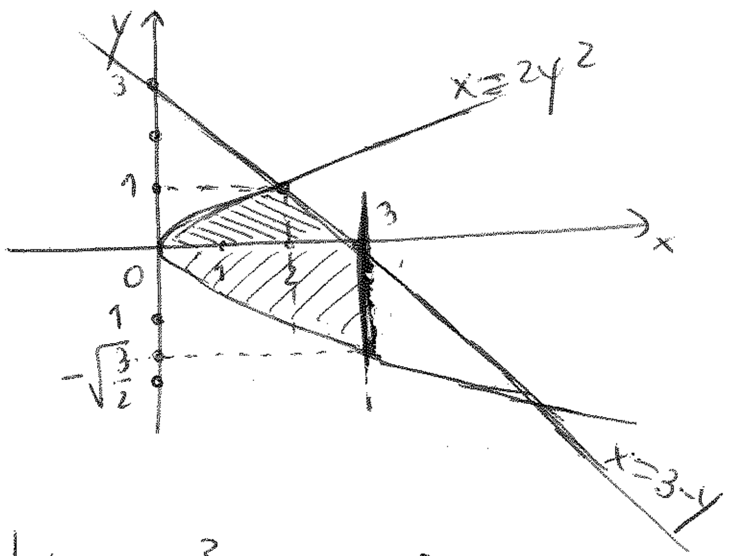
Ficha extra

1.a) $-\sqrt{\frac{3}{2}} \leq y \leq 0$ $\quad\cup\quad$ $0 \leq y \leq 1$

$2y^2 \leq x \leq 3$ $\qquad\qquad$ $2y^2 \leq x \leq 3-y$



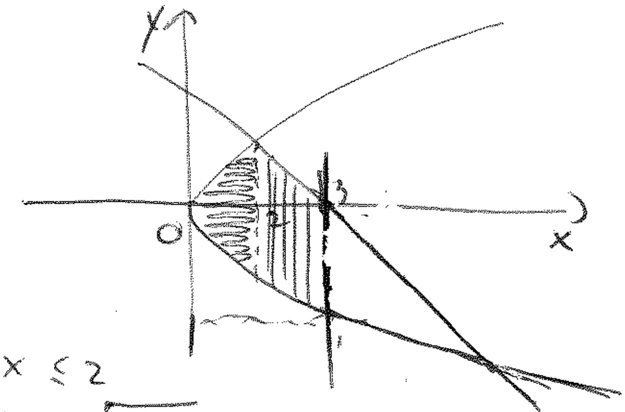
Intersecção entre

$\begin{cases} x = 2y^2 \\ x = 3-y \end{cases}$ $\begin{cases} 3-y = 2y^2 \end{cases}$

$2y^2 + y - 3 = 0$

$D = \{(x,y) \in \mathbb{R}^2 : x \geq 2y^2 \wedge y \leq 3-x \wedge x \leq 3\}$

b)



$0 \leq x \leq 2$ $\qquad\qquad$ $2 \leq x \leq 3$

$-\sqrt{\frac{x}{2}} \leq y \leq \sqrt{\frac{x}{2}}$ $\quad\cup\quad$ $-\sqrt{\frac{x}{2}} \leq y \leq 3-x$

$$\int_0^2 \int_{-\sqrt{\frac{x}{2}}}^{\sqrt{\frac{x}{2}}} xy \, dy \, dx + \int_2^3 \int_{-\sqrt{\frac{x}{2}}}^{3-x} xy \, dy \, dx =$$

$$= \int_0^2 x \left[\frac{y^2}{2}\right]_{-\sqrt{\frac{x}{2}}}^{\sqrt{\frac{x}{2}}} dx + \int_2^3 x \left[\frac{y^2}{2}\right]_{-\sqrt{\frac{x}{2}}}^{3-x} dx =$$

$$= \int_0^2 \frac{x}{2}\left[\frac{x}{2} - \frac{x}{2}\right] dx + \int_2^3 \frac{x}{2}\left[(3-x)^2 - \frac{x}{2}\right] dx =$$

$$= \int_2^3 \frac{x}{2}\left(9 + x^2 - 6x - \frac{x}{2}\right) dx = \int_2^3 \left(\frac{9x}{2} + \frac{x^3}{2} - \frac{13x^2}{4}\right) dx =$$

$$= \left[\frac{9x^2}{4} + \frac{x^4}{8} - \frac{13x^3}{12}\right]_2^3 = \frac{9\times 9}{4} + \frac{3^4}{8} - \frac{13\times 3^3}{12} -$$

$$- 9 - \frac{16}{8} + \frac{13\times 2^3}{12} = \frac{165}{8} + \frac{26}{3} - 11.$$

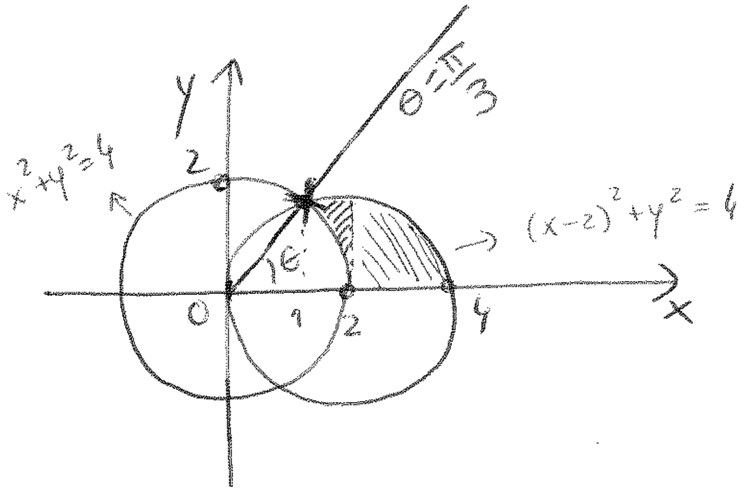
2

$$1 \leq x \leq 2 \qquad\qquad 2 \leq x \leq 4$$

$$\sqrt{4-x^2} \leq y \leq \sqrt{4-(x-2)^2} \quad \cup \quad 0 \leq y \leq \sqrt{4-(x-2)^2}$$

A curva $y = \sqrt{4-(x-2)^2}$ é a semi-circunferência superior centrada em $(2,0)$ e raio 2. Faz parte da circunf. $(x-2)^2+y^2=4$.

A curva $y=\sqrt{4-x^2}$ é a semi-circunf. sup. centrada em $(0,0)$ e raio 2. Faz parte da circunf. $x^2+y^2=4$.



$$D=\{(x,y)\in\mathbb{R}^2 : x^2+y^2\geq 4 \wedge (x-2)^2+y^2\leq 4 \wedge y\geq 0\}$$

b) $x = R\cos\theta$
$y = R \operatorname{sen}\theta$

A circunf. $(x-2)^2+y^2=4$ escreve-se em coordenadas polares da forma $R = 4\cos\theta$.

A circunf. $x^2+y^2=4$ escreve-se em coordenadas polares da forma $R=2$.

Assim, $2\leq R\leq 4\cos\theta$

Para saber o valor de θ no ponto de intersecção das duas circunf. no 1º Q, faz-se

$$\begin{cases} R=2 \\ R=4\cos\theta \end{cases} \quad \{ 2=4\cos\theta \Leftrightarrow \cos\theta = \frac{1}{2}$$

$$\Leftrightarrow \theta=\frac{\pi}{3}$$

$$0\leq\theta\leq\frac{\pi}{3}$$

$$\int_0^{\pi/3}\int_2^{4\cos\theta} (R\operatorname{sen}\theta)\, R\, dR\, d\theta =$$

$$= \int_0^{\pi/3} \left[\frac{R^3}{3}\right]_2^{4\cos\theta} \cdot \operatorname{sen}\theta \, d\theta = \int_0^{\pi/3} \frac{\operatorname{sen}\theta}{3} \left[4^3 \cdot \cos^3\theta - 8\right] d\theta =$$

$$= \frac{1}{3} \int_0^{\pi/3} \left(4^3 \cdot \cos^3\theta \cdot \operatorname{sen}\theta - 8 \cdot \operatorname{sen}\theta\right) d\theta =$$

$$= \frac{1}{3}\left[-4^3 \cdot \frac{\cos^4\theta}{4} - 8 \cdot \cos\theta\right]_0^{\pi/3} =$$

$$= \frac{1}{3}\left[-4^2\left(\cos^4\frac{\pi}{3} - \cos^4 0\right) - 8\left(\cos\frac{\pi}{3} - \cos 0\right)\right] =$$

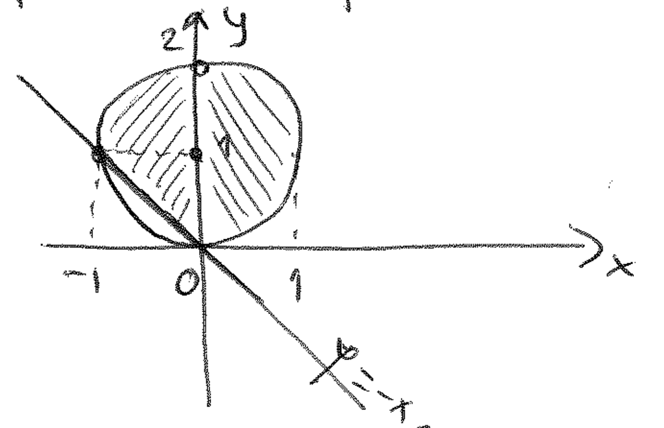
$$= \frac{1}{3}\left[-16\left(\frac{1}{16} - 1\right) - 8\left(\frac{1}{2} - 1\right)\right] = 1$$

3

$-1 \le x \le 0$ $\quad\cup\quad$ $0 \le x \le 1$

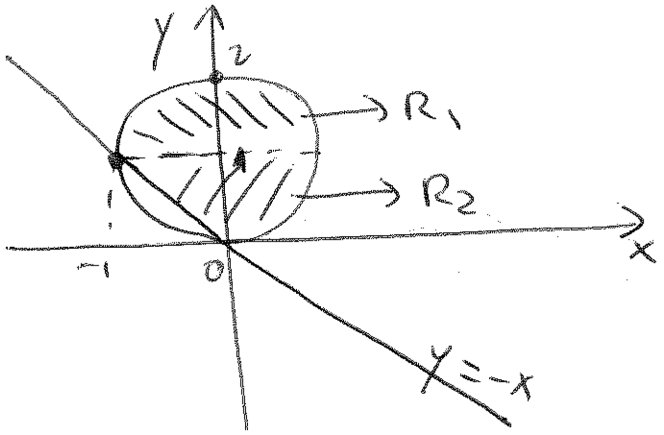
$-x \le y \le 1 + \sqrt{1-x^2}$ $\qquad$ $1 - \sqrt{1-x^2} \le y \le 1 + \sqrt{1-x^2}$

As curvas $y = 1 + \sqrt{1-x^2}$ e $y = 1 - \sqrt{1-x^2}$ fazem parte da circunferência $x^2 + (y-1)^2 = 1$.



$$D = \{(x,y) \in \mathbb{R}^2 : x^2 + (y-1)^2 \le 1 \wedge y \ge -x\}$$

b)



R_1 $\boxed{\diagdown}$

$1 \leq y \leq 2$

$-\sqrt{1-(y-1)^2} \leq x \leq \sqrt{1-(y-1)^2}$

$\cup$

R_2 $\boxed{/\!/\!/}$

$0 \leq y \leq 1$

$-y \leq x \leq \sqrt{1-(y-1)^2}$

$$\int_1^2 \int_{-\sqrt{1-(y-1)^2}}^{\sqrt{1-(y-1)^2}} f(x,y)\,dx\,dy + \int_0^1 \int_{-y}^{\sqrt{1-(y-1)^2}} f(x,y)\,dx\,dy.$$

4.

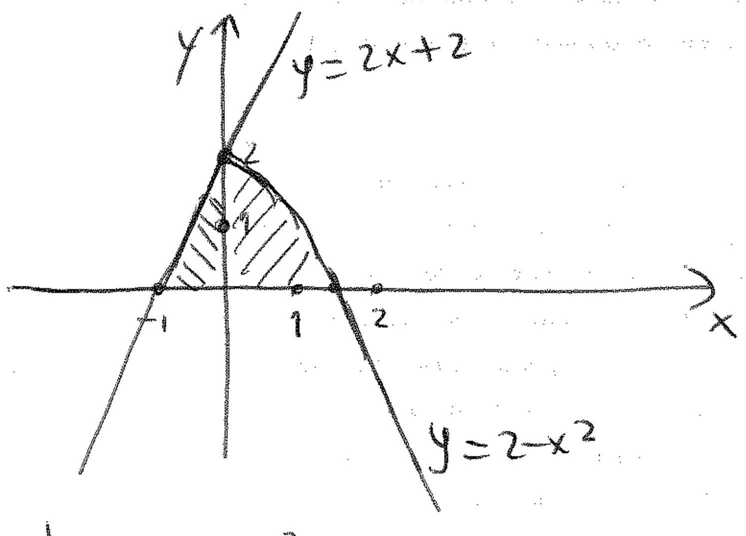
$-1 \leq x \leq 0$

$0 \leq y \leq 2x+2$ $\boxed{\diagdown}$

$\cup$

$0 \leq x \leq \sqrt{2}$

$0 \leq y \leq 2-x^2$ $\boxed{/\!/\!/}$



$D = \{(x,y) \in \mathbb{R}^2 : 0 \leq y \leq 2x+2 \ \wedge \ y \leq 2-x^2\}.$

b) $0 \leq y \leq 2$

$\frac{y}{2} - 1 \leq x \leq \sqrt{2-y}$

$$\int_0^2 \int_{\frac{y}{2}-1}^{\sqrt{2-y}} y \, dx \, dy = \int_0^2 y \left[x\right]_{\frac{y}{2}-1}^{\sqrt{2-y}} dy =$$

$$= \int_0^2 y \left[\sqrt{2-y} - \frac{y}{2} + 1\right] dy = \int_0^2 \left(y\sqrt{2-y} - \frac{y^2}{2} + y\right) dy =$$

$$= \int_0^2 \underbrace{y\sqrt{2-y} \, dy}_{\text{Aplica-se}} + \int_0^2 \left(-\frac{y^2}{2} + y\right) dy =$$

uma substituição

$2 - y = u \Leftrightarrow y = 2 - u$

$dy = -du$

$$\begin{cases} y = 0 \Rightarrow u = 2 \\ y = 2 \Rightarrow u = 0 \end{cases}$$

$$= \int_2^0 -(2-u)\sqrt{u} \, du + \int_0^2 \left(-\frac{y^2}{2} + y\right) dy =$$

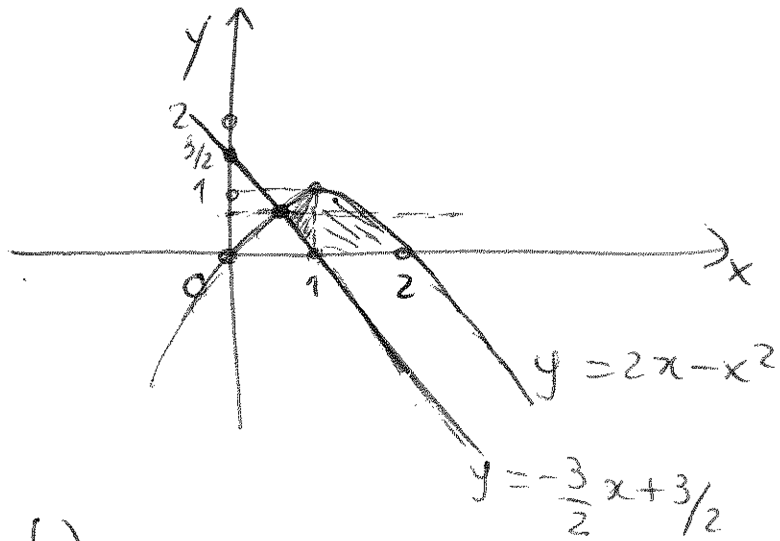
$$= \int_0^2 \left(2u^{1/2} - u^{3/2}\right) du + \left[-\frac{y^3}{6} + \frac{y^2}{2}\right]_0^2 dy =$$

$$= \left[2\frac{u^{3/2}}{\frac{3}{2}} - \frac{u^{5/2}}{5/2}\right]_0^2 + \frac{-2^3}{2 \times 3} + \frac{2^2}{2} =$$

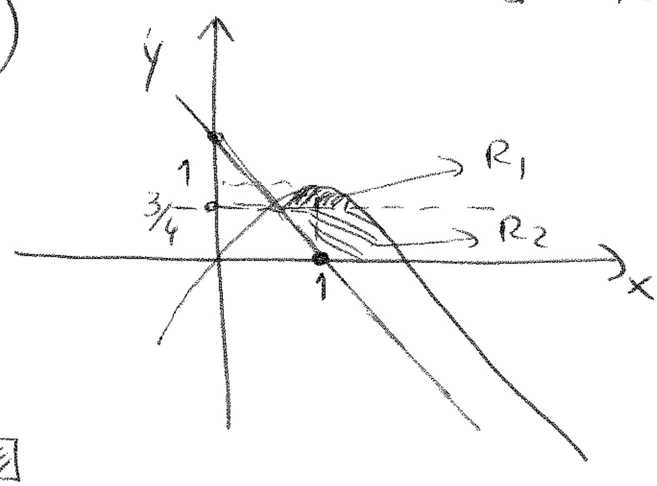
$$= 4 \times \frac{2^{3/2}}{3} - \frac{2}{5} \cdot 2^{5/2} - \frac{2^2}{3} + \frac{2^2}{2} = \frac{8\sqrt{2} - 4}{3} - \frac{8\sqrt{2}}{5} + 2 = \frac{16\sqrt{2} + 10}{15}$$

5. $\frac{1}{2} \leq x \leq 1$ $\qquad\cup\qquad$ $1 \leq x \leq 2$

$-\frac{3}{2}x + \frac{3}{2} \leq y \leq 2x - x^2$ $\qquad\qquad$ $0 \leq y \leq 2x - x^2$



b)



Pt° de intersecção

$$\begin{cases} y = -\frac{3}{2}x + \frac{3}{2} \\ y = 2x - x^2 \end{cases} \Leftrightarrow \quad x = 3 \vee x = +1/2$$

$$y = \frac{3}{4}$$

$$\boxed{\begin{array}{l} y = 2x - x^2 \Leftrightarrow \\ \Leftrightarrow y - 1 = -(x-1)^2 \end{array}}$$

R_1

$\frac{3}{4} \leq y \leq 1$

$1 - \sqrt{1-y} \leq x \leq 1 + \sqrt{1-y}$ $\qquad\cup$

$\cup$ R_2

$0 \leq y \leq \frac{3}{4}$

$1 - \frac{2}{3}y \leq x \leq 1 + \sqrt{1-y}$

$$\int_{\frac{3}{4}}^{1}\int_{1-\sqrt{1-y}}^{1+\sqrt{1-y}} dx\,dy + \int_{0}^{3/4}\int_{1-\frac{2}{3}y}^{1+\sqrt{1-y}} dx\,dy =$$

$$= \int_{\frac{3}{4}}^{1}\left(\cancel{1}+\sqrt{1-y}-\cancel{1}+\sqrt{1-y}\right)dy + \int_{0}^{3/4}\left(\cancel{1}+\sqrt{1-y}-\cancel{1}+\frac{2}{3}y\right)dy =$$

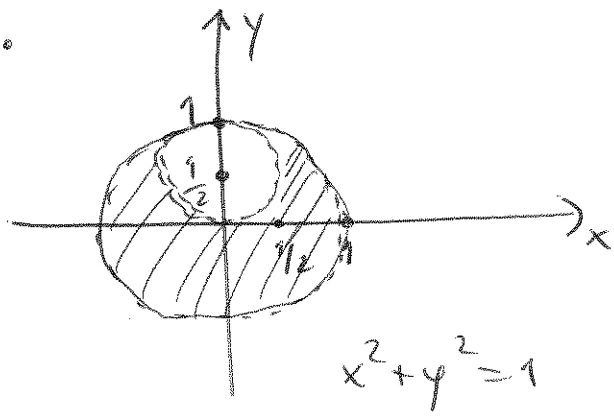
$$= \left[-\frac{4}{3}(1-y)^{3/2}\right]_{3/4}^{1} + \left[\left(-\frac{2}{3}\right)(1-y)^{3/2} + \frac{y^2}{3}\right]_{0}^{3/4} =$$

$$= -\frac{4}{3}\left[0-\left(1-\frac{3}{4}\right)^{3/2}\right] + \left[-\frac{2}{3}\left(1-\frac{3}{4}\right)^{3/2} + \frac{2}{3}\times 1 + \frac{1}{3}\left(\frac{3}{4}\right)^{2} - 0\right] =$$

$$= \frac{4}{3}\times\left(\frac{1}{4}\right)^{3/2} - \frac{2}{3}\times\left(\frac{1}{4}\right)^{3/2} + \frac{2}{3} + \frac{3}{4} = \frac{4}{3}\times\frac{1}{2^3} - \frac{2}{3}\times\frac{1}{2^3} + \frac{2}{3} + \frac{3}{4} =$$

$$= \frac{1}{6}_{(2)} - \frac{1}{12} + \frac{2}{3}_{(4)} + \frac{3}{4}_{(3)} = \frac{3}{2} \;//$$

6.



$$x^2+y^2=y \Leftrightarrow x^2+y^2-y=0$$

$$\Leftrightarrow x^2+\underbrace{y^2-2\cdot\frac{1}{2}y+\frac{1}{4}}-\frac{1}{4}=0$$

$$\Leftrightarrow x^2+\left(y-\frac{1}{2}\right)^2=\frac{1}{4}$$

$x^2+y^2=1 \iff R=1 \rightarrow$ Em coordenadas polares

A circunf. $x^2+\left(y-\frac{1}{2}\right)^2=\frac{1}{4}$ escreve-se em coordenadas polares da forma $x = \operatorname{sen}\theta$

Em coordenadas polares, D escreve-se

$\operatorname{sen}\theta \leq R \leq 1$

$0 \leq \theta \leq 2\pi$

$$\int_0^{2\pi} \int_{\operatorname{sen}\theta}^{1} \frac{R \operatorname{sen}\theta}{\cancel{\sqrt{R^2}}} \cancel{R}\, dR\, d\theta = \int_0^{2\pi} \int_{\operatorname{sen}\theta}^{1} R \operatorname{sen}\theta \, dR\, d\theta =$$

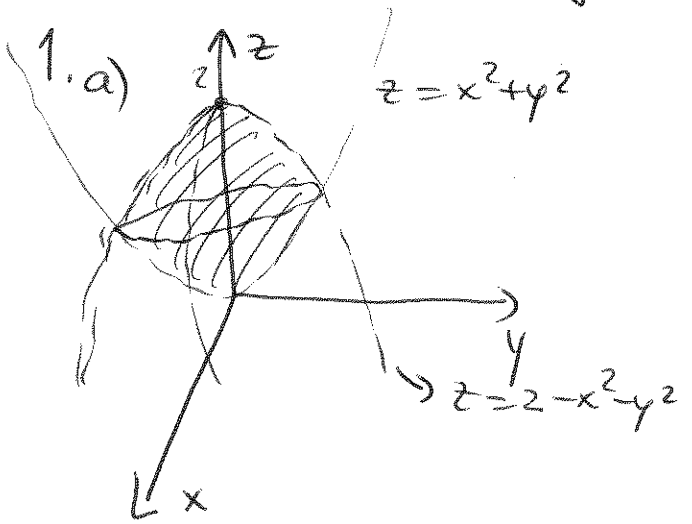
$$= \int_0^{2\pi} \left[\frac{R^2}{2}\right]_{\operatorname{sen}\theta}^{1} \operatorname{sen}\theta \, d\theta = \int_0^{2\pi} \left(\frac{1}{2} - \operatorname{sen}\theta\right) \operatorname{sen}\theta \, d\theta = \int_0^{2\pi} \frac{\operatorname{sen}\theta}{2} - \operatorname{sen}^2\theta \, d\theta$$

$$\int_0^{2\pi} \frac{\operatorname{sen}\theta}{2} - \frac{1-\cos(2\theta)}{2} \, d\theta = \frac{1}{2} \left[\operatorname{sen}^2\theta - \theta - \frac{\operatorname{sen}(2\theta)}{2}\right]_0^{2\pi} =$$

$$= \frac{1}{2}\left[\operatorname{sen}^2(2\pi) - 2\pi - \frac{\operatorname{sen}(4\pi)}{2}\right] = \frac{1}{2}\left[0 - 2\pi - 0\right] = -\pi.$$

Integrais triplos

1. a)

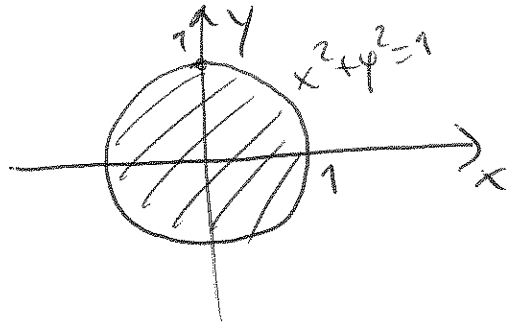


b) $2-x^2-y^2 \leq z \leq x^2+y^2$

Projeção no plano xOy

Intenseção dos paraboloides:

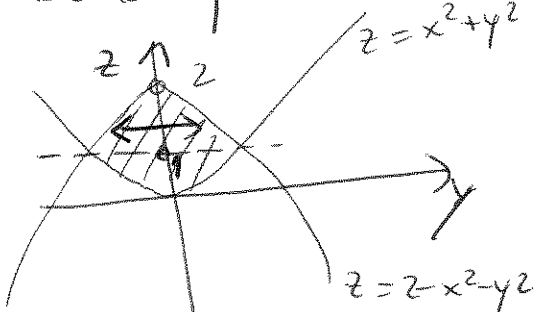
$$\begin{cases} z = 2-x^2-y^2 \\ z = x^2+y^2 \end{cases} \Leftrightarrow x^2+y^2=1$$



$$-\sqrt{1-x^2} \leq y \leq \sqrt{1-x^2}$$
$$-1 \leq x \leq 1$$

$$\int_{x=-1}^{x=1} \int_{y=-\sqrt{1-x^2}}^{y=\sqrt{1-x^2}} \int_{z=2-x^2-y^2}^{z=x^2+y^2} dz\,dy\,dx$$

c) Projeção no Plano yOz



$z = x^2+y^2 \Leftrightarrow y^2 = z-x^2 \Leftrightarrow$
$y = \pm\sqrt{z-x^2}$

$z = 2-x^2-y^2 \Leftrightarrow y^2 = 2-z-x^2$
$y = \pm\sqrt{2-z-x^2}$

Acima do plano $z=1$

$-\sqrt{2-x^2-z} \leq y \leq \sqrt{\underbrace{2-x^2-z}_{\geq 0}}$

$z \leq 2-x^2$

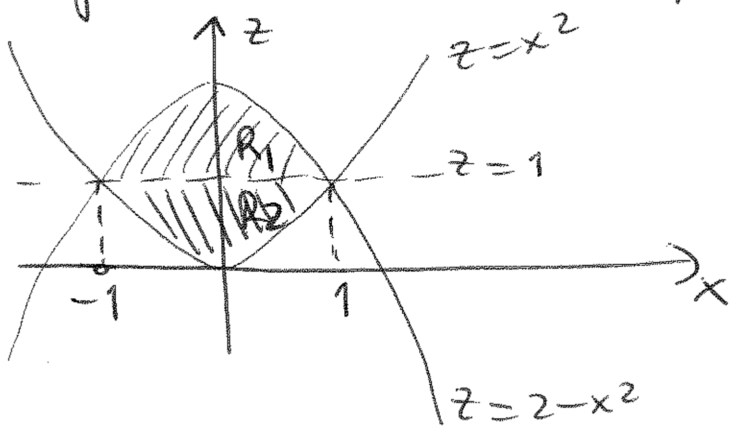
$\cup$

Abaixo do plano $z=1$

$-\sqrt{z-x^2} \leq y \leq \sqrt{\underbrace{z-x^2}_{\geq 0}}$

$z \geq x^2$

Projeção no plano xOz $(y=0)$



R_1

$$1 \leq z \leq 2-x^2$$
$$-1 \leq x \leq 1$$

R_2:

$$x^2 \leq z \leq 1$$
$$-1 \leq x \leq 1$$

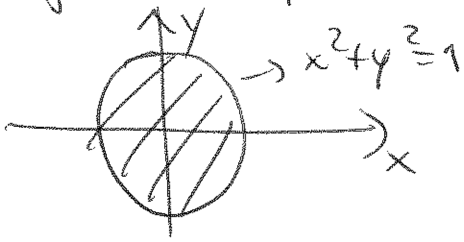
c) $$\int_{x=-1}^{x=1}\int_{z=1}^{z=2-x^2}\int_{y=-\sqrt{2-x^2-z}}^{y=\sqrt{2-x^2-z}} dy\,dz\,dx + \int_{x=-1}^{x=1}\int_{z=x^2}^{z=1}\int_{y=-\sqrt{z-x^2}}^{y=\sqrt{z-x^2}} dy\,dz\,dx$$

d) $x = R\cos\theta$
$y = R\,\text{sen}\,\theta$
$z = z$

Do enquadramento
$x^2+y^2 \leq z \leq 2-x^2-y^2$

Vem
$R^2 \leq z \leq 2-R^2$

Projectando no plano xoy



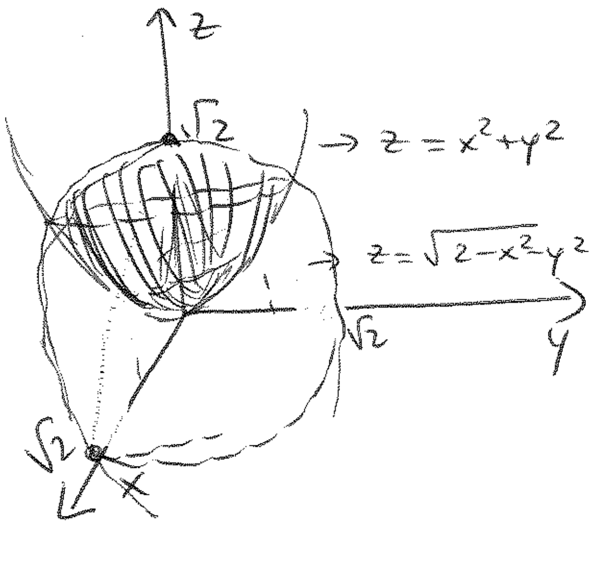
$0 \leq R \leq 1$
$0 \leq \theta \leq 2\pi$

$$\int_0^{2\pi}\int_0^1\int_{R^2}^{2-R^2} R\,dz\,dR\,d\theta = \int_0^{2\pi}\int_0^1 (2R-2R^3)\,dR\,d\theta$$

$= \pi$.

2. a) $x^2+y^2+z^2=2 \Leftrightarrow z^2 = 2-x^2-y^2 \Leftrightarrow z = \pm\sqrt{2-x^2-y^2}$

superfície esférica.



b) $x^2+y^2 \leq z \leq \sqrt{2-x^2-y^2}$

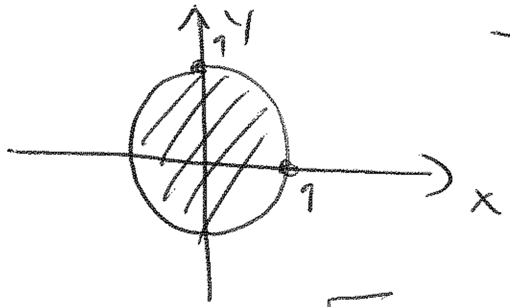
Projeção no plano xOy ~~vem~~ da interseção das superfícies

$$\begin{cases} z^2 = 2-x^2-y^2 \\ z = x^2+y^2 \end{cases} \quad \begin{cases} z^2 = 2-z \Leftrightarrow z^2+z-2=0 \Leftrightarrow z = \frac{-1\pm\sqrt{1+8}}{2} \\ \text{—} \end{cases}$$

$$z = \frac{-1\pm 3}{2} \Leftrightarrow$$

$$z=-2 \vee z=1$$

$z=1 \Leftrightarrow x^2+y^2=1$

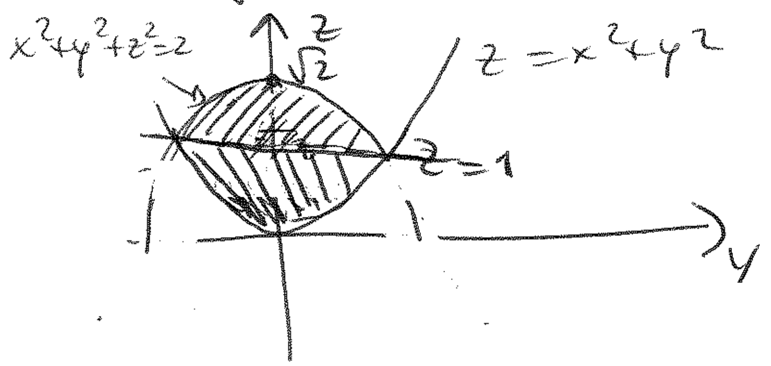


$$-\sqrt{1-x^2} \leq y \leq \sqrt{1-x^2}$$

$$-1 \leq x \leq 1$$

$$\int_{x=-1}^{x=1} \int_{y=-\sqrt{1-x^2}}^{y=\sqrt{1-x^2}} \int_{z=x^2+y^2}^{z=2-x^2-y^2} dz\,dy\,dx.$$

c) Projeção no plano yOz



Acima de $z=1$

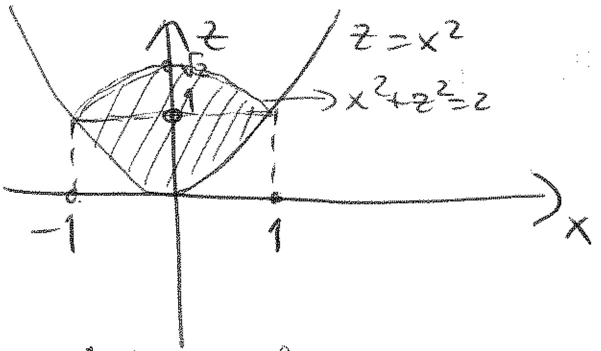
$$-\sqrt{2-z^2-x^2} \leq y \leq \sqrt{\underbrace{2-x^2-z^2}_{\geq 0}}$$

$$z^2 \leq 2-x^2$$

Abaixo de $z=1$

$$-\sqrt{z-x^2} \leq y \leq \sqrt{\underbrace{z-x^2}_{\geq 0}}$$

Projeção no plano xOz $(y=0)$



$\rightarrow z = 1 \Rightarrow x^2+1=2$
$x^2 = 1$

Acima de $z=1$

$-\sqrt{2-x^2} \leq z \leq \sqrt{2-x^2}$

$-1 \leq x \leq 1$

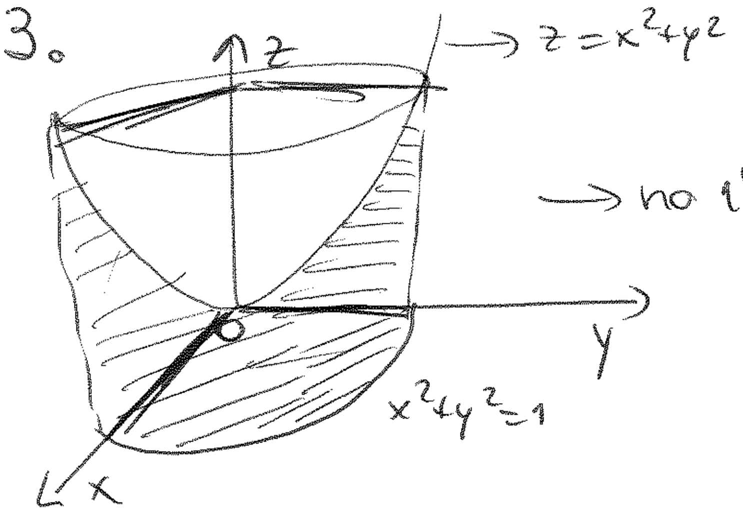
Abaixo de $z=1$

$x^2 \leq z \leq 1$

$-1 \leq x \leq 1$

$$\int_{x=-1}^{x=1} \int_{z=-\sqrt{2-x^2}}^{z=\sqrt{2-x^2}} \int_{y=-\sqrt{2-z^2-x^2}}^{y=\sqrt{2-z^2-x^2}} dy\,dz\,dx + \int_{-1}^{1} \int_{x^2}^{1} \int_{-\sqrt{z-x^2}}^{\sqrt{z-x^2}} dy\,dz\,dx.$$

3.



$z = x^2+y^2 \rightarrow$ paraboloide

$\rightarrow$ no 1º octante

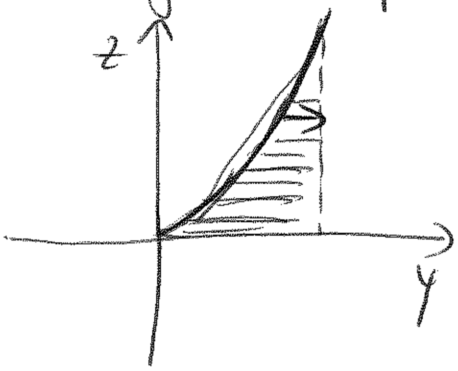
$0 \leq z \leq x^2+y^2$

$0 \leq y \leq \sqrt{1-x^2}$

$0 \leq x \leq 1$

$$\int_0^1 \int_0^{\sqrt{1-x^2}} \int_0^{x^2+y^2} dz\,dy\,dx.$$

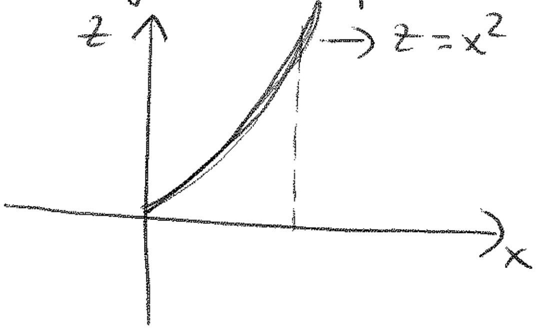
b) Projeção no plano yOz



$$\sqrt{z-x^2} \leq y \leq \sqrt{1-x^2}$$
$$x^2 \leq z \leq 1$$

$z - x^2 \geq 0$
$1 - x^2 \geq 0$

Projeção no plano xOz



$0 \leq x \leq 1$

Interseção das superfícies

$$\sqrt{1-x^2} = \sqrt{\underbrace{z-x^2}} \Leftrightarrow z - x^2 = 1 - x^2 \Leftrightarrow z = 1$$

$\rightarrow z - x^2 \geq 0$

$\underline{z \geq x^2}$

$$x^2 \leq z \leq 1$$

$$\int_0^1 \int_{x^2}^1 \int_{\sqrt{z-x^2}}^{\sqrt{1-x^2}} dy\, dz\, dx$$

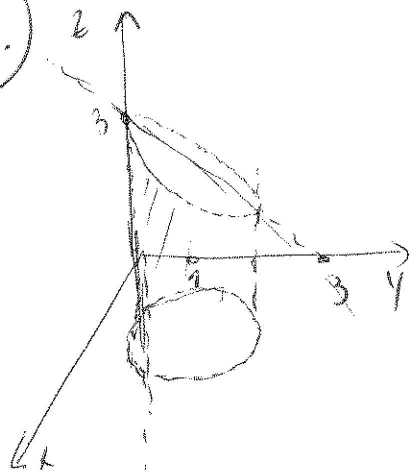
d) $x = R\cos\theta$
$y = R\,\text{sen}\,\theta$
$z = z$

$0 \leq z \leq R^2$
$0 \leq R \leq 1$
$0 \leq \theta \leq \frac{\pi}{2}$

$$\int_0^{\pi/2} \int_0^1 \int_0^{R^2} R\, dz\, dR\, d\theta = \frac{\pi}{8}$$

4. 15

a)



$x^2+y^2=2y \Leftrightarrow x^2+(y-1)^2=1$

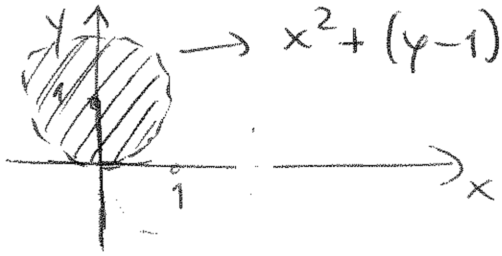
$\to$ superfície cilíndrica.

$z=-1 \to$ plano horizontal

$z=3-y \to$ plano inclinado

b) $-1 \leq z \leq 3-y$

Projeção no plano xOy



$\to x^2+(y-1)^2=1$

$1-\sqrt{1-x^2} \leq y \leq 1+\sqrt{1-x^2}$

$-1 \leq x \leq 1$

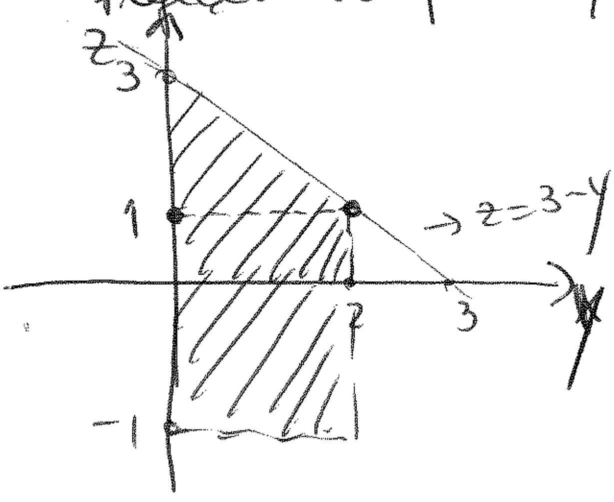
ou $-\sqrt{1-(y-1)^2} \leq x \leq \sqrt{1-(y-1)^2}$

$-1 \leq y \leq 1$

$$\int_{-1}^{1}\int_{1-\sqrt{1-x^2}}^{1+\sqrt{1-x^2}}\int_{-1}^{3-y} dz\,dy\,dx = \int_{-1}^{1}\int_{-\sqrt{1-(y-1)^2}}^{\sqrt{1-(y-1)^2}}\int_{-1}^{3-y} dz\,dx\,dy$$

c) $-\sqrt{1-(y-1)^2} \leq x \leq \sqrt{1-(y-1)^2}$

Projeção no plano yOz



$\to z=3-y \Leftrightarrow y=3-z$

$0 \leq y \leq 3-z$

$1 \leq z \leq 3$

$\cup$ $0 \leq y \leq 2$

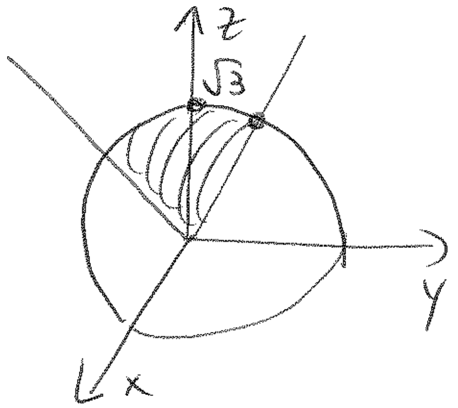
$-1 \leq z \leq 1$

$$\int_1^3 \int_0^{3-z} \int_{-\sqrt{1-(y-1)^2}}^{\sqrt{1-(y-1)^2}} dx\,dy\,dz + \int_{-1}^{1} \int_0^{2} \int_{-\sqrt{1-(y-1)^2}}^{\sqrt{1-(y-1)^2}} dx\,dy\,dz$$

5.
$$\sqrt{\frac{1}{2}(x^2+y^2)} \leq z \leq \sqrt{3-x^2-y^2}$$
$$0 \leq y \leq \sqrt{2-x^2}$$
$$0 \leq x \leq \sqrt{2}$$

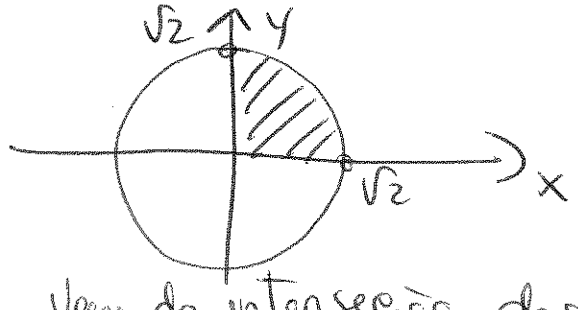
$z = \sqrt{3-x^2-y^2}$ → parte superior da superfície esférica $x^2+y^2+z^2=3$

$z = \sqrt{\frac{1}{2}(x^2+y^2)}$ → parte superior da superfície cónica. $z^2 = \frac{1}{2}(x^2+y^2)$



A projeção no plano xoy

$$0 \leq y \leq \sqrt{2-x^2}$$
$$0 \leq x \leq \sqrt{2}$$



Vem da intersecção das duas superfícies.

$$D = \left\{ (x,y,z) \in \mathbb{R}^3 : x^2+y^2+z^2 \leq 3 \ \wedge\ z^2 \geq \frac{1}{2}(x^2+y^2) \right\}$$

b) $x = R \operatorname{sen}\varphi \cos\theta$

$y = R \operatorname{sen}\varphi \operatorname{sen}\theta$

$z = R \cos\varphi$

$$0 \leq R \leq \sqrt{2}$$

$$0 \leq \theta \leq \frac{\pi}{2}$$

A superfície cónica $z^2 = \frac{1}{2}(x^2+y^2)$ escreve-se em coordenadas esféricas da forma

$$R^2\cos^2\varphi = \frac{R^2 \operatorname{sen}^2\varphi}{2} \Leftrightarrow 2 - 3\operatorname{sen}^2\varphi = 0 \Leftrightarrow$$

$$\operatorname{sen}\varphi = \pm\sqrt{\frac{2}{3}}$$

Como ~~φ~~ φ ~~está~~ no 1º Octante, $\operatorname{sen}\varphi = \sqrt{\frac{2}{3}}$

$$0 < \varphi < \operatorname{arcsen}\left(\sqrt{\frac{2}{3}}\right)$$

Assim, o integral fica.

$$\int_0^{\operatorname{arcsen}\sqrt{\frac{2}{3}}} \int_0^{\pi/2} \int_0^{\sqrt{3}} R\cos\varphi \sqrt{R^2}\, R^2 \operatorname{sen}\varphi \cdot dR\, d\theta\, d\varphi =$$

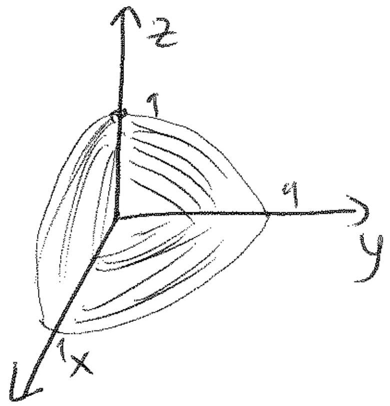
$$= \int_0^{\operatorname{arcsen}\sqrt{\frac{2}{3}}} \int_0^{\pi/2} \left[\frac{R^5}{5}\right]_0^{\sqrt{3}} \operatorname{sen}\varphi\cos\varphi\, d\theta\, d\varphi =$$

$$= \frac{9\sqrt{3}}{5}\int_0^{\operatorname{arcsen}\sqrt{\frac{2}{3}}} \int_0^{2\pi} \operatorname{sen}\varphi\cos\varphi\, d\theta\, d\varphi =$$

$$= \frac{9\pi\sqrt{3}}{10}\int_0^{\operatorname{arcsen}\sqrt{\frac{2}{3}}} \operatorname{sen}\varphi\cos\varphi\, d\varphi = \frac{9\pi\sqrt{3}}{10}\left[\operatorname{sen}^2\varphi\right]_0^{\operatorname{arcsen}\sqrt{\frac{2}{3}}} =$$

$$= \frac{3\pi\sqrt{3}}{5}.$$

6.



$z = \sqrt{1-x^2-y^2} \rightarrow$ parte superior da sup. esférica $x^2+y^2+z^2=1$.

$x = \sqrt{1-y^2}$ com $0 \leq y \leq 1$

↳ parte da circunferência que se encontra no 1º quadrante do plano xoy

$D = \{(x,y,z) \in \mathbb{R}^3 : 0 \leq z \leq \sqrt{1-x^2-y^2} \wedge x \geq 0 \wedge y \geq 0\}$

b)
$x = R\cos\theta$
$y = R \,\text{sen}\,\theta$
$z = z$

$0 \leq z \leq \sqrt{1-R^2}$
$0 \leq R \leq 1$
$0 \leq \theta \leq \frac{\pi}{2}$

$$\int_0^{\pi/2} \int_0^1 \int_0^{\sqrt{1-R^2}} R z \sqrt{R^2+z^2} \, dz \, dR \, d\theta$$

c)
$x = R \,\text{sen}\,\varphi \cos\theta$
$y = R \,\text{sen}\,\varphi \,\text{sen}\,\theta$
$z = R\cos\varphi$

$0 \leq R \leq 1$
$0 \leq \theta \leq \frac{\pi}{2}$
$0 \leq \varphi \leq \frac{\pi}{2}$

$$\int_0^{\pi/2} \int_0^{\pi/2} \int_0^1 R\cos\varphi \sqrt{R^2} \, R^2 \,\text{sen}\,\varphi \, dR \, d\varphi \, d\theta.$$

$$= \int_0^{\pi/2} \int_0^{\pi/2} \left[\frac{R^5}{5}\right]_0^1 \text{sen}\,\varphi \cos\varphi \, d\varphi \, d\theta =$$

$$= \frac{1}{5} \int_0^{\pi/2} \cancel{\int_0^{\pi/2}} \left[\frac{\text{sen}^2\varphi}{2}\right]_0^{\pi/2} d\theta = \frac{1}{10} \int_0^{\pi/2} 1 \, d\theta = \frac{\pi}{20}.$$

7.a) $\sqrt{x^2+y^2} \leq z \leq \sqrt{8-x^2-y^2}$

$0 \leq x \leq \sqrt{4-y^2}$

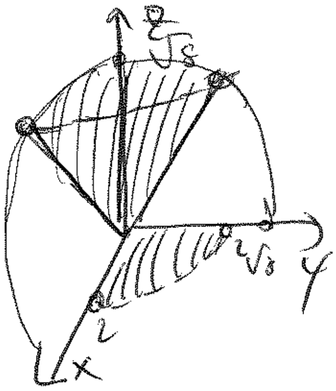
$0 \leq y \leq 2$

$z=\sqrt{8-x^2-y^2} \rightarrow$ parte superior da ~~esfera~~ sup. esférica de raio $\sqrt{8}$
$x^2+y^2+z^2=8$

$z=\sqrt{x^2+y^2} \longrightarrow$ parte superior da sup. cónica $z^2=x^2+y^2$

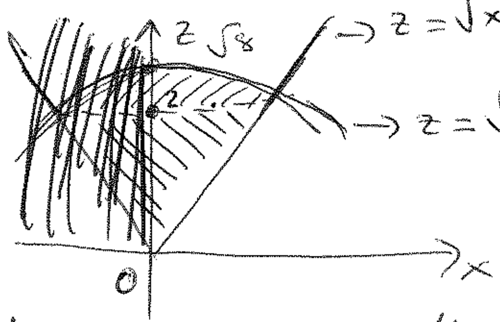
$\left.\begin{array}{l} 0 \leq x \leq \sqrt{4-y^2} \\ 0 \leq y \leq 2 \end{array}\right\} \rightarrow$ projeção no plano xOy no 1º Quadrante da intersecção dos superfícies

$$\begin{cases} z=\sqrt{x^2+y^2} \\ z=\sqrt{8-x^2-y^2} \end{cases} \Leftrightarrow \underbrace{x^2+y^2=4}_{\text{circunf. de raio 2 C}\to(0,0) \text{ no plano } xOy}$$



$D=\{(x,y,z)\in\mathbb{R}^3:\ x^2+y^2+z^2\leq 8 \ \wedge\ z\geq\sqrt{x^2+y^2} \ \wedge\ x\geq 0 \ \wedge\ y\geq 0\}$

b) No plano xOz



$\rightarrow z=\sqrt{x^2+y^2} \Rightarrow x^2=z^2-y^2 \Rightarrow x=\sqrt{z^2-y^2}$

$\rightarrow z=\sqrt{8-x^2-y^2} \Rightarrow x^2=8-z^2-y^2 \Rightarrow z=\sqrt{8-z^2-y^2}$

Abaixo do plano $z=2$

$0 \leq x \leq \sqrt{z^2-y^2}$

$z^2 \geq y^2$

$z\geq y \ \vee\ z\leq -y$

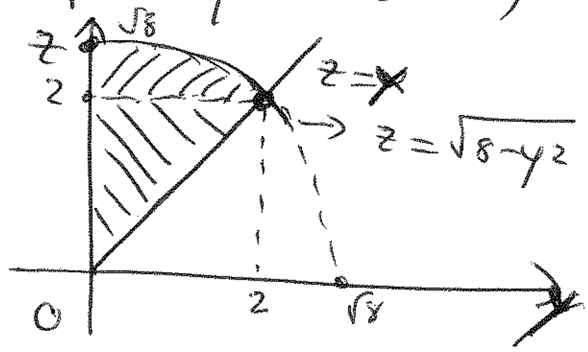
~~Abaixo~~ Acima do plano $z=2$

$0 \leq x \leq \sqrt{8-z^2-y^2}$

$8-z^2-y^2\geq 0$

$z^2+y^2\leq 8$

b) No plano yOz $(x=0)$



$\Rightarrow$ Intersectando

$\begin{cases} z = y \\ z = \sqrt{8-y^2} \end{cases} \begin{cases} y^2 = 8 - y^2 \Leftrightarrow \\ y^2 = 4 \Rightarrow y = 2 \end{cases}$

Acima do plano $z=2$

$2 \leq z \leq \sqrt{8-y^2}$

$0 \leq y \leq 2$

Abaixo do plano $z=2$

$y \leq z \leq 2$

$0 \leq y \leq 2$

$$\int_{y=0}^{y=2} \int_{z=2}^{z=\sqrt{8-y^2}} \int_{x=0}^{x=\sqrt{8-z^2-y^2}} z \, dx\,dz\,dy + \int_{y=0}^{y=2} \int_{z=y}^{z=2} \int_{x=0}^{x=\sqrt{z^2-y^2}} z \, dx\,dz\,dy$$

c) $\begin{cases} x = R\cos\theta \\ y = R\,\text{sen}\,\theta \\ z = z \end{cases}$

$R \leq z \leq \sqrt{8-R^2}$

$0 \leq \theta \leq \frac{\pi}{2}$

$0 \leq R \leq 2$

$$\int_0^2 \int_0^{\pi/2} \int_R^{\sqrt{8-R^2}} z \cdot R \, dz\, d\theta\, dR$$

d) $\begin{cases} x = R\,\text{sen}\,\varphi \cdot \cos\theta \\ y = R\,\text{sen}\,\varphi \cdot \text{sen}\,\theta \\ z = R \cdot \cos\varphi \end{cases}$

$0 \leq R \leq \sqrt{8}$

$0 \leq \theta \leq \frac{\pi}{2}$

$0 \leq \varphi \leq ??$

?? Da superfície esférica $z^2 = x^2 + y^2$ vem

$R^2\cos^2\varphi = R^2\text{sen}^2\varphi \Leftrightarrow \cos^2\varphi - \text{sen}^2\varphi = 0 \Leftrightarrow 1 - 2\,\text{sen}^2\varphi = 0 \Leftrightarrow$

$\text{sen}^2\varphi = \frac{1}{2} \Leftrightarrow \text{sen}\,\varphi = \frac{\sqrt{2}}{2} \wedge 0 \leq \varphi \leq \pi \Rightarrow \varphi = \frac{\pi}{4}$

$$\int_0^{\sqrt{8}} \int_0^{\pi/2} \int_0^{\pi/4} R\cos\varphi \cdot R^2 \operatorname{sen}\varphi \, d\varphi \, d\theta \, dR =$$

$$= \int_0^{\sqrt{8}} \int_0^{\pi/2} R^3 \left[\frac{\operatorname{sen}^2\varphi}{2}\right]_0^{\pi/4} d\theta \, dR = \int_0^{\sqrt{8}} \int_0^{\pi/2} \frac{R^3}{2}\left[\frac{1}{2}\right] d\theta \, dR =$$

$$= \frac{1}{4} \times \frac{\pi}{2} \int_0^{\sqrt{8}} R^3 \, dR = \frac{\pi}{8}\left[\frac{R^4}{4}\right]_0^{\sqrt{8}} = 2\pi //$$