$\boxed{F.1}$

1. a) $a=0$; $R=1$; $I=[-1,1[$

b) $a=-3$; $R=3$; $I=]-6,0[$

c) $a=10$; $R=+\infty$; $I=]-\infty,+\infty[$

d) $a=0$; $R=0$; $I=\{0\}$

e) $a=4$; $R=1/\sqrt{3}$; $I=\left[4-\frac{1}{\sqrt{3}}, 4+\frac{1}{\sqrt{3}}\right]$

f) $a=3$; $R=2$; $I=]1,5[$

g) $a=0$; $R=1$; $I=[-1,1]$

2. $b=2$

3. a) $\frac{1}{1-x^4}=\sum_{n=0}^{\infty} x^{4n}$, $|x|<1$

b) $\frac{1}{2+x}=\sum_{n=0}^{\infty} \frac{(-1)^n x^n}{2^{n+1}}$, $|x|<2$

NOTA: $\frac{1}{2+x}=\frac{1}{2}\frac{1}{1-\left(-\frac{x}{2}\right)}=\frac{1}{2}\sum\left(-\frac{x}{2}\right)^n=\sum\frac{(-1)^n x^n}{2^{n+1}}$,

$\left|\frac{x}{2}\right|<1 \Leftrightarrow |x|<2$

c) $\frac{x}{1-x^2}=\sum_{n=0}^{\infty} x^{2n+1}$, $|x|<1$

d) $\frac{1}{6-x-x^2}=\frac{1}{-(x+3)(x-2)}=\frac{A}{-(x+3)}+\frac{B}{(x-2)}=\dots=\sum_{n=0}^{\infty}\left(\frac{2^{n+1}(-1)^n+3^{n+1}}{5\times 6^{n+1}}\right)x^n$

e) $\ln(1-x)=-\sum_{n=0}^{+\infty}\frac{x^{n+1}}{n+1}$, $|x|<1$

NOTA: $\ln(1-x)=-\int\frac{dx}{1-x}=-\int\sum x^n\,dx=-\sum\int x^n\,dx$

f) $\ln\left(\frac{1+x}{1-x}\right) = \sum_{n=0}^{\infty} \frac{2x^{n+1}}{n+1}$

Nota: $\ln\left(\frac{1+x}{1-x}\right) = \ln(1+x) - \ln(1-x)$

g) $\int_0^x \frac{dt}{6-t-t^2} = \sum_{n=0}^{\infty} \frac{\left[(-1)^n 2^{n+1} + 3^{n+1}\right]}{5(n+1)\, 6^{n+1}} x^{n+1}$, $|x|<2$

(4)

a) $f'(x) = \sum_{n=1}^{\infty} n^3 x^{n-1}$, $R = 1$

b) $f'(x) = \sum_{n=0}^{\infty} \frac{x^n}{n!}$, $R = +\infty$

e) $f'(x) = \sum_{n=1}^{\infty} n \cdot 2^{(n+2)/2} (x+1)^{2n-1}$, $R = 2^{-1/4}$

(5)

a) $\sum_{n=0}^{+\infty} \frac{(-1)^n x^{2n+1}}{(2n+1)!}$; $R = +\infty$

b) $\sum_{n=0}^{+\infty} \frac{x^{n+1}}{2^{n+1}}$; $R = 2$

c) $\sum_{n=0}^{+\infty} \frac{x^{2n+2}}{(2n+2)!}$; $R = +\infty$

(6)

a) Como $(\operatorname{sen} x)' = \cos x$

$\cos x = D_x\left(x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \dots\right) = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \dots$

b) $\cos x = \int -\operatorname{sen} x \, dx = -\int \left(x - \frac{x^3}{3!} + \frac{x^5}{5!} - \dots\right) dx =$

$= -\frac{x^2}{2} + \frac{x^4}{4!} + \frac{x^6}{6!} + \dots$