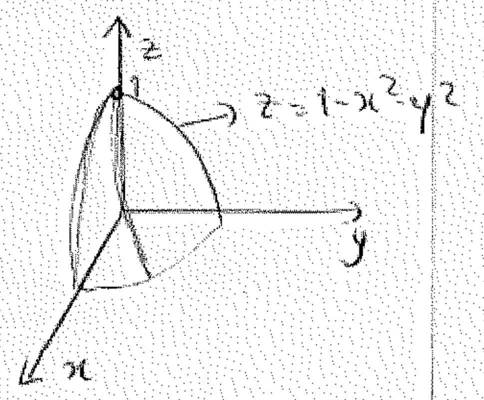
Ficha 10 B

(1)

1 – igual ao ex. 2a) da ficha 10A

2. $\iiint_R f(x,y,z)\,dx\,dy\,dz$



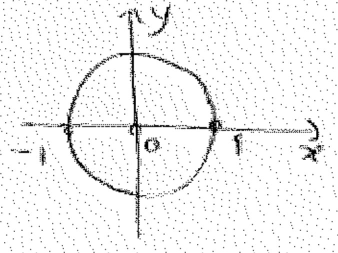
$z = 1-x^2-y^2 \rightarrow$ parabolóide

$z = 0$ plano

$0 \leq z \leq 1-x^2-y^2$

Projecção no plano xoy

$\begin{cases} z = 0 \\ z = 1-x^2-y^2 \end{cases} \quad \begin{cases} x^2+y^2 = 1 \end{cases} \longrightarrow$



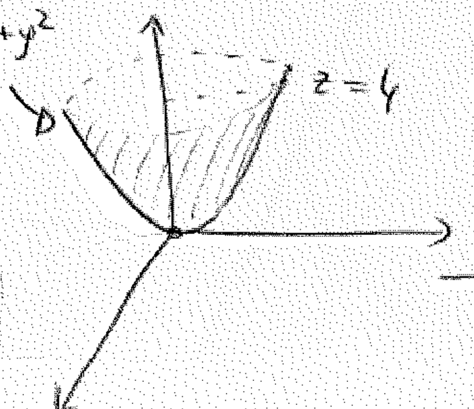
$-\sqrt{1-x^2} \leq y \leq \sqrt{1-x^2}$

$-1 \leq x \leq 1$

$$\int_{x=-1}^{x=1} \int_{y=-\sqrt{1-x^2}}^{y=\sqrt{1-x^2}} \int_{z=0}^{z=1-x^2-y^2} f(x,y,z)\,dz\,dy\,dx$$

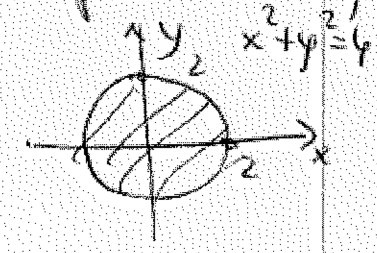
3. Volume de R , onde R é limitado por

a) $z = x^2+y^2$ parabolóide



$x^2+y^2 \leq z \leq 4$

Projecção em xoy



$-\sqrt{4-x^2} \le y \le \sqrt{4-x^2}$

$-2 \le x \le 2$

$$\int_{x=-2}^{x=2} \int_{y=-\sqrt{4-x^2}}^{y=\sqrt{4-x^2}} \int_{z=x^2+y^2}^{z=4} 1 \, dz\, dy\, dx$$

$$= \int_{-2}^{2} \int_{-\sqrt{4-x^2}}^{\sqrt{4-x^2}} \left(4-x^2-y^2\right) dy\, dx$$

Usando aqui coordenadas polares, o círculo $x^2+y^2 \le 4$, escreve-se da forma $\rho = 2$, com $0 \le \theta \le 2\pi$

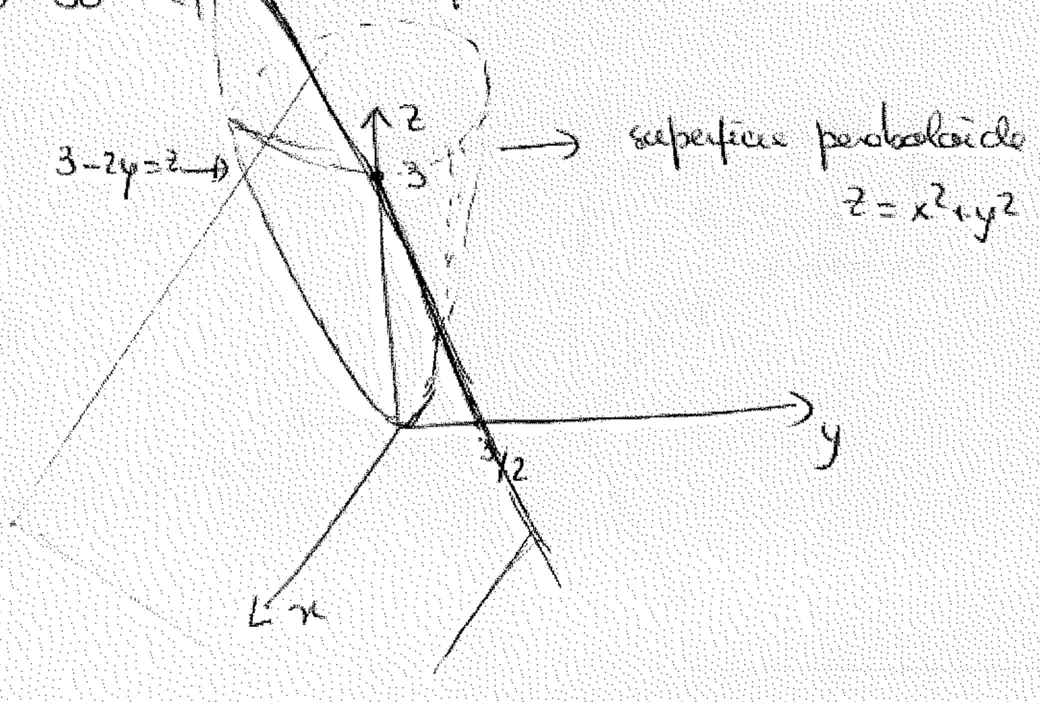
$$\begin{cases} x = \rho\cos\theta \\ y = \rho \operatorname{sen}\theta \end{cases} \quad \begin{matrix} 0 \le \rho \le 2 \\ 0 \le \theta \le 2\pi \end{matrix} \quad \text{e } |J| = \rho$$

Assim

$$\int_0^2 \int_0^{2\pi} \left(4 - \rho^2\cos^2\theta - \rho^2 \operatorname{sen}^2\theta\right) \rho \, d\theta\, d\rho =$$

$$= \int_0^2 \int_0^{2\pi} \left(4\rho - \rho^3\right) d\theta\, d\rho = 8\pi$$

b)



$x^2+y^2 \le z \le 3-2y$

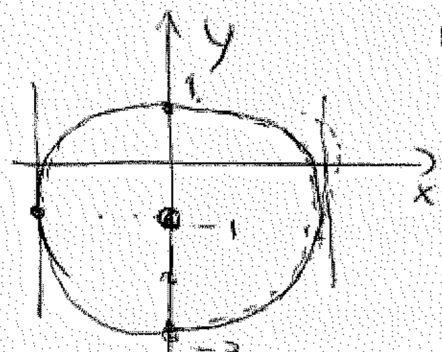
Projeção no plano xoy $\rightarrow$ intersecção das duas superfícies e sua projeção no plano xoy

$$\begin{cases} z = 3-2y \\ z = x^2+y^2 \end{cases} \qquad \begin{cases} 3-2y = x^2+y^2 \end{cases} \qquad \begin{cases} x^2+(y+1)^2 = 4 \end{cases}$$

$C(0,-1)$
$R=2$

$-1-\sqrt{4-x^2} \le y \le -1+\sqrt{4-x^2}$

$-2 \le x \le 2$



$$\int_{x=-2}^{x=2} \int_{y=-1-\sqrt{4-x^2}}^{y=-1+\sqrt{4-x^2}} \int_{z=x^2+y^2}^{z=3-2y} 1 \, dz \, dy \, dx =$$

$$= \int_{-2}^{2} \int_{-1-\sqrt{4-x^2}}^{-1+\sqrt{4-x^2}} (3-2y-x^2-y^2) \, dy \, dx = \int_{-2}^{2} \int_{-1-\sqrt{4-x^2}}^{-1+\sqrt{4-x^2}}$$

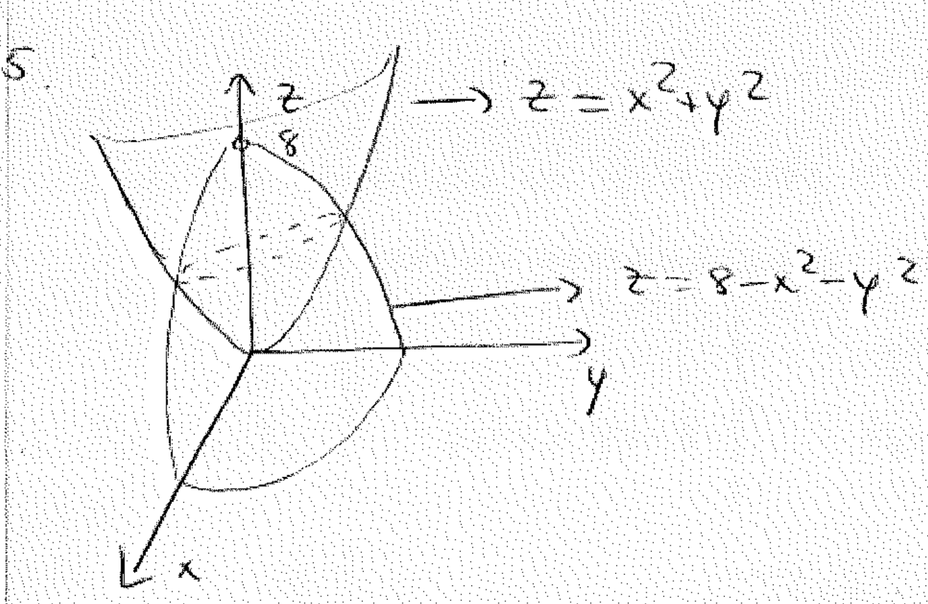
Escrevendo o círculo $x^2+(y+1)^2=4$ em coordenadas polares

$$\begin{cases} x = \rho \cos\theta \\ y+1 = \rho \sin\theta \end{cases}, \text{ tem-se } \quad \begin{array}{l} 0 \le \rho \le 2 \\ 0 \le \theta \le 2\pi \end{array} \quad e \ |J| = \rho$$

$$\int_0^2 \int_0^{2\pi} (4 - 4\rho \sin\theta - \rho^2) \rho \, d\theta \, d\rho$$

$$= \int_0^2 (8\pi\rho - 2\pi\rho^3) \, d\rho = 8\pi.$$

5.



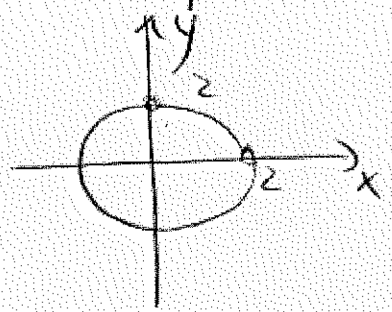
$$x^2+y^2 \le z \le 8-x^2-y^2$$

Projecção em xoy → intersecção dos parabolóides e projecção em xoy

$$\begin{cases} z = x^2+y^2 \\ z = 8-x^2-y^2 \end{cases} \quad \Big\{ \; x^2+y^2 = 8-x^2-y^2 \quad \Big\{ \; x^2+y^2 = 4$$

$$-\sqrt{4-x^2} \le y \le \sqrt{4-x^2}$$

$$-2 \le x \le 2$$



Em coordenadas cilíndricas,

$$\begin{cases} x = \rho\cos\theta \\ y = \rho\,\text{sen}\,\theta \\ z = z \end{cases} \qquad \begin{array}{l} 0 \le \rho \le 2 \\ 0 \le \theta \le 2\pi \\ \rho^2 \le z \le 8-\rho^2 \end{array}$$

$$\int_0^2 \int_0^{2\pi} \int_{\rho^2}^{8-\rho^2} \rho \,.\, dz\, d\theta\, d\rho = \frac{80\pi}{3}$$

4)

a) $$\int_0^a \int_0^{2\pi} \int_{-\sqrt{a^2-\rho^2}}^{\sqrt{a^2-\rho^2}} \rho \, dz \, d\theta \, d\rho = \frac{4\pi}{3} a^3$$

b) $$\int_0^{\sqrt{2}/2} \int_0^{2\pi} \int_{\rho}^{\sqrt{1-\rho^2}} \rho \, dz \, d\theta \, d\rho = -\frac{\sqrt{2}}{6} + \frac{1}{3}$$