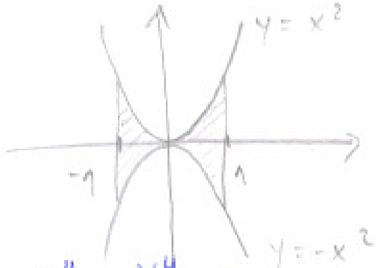
1. a) $$\int_0^2\int_0^1 \frac{x^2}{1+y^2}\,dx\,dy = \int_0^2 \frac{1}{1+y^2}\left[\frac{x^3}{3}\right]_{x=0}^{x=1} dy =$$

$$= \int_0^2 \frac{1}{1+y^2}\cdot\frac{1}{3}\,dy = \frac{1}{3}\left[\arctan(y)\right]_{y=0}^{y=2} = \frac{1}{3}\arctan(2)$$

b) $$\iint_D (x^2-y)\,dx\,dy \qquad D=\{(x,y)\in\mathbb{R}^2 : -x^2\le y\le x^2,\ -1\le x\le 1\}$$



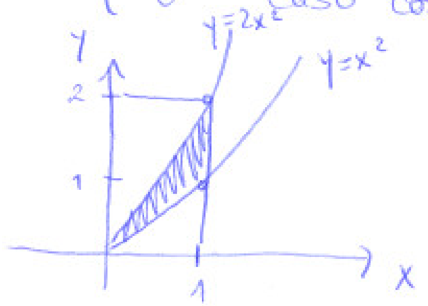
$$= \int_{-1}^{1}\int_{-x^2}^{x^2} (x^2-y)\,dy\,dx$$

$$= \int_{-1}^{1}\left[x^2y-\frac{y^2}{2}\right]_{y=-x^2}^{y=x^2} dx = \int_{-1}^{1} x^4-\frac{x^4}{2}+x^4+\frac{x^4}{2}\,dx$$

$$= \int_{-1}^{1} 2x^4\,dx = 2\left[\frac{x^5}{5}\right]_{x=-1}^{x=1} = 2\left(\frac{1}{5}+\frac{1}{5}\right) = \frac{4}{5}$$

c) $$\iint_D f(x,y)\,dx\,dy \qquad D=[0,1]\times[0,2]$$

$$f(x,y)=\begin{cases} x^3y & \text{se } x^2<y<2x^2 \\ 0 & \text{caso contrário}\end{cases}$$



$x^2<y \ \wedge\ y<2x^2$

$$= \int_0^1\int_{x^2}^{2x^2} x^3y\,dy\,dx$$

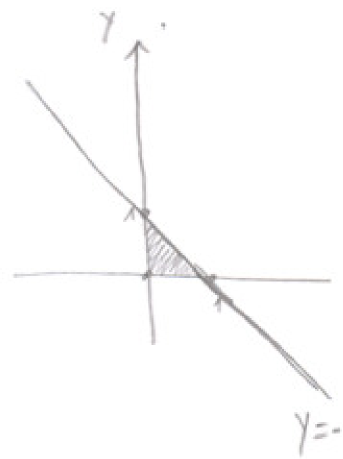
$$= \int_0^1 x^3\left[\frac{y^2}{2}\right]_{y=x^2}^{y=2x^2} dx$$

$$= \int_0^1 x^3\left[2x^4-\frac{x^4}{2}\right] dx$$

$$= \int_0^1\left(2x^7-\frac{x^7}{2}\right) dx = \left[2\frac{x^8}{8}-\frac{x^8}{16}\right]_0^1 = \frac{2}{8}-\frac{1}{16} = \frac{3}{16}$$

a) $x \ge 0 \wedge y \ge 0 \wedge x + y \le 1$

$y \le -x + 1$

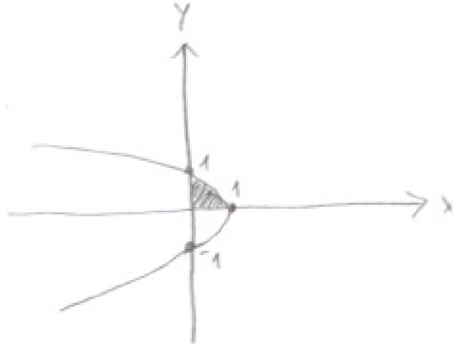


$$\int_0^1 \int_0^{-y+1} f(x,y)\,dx\,dy \quad \text{ou} \quad \int_0^1 \int_0^{-x+1} f(x,y)\,dy\,dx$$

b) $x \ge 0 \wedge y \ge 0 \wedge x \le 1 - y^2$

$y^2 = 1 - x$

$y = \pm\sqrt{1-x}$



$$\int_0^1 \int_0^{1-y^2} f(x,y)\,dx\,dy$$

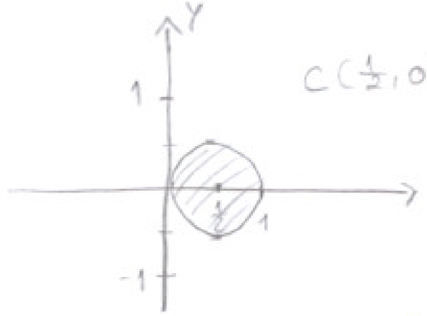
ou

$$\int_0^1 \int_0^{\sqrt{1-x}} f(x,y)\,dy\,dx$$

c) $x^2 + y^2 \le x$

$x^2 - x + y^2 \le 0$

$\left(x - \frac{1}{2}\right)^2 + y^2 \le \frac{1}{4}$



$C\left(\frac{1}{2}, 0\right) \quad r = \frac{1}{2}$

$$\int_0^1 \int_{-\sqrt{\frac{1}{4} - (x-\frac{1}{2})^2}}^{\sqrt{\frac{1}{4} - (x-\frac{1}{2})^2}} f(x,y)\,dy\,dx$$

$y^2 = \frac{1}{4} - \left(x - \frac{1}{2}\right)^2$

$y = \pm\sqrt{\frac{1}{4} - \left(x - \frac{1}{2}\right)^2}$

ou

$$\int_{-\frac{1}{2}}^{\frac{1}{2}} \int_{\frac{1}{2} - \sqrt{\frac{1}{4} - y^2}}^{\frac{1}{2} + \sqrt{\frac{1}{4} - y^2}} f(x,y)\,dx\,dy$$

$\left(x - \frac{1}{2}\right)^2 = \frac{1}{4} - y^2$

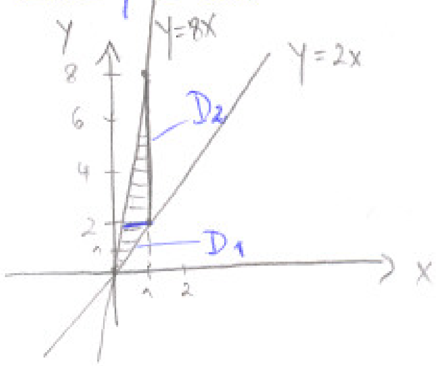
$x - \frac{1}{2} = \pm\sqrt{\frac{1}{4} - y^2}$

$x = \frac{1}{2} \pm \sqrt{\frac{1}{4} - y^2}$

3. a) $\int_0^1 \int_{2x}^{8x} f(x,y)\, dy\, dx$

$0 \le x \le 1$

$2x \le y \le 8x$



$$\int_0^2 \int_{\frac{y}{8}}^{\frac{y}{2}} f(x,y)\, dx\, dy +$$

$$\int_2^8 \int_{\frac{y}{8}}^{1} f(x,y)\, dx\, dy$$

$D = \{ (x,y) \in \mathbb{R}^2 \mid 0 \le x \le 1 ,\ 2x \le y \le 8x \}$

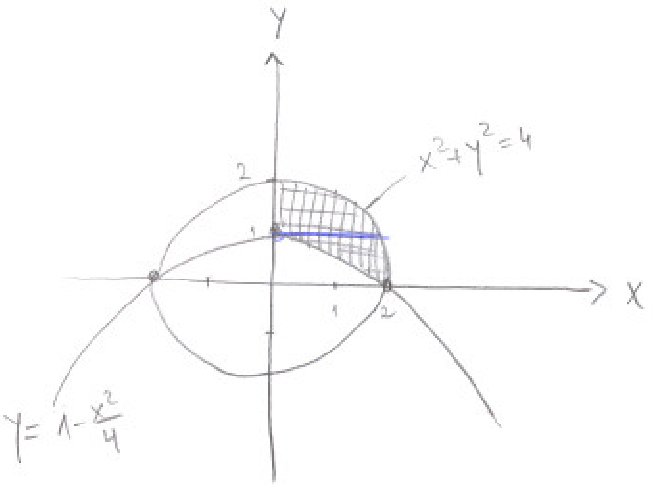
$D_1 = \{ (x,y) \in \mathbb{R}^2 \mid 0 \le y \le 2 ,\ \frac{y}{8} \le x \le \frac{y}{2} \}$

$D_2 = \{ (x,y) \in \mathbb{R}^2 \mid 2 \le y \le 8 ,\ \frac{y}{8} \le x \le 1 \}$

b) $\int_0^2 \int_{1-\frac{x^2}{4}}^{\sqrt{4-x^2}} f(x,y)\, dy\, dx$

$D = \{ (x,y) \in \mathbb{R}^2 \mid 0 \le x \le 2 ,\ 1-\frac{x^2}{4} \le y \le \sqrt{4-x^2} \}$

$$\int_0^1 \int_{2\sqrt{1-y}}^{\sqrt{4-y^2}} f(x,y)\, dx\, dy +$$



$$\int_1^2 \int_0^{\sqrt{4-y^2}} f(x,y)\, dx\, dy$$

$1 - \frac{x^2}{4} \le y$

$x = \pm\sqrt{4-4y}$

$x = \pm 2\sqrt{1-y}$

$y \le \sqrt{4-x^2}$

$x^2+y^2 \le 4$

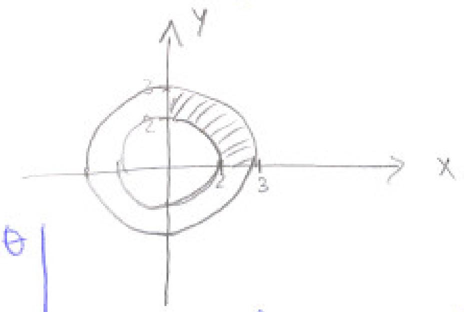
$x = \pm\sqrt{4-y^2}$

4.

a) $\iint_D \sqrt{x^2+y^2}\,dx\,dy \quad D=\{(x,y)\in\mathbb{R}^2 : 4\le x^2+y^2\le 9 \wedge x\ge 0 \wedge y\ge 0\}$

Usando coordenadas polares:

$$\begin{cases} x=\rho\cos\theta = x(\rho,\theta) \\ y=\rho\sin\theta = y(\rho,\theta)\end{cases}$$



$$|J| = \begin{vmatrix} \frac{\partial x}{\partial\rho} & \frac{\partial x}{\partial\theta} \\ \frac{\partial y}{\partial\rho} & \frac{\partial y}{\partial\theta}\end{vmatrix} = \begin{vmatrix} \cos\theta & -\rho\sin\theta \\ \sin\theta & \rho\cos\theta\end{vmatrix} = \rho\cos^2\theta+\rho\sin^2\theta=\rho$$

$$x^2+y^2=\rho^2$$

$$4\le x^2+y^2\le 9 \longrightarrow 4\le\rho^2\le 9 \longrightarrow \underline{2\le\rho\le 3}$$

$$x\ge 0 \wedge y\ge 0 \longrightarrow \underline{0\le\theta\le\tfrac{\pi}{2}}$$

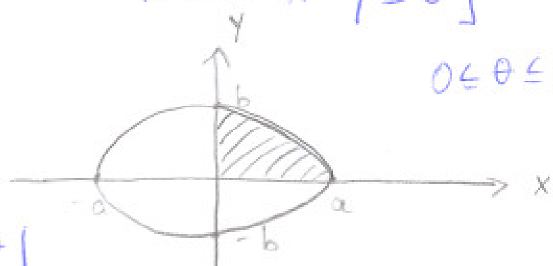
$$\int_2^3\int_0^{\frac{\pi}{2}} \sqrt{\rho^2}\,\rho\,d\theta\,d\rho = \int_2^3\int_0^{\frac{\pi}{2}} \rho^2\,d\theta\,d\rho = \int_2^3 \rho^2[\theta]_0^{\frac{\pi}{2}}\,d\rho =$$

$$= \int_2^3 \rho^2\frac{\pi}{2}\,d\rho = \frac{\pi}{2}\left[\frac{\rho^3}{3}\right]_2^3 = \frac{\pi}{2}\left[\frac{27}{3}-\frac{8}{3}\right] = \frac{\pi}{2}\cdot\frac{19}{3} = \frac{19}{6}\pi$$

b) $\iint_D \sqrt{1-\frac{x^2}{a^2}-\frac{y^2}{b^2}}\,dx\,dy \quad D=\{(x,y)\in\mathbb{R}^2 : \frac{x^2}{a^2}+\frac{y^2}{b^2}\le 1 \wedge x\ge 0 \wedge y\ge 0\}$

$$\begin{cases} x=a\rho\cos\theta = x(\rho,\theta) \\ y=b\rho\sin\theta = y(\rho,\theta)\end{cases}$$

$0\le\theta\le\frac{\pi}{2}$



$$|J| = \begin{vmatrix} \frac{\partial x}{\partial\rho} & \frac{\partial x}{\partial\theta} \\ \frac{\partial y}{\partial\rho} & \frac{\partial y}{\partial\theta}\end{vmatrix} = \begin{vmatrix} a\cos\theta & -a\rho\sin\theta \\ b\sin\theta & b\rho\cos\theta\end{vmatrix} = ab\rho\cos^2\theta+ab\rho\sin^2\theta$$

$$=ab\rho$$

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}\le 1 \longrightarrow \frac{a^2}{a^2}\rho^2\cos^2\theta+\frac{b^2}{b^2}\rho^2\sin^2\theta\le 1$$

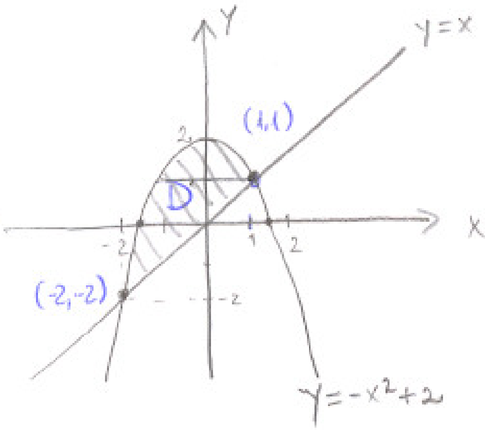
$$\rho^2\le 1 \longrightarrow \underline{0\le\rho\le 1}$$

$$\int_0^1 \int_0^{\frac{\pi}{2}} \sqrt{1-\rho^2}\, ab\rho\, d\theta\, d\rho = \int_0^1 \sqrt{1-\rho^2}\, [\theta]_0^{\frac{\pi}{2}}\, ab\rho\, d\rho =$$

$$\int_0^1 \frac{\pi}{2} \sqrt{1-\rho^2}\, ab\rho\, d\rho = \frac{\pi}{2}\, ab \left[-\frac{(1-\rho^2)^{3/2}}{3}\right]_0^1 = \frac{\pi}{2}\, ab\left(+\frac{1}{3}\right)$$

$$= \frac{\pi}{6} ab$$

5.



$$\begin{cases} Y = 2-x^2 \\ Y = X \end{cases} \qquad \begin{cases} x^2+X-2=0 \\ Y=X \end{cases}$$

$$X_{1,2} = \frac{-1 \pm \sqrt{1+8}}{2} = \frac{-1\pm 3}{2} \begin{cases} 1 \\ -2 \end{cases}$$

$$\begin{cases} X_1 = 1 \\ Y_1 = 1 \end{cases} \qquad \begin{cases} X_2 = -2 \\ Y_2 = -2 \end{cases}$$

$$x^2 = 2-Y$$
$$X = \pm\sqrt{2-Y}$$

$$A = \iint_D 1\, dA$$

$$A = \int_{-2}^{1} \int_{X}^{-x^2+2} 1 \; dy\, dx = \int_{-2}^{1} [Y]_{Y=X}^{Y=-x^2+2}\, dx =$$

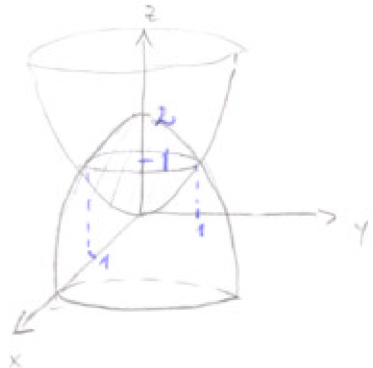
$$= \int_{-2}^{1} (-x^2+2-x)\, dx = \left[-\frac{x^3}{3} + 2x - \frac{x^2}{2}\right]_{-2}^{1} = -\frac{1}{3} + 2 - \frac{1}{2}$$

$$-\frac{8}{3} + 4 + 2 = 8 - 3 - \frac{1}{2} = 5 - \frac{1}{2} = \frac{9}{2}$$ //

6. $S = \{ (x,y,z) \in \mathbb{R}^3 : \; x^2 + y^2 \leq z \leq 2 - x^2 - y^2 \}$

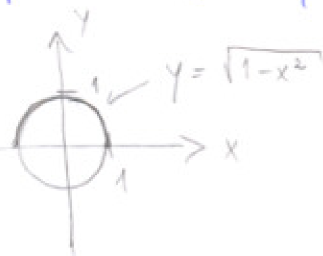
$z = x^2 + y^2$
parabolóide

$z = 2 - x^2 - y^2$
parabolóide



$$\begin{cases} z = x^2 + y^2 \\ z = 2 - x^2 - y^2 \end{cases} \quad \begin{cases} z = x^2 + y^2 \\ x^2 + y^2 = 2 - x^2 - y^2 \end{cases} \quad \begin{cases} z = x^2 + y^2 \\ 2x^2 + 2y^2 = 2 \end{cases} \quad \begin{cases} z = 1 \\ x^2 + y^2 = 1 \end{cases}$$

$$\boxed{V = 4 \cdot \int_0^1 \int_0^{\sqrt{1-x^2}} \left[2 - x^2 - y^2 - (x^2 + y^2) \right] dy\, dx}$$



$$= 4 \int_0^1 \int_0^{\sqrt{1-x^2}} (2 - 2x^2 - 2y^2)\, dy\, dx = 4 \int_0^1 \left[2y - 2x^2 y - 2\frac{y^3}{3} \right]_{y=0}^{y=\sqrt{1-x^2}} dx$$

$$= 4 \int_0^1 \left[2\sqrt{1-x^2} - 2x^2\sqrt{1-x^2} - \frac{2}{3}(1-x^2)\sqrt{1-x^2} \right] dx$$

$= \ldots$

ou $$V = 4 \int_0^1 \int_0^{\sqrt{1-x^2}} (2 - x^2 - y^2)\, dy\, dx - 4 \int_0^1 \int_0^{\sqrt{1-x^2}} (x^2 + y^2)\, dy\, dx$$