Table of Fourier Transform Pairs

Function, f(t)	**Fourier Transform, F(ω)**
Definition of Inverse Fourier Transform $$f(t)=\frac{1}{2\pi}\int_{-\infty}^{\infty}F(\omega)e^{j\omega t}\,d\omega$$	***Definition of Fourier Transform*** $$F(\omega)=\int_{-\infty}^{\infty}f(t)e^{-j\omega t}\,dt$$
$f(t-t_0)$	$F(\omega)e^{-j\omega t_0}$
$f(t)e^{j\omega_0 t}$	$F(\omega-\omega_0)$
$f(\alpha t)$	$\frac{1}{\lvert\alpha\rvert}F(\frac{\omega}{\alpha})$
$F(t)$	$2\pi f(-\omega)$
$\frac{d^n f(t)}{dt^n}$	$(j\omega)^n F(\omega)$
$(-jt)^n f(t)$	$\frac{d^n F(\omega)}{d\omega^n}$
$\int_{-\infty}^{t} f(\tau)d\tau$	$\frac{F(\omega)}{j\omega}+\pi F(0)\delta(\omega)$
$\delta(t)$	1
$e^{j\omega_0 t}$	$2\pi\delta(\omega-\omega_0)$
sgn *(t)*	$\frac{2}{j\omega}$

Fourier Transform Table

$F(t)$	$\widehat{F}(\omega)$	Notes	(0)
$f(t)$	$\int_{-\infty}^{\infty} f(t)e^{-i\omega t}\,dt$	Definition.	(1)
$\frac{1}{2\pi}\int_{-\infty}^{\infty} \widehat{f}(\omega)e^{i\omega t}\,d\omega$	$\widehat{f}(\omega)$	Inversion formula.	(2)
$\widehat{f}(-t)$	$2\pi f(\omega)$	Duality property.	(3)
$e^{-at}u(t)$	$\frac{1}{a+i\omega}$	a constant, $\Re e(a) > 0$	(4)
$e^{-a\lvert t\rvert}$	$\frac{2a}{a^2+\omega^2}$	a constant, $\Re e(a) > 0$	(5)
$\beta(t) = \begin{cases} 1, & \text{if } \lvert t\rvert < 1, \\ 0, & \text{if } \lvert t\rvert > 1 \end{cases}$	$2\,\mathrm{sinc}(\omega) = 2\frac{\sin(\omega)}{\omega}$	Boxcar in time.	(6)
$\frac{1}{\pi}\,\mathrm{sinc}(t)$	$\beta(\omega)$	Boxcar in frequency.	(7)
$f'(t)$	$i\omega\widehat{f}(\omega)$	Derivative in time.	(8)
$f''(t)$	$(i\omega)^2\widehat{f}(\omega)$	Higher derivatives similar.	(9)
$tf(t)$	$i\frac{d}{d\omega}\widehat{f}(\omega)$	Derivative in frequency.	(10)
$t^2f(t)$	$i^2\frac{d^2}{d\omega^2}\widehat{f}(\omega)$	Higher derivatives similar.	(11)
$e^{i\omega_0 t}f(t)$	$\widehat{f}(\omega-\omega_0)$	Modulation property.	(12)
$f\left(\frac{t-t_0}{k}\right)$	$ke^{-i\omega t_0}\widehat{f}(k\omega)$	Time shift and squeeze.	(13)
$(f*g)(t)$	$\widehat{f}(\omega)\widehat{g}(\omega)$	Convolution in time.	(14)
$u(t) = \begin{cases} 0, & \text{if } t < 0 \\ 1, & \text{if } t > 0 \end{cases}$	$\frac{1}{i\omega} + \pi\delta(\omega)$	Heaviside step function.	(15)
$\delta(t-t_0)f(t)$	$e^{-i\omega t_0}f(t_0)$	Assumes f continuous at t_0.	(16)
$e^{i\omega_0 t}$	$2\pi\delta(\omega-\omega_0)$	Useful for $\sin(\omega_0 t)$, $\cos(\omega_0 t)$.	(17)

Convolution: $$(f*g)(t) = \int_{-\infty}^{\infty} f(t-u)g(u)\,du = \int_{-\infty}^{\infty} f(u)g(t-u)\,du.$$

Parseval: $$\int_{-\infty}^{\infty} \lvert f(t)\rvert^2\,dt = \frac{1}{2\pi}\int_{-\infty}^{\infty} \left\lvert \widehat{f}(\omega)\right\rvert^2\,d\omega.$$

$j\dfrac{1}{\pi t}$	$\mathrm{sgn}(\omega)$
$u(t)$	$\pi\delta(\omega)+\dfrac{1}{j\omega}$
$\sum\limits_{n=-\infty}^{\infty}F_n e^{jn\omega_0 t}$	$2\pi\sum\limits_{n=-\infty}^{\infty}F_n\delta(\omega-n\omega_0)$
$rect(\dfrac{t}{\tau})$	$\tau Sa(\dfrac{\omega\tau}{2})$
$\dfrac{B}{2\pi}Sa(\dfrac{Bt}{2})$	$rect(\dfrac{\omega}{B})$
$tri(t)$	$Sa^2(\dfrac{\omega}{2})$
$A\cos(\dfrac{\pi t}{2\tau})rect(\dfrac{t}{2\tau})$	$\dfrac{A\pi}{\tau}\dfrac{\cos(\omega\tau)}{(\pi/2\tau)^2-\omega^2}$
$\cos(\omega_0 t)$	$\pi[\delta(\omega-\omega_0)+\delta(\omega+\omega_0)]$
$\sin(\omega_0 t)$	$\dfrac{\pi}{j}[\delta(\omega-\omega_0)-\delta(\omega+\omega_0)]$
$u(t)\cos(\omega_0 t)$	$\dfrac{\pi}{2}[\delta(\omega-\omega_0)+\delta(\omega+\omega_0)]+\dfrac{j\omega}{\omega_0^2-\omega^2}$
$u(t)\sin(\omega_0 t)$	$\dfrac{\pi}{2j}[\delta(\omega-\omega_0)-\delta(\omega+\omega_0)]+\dfrac{\omega^2}{\omega_0^2-\omega^2}$
$u(t)e^{-\alpha t}\cos(\omega_0 t)$	$\dfrac{(\alpha+j\omega)}{\omega_0^2+(\alpha+j\omega)^2}$

$u(t)e^{-\alpha t}\,\sin(\omega_0 t)$	$\dfrac{\omega_0}{\omega_0^2+(\alpha+j\omega)^2}$
$e^{-\alpha\lvert t\rvert}$	$\dfrac{2\alpha}{\alpha^2+\omega^2}$
$e^{-t^2/(2\sigma^2)}$	$\sigma\sqrt{2\pi}\;e^{-\sigma^2\omega^2/2}$
$u(t)e^{-\alpha t}$	$\dfrac{1}{\alpha+j\omega}$
$u(t)te^{-\alpha t}$	$\dfrac{1}{(\alpha+j\omega)^2}$

- ***Trigonometric Fourier Series***

$$f(t)=a_0+\sum_{n=1}^{\infty}\left(a_n\cos(\omega_0 nt)+b_n\sin(\omega_0 nt)\right)$$

where

$$a_0=\frac{1}{T}\int_0^T f(t)dt \quad , \quad a_n=\frac{2}{T}\int_0^T f(t)\cos(\omega_0 nt)dt \quad \text{, and}$$

$$b_n=\frac{2}{T}\int_0^T f(t)\sin(\omega_0 nt)dt$$

- ***Complex Exponential Fourier Series***

$$f(t)=\sum_{n=-\infty}^{\infty}F_n e^{j\omega nt} \quad \text{, where} \qquad F_n=\frac{1}{T}\int_0^T f(t)e^{-j\omega_0 nt}dt$$

Some Useful Mathematical Relationships

$\cos(x) = \dfrac{e^{jx} + e^{-jx}}{2}$
$\sin(x) = \dfrac{e^{jx} - e^{-jx}}{2j}$
$\cos(x \pm y) = \cos(x)\cos(y) \mp \sin(x)\sin(y)$
$\sin(x \pm y) = \sin(x)\cos(y) \pm \cos(x)\sin(y)$
$\cos(2x) = \cos^2(x) - \sin^2(x)$
$\sin(2x) = 2\sin(x)\cos(x)$
$2\cos^2(x) = 1 + \cos(2x)$
$2\sin^2(x) = 1 - \cos(2x)$
$\cos^2(x) + \sin^2(x) = 1$
$2\cos(x)\cos(y) = \cos(x - y) + \cos(x + y)$
$2\sin(x)\sin(y) = \cos(x - y) - \cos(x + y)$
$2\sin(x)\cos(y) = \sin(x - y) + \sin(x + y)$

Useful Integrals

$\int \cos(x)dx$	$\sin(x)$
$\int \sin(x)dx$	$-\cos(x)$
$\int x\cos(x)dx$	$\cos(x) + x\sin(x)$
$\int x\sin(x)dx$	$\sin(x) - x\cos(x)$
$\int x^2 \cos(x)dx$	$2x\cos(x) + (x^2 - 2)\sin(x)$
$\int x^2 \sin(x)dx$	$2x\sin(x) - (x^2 - 2)\cos(x)$
$\int e^{\alpha x} dx$	$\dfrac{e^{\alpha x}}{a}$
$\int xe^{\alpha x} dx$	$e^{\alpha x}\left[\dfrac{x}{a} - \dfrac{1}{a^2}\right]$
$\int x^2 e^{\alpha x} dx$	$e^{\alpha x}\left[\dfrac{x^2}{a} - \dfrac{2x}{a^2} - \dfrac{2}{a^3}\right]$
$\int \dfrac{dx}{\alpha + \beta x}$	$\dfrac{1}{\beta}\ln\lvert\alpha + \beta x\rvert$
$\int \dfrac{dx}{\alpha^2 + \beta^2 x^2}$	$\dfrac{1}{\alpha\beta}\tan^{-1}(\dfrac{\beta x}{\alpha})$

Engineering Tables/Fourier Transform Table 2

Signal	Fourier transform unitary, angular frequency	Fourier transform unitary, ordinary frequency	Remarks
$g(t) \equiv$ $\frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} G(\omega) e^{i\omega t} d\omega$	$G(\omega) \equiv$ $\frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} g(t) e^{-i\omega t} dt$	$G(f) \equiv$ $\int_{-\infty}^{\infty} g(t) e^{-i2\pi f t} dt$	
10 $\mathrm{rect}(at)$	$\frac{1}{\sqrt{2\pi a^2}} \cdot \mathrm{sinc}\left(\frac{\omega}{2\pi a}\right)$	$\frac{1}{\|a\|} \cdot \mathrm{sinc}\left(\frac{f}{a}\right)$	The rectangular pulse and the normalized sinc function
11 $\mathrm{sinc}(at)$	$\frac{1}{\sqrt{2\pi a^2}} \cdot \mathrm{rect}\left(\frac{\omega}{2\pi a}\right)$	$\frac{1}{\|a\|} \cdot \mathrm{rect}\left(\frac{f}{a}\right)$	Dual of rule 10. The rectangular function is an idealized low-pass filter, and the sinc function is the non-causal impulse response of such a filter.
12 $\mathrm{sinc}^2(at)$	$\frac{1}{\sqrt{2\pi a^2}} \cdot \mathrm{tri}\left(\frac{\omega}{2\pi a}\right)$	$\frac{1}{\|a\|} \cdot \mathrm{tri}\left(\frac{f}{a}\right)$	*tri* is the triangular function
13 $\mathrm{tri}(at)$	$\frac{1}{\sqrt{2\pi a^2}} \cdot \mathrm{sinc}^2\left(\frac{\omega}{2\pi a}\right)$	$\frac{1}{\|a\|} \cdot \mathrm{sinc}^2\left(\frac{f}{a}\right)$	Dual of rule 12.
14 $e^{-\alpha t^2}$	$\frac{1}{\sqrt{2\alpha}} \cdot e^{-\frac{\omega^2}{4\alpha}}$	$\sqrt{\frac{\pi}{\alpha}} \cdot e^{-\frac{(\pi f)^2}{\alpha}}$	Shows that the Gaussian function exp(- at^2) is its own Fourier transform. For this to be integrable we must have Re(a) > 0.

$e^{iat^2} = e^{-\alpha t^2}\Big\vert_{\alpha=-ia}$	$\frac{1}{\sqrt{2a}} \cdot e^{-i\left(\frac{\omega^2}{4a} - \frac{\pi}{4}\right)}$	$\sqrt{\frac{\pi}{a}} \cdot e^{-i\left(\frac{\pi^2 f^2}{a} - \frac{\pi}{4}\right)}$	common in optics
$\cos(at^2)$	$\frac{1}{\sqrt{2a}} \cos\left(\frac{\omega^2}{4a} - \frac{\pi}{4}\right)$	$\sqrt{\frac{\pi}{a}} \cos\left(\frac{\pi^2 f^2}{a} - \frac{\pi}{4}\right)$	
$\sin(at^2)$	$\frac{-1}{\sqrt{2a}} \sin\left(\frac{\omega^2}{4a} - \frac{\pi}{4}\right)$	$-\sqrt{\frac{\pi}{a}} \sin\left(\frac{\pi^2 f^2}{a} - \frac{\pi}{4}\right)$	
$e^{-a\vert t\vert}$	$\sqrt{\frac{2}{\pi}} \cdot \frac{a}{a^2 + \omega^2}$	$\frac{2a}{a^2 + 4\pi^2 f^2}$	*a>0*
$\frac{1}{\sqrt{\vert t\vert}}$	$\frac{1}{\sqrt{\vert\omega\vert}}$	$\frac{1}{\sqrt{\vert f\vert}}$	the transform is the function itself
$J_0(t)$	$\sqrt{\frac{2}{\pi}} \cdot \frac{\mathrm{rect}\left(\frac{\omega}{2}\right)}{\sqrt{1-\omega^2}}$	$\frac{2 \cdot \mathrm{rect}(\pi f)}{\sqrt{1 - 4\pi^2 f^2}}$	$J_0(t)$ is the Bessel function of first kind of order 0, *rect* is the rectangular function
$J_n(t)$	$\sqrt{\frac{2}{\pi}} \frac{(-i)^n T_n(\omega) \mathrm{rect}\left(\frac{\omega}{2}\right)}{\sqrt{1-\omega^2}}$	$\frac{2(-i)^n T_n(2\pi f) \mathrm{rect}(\pi f)}{\sqrt{1 - 4\pi^2 f^2}}$	it's the generalization of the previous transform; $T_n(t)$ is the Chebyshev polynomial of the first kind.
$\frac{J_n(t)}{t}$	$\sqrt{\frac{2}{\pi}} \frac{i}{n} (-i)^n \cdot U_{n-1}(\omega)$ $\cdot \sqrt{1-\omega^2} \mathrm{rect}\left(\frac{\omega}{2}\right)$	$\frac{2i}{n} (-i)^n \cdot U_{n-1}(2\pi f)$ $\cdot \sqrt{1-4\pi^2 f^2} \mathrm{rect}(\pi f)$	$U_n(t)$ is the Chebyshev polynomial of the second kind