Teste 1 2019/2022

1- Diagrama de radiação

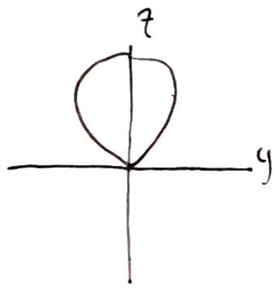
$$K = \frac{2\pi}{\lambda}$$

$$\lambda = \frac{c}{f} =$$

$$\frac{\cos\left(\frac{Kl}{2} \cdot \cos\theta\right) - \cos\left(\frac{Kl}{2}\right)}{\sin\theta}$$

De forma a eliminar interferência é necessário haver zona em $\theta = \frac{\pi}{2}$

E para não existir Interrupções é necessário não existirem mais zeros Portanto



$\theta = \pi/2$

2- 1 Km é Campo Distante $\quad Kr >> 1$

$$Kr = 100 \Leftrightarrow \underset{(1\,km)}{K \times 1000} = 100 \Leftrightarrow \frac{2\pi}{\lambda} \cdot 1000 = 100$$

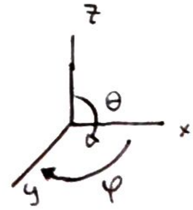
$$\Leftrightarrow \frac{200\cancel{0} \cdot \pi}{10\cancel{0}} = \lambda \Leftrightarrow \lambda = 20\pi$$

$$\lambda = \frac{c}{f} \Leftrightarrow 20\pi = \frac{3 \times 10^8}{f}$$

$$\Leftrightarrow f = \frac{3 \times 10^8}{20\pi} \; Hz$$

3-

No eixo OY



$$E_\varphi = j\eta \cdot \frac{I_0 \cdot e^{-jkr}}{2\pi r} \cdot \left(\frac{\cos\left(\frac{kl}{2} \cdot \overset{\psi}{\cos\theta}\right) - \cos\left(\frac{kl}{2}\right)}{\underset{\psi}{\sin\theta}} \right)$$

$$\hat{r} = \underbrace{\sin\theta \cdot \cos\varphi\, \hat{i}}_{OX} + \underbrace{\sin\theta \cdot \sin\varphi \cdot \hat{j}}_{OY} + \underbrace{\cos\theta\, \hat{k}}_{OZ}$$

$$\cos\psi = \sin\theta \cdot \sin\varphi$$

$$\cos^2\psi + \sin^2\psi = 1 \Leftrightarrow \sin^2\psi = 1 - \cos^2\psi \Leftrightarrow \sin\psi = \sqrt{1 - \sin^2\theta \cdot \sin^2\varphi}$$

$$E_\varphi = j\eta \cdot \frac{I_0 \cdot e^{-jkr}}{2\pi r} \cdot \left(\frac{\cos\left(\frac{kl}{2} \cdot \sin\theta \cdot \sin\varphi\right) - \cos\left(\frac{kl}{2}\right)}{\sqrt{1 - \sin^2\theta \cdot \sin^2\varphi}} \right)$$

eixo OY $\varphi = 0\ 90 \to \pi/2$

$$\cos\left(\frac{kl}{2} \cdot \sin\theta\right) - \cos\left(\frac{kl}{2}\right) = 0 \quad \wedge \quad \sqrt{1 - \sin^2\theta} \neq 0$$

$$\frac{kl}{2} \sin\theta = \pm \frac{kl}{2} \pm 2m\pi$$

$$\sin\theta = \pm 1 \pm \frac{\frac{2m\pi}{1}}{\frac{kl}{2}} \left(= \frac{\frac{2 \cdot m \cdot \pi}{1}}{\frac{\frac{2\pi}{\lambda} \cdot l}{2}} = \frac{\frac{2m\pi}{1}}{\left(\frac{\frac{2\pi l}{\lambda}}{\frac{2}{1}}\right)} = \frac{\frac{2m\pi}{1}}{\frac{\cancel{2}\pi l}{\cancel{2}\lambda}} \right) = \frac{2m\pi\lambda}{\cancel{\pi} l} = \frac{2m\lambda}{l}$$

$$\sin\theta = \pm 1 \pm \frac{2m\lambda}{l}$$

$$1 = -1 + \frac{2m\lambda}{l}\Big|_{m=1} \Leftrightarrow 2 = \frac{2\lambda}{l} \Leftrightarrow l = 20\pi$$

d) $\vec{S} = \frac{1}{2}\cdot \vec{E_\psi} \wedge \vec{H_x}$

$$\vec{S} = S_r = \frac{1}{2}\cdot \vec{E_\psi}\cdot \frac{\vec{E_\psi}}{\eta} = \frac{1}{2\eta}\cdot |\vec{E_\psi}|^2 = 20\times10^{-6}\ W/m^2$$

$$E_\psi = j\eta\, \frac{I_0 \cdot e^{-jkr}}{2\pi r}\cdot\left(\frac{\cos(\frac{kl}{2}\cdot\sin\theta) - \cos(\frac{kl}{2})}{\cos\theta}\right)$$

$$\hookrightarrow \frac{\cos(\frac{kl}{2}\cdot\frac{\sqrt{2}}{2}) - \cos(\frac{kl}{2})}{\frac{\sqrt{2}}{2}}$$

$$\vec{S} = \frac{1}{2\cdot\eta}\cdot(-1)\,\eta^2\, \frac{I_0^2\cdot(1)}{4\pi^2\cdot r^2}\left(\frac{\cos(\frac{kl}{2}\cdot\frac{\sqrt{2}}{2}) - \cos(\frac{kl}{2})}{\frac{\sqrt{2}}{2}}\right)^2$$

$$K = \frac{2\pi}{\lambda} = \frac{2\pi}{20\pi} = \frac{1}{10}$$

$$\frac{kl}{2} = \frac{\frac{1}{10}\,l}{2} = \frac{\frac{\cancel{20}\pi}{\cancel{10}}}{\cancel{2}} = \pi$$

$$S = -\frac{1}{2\cdot\eta}\cdot\eta^2\cdot\frac{I_0^2}{4\pi\cdot r^2}\cdot\underbrace{\left(\frac{-0{,}606-(-1)}{\frac{\sqrt{2}}{2}}\right)^2}_{0{,}311}$$

$$20\times10^{-6} = -\eta\cdot\frac{I_0^2}{8\pi\cdot r^2}\cdot(0{,}311)$$

$$20\times10^{-6} = \frac{-120\pi\cdot I_0^2}{8\pi(30.000)^2}\cdot 0{,}31 \Leftrightarrow 4053667.94 = -120\pi\cdot I_0^2$$

$$\Leftrightarrow I_0^2 = 10752.688\ A$$

e-

60%

$$W = \iint Re\{\vec{S}\}\, ds = \int_0^{2\pi}\int_0^{\pi} 20\times10^{-6} . r^2 . \sin\theta \; d\theta\, d\varphi$$

$Re\{\vec{S}_n\}$ -> densidade Potencia média no temp

$$W = \iint Re\{S\}\ \hat{n} . dS = \int_0^{2\pi}\int_0^{\pi} 20\times10^{-6} . r^2 . \sin\theta$$

$$= 20\times10^{-6} \times 2\pi \times r^2 . \int_0^{\pi} \sin\theta$$

$$= 20\times10^{-6} \times 2\pi \times r^2 . [-\cos\theta]_0^{\pi}$$

$$= \quad [1-(-1)] (=2)$$

$$= 20\times10^{-6} \times 4\pi \times r^2$$

$$= 400\pi \times 10^{2}\ W \rightarrow$$

eficiencia do sistema

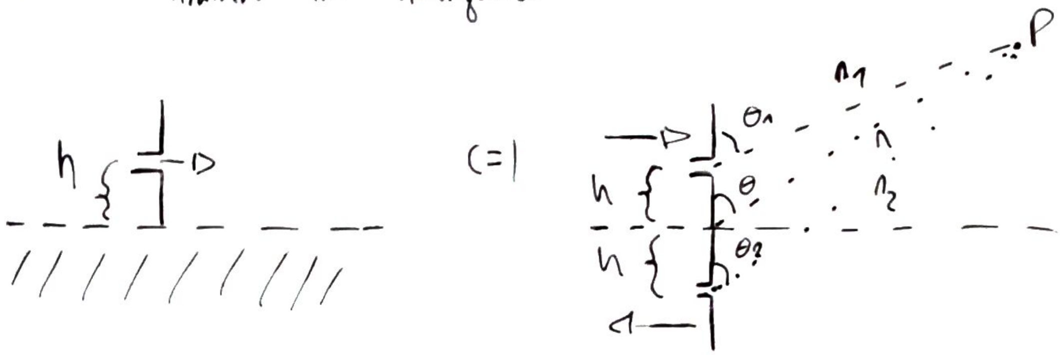
$$\frac{W}{W+P_p} = 0,6 \Leftrightarrow P_p = \frac{2}{3} . W$$

Potencia a gerar é $W + P_p = W + \frac{2}{3} W = W\left(1+\frac{2}{3}\right) = W . \frac{5}{3}$

$$= \frac{5}{3} . (400\pi \times 10^{2})$$

P)

Antena na horizontal



$$E_\psi^t = E_\psi^d + Rh \cdot E_\psi^r \qquad \text{com } Rh = -1$$

$$E_\psi^d = j\eta \cdot \frac{I_0 \cdot e^{-jkr_1}}{2\pi r_1} \cdot \left(\frac{\cos\left(\frac{kl}{2} \cdot \sin\theta_1 \sin\varphi\right) - \cos\left(\frac{kl}{2}\right)}{\cos(\theta_1)} \right)$$

$$E_\psi^r = j\eta \cdot \frac{I_0 \cdot e^{-jkr_2}}{2\pi \cdot r_2} \cdot \left(\frac{\cos\left(\frac{kl}{2} \cdot \sin\theta_2 \cdot \sin\varphi\right) - \cos\left(\frac{kl}{2}\right)}{\cos(\theta_1)} \right)$$

As aproximações são as seguintes

módulo: $\theta = \theta_1 = \theta_2$
$r_0 = r_1 = r_2$

Fase: $r_1 = r - h \cdot \cos\theta$
$r_2 = r + h \cdot \cos\theta$

$$E_\psi^t = j\eta \cdot \frac{I_0}{2\pi r} \cdot \left(\frac{\cos\left(\frac{kl}{2} \sin\theta\right) - \cos\left(\frac{kl}{2}\right)}{\cos\theta} \right) \cdot \left(e^{-jk(r - h\cos\theta)} - e^{-jk(r + h\cos\theta)} \right)$$

$$\left(e^{-jkr} \cdot e^{+jkh \cdot \cos\theta} - e^{-jkr - jkh\cos\theta} \right)$$

$$= j\eta \cdot \frac{I_0}{2\pi r} \cdot \left(\frac{\cos\left(\frac{kl}{2} \cdot \sin\theta\right) - \cos\left(\frac{kl}{2}\right)}{\cos\theta} \right) \cdot e^{jkr} \cdot \Big(\underbrace{e^{jkh\cos\theta} - e^{-jkh \cdot \cos\theta}}_{2j \cdot \sin(kh \cdot \cos\theta)} \Big)$$

Zero da Aprox Fo u

$$\sin(Kh.\cos\theta) = 0 \iff Kh.\cos\theta = \pm n\pi$$

$$\iff \cos\theta = \pm \frac{n}{K.h}\pi \quad \left(\to \frac{n\cancel{\pi}}{\frac{2\cancel{\pi}.h}{\lambda}} = \frac{n\lambda}{2h} \right.$$

$$\theta_0 = \cos^{-1}(\phi) = \frac{\pi}{2}$$

$$\theta_1 = \cos^{-1}\left(\pm\frac{\lambda}{2h}\right)$$ → Como ~~não~~ se pretende manter o diagrama

$$\frac{\lambda}{2h} > 1 \quad e \quad -\frac{\lambda}{2h} < -1 \quad \Rightarrow \frac{\lambda}{2h} > 1 \iff h < \frac{\lambda}{2} \to 20\pi$$

ent $h < 10\pi$ metros

6- O facto da terra não ser ideal causa perdas devido a dois fatores

1º - Devido à condutividade finita, dispersa o feixe electromagnética causando perdas significativas

2º - condutividade finita significa que nem toda a onda incidente é refletida havendo energia perdida para a terra (Cerca 23% de perdas) h)

Para que o diagrama assuma uma forma trilobada é necessário mais 1 zero

$$\cos\theta = \pm m \frac{\lambda}{2h}$$

$$\theta_1 = \cos^{-1}\left(\frac{\lambda}{2h}\right) \Rightarrow \frac{\lambda}{2h} < 1 \quad e \quad \frac{\lambda}{2h} > 1$$