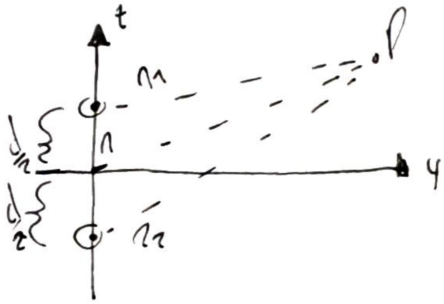
Teste 2 17/18

1- a)



$r_1 = r - \frac{d}{2}.\cos\theta$

$r_2 = r - \frac{d}{2}.\cos\theta$

$$E_\varphi = \frac{a.w.\mu.I_0.e^{-jkr}}{2r} . j_1(k.a.\sin\theta)$$

$$\psi = kd.\cos\theta + \beta$$

$$(AF) = e^{-j(k\frac{d}{2}.\cos\theta + \beta/2)} + e^{-j(k(-\frac{d}{2}).\cos\theta + \beta/2)}$$

$$= e^{-j(\frac{kd}{2}.\cos\theta \ \frac{\beta}{2})} + e^{j(\frac{kd}{2}.\cos\theta + \beta/2)}$$

$$= 2.\cos(\frac{kd}{2}.\cos\theta + \beta/2)$$

O campo do agrupado é igual à multiplicação do campo criado pelo elemento radiante,

$$E_\varphi = \frac{a.w.\mu.I_0.e^{-jkr}}{2r} . J_1(ka.\sin(\theta)) . 2.\cos(\frac{kd}{2}.\cos\theta + \beta/2)$$

---

b)

$\beta = \pi/2$

max radiação em $\theta = \pi/2$

$$\cos(\frac{kd}{2}.\cos\theta + \frac{\pi}{4})\Big|_{\theta=\frac{\pi}{2}} = \pm 1 \quad , \text{Impossível}$$

$$\cos(\frac{kd}{2}.\cos\theta + \beta)\Big|_{\theta=\pi/2} = 1 \qquad \text{se } \beta = \emptyset$$

Ent para o agregado transversal os elementos necessitam de ser excitados em fase. x c)

$$\cos\left(\frac{kd}{2}\cdot\cos\theta + \frac{\beta}{2}\right)$$

$\vee\ \beta = \pi/2$

$d = \lambda$

$c/\lambda = 4$

$y_1(x) = 0$ Para $x = 3{,}84\ ;\ 7{,}01$

$$ka \cdot \sin\theta = 3{,}84 \Leftrightarrow \sin\theta = \frac{3{,}84}{\underbrace{k.a}} = \frac{3{,}84}{4} \Leftrightarrow$$

$$\frac{2\pi . a}{\lambda} = \frac{c}{\lambda} = 4$$

$$\Leftrightarrow \arcsin\left(\frac{3{,}84}{4}\right) = \begin{pmatrix} 1{,}287 \text{ rad} - x \\ 2\pi \quad - 360 \end{pmatrix} = 73{,}73^{\circ}$$

Para o (AF) temos

$$\cos\left(\frac{kd}{2}\cdot\cos\theta + \frac{\pi}{4}\right) = 0 \Leftrightarrow \frac{kd}{2}\cdot\cos\theta + \frac{\pi}{4} = \pm\frac{\pi}{2} \pm m\pi$$

$$\Leftrightarrow \frac{kd}{2}\cdot\cos\theta = \pm\frac{\pi}{2} \pm m\pi - \frac{\pi}{4}$$

$$\Leftrightarrow \left(\frac{\frac{2\pi . \cancel{\lambda}}{\cancel{\lambda}}}{2}\right)\cdot\cos\theta = \pm\frac{\pi}{2} \pm m\pi - \frac{\pi}{4} \Leftrightarrow \pi . \cos\theta = \pm\frac{\pi}{2} \pm m\pi - \frac{\pi}{4}$$

$$\Leftrightarrow \cos\theta = \pm\frac{1}{2} \pm m - \frac{1}{4}$$

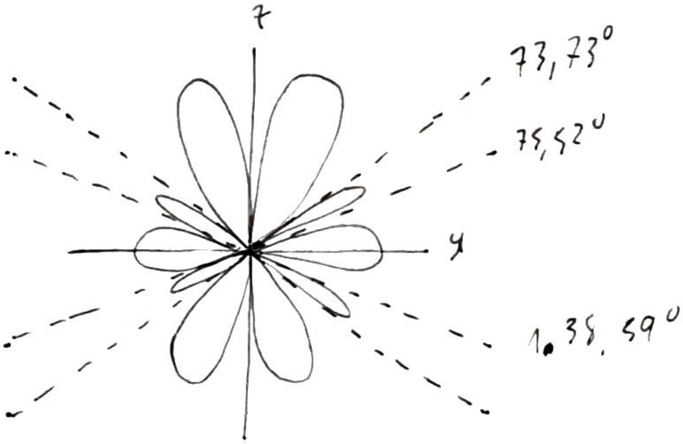
$$\theta = \arccos\left(\pm\frac{1}{2} \pm m - \frac{1}{4}\right)$$

$$\theta_0 = \arccos\left(\pm\frac{1}{2} - \frac{1}{4}\right) = \rightarrow \theta_{01} = \arccos(1/4) = 75{,}52^{\circ}$$

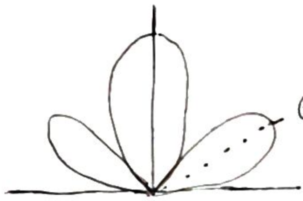
$$\rightarrow \theta_{02} = \arccos(-3/4) = 138{,}59^{\circ}$$

$$\theta_1 = \arccos\left(\pm\frac{1}{2} \pm 1 - 1/4\right) = \rightarrow \theta_{11} = \arccos(5/4) = \times$$

$$\rightarrow \theta_{12} = \arccos(-7/4) = \times$$



c))



$$|AF| = \cos\left(\frac{kd}{2}\cdot\cos\theta + \beta/2\right)$$

$$\cos\left(\frac{kd}{2}\cdot\cos\theta + \beta/2\right) = \pm 1$$

$$\left(\frac{kd}{2}\cdot\cos\theta + \beta/2\right) = \pm m\pi$$

$$\frac{kd}{2}\cdot\cos\left(\frac{\pi}{4}\right) + \beta/2 = \pm m\pi \Leftrightarrow \frac{kd}{2}\cdot\cos\left(\frac{\pi}{4}\right) = \pm m\pi - \frac{\beta}{2}$$

$$\Leftrightarrow \left(\frac{\frac{2\pi . d}{\lambda}}{2}\right)\cdot\cos\left(\frac{\pi}{4}\right) = \pm m\pi - \frac{\beta}{2} \Leftrightarrow \frac{\pi . d}{\lambda}\cdot\cos\left(\frac{\pi}{4}\right) = \pm m\pi - \frac{\beta}{2}$$

$$\Leftrightarrow \frac{\pi . d . \frac{\sqrt{2}}{2}}{\lambda} = \pm m\pi - \frac{\beta}{2} \Leftrightarrow \frac{\pi . d . \sqrt{2}}{2} = \pm m\lambda . \pi - \frac{\beta\lambda}{2}$$

$$\Leftrightarrow \pi . d . \sqrt{2} = \pm 2m\lambda . \pi - \beta\lambda \Leftrightarrow d . \sqrt{2} = \pm 2m\lambda - \frac{\beta\lambda}{\pi}$$

$$\Leftrightarrow d = \pm\frac{2m\lambda}{\sqrt{2}} - \frac{\beta\lambda}{\pi . \sqrt{2}} = \lambda\left(\pm\frac{2m}{\sqrt{2}} - \frac{\beta}{\pi . \sqrt{2}}\right) \rightarrow$$ garante a radiação max na direção $\theta = \pi/4$

Esboço do diagrama

$$\cos\left(\frac{kd}{2}\cdot\cos\theta + \beta/2\right) = 0$$

$$\frac{kd}{2}\cdot\cos\theta + \beta/2 = \pm\frac{\pi}{2} \pm m\pi$$

$$\frac{kd}{2}\cdot\cos\theta = \pm\frac{\pi}{2} \pm m\pi - \frac{\beta}{2}$$

$$\cos\theta = \frac{2}{kd}\cdot\left(\pm\frac{\pi}{2} \pm m\pi - \frac{\beta}{2}\right) = \left(\frac{2}{\frac{2\pi}{\lambda}\cdot d} = \frac{2\lambda}{2\pi d}\right)\cdot\left(\pm\frac{\pi}{2}\cdot \pm m\pi - \frac{\beta}{2}\right)$$

$$\cancel{\cos}\,\theta = \lambda$$

$$\left.\begin{aligned} \theta &= \arccos\left(\frac{\lambda}{\pi d}\cdot\left(\pm\frac{\pi}{2} \pm m\pi - \beta/2\right)\right) \\ d &= \lambda\cdot\left(\pm\frac{2m}{\sqrt{2}} - \frac{\beta}{\pi\cdot\sqrt{2}}\right) \end{aligned}\right\}$$ Eq. para localizar as zonas do AF

$$d = \lambda\left(\pm\frac{2m\pi}{\pi\cdot\sqrt{2}} - \frac{\beta}{\pi\cdot\sqrt{2}}\right) = \lambda\left(\frac{\pm 2m\pi - \beta}{\pi\cdot\sqrt{2}}\right) \Big/ \lambda\left(\frac{2\pi - \beta}{2\pi\sqrt{2}}\right) = \lambda\left(\frac{2\pi(1-\beta)}{2\pi\sqrt{2}}\right)$$

$$d = \lambda\left(\frac{1-\beta}{\sqrt{2}}\right)$$

$$m=0 \Rightarrow \quad \theta_0 = \arccos\left(\frac{\lambda}{\pi\cdot\left(\lambda\cdot\left(\frac{1-\beta}{\sqrt{2}}\right)\right)}\cdot\left(\frac{\pi-\beta}{2}\right)\right) = \arccos\left(\frac{\sqrt{2}}{2}\cdot\frac{\pi-\beta}{\pi(1-\beta)}\right)$$

$$\theta_1 = \arccos\left(\frac{\lambda}{\pi\cdot\left(\lambda\cdot\left(\frac{1-\beta}{\sqrt{2}}\right)\right)}\cdot\left(\frac{\pm(-\pi)-\beta}{2}\right)\right) =$$

$$\frac{\pi}{2} > 4\theta_0 \Leftrightarrow \theta_0 < \frac{\pi}{8}$$

$$\cos\left(\frac{\pi}{10}\right) = \frac{\sqrt{2}}{2}\cdot\frac{\pi-\beta}{\pi(1-\beta)} \Leftrightarrow \frac{\pi-\beta}{\pi(1-\beta)} = \sqrt{2}\cdot\cos\left(\frac{\pi}{10}\right) \Leftrightarrow (\pi-\beta) = \sqrt{2}\cdot\cos\left(\frac{\pi}{10}\right)\cdot\pi(1-\beta)$$

$$\Leftrightarrow \pi-\beta = \sqrt{2}\cdot\cos(\pi/10)\cdot\pi - \sqrt{2}\cdot\cos(\pi/10)\cdot\pi\cdot\beta$$

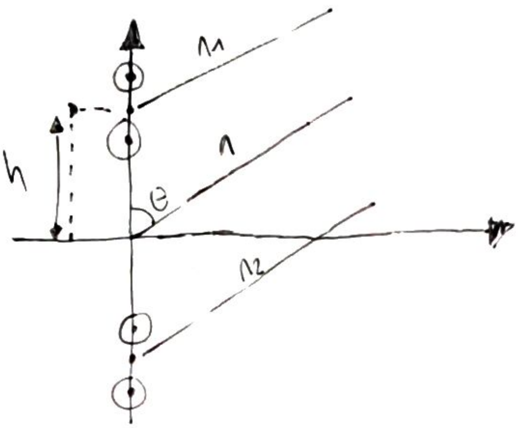
$$\pi - \beta + \sqrt{2}.\cos\left(\pi/10\right).\pi.\beta = \sqrt{2}.\cos(\pi/10).\pi$$

$$\Leftrightarrow \beta(\sqrt{2}.\cos(\pi/10).\pi - 1) = \sqrt{2}\cos(\pi/10).\pi - \pi$$

$$\beta = \simeq \frac{\pi}{10}$$

---

e)



$$E^t_\psi = E^d_\varphi + R_h.E^r_\varphi \qquad R_h = -1 \ \to \text{horizontal}$$

$$E^d_\psi = \left(\frac{a.\omega.\mu.I_0.e^{-jkr_1}}{2r_1}\right).J_1.(k.a.\sin\theta_1).2.\left(\cos\left(\frac{kd}{2}.\cos\theta_1 + \beta/2\right)\right)$$

$$E^r_\psi = \left(\frac{a.\omega.\mu.I_0.e^{-jkr_2}}{2.r_2}\right).J_1.(k.a.\sin\theta_2).2.\left(\cos\left(\frac{kd}{2}.\cos\theta_2 + \beta/2\right)\right)$$

Aprox. à amplitude $\theta = \theta_1 = \theta_2$ / $r_1 = r_2$

$$E^t_\psi = \left(\frac{a.\omega.\mu.I_0.}{2.r}\right).J_1.(ka.\sin\theta).2.\cos\left(\frac{kd}{2}.\cos\theta + \beta/2\right).\left(e^{-jkr_1} + R_h.e^{-jkr_2}\right)$$

Aprox. de Fase $\Rightarrow$ $r = r_1 + h.\cos\theta$

$r = r_2 - h.\cos\theta$

$$\Leftrightarrow e^{-jk(r-h.\cos\theta)} - e^{-jk(r+h\cos\theta)} = e^{-jkr}.e^{jkh.\cos\theta} - e^{-jkr}.e^{-jkh\cos\theta}$$

$$= e^{-jkr}.\left(e^{jkh.\cos\theta} - e^{-jkh.\cos\theta}\right) = e^{-jkr}.2.j.\sin(k.h.\cos\theta)$$

$$E^{+}_{\varphi} = \frac{a.\omega.\mu.I_0.e^{jkn}}{2.n} \cdot J_1 \cdot (K.a.\sin\theta) \cdot 2 \cdot \cos\left(\frac{K.d}{2}.\cos\theta + \frac{\beta}{2}\right) \cdot 2.j.\sin(Kh.\cos\theta)$$

2-

b)1 $\beta \simeq \left(-Kd + \frac{\pi}{N}\right)$ pretende-se radiação máxima para $\theta = \phi$

$\beta \simeq \left(Kd + \frac{\pi}{N}\right)$ para $\theta = \pi$

$D = \pi.\cos\theta_0 \; D_x.D_y$ com $D_x = 1{,}789\left(4\frac{L_x}{\lambda}\right)$

e $D_y = 1{,}784\left(4.\frac{L_y}{\lambda}\right)$

$L_x = (N-1).d_x = (6-1).\frac{\lambda}{6}$

$L_y = (N-1).d_y = (8-1).\frac{\lambda}{4}$

$D_x = 1{,}784.\left(4.\frac{5\frac{\lambda}{6}}{\lambda}\right)$ $D_y = 1{,}784.\left(4.\frac{7.\frac{\lambda}{4}}{\lambda}\right)$

$= 1{,}784.\left(4.\frac{5}{6}\right) \frac{10}{3}$ $= 1{,}784.\left(4.\frac{7}{4}\right) 7$

$= 5{,}963$ $= 12{,}523$

$D = \pi.\cos(\pi/6).(1{,}789)^2.\frac{20}{6}.7 = \pi.\frac{1}{2}.(1{,}784)^2.\frac{140}{6}$

$= 117.31$

b)2 $D = \frac{2.R_0^2}{1+(R_0^2-1)f.\frac{\lambda}{L+d}} \Rightarrow D_x = \frac{2.R_0^2}{1+(R_0^2-1).f.\frac{\lambda}{L+d}}$

$20.\log_{10}(R_0) = R_0|_{dB} \Leftrightarrow 50 = 20.\log_{10}(R_0) \Leftrightarrow R_0 = 316.2$

$f = 1{,}5$ para $R_0 = -50\,dB$

$$D_x = \frac{2.(316.2)^2}{1+((316,2)^2-1).1,5.\frac{\lambda}{\frac{5\lambda}{6}+\frac{\lambda}{6}}} = \frac{200\,000}{1+\frac{149998.5}{1}} = 1,33$$

$$D_y = \frac{2.(316,2)^2}{1+((316,2)^2-1).1,5.\frac{\lambda}{\frac{7.\lambda}{4}+\frac{\lambda}{4}}} = \frac{200\,000}{1+(99499)\times 1,5\times \underbrace{\frac{\lambda}{\frac{8\lambda}{4}}}_{2}} = 2,67$$

$$D = \pi.(\cos\frac{\pi}{6}).D_x.D_y = 4,6$$

c) 1

$$\Omega_A = \Theta_h.\Psi_h$$

H-W

$$\theta_{x_0} = 2.\arccos\left(1-0,1398.\frac{\lambda}{N.d}\right) = 2.\arccos\left(1-0,1398.\frac{\lambda}{6.\frac{\lambda}{6}}\right) = 61,32^0$$

$$\theta_{y_0} = 2.\arccos\left(1-0,1398.\frac{\lambda}{N_y.d_y}\right) = 2.\arccos\left(1-0,1398.\frac{\lambda}{8.\frac{\lambda}{4}}\right) = 43,10^0$$

$$\theta_h = \sqrt{\frac{1}{\cos^2\theta_0\left(\theta_{x_0}^{-2}.\cos^2\varphi_0 + \theta_{y_0}^{-2}.\sin^2\varphi_0\right)}}$$

$$= \sqrt{\frac{1}{\left(\frac{\sqrt{3}}{2}\right)^2.\left(0,87.0^2+1,767.1^2\right)}} = \sqrt{\frac{1}{\left(\frac{\sqrt{3}}{2}\right)^2.(1,767)}}$$

$$= 0,8686 = 49,77^0$$

$$\Psi_h = \sqrt{\frac{1}{\theta_{x_0}^{-2}.\sin^2\varphi_0+\theta_{y_0}^{-2}.\cos^2\varphi_0}} = \sqrt{\frac{1}{0,87.1+0}} = 1,07 = 61,3$$

$$\Omega_A = 49,77\times 61,3 = 3050^{0\,2}$$

$$D = \frac{32400}{30050,4} = 10$$

2

$$\theta_{x0} = \arccos\left(-0{,}443 \cdot \frac{\lambda}{Lx + dx}\right) = \arccos\left(-0{,}443 \cdot \frac{\lambda}{\frac{5\lambda}{6} + \frac{\lambda}{6}}\right) = \arccos\left(-0{,}443 \cdot 1\right) = 2{,}0297$$

$$= 116.24$$

$$\theta_{y0} = \arccos\left(0{,}443 \cdot \frac{\lambda}{Ly + dy}\right) = \arccos\left(0{,}443 \cdot \frac{\lambda}{\frac{7\lambda}{4} + \frac{\lambda}{4}}\right) = \arccos\left(\frac{0{,}443}{2}\right) =$$

Teste 14/10

1- a) Zero para $\theta = \pi/3$

$$\lambda = \frac{c}{f} = \frac{3 \times 10^8}{300 \times 10^6} = 1\,m$$

1º Zero:

$\theta = \phi$

$J_1(k.a.\sin\theta) = 0 \Leftrightarrow$

$\Leftrightarrow k\,a.\sin\theta = 0;\ 3,85;\ 7,05$

2º Zero:

$$k a.\sin\theta = 3,85 \Leftrightarrow k.a.\sin(\pi/3) = 3,85$$

$$\Leftrightarrow \frac{2\pi}{\lambda}.a.\sin(\pi/3) = 3,85$$

$$\Leftrightarrow a = \frac{3,85}{2\pi.\sin(\pi/3)} \Leftrightarrow a = \frac{3,85}{\cancel{2}\pi.\frac{\sqrt{3}}{\cancel{2}}} \Leftrightarrow a = 0,7075$$

Verificar se existem mais zeros:

$$Ka.\sin\theta = 7,05 \Leftrightarrow \sin\theta = \frac{7,05}{2\pi.\left(\frac{3,85}{\pi.\sqrt{3}}\right)} = \frac{7,05 \times \sqrt{3}}{2.3,85} = (1,585 > 1)$$

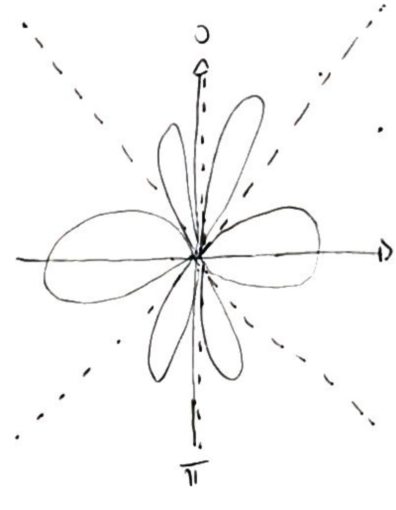
Como não existem mais zeros pela função de bessel

$$\sin(\pi/3) = \sin\left(\frac{2\pi}{3}\right)$$

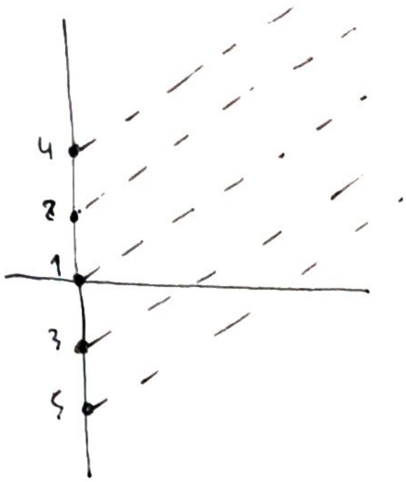
$\theta = \phi$

$\theta = \pi/3$

$\theta = 2\pi/3$



b)



$$E_\varphi^t = E_{\varphi 1} + E_{\varphi 2} + E_{\varphi 3} + E_{\varphi 4} + E_{\varphi 5}$$

$$E_\varphi^t = \frac{a.\omega.\mu.I_0}{2} . J_1(k.a.\sin\theta) . \left(\frac{e^{-jkr_1}}{r_1} + \frac{e^{-jkr_2}}{r_2} + \frac{e^{-jk(-r_3)}}{r_3} + \frac{e^{-jk(r_4)}}{r_4} + \frac{e^{-jk(-r_5)}}{r_5} \right)$$

Aprox. de módulo, $r_1 = r_2 = r_3 = r_4 = r_5$

$r_1 = r$

$r_2 = r - d.\cos\theta$

$r_4 = r - 2d.\cos\theta$

$r_3 = r + d.\cos\theta$

$r_5 = r + 2d.\cos\theta$

$$E_\varphi^t = (\ldots).\left(\frac{e^{-jkr}}{r} + \frac{e^{-jk(r-d.\cos\theta)}}{r} + \frac{e^{jk(r+d.\cos\theta)}}{r} + \frac{e^{-jk(r-2d.\cos\theta)}}{r} + \frac{e^{jk(r+2.d.\cos\theta)}}{r} \right)$$

$$= (\ldots). \frac{e^{-jkr}}{r} . \left(1 + e^{jkd.\cos\theta} + e^{-jkd.\cos\theta} + e^{jk2d\cos\theta} + e^{-jk2d\cos\theta} \right)$$

$$(AF) = 1 + e^{-jkd\cos\theta} + e^{-jkd.\cos\theta} + e^{jk2d.\cos\theta} + e^{-jk2.d\cos\theta}$$

Se existe uma diferença de fase ent

$$(AF) = 1 + e^{jkd(\cos\theta + \beta)} + e^{-j(kd.\cos\theta + \beta)} + e^{-j(k2.d.\cos\theta + \beta)} + e^{j(k2.d\cos\theta + \beta)}$$

$$= 1 + 2.\cos(kd.\cos\theta + \beta) + 2.\cos(kd2\cos\theta + \beta)$$

Progressão geométrica

$$(AF) = \sum_{m=-2}^{2} \left(e^{jm(kd.\cos\theta + \beta)} \right) = \sum_{n=-2}^{2} e^{jm\psi} \qquad c/ \left(\psi = kd\cos\theta + \beta \right)$$

$$\text{mv. } m = m+2 \Rightarrow \sum_{m=0}^{4} \left(e^{j(m-2)\psi} \right) = e^{j(-2).\psi} + e^{j(-1.)\psi} + e^{j(0).\psi} + e^{j(1).\psi} + e^{j(2)\psi}$$

$$= 1 + \frac{1}{e^{j2\psi}} + \frac{1}{e^{j\psi}} + e^{j\psi} + e^{j2\psi}$$

2. 18/19

10×8

$(\theta, \varphi) = (\pi/6, \pi/2)$

$d_x = \frac{3\lambda}{4}$

$d_y = \frac{\lambda}{2}$

b) $D = \pi \cdot \cos\theta_0 \cdot D_x \cdot D_y$

1 - H-W

$D_x = 1{,}789\left(4 \frac{L_x}{\lambda}\right)$ $\qquad L_x = (N-1) \cdot d_x = 9 \cdot \frac{3}{4}\lambda = \frac{27 \cdot \lambda}{4}$

$D_x = 1{,}784\left(4 \cdot \frac{\frac{27\lambda}{4}}{\lambda}\right) = 1{,}784 \times 27$

$D_y = 1{,}789\left(4 \cdot \frac{L_y}{\lambda}\right)$ $\qquad L_y = (N-1) \cdot d_y = 7 \cdot \frac{\lambda}{2}$

$D_y = 1{,}789\left(4 \times \frac{\frac{7 \cdot \lambda}{2}}{\lambda}\right) = 1{,}784 \times 14$

$D = \pi \cdot \cos\left(\frac{\pi}{6}\right) \cdot (1{,}784)^2 \times 27 \times 14 \approx 3241$

2. Tchebyschev

$$D_x = \frac{2 \cdot R_0^2}{1 + (R_0^2 - 1) \cdot f \cdot \frac{\lambda}{L_x + d_x}}$$

$$R_{0\,dB} = 20 \cdot \log_{10} R_0 = -40$$

$$\Leftrightarrow \log_{10} R_0 = 2 \Leftrightarrow R_0 = 100$$

$$L_x = (N-1) \cdot d_x = \frac{27 \cdot \lambda}{4}$$

$$d_x = \frac{3}{4}\lambda$$

$$D_x = \frac{2(100)^2}{1 + (100^2 - 1) \cdot \underset{1{,}32}{f} \cdot \frac{\lambda}{\frac{(27+3)\lambda}{4}}} \left(= \frac{4}{30}\right) = 11{,}36$$

$$D_y = \frac{2(100)^2}{1 + (100^2 - 1) \cdot (1{,}32) \cdot \frac{\lambda}{\left(\frac{7+1}{2}\right)\lambda}} \left(\frac{2}{8}\right) = 6{,}06$$

$$D = \pi \cdot \cos(\theta_0) \cdot D_x \cdot D_y = 187{,}3$$

c) $\Omega_A = \theta_h \cdot \psi_h$

$$\theta_h = \sqrt{\frac{1}{\cos^2\theta_0 \cdot \left(\theta_{x0}^{-2} \cdot \cos^2\varphi_0 + \theta_{y0}^{-2} \cdot \sin^2\varphi_0\right)}} = \sqrt{\frac{1}{\cos^2\theta_0 \cdot \left(0 + \theta_{y0}^{-2}\right)}}$$

1 - H - W

$$\theta_{x0} = 2 \cdot \cos^{-1}\left(1 - 0{,}1398 \cdot \frac{\lambda}{N_x d_x}\right) = 2 \cdot \cos^{-1}\left(1 - 0{,}1398 \cdot \frac{\lambda}{10 \cdot \frac{3}{4}\lambda}\right)$$

$$= 2 \cdot \arccos\left(1 - 0{,}1398 \cdot \frac{4}{30}\right)$$

$$\theta_{y0} = 2 \cdot \arccos\left(1 - 0{,}1398 \cdot \frac{\lambda}{\frac{8 \times \lambda}{2}}\right) = 2 \cdot \arccos\left(1 - 0{,}1398 \cdot \frac{2}{8}\right)$$

$$\theta_h = \sqrt{\frac{1}{\cos^2(\pi/6) \cdot \theta_{y0}^{-2}}} = \frac{\theta_{y0}}{\cos(\pi/6)} = 35{,}04$$

$$\psi_h = \sqrt{\frac{1}{\theta_{x0}^{-2} \cdot \sin^2\varphi_0 + \theta_{y0}^{-2} \cdot \cos^2\varphi_0}} = \sqrt{\frac{1}{\theta_{x0}^{-2}}} = \theta_{x0} = 22{,}16^\circ$$

$$\Omega_A = 35{,}09 \times 22{,}16 = 777{,}6^\circ$$

i- Tchib[illegible]

$$\Omega_A = \theta_h \cdot \psi_h$$

$$\theta_{x_0} = \rho \cdot \left(\arccos\left(-0{,}443 \cdot \frac{\lambda}{L_x + d_x}\right) - \arccos\left(+0{,}443 \cdot \frac{\lambda}{L_x + d_x}\right)\right)$$

$$= 1{,}32 \cdot \left(\arccos\left(-0{,}443 \cdot \frac{4}{30}\right) - \arccos\left(0{,}443 \cdot \frac{4}{30}\right)\right) = 8{,}94^\circ$$

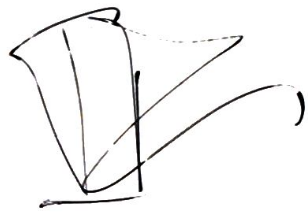
$$\theta_{y_0} = \rho \cdot \left(\arccos\left(-0{,}443 \cdot \frac{\lambda}{L_y + d_y}\right) - \arccos\left(0{,}443 \cdot \frac{\lambda}{L_y + d_y}\right)\right)$$

$$= 1{,}32 \left(\arccos\left(-0{,}443 \cdot \frac{2}{8}\right) - \arccos\left(0{,}443 \cdot \frac{2}{8}\right)\right) = 16{,}79^\circ$$

$$\theta_h = \frac{\theta_{y_0}}{\cos(\pi/6)} = \frac{16{,}79}{\cos(\pi/6)} = 19{,}38$$

$$\psi_h = \theta_{x_0} = 8{,}94$$

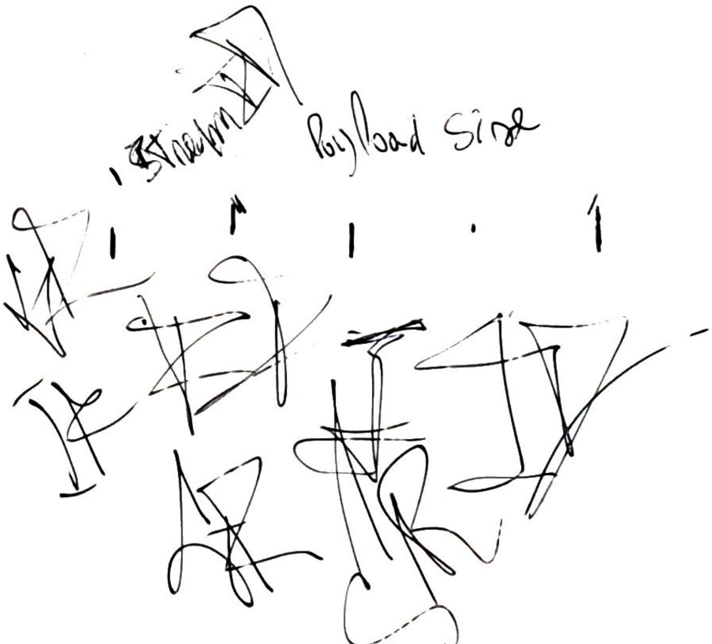
$$\Omega_A = \theta_h \cdot \psi_h = 19{,}38 \times 8{,}94 = 173{,}29^\circ$$



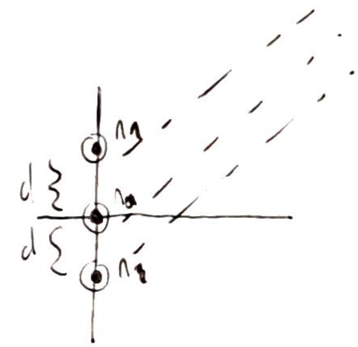
Directividade para Snodd Sick

$$D = \frac{32400}{\Omega_A = 777{,}6} = 41{,}67 \neq 3291$$

$$D = \frac{32400}{173{,}89} = 186 \simeq 187$$



1- a)



$$E_\varphi^t = E_\varphi^1 + E_\varphi^2 + E_\varphi^3$$

$$= (\dots) \cdot \left(\frac{e^{-jkr}}{r} + \frac{e^{-jkr_1}}{r_1} + \frac{e^{-jkr_2}}{r_2} \right)$$

pa $\begin{cases} r_1 = r - d\cos\theta \\ r_2 = r + d\cos\theta \end{cases}$

$$E_\varphi^t = (\dots) \cdot \left(\frac{e^{-jkr}}{r} + \frac{e^{-jk(r - d\cos\theta)}}{r} + \frac{e^{-jk(r + d\cos\theta)}}{r} \right)$$

$$= (\dots) \cdot \left(\frac{e^{-jkr}}{r} \left(e^{-jkd\cos\theta} + e^{jkd\cos\theta} \right) \right)$$

$$= (\dots) \cdot \left(\frac{e^{-jkr}}{r} \cdot 2 \cdot \cos(kd \cdot \cos\theta) \right) + B$$

$\begin{cases} \theta = \theta_1 = \theta_2 = \theta_3 \\ r = r_1 = r_2 \end{cases}$

$$\frac{\sin\left(N \cdot \frac{\psi}{2}\right)}{N \cdot \frac{\psi}{2}}$$

b)

max para $\theta = \pi/6$

e zeros em simétricos $\theta = \frac{\pi}{3}, \frac{2\pi}{3}$

zeros de J_1 são $0 ; 3,84 ; 7,01$

$ka \cdot \sin\theta = \emptyset \Rightarrow \theta = 0, \pi$

$ka \cdot \sin\theta \big|_{\pi/6} = 3,84 \Leftrightarrow k \cdot a \cdot \sin(\pi/6) = 3,84 \Leftrightarrow a = \frac{3,84}{k \cdot \sin(\pi/3)} = \frac{3.84}{\frac{2\pi}{\lambda} \cdot \frac{\sqrt{3}}{2}}$

$$(AF)_N = \frac{1 + 2 \cdot \cos(kd \cdot \cos\theta + B)}{3} = \frac{\sin\left(3 \cdot \frac{\psi}{2}\right)}{3 \cdot \frac{\psi}{2}}$$

$\psi = kd\cos\theta \big|_{\pi/6} + B = 0 \Leftrightarrow \frac{2\pi}{\lambda} \cdot d \cdot \frac{\sqrt{3}}{2} + B = 0 \Leftrightarrow B = -d \cdot \sqrt{3} \cdot \pi$ (rad)

c) $\vec{d}$ O agrupado seguindo o eixo OY

$$E^t_\phi = (\dots) \frac{a.\omega.I_o}{2r} \cdot J_1(ka.\sin\theta).e^{-jk\theta} \cdot \frac{\sin(3.\frac{\psi}{2})}{3.\frac{\psi}{2}} \quad (c/ \ \psi = kd.\cos\theta + \beta)$$

$$\cos\theta = \hat{j}.\hat{d_r} = \hat{j}.(\sin\theta.\cos\varphi.\hat{i} + \sin\theta.\sin\varphi.\hat{j} + \cos\theta.\hat{k}) = \sin\theta.\sin\varphi$$

$$\sin\theta = \sqrt{1.\sin^2\theta.\sin^2\varphi}$$

$$H-W \Rightarrow \underset{0°}{\beta = -kd - \frac{\pi}{N}} \vee \underset{\pi}{\beta = kd + \frac{\pi}{N}} \Rightarrow d \simeq \frac{\lambda}{4}$$

Para termo de agrupado $\beta = \frac{\sqrt{3}}{\cancel{\lambda}}.\pi\frac{\cancel{\lambda}}{4} \vee \beta = -\frac{\sqrt{3}}{4}\pi$ rad