Soluções da ficha 4B

1. (a) $\dfrac{x+2}{2x(x-1)^2(x^2+1)} = \dfrac{1}{x} - \dfrac{5}{4(x-1)} + \dfrac{3}{4(x-1)^2} + \dfrac{\frac{1}{2}x+1}{2(x^2+1)}$

$$P\left[\frac{x+2}{2x(x-1)^2(x^2+1)}\right] = \ln\left[\frac{|x|\left(x^2+1\right)^{\frac{1}{8}}}{|x-1|^{\frac{5}{4}}}\right] - \frac{3}{4(x-1)} + \frac{1}{2}\arctan x + C.$$

(b) $\dfrac{27}{x^4-3x^3} = \dfrac{1}{x-3} - \dfrac{3}{x^2} - \dfrac{9}{x^3} - \dfrac{1}{x}$

$$P\left[\frac{27}{x^4-3x^3}\right] = \ln\left|\frac{x-3}{x}\right| + \frac{3}{x} + \frac{9}{2x^2} + C$$

(c) $\dfrac{u^4-8}{u^3-2u^2} = u + \dfrac{2}{u} + \dfrac{4}{u^2} + \dfrac{2}{u-2} + 2$

$$P\left[\frac{u^4-8}{u^3-2u^2}\right] = \frac{u^2}{2} + 2u + 2\ln|u(u-2)| - \frac{4}{u} + C$$

(d) $\dfrac{y^4}{y^4-1} = \dfrac{1}{4(y-1)} - \dfrac{1}{4(y+1)} - \dfrac{1}{2(y^2+1)} + 1$

$$P\left[\frac{y^4}{y^4-1}\right] = y + \ln\sqrt[4]{\frac{y-1}{y+1}} - \frac{1}{2}\arctan y + C$$

(e) $\dfrac{3x^3+x^2-x-1}{x^2(x^2-1)} = \dfrac{1}{x} + \dfrac{1}{x^2} + \dfrac{1}{x-1} + \dfrac{1}{x+1}$

$$P\left[\frac{3x^3+x^2-x-1}{x^2(x^2-1)}\right] = \ln\left|x\left(x^2-1\right)\right| - \frac{1}{x} + C$$

(f) $\dfrac{x^2+x-1}{x^2(x-1)} = \dfrac{1}{x^2} + \dfrac{1}{x-1}$

$$P\left[\frac{x^2+x-1}{x^2(x-1)}\right] = -\frac{1}{x} + \ln|x-1| + C$$

(g) $\dfrac{x}{(x-1)^2(x^2+1)} = \dfrac{1}{2(x-1)^2} - \dfrac{1}{2(x^2+1)}$

$$P\left[\frac{x}{(x-1)^2(x^2+1)}\right] = -\frac{1}{2(x-1)} - \frac{1}{2}\arctan x + C$$

(h) $\dfrac{2x^2+x+1}{(x+1)^2(x-1)} = \dfrac{1}{x-1} + \dfrac{1}{x+1} - \dfrac{1}{(x+1)^2}$

$$P\left[\frac{2x^2+x+1}{(x+1)^2(x-1)}\right] = \ln\left|\left(x^2-1\right)\right| + \frac{1}{x+1} + C$$

2. (a) $P\dfrac{1}{x^2\sqrt{4-x^2}} = -\dfrac{1}{4}\dfrac{\sqrt{4-x^2}}{x} + C$

(b) $P\dfrac{3e^u}{1+e^{2u}} = 3\arctan\left(e^u\right) + C$

(c) $P\dfrac{\sin x}{\cos^2 x + \cos x} = \ln\left|\dfrac{\cos x + 1}{\cos x}\right| + C$

(d) $Pt\sqrt{1+t} = \dfrac{2}{5}(t+1)^{\frac{5}{2}} - \dfrac{2}{3}(t+1)^{\frac{3}{2}} + C$

(e) $P\sin\sqrt{1+x} = 2\left[\sin\sqrt{1+x} - \sqrt{1+x}\cos\sqrt{1+x}\right] + C$

(f) $P2y\sqrt{4-y} = \dfrac{4}{5}(4-y)^{\frac{5}{2}} - \dfrac{16}{3}(4-y)^{\frac{3}{2}} + C$

(g) $P\dfrac{\sqrt{t}}{t-\sqrt[3]{t}} = 2\sqrt{t} + \dfrac{3}{2}\ln\left|\dfrac{\sqrt[6]{t}-1}{\sqrt[6]{t}+1}\right| + 3\arctan\sqrt[6]{t} + C$

(h) $P\dfrac{7^x}{7^{3x}-7^{-x}} = \dfrac{1}{4\ln 7}\ln\left|\dfrac{7^{2x}-1}{7^{2x}+1}\right| + C$