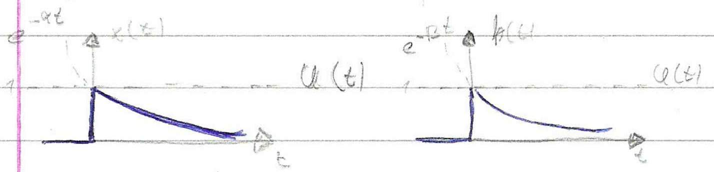
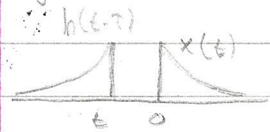
Folha exercícios 2 (Sistemas LIT)

1) a → $x(t)=e^{-\alpha t}u(t)$

$h(t)=e^{-\beta t}u(t)$

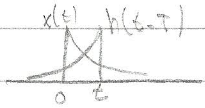


Fazendo para todos os casos:



$y(t)=0$ enquanto $t<0$

Agora quando



$y(t)=x(t)*h(t)$ enquanto $t>0$

$y(t)=\int_0^t x(\tau)\cdot h(t-\tau)\,d\tau \Leftrightarrow y(t)=\int_0^t e^{-\alpha\tau}\cdot e^{-\beta(t-\tau)}\,d\tau$

$\Leftrightarrow y(t)=\int_0^t e^{(\beta-\alpha)\tau-\beta t}\,d\tau = \frac{1}{\beta-\alpha}\,e^{(\beta-\alpha)\tau-\beta t}\Big|_0^t = \frac{1}{\beta-\alpha}\,e^{(\beta-\alpha)t-\beta t} - \frac{e^{-\beta t}}{\beta-\alpha}$

$=\frac{1}{\beta-\alpha}\,e^{-\alpha t}-\frac{e^{-\beta t}}{\beta-\alpha}$

Logo, $y(t)=\begin{cases}0, & t<0\\ \frac{1}{\beta-\alpha}\left(e^{-\alpha t}-e^{-\beta t}\right), & t>0\end{cases}$

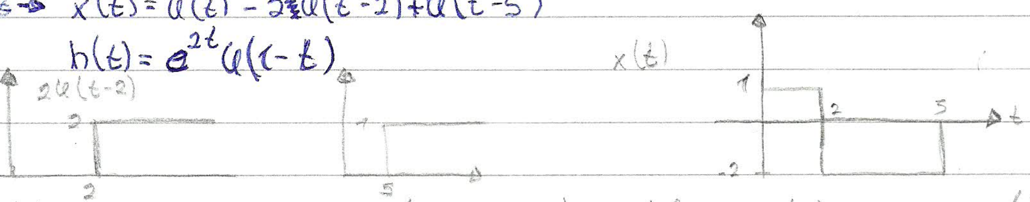
Quando $\alpha=\beta$ então

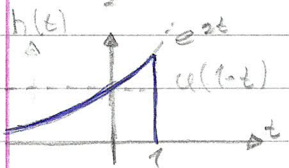
$y(t)=\int_0^t e^{-\alpha\tau}\cdot e^{-\beta(t-\tau)}\,d\tau=\int_0^t e^{-\alpha\tau+\alpha\tau-\alpha t}\,d\tau=\int_0^t e^{-\alpha t}\,d\tau=e^{-\alpha t}\int_0^t e^{0}\,d\tau$

$=e^{-\alpha t}(\tau)_0^t = t\,e^{-\alpha t}$, $t>0$, logo $y(t)=\begin{cases}0, & t<0\\ t\,e^{-\alpha t}, & t>0\end{cases}$

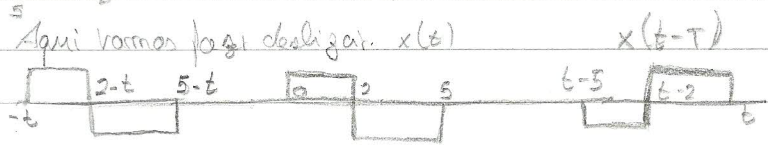
b → $x(t)=u(t)-2u(t-2)+u(t-5)$

$h(t)=e^{2t}u(1-t)$

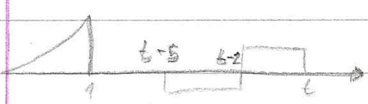




Aqui vamos fazer deslizar $x(t)$



1º caso



$y(t)=0$, para $t-5\geq 1 \Leftrightarrow t\geq 6$

2º caso: 

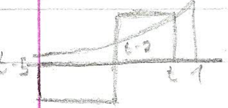
$t-2 \geq 1 \Leftrightarrow 3 \leq t \leq 6$

$$y(t)=\int_{t-5}^{1} e^{-2\tau}\,d\tau = \left(-\frac{1}{2}e^{2\tau}\right)\Big|_{t-5}^{1} = -\frac{1}{2}e^{2}+\frac{1}{2}e^{2t-10}$$

$t \geq 1$ e $t-2 < 1 \Rightarrow 1 \leq t < 3$

$$y(t)=\int_{t-5}^{t-2} e^{2\tau}\,d\tau + \int_{t-2}^{1} e^{2\tau}\,d\tau = \left(-\frac{1}{2}e^{2\tau}\right)\Big|_{t-5}^{t-2} + \left(\frac{1}{2}e^{2\tau}\right)\Big|_{t-2}^{1}$$

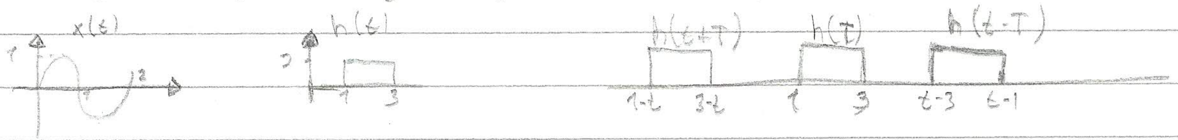
$$= -\frac{1}{2}e^{2t-4}+\frac{1}{2}e^{2t-10}+\frac{1}{2}e^{2}-\frac{1}{2}e^{2t-4} = -e^{2t-4}+\frac{1}{2}e^{2t-10}+\frac{1}{2}e^{2}$$



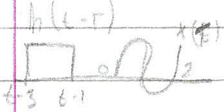
, $t < 1$

$$y(t)=\int_{t-5}^{t-2} e^{2\tau}\,d\tau + \int_{t-2}^{t} e^{2\tau}\,d\tau = \left(-\frac{1}{2}e^{2\tau}\right)\Big|_{t-5}^{t-2} + \left(\frac{1}{2}e^{\tau}\right)\Big|_{t-2}^{t}$$

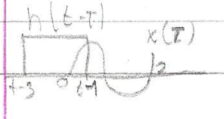
$$= -\frac{1}{2}e^{2t-4}+\frac{1}{2}e^{2t-10}+\frac{1}{2}e^{2t}-\frac{1}{2}e^{2t-4} = e^{-2t-4}+\frac{1}{2}e^{2t-10}+\frac{1}{2}e^{2t}$$

C → 

Verificando todos os casos



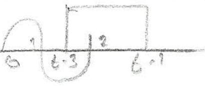
$t-1 \leq 0 \Leftrightarrow t \leq 1 \rightarrow y(t)=0$



$$y(t)=\int_{0}^{t-1} \operatorname{sen}(\pi\tau)\cdot 2\,d\tau = \frac{2}{\pi}\left(-\cos(\pi\tau)\right)\Big|_{0}^{t-1} = \frac{2}{\pi}\left(-\cos(\pi(t-1))+\cos(0)\right)$$

$$= \frac{2}{\pi}\left(1-\cos(\pi(t-1))\right)$$

$t-1 \leq 2$ e $t-1 > 0$ então, $0 < t-1 \leq 2 \Leftrightarrow 1 < t \leq 3$, porque um cabe dentro do outro



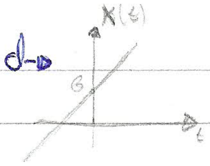
$$y(t)=\int_{t-3}^{2} \operatorname{sen}(\pi\tau)\cdot 2\,d\tau = \frac{2}{\pi}\left(-\cos(\pi\tau)\right)\Big|_{t-3}^{2} = \frac{2}{\pi}\left(-\cos(2\pi)+\cos(\pi(t-3))\right)$$

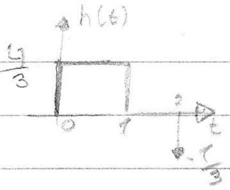
$$= \frac{2}{\pi}\left(\cos(\pi(t-3))-1\right)$$

$0 \leq t-3 \leq 2 \Leftrightarrow 3 \leq t \leq 5$

Para $t > 5$ $y(t)=0$

d)





$h_2(t) = -\frac{1}{3}\delta(t-2)$

$x(t) = at + B \qquad h(t) = h_1(t) + h_2(t)$

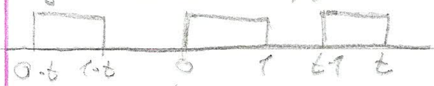
$y(t) = x(t) * (h_1(t) + h_2(t)) = \underbrace{x(t)*h_1(t)}_{y_1(t)} + \underbrace{x(t)*h_2(t)}_{y_2(t)}$

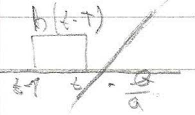
$y(t) = y_1(t) + x(t) * \left[-\frac{1}{3}\delta(t-2)\right]$

Sabendo que $x(t)*\delta(t) = x(t)$ e que $x(t) * A\delta(t-t_0) = Ax(t-t_0)$

então: $y(t) = y_1(t) - \frac{1}{3}x(t-2)$

Agora calculando $y_1(t)$:





$y_1(t) = \int_{-\infty}^{+\infty} x(\tau)h(t-\tau)\,d\tau = \int_{t-1}^{t} x(\tau)\frac{4}{3}\,d\tau = \frac{4}{3}\int_{t-1}^{t}(a\tau + B)\,d\tau$

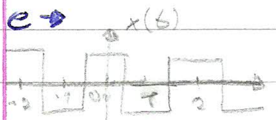
$= \frac{4}{3}\left[\frac{a\tau^2}{2} + B\tau\right]_{t-1}^{t} = \frac{4}{3}\left(at - \frac{a}{2} + B\right)$, é valido para todos os instantes

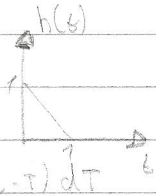
$y(t) = y_1(t) + y_2(t) = \frac{4}{3}\left(at - \frac{a}{2} + B\right) - \frac{1}{3}x[t-2]$, mas como $x(t) = at + B$ então

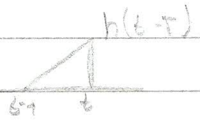
$y(t) = \frac{4}{3}\left(at - \frac{a}{2} + B\right) - \frac{1}{3}(a(t-2) + B) = \frac{4}{3}\left(at - \frac{a}{2} + B\right) - \frac{1}{3}(at - 2a + B)$

$= \frac{4}{3}at - \frac{4}{3}\left(\frac{a}{2}\right) + \frac{4}{3}B - \frac{1}{3}at + \frac{2}{3}a - \frac{1}{3}B = at + B = x(t)$

e)



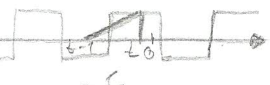




$y(t) = \int_{-\infty}^{+\infty} x(\tau)h(t-\tau)\,d\tau$

$h(t) = -t + 1$

$h(t-\tau) = -t + \tau + 1$

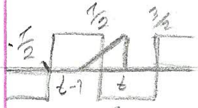


$y(t) = \int_{t-1}^{-\frac{1}{2}} (-1)(-t+\tau+1)\,d\tau + \int_{-\frac{1}{2}}^{t} (1)(-t+\tau+1)\,d\tau$

$= \int_{t-1}^{-\frac{1}{2}} (t-\tau-1)\,d\tau + \int_{-\frac{1}{2}}^{t} (-t+\tau+1)\,d\tau = \left(-t\tau + \frac{\tau^2}{2} + \tau\right)\Big|_{t-1}^{-\frac{1}{2}} + \left(-t\tau + \frac{\tau^2}{2} + \tau\right)\Big|_{-\frac{1}{2}}^{t}$

$= \frac{t}{2} - \frac{1}{8} - \frac{1}{2} - t(t-1) + \frac{(t-1)^2}{2} + (t-1) + \left(-t^2 + \frac{t^2}{2} + t\right) - \left(+\frac{t}{2} - \frac{1}{8} - \frac{1}{2}\right) = -t^2 - 3t + \frac{1}{4}$

enquanto $-\frac{1}{2} < t < \frac{1}{2}$



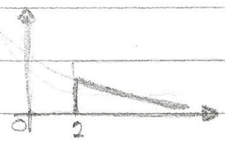
$$y(t) = \int_{t-1}^{\frac{1}{2}} x(\tau)\,h(t-\tau)\,d\tau + \int_{\frac{1}{2}}^{t} x(\tau)\,h(t-\tau)\,d\tau$$

$$= \int_{t-1}^{\frac{1}{2}} (\tau)(-t+\tau+1)\,d\tau + \int_{\frac{1}{2}}^{t} (-\tau)(-t+\tau+1) = t^2 - 3t + \frac{7}{4} \text{ enquanto } -\frac{1}{2} < t < \frac{3}{2}$$

$$y(t) = \begin{cases} -t^2 + t + \frac{1}{4}, & -\frac{1}{2} < t \leq \frac{1}{2} \\ t^2 - 3t + \frac{7}{4}, & \frac{1}{2} < t \leq \frac{3}{2} \end{cases}$$

2)

a -> $h(t) = e^{-4t}u(t-2)$

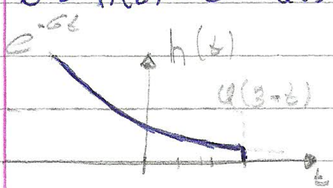


- Como para $t \neq 0$, $h(t) \neq 0$, tem memória.
- Como para $t < 0$, $h(t) = 0$, então o sistema é causal

$$\lim_{T\to\infty} \int_2^T e^{-4t}\,dt = \lim_{T\to\infty} \left(-\frac{1}{4}e^{-4t}\right)_2^T = \lim_{T\to\infty} -\frac{1}{4}e^{-4T} + \frac{e^{-8}}{4}$$

$= 0 + \frac{e^{-8}}{4}$, logo é estável.

b -> $h(t) = e^{-6t}u(3-t)$

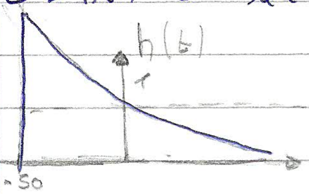


- Como para $t \neq 0$ $h(t)$ não é igual a zero, tem memória.
- Como para $t < 0$, $h(t) \neq 0$, então é não causal
- também podemos afirmar que o sistema é instável pois à medida que t aumenta e tende para $-\infty$ a função é crescente e tende para $+\infty$.

$$\lim_{T\to-\infty} \int_T^3 e^{-6t} = \lim_{T\to-\infty} \int_T^3 -\frac{1}{6}e^{-6t} = \lim_{T\to\infty} -\frac{1}{6}e^{-18} + \frac{1}{6}e^{-6T}$$

$= -\frac{1}{6}e^{-18} + +\infty$

c -> $h(t) = e^{-2t}u(t+50)$



- Como para $t \neq$, $h(t) \neq 0$, tem memória
- Como para $t < 0$, $h(t) \neq 0$ é não causal
- $\lim_{T\to+\infty} \int_{-50}^{T} e^{-2t}\,dt = \lim_{T\to+\infty} \int_{-50}^{T} -\frac{1}{2}e^{-2t}\,dt$

$= \lim_{T\to+\infty} -\frac{1}{2}e^{-2T} + \frac{1}{2}e^{+100} = 0 + \frac{1}{2}e^{+100}$, logo é estável