O campo elétrico criado por um dipolo vertical de comprimento l num meio exato de materiais condutores e infinito é dado por:

$$E_\Theta = j \cdot \eta \cdot \frac{I_0 \cdot e^{-jkr}}{2\pi \cdot r} \cdot \left(\frac{\cos(kl/2 \cdot \cos\Theta) - \cos(kl/2)}{\sin\Theta} \right) \qquad H = \frac{E_\Theta}{\eta}$$

Suponha que uma antena deste tipo com 2 m de comprimento e situada a uma altura de 1m do solo está a servir uma emissora de rádio com uma portadora de 200 mhz.

Supondo que o solo é um condutor perfeito, pode ser considerado plano, calcule:

a) Campo eletromagnético (E_Θ / H = ?)

b) Intensidade e diagrama de radiação (U = ? / diag. rad)

c) Potência irradiada, resistência de radiação e diretividade da antena nestas condições. (W, R_r, D = ?)

d) Amplitude da corrente sobre a antena para que a potência de densidade média a 100km da antena, numa direção que faz um ângulo de 45° com o eixo da mesma que seja de 20 mW/m²?
I = ?

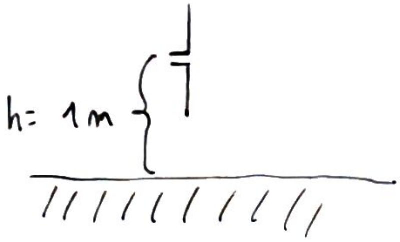
e) Que alterações seriam produzidas nos parâmetros de radiação da antena se o plano condutor infinito fosse substituído pelo planeta Terra com condutividade finita e curvatura com raio finito.

campo criado vai ser menor

não reflete e dispersa pela curvatura

medida de observação menor

a)



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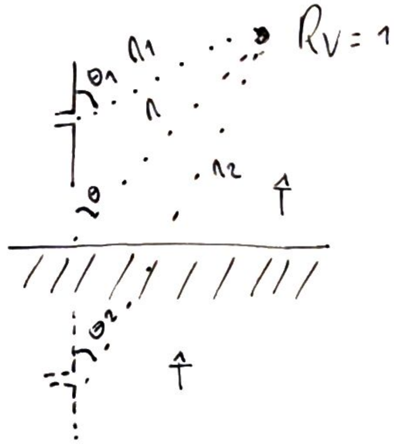


$$\frac{Kl}{2}\cdot\cos\theta = \frac{2\pi}{\lambda}\cdot\frac{2}{2}\cdot\cos\theta$$

$$= \frac{2\pi}{\lambda}\cdot\cos\theta$$

$$\lambda = \frac{c}{f} = \frac{3\times10^{8}}{200\times10^{6}} = \frac{3}{2}\ m$$

$$\frac{Kl}{2}\cdot\cos\theta = \frac{2\pi}{\frac{3}{2}}\cdot\cos\theta$$

$$= \frac{4\pi}{3}\cdot\cos\theta$$

$$E_\theta^t = E_\theta^d + D.R_V.E_\theta^r$$

$$E_\theta^d = j\eta\cdot\frac{I_0\cdot e^{-jkr_1}}{2\pi r_1}\cdot\left(\frac{\cos\left(\frac{4\pi}{3}\cdot\cos\theta_1\right)-\cos\left(\frac{4\pi}{3}\right)}{\sin(\theta_1)}\right)$$

$$E_\theta^r = j.\eta.\frac{I_0.e^{-jkr_2}}{2\pi.r_2}\cdot\left(\frac{\cos\left(\frac{4\pi}{3}\cdot(\cos\theta_2)\right)-\cos\left(\frac{4\pi}{3}\right)}{\sin\theta_2}\right)$$

$$\left.\begin{array}{l}\theta = \theta_1 = \theta_2\\ r = r_1 = r_2\end{array}\right\}\text{ módulo}$$

$$E_\theta^t = j.\eta.\frac{I_0}{2\pi r}\cdot\left(\frac{\cos\left(\frac{4\pi}{3}\cdot\cos\theta\right)-\cos\left(\frac{4\pi}{3}\right)}{\sin(\theta)}\right)\cdot\left(e^{-jkr_1}+e^{-jkr_2}\right)\ \text{Aprox de fase}$$

$$\begin{cases} r_1 = r - h.\cos\theta \\ r_2 = r + h.\cos\theta \end{cases}$$

$$E_\theta^t = j.\eta.\frac{I_0}{2\pi.r}\cdot\left(\frac{\cos\left(\frac{4\pi}{3}.\cos\theta\right)+\frac{1}{2}}{\sin\theta}\right).e^{-jkr}\cdot\underbrace{\left(e^{jkh.\cos\theta}+e^{-jkh.\cos\theta}\right)}_{2.\cos(k.h.\cos\theta)}$$

$$H_\varphi^t = \frac{E_\theta}{\eta} = j.\frac{I_0}{2\pi.r}\cdot\left(\frac{\cos\left(\frac{4\pi}{3}.\cos\theta\right)+\frac{1}{2}}{\sin\theta}\right).e^{-jkr}\cdot\left(2.\cos(k.h.\cos\theta)\right)$$

b) $U = \mathrm{Re}\{\vec{S_n}\} \cdot r^2$ $\qquad (e^{-jkr})^2 = 1$

$$\mathrm{Re}\{\vec{S_n}\} = \frac{1}{2\eta} \cdot E_\theta^2 \Rightarrow U = \frac{\eta \cdot I_0^2}{2 \cdot 4\pi r^2} \cdot \left(\frac{\cos(\frac{4}{3}\pi \cdot \cos\theta) + \frac{1}{2}}{\sin\theta}\right)^2 \cdot 4 \cdot \cos^2(kh \cdot \cos\theta)$$

~~c)~~ d)
diagrama de radiação

calcular $\varnothing$'s

$$\frac{\cos\left(\frac{4}{3}\pi \cdot \cos\theta\right) - \cos\left(\frac{4\pi}{3}\right)}{\sin\theta} = 0$$

$\Rightarrow \cos\left(\frac{4\pi}{3} \cdot \cos\theta\right) - \cos\left(\frac{4\pi}{3}\right) = 0 \qquad \wedge \quad \sin\theta \neq 0$

$\cos\left(\frac{4\pi}{3} \cdot \cos\theta\right) = \cos\left(\frac{4\pi}{3}\right) \qquad \wedge \quad$ "

$\frac{4\pi}{3} \cdot \cos\theta = \pm\frac{4\pi}{3} \pm 2\pi m$

$\cos\theta = \pm 1 \pm \frac{3}{2} m$

$\underline{m = \varnothing}$ $\quad \cos\theta = \pm 1 \Rightarrow \theta = 0, \theta = \pi$

$\underline{m = 1}$

$\cos\theta = \pm 1 \pm \frac{3}{2} \Rightarrow \theta = \pi - \theta_1$

$\underline{m = 2}$ $\quad \cos\theta = \pm 1 \pm 3 \Rightarrow$ X

Em $m = \varnothing$ temos uma indeterminação pois $\sin\theta = 0$ para $\theta = \{0, \pi\}$

$$\lim_{\theta \to \phi} \frac{\cos\left(\frac{kl}{2}\cdot\cos\theta\right)-\cos\left(\frac{kl}{2}\right)}{\sin\theta} = \lim_{\theta \to 0} \frac{\left(\cos\left(\frac{kl}{2}\right)\cdot\cos\theta)-\cos\left(\frac{kl}{2}\right)\right)'}{(\sin\theta)'}$$

$$\lim_{\theta \to \phi} \frac{-\frac{kl}{2}\cdot\sin\theta\cdot\sin\left(\frac{kl}{2}\cdot\cos\theta\right)}{\cos\theta} = \frac{0}{1} = 0$$ logo $\{0, \pi\}$ são zeros do diagrama

2) AF

$$\cos(kh\cdot\cos\theta)=\phi \Rightarrow \cos\left(\frac{2\pi}{\lambda}\cdot h\cdot\cos\theta\right)=\phi \Rightarrow \frac{2\pi}{\lambda}h\cdot\cos\theta = \pm\frac{\pi}{2} \pm m\pi$$

$$\Rightarrow \cos\theta = \pm\frac{\lambda}{4h} \pm \frac{\lambda}{2h}\cdot m$$

$\lambda = \frac{3}{2}$

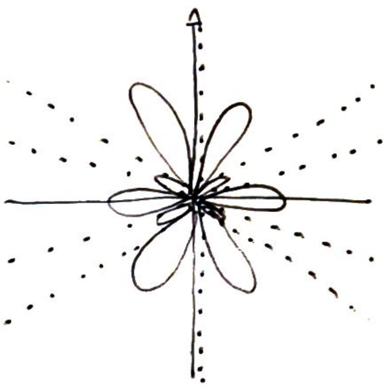
$h = 1$

$$\cos\theta = \pm\frac{3}{8} \pm \frac{3}{4}$$

$\xrightarrow{m=\phi}$ $\theta_0 = \arccos\left(\frac{3}{8}\right) = \pi - \theta_{l0} = \theta_{l1}$ 68°

$\xrightarrow{m=1}$ $\theta_1 = \arccos\left(\pm\frac{3}{8}\right) = \theta_{l3} = \theta_{l1}$

$\xrightarrow{m=2}$ $\theta_2 = \nexists$ há zero



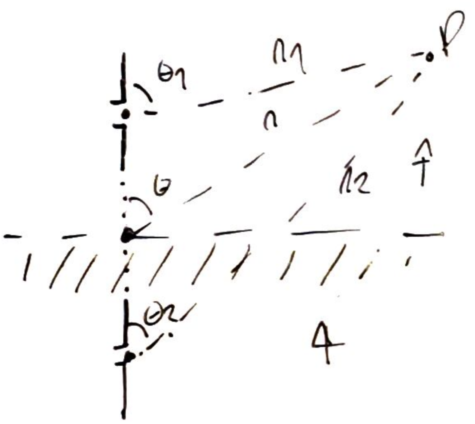
a) $l = 2\,m$

$$\lambda = \frac{c}{200\times10^6} = \frac{300\times10^6}{200\times10^6} = \frac{3}{2} = 1{,}5\,m$$

$$E = j\eta \cdot \frac{I_0\, e^{-jkr}}{2\pi \cdot r} \cdot \left(\frac{\cos\left(\frac{kl}{2} \cdot \cos\theta\right) - \cos\left(\frac{kl}{2}\right)}{\sin\theta} \right)$$

$$E_\psi^t = E_\psi^d + R_V \cdot E_\psi^r$$

↳ vertical $R_V = 1$



Aproximasión campo distante

$\theta = \theta_1 = \theta_2$

Aprox. Módulo

$r = r_1 = r_2$

Aprox. Fase (no exponencial)

$r_1 = r - h\cos\theta$

$r_2 = r + h\cos\theta$

$$E_\psi^d = j \cdot \eta \cdot \frac{I_0 \cdot e^{-jkr_1}}{2\pi r_1} \cdot \left(\frac{\cos\left(\frac{kl}{2} \cdot \cos\theta_1\right) - \cos\left(\frac{kl}{2}\right)}{\sin\theta_1} \right)$$

$$E_\psi^r = j \cdot \eta \cdot \frac{I_0 \cdot e^{-jkr_2}}{2\pi r_2} \cdot \left(\frac{\cos\left(\frac{kl}{2} \cdot \cos\theta_2\right) - \cos\left(\frac{kl}{2}\right)}{\sin\theta_2} \right)$$

$$E_\psi^t = j\eta \cdot \frac{I_0}{2\pi r} \cdot \left(\frac{\cos\left(\frac{kl}{2} \cdot \cos\theta\right) - \cos\left(\frac{kl}{2}\right)}{\sin\theta} \right) \cdot \left(\underbrace{e^{-jk(r - h\cos\theta)} + e^{-jk(r + h\cos\theta)}}_{\text{Resolviendo}} \right)$$

$e^{-jk(\cancel{h}Acos}$

$$\left(e^{-jk(r-h\cos\theta)}+e^{-jk(r+h\cos\theta)}\right)=e^{-jkr+jkh\cos\theta}+e^{-jkr-jkh\cos\theta}$$

$$=e^{-jkr}\cdot\left(e^{+jkh\cos\theta}+e^{-jkh.\cos\theta}\right)$$

$$E_\psi^t=j\eta\cdot\frac{I_0.e^{-jkr}}{2\pi r}\cdot\left(\frac{\cos(\frac{kl}{2}.\cos\theta)-\cos(\frac{kl}{2})}{\sin\theta}\right)\cdot\left(2.\cos(kh\cos\theta)\right)$$

$$H=\frac{E_0}{\eta}=j.\frac{I_0.e^{-jkr}}{2\pi r}\cdot\left(\frac{\cos(\frac{kl}{2}.(\cos\theta)-\cos(\frac{kl}{2})}{\sin\theta}\right).2.\cos(kh.\cos\theta)$$

---

b) $u=?$

$$u=\vec{S}_r.r^2$$

$$\vec{S}_r=\frac{1}{2}.\vec{E}\wedge\vec{H}=\frac{1}{2\eta}|\vec{E}|^2$$

$$=\frac{1}{2\cancel{\eta}}\cdot\eta^{\cancel{2}}\cdot\frac{I_0^2}{4\pi^2r^2}\cdot\left(\frac{\cos(\frac{kl}{2}.\cos\theta)-\cos(\frac{kl}{2})}{\sin\theta}\right)^2\cdot(2.\cos(kh.\cos\theta))^2$$

$$=\frac{\eta I_0^2}{8\pi^2r^2}\cdot\left(\frac{\cos(\frac{kl}{2}.\cos\theta)-\cos(\frac{kl}{2})}{\sin\theta}\right)^2.4.\cos(kh.\cos\theta)^2$$

$$U=\frac{\eta I_0^2}{2\pi^2r^2}\cdot\left(\frac{\cos(\frac{kl}{2}.\cos\theta)-\cos(\frac{kl}{2})}{\sin\theta}\right)^2.\cos^2(kh.\cos\theta).\cancel{r^2}$$

$$=\frac{\eta I_0^2}{2\pi^2r^2}\cdot\left(\frac{\cos(\frac{kl}{2}.\cos\theta)-\cos(\frac{kl}{2})}{\sin\theta}\right).\cos^2(kh.\cos\theta)$$

Diagrama de Radiação

$$\lambda = \frac{3}{2}$$

$$\cos\left(\frac{kl}{2}\cdot\cos\theta\right) - \cos\left(\frac{kl}{2}\right) = 0 \quad \wedge \quad \sin\theta \neq 0$$

$$k = \frac{2\pi}{\lambda} = \frac{2\pi}{\frac{3}{2}} = \frac{4\pi}{3}$$

$$\frac{kl}{2} = \frac{4\pi}{3}$$

$$\cos\left(\frac{4\pi}{3}\cdot\cos\theta\right) = \cos\left(\frac{4\pi}{3}\right) \quad \wedge \quad \sin\theta \neq 0$$

$$\frac{4\pi}{3}\cdot\cos\theta = \pm\frac{4\pi}{3} \pm 2\pi m$$

$$\cos\theta = \pm 1 \pm \frac{2\pi m}{\frac{4\pi}{3}} \left(= \frac{6m}{4} = \frac{3}{2}m\right)$$

$$\theta_0 = \arccos(\pm 1) = 0 \,/\, \pi$$

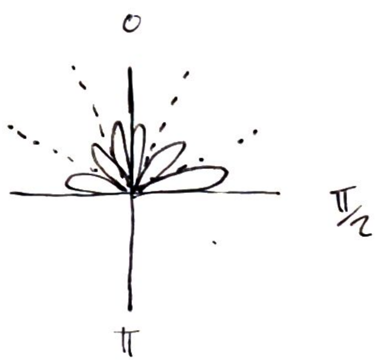
$$\theta_1 = \arccos\left(\pm 1 \pm \frac{3}{2}\right) = \frac{2}{3}\pi \,/\, \frac{1}{3}\pi$$

$$\theta_2 = \arccos(\pm 1 \pm 3) \rightarrow \text{fora do intervalo } [-1, 1]$$

$$\sin 0 = 0$$
$$\sin \pi = 0$$
$\rightarrow$ Indeterminação

$$\lim_{\theta \to 0} \frac{\left[\cos\left(\frac{kl}{2}\cdot\cos\theta\right) - \cos\left(\frac{kl}{2}\right)\right]'}{(\sin\theta)'} = \frac{\left(\frac{kl}{2}\cdot\cos\theta\right)'\cdot\left(-\sin\left(\frac{kl}{2}\cdot\cos\theta\right)\right)}{\cos\theta} = \frac{\frac{kl}{2}\cdot\sin\theta\cdot\sin\left(\frac{kl}{2}\cos\theta\right)}{\cos\theta}$$

$$= \frac{0}{1} = 0$$ portanto é zero



c)

$$W = \int_S \int \mathrm{Re}\{S\} \cdot ds$$

$$= \int_0^{2\pi} \int_0^{\pi} \mathrm{Re}\{S_r\} \cdot r^2 \cdot \sin\theta \cdot d\theta \cdot d\phi$$

$$S_r = \eta \cdot \frac{I_0^2}{2\pi^2 r^2} \cdot \left(\frac{\cos\left(\frac{kl}{2} \cdot \cos\theta\right) - \cos\left(\frac{kl}{2}\right)}{\sin\theta} \right)^2 \cdot \cos(kh \cdot \cos\theta)^2 \cdot \sin\theta$$

$$W = \eta \cdot \frac{I_0^2}{2\pi^2 r^2} \cdot 2\pi \cdot \int_0^{\pi} \left(\frac{\cos\left(\frac{kl}{2} \cdot \cos\theta\right) - \cos\left(\frac{kl}{2}\right)}{\sin\theta} \right)^2 \cdot \cos(kh \cdot \cos\theta)^2 \cdot d\theta$$

$$= \eta \cdot \frac{I_0^2}{\pi \cdot r^2} \cdot \int_0^{\pi} \left(\frac{\cos\left(\frac{4\pi}{3} \cdot \cos\theta\right) + \frac{1}{2}}{\sin\theta} \right)^2 \cdot \cos\left(\frac{4\pi}{3} \cdot h \cdot \cos\theta\right)^2 \cdot d\theta$$

$$= \eta \cdot \frac{I_0^2}{\pi} \cdot \int_0^{\pi} \frac{\cos^2\left(\frac{4\pi}{3} \cdot \cos\theta\right) + \cos^2\left(\frac{4\pi}{3}\right)}{\sin^2\theta} \cdot \cos\left(\frac{4\pi}{3} h \cdot \cos\theta\right)^2 \cdot \sin\theta$$

$$= \eta \cdot \frac{I_0^2}{\pi} \cdot \int_0^{\pi} \frac{\cos^2\left(\frac{4\pi}{3} \cdot \cos\theta\right) + \cos^2\left(\frac{4\pi}{3}\right)}{\sin\theta} \cdot \cos\left(\frac{4\pi}{3} \cdot h \cdot \cos\theta\right)^2 \cdot d\theta$$

$$R_r = \frac{2W}{|I_0|^2} = \frac{2\eta}{\pi} \cdot \int_0^{\pi} \frac{\cos\left(\frac{4\pi}{3} \cdot \cos\theta\right) + \cos^2\left(\frac{4\pi}{3}\right)}{\sin\theta} \cdot \cos^2\left(\frac{4\pi}{3} \cdot h \cdot \cos\theta\right) \cdot d\theta$$

d)

100Km -> Campo distante

$\theta = 45° = \pi/2$

$S = 20\ m\ W/m^2 = 20 \times 10^{-6}\ W/m^2$

$S = \frac{1}{2} \cdot \vec{E} \wedge \vec{H}$ Sabendo que $H = \frac{\vec{E}}{\eta}$

$= \frac{1}{2\eta} \cdot |\vec{E}|^2$

$$\vec{S_r} = \frac{1}{2\eta} \cdot \eta^2 \cdot \frac{I_0^2}{4\pi^2 r^2} \cdot \left(\frac{\cos(\frac{kl}{2} \cdot \cos\theta) - \cos(\frac{kl}{2})}{\sin\theta} \right)^2 \cdot \cos(kh \cdot \cos\theta)^2$$

$$= \frac{I_0^2}{2\pi^2 \cdot r^2} \cdot \left(\frac{\cos(0) - (-\frac{1}{2})}{1} \right)^2 \cdot \cos(0) \left(= (1 + \frac{1}{2})^2 \cdot 1 = \frac{9}{4} \right)$$

$$= \frac{I_0^2 \cdot \frac{9}{4}}{2\pi^2 \cdot r^2} = \frac{I_0^2 \cdot 9}{8\pi^2 \cdot r^2}$$

$$20 \times 10^{-6} = \frac{I_0^2 \cdot 9}{8\pi^2 \cdot (100.000)^2} \Leftrightarrow 1,6 \times 10^6 = I_0^2 \cdot 9 \Leftrightarrow I_0^2 = 1,778 \times 10^5$$

$$I_0 = \sqrt{1,778 \times 10^5} = 421.64\ A$$