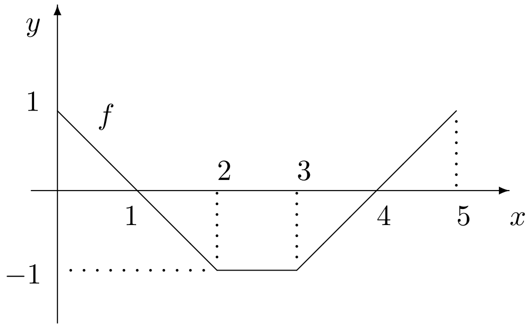
Folha 6B – Integral de Riemann

1. Sabendo que $\int_1^4 f(x)\,dx = 3$ e que $\int_2^4 f(x)\,dx = 5$, determine:

   (a) $\int_1^4 f(t)\,dt$; (b) $\int_4^2 f(t)\,dt$; (c) $\int_1^2 f(x)\,dx$; (d) $\int_{1/2}^2 f(2x)\,dx$.

2. Seja $f\colon [0,5] \longrightarrow \mathbb{R}$ a função representada na figura ao lado. Recorrendo ao significado geométrico do integral em termos de área, calcule

   $$\int_0^1 f(x)dx,\ \int_1^2 f(x)dx,\ \int_0^5 f(x)dx.$$



3. Apresente um exemplo de:

   (a) uma função $f\colon [0,2] \longrightarrow \mathbb{R}$ tal que $\int_0^2 f(x)\,dx = 0$ e $f(x) \neq 0, \forall x \in [0,2]$;

   (b) duas funções $f, g\colon [0,2] \longrightarrow \mathbb{R}$ tais que $\int_0^2 f(x)\,dx = \int_0^2 g(x)\,dx$ e $f(x) \neq g(x)$, $\forall x \in [0,2]$.

4. Calcule os seguintes integrais definidos:

   (a) $\int_0^3 \sqrt{9-x^2}\ dx$; (b) $\int_0^1 \ln(x^2+1)dx$;

   (c) $\int_0^\pi x \operatorname{sen} x\ dx$; (d) $\int_0^{\sqrt{2}/2} \operatorname{arcsen} x\ dx$;

   (e) $\int_{-3}^2 \sqrt{|x|}\ dx$; (f) $\int_{-1}^1 \frac{1}{1+x^2}\ dx$;

(g) $\displaystyle\int_3^4 \frac{1-4x^3}{x-x^4}\ dx$;

(h) $\displaystyle\int_0^{\frac{\pi}{2}} \operatorname{sen} 2x \cos 5x\ dx$;

(i) $\displaystyle\int_0^1 x \operatorname{arctg} x^2\ dx$;

(j) $\displaystyle\int_0^3 2-|x|\ dx$;

(k) $\displaystyle\int_0^2 \frac{2x-1}{(x-3)(x+1)}\ dx$;

(l) $\displaystyle\int_{-3}^2 \sqrt{|x|}\ dx$;

(m) $\displaystyle\int_e^{e^2} \frac{\ln(\ln x^2)}{x}\ dx$;

(n) $\displaystyle\int_0^8 \frac{\sqrt[3]{x}}{\sqrt[3]{x}^2+1}\ dx$;

(o) $\displaystyle\int_0^{2\pi} |\cos x|\ dx$;

(p) $\displaystyle\int_{-1}^2 x|x|\ dx$;

(q) $\displaystyle\int_0^1 g(x)\ dx$, com $g(x) = \begin{cases} x & \text{se} \quad 0 \leq x \leq \dfrac{1}{2}, \\ -x & \text{se} \quad \dfrac{1}{2} < x \leq 1; \end{cases}$

(r) $\displaystyle\int_0^1 \frac{1}{(1+x^2)^2}\ dx$, utilizando a mudana de varivel definida por $x = \operatorname{tg} t$.