INSTITUTO SUPERIOR DE ENGENHARIA DE COIMBRA

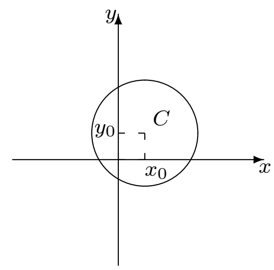
Cálculo II – 2011/2012 **Engenharia Biomédica**

Formulário

- Equações reduzidas das cónicas:

(A) **Circunferência** de centro $C(x_0, y_0)$ e raio r

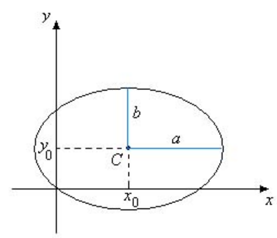
$$(x - x_0)^2 + (y - y_0)^2 = r^2$$



(B) **Elipse** de centro $C(x_0, y_0)$ e semieixos a e b

$$\frac{(x - x_0)^2}{a^2} + \frac{(y - y_0)^2}{b^2} = 1$$

(Nota: quando $a = b$, a elipse é uma circunferência)



(C) **Hipérbole** de centro $C(x_0, y_0)$ e semieixos a e b

1º caso

$$\frac{(x - x_0)^2}{a^2} - \frac{(y - y_0)^2}{b^2} = 1$$

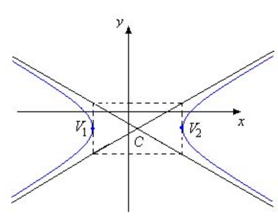
Vértices:

$V_1(x_0 - a, y_0)$

$V_2(x_0 + a, y_0)$

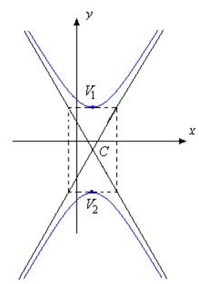
Assímptotas:

$$y - y_0 = \pm \frac{b}{a}(x - x_0)$$



2º caso

$$\frac{(y-y_0)^2}{b^2} - \frac{(x-x_0)^2}{a^2} = 1$$



Vértices:
$V_1(x_0, y_0 + b)$
$V_2(x_0, y_0 - b)$

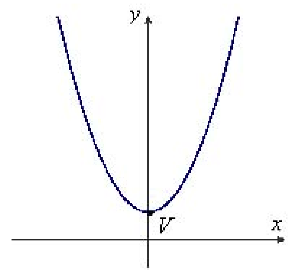
Assímptotas:

$$x - x_0 = \pm \frac{b}{a}\,(y - y_0)$$

(D) **Parábola**

1º caso

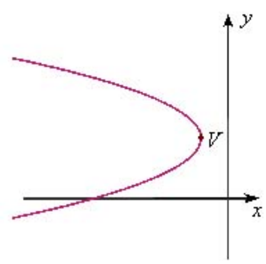
$$(x-x_0)^2 = \alpha(y-y_0)$$



Vértice da parábola: $V(x_0, y_0)$

2º caso

$$(y-y_0)^2 = \beta(x-x_0)$$



Vértice da parábola: $V(x_0, y_0)$

- Desigualdades

$$|x| \leq \sqrt{x^2+y^2}, \quad |y| \leq \sqrt{x^2+y^2}, \qquad |x-x_0| \leq \sqrt{(x-x_0)^2+(y-y_0)^2}$$

$$x^2 \leq x^2+y^2, \qquad y^2 \leq x^2+y^2$$

$$|xy| \leq x^2+y^2, \quad |x \pm y| \leq |x|+|y|, \quad \sqrt{x^2+y^2} \leq |x|+|y|$$

- Área de uma superfície tridimensional

$$S = \iint_\Omega \sqrt{1 + \left(\frac{\partial z}{\partial x}\right)^2 + \left(\frac{\partial z}{\partial y}\right)^2}\; dA$$

- Coordenadas Polares

$$\begin{array}{rclcrcl} x &=& r\cos\theta & \qquad & r &=& \sqrt{x^2+y^2} \\ y &=& r\sin\theta & \qquad & \operatorname{tg}\theta &=& y/x, \quad \theta \in [0\,, 2\pi[\end{array}$$

- Integrais duplos em coordenadas polares

$$\iint_\Omega f(x,y)\; dA = \int_{\theta_1}^{\theta_2} \int_{g_1(\theta)}^{g_2(\theta)} f(r\cos\theta,\ r\sin\theta)\; r\; dr\, d\theta$$

- Integrais triplos

$$\iiint_G f(x,y,z)\ dV = \iint_\Omega \left[\int_{\eta_1(x,y)}^{\eta_2(x,y)} f(x,y,z)\ dz\right]\ dA$$

- Coordenadas Cilíndricas

$$\begin{aligned} x &= r\cos\theta \\ y &= r\sin\theta \\ z &= z \end{aligned} \qquad \begin{aligned} r &= \sqrt{x^2+y^2} \\ \operatorname{tg}\theta &= y/x, \quad \theta \in [0\,,2\pi[\\ z &= z \end{aligned}$$

- Coordenadas Esféricas

$$\begin{aligned} x &= \rho\sin\phi\cos\theta \\ y &= \rho\sin\phi\sin\theta \\ z &= \rho\cos\phi \end{aligned} \qquad \begin{aligned} \rho &= \sqrt{x^2+y^2+z^2} \\ \phi &= \arccos(z/\rho) \ \in [0,\pi] \\ \cos\theta &= \tfrac{x}{\rho\sin\phi} \\ \sin\theta &= \tfrac{y}{\rho\sin\phi}, \quad \theta \in [0, 2\pi[\end{aligned}$$

- Integrais triplos em coordenadas cilíndricas

$$\iiint_G f(x,y,z)\ dV = \int_{\theta_1}^{\theta_2} \int_{g_1(\theta)}^{g_2(\theta)} \int_{h_1(\theta,r)}^{h_2(\theta,r)} f(r\cos\theta,\ r\sin\theta,\ z)\ r\ dz\, dr\, d\theta$$

- Integrais triplos em coordenadas esféricas

$$\iiint_G f(x,y,z)\ dV =$$

$$= \int_{\theta_1}^{\theta_2} \int_{g_1(\theta)}^{g_2(\theta)} \int_{h_1(\theta,\phi)}^{h_2(\theta,\phi)} f(\rho\sin\phi\cos\theta,\ \rho\sin\phi\sin\theta, \rho\cos\phi)\ \rho^2\sin\phi\ d\rho\ d\phi\ d\theta$$

- Integrais de linha

$$\int_\gamma f(x,y,z)\ ds = \int_a^b f\left(x(t),y(t),z(t)\right)\sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2 + \left(\frac{dz}{dt}\right)^2}\ dt$$

- Campos vectoriais
  - Rotacional: $\operatorname{rot} F(x,y,z) = \nabla \times F(x,y,z)$
  - Divergente:

$$\begin{aligned} \operatorname{div} F(x,y) &= \nabla\cdot F(x,y) \\ \operatorname{div} F(x,y,z) &= \nabla\cdot F(x,y,z) \end{aligned}$$