Ficha 2A

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a) $P\; x^2\, ch(x^3) + x \cdot 4^{x^2}$

$P\; x^2\, ch(x^3) \quad + P\; x\, 4^{x^2}$

$\frac{1}{3} P\, 3x^2\, ch(x^3) \quad + \frac{1}{2} P\; 2x\, 4^{x^2}$

$\frac{1}{3} P\; ch(u)\, u' \quad + \frac{1}{2} P\; 4^{u}\; u'$

$\frac{1}{3}\, sh(u) \quad + \frac{1}{2} \cdot \frac{4^{u}}{\ln 4} \quad + C$

$\frac{1}{3}\, sh(x^3) \quad + \frac{1}{2}\; \frac{4^{x^2}}{\ln 4} + C$

R_1: $\frac{d\, sh\, u}{du} = ch\, u\; \frac{du}{dx}$

R_2: $\frac{d}{dx}\left(\frac{a^{u}}{\ln a}\right) = a^{u}\, \frac{du}{dx}$

b)

$$P\, \frac{sh(5x)}{\sqrt[3]{ch^4(5x)}} = \frac{1}{5}\; P\; \frac{5\, sh(5x)}{ch^{4/3}(5x)} = \frac{1}{5}\; P\; \frac{u'}{u^{4/3}} =$$

$u = ch(5x)$
$u' = 5\, sh(5x)$

$$= \frac{1}{5}\; P\; u^{-4/3}\, u' = \frac{1}{5}\; \frac{u^{-1/3}}{\left(-\frac{1}{3}\right)} + C \quad = -\frac{3}{5}\left[ch(5x)\right]^{-1/3} + C$$

R: $P\; u^{\alpha} u' = \frac{u^{\alpha+1}}{\alpha+1} + C$

c)

$$P \frac{1}{\sqrt{4-9x^2}} = P \frac{1}{\sqrt{4\left(1-\frac{9x^2}{4}\right)}} = \frac{1}{2} P \frac{1}{\sqrt{1-\left(\frac{3}{2}x\right)^2}} =$$

$u = \frac{3}{2}x$

$u' = \frac{3}{2}$

$$= \frac{1}{\cancel{2}} \cdot \frac{\cancel{2}}{3} P \frac{3/2}{\sqrt{1-\left(\frac{3}{2}x\right)^2}} = \frac{1}{3} P \frac{u'}{\sqrt{1-u^2}} =$$

R: $\frac{d}{dx}(\text{arc sen } u) = \frac{1}{\sqrt{1-u^2}} \frac{du}{dx}$

$$= \frac{1}{3} \text{arc sen } u + C = \frac{1}{3} \text{arc sen}\left(\frac{3}{2}x\right) + C$$

d)

$$P \frac{(\ln x + e)^4}{x} = P\, u^4 u' = \frac{u^5}{5} + C = \frac{(\ln x + e)^5}{5} + C$$

$u = \ln x + e$

$u' = \frac{1}{x}$

e) $P \text{ tg } x = P \frac{\text{sen } x}{\cos x} = - P \frac{-\text{sen } x}{\cos x} = - P \frac{u'}{u} = - \ln|u| + C$

$u = \cos x$

$u' = -\text{sen } x$

$= - \ln|\cos x| + C$

R: $\boxed{\frac{d}{dx}(\log_a u) = \frac{\log_a e}{u} \frac{du}{dx} \quad c/ \quad a = e}$

f)

$$P \frac{5x}{4+4x^2} = 5 P \frac{x}{4(1+x^2)} = \frac{5}{4} P \frac{x}{1+x^2} =$$

$u = x^2$

$u' = 2x$

$$= \frac{5}{4} \cdot \frac{1}{2} P \frac{2x}{1+x^2} = \frac{5}{8} P \frac{u'}{1+u^2}$$

$$= \frac{5}{8} \ln u + C = \frac{5}{8} \text{arctg}(2x) + C$$

R $\boxed{\frac{d}{dx}(\text{arctg } u) = \frac{1}{1+u^2} \frac{du}{dx}}$

g)

$$P \frac{3x}{\sqrt{1+5x^2}} = 3 P \frac{x}{\sqrt{1+(\sqrt{5}x)^2}} = \frac{3}{10} P \frac{10x}{(1+5x^2)^{1/2}}$$

$u = 1+5x^2 \Rightarrow u' = 10x$

$$= \frac{3}{10} P \frac{u'}{u^{1/2}} = \frac{3}{10} P u^{-1/2} u' = \frac{3}{10} \frac{u^{1/2}}{\frac{1}{2}} + C =$$

$$= \frac{3}{5} \sqrt{5x^2+1} + C$$

(2)

$$f'(x) = \frac{x}{(1+x^2)^2} \qquad f(0) = 2$$

$$P \frac{x}{(1+x^2)^2} = \frac{1}{2} P \frac{2x}{(1+x^2)^2} = \frac{1}{2} P \frac{u'}{u^2}$$

$u = 1+x^2$
$u' = 2x$

$$= \frac{1}{2} P u^{-2} u' = \frac{1}{2} \frac{u^{-1}}{-1} = -\frac{1}{2} \cdot \frac{1}{(1+x^2)} + C$$

$$f(x) = -\frac{1}{2} \cdot \frac{1}{(1+x^2)} + C$$

$$f(0) = -\frac{1}{2} \cdot \frac{1}{1} + C = 2$$

$$-\frac{1}{2} + C = 2$$

$$C = \frac{5}{2}$$

$\therefore \quad f(x) = -\frac{1}{2(1+x^2)} + \frac{5}{2}$