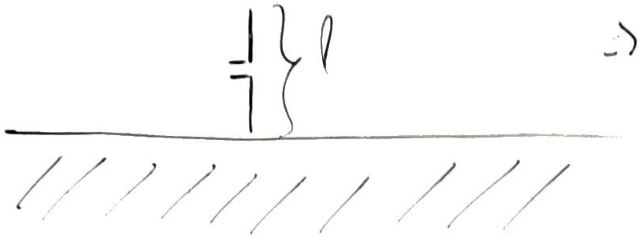
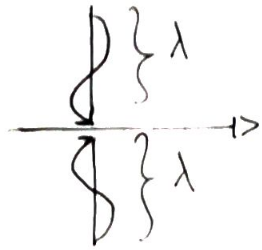
Teste 2018/2019

a)



$\Rightarrow$



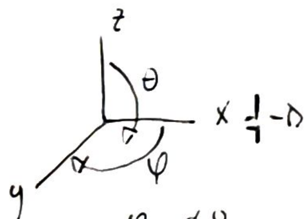
$$\lambda = \frac{c}{f} = \frac{3\times10^{8}}{300\times10^{6}} = 1\,m$$

$$2\lambda = l \Leftrightarrow l = 2\,m$$

b) Antena colocada no eixo OX

$$\cos\psi = \hat{i}_r \cdot \hat{\imath} = (\underbrace{\sin\theta \cdot \cos\varphi \cdot \hat{\imath}}_{Ox} + \underbrace{\sin\theta \cdot \sin\varphi \cdot \hat{\jmath}}_{Oy} + \underbrace{\cos\theta \cdot \hat{k}}_{Oz}) \cdot \hat{\imath}$$

$$\cos\psi = \sin\theta \cdot \cos(\varphi)$$



$\varphi = 0^\circ$

$$\cos\psi^2 + \sin^2\psi = 1 \Leftrightarrow \sin\psi = \sqrt{1-\cos^2\psi}$$

$$\Leftrightarrow \sin\psi = \sqrt{1-\sin^2\theta \cdot \cos^2\varphi}$$

$$E_\psi = j\eta \cdot \frac{I_0\, e^{-jkr}}{2\pi r} \cdot \left(\frac{\cos\left(\frac{kl}{2} \cdot \cos\psi\right) - \cos\left(\frac{kl}{2}\right)}{\sin\theta} \right)$$

$$= j \cdot \eta \cdot \frac{I_0 \cdot e^{-jkr}}{2\pi r} \cdot \left(\frac{\cos\left(\frac{kl}{2} \cdot \sin\theta \cos\varphi\right) - \cos\left(\frac{kl}{2}\right)}{\sqrt{1-\sin^2\theta - \cos^2\varphi}} \right)$$

$$= j\eta \cdot \frac{I_0 \cdot e^{-jkr}}{2\pi r} \left(\frac{\cos\left(\frac{kl}{2} \cdot \sin\theta\right) - \cos\left(\frac{kl}{2}\right)}{\underbrace{\sqrt{1-\sin^2\theta}}_{\cos\theta}} \right)$$

zeros E_φ

$$\cos\left(\frac{kl}{2} . \sin\theta\right) = \cos\left(\frac{kl}{2}\right) \quad \wedge \quad \cos\theta \neq 0$$

$$\frac{kl}{2} \sin\theta = \pm \frac{kl}{2} \pm 2\pi m \quad \wedge \quad ''$$

$$\sin\theta = \pm 1 \pm \frac{2\pi m}{\frac{kl}{2}} \left(= \frac{2\pi m}{\frac{\frac{2\pi}{\lambda} . l}{2}} = \frac{2\pi m}{\frac{\pi l}{\lambda}} = \frac{2\lambda m}{l} = \frac{2m}{2} = m\right.$$

$$\sin\theta = \pm 1 \pm m$$

$$\theta_0 = \sin^{-1}(1) = \frac{\pi}{2} \,/\, -\frac{\pi}{2}$$

$$\theta_1 = \sin^{-1}(0) = 0$$

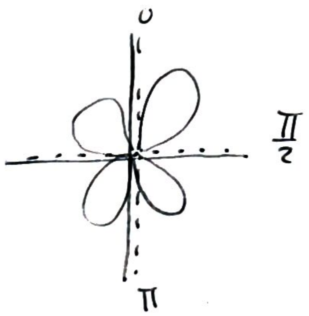
$$\theta_2 = \sin^{-1}(1-2) = -\frac{\pi}{2}$$
$$(-1+2) = \frac{\pi}{2}$$

$$\left.\begin{array}{l} \cos\left(\frac{\pi}{2}\right) = 0 \\ \cos\left(-\frac{\pi}{2}\right) = 0 \end{array}\right\} \text{Regra L'hopital}$$

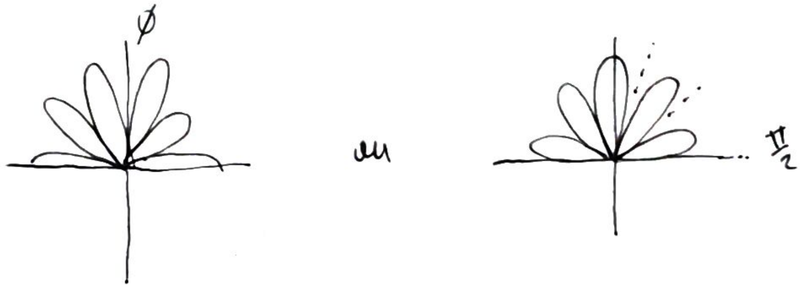
$$\lim_{\theta \to \frac{\pi}{2}} \frac{\left(\cos\left(\frac{kl}{2} \sin\theta\right) - \cos\left(\frac{kl}{2}\right)\right)'}{(\cos\theta)'} = \lim \frac{-\left(\frac{kl}{2} . \sin\theta\right)' . \sin\left(\frac{kl}{2} . \sin\theta\right)}{-\sin\theta}$$

$$= \lim_{\theta \to \frac{\pi}{2}} \frac{-\frac{kl}{2} . \cos\theta . \sin\left(\frac{kl}{2} . \sin\theta\right)}{-\sin\theta} = \frac{0}{1} = 0$$

portanto
São zeros do diagrama



c) Ponto lobular



cumprirem:

$$\sin\theta = \pm 1 \pm \frac{2\lambda m}{l} \quad \wedge \quad m \neq 0 \qquad (\lambda = 1\,m)$$

$$\sin\theta = \pm 1 \pm \frac{2m}{l}$$

$$\theta_0 = \arcsin(1) = \pi/2$$
$$\arcsin(-1) = -\pi/2$$

$$\theta_1 = \arcsin\left(\pm 1 \pm \frac{2}{l}\right) \to +1 - \frac{2}{l} > 0 \Rightarrow \frac{2}{l} < 1 \Leftrightarrow l > 1$$
$$\to -1 + \frac{2}{l} < 1 \Rightarrow \frac{2}{l} < 2 \Leftrightarrow l > 1$$

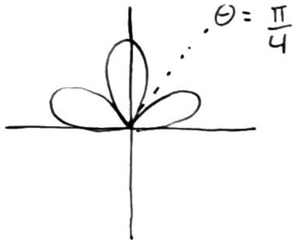
$$\theta_2 = \arcsin\left(\pm 1 \pm \frac{4}{l}\right) \to +1 - \frac{4}{l} > 0 \Rightarrow -\frac{4}{l} > -1 \Leftrightarrow l > 4 -$$
$$-1 + \frac{4}{l} < 1 \Rightarrow \frac{4}{l} < 2 \Leftrightarrow l > 2$$

Agora como temos a Zona nucleonica, θ_3 não pode existir

$$\theta_3 = \arcsin\left(\pm 1 \pm \frac{6}{l}\right) \to \pm 1 - \frac{6}{l} < 0 \Leftrightarrow \frac{6}{l} > 1 \Leftrightarrow l < 6 -$$

$4 < l < 6$ metros

d)



Expressão:

$$\sin\theta = \pm 1 \pm \frac{2\lambda m}{l}$$

Se $\lambda = 1$

$l = 2$

$\sin\theta = \pm 1 \pm m$ → para que verifique-se $\sin\theta = \frac{\sqrt{2}}{2}$ nunca vai acontecer, portanto teremos de adotar outro comprimento

$$\sin\theta = \pm 1 \pm \frac{2\lambda m}{l}$$

$$\theta_0 = \arcsin(\pm 1) = [-\pi/2 \,;\, \pi/2]$$

Para $m = 1$

$$\pm 1 \pm \frac{2\lambda m}{l} = \frac{\sqrt{2}}{2} \iff 1 - \frac{2\lambda}{l} = \frac{\sqrt{2}}{2} \iff \frac{2\lambda}{l} = 1 - \frac{\sqrt{2}}{2}$$

$$\iff l = \frac{2\lambda}{1-\sqrt{2}} \quad (\text{se } \lambda = 1\,m)$$

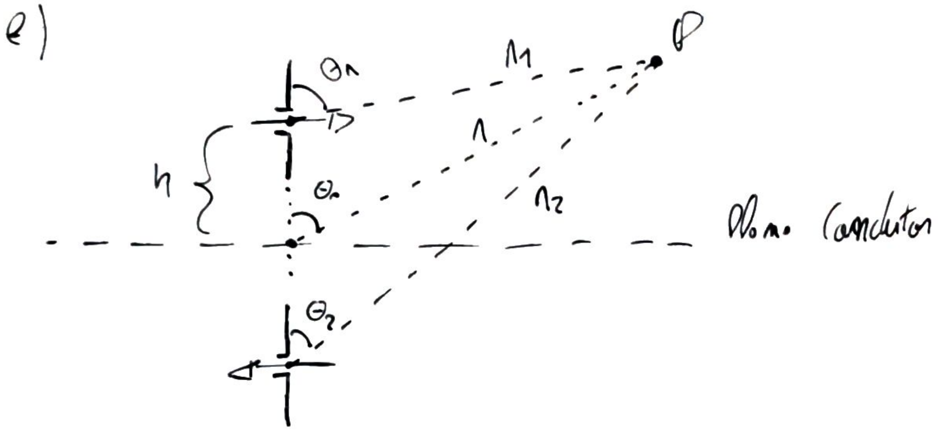
$$\iff l = 6{,}83\ m$$

~~Para 2m=2~~

Para $m = 2$, não é necessário para já adquirirmos os zeros para o diagrama tubular

$$1 - \frac{4\lambda}{l} < 0 \iff -4\lambda < -l \iff 4\lambda > l$$

e)



$$E^t_\psi = E^d_\psi + Rh \cdot E^r_\psi$$

↳ Para a polarização horizontal temos $Rh = -1$

$$E^d_\psi = j\eta \cdot \frac{I_0 \cdot e^{-jkr_1}}{2\pi r_1} \left(\frac{\cos\left(\frac{kl}{2} \cdot \sin\theta_1 \cdot \cos\varphi\right) - \cos\left(\frac{kl}{2}\right)}{\sqrt{1 - \sin^2\theta_1 \cdot \cos^2\varphi}} \right)$$

$$E^r_\psi = j \cdot \eta \cdot \frac{I_0 \cdot e^{-jkr_2}}{2\pi r_2} \cdot \left(\frac{\cos\left(\frac{kl}{2} \cdot \sin\theta_2 \cdot \cos\varphi\right) - \cos\left(\frac{kl}{2}\right)}{\sqrt{1 - \sin^2\theta_2 \cdot \cos^2\varphi}} \right)$$

Aprox. de campo Distante:

$$\theta = \theta_1 = \theta_2$$

e em módulo.

$$r = r_1 = r_2$$

e em fase:

$$r_1 = r - h\cos\theta$$
$$r_2 = r + h\cos\theta$$

$$E^t_\psi = j\eta \, \frac{I_0}{2\pi r} \cdot \left(\frac{\cos(kl/2 \cdot \sin\theta \cdot \cos\varphi) - \cos(kl/2)}{\sqrt{1 - \sin^2\theta \cdot \cos^2\varphi}} \right) \cdot \left(e^{-jk(r - h\cos\theta)} - e^{-jk(r+h\cos\theta)} \right.$$

$$\Rightarrow \left(e^{-jkr + jkh\cos\theta} - e^{-jkr - jkh\cos\theta} \right) \Leftrightarrow$$

$$\Leftrightarrow e^{-jkr} \cdot \left(e^{jkh\cos\theta} - e^{-jkh\cos\theta} \right) \Leftrightarrow$$

$$\Leftrightarrow e^{-jkr} \cdot \left(2 \cdot j \cdot \sin(kh \cdot \cos\theta) \right)$$

$$E^t_\psi = j\eta \cdot \frac{I_0 \cdot e^{-jkr}}{2\pi r} \cdot \left(\frac{\cos(kl/2 \cdot \sin\theta \cos\varphi) - \cos(kl/2)}{\sqrt{1 - \sin^2\theta \cdot \cos^2\varphi}} \right) \cdot \left(2\, j \sin(kh \cos\theta) \right)$$

l) Densidade de potência média irradiada

$$\vec{S} = \frac{1}{2} \cdot \vec{E} \wedge \vec{H} \qquad H_x = \frac{E_\varphi}{\eta}$$

$$\vec{S} = \frac{1}{2\eta} \cdot |\vec{E}|^2$$

$$\vec{S} = \frac{1}{2\eta} \cdot \eta^2 \cdot \frac{I_0^2}{4\pi^2 r^2} \cdot \left(\frac{\cos(\frac{kl}{2} \cdot \sin\theta \cdot \cos\varphi) - \cos(\frac{kl}{2})}{\sqrt{1 - \sin^2\theta \cdot \cos^2\varphi}} \right)^2$$

Sem plano Condutor

com plano condutor

$$|\vec{E_p}| = |\vec{E}| \cdot 2 \cdot \sin(kh \cdot \cos\theta)$$

$$|\vec{E_p}|^2 = |\vec{E}|^2 \cdot 4 \cdot \sin^2(kh \cos\theta)$$

g)

$$2\sin(kh.\cos\theta)=0$$

$$k.h.\cos\theta = \pm m\pi$$

$$\cos\theta = \pm\frac{m\pi}{k.h}\left(=\frac{m\pi}{\frac{2\pi h}{\lambda}} = \frac{m\pi\lambda}{2\pi h} = m.\frac{\lambda}{2h}\right)$$

$m=0$

$\theta = \arccos(0) = \frac{1}{2}\pi$

$m=1$

$\theta = \arccos\left(\pm\frac{\lambda}{2h}\right) \Rightarrow \frac{\lambda}{2h} < 1 \Leftrightarrow \lambda < 2h \Leftrightarrow h > \frac{\lambda}{2}$

$m=2$

$\theta = \arccos\left(\pm\frac{\lambda}{h}\right) \Rightarrow \frac{\lambda}{h} < 1 \Leftrightarrow \lambda < h$

$m=3$ → já não queremos zeros

$\theta = \arccos\left(\pm\frac{3\lambda}{2h}\right) \Rightarrow \frac{3\lambda}{2h} > 1 \Leftrightarrow 3\lambda > 2h \Leftrightarrow h < \frac{3\lambda}{2}$

se $\lambda = 1\,m$

$1 < h < \frac{3}{2}$ metros

h)

$h = ?$

$\theta = 0$

$\sin(kh.\cos\theta) = 0$

$\sin(kh) = 0$

$kh = \pm n\pi$

$h = \pm \frac{n\pi}{k} \left(= \frac{n\pi}{\frac{2\pi}{\lambda}} = n\frac{\lambda}{2} \right)$

$n = 0$

$h = 0\,m$

$n = 1$

$h = 0{,}5\,m$

$n = 2$

$h = 1m$