ficha 6A

6.1

1 $f(x,y) = \begin{cases} x+y & \text{se } xy=0 \\ 1 & \text{se } xy\neq 0 \end{cases}$

a) $\frac{\partial f}{\partial x}(0,0) = \lim_{h\to 0} \frac{f(h,0)-f(0,0)}{h} = \lim_{h\to 0} \frac{h+0-0}{h} = 1$

$\frac{\partial f}{\partial y}(0,0) = \lim_{h\to 0} \frac{f(0,0+h)-f(0,0)}{h} = \lim_{h\to 0} \frac{f(0,h)-0}{h} =$

$= \lim_{h\to 0} \frac{0+h}{h} = \lim_{h\to 0} 1 = 1$

As derivadas parciais existem no ponto (0,0).

b) $\lim_{(x,y)\to(0,0)} f(x,y) \overset{?}{=} \underbrace{f(0,0)}_{0}$

limites iterados: $\lim_{x\to 0}\left(\lim_{y\to 0} 1\right) = 1$; $\lim_{y\to 0}\left(\lim_{x\to 0} 1\right) = 1$. Se o limite existir é 1. Como $f(0,0)=0$

logo f não é contínua no pto (0,0).

c) Como f não é contínua no pto (0,0) então f não é diferenciável nesse pto!

2 - Rev:

- Se f admitir derivadas parciais de 1ª ordem $\frac{\partial f}{\partial x}$, $\frac{\partial f}{\partial y}$ contínuas numa vizinhança de (a,b) então f é diferenciável em (a,b).

- Se f é diferenciável em (a,b) então: $a \Rightarrow b$ ($\equiv \sim b \Rightarrow \sim a$)
  - f é contínua em (a,b)
  - f admite derivadas parciais de 1ª ordem em (a,b).
  - f

- $dz \equiv df \equiv \frac{\partial f}{\partial x}(a,b)\,dx + \frac{\partial f}{\partial y}(a,b)\,dy$ ← diferencial de função

$\Delta f \equiv \Delta z = f(a+dx, b+dy) - f(a,b) \Leftrightarrow f(a+dx,b+dy) = f(a,b) + \Delta z$

$$\boxed{f(a+dx,b+dy) \simeq f(a,b) + dz}$$

$$df = \frac{\partial f}{\partial x} \cdot dx + \frac{\partial f}{\partial y} \cdot dy + \frac{\partial f}{\partial z} dz + \frac{\partial f}{\partial t} \cdot dt$$

$$\Leftrightarrow df = 3 \cdot dx + 4y \cdot dy - 3z^2 \cdot dz + dt$$

$\boxed{3.}$ $\ln(1.01^2 + 0.02^3)$

Seja $f(x,y) = \ln(x^2+y^3)$

- $(a,b) = (1,0)$
- $dx = 1.01 - 1 = 0.01$
- $dy = 0.02 - 0 = 0.02$

Assim

$$f(\overset{a}{1} + \overset{dx}{0.01}, \overset{b}{0} + \overset{dx}{0.02}) \simeq \overset{0}{\overset{\shortparallel}{f(1,0)}} + df(1,0)$$

$$\Leftrightarrow f(1.01, 0.02) \simeq 0 + 0.02 = 0.02$$

Valor exacto:
$f(1.01, 0.02) = 0.0199085$

C.A:

- $f(1,0) = \ln(1^2 + 0^3) = \ln 1 = 0$
- $df(1,0) = \frac{\partial f}{\partial x}(1,0) \cdot dx + \overset{=0}{\frac{\partial f}{\partial y}(1,0)} \cdot dy = 2 \times 0.01 + 0 \times 0.02$
$= 0.02$
- $\frac{\partial f}{\partial x} = \frac{2x}{x^2+y^3} \rightarrow \frac{\partial f}{\partial x}(1,0) = \frac{2}{1+0} = 2$
- $\frac{\partial f}{\partial y} = \frac{3y^2}{x^2+y^3} \rightarrow \frac{\partial f}{\partial y}(1,0) = \frac{0}{1+0} = 0$

$\boxed{4}$ $A(c,l) = c \times l$ $\qquad dc = dl = 0{,}1$

C.A:

$\frac{\partial A}{\partial c} = l \rightarrow \frac{\partial A}{\partial c}(10,5) = 5$

$\frac{\partial A}{\partial l} = c \rightarrow \frac{\partial A}{\partial c}(10,5) = 10$

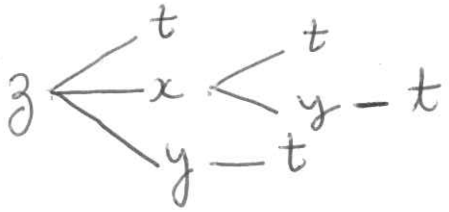
- $dA^{(10,5)} = \frac{\partial A}{\partial c}^{(10,5)} \cdot dc + \frac{\partial A}{\partial l}^{(10,5)} \cdot dl$
- $dA(10,5) = 5 \times 0{,}1 + 10 \times 0{,}1 = 0{,}5 + 1 = 1{,}5$
- erro maximo e' $1{,}5 \text{ cm}^2$

$\hookrightarrow$ erro $= A(c+0{,}1, l+0{,}1) - A(c,l) \simeq dA(10,5)$

5 - $z = txy^2$ em que:

$x = t + \ln(y+t^2)$

$y = e^t$



$$+ \frac{\partial z}{\partial x} \cdot \frac{\partial x}{\partial y} \cdot \frac{dy}{dt}$$

$$+ ty^2 \cdot \left(1 + \frac{2t}{y+t^2}\right) +$$

$$\frac{\partial z}{\partial t} = \frac{\partial z}{\partial t} + \frac{\partial z}{\partial x} \cdot \frac{\partial x}{\partial t} + \frac{\partial z}{\partial y} \cdot \frac{dy}{dt} = xy^2 + \cancel{ty^2\left(1+\frac{2t}{y+t^2}\right)+t}$$

C.A.

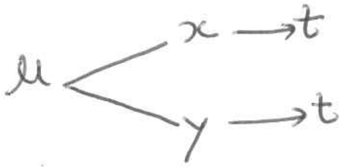
$$ty^2 \cdot \left(\frac{1}{y+t^2} \cdot e^t\right) + 2txy \cdot e^t$$

CA:

- $\frac{\partial z}{\partial t} = xy^2$; $\frac{\partial z}{\partial x} = ty^2$; $\frac{\partial z}{\partial y} = 2txy$
- $\frac{\partial x}{\partial t} = 1 + \frac{2t}{y+t^2}$ $\qquad \frac{\partial x}{\partial y} = \frac{1}{y+t^2}$
- $\frac{dy}{dt} = e^t$

$\boxed{6.}$ $u = e^{x-2y}$ $\quad$ • $x = \sin t$ $\quad$ • $y = t^3$

$u(x,y)$



- $$\frac{du}{dt} = \frac{\partial u}{\partial x} \cdot \frac{dx}{dt} + \frac{\partial u}{\partial y} \cdot \frac{dy}{dt} = e^{x-2y} \cdot \cos t - 2e^{x-2y} \cdot 3t^2$$

$$= e^{x-2y} \cdot \cos t - 6e^{x-2y} \cdot t^2 = (\cos t - 6t^2)e^{x-2y}$$

$$\frac{d^2u}{dt^2} = \frac{d}{dt}\left(\frac{du}{dt}\right) = \frac{d}{dt}\left(e^{x-2y} \cdot \cos t - 6e^{x-2y} \cdot t^2\right)$$

$$= \frac{d}{dt}(\underbrace{e^{x-2y}}_{u}) \cdot \cos t + e^{x-2y} \cdot \frac{d}{dt}(\cos t) - 6 \cdot \frac{d}{dt}(\underbrace{e^{x-2y}}_{u})t^2 - 6e^{x-2y} \cdot \frac{d}{dt}(t^2)$$

$$= (e^{x-2y} \cdot \cos t - 6t^2 \cdot e^{x-2y}) \cdot \cos t - \sin t \cdot e^{x-2y} - 6t^2 (e^{x-2y} \cos t - 6t^2 e^{x-2y})$$
$$- 12t\, e^{x-2y}$$

$$= (\cos t - 6t^2)(e^{x-2y} \cdot \cos t - 6t^2 e^{x-2y}) - e^{x-2y}(\sin t + 12t)$$

$$= (\cos t - 6t^2)(\cos t - 6t^2)e^{x-2y} - e^{x-2y}(\sin t + 12t)$$

$$= \left[(\cos t - 6t^2)^2 - (\sin t + 12t)\right] e^{x-2y}$$

$$\frac{\partial u}{\partial x} \begin{array}{l} \nearrow x - t \\ \searrow y - t \end{array} \qquad \frac{\partial u}{\partial y} \begin{array}{l} \nearrow x - t \\ \searrow y \to t \end{array}$$

$$\frac{d^2u}{dt^2} = \frac{d}{dt}\left[\frac{du}{dt}\right] = \frac{d}{dt}\left[\frac{\partial u}{\partial x}\cdot\frac{dx}{dt} + \frac{\partial u}{\partial y}\cdot\frac{dy}{dt}\right]$$

$$= \frac{d}{dt}\left[\frac{\partial u}{\partial x}\cdot\frac{dx}{dt}\right] + \frac{d}{dt}\left[\frac{\partial u}{\partial y}\cdot\frac{dy}{dt}\right]$$

$$= \left[\frac{\partial}{\partial x}\left(\frac{\partial u}{\partial x}\right)\cdot\frac{dx}{dt} + \frac{\partial}{\partial y}\left(\frac{\partial u}{\partial x}\right)\cdot\frac{dy}{dt}\right]\cdot\frac{dx}{dt} + \frac{\partial u}{\partial x}\cdot\frac{d^2x}{dt^2} +$$

$$+ \left[\frac{\partial}{\partial x}\left(\frac{\partial u}{\partial y}\right)\cdot\frac{dx}{dt} + \frac{\partial}{\partial y}\left(\frac{\partial u}{\partial y}\right)\cdot\frac{dy}{dt}\right]\cdot\frac{dy}{dt} + \frac{\partial u}{\partial y}\cdot\frac{d^2y}{dt^2}$$

$$= \left[\frac{\partial^2 u}{\partial x^2}\cdot\frac{dx}{dt} + \frac{\partial^2 u}{\partial y\partial x}\cdot\frac{dy}{dt}\right]\cdot\frac{dx}{dt} + \frac{\partial u}{\partial x}\cdot\frac{d^2x}{dt^2} +$$

$$+ \left[\frac{\partial^2 u}{\partial x\partial y}\cdot\frac{dx}{dt} + \frac{\partial^2 u}{\partial y^2}\cdot\frac{dy}{dt}\right]\cdot\frac{dy}{dt} + \frac{\partial u}{\partial y}\cdot\frac{d^2y}{dt^2}$$

$$= \left(e^{x-2y}\cdot\cos t - 2e^{x-2y}\cdot 3t^2\right)\cdot\cos t + e^{x-2y}\cdot(-\sin t) +$$

C.A. $+ \left[-2e^{x-2y}\cdot\cos t + 4e^{x-2y}\cdot 3t^2\right]\cdot 3t^2 - 2e^{x-2y}\cdot 6t$

$$= (\cos t - 6t^2)\cos t\cdot e^{x-2y} + (-6t^2\cdot\cos t + 4\times 3\times 3t^4 - \sin t - 12t]\,e^{x-2y}$$

$$= (\cos^2 t - 6t^2\cos t - 6t^2\cos t + 36t^4 - \sin t - 12t)\,e^{x-2y} = \left[(\cos t - 6t^2)^2 - (\sin t + 12t)\right]e^{x-2y}$$

C.A:

- $\frac{\partial^2 u}{\partial x^2} = \frac{\partial}{\partial x}\left(e^{x-2y}\right) = e^{x-2y}$

$\frac{dx}{dt} = \cos t \to \frac{d^2x}{dt^2} = -\sin t$

- $\frac{\partial^2 u}{\partial y\partial x} = \frac{\partial}{\partial y}\left(e^{x-2y}\right) = -2e^{x-2y}$

$\frac{dy}{dt} = 3t^2 \to \frac{d^2y}{dt^2} = 6t$

- $\frac{\partial^2 u}{\partial x\partial y} = \frac{\partial}{\partial x}\left(-2e^{x-2y}\right) = -2e^{x-2y}$; $\frac{\partial^2 u}{\partial y^2} = 4e^{x-2y}$

7.

$z = f(x, y)$ $\quad x = 2v + \ln t$ $\quad \dfrac{\partial x}{\partial v} = 2$

$y = \dfrac{1}{t}$

$z \begin{cases} x \begin{cases} v \\ t \end{cases} \\ y \to t \end{cases}$ $\qquad \dfrac{\partial z}{\partial x} \begin{cases} x \begin{cases} v \\ t \end{cases} \\ y - t \end{cases}$

$\dfrac{\partial x}{\partial t} = \dfrac{1}{t}$

$\dfrac{\partial^2 x}{\partial t^2} = -\dfrac{1}{t^2}$

- $\dfrac{\partial z}{\partial v} = \dfrac{\partial z}{\partial x} \cdot \dfrac{\partial x}{\partial v} = \dfrac{\partial z}{\partial x} \cdot 2$

- $\dfrac{\partial z}{\partial t} = \dfrac{\partial z}{\partial x} \cdot \dfrac{\partial x}{\partial t} + \dfrac{\partial z}{\partial y} \cdot \dfrac{dy}{dt} = \dfrac{\partial z}{\partial x} \cdot \dfrac{1}{t} + \dfrac{\partial z}{\partial y} \cdot \left(-\dfrac{1}{t^2}\right)$

$= \dfrac{1}{t} \cdot \dfrac{\partial z}{\partial x} - \dfrac{1}{t^2} \cdot \dfrac{\partial z}{\partial y}$

- $\dfrac{\partial^2 z}{\partial v^2} = \dfrac{\partial}{\partial v}\left(\dfrac{\partial z}{\partial v}\right) = \dfrac{\partial}{\partial v}\left(2 \cdot \dfrac{\partial z}{\partial x}\right) = 2 \cdot \dfrac{\partial}{\partial x}\left(\dfrac{\partial z}{\partial x}\right) \cdot \dfrac{\partial x}{\partial v}$

$= 2 \cdot \dfrac{\partial^2 z}{\partial x^2} \cdot 2 = 4 \dfrac{\partial^2 z}{\partial x^2}$

$\dfrac{\partial^2 z}{\partial t \partial v} = \dfrac{\partial}{\partial t}\left(\dfrac{\partial z}{\partial v}\right) = \dfrac{\partial}{\partial t}\left(\dfrac{\partial z}{\partial x} \cdot \dfrac{\partial x}{\partial v}\right) = \left[\dfrac{\partial}{\partial x}\left(\dfrac{\partial z}{\partial x}\right) \cdot \dfrac{\partial x}{\partial t} + \dfrac{\partial}{\partial y}\left(\dfrac{\partial z}{\partial x}\right) \cdot \dfrac{dy}{dt}\right] \cdot \dfrac{\partial x}{\partial v}$

$+ \dfrac{\partial z}{\partial x} \cdot \dfrac{\partial^2 x}{\partial v^2} = \left[\dfrac{\partial^2 z}{\partial x^2} \cdot \left(\dfrac{\partial x}{\partial t}\right) + \dfrac{\partial^2 z}{\partial y \partial x} \cdot \dfrac{dy}{dt}\right] \dfrac{\partial x}{\partial v}$

$+ \dfrac{\partial z}{\partial x} \cdot 0$

$= \left[\dfrac{\partial^2 z}{\partial x^2} \cdot \left(\dfrac{1}{t}\right) + \dfrac{\partial^2 z}{\partial y \partial x} \cdot \left(-\dfrac{1}{t^2}\right)\right] \times 2$

6.4

$$\frac{\partial^2 z}{\partial t^2} = \frac{\partial}{\partial t}\left(\frac{\partial z}{\partial t}\right) = \frac{\partial}{\partial t}\left[\frac{\partial z}{\partial x}\cdot\frac{\partial x}{\partial t} + \frac{\partial z}{\partial y}\cdot\frac{dy}{dt}\right]$$

$$= \frac{\partial}{\partial t}\left[\frac{\partial z}{\partial x}\cdot\frac{\partial x}{\partial t}\right] + \frac{\partial}{\partial t}\left[\frac{\partial z}{\partial y}\cdot\frac{dy}{dt}\right]$$

$$= \left[\frac{\partial^2 z}{\partial x^2}\cdot\frac{\partial x}{\partial t} + \frac{\partial^2 z}{\partial y\partial x}\cdot\frac{dy}{dt}\right]\cdot\frac{\partial x}{\partial t} + \frac{\partial z}{\partial x}\cdot\frac{\partial^2 x}{\partial t^2} +$$

$$+ \left[\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right)\cdot\frac{\partial x}{\partial t} + \frac{\partial^2 z}{\partial y^2}\cdot\frac{dy}{dt}\right]\frac{dy}{dt} + \frac{\partial z}{\partial y}\cdot\frac{d^2 y}{dt^2}$$

$$= \left[\frac{\partial^2 z}{\partial x^2}\cdot\frac{1}{t}\cdot\frac{\partial^2 z}{\partial y\partial x}\cdot\left(-\frac{1}{t^2}\right)\right]\cdot\frac{1}{t} + \frac{\partial z}{\partial x}\cdot\left(-\frac{1}{t^2}\right) +$$

$$+ \left[\frac{\partial^2 z}{\partial x\partial y}\cdot\frac{1}{t} + \frac{\partial^2 z}{\partial y^2}\cdot\left(-\frac{1}{t^2}\right)\right]\cdot\left(-\frac{1}{t^2}\right) + \frac{\partial z}{\partial y}\left(+\frac{1}{t^3}\right)$$

$$= \frac{1}{t^2}\frac{\partial^2 z}{\partial x^2} - \frac{1}{t^3}\cdot\underline{\frac{\partial^2 z}{\partial y\partial x}} - \frac{1}{t^2}\cdot\frac{\partial z}{\partial x} - \frac{1}{t^3}\underline{\frac{\partial^2 z}{\partial x\partial y}} + \frac{1}{t^4}\frac{\partial^2 z}{\partial y^2}$$

$$+ \frac{1}{t^3}\cdot\frac{\partial z}{\partial y}$$

$$= \frac{1}{t^2}\frac{\partial^2 z}{\partial x^2} \checkmark - \frac{2}{t^3}\cdot\frac{\partial^2 z}{\partial y\partial x} + \frac{1}{t^4}\frac{\partial^2 z}{\partial y^2} - \frac{1}{t^2}\frac{\partial z}{\partial x} \checkmark + \frac{1}{t^3}\frac{\partial z}{\partial y}$$

8- $T = e^{-t} \cdot z$

$$T \begin{cases} t \\ z - t \end{cases}$$

a) $z = f(t)$

$$\frac{dT}{dt} = \frac{\partial T}{\partial t} + \frac{\partial T}{\partial z} \cdot \frac{dz}{dt} = -e^{-t} \cdot z + e^{-t} \cdot \frac{dz}{dt}$$

b) $f(t) = e^{t}$

$$\frac{dT}{dt} = -e^{-t} \cdot e^{t} + e^{-t} \cdot e^{t} = -e^{0} + e^{0} = -1 + 1 = 0$$