# Colectânea de Problemas

## Parte 1 - Teoria dos Circuitos

1.1

a) $R_{12} + R_{34} + R_{56} + R_{78}$

b) $(R_{12} + R_{34}) // (R_{56} + R_{78})$

c) $R_{12} // (R_{34} + R_{56} + R_{78})$

d) $R_{12} // R_{34} // R_{56} // R_{78}$

1.2 $V = RI$

$R = R_1 + (R_2 // R_3 // R_4)$

$= 12\,\Omega + 3\,\Omega = 15\,\Omega$

$\frac{1}{R_{234}} = \frac{1}{18} + \frac{1}{9} + \frac{1}{6} = \frac{6}{18} = \frac{1}{3}$

$I = \frac{V}{R} = \frac{30}{15} = 2A$

1.3

a) nº de ramos = nº de componentes = 6

nº de nós = 4

b)

KCL - $I_5 = I_4 + I_1$

$I_4 = 6 - 10 = -4A$ //

KCL - $I_1 + I_3 = I_5$

$10 + I_3 = 12$

$I_3 = 2A$

KVL : $-V_2 + V_5 - R_3 I_3 = 0$

$-V_2 + 60 - 10 = 0$

$V_2 = 50\,V$

c) KVL - $-R_4 I_4 + R_1 I_1 - V_5 = 0$

$-5 \times (-4) + 10 R_1 - 60 = 0$

$10 R_1 = 60 - 20$

$R_1 = 4\,\Omega$

KCL - $I_4 + I_5 = I_2$

$-4A + 12A = I_2$

$I_2 = 8A$

$V = RI \Leftrightarrow R = \frac{V}{I} = \frac{50}{8} = 6{,}25\,\Omega$

a) $P = VI \Leftrightarrow V = \frac{P}{I} \Leftrightarrow V = \frac{125}{5} = 25\,V$

KVL — $-25 + V'' - 110 + 40 = 0$

$V = 110 + 25 - 40$

$= 95$

$V = RI \Leftrightarrow R = \frac{V}{I} \Leftrightarrow R = \frac{95}{5} = 19\,\Omega$

KCL (nó a) — $I_{R2} = 5 + 6 + 7$

$= 18\,A$

KVL (malha elem maior) — $-25 + 95 - 110 + 18R_2 = 0$

$R_2 = 2{,}22\,\Omega$

b) $P = VI$

$= 40 \times -7$

$= -280\,W$

1.5 a) KVL (malha exterior) $-V_{IN} + R_1 I_{R1} - V_1 - R_3 I_3 + 4I_x = 0$

$V_{IN} = 10 - 60 - 16 + 40$

$= -26\,V$

KCL — $I_{R1} + I_{R3} = I_x$

$2 + 8 = I_x$

$I_x = 10\,A$

KVL (malha elem peq) $-16 + 40 + V_S - 30 = 0$

$V_S = 6V$

b) $P = VI$

$= 40 \times (-8)$

$= -320$

1.6 $R_{23} = R_2 + R_3 = 41\,\Omega$

KCL : $I_1 = I_{R4} + I_{23}$

$I_{23} = I_1 - 5$

KVL (para a malha de fora) — $-V_1 + R_1 I_1 + R_{23}(I_1 - 5) = 0$

$-50 + 1{,}5 I_1 + 41(I_1 - 5) = 0$

$42{,}5 I_1 = 50 + 205$

$I_1 = 6\,A$

KVL (esq) = $-V_1 + R_1 I_1 + R_4 I_{R4} = 0$

$5R_4 = 59$

$R_4 = 8,2\ \Omega$

1.7 :

a) R6 : $V = RI = 20V$

R5 : KCL a $I_5 = 1A$

$V = RI \Leftrightarrow V_5 = 30V$

R4 : KVL (dta) : $20 + 30 + V_4 = 0 \Leftrightarrow V_4 = -50V$

$I_4 = \frac{-50}{10} = -5A$

R3 :

KCL b : $I_3 + I_4 = I_5$

$I_3 = 6A$

$V = RI \Leftrightarrow V_3 = 30V$

R2 KVL (meo) : $30 + 50 + V_2 = 0$

$V_2 = -80V$

$V = RI \Leftrightarrow I_2 = -5A$

R1 : KCL c : $I_1 + I_2 = I_3$

$I_1 = 11A$

$V = RI \Leftrightarrow V_1 = 220V$

KVL (esq) : $220 + 80 - V_S = 0$

$V_S = 300V$

b) R6 : $V = RI = 8V$

R5 : KCL a $I_5 = 0,4A$

$V = RI \Leftrightarrow V_5 = 12V$

R4 : KVL (dta) $8 + 12 + V_4 = 0 \Leftrightarrow V_4 = -20V$

$I_4 = \frac{-20}{10} = -2A$

R3. KCL (b) : $I_3 + I_4 = I_5 \Leftrightarrow I_3 = 2,4$

$V = RI \Leftrightarrow V_3 = 12V$

R2: KVL (meio): $V_3 - V_4 + V_2 = 0$

$V_2 = -32\ V$

$V = RI \Leftrightarrow I_2 = -2\ V$

R1: KCL ⓪: $I_1 + I_2 = I_3$

$I_1 = 2{,}4 + 2 = 4{,}4\ A$

$V = RI \Leftrightarrow V_1 = 88\ V$

KVL (esq): $88 + 32 - V_5 = 0$

$V_5 = 120\ V$

c) Pela homogeneidade:

$1 - 300$

$x - 100$

$x = \frac{1}{3}$

R: $I_0 = \frac{1}{3}$

1.8

a) $R_L = ((R_2 // R_4) + R_3) // (R_6 // R_5)$

$= (6 + 2) // 24$

$= 8 // 24$

$= 6\ \Omega$

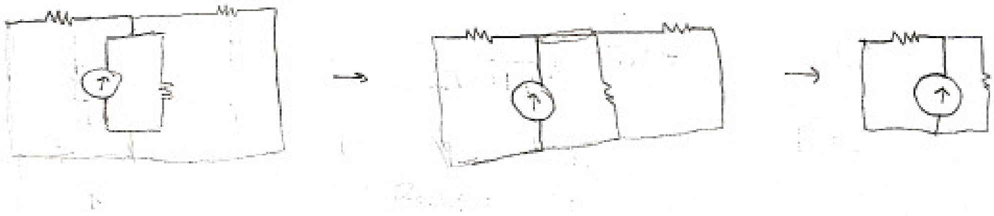
b) $R_L = 6 \quad R_1 = 10 \quad R = 16$

$V = RI \Leftrightarrow I = \frac{V}{R} \Leftrightarrow I = \frac{10}{16} = 0{,}625\ A$

1.9

$I = \frac{V}{R} \Leftrightarrow I = \frac{72}{35} \simeq 2{,}057$

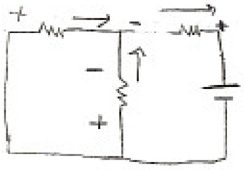
$R = 35/3$



$-I_{x2} = \frac{6{,}67}{11{,}67} \times 2$

$-I_{x2} = 1{,}14$

$I_{x2} = -1{,}14$




KVL: $36 + 20I + 10(I + Ix) = 0$

KVL: $36 + 5Ix + 10(I + Ix) = 0$

$36 = -30I - 10Ix$

$36 = -7,5Ix - 10Ix$

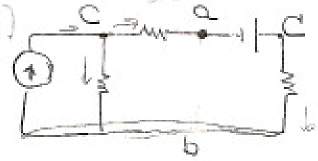
$Ix3 = -2,06$

$Ix = 4I$

$2,06 - 1,14 - 2,06 = -1,14V$

10 a)

VOC




Nó de referência: $Va = 0V$

$VC = 60V$

Nó C: $2 = \frac{VC - VB}{20} + \frac{VC}{5}$

Nó B: $2 = \frac{VC - VB}{20} + \frac{VC - VB}{15}$

$$\begin{bmatrix} \frac{-1}{20} & \frac{1}{20} + \frac{1}{5} \\ \frac{-1}{20} \cdot \frac{1}{15} & \frac{1}{20} \end{bmatrix} \begin{bmatrix} Vb \\ VC \end{bmatrix} = \begin{bmatrix} 2 \\ -2 \end{bmatrix}$$

$Vb = 22,5$

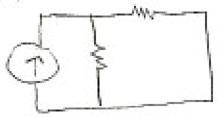
$VC = 12,5$

$VOC = Va - Vb = -22,5$

ISC

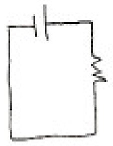
$ISC = I1 + I2$

I1:



$I = \frac{R1}{R1 + R2}$ $I1 = 1,6A$

I2:



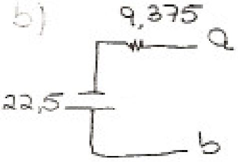
$I_2 = \frac{V}{R3} = \frac{-60}{15} = -4$

$ISC = 1,6 - 4 = -2,4A$

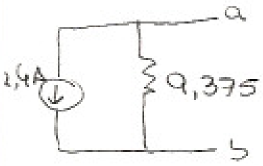
RTH

$RTH = \frac{VOC}{ISC} = \frac{-22,5}{-2,4} = 9,375\ \Omega$

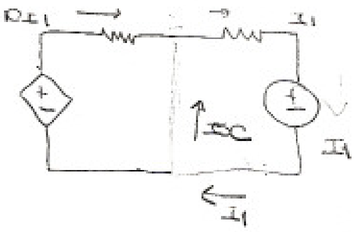
b)




Thévenin




Norton



$I1 = \frac{50}{40} = 1{,}25$

$\frac{10 \times 1{,}25}{20} + ISC = 1{,}25$

$ISC = 0{,}625$

NÓ $c = 50$

NÓ $c = 10I = \frac{va - 50}{4}$

NÓ a $\frac{\frac{va-50}{4} - va}{20} = \frac{va - 50}{40}$

$RTH = \frac{VOC}{ISC} = \frac{10}{0{,}625} = 16\,\Omega$

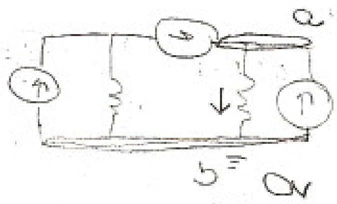
$va - 50 - 4va = 2va - 100$

$va = 10\,V$

.12

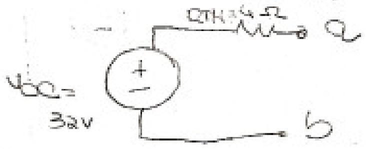
$vab = 10 - 0 = 10$

a) $RTH = R2 = 4\,\Omega$

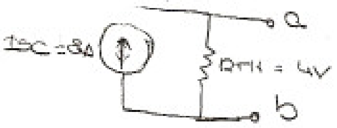


KCL a : $I2 + I3 = \frac{va}{4} \Leftrightarrow va = 32\,V$

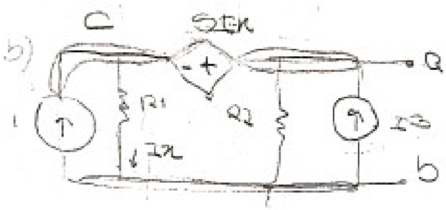
$VOC = vab = 32 - 0 = 32\,V$



$ISC = \frac{32}{4} = 8\,A$



b)



super nó ac : $I1 + I3 = Ix + \frac{va}{R2}$

$va - vc = 5Ix$

$15 = \frac{vc}{12} + \frac{va}{4}$

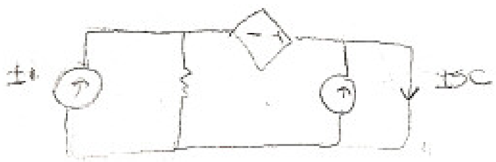
$Ix = \frac{vc}{12}$

$\begin{cases} 180 = vc + 3va \\ va - vc = \frac{5vc}{12} \end{cases}$

$\begin{cases} 180 = \frac{12}{17} va + 3va \\ vc = \frac{12}{17} va \end{cases}$

$\begin{cases} va = 48{,}571 \\ Ix = 2{,}857 \end{cases}$

$VOC = vab = 48{,}571 - 0 = 48{,}571\,V$



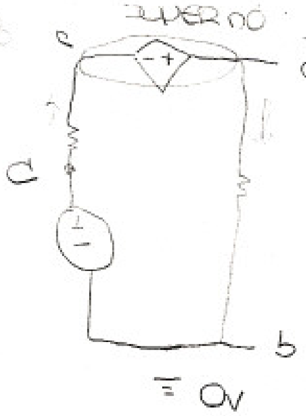
$\begin{cases} Ix + ISC = I3 + I1 \\ IxR1 - 5Ix = 0 \end{cases}$

$\begin{cases} ISC = 15 \\ -Ix(R1 + 5) = 0 \end{cases}$

$RtH = \frac{48{,}571}{15} = 3{,}238$

1.13

a)




$$\begin{cases} V_a - V_c = \alpha I_a \\ I_a = \dfrac{V_a}{R_2} \end{cases}$$

KCL super nó: $\dfrac{V_S - V_C}{R_1} = \dfrac{V_a}{R_2}$

$$\begin{cases} V_a - V_c = \alpha \dfrac{V_a}{R_2} \\ \dfrac{V_S - V_C}{R_1} = \dfrac{V_a}{R_2} \end{cases} \qquad \begin{cases} V_C = V_a - \dfrac{\alpha V_a}{R_2} \\ \dfrac{V_S - V_a + \dfrac{\alpha V_a}{R_2}}{R_1} = \dfrac{V_a}{R_2} \end{cases}$$

$R_2 V_S - R_2 V_a + \alpha V_a = V_a R_1$

$R_2 V_S = V_a (R_1 + R_2 - \alpha)$

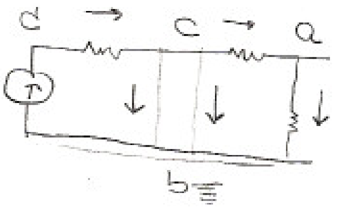
$V_a = \dfrac{R_2 V_S}{R_1 + R_2 - \alpha}$

b)

$R_{TH} = \dfrac{\dfrac{R_2 V_S}{R_1+R_2-\alpha}}{\dfrac{V_a}{R_2}} = \dfrac{R_2^2 V_S}{V_a(R_1+R_2-\alpha)}$ RTH fica negativa porque $R_1+R_2-\alpha$ fica negativo.

1.14

a) $V_1 = V_2$




$$\begin{cases} \text{Nó d: } I_S = \dfrac{V_S - V_1}{2R} \\ \text{Nó c: } \dfrac{V_S - V_1}{2R} = \dfrac{V_1}{R} + \dfrac{V_1}{2R} + \dfrac{V_1 - V_3}{R} \\ \text{Nó a: } \dfrac{V_1 - V_3}{R} = \dfrac{V_3}{R} \end{cases}$$

$$\begin{cases} 4 = V_S - V_1 \\ \dfrac{V_S - V_1}{2} = V_1 + \dfrac{V_1}{2} + V_1 - \dfrac{V_1}{2} \\ V_1 = 2V_3 \end{cases} \qquad \begin{cases} 4 = 4V_1 \\ V_S = 5V_1 \\ V_1 = 2V_3 \end{cases}$$

$V_1 = 1$

$V_2 = 1$

$V_S = 5$

$V_3 = 0{,}5$

b) $V_{TH} = 0{,}5$

$R_{TH}$:


$(((R_4 + R_5) // R_3) // R_2) + R_1$

$((2 // 1) // 1) + 1 = \dfrac{5}{8} = 0{,}625 \text{ k}\Omega$

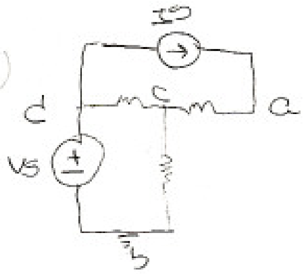
c)


Pelo divisor de tensão, se a tensão se divide em duas iguais, as resistências são iguais, $R_x = 0,625\,\Omega$

15

a)

TH:



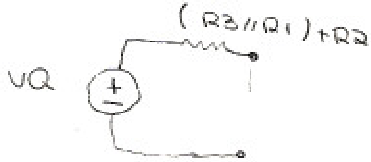

$$\begin{cases} \text{Nó d:} \quad V_C = V_S \\ \text{Nó c:} \quad \dfrac{V_S - V_C}{R_1} = \dfrac{V_C - V_a}{R_2} + \dfrac{V_C}{R_3} \\ \text{Nó a:} \quad I_S = -\dfrac{V_C - V_a}{R_2} \end{cases} \Leftrightarrow \begin{cases} \text{—} \\ \dfrac{R_3(V_S - V_C)}{R_1} = R_3 I_S + V_C \\ \text{—} \end{cases}$$

$$\Leftrightarrow \begin{cases} \text{—} \\ \dfrac{R_3 V_S}{R_1} - R_3 I_S = V_C + \dfrac{R_3 V_C}{R_1} \\ \text{—} \end{cases} \Leftrightarrow \begin{cases} \text{—} \\ \dfrac{R_3 V_S - R_1 R_3 I_S}{R_1 + R_3} = V_C \\ -I_S = \dfrac{R_3 V_S - R_1 R_3 I_S}{R_2(R_1+R_3)} - \dfrac{V_a}{R_2} \end{cases}$$

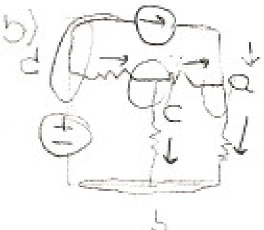
$$\Leftrightarrow \begin{cases} \text{—} \\ \text{—} \\ + I_S R_2 + \dfrac{R_3 V_S}{R_1 + R_3} + \dfrac{R_1 R_3 I_S}{R_1 + R_3} = V_a \end{cases} \Leftrightarrow \begin{cases} \text{—} \\ \text{—} \\ V_a = \dfrac{R_3 V_S}{R_1 + R_3} + I_S\left(R_2 + (R_1 /\!/ R_3)\right) \end{cases}$$

RTH:

$(R_3 /\!/ R_1) + R_2$




b)




$$\begin{cases} \text{Nó d:} \quad V_d = V_S \\ \text{Nó c:} \quad \dfrac{V_S - V_C}{R_1} = \dfrac{V_C - V_a}{R_2} + \dfrac{V_C}{R_3} \\ \text{Nó a:} \quad I_S + \dfrac{V_C - V_a}{R_2} = \dfrac{V_a}{R_L} \end{cases}$$

$$\frac{V_S - V_C}{R_1} - \frac{V_C}{R_3} + I_S = \frac{V_a}{R_L} \Leftrightarrow V_a = \frac{V_S R_L}{R_1} + I_S R_L - \frac{R_L R_3 V_C + R_L R_1 V_C}{R_1 R_3}$$

$$\cancel{I_S} + \frac{V_C}{R_2} + \frac{V_S R_L}{R_1 R_2} + R_2 I_S R_L - \frac{R_2 R_L R_3 V_C + R_L R_2 R_1 V_C}{R_1 R_3} = \frac{V_S}{R_1} + \cancel{I_S} - \frac{R_3 V_C + R_1 V_C}{R_1 R_3}$$

$$\frac{R_1 R_3 V_C - R_2 R_L R_3 V_C R_2 + R_L R_2 R_1 V_C R_2 + R_2 R_3 V_C + R_1 V_C R_2}{R_1 R_2 R_3} = \frac{V_S}{R_1} - \frac{V_S R_L}{R_1 R_2} - R_2 I_S R_L$$

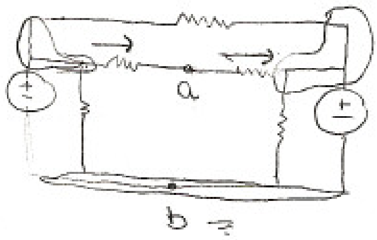
$$V_C = \frac{R_2 R_3 V_S - V_S R_L R_3 - R_1 R_2 R_3 R_2 I_S R_L}{R_1 R_2 - R_2 R_L R_3 R_2 - R_L R_2 R_1 R_2 + R_2 R_3 + R_1 R_2}$$

$$V_a = \frac{V_S R_L}{R_1} + I_S R_L - R_2 V_S$$

1.16.

VOC

a)



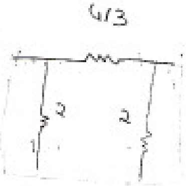

$$\frac{2-va}{2} = \frac{va-2}{2}$$

$$2 - va = va - 2$$

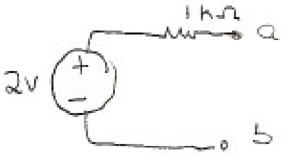
$$va = 2$$

RTH:

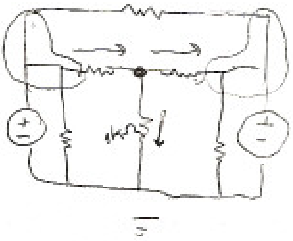
$\rightarrow$




$$\frac{4}{3} \,//\, 4 = \frac{4 \times 4/3}{4+4/3} = \frac{\frac{16}{3}}{\frac{16}{3}} = 1$$




b)




$$\frac{2-va}{2} = \frac{va-2}{2} + va$$

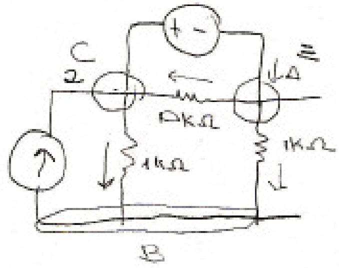
$$2 - va = va - 2 + 2va$$

$$4va = 4 \Leftrightarrow va = 1$$

$$V = RI \Leftrightarrow I = \frac{V}{R} = \frac{1}{1} = 1mA$$

1.17

a)

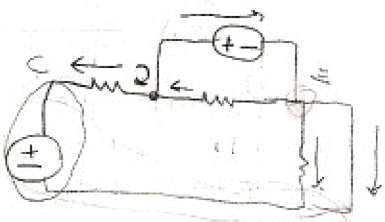



NÓ B: $1 = 2 - vB - vB$

$vB = 0{,}5$

$VAB = 0 - 0{,}5 = -0{,}5V$

b)




$$vC - vB = 1 \Leftrightarrow vC = 1 + vB$$

$$2 - vC - vB + ICC = 0$$

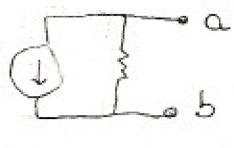
$$IC = -2 + 2 + 2vB$$

$$I = 2vB \Leftrightarrow I = -1mA$$

c) NORTON:

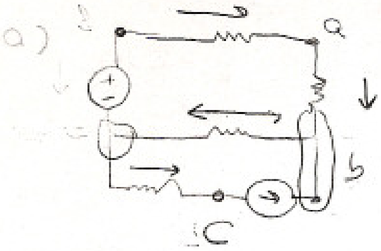
$ICC = -1mA$

$$RTH = \frac{-0{,}5}{-1} = 0{,}5$$



1.18

a)



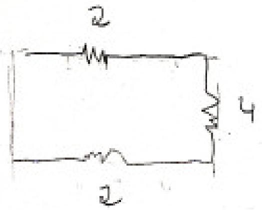
Nó g : 4v

Nó a : $\dfrac{4 - va}{2} = \dfrac{va - vb}{4}$  $va = 3,5$

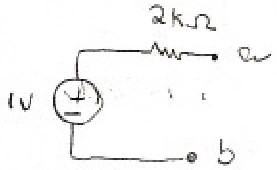
Nó b : $\dfrac{va - vb}{4} + 1 = \dfrac{vb}{2}$  $vb = 2,5$

Nó c : $-\dfrac{vc}{1} = 1$

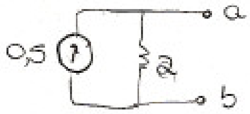
$VTH = 3,5 - 2,5 = 1$



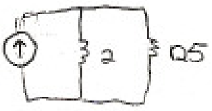
$(R1 + R3) // R2 = 2$



b) $I = \dfrac{V}{R} \Leftrightarrow I = \dfrac{1}{2} = 0,5mA$



c)

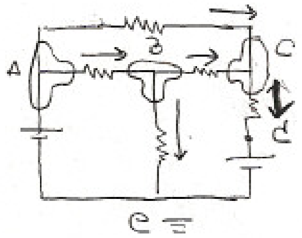


Pelo divisor de corrente R5 tem que ser igual a RTH = 2v

$IR5 = \dfrac{RTH}{RTH + R5} I \Leftrightarrow 0,25\,RTH + 0,25\,R5 = 0,5\,RTH$

$\Leftrightarrow R5 = \dfrac{0,25\,RTH}{0,25} \Leftrightarrow R5 = RTH = 2k\Omega$

1.19



Nó a : $va = 28V$

Nó d : $vd = 24V$

Nó b : $\dfrac{28 - vB}{6} = \dfrac{vB - vC}{5} + \dfrac{vB}{2}$

Nó c : $\dfrac{28 - vc}{8} + \dfrac{vB - vc}{5} = \dfrac{vc - 24}{4}$

$$\begin{cases} \dfrac{14}{3} = \dfrac{vB}{6} + \dfrac{vB}{5} + \dfrac{vB}{2} - \dfrac{vc}{5} \\ \dfrac{19}{2} = \dfrac{vc}{4} + \dfrac{vc}{5} - \dfrac{vB}{5} + \dfrac{vc}{8} \end{cases}$$

$vb = 10$

$vc = 20$

$IR1 = \dfrac{va - vc}{R1} = \dfrac{28 - 20}{8} = 1A$

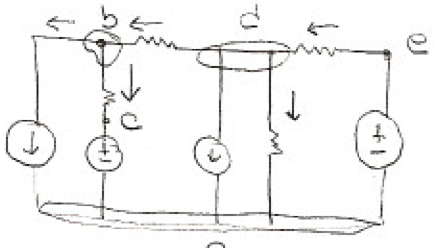
$IR4 = \dfrac{vB}{R4} = \dfrac{10}{2} = 5A$

$IR2 = \dfrac{va - vB}{R2} = \dfrac{28 - 10}{6} = \dfrac{18}{6} = 3A$

$IR5 = \dfrac{vc - vc}{R5} = \dfrac{20 - 24}{4} = -1A$

$IR3 = \dfrac{vB - vc}{R3} = \dfrac{10 - 20}{5} = -2A$

a)



Nó e: $V_e = 3V$

Nó c: $V_c = 3V$

Nó b: $4 + V_B - 3 = \frac{V_d - V_b}{2}$

Nó c: $\frac{3 - V_d}{2} = 5 + \frac{V_d}{1} + \frac{V_d - V_b}{2}$

Nó b: $1 = \frac{V_d}{2} - \frac{V_b}{2} - V_B$

Nó d: $\frac{3}{2} - 5 = V_d + \frac{V_d}{2} - \frac{V_b}{2} + \frac{V_d}{2}$

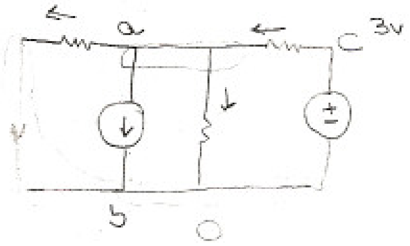
$V_d = -2{,}0909\ V$

$V_b = -1{,}3636\ V$

$V_x = V_d - 0 = -2{,}0909\ V$

$I_Y = \frac{V_b - 3}{1} = -4{,}3636A$

b)



$\frac{3 - V_a}{2} = 5 + \frac{V_a}{1}$

$5 - \frac{3}{2} = -\frac{3}{2} V_a$

$10 - 3 = -3V_a$

$V_a = -2{,}33\ V$

$\frac{3 - V_a}{2} = 5 + V_a + \frac{V_a}{2}$

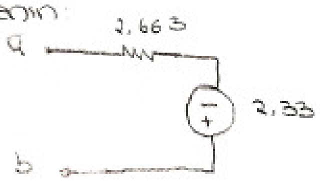
$\frac{3}{2} - 5 = 2V_a$

$\frac{-7}{4} = V_a \Leftrightarrow V_a = -1{,}75$

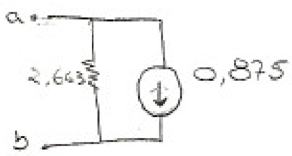
$I = \frac{V}{R} = \frac{-1{,}75}{2} = -0{,}875$

$R_{TH} = \frac{-2{,}33}{-0{,}875} = 2{,}663$

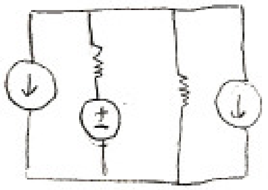
Thevenin:

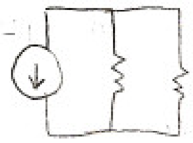



Norton



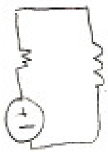

c)





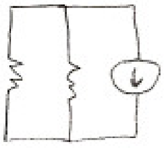
divisor de corrente:

$\frac{2{,}663}{3{,}663} \times (-4) = -2{,}9$



divisor de tensão:

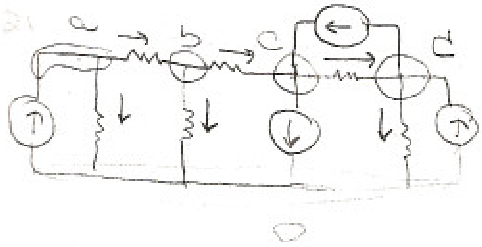
$-3 \times \frac{1}{3{,}663} = -0{,}82$



divisor de corrente:

$\frac{2{,}663}{3{,}663} \times (-0{,}875) = -0{,}636$

1.21




Nó a: $20 = \frac{va - vb}{20} + \frac{va}{10}$

Nó b: $\frac{va - vb}{20} = \frac{vb}{50} + \frac{vb - vc}{25}$

Nó c: $\frac{vb - vc}{25} + 4 = \frac{vc - vd}{5} + 5$

Nó d: $\frac{vc - vd}{5} + 10 = \frac{vd}{100} + 4$

va = 190,546

vb = 171,6387

vc = 233,824

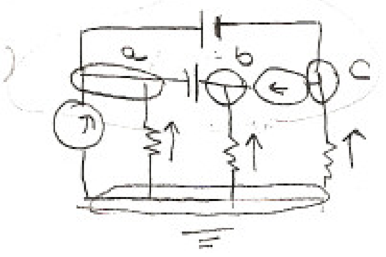
vd = 251,26

$vy = vc - vd = 233,824 - 251,26 = -17,43$

$vx = vb - 0 = 171,64$

1.22

a)




$\begin{cases} va - vb = -60 \\ va - vc = 100 \end{cases}$ $\begin{cases} va = vb - 60 \\ va = vc + 100 \end{cases}$

va = 39,13 v

vb = 99,13 v

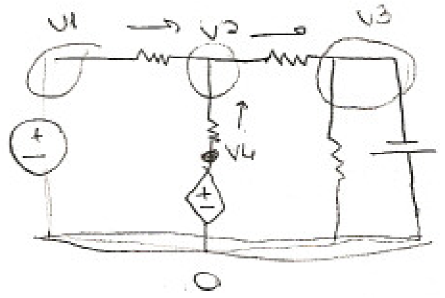
vc = -60,87 v

super nó: $5 - \frac{va}{25} - \frac{vb}{20} - \frac{vc}{60} = 0$

b) $P = VI = -39,13 \times 5 = -195,65$ w

1.23

a)




v1 = 6v

v3 = -8v

$\frac{v1 - v2}{} + \frac{5vx - v2}{2} = \frac{v2 - v3}{3}$

$vx = v1 - v2 = 6 - v2$

$6 - v2 + \frac{5(6 - v2) - v2}{2} = \frac{v2 + 8}{3}$

$\frac{55}{3} = \frac{v2}{3} + 3v2 + v2$

$v2 = \frac{55}{13} = 4,23$

$v4 = 30 - 21,154 = 8,846$

1.24

a) Para $t<0$, regime estacionário, $v_{C_0} = 10V$

Para $t>0$

$\tau = RC = 1$ segundo

$v_C(0) = k_1 + k_2 = 10V$

$v_C(+\infty) = k_1 = 0V$

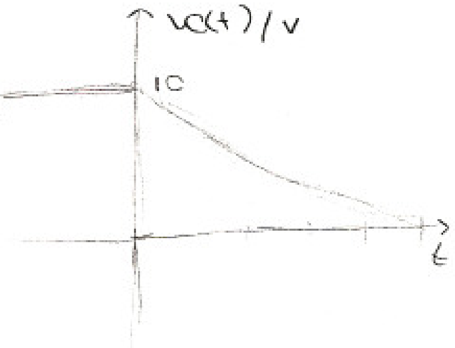
$v_C(t) = 10e^{-t/\tau}$

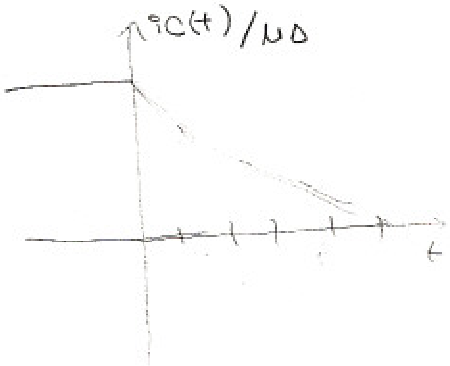
$v_C(1) = \frac{10}{e}$

$i(t) = \frac{v_C(t)}{R} = 10e^{-t/\tau}$ [$\mu A$]

$v_C(2) = \frac{10}{e^2}$

$i(1) = \frac{10}{e}\ \mu A \quad i(2) = \frac{10}{e^2}\ \mu A$




b) $5 = 10e^{-t/\tau} \qquad \frac{1}{2} = e^{-t} \Leftrightarrow t = \ln 2$

c) $2 = 10e^{-t/1 \times R\,\mu F} \Leftrightarrow \frac{1}{R \times 1\mu F} = \ln 5 \Leftrightarrow R = \frac{1}{\ln 5 \times 10^{-6}} = 621k\Omega$

1.25

a) antes do interruptor fechar: regime estacionário: condensador comporta-se como curto circuito ($V = 0v$)

$v_C(0) = 0 = k_1 + k_2$

$v_C(t) = 10 - 10e^{-10^6 t}$

$v_C(+\infty) = 10V = k_1$

$i(t) = 10 - 10 + 10e^{-10^6 t} = 10e^{-10^6 t}$

$v_C(0{,}5\tau) = 10 - 10e^{-0,5} = 3{,}93$

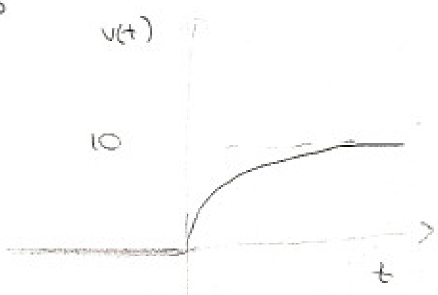
$v_C(\tau) = 10 - 10e^{-1} = 6{,}321$

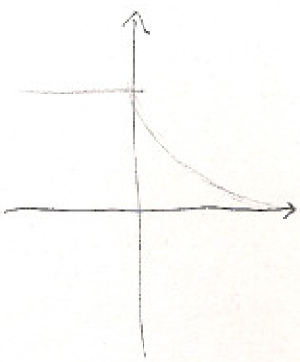
$v_C(2\tau) = 8{,}6466$

$v_C(3\tau) = 9{,}5$

$v_C(4\tau) = 9{,}8$

$v_C(5\tau) = 9{,}93$



$V_R = 10 - 10 + 10e^{-t/\tau}$


b) valor final = 10 entre 10% e 90% $\Rightarrow$ entre 1 e 9

$v(t) = 9$

$9 = 10 - 10e^{-10^6 t} \Leftrightarrow e^{-10^6 t} = \frac{1}{10} \Leftrightarrow t = \frac{\ln 10}{10^6} = 2{,}3 \times 10^{-6}$

$v(t) = 1$

$1 = 10 - 10e^{-10^6 t} \Leftrightarrow e^{-10^6 t} = \frac{9}{10} \Leftrightarrow t = \frac{\ln(10/9)}{10^6} = 1{,}054 \times 10^{-7}$

entre 10% e 90% $= 2{,}3 \times 10^{-6} - 1{,}054 \times 10^{-7} = 2{,}1946 \times 10^{-6}$ s

1.26

a) com $t < 0$, regime estacionário.

$v_C(t)$ é constante e igual a 1, logo $i_C(t) = 0$

com $t > 0$, se v não mudar

$f(0) = K_1 + K_2 = 1$

$f(+\infty) = 5 = K_1$

$\tau = 2 \times 10^3 \times 0{,}5 \times 10^{-6} = 1 \times 10^{-4}$

$v_C(t) = 5 - 4e^{-t/\tau} = 5 - 4e^{-10^4 t}$

$t > 150\ \mu s$

$f(t_1) = 5 - 4e^{-10^4 \times 150 \times 10^{-6}} = 4{,}1 = K_1 + K_2$

$f(+\infty) = -2 = K_1$

$v_C(t) = -2 + 6{,}1e^{-\frac{t - 1{,}5 \times 10^{-4}}{10^{-4}}}$

$V_R$ $t < 0$

$v = 0$

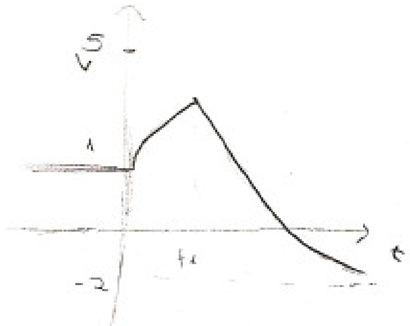
$0 < t < t_1$

KCL: $5 - 5 + 4e^{-t/\tau} - V_R = 0 \Leftrightarrow V_R = 4e^{-t/\tau}$

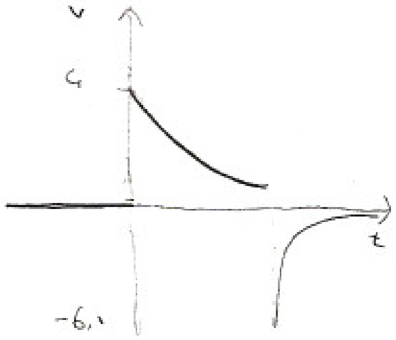
$t > t_1$

KCL: $-2+2-6{,}1\,e^{\frac{-t-1{,}5\times10^{-4}}{10^{-4}}} - V_R = 0 \Leftrightarrow V_R = -6{,}1\,e^{\frac{-t-1{,}5\times10^{-4}}{10^{-4}}}$

$$v_C(t) = \begin{cases} 1, & t<0 \\ 5-4e^{-10^4 t}, & 0<t<t_1 \\ -2+6{,}1\,e^{\frac{-t-1{,}5\times10^{-4}}{10^{-4}}}, & t>t_1 \end{cases}$$



$$v_R(t) = \begin{cases} 0, & t<0 \\ 4e^{-10^4 t}, & 0<t<t_1 \\ -6{,}1\,e^{\frac{-t-1{,}5\times10^{-4}}{10^{-4}}} & t>t_1 \end{cases}$$



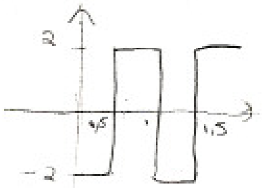
1.27 $i_1 = I_R = I_C(t) = C \times \frac{dv_C(t)}{dt}$

$-1 = 0{,}5 \times x$ $\qquad 1 = 0{,}5 \times x$

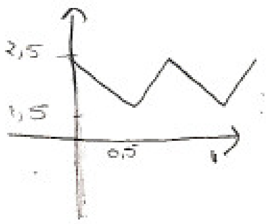
$x = -0{,}5$ $\qquad x = 0{,}5$

A tensão varia $\pm 0{,}5$ V. Sendo o valor médio é 2, a tensão varia entre 1,5 e 2,5 V

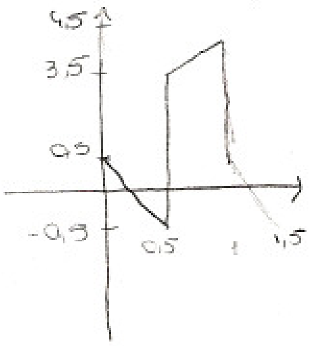
$V_R(t) = i \times 2$



$v_C(t)$



$v_1(t) = v_C(t) + v_R(t)$



1.28

a) $\tau = 1 \times 1 = 1s$

b)

$vc(0) = 1 = k_1 + k_2$

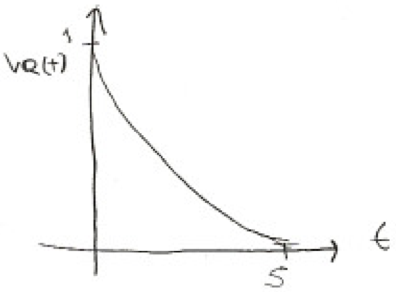
$vc(+\infty) = 0 = k_1$

$vc(t) = e^{-t}$

c) $vc(1) = \frac{1}{e}$ $vc(4) = \frac{1}{4}$

d) $vc(t) - v_R(t) = 0$

$vc(t) = v_R(t) = e^{-t}$




e) se $R \gg 1\Omega$, o $\tau$ aumenta, logo a descida é mais lenta
se $R \ll 1\Omega$, o $\tau$ diminui, logo a descida é mais rápida

1.29 $vc(0) = 0 = k_1 + k_2$

com $vs = 1V$, $k_1 = 1V$

?

1.30 b) em a) $C = \frac{1}{j\omega C}$ $L = j\omega L$ como passar em a) ?

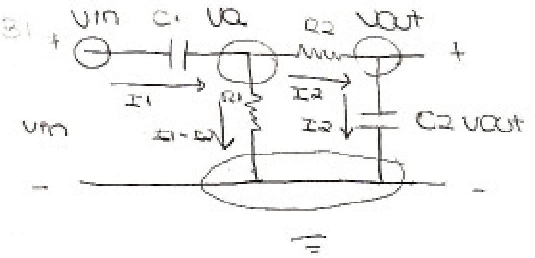
$(Z_2 // Z_3) + Z_1$

$$Z = \frac{\frac{1}{j\omega C_3} \times \frac{1}{j\omega C_2}}{\frac{1}{j\omega C_3} + \frac{1}{j\omega C_2}} + \frac{1}{j\omega_1} = \frac{\cancel{j\omega C_2}\, \cancel{j\omega C_3}}{(C_3+C_2)\, \cancel{j\omega C_3 C_2}} + \frac{1}{j\omega C_1} = \frac{C_1 + (C_2 + C_3)}{(C_3 + C_2)\, j\omega C_1}$$

em b)

$$Z = \frac{j\omega L_2 \times j\omega L_3}{j\omega L_2 + j\omega L_3} + j\omega L_1 = j\omega\left(\frac{L_2 \times L_3}{L_2 + L_3} + L_1\right)$$

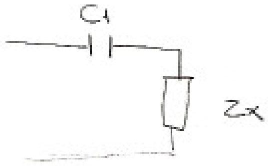
1.31




$$Vout = \frac{ZC2}{ZR2 + ZC2} Va$$

$$Z2 = R2 + ZC2$$

$$Zx = R1 // R2$$
$$= R1 // (R2 + ZC2)$$




$$Va = \frac{Zx}{ZC1 + Zx} vin$$

$$Vout = \frac{ZC2}{ZR2 + ZC2} \times \frac{Zx}{ZC1 + Zx} vin$$

$$\frac{Vout}{Vin} = \frac{ZC2}{ZC2 + R2} \cdot \frac{Zx}{ZC1 + Zx} \qquad ZC1 = \frac{1}{j\omega C_1} = \frac{1}{SC_1}$$

$$\frac{Vout}{Vin} = \frac{1}{1 + SR2C2} \times \frac{\dfrac{\cancel{R1}(R2SC2+1)}{(R1+R2)\cancel{SC2}+1}}{\dfrac{\cancel{R1}(R2SC2+1)}{1+\cancel{SC2}(R1+R2)} + \dfrac{1}{SC1R1}} \times (R1+R2)SC2+1$$

$$= \frac{SC1R1}{1 + S\underbrace{\left[C1R1 + C2(R1+R2)\right]}_{A} + S^2\underbrace{\left[C1C2R1R2\right]}_{B}}$$

$$= \frac{j\omega C1R1}{1 - \omega^2 B + j\omega A}$$

$$\left|\frac{Vout}{Vin}\right| = \frac{\omega C1R1}{\sqrt{(1-\omega^2 B)^2 + (\omega A)^2}} \qquad \text{Passa baixo}$$

1.32

a) $$Vo = \frac{ZC}{ZC + ZR} \times VS = \frac{\frac{1}{j\omega C}}{\frac{1}{j\omega C} + R} \times VS = \frac{1}{1 + j\omega CR} VS$$

b) $$\left|\frac{VO}{VS}\right| = \left|\frac{1}{1 + j\omega RC}\right| = \frac{1}{\sqrt{1 + (j\omega RC)^2}} = \frac{1}{\sqrt{2}}$$

$$1 = \omega RC$$

$$f = \frac{1}{2\pi} \times \frac{1}{RC} = 159\,Hz$$

1.33

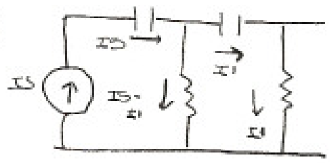
a) $I_{bobine} = \frac{Z_R}{Z_R + Z_L} \times I_S = \frac{R}{R + j\omega L} \times I_S$

$V = Z I \iff V_O = \frac{R^2 I_S}{R + j\omega L}$ ?

b) $\left| \frac{V_O}{I_S} \right| = \left| \frac{R^2}{R + j\omega L} \right|$ ?

1.34

a)



$I_1 \times Z_{C2} + I_1 \times Z_{R2} - (I_S - I_1) Z_{R1} = 0$

$I_1 (Z_{C2} + Z_{R2} + Z_{R1}) = I_S Z_{R1}$

$I_1 = \frac{I_S Z_{R1}}{Z_{C2} + Z_{R2} + Z_{R1}}$

$V_{ab} = Z_{R2} \times \frac{I_S Z_{R1}}{Z_{C2} + Z_{R2} + Z_{R1}}$

$\iff V_{ab} = \frac{R_2 R_1}{\frac{1}{j\omega C_2} + R_2 + R_1} \times I_S$

$\iff V_{ab} = \frac{j\omega C_2 R_2 R_1}{1 + j\omega C_2 (R_2 + R_1)} I_S$

b)

$(Z_{R1} + Z_{C2}) // R_2 = \frac{\left(Z_{R1} + \frac{1}{j\omega C_2}\right) Z_{R2}}{Z_{R2} + \frac{1}{j\omega C_2} + Z_{R1}}$

$= \frac{j\omega C_2 (R_2 + R_1) + R_2}{j\omega C_2 (R_2 + R_1) + 1}$

35

b) figura a)

$$((Z_{L2} + Z_{C2}) \parallel Z_{C1}) + Z_{L1}$$

$$= \frac{(Z_{L2} + Z_{C2}) \cdot Z_{C1}}{Z_{L2} + Z_{C2} + Z_{C1}} + Z_{L1}$$

$$= \frac{\left(j\omega L_2 + \frac{1}{j\omega C_2}\right) \frac{1}{j\omega C_1}}{j\omega L_2 + \frac{1}{j\omega C_2} + \frac{1}{j\omega C_1}} + j\omega L_1 = \frac{\frac{\omega^2 C_2 L_2 - 1}{\omega^2 \cancel{C_1 C_2}}}{\frac{C_1 + C_2 - \omega^2 L_2 C_1 C_2}{\cancel{j\omega C_2 C_1}}} + j\omega L_1 = \frac{j\omega^2 C_2 L_2 - j}{C_1 + C_2 - \omega^2 L_2 C_1 C_2} + j\omega L_1$$

?

figura b)

$$((Z_{L2} + Z_{C1}) \parallel Z_{L1}) + Z_{C1}$$

$$\frac{\left(j\omega L_2 + \frac{1}{j\omega C_1}\right) \cdot j\omega L_1}{j\omega L_2 + \frac{1}{j\omega C_1} + j\omega L_1} + \frac{1}{j\omega C_1}$$

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a)