

Laplace Transform Table

Largely modeled on a table in D'Azzo and Houpis, *Linear Control Systems Analysis and Design*, 1988

$F(s)$	$f(t) \quad 0 \leq t$
1. 1	$\delta(t)$ unit impulse at $t = 0$
2. $\frac{1}{s}$	1 or $u(t)$ unit step starting at $t = 0$
3. $\frac{1}{s^2}$	$t \cdot u(t)$ or t ramp function
4. $\frac{1}{s^n}$	$\frac{1}{(n-1)!} t^{n-1}$ $n = \text{positive integer}$
5. $\frac{1}{s} e^{-as}$	$u(t - a)$ unit step starting at $t = a$
6. $\frac{1}{s} (1 - e^{-as})$	$u(t) - u(t - a)$ rectangular pulse
7. $\frac{1}{s + a}$	e^{-at} exponential decay
8. $\frac{1}{(s + a)^n}$	$\frac{1}{(n-1)!} t^{n-1} e^{-at}$ $n = \text{positive integer}$
9. $\frac{1}{s(s + a)}$	$\frac{1}{a} (1 - e^{-at})$
10. $\frac{1}{s(s + a)(s + b)}$	$\frac{1}{ab} (1 - \frac{b}{b-a} e^{-at} + \frac{a}{b-a} e^{-bt})$
11. $\frac{s + \alpha}{s(s + a)(s + b)}$	$\frac{1}{ab} [\alpha - \frac{b(\alpha - a)}{b-a} e^{-at} + \frac{a(\alpha - b)}{b-a} e^{-bt}]$
12. $\frac{1}{(s + a)(s + b)}$	$\frac{1}{b-a} (e^{-at} - e^{-bt})$
13. $\frac{s}{(s + a)(s + b)}$	$\frac{1}{a-b} (ae^{-at} - be^{-bt})$

$F(s)$	$f(t) \quad 0 \leq t$
14. $\frac{s + \alpha}{(s + a)(s + b)}$	$\frac{1}{b - a} [(\alpha - a)e^{-at} - (\alpha - b)e^{-bt}]$
15. $\frac{1}{(s + a)(s + b)(s + c)}$	$\frac{e^{-at}}{(b - a)(c - a)} + \frac{e^{-bt}}{(c - b)(a - b)} + \frac{e^{-ct}}{(a - c)(b - c)}$
16. $\frac{s + \alpha}{(s + a)(s + b)(s + c)}$	$\frac{(\alpha - a)e^{-at}}{(b - a)(c - a)} + \frac{(\alpha - b)e^{-bt}}{(c - b)(a - b)} + \frac{(\alpha - c)e^{-ct}}{(a - c)(b - c)}$
17. $\frac{\omega}{s^2 + \omega^2}$	$\sin \omega t$
18. $\frac{s}{s^2 + \omega^2}$	$\cos \omega t$
19. $\frac{s + \alpha}{s^2 + \omega^2}$	$\frac{\sqrt{\alpha^2 + \omega^2}}{\omega} \sin(\omega t + \phi) \quad \phi = \text{atan2}(\omega, \alpha)$
20. $\frac{s \sin \theta + \omega \cos \theta}{s^2 + \omega^2}$	$\sin(\omega t + \theta)$
21. $\frac{1}{s(s^2 + \omega^2)}$	$\frac{1}{\omega^2} (1 - \cos \omega t)$
22. $\frac{s + \alpha}{s(s^2 + \omega^2)}$	$\frac{\alpha}{\omega^2} - \frac{\sqrt{\alpha^2 + \omega^2}}{\omega^2} \cos(\omega t + \phi) \quad \phi = \text{atan2}(\omega, \alpha)$
23. $\frac{1}{(s + a)(s^2 + \omega^2)}$	$\frac{e^{-at}}{a^2 + \omega^2} + \frac{1}{\omega \sqrt{a^2 + \omega^2}} \sin(\omega t - \phi)$ $\phi = \text{atan2}(\omega, \alpha)$
24. $\frac{1}{(s + a)^2 + b^2}$	$\frac{1}{b} e^{-at} \sin(bt)$
24a. $\frac{1}{s^2 + 2\zeta\omega_n s + \omega_n^2}$	$\frac{1}{\omega_n \sqrt{1 - \zeta^2}} e^{-\zeta\omega_n t} \sin(\omega_n \sqrt{1 - \zeta^2} t)$
25. $\frac{s + a}{(s + a)^2 + b^2}$	$e^{-at} \cos(bt)$

$F(s)$	$f(t) \quad 0 \leq t$
26. $\frac{s + \alpha}{(s + a)^2 + b^2}$	$\frac{\sqrt{(\alpha - a)^2 + b^2}}{b} e^{-at} \sin(bt + \phi) \quad \phi = \text{atan2}(b, \alpha - a)$
26a. $\frac{s + \alpha}{s^2 + 2\zeta\omega_n s + \omega_n^2}$	$\frac{\sqrt{\left(\frac{\alpha}{\omega_n} - \zeta\omega_n\right)^2 + 1}}{1 - \zeta^2} \cdot e^{-\zeta\omega_n t} \sin(\omega_n \sqrt{1 - \zeta^2} t + \phi)$ $\phi = \text{atan2}(\omega_n \sqrt{1 - \zeta^2}, \alpha - \zeta\omega_n)$
27. $\frac{1}{s[(s + a)^2 + b^2]}$	$\frac{1}{a^2 + b^2} + \frac{1}{b\sqrt{a^2 + b^2}} e^{-at} \sin(bt - \phi) \quad \phi = \text{atan2}(b, -a)$
27a. $\frac{1}{s(s^2 + 2\zeta\omega_n s + \omega_n^2)}$	$\frac{1}{\omega_n^2} - \frac{1}{\omega_n^2 \sqrt{1 - \zeta^2}} e^{-\zeta\omega_n t} \sin(\omega_n \sqrt{1 - \zeta^2} t + \phi)$ $\phi = \cos^{-1} \zeta$
28. $\frac{s + \alpha}{s[(s + a)^2 + b^2]}$	$\frac{\alpha}{a^2 + b^2} + \frac{1}{b} \sqrt{\frac{(\alpha - a)^2 + b^2}{a^2 + b^2}} e^{-at} \sin(bt + \phi)$ $\phi = \text{atan2}(b, \alpha - a) - \text{atan2}(b, -a)$
28a. $\frac{s + \alpha}{s(s^2 + 2\zeta\omega_n s + \omega_n^2)}$	$\frac{\alpha}{\omega_n^2} + \frac{1}{\omega_n \sqrt{1 - \zeta^2}} \sqrt{\left(\frac{\alpha}{\omega_n} - \zeta\right)^2 + (1 - \zeta^2)} \cdot e^{-\zeta\omega_n t} \sin(\omega_n \sqrt{1 - \zeta^2} t + \phi)$ $\phi = \text{atan2}(\omega_n \sqrt{1 - \zeta^2}, \alpha - \omega_n \zeta) - \text{atan2}(\sqrt{1 - \zeta^2}, -\zeta)$
29. $\frac{1}{(s + c)[(s + a)^2 + b^2]}$	$\frac{e^{-ct}}{(c - a)^2 + b^2} + \frac{e^{-at} \sin(bt - \phi)}{b\sqrt{(c - a)^2 + b^2}} \quad \phi = \text{atan2}(b, c - a)$

$F(s)$	$f(t) \quad 0 \leq t$
30. $\frac{1}{s(s+c)[(s+a)^2+b^2]}$	$\frac{1}{c(a^2+b^2)} - \frac{e^{-ct}}{c[(c-a)^2+b^2]} + \frac{e^{-at} \sin(bt-\phi)}{b\sqrt{a^2+b^2}\sqrt{(c-a)^2+b^2}}$ $\phi = \text{atan2}(b, -a) + \text{atan2}(b, c-a)$
31. $\frac{s+\alpha}{s(s+c)[(s+a)^2+b^2]}$	$\frac{\alpha}{c(a^2+b^2)} + \frac{(c-\alpha)e^{-ct}}{c[(c-a)^2+b^2]}$ $+ \frac{\sqrt{(\alpha-a)^2+b^2}}{b\sqrt{a^2+b^2}\sqrt{(c-a)^2+b^2}} e^{-at} \sin(bt+\phi)$ $\phi = \text{atan2}(b, \alpha-a) - \text{atan2}(b, -a) - \text{atan2}(b, c-a)$
32. $\frac{1}{s^2(s+a)}$	$\frac{1}{a^2}(at-1+e^{-at})$
33. $\frac{1}{s(s+a)^2}$	$\frac{1}{a^2}(1-e^{-at}-ate^{-at})$
34. $\frac{s+\alpha}{s(s+a)^2}$	$\frac{1}{a^2}[\alpha - \alpha e^{-at} + a(a-\alpha)te^{-at}]$
35. $\frac{s^2+\alpha_1s+\alpha_0}{s(s+a)(s+b)}$	$\frac{\alpha_0}{ab} + \frac{a^2-\alpha_1a+\alpha_0}{a(a-b)}e^{-at} - \frac{b^2-\alpha_1b+\alpha_0}{b(a-b)}e^{-bt}$
36. $\frac{s^2+\alpha_1s+\alpha_0}{s[(s+a)^2+b^2]}$	$\frac{\alpha_0}{c^2} + \frac{1}{bc}[(a^2-b^2-\alpha_1a+\alpha_0)^2$ $+ b^2(\alpha_1-2a)^2]^{\frac{1}{2}} e^{-at} \sin(bt+\phi)$ $\phi = \text{atan2}[b(\alpha_1-2a), a^2-b^2-\alpha_1a+\alpha_0] - \text{atan2}(b, -a)$ $c^2 = a^2 + b^2$

$F(s)$	$f(t) \quad 0 \leq t$
37. $\frac{1}{(s^2 + \omega^2)[(s+a)^2 + b^2]}$	$\frac{(1/\omega) \sin(\omega t + \phi_1) + (1/b) e^{-at} \sin(bt + \phi_2)}{[4a^2 \omega^2 + (a^2 + b^2 - \omega^2)^2]^{\frac{1}{2}}}$ $\phi_1 = \text{atan2}(-2a\omega, a^2 + b^2 - \omega^2)$ $\phi_2 = \text{atan2}(2ab, a^2 - b^2 + \omega^2)$
38. $\frac{s + \alpha}{(s^2 + \omega^2)[(s+a)^2 + b^2]}$	$\frac{1}{\omega} \left(\frac{\alpha^2 + \omega^2}{c} \right)^{\frac{1}{2}} \sin(\omega t + \phi_1)$ $+ \frac{1}{b} \left[\frac{(\alpha - a)^2 + b^2}{c} \right]^{\frac{1}{2}} e^{-at} \sin(bt + \phi_2)$ $c = (2a\omega)^2 + (a^2 + b^2 - \omega^2)^2$ $\phi_1 = \text{atan2}(\omega, \alpha) - \text{atan2}(2a\omega, a^2 + b^2 + \omega^2)$ $\phi_2 = \text{atan2}(b, \alpha - a) + \text{atan2}(2ab, a^2 - b^2 - \omega^2)$
39. $\frac{s + \alpha}{s^2[(s+a)^2 + b^2]}$	$\frac{1}{c} (\alpha t + 1 - \frac{2\alpha a}{c}) + \frac{[b^2 + (\alpha - a)^2]^{\frac{1}{2}}}{bc} e^{-at} \sin(bt + \phi)$ $c = a^2 + b^2$ $\phi = 2\text{atan2}(b, a) + \text{atan2}(b, \alpha - a)$
40. $\frac{s^2 + \alpha_1 s + \alpha_0}{s^2(s+a)(s+b)}$	$\frac{\alpha_1 + \alpha_0 t}{ab} - \frac{\alpha_0(a+b)}{(ab)^2} - \frac{1}{a-b} \left(1 - \frac{\alpha_1}{a} + \frac{\alpha_0}{a^2} \right) e^{-at}$ $- \frac{1}{b-a} \left(1 - \frac{\alpha_1}{b} + \frac{\alpha_0}{b^2} \right) e^{-bt}$