Análise Matemática - Soluções da Ficha 2B

1.

(a) $f(x) = ln2 + \sum_{n=1}^{\infty} (-1)^{n+1} \frac{(x-2)^n}{n2^n}$ e $x \in]0,4]$;

(b) $g(x) = e^{-\frac{1}{2}} \times \sum_{n=0}^{\infty} \frac{(x+1)^n}{n!2^n}$ e $x \in \mathbb{R}$;

(c) $i(x) = \sum_{n=0}^{\infty} \frac{(x+2)^n(n+1)}{2^{n+2}}$ e $x \in]-4,0[$;

(d) $b(x) = \sqrt{2} \sum_{n=1}^{\infty} (-1)^{n+1} \frac{(x-\frac{\pi}{4})^{2n-1}}{(2n-1)!}$ e $x \in \mathbb{R}$.

2.

(a) $h(x) = \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n}}{(2n+1)!}$ e $x \in \mathbb{R} \setminus \{0\}$;

(b) $j(x) = 1 + \frac{x}{2} - \frac{x^2}{2^2 2!} - \frac{x^3}{2^3 3!} + \frac{x^4}{2^4 4!} + \frac{x^5}{2^5 5!} - ...$ e $x \in \mathbb{R}$;

(c) $l(x) = x^2 - \frac{2x^4}{3!} + x^6(\frac{2}{5!} + \frac{1}{3!3!}) - x^8(\frac{2}{7!} + \frac{2}{3!5!}) + ...$ e $x \in \mathbb{R}$.

3.

(a) $e^x \times \sin x = (1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + ...) \times (x - \frac{x^3}{3!} + \frac{x^5}{5!} - ...) = x + x^2 + \frac{x^3}{3} - \frac{x^5}{30} - \frac{x^6}{90} + ...$;

(b) $\frac{d}{dx}(e^x \sin x) = e^x \sin x + e^x \cos x$

$e^x \cos x = \frac{d}{dx}(e^x \sin x) - e^x \sin x = 1 + x - \frac{x^3}{3} - \frac{x^4}{6} - \frac{x^5}{30} + ...$

4.

(a) $\int_0^x \frac{e^t - 1}{t} dt = \sum_{n=1}^{\infty} \frac{x^n}{n\, n!}$;

(b) $\int_0^x \frac{\sin t}{t} dt = \sum_{n=1}^{\infty} \frac{(-1)^{n+1} x^{2n-1}}{(2n-1)(2n-1)!}$.

5.

(a) 2; (b) 2 e (c) $\frac{1}{2}$.

6.

$\cos x - 2x^2 = 0 \Longleftrightarrow 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + ... - 2x^2 = 0 \Longleftrightarrow 1 - \frac{5}{2}x^2 + \frac{x^4}{4!} - \frac{x^6}{6!} + ... = 0 \approx$

$\approx 1 - \frac{5}{2}x^2 = 0 \Longleftrightarrow x = \pm\frac{\sqrt{2}}{5}$