Ficha 1 - B
Funções trigonométricas inversas

1. (a) $-\pi$;

   (b) $\dfrac{\sqrt{3}}{2}$

   (c) $-\dfrac{\sqrt{3}}{3}$

   (d) $-\dfrac{5}{13}$

   (e) $\dfrac{5\sqrt{123}-4\sqrt{41}}{82}$

2. (a) $-\dfrac{2\pi}{3}$. A resposta certa seria $\dfrac{11\pi}{4}$.

   (b) $-\dfrac{175}{144}$. A resposta certa seria $\dfrac{3\sqrt{3}}{16}$.

3. (a) $D_f = [0,1]\,;\;\; D'_f = [0, 2\pi]$

   (b) $D_f = \left]-\infty, -5\right] \cup \left[1, +\infty\right[\,;\;\; D'_f = \left[\dfrac{\pi}{2}, \dfrac{5\pi}{2}\right] \setminus \left\{\dfrac{3\pi}{2}\right\}$

   Para o domínio, notar que tem que ser $x \neq -2$ e $-1 \leq \dfrac{3}{x+2} \leq 1$.

   Depois basta ver que

   - $x > -2 \Rightarrow x+2 > 0$ e, para ter $\dfrac{3}{x+2} \leq 1$, tem que ser $x \geq 1$;
   - $x < -2 \Rightarrow x+2 < 0$ e, para ter $\dfrac{3}{x+2} \geq -1$, tem que ser $x \leq -5$.

   Para o contradomínio, notar que

   - $x \geq 1 \Rightarrow 0 < \dfrac{3}{x+2} \leq 1 \Rightarrow \dfrac{\pi}{2} \leq f(x) < \dfrac{3\pi}{2}$, porque arc cos é decrescente;
   - $x \leq -5 \Rightarrow -1 \leq \dfrac{3}{x+2} < 0 \Rightarrow \dfrac{3\pi}{2} < f(x) \leq \dfrac{5\pi}{2}$.

   (c) $D_f = \left[0, \dfrac{1}{2}\right]\,;\;\; D'_f = [-1, 3\pi - 1]$

   (d) $D_f = \mathbb{R} \setminus \{-5\}\,;\;\; D'_f = \left]-\dfrac{\pi}{6}, \dfrac{5\pi}{6}\right[\setminus \left\{\dfrac{\pi}{3}\right\}$. Semelhante a (b).

4. (a) $\dfrac{df}{dx}(x) = \arcsin(4x) + \dfrac{4x}{\sqrt{1-16x^2}}$

   (b) $\dfrac{dg}{dt}(t) = \dfrac{14\arctan(7t)}{1+49t^2}$

   (c) $\dfrac{dh}{dy}(y) = \dfrac{\cos y}{2\sqrt{\sin y}} + \dfrac{1}{y^2\sqrt{1-\frac{1}{y^2}}}$

(d) $\dfrac{di}{dx}(x) = -\dfrac{3\sin(\arctan(3x))}{1+9x^2}$

5. (a) $\dfrac{\pi}{2}$

   (b) $D_t = \mathbb{R}\setminus\{-1\}$; $D'_t = \left]-\dfrac{\pi}{4}, \dfrac{3\pi}{4}\right[\setminus \left\{\dfrac{\pi}{4}\right\}$

   (c) $A = \left]-\infty, -2\right[\cup \left]-1, +\infty\right[$

   (d) $D_{t^{-1}} = \left]-\dfrac{\pi}{4}, \dfrac{3\pi}{4}\right[\setminus \left\{\dfrac{\pi}{4}\right\}$; $D'_{t^{-1}} = \mathbb{R}\setminus\{-1\}$; $t^{-1}(y) = \cot\left(y - \dfrac{\pi}{4}\right) - 1$

6. (a) $\dfrac{4\pi}{3}$

   (b) $D_g = \left]-\infty, -1\right] \cup \left[1, +\infty\right[$; $D'_g = \left]-\dfrac{2\pi}{3}, \dfrac{4\pi}{3}\right[\setminus \left\{\dfrac{\pi}{3}\right\}$

   (c) $A = [1, 2]$

   (d) $D_{g^{-1}} = \left]-\dfrac{2\pi}{3}, \dfrac{4\pi}{3}\right[\setminus \left\{\dfrac{\pi}{3}\right\}$; $D'_{g^{-1}} = \left]-\infty, -1\right] \cup \left[1, +\infty\right[$; $g^{-1}(y) = \csc\left(\dfrac{y}{2} - \dfrac{\pi}{6}\right)$.