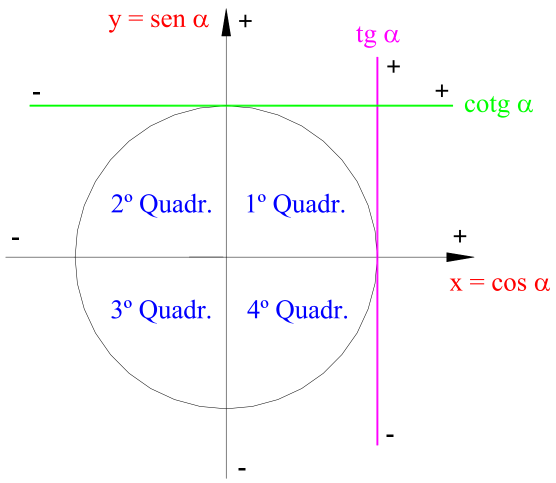
Teoria Relevante



sen α > 0 $\Longrightarrow$ 1º Quadr. 2º Quadr.

sen α < 0 $\Longrightarrow$ 3º Quadr. 4º Quadr.

cos α > 0 $\Longrightarrow$ 1º Quadr. 4º Quadr.

cos α < 0 $\Longrightarrow$ 2º Quadr. 3º Quadr.

Equação fundamental da trigonometria: $sen^2\alpha + \cos^2\alpha = 1$

$$tg(\alpha) = \frac{sen(\alpha)}{\cos(\alpha)} \quad ; \quad \text{cotg}(\alpha) = \frac{1}{tg(\alpha)} = \frac{\cos(\alpha)}{sen(\alpha)}$$

Domínios $D_{f^{-1}}$ e Contradomínios $D'_{f^{-1}}$ das funções inversas mais usuais

$f^{-1}(x) = arcsen(x)$	$f^{-1}(x) = \arccos(x)$	$f^{-1}(x) = arctg(x)$	$f^{-1}(x) = arccotg(x)$
$D_{f^{-1}} = [-1;1]$ $D'_{f^{-1}} = \left[-\frac{\pi}{2};\frac{\pi}{2}\right]$	$D_{f^{-1}} = [-1;1]$ $D'_{f^{-1}} = [0;\pi]$	$D_{f^{-1}} = \Re$ $D'_{f^{-1}} = \left]-\frac{\pi}{2};\frac{\pi}{2}\right[$	$D_{f^{-1}} = \Re$ $D'_{f^{-1}} =]0;\pi[$

Tabela que relaciona o seno e o co-seno dos ângulos mais usuais

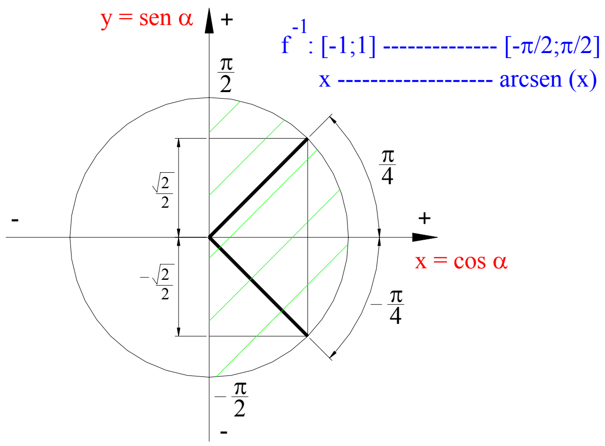
α	$\pi/6$	$\pi/4$	$\pi/3$	$0 \equiv 2\pi$	$\pi/2$	π	$3\pi/2$
sen α	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	0	1	0	-1
cos α	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	1	0	-1	0
tg α	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	0	---	0	---

1. Calcule:

a) $arcsen\left(-\frac{\sqrt{2}}{2}\right)$

R:

$$arcsen\left(-\frac{\sqrt{2}}{2}\right) = -\underbrace{arcsen\left(\frac{\sqrt{2}}{2}\right)}_{=\pi/4} = -\frac{\pi}{4}$$ [1]



b) $2 \cdot arcsen(-1)$

R:

$$2 \cdot arcsen(-1) = 2 \cdot \left[-\underbrace{arcsen(1)}_{=\pi/2}\right] = 2 \cdot \left(-\frac{\pi}{2}\right) = -\pi$$

c) $\cos\left[arcsen\left(\frac{1}{2}\right)\right]$

R:

$$\cos\left[\underbrace{arcsen\left(\frac{1}{2}\right)}_{=\alpha}\right] \Rightarrow \alpha = arcsen\left(\frac{1}{2}\right) \Leftrightarrow sen(\alpha) = sen\left[arcsen\left(\frac{1}{2}\right)\right] \Leftrightarrow sen(\alpha) = \frac{1}{2}$$

Recorrendo à equação fundamental da trigonometria, teremos:

$$sen^2\alpha + \cos^2\alpha = 1 \Leftrightarrow \left(\frac{1}{2}\right)^2 + \cos^2\alpha = 1 \Leftrightarrow \cos^2\alpha = 1 - \frac{1}{4} \Leftrightarrow \cos(\alpha) = \pm\sqrt{\frac{4-1}{4}} \Leftrightarrow$$

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[1] Como a restrição a $f^{-1}(x) = arcsen(x)$ é dada por: $D'_{f^{-1}} = \left[-\frac{\pi}{2};\frac{\pi}{2}\right] \Rightarrow 1^{\text{o}}$ e 4^{o} Quadrantes, então isto significa que: $arcsen(-x) = -arcsen(x)$. Esta regra também é válida para $f^{-1}(x) = arctg(x)$.

$$\Leftrightarrow \cos(\alpha)=\pm\sqrt{\frac{3}{4}} \Leftrightarrow \cos(\alpha)=\pm\frac{\sqrt{3}}{2} \Rightarrow \left\{\begin{array}{l}\text{Uma vez que}: sen(\alpha)=\frac{1}{2}>0 \Rightarrow 1^{\text{o}}\text{ Quadr., então, isto}\\ \text{implica que no } 1^{\text{o}}\text{ Quadr., o coseno é} > 0\text{, logo}:\\ cos(\alpha)=+\frac{\sqrt{3}}{2}\end{array}\right\}$$

Então: $\cos(\alpha)=\frac{\sqrt{3}}{2}$

d) $tg\left[arcsen\left(-\frac{\sqrt{3}}{2}\right)\right]$

R:

$$tg\left[\underbrace{arcsen\left(-\frac{\sqrt{3}}{2}\right)}_{=\alpha}\right] \Rightarrow \alpha = arcsen\left(-\frac{\sqrt{3}}{2}\right) \Leftrightarrow sen(\alpha)=sen\left[arcsen\left(-\frac{\sqrt{3}}{2}\right)\right] \Leftrightarrow sen(\alpha)=-\frac{\sqrt{3}}{2}$$

Recorrendo à equação fundamental da trigonometria, teremos:

$$sen^2\alpha+\cos^2\alpha=1 \Leftrightarrow \left(-\frac{\sqrt{3}}{2}\right)^2+\cos^2\alpha=1 \Leftrightarrow \cos^2\alpha=1-\frac{3}{4} \Leftrightarrow \cos(\alpha)=\pm\sqrt{\frac{4-3}{4}} \Leftrightarrow$$

$$\Leftrightarrow \cos(\alpha)=\pm\sqrt{\frac{2}{4}} \Leftrightarrow \cos(\alpha)=\pm\frac{1}{2} \Rightarrow \left\{\begin{array}{l}\text{Uma vez que}: sen(\alpha)=-\frac{\sqrt{3}}{2}<0 \Rightarrow 4^{\text{o}}\text{Quadr., então, isto}\\ \text{implica que no } 4^{\text{o}}\text{ Quadr., o coseno é} > 0\text{, logo}:\\ cos(\alpha)=+\frac{1}{2}\end{array}\right\}$$

Sabendo ainda que: $tg(\alpha)=\frac{sen(\alpha)}{\cos(\alpha)}$. Então: $tg(\alpha)=\frac{-\frac{\sqrt{3}}{2}}{\frac{1}{2}} \Leftrightarrow tg(\alpha)=-\frac{2\cdot\sqrt{3}}{2\cdot 1} \Leftrightarrow tg(\alpha)=-\sqrt{3}$

e) $\operatorname{cotg}\left[arcsen\left(-\frac{4}{5}\right)\right]$

R:

$$\operatorname{cotg}\left[\underbrace{arcsen\left(-\frac{4}{5}\right)}_{=\alpha}\right] \Rightarrow \alpha = arcsen\left(-\frac{4}{5}\right) \Leftrightarrow sen(\alpha) = sen\left[arcsen\left(-\frac{4}{5}\right)\right] \Leftrightarrow sen(\alpha) = -\frac{4}{5}$$

Recorrendo à equação fundamental da trigonometria, teremos:

$$sen^2\alpha + \cos^2\alpha = 1 \Leftrightarrow \left(-\frac{4}{5}\right)^2 + \cos^2\alpha = 1 \Leftrightarrow \cos^2\alpha = 1 - \frac{16}{25} \Leftrightarrow \cos(\alpha) = \pm\sqrt{\frac{25-16}{25}} \Leftrightarrow$$

$$\Leftrightarrow \cos(\alpha) = \pm\sqrt{\frac{9}{25}} \Leftrightarrow \cos(\alpha) = \pm\frac{3}{5} \Rightarrow \left\{\begin{array}{l} \text{Uma vez que}: sen(\alpha) = -\frac{4}{5} < 0 \Rightarrow 4^{\text{o}} \text{ Quadr., então isto} \\ \text{implica que no } 4^{\text{o}} \text{ Quadr., o coseno é} > 0\text{, logo}: \\ \cos(\alpha) = +\frac{3}{5} \end{array}\right\}$$

Sabendo ainda que: $\operatorname{cotg}(\alpha) = \dfrac{1}{tg(\alpha)} = \dfrac{\cos(\alpha)}{sen(\alpha)}$.

Então: $\operatorname{cotg}(\alpha) = \dfrac{\frac{3}{5}}{-\frac{4}{5}} \Leftrightarrow \operatorname{cotg}(\alpha) = -\dfrac{3\cdot 5}{4\cdot 5} \Leftrightarrow \operatorname{cotg}(\alpha) = -\dfrac{3}{4}$

f) $sen\left[arcsen\left(-\frac{5}{13}\right)\right]$

R:

$$sen\left[arcsen\left(-\frac{5}{13}\right)\right] = -\frac{5}{13}$$

g) $sen\left[\underbrace{\frac{\pi}{3}}_{=\alpha}-\underbrace{arctg\left(\frac{4}{5}\right)}_{=\beta}\right]$ [2]

R:

$$sen\left[\frac{\pi}{3}-arctg\left(\frac{4}{5}\right)\right]=sen\left[\frac{\pi}{3}\right]\cdot\cos\left[arctg\left(\frac{4}{5}\right)\right]-\cos\left[\frac{\pi}{3}\right]\cdot sen\left[arctg\left(\frac{4}{5}\right)\right]\Leftrightarrow$$

$$\Leftrightarrow sen\left[\frac{\pi}{3}-arctg\left(\frac{4}{5}\right)\right]=\frac{\sqrt{3}}{2}\cdot\cos\left[arctg\left(\frac{4}{5}\right)\right]-\frac{1}{2}\cdot sen\left[arctg\left(\frac{4}{5}\right)\right]\Leftrightarrow ☼$$

Cálculos Auxiliares:

$$\cos\left[\underbrace{arctg\left(\frac{4}{5}\right)}_{=\alpha}\right]\Rightarrow\alpha=arctg\left(\frac{4}{5}\right)\Leftrightarrow tg(\alpha)=tg\left[arctg\left(\frac{4}{5}\right)\right]\Leftrightarrow tg(\alpha)=\frac{4}{5}$$

Recorrendo à equação fundamental da trigonometria, teremos:

$$sen^2\alpha+\cos^2\alpha=1\Leftrightarrow\frac{sen^2\alpha}{\cos^2\alpha}+\frac{\cos^2\alpha}{\cos^2\alpha}=\frac{1}{\cos^2\alpha}\Leftrightarrow tg^2\alpha+1=\frac{1}{\cos^2\alpha}\Leftrightarrow\left(\frac{4}{5}\right)^2+1=\frac{1}{\cos^2\alpha}\Leftrightarrow$$

$$\Leftrightarrow\frac{16}{25}+1=\frac{1}{\cos^2\alpha}\Leftrightarrow\frac{16+25}{25}=\frac{1}{\cos^2\alpha}\Leftrightarrow\cos^2\alpha=\frac{25}{41}\Leftrightarrow\cos(\alpha)=\pm\sqrt{\frac{25}{41}}\Leftrightarrow\cos(\alpha)=\pm\frac{5}{\sqrt{41}}\Leftrightarrow$$

$$\Leftrightarrow\cos(\alpha)=\pm\frac{5\cdot\sqrt{41}}{\sqrt{41}\cdot\sqrt{41}}\Leftrightarrow\cos(\alpha)=\pm\frac{5\cdot\sqrt{41}}{41}\Rightarrow$$

$$\Rightarrow\left\{\begin{array}{l}\text{Uma vez que}: tg(\alpha)=\frac{4}{5}>0\Rightarrow 1^{\text{o}}\text{ Quadr., então isto implica que no } 1^{\text{o}}\text{ Quadr.,}\\ \text{o coseno é} > 0\text{, logo}: \cos(\alpha)=+\frac{5\cdot\sqrt{41}}{41}\end{array}\right\}$$

Então: $\cos\left[arctg\left(\frac{4}{5}\right)\right]=\frac{5\cdot\sqrt{41}}{41}$

[2] A expressão trigonométrica a aplicar aqui é: $sen(\alpha\pm\beta)=sen(\alpha)\cdot\cos(\beta)\pm\cos(\alpha)\cdot sen(\beta)$

Sabendo que: $\cos\left[arctg\left(\frac{4}{5}\right)\right]=\frac{5\cdot\sqrt{41}}{41}$

E recorrendo à equação fundamental da trigonometria, teremos que:

$$sen^2\alpha+\cos^2\alpha=1 \Leftrightarrow sen^2\left[arctg\left(\frac{4}{5}\right)\right]+\cos^2\left[arctg\left(\frac{4}{5}\right)\right]=1 \Leftrightarrow$$

$$\Leftrightarrow sen^2\left[arctg\left(\frac{4}{5}\right)\right]+\left(\frac{5\sqrt{41}}{41}\right)^2=1 \Leftrightarrow sen^2\left[arctg\left(\frac{4}{5}\right)\right]=1-\frac{5^2\cdot\left(\sqrt{41}\right)^2}{41^2} \Leftrightarrow$$

$$\Leftrightarrow sen^2\left[arctg\left(\frac{4}{5}\right)\right]=1-\frac{1025}{1681} \Leftrightarrow sen^2\left[arctg\left(\frac{4}{5}\right)\right]=\frac{1681-1025}{1681} \Leftrightarrow sen^2\left[arctg\left(\frac{4}{5}\right)\right]=\frac{656}{1681} \Leftrightarrow$$

$$\Leftrightarrow sen\left[arctg\left(\frac{4}{5}\right)\right]=\pm\sqrt{\frac{656}{1681}} \Leftrightarrow sen\left[arctg\left(\frac{4}{5}\right)\right]=\pm\frac{\sqrt{656}}{41} \Rightarrow$$

$$\Rightarrow \left\{\begin{array}{l}\text{Uma vez que}: tg(\alpha)=\frac{4}{5}>0 \Rightarrow 1^{\text{o}}\ \text{Quadr., então isto implica que no } 1^{\text{o}}\ \text{Quadr.,}\\ \text{o seno é} > 0,\ \text{logo}: sen(\alpha)=+\frac{\sqrt{656}}{41}\end{array}\right\}$$

Então: $sen\left[arctg\left(\frac{4}{5}\right)\right]=\frac{\sqrt{656}}{41}$

Assim sendo teremos então que:

$$☼\Leftrightarrow sen\left[\frac{\pi}{3}-arctg\left(\frac{4}{5}\right)\right]=\frac{\sqrt{3}}{2}\cdot\frac{5\cdot\sqrt{41}}{41}-\frac{1}{2}\cdot\frac{\sqrt{656}}{41} \Leftrightarrow sen\left[\frac{\pi}{3}-arctg\left(\frac{4}{5}\right)\right]=\frac{5\cdot\sqrt{3\cdot41}}{2\cdot41}-\frac{\sqrt{656}}{2\cdot41} \Leftrightarrow$$

$$\Leftrightarrow sen\left[\frac{\pi}{3}-arctg\left(\frac{4}{5}\right)\right]=\frac{5\cdot\sqrt{123}-\sqrt{656}}{82}$$

h) $\cos\left[\underbrace{arcsen\left(\frac{1}{2}\right)}_{=\alpha}-\underbrace{\arccos\left(\frac{3}{5}\right)}_{=\beta}\right]$[3]

R:

$$\cos\left[arcsen\left(\frac{1}{2}\right)-\arccos\left(\frac{3}{5}\right)\right]=\cos\left[arcsen\left(\frac{1}{2}\right)\right]\cdot\cos\left[\arccos\left(\frac{3}{5}\right)\right]+sen\left[arcsen\left(\frac{1}{2}\right)\right]\cdot sen\left[\arccos\left(\frac{3}{5}\right)\right]\Leftrightarrow$$

$$\Leftrightarrow\cos\left[arcsen\left(\frac{1}{2}\right)-\arccos\left(\frac{3}{5}\right)\right]=\cos\left[arcsen\left(\frac{1}{2}\right)\right]\cdot\frac{3}{5}+\frac{1}{2}\cdot sen\left[\arccos\left(\frac{3}{5}\right)\right]\Leftrightarrow☼$$

Cálculos Auxiliares:

$$\cos\left[\underbrace{arcsen\left(\frac{1}{2}\right)}_{=\alpha}\right]\Rightarrow\alpha=arcsen\left(\frac{1}{2}\right)\Leftrightarrow sen(\alpha)=sen\left[arcsen\left(\frac{1}{2}\right)\right]\Leftrightarrow sen(\alpha)=\frac{1}{2}$$

Recorrendo à equação fundamental da trigonometria, teremos:

$$sen^2\alpha+\cos^2\alpha=1\Leftrightarrow\left(\frac{1}{2}\right)^2+\cos^2\alpha=1\Leftrightarrow\cos^2\alpha=1-\frac{1}{4}\Leftrightarrow\cos(\alpha)=\pm\sqrt{\frac{4-1}{4}}\Leftrightarrow$$

$$\Leftrightarrow\cos(\alpha)=\pm\sqrt{\frac{3}{4}}\Leftrightarrow\cos(\alpha)=\pm\frac{\sqrt{3}}{2}\Rightarrow\left\{\begin{array}{l}\text{Uma vez que}: sen(\alpha)=\frac{1}{2}>0\Rightarrow 1^{\text{o}}\text{ Quadr., então, isto}\\ \text{implica que no }1^{\text{o}}\text{ Quadr., o coseno é}>0\text{, logo}:\\ \cos(\alpha)=+\frac{\sqrt{3}}{2}\end{array}\right\}$$

Então: $\cos\left[arcsen\left(\frac{1}{2}\right)\right]=\frac{\sqrt{3}}{2}$

$$sen\left[\underbrace{\arccos\left(\frac{3}{5}\right)}_{=\alpha}\right]\Rightarrow\alpha=\arccos\left(\frac{3}{5}\right)\Leftrightarrow\cos(\alpha)=\cos\left[\arccos\left(\frac{3}{5}\right)\right]\Leftrightarrow\cos(\alpha)=\frac{3}{5}$$

[3] A expressão trigonométrica a aplicar aqui é: $\cos(\alpha\pm\beta)=\cos(\alpha)\cdot\cos(\beta)\mp sen(\alpha)\cdot sen(\beta)$

E recorrendo à equação fundamental da trigonometria, teremos que:

$$sen^2\alpha + \cos^2\alpha = 1 \Leftrightarrow sen^2(\alpha) + \left(\frac{3}{5}\right)^2 = 1 \Leftrightarrow sen^2(\alpha) = 1 - \frac{9}{25} \Leftrightarrow sen^2(\alpha) = \frac{25-9}{25} \Leftrightarrow$$

$$\Leftrightarrow sen^2(\alpha) = \frac{16}{25} \Leftrightarrow sen(\alpha) = \pm\sqrt{\frac{16}{25}} \Leftrightarrow sen(\alpha) = \pm\frac{4}{5} \Rightarrow$$

$$\Rightarrow \left\{ \begin{array}{l} \text{Uma vez que} : \cos(\alpha) = \frac{3}{5} > 0 \Rightarrow 1^{\text{o}} \text{ Quadr., então isto implica que no } 1^{\text{o}} \text{ Quadr.,} \\ \text{o seno é} > 0, \text{ logo} : sen(\alpha) = +\frac{4}{5} \end{array} \right\}$$

Então: $sen\left[\arccos\left(\frac{3}{5}\right)\right] = \frac{4}{5}$

Assim sendo teremos então que:

$$☼ \Leftrightarrow \cos\left[arcsen\left(\frac{1}{2}\right) - \arccos\left(\frac{3}{5}\right)\right] = \frac{\sqrt{3}}{2}\cdot\frac{3}{5} + \frac{1}{2}\cdot\frac{4}{5} \Leftrightarrow \cos\left[arcsen\left(\frac{1}{2}\right) - \arccos\left(\frac{3}{5}\right)\right] = \frac{3\sqrt{3}}{10} + \frac{4}{10} \Leftrightarrow$$

$$\Leftrightarrow \cos\left[arcsen\left(\frac{1}{2}\right) - \arccos\left(\frac{3}{5}\right)\right] = \frac{3\sqrt{3}+4}{10}$$

2. Determine o número real designado por:

a) $arcsen\left[sen\left(\frac{\pi}{2}\right)\right]+4\cdot arcsen\left(-\frac{1}{2}\right)+2\cdot \arccos\left(-\frac{\sqrt{2}}{2}\right)$

R:

Cálculos Auxiliares:

$$arcsen\left[sen\left(\frac{\pi}{2}\right)\right]=\frac{\pi}{2}$$

$$4\cdot arcsen\left(-\frac{1}{2}\right)=4\cdot\left[-\underbrace{arcsen\left(\frac{1}{2}\right)}_{=\pi/6}\right]=4\cdot\left(-\frac{\pi}{6}\right)=-\frac{2\pi}{3}$$

$$2\cdot \arccos\left(-\frac{\sqrt{2}}{2}\right)\overset{4}{=}2\cdot\left[\pi-\underbrace{\arccos\left(\frac{\sqrt{2}}{2}\right)}_{=\pi/4}\right]=2\cdot\left(\pi-\frac{\pi}{4}\right)=\frac{6\pi}{4}=\frac{3\pi}{2}$$

Assim sendo, então: $arcsen\left[sen\left(\frac{\pi}{2}\right)\right]+4\cdot arcsen\left(-\frac{1}{2}\right)+2\cdot \arccos\left(-\frac{\sqrt{2}}{2}\right)=$

$$=\frac{\pi}{2}+\left(-\frac{2\pi}{3}\right)+\left(\frac{3\pi}{2}\right)=\frac{\pi}{2}-\frac{2\pi}{3}+\frac{3\pi}{2}=\frac{3\pi-4\pi+6\pi}{6}=\frac{5\pi}{6}$$

b) $\cos^2\left[\underbrace{\frac{1}{2}\arccos\left(\frac{1}{3}\right)}_{=\alpha}\right]-sen^2\left[\underbrace{\frac{1}{2}\arccos\left(\frac{1}{3}\right)}_{=\beta}\right]$

R:

Uma vez que α = β, então teremos que:

$$\cos^2\alpha-sen^2\alpha=\cos(\alpha)\cdot\cos(\alpha)-sen(\alpha)\cdot sen(\alpha)=\cos(\alpha+\alpha)=\cos(2\alpha)$$

[4] Como a restrição a $f^{-1}(x)=\arccos(x)$ é dada por: $D'_{f^{-1}}=[0;\pi]\Rightarrow 1^{\text{o}}\text{ e }2^{\text{o}}\text{ Quadrantes}$, então isto significa que: $\arccos(-x)=\pi-\arccos(x)$. Esta regra também é válida para $f^{-1}(x)=arc\text{cotg}(x)$.

Assim sendo teremos então que:

$$\left.\begin{cases}\cos(2\alpha)\\ \alpha=\frac{1}{2}\arccos\left(\frac{1}{3}\right)\end{cases}\right\}\Rightarrow \cos\left(2\cdot\frac{1}{2}\arccos\left(\frac{1}{3}\right)\right)=\cos\left(\arccos\left(\frac{1}{3}\right)\right)=\frac{1}{3}$$

c) $tg^2\left[arcsen\left(\frac{3}{5}\right)\right]-\text{cotg}^2\left[\arccos\left(\frac{4}{5}\right)\right]$

R:

Estudando ambos os membros independentemente teremos o seguinte:

$tg^2\left[arcsen\left(\frac{3}{5}\right)\right]$	$\text{cotg}^2\left[\arccos\left(\frac{4}{5}\right)\right]$
$\alpha=arcsen\left(\frac{3}{5}\right)\Leftrightarrow sen(\alpha)=\frac{3}{5}$ Pela equação fundamental, teremos então: $sen^2\alpha+\cos^2\alpha=1\Leftrightarrow\left(\frac{3}{5}\right)^2+\cos^2\alpha=1\Leftrightarrow$ $\Leftrightarrow\cos^2\alpha=1-\frac{9}{25}\Leftrightarrow\cos^2\alpha=\frac{16}{25}$ Sabendo que: $tg(\alpha)=\frac{sen(\alpha)}{\cos(\alpha)}\Rightarrow tg^2(\alpha)=\frac{sen^2(\alpha)}{\cos^2(\alpha)}$ Então: $tg^2(\alpha)=\frac{sen^2(\alpha)}{\cos^2(\alpha)}\Leftrightarrow tg^2(\alpha)=\frac{\left(\frac{3}{5}\right)^2}{\frac{16}{25}}\Leftrightarrow$ $\Leftrightarrow tg^2(\alpha)=\frac{\frac{9}{25}}{\frac{16}{25}}\Leftrightarrow tg^2(\alpha)=\frac{9}{16}$	$\alpha=\arccos\left(\frac{4}{5}\right)\Leftrightarrow\cos(\alpha)=\frac{4}{5}$ Pela equação fundamental, teremos então: $sen^2\alpha+\cos^2\alpha=1\Leftrightarrow sen^2\alpha+\left(\frac{4}{5}\right)^2=1\Leftrightarrow$ $\Leftrightarrow sen^2\alpha=1-\frac{16}{25}\Leftrightarrow sen^2\alpha=\frac{9}{25}$ Sabendo que: $\cot g(\alpha)=\frac{\cos(\alpha)}{sen(\alpha)}\Rightarrow\text{cotg}^2(\alpha)=\frac{\cos^2(\alpha)}{sen^2(\alpha)}$ Então: $\text{cotg}^2(\alpha)=\frac{\cos^2(\alpha)}{sen^2(\alpha)}\Leftrightarrow\text{cotg}^2(\alpha)=\frac{\left(\frac{4}{5}\right)^2}{\frac{9}{25}}\Leftrightarrow$ $\Leftrightarrow\text{cotg}^2(\alpha)=\frac{\frac{16}{25}}{\frac{9}{25}}\Leftrightarrow\text{cotg}^2(\alpha)=\frac{16}{9}$

Assim sendo, teremos finalmente que:

$$tg^2\left[arcsen\left(\frac{3}{5}\right)\right]-\mathrm{cotg}^2\left[\mathrm{arccos}\left(\frac{4}{5}\right)\right]=\frac{9}{16}-\frac{16}{9}=\frac{9\times 9-16\times 16}{16\times 9}=\frac{81-256}{144}=-\frac{175}{144}$$

3. Considere as seguintes funções reais de variável real. Determine o domínio e o contradomínio das funções indicadas. Caracterize as suas funções inversas.

a) $f(x) = 2 \cdot arcsen(2x-1) + \pi$

R:

Sabendo que para: $f^{-1}(x) = arcsen(x) \Rightarrow \left\{ \begin{array}{l} D = [-1;1] \Rightarrow -1 \leq x \leq 1 \\ D' = \left[-\frac{\pi}{2}; \frac{\pi}{2}\right] \Rightarrow -\frac{\pi}{2} \leq arcsen(x) \leq \frac{\pi}{2} \end{array} \right\}$, então:

- **Domínio de f(x):** $D = \{x \in \Re : -1 \leq 2x-1 \leq 1\}$

$$-1 \leq 2x-1 \leq 1 \Leftrightarrow \left\{ \begin{array}{l} 2x-1 \geq -1 \\ 2x-1 \leq 1 \end{array} \right\} \Leftrightarrow \left\{ \begin{array}{l} 2x-1+1 \geq 0 \\ 2x-1-1 \leq 0 \end{array} \right\} \Leftrightarrow \left\{ \begin{array}{l} x \geq 0 \\ x \leq 1 \end{array} \right\} \Rightarrow D = [0;1]$$

- **Contradomínio de f(x):**

$$-\frac{\pi}{2} \leq arcsen(2x-1) \leq \frac{\pi}{2} \Leftrightarrow -2 \cdot \frac{\pi}{2} \leq 2 \cdot arcsen(2x-1) \leq 2 \cdot \frac{\pi}{2} \Leftrightarrow$$

$$\Leftrightarrow -2 \cdot \frac{\pi}{2} + \pi \leq 2 \cdot arcsen(2x-1) + \pi \leq 2 \cdot \frac{\pi}{2} + \pi \Leftrightarrow$$

$$\Leftrightarrow 0 \leq 2 \cdot arcsen(2x-1) + \pi \leq 2\pi \Rightarrow D' = [0;2\pi] \backslash \{????\}$$

Para se determinar a restrição normalmente associada ao contradomínio, teremos que começar por determinar a inversa da função, para em seguida determinar o seu domínio. Será então o domínio da inversa a restrição a colocar no contradomínio.
Assim sendo teremos então que:

$$y = 2 \cdot arcsen(2x-1) + \pi \Leftrightarrow \frac{y-\pi}{2} = arcsen(2x-1) \Leftrightarrow sen\left(\frac{y-\pi}{2}\right) = sen[arcsen(2x-1)] \Leftrightarrow$$

$$\Leftrightarrow sen\left(\frac{y-\pi}{2}\right) = 2x-1 \Leftrightarrow x = \frac{sen\left(\frac{y-\pi}{2}\right)+1}{2} \Rightarrow D_{inversa} = \Re$$

Como: $D_{inversa} = \Re$, então não é aplicável qualquer restrição ao contradomínio, por isso teremos que: $D' = [0;2\pi]$

- **Caracterização da função inversa:**

$$f^{-1} : [0;2\pi] \quad \rightarrow \quad [0;1]$$

$$x \quad \rightarrow \quad \frac{sen\left(\frac{x-\pi}{2}\right)+1}{2}$$

b) $g(x) = \cos(\pi) + 3 \cdot \arccos(1-4x)$

R:

Re-arranjando a função temos: $g(x) = \underbrace{\cos(\pi)}_{=-1} + 3 \cdot \arccos(1-4x) \Leftrightarrow g(x) = -1 + 3 \cdot \arccos(1-4x)$

Sabendo que para: $f^{-1}(x) = \arccos(x) \Rightarrow \left\{ \begin{array}{l} D = [-1;1] \Rightarrow -1 \le x \le 1 \\ D' = [0;\pi] \Rightarrow 0 \le \arccos(x) \le \pi \end{array} \right\}$, então:

- **Domínio de g(x):** $D = \{x \in \Re : -1 \le 1-4x \le 1\}$

$$-1 \le 1-4x \le 1 \Leftrightarrow \left\{ \begin{array}{l} 1-4x \ge -1 \\ 1-4x \le 1 \end{array} \right\} \Leftrightarrow \left\{ \begin{array}{l} 1-4x+1 \ge 0 \\ 1-4x-1 \le 0 \end{array} \right\} \Leftrightarrow \left\{ \begin{array}{l} 2-4x \ge 0 \\ -4x \le 0 \end{array} \right\} \Leftrightarrow \left\{ \begin{array}{l} 2 \ge 4x \\ 0 \le 4x \end{array} \right\} \Leftrightarrow \left\{ \begin{array}{l} x \le \frac{1}{2} \\ x \ge 0 \end{array} \right\}$$

$$\Rightarrow D = \left[0;\frac{1}{2}\right]$$

- **Contradomínio de g(x):**

$$0 \le \arccos(1-4x) \le \pi \Leftrightarrow 3 \cdot 0 \le 3 \cdot \arccos(1-4x) \le 3\pi \Leftrightarrow$$

$$\Leftrightarrow -1 + 3 \cdot 0 \le -1 + 3 \cdot \arccos(1-4x) \le -1 + 3\pi \Leftrightarrow$$

$$\Leftrightarrow -1 \le -1 + 3 \cdot \arccos(1-4x) \le 3\pi - 1 \Rightarrow D' = [-1;3\pi-1] \backslash \{????\}$$

Para se determinar a restrição normalmente associada ao contradomínio, teremos que começar por determinar a inversa da função, para em seguida determinar o seu domínio. Será então o domínio da inversa a restrição a colocar no contradomínio.
Assim sendo teremos então que:

$$y=-1+3\cdot\arccos(1-4x)\Leftrightarrow\frac{y+1}{3}=\arccos(1-4x)\Leftrightarrow\cos\left(\frac{y+1}{3}\right)=\cos[\arccos(1-4x)]\Leftrightarrow$$

$$\Leftrightarrow\cos\left(\frac{y+1}{3}\right)=1-4x\Leftrightarrow x=\frac{\cos\left(\frac{y+1}{3}\right)-1}{-4}\Rightarrow D_{inversa}=\Re$$

Como: $D_{inversa}=\Re$, então não é aplicável qualquer restrição ao contradomínio, por isso teremos que: $D^{'}=[-1;3\pi-1]$

- **Caracterização da função inversa:**

$$g^{-1}:[-1;3\pi-1]\quad\to\quad\left[0;\frac{1}{2}\right]$$

$$x\quad\to\quad\frac{\cos\left(\frac{x+1}{3}\right)-1}{-4}$$

c) $h(x)=2\arccos\left(\frac{3}{x+2}\right)+\frac{\pi}{2}$

R:

Sabendo que para: $f^{-1}(x)=\arccos(x)\Rightarrow\begin{cases}D=[-1;1]\Rightarrow-1\le x\le1\\ D^{'}=[0;\pi]\Rightarrow0\le\arccos(x)\le\pi\end{cases}$, então:

- **Domínio de h(x):** $D=\left\{x\in\Re:-1\le\frac{3}{x+2}\le1\right\}$

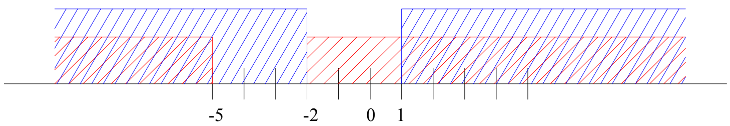
$$-1\le\frac{3}{x+2}\le1\Leftrightarrow\begin{cases}\frac{3}{x+2}\ge-1\\ \frac{3}{x+2}\le1\end{cases}\Leftrightarrow\begin{cases}\frac{3}{x+2}+1\ge0\\ \frac{3}{x+2}-1\le0\end{cases}\Leftrightarrow\begin{cases}\frac{x+5}{x+2}\ge0\\ \frac{1-x}{x+2}\le0\end{cases}$$

	$-\infty$	-5		-2	$+\infty$
$x+5$	-	0	+	+	+
$x+2$	-	-	-	0	+
$\dfrac{x+5}{x+2}$	+	0	-	s.s.	+

	$-\infty$	-2		1	$+\infty$
$1-x$	+	+	+	0	-
$x+2$	-	0	+	+	+
$\dfrac{1-x}{x+2}$	-	s.s.	+	0	-

$$\frac{x+5}{x+2} \geq 0 \Rightarrow D_A = \left]-\infty;-5\right] \cup \left]-2;+\infty\right[$$

$$\frac{1-x}{x+2} \leq 0 \Rightarrow D_B = \left]-\infty;-2\right[\cup \left[1;+\infty\right[$$



$$\Rightarrow D = D_A \cap D_B \Leftrightarrow D = \left]-\infty;-5\right] \cup \left[1;+\infty\right[$$

- **Contradomínio de h(x):**

$$0 \leq \arccos\left(\frac{3}{x+2}\right) \leq \pi \Leftrightarrow 2 \cdot 0 \leq 2 \cdot \arccos\left(\frac{3}{x+2}\right) \leq 2\pi \Leftrightarrow$$

$$\Leftrightarrow 2 \cdot 0 + \frac{\pi}{2} \leq 2 \cdot \arccos\left(\frac{3}{x+2}\right) + \frac{\pi}{2} \leq 2\pi + \frac{\pi}{2} \Leftrightarrow$$

$$\Leftrightarrow \frac{\pi}{2} \leq 2 \cdot \arccos\left(\frac{3}{x+2}\right) + \frac{\pi}{2} \leq \frac{5\pi}{2} \Rightarrow D' = \left[\frac{\pi}{2}; \frac{5\pi}{2}\right] \setminus \{????\}$$

Para se determinar a restrição normalmente associada ao contradomínio, teremos que começar por determinar a inversa da função, para em seguida determinar o seu domínio. Será então o domínio da inversa a restrição a colocar no contradomínio.

Assim sendo teremos então que:

$$y = 2\arccos\left(\frac{3}{x+2}\right) + \frac{\pi}{2} \Leftrightarrow \frac{y - \frac{\pi}{2}}{2} = \arccos\left(\frac{3}{x+2}\right) \Leftrightarrow \frac{y}{2} - \frac{\pi}{4} = \arccos\left(\frac{3}{x+2}\right) \Leftrightarrow$$

$$\Leftrightarrow \cos\left(\frac{y}{2} - \frac{\pi}{4}\right) = \cos\left[\arccos\left(\frac{3}{x+2}\right)\right] \Leftrightarrow \cos\left(\frac{y}{2} - \frac{\pi}{4}\right) = \frac{3}{x+2} \Leftrightarrow x+2 = \frac{3}{\cos\left(\frac{y}{2} - \frac{\pi}{4}\right)} \Leftrightarrow$$

$$\Leftrightarrow x=\frac{3}{\cos\left(\frac{y}{2}-\frac{\pi}{4}\right)}-2\Rightarrow D=\left\{x\in\Re:\cos\left(\frac{y}{2}-\frac{\pi}{4}\right)\neq 0\right\}$$

➢ $\cos\left(\frac{y}{2}-\frac{\pi}{4}\right)\neq 0\Leftrightarrow \frac{y}{2}-\frac{\pi}{4}\neq \underbrace{\arccos(0)}_{=\pi/2}\Leftrightarrow \frac{y}{2}-\frac{\pi}{4}\neq\frac{\pi}{2}\Leftrightarrow\frac{y}{2}\neq\frac{\pi}{2}+\frac{\pi}{4}\Leftrightarrow\frac{y}{2}\neq\frac{3\pi}{4}\Leftrightarrow y\neq\frac{3\pi}{2}\Rightarrow$

$$\Rightarrow D_{inversa}=\Re\setminus\left\{\frac{3\pi}{2}\right\}$$

Como: $D_{inversa}=\Re\setminus\left\{\frac{3\pi}{2}\right\}$, então a restrição ao contradomínio será: $D^{'}=\left[\frac{\pi}{2};\frac{5\pi}{2}\right]\setminus\left\{\frac{3\pi}{2}\right\}$

- **Caracterização da função inversa:**

$$h^{-1}:\left[\frac{\pi}{2};\frac{5\pi}{2}\right]\setminus\left\{\frac{3\pi}{2}\right\}\quad\rightarrow\quad]-\infty;-5]\cup[1;+\infty[$$
$$x\quad\rightarrow\quad\frac{3}{\cos\left(\frac{x}{2}-\frac{\pi}{4}\right)}-2$$

d) $i(x)=\frac{\pi}{3}+arctg\left(\frac{1}{x+5}\right)$

R:

Sabendo que para: $f^{-1}(x)=arctg(x)\Rightarrow\begin{cases}D=\Re\\ D^{'}=\left]-\frac{\pi}{2};\frac{\pi}{2}\right[\Rightarrow-\frac{\pi}{2}<arctg(x)<\frac{\pi}{2}\end{cases}$, então:

- **Domínio de i(x):** $D=\{x\in\Re:x+5\neq 0\}$

$$x+5\neq 0\Leftrightarrow x\neq -5\Rightarrow D=\Re\setminus\{-5\}$$

- **Contradomínio de i(x):**

$$-\frac{\pi}{2}<arctg\left(\frac{1}{x+5}\right)<\frac{\pi}{2}\Leftrightarrow-\frac{\pi}{2}+\frac{\pi}{3}<\frac{\pi}{3}+arctg\left(\frac{1}{x+5}\right)<\frac{\pi}{3}+\frac{\pi}{2}\Leftrightarrow$$

$$\Leftrightarrow \frac{2\pi - 3\pi}{6} < \frac{\pi}{3} + arctg\left(\frac{1}{x+5}\right) < \frac{2\pi + 3\pi}{6} \Leftrightarrow$$

$$\Leftrightarrow -\frac{\pi}{6} < \frac{\pi}{3} + arctg\left(\frac{1}{x+5}\right) < \frac{5\pi}{6} \Rightarrow D' = \left]-\frac{\pi}{6};\frac{5\pi}{6}\right[\setminus \{????\}$$

Para se determinar a restrição normalmente associada ao contradomínio, teremos que começar por determinar a inversa da função, para em seguida determinar o seu domínio. Será então o domínio da inversa a restrição a colocar no contradomínio.
Assim sendo teremos então que:

$$y = \frac{\pi}{3} + arctg\left(\frac{1}{x+5}\right) \Leftrightarrow y - \frac{\pi}{3} = arctg\left(\frac{1}{x+5}\right) \Leftrightarrow tg\left(y - \frac{\pi}{3}\right) = tg\left[arctg\left(\frac{1}{x+5}\right)\right] \Leftrightarrow$$

$$\Leftrightarrow tg\left(y - \frac{\pi}{3}\right) = \frac{1}{x+5} \Leftrightarrow x + 5 = \frac{1}{tg\left(y - \frac{\pi}{3}\right)} \Leftrightarrow x = \frac{1}{tg\left(y - \frac{\pi}{3}\right)} - 5 \Rightarrow D = \left\{x \in \Re : tg\left(y - \frac{\pi}{3}\right) \neq 0\right\}$$

- $tg\left(y - \frac{\pi}{3}\right) \neq 0 \Leftrightarrow y - \frac{\pi}{3} \neq \underbrace{arctg(0)}_{=0} \Leftrightarrow y - \frac{\pi}{3} \neq 0 \Leftrightarrow y \neq \frac{\pi}{3} \Rightarrow D_{inversa} = \Re \setminus \left\{\frac{\pi}{3}\right\}$

Como: $D_{inversa} = \Re \setminus \left\{\frac{\pi}{3}\right\}$, então a restrição ao contradomínio será: $D' = \left]-\frac{\pi}{6};\frac{5\pi}{6}\right[\setminus \left\{\frac{\pi}{3}\right\}$

- **Caracterização da função inversa:**

$$\begin{array}{lcl} i^{-1} : \left]-\frac{\pi}{6};\frac{5\pi}{6}\right[\setminus \left\{\frac{\pi}{3}\right\} & \to & \Re \setminus \{-5\} \\ x & \to & \dfrac{1}{tg\left(x - \frac{\pi}{3}\right)} - 5 \end{array}$$

4. Considere a função real de variável real definida por:

$$p(x)=\frac{\pi}{3}-2\cdot\arccos(x+1)$$

a) Calcule: $p(-1)-p\left(-\frac{3}{2}\right)$

R:

$$p(-1)=\frac{\pi}{3}-2\cdot\arccos(-1+1)\Leftrightarrow p(-1)=\frac{\pi}{3}-2\cdot\underbrace{\arccos(0)}_{=\pi/2}\Leftrightarrow p(-1)=\frac{\pi}{3}-2\cdot\frac{\pi}{2}\Leftrightarrow p(-1)=-\frac{2\pi}{3}$$

$$p\left(-\frac{3}{2}\right)=\frac{\pi}{3}-2\cdot\arccos\left(-\frac{3}{2}+1\right)\Leftrightarrow p\left(-\frac{3}{2}\right)=\frac{\pi}{3}-2\cdot\underbrace{\arccos\left(-\frac{1}{2}\right)}_{=\pi-\pi/3}{}^{5}\Leftrightarrow p\left(-\frac{3}{2}\right)=\frac{\pi}{3}-2\cdot\left(\pi-\frac{\pi}{3}\right)\Leftrightarrow$$

$$\Leftrightarrow p\left(-\frac{3}{2}\right)=\frac{\pi}{3}-2\cdot\frac{2\pi}{3}\Leftrightarrow p\left(-\frac{3}{2}\right)=\frac{\pi-4\pi}{3}\Leftrightarrow p\left(-\frac{3}{2}\right)=-\pi$$

Então: $p(-1)-p\left(-\frac{3}{2}\right)=-\frac{2\pi}{3}-(-\pi)=\frac{\pi}{3}$

b) Determine o domínio e o contradomínio da função.

R:

Sabendo que para: $f^{-1}(x)=\arccos(x)\Rightarrow\left\{\begin{array}{l}D=[-1;1]\Rightarrow -1\leq x\leq 1\\ D^{'}=[0;\pi]\Rightarrow 0\leq\arccos(x)\leq\pi\end{array}\right\}$, então:

- **Domínio:** $D=\{x\in\Re:-1\leq x+1\leq 1\}$

$$-1\leq x+1\leq 1\Leftrightarrow\left\{\begin{array}{l}x+1\geq -1\\ x+1\leq 1\end{array}\right\}\Leftrightarrow\left\{\begin{array}{l}x\geq -2\\ x\leq 0\end{array}\right\}\Rightarrow D=[-2;0]$$

[5] Como a restrição a $f^{-1}(x)=\arccos(x)$ é dada por: $D^{'}_{f^{-1}}=[0;\pi]\Rightarrow 1^{o}\text{ e }2^{o}\text{ Quadrantes}$, então isto significa que: $\arccos(-x)=\pi-\arccos(x)$. Esta regra também é válida para $f^{-1}(x)=arc\cot g(x)$.

- **Contradomínio:**

$$0 \le \arccos(x+1) \le \pi \Leftrightarrow^{6} -2\cdot 0 \ge -2\cdot\arccos(x+1) \ge -2\pi \Leftrightarrow$$

$$\Leftrightarrow \frac{\pi}{3} - 2\cdot 0 \ge \frac{\pi}{3} - 2\cdot\arccos(x+1) \ge \frac{\pi}{3} - 2\pi \Leftrightarrow$$

$$\Leftrightarrow \frac{\pi}{3} \ge \frac{\pi}{3} - 2\cdot\arccos(x+1) \ge -\frac{5\pi}{3} \Rightarrow D' = \left[-\frac{5\pi}{3};\frac{\pi}{3}\right] \setminus \{????\}$$

Para se determinar a restrição normalmente associada ao contradomínio, teremos que começar por determinar a inversa da função, para em seguida determinar o seu domínio. Será então o domínio da inversa a restrição a colocar no contradomínio.
Assim sendo teremos então que:

$$y = \frac{\pi}{3} - 2\cdot\arccos(x+1) \Leftrightarrow \frac{y-\frac{\pi}{3}}{-2} = \arccos(x+1) \Leftrightarrow \frac{\pi}{6} - \frac{y}{2} = \arccos(x+1) \Leftrightarrow$$

$$\Leftrightarrow \cos\left(\frac{\pi}{6} - \frac{y}{2}\right) = \cos[\arccos(x+1)] \Leftrightarrow \cos\left(\frac{\pi}{6} - \frac{y}{2}\right) = x+1 \Leftrightarrow x = \cos\left(\frac{\pi}{6} - \frac{y}{2}\right) - 1 \Rightarrow D_{inversa} = \Re$$

Como: $D_{inversa} = \Re$, então não é aplicável qualquer restrição ao contradomínio, por isso teremos que: $D' = \left[-\frac{5\pi}{3};\frac{\pi}{3}\right]$

c) Calcule, caso existam, os zeros de p .

R:

$$p(x) = 0 \Leftrightarrow \frac{\pi}{3} - 2\cdot\arccos(x+1) = 0 \Leftrightarrow \frac{\pi}{3} = 2\cdot\arccos(x+1) \Leftrightarrow \frac{\pi}{6} = \arccos(x+1) \Leftrightarrow$$

$$\Leftrightarrow \cos\left(\frac{\pi}{6}\right) = \cos[\arccos(x+1)] \Leftrightarrow \cos\left(\frac{\pi}{6}\right) = x+1 \Leftrightarrow \frac{\sqrt{3}}{2} = x+1 \Leftrightarrow x = \frac{\sqrt{3}}{2} - 1 \Leftrightarrow x = \frac{\sqrt{3}-2}{2}$$

$$S = \left\{\frac{\sqrt{3}-2}{2}\right\}$$

[6] Sempre que se **multiplica** uma expressão deste tipo **por um número negativo**, é obrigatório **mudar o sentido dos sinais**.

d) Caracterize a função inversa de p.

R:

$$p^{-1}:\left[-\frac{5\pi}{3};\frac{\pi}{3}\right] \to [-2;0]$$

$$x \to \cos\left(\frac{\pi}{6}-\frac{x}{2}\right)-1$$

e) Resolva a seguinte inequação: $p(x)\leq-\frac{\pi}{3}$.

R:

$$p(x)\leq-\frac{\pi}{3} \Leftrightarrow \frac{\pi}{3}-2\cdot\arccos(x+1)\leq-\frac{\pi}{3} \Leftrightarrow \frac{\pi}{3}+\frac{\pi}{3}\leq 2\cdot\arccos(x+1) \Leftrightarrow \frac{\frac{2\pi}{3}}{2}\leq\arccos(x+1) \Leftrightarrow$$

$$\Leftrightarrow \frac{2\pi}{6}\leq\arccos(x+1) \Leftrightarrow \frac{\pi}{3}\leq\arccos(x+1) \Leftrightarrow^{7} \cos\left(\frac{\pi}{3}\right)\geq\cos[\arccos(x+1)] \Leftrightarrow \cos\left(\frac{\pi}{3}\right)\geq x+1 \Leftrightarrow$$

$$\Leftrightarrow \cos\left(\frac{\pi}{3}\right)-1\geq x \Leftrightarrow \frac{1}{2}-1\geq x \Leftrightarrow x\leq-\frac{1}{2} \Rightarrow x\in\left]-\infty;-\frac{1}{2}\right]$$

Assim sendo, então: $C.S.=[-2;0]\cap\left]-\infty;-\frac{1}{2}\right] \Leftrightarrow C.S.=\left[-2;-\frac{1}{2}\right]$

[7] O **sentido do sinal inverte-se sempre** para as funções: $\cos(\alpha)$ e $\operatorname{cotg}(\alpha)$, porque são funções decrescentes.

5. Determine a expressão das derivadas das funções:

a) $f(x) = x \cdot arcsen(4x)$

R:

$$f'(x) = (x \cdot arcsen(4x))' = \underbrace{(x)'}_{=1} \cdot arcsen(4x) + x \cdot \underbrace{(arcsen(4x))'}_{(arcsen(u))' = \frac{u'}{\sqrt{1-u^2}}} = arcsen(4x) + x \cdot \frac{(4x)'}{\sqrt{1-(4x)^2}} =$$

$$= arcsen(4x) + x \cdot \frac{4}{\sqrt{1-(4x)^2}} = arcsen(4x) + \frac{4x}{\sqrt{1-(4x)^2}}$$

b) $g(t) = arctg^2(7t)$

R:

$$g'(t) = \left(arctg^2(7t)\right)' = \underbrace{\left((arctg(7t))^2\right)'}_{(u^\alpha)' = \alpha \times u^{\alpha-1} \times u'} = 2 \cdot (arctg(7t))^{2-1} \cdot \underbrace{(arctg(7t))'}_{(arctg(u))' = \frac{u'}{1+u^2}} = 2 \cdot arctg(7t) \cdot \frac{(7t)'}{1+(7t)^2} =$$

$$= 2 \cdot arctg(7t) \cdot \frac{7}{1+(7t)^2} = \frac{14 \cdot arctg(7t)}{1+(7t)^2}$$

c) $h(y) = \sqrt{sen(y)} + \arccos\left(\frac{1}{y}\right)$

R:

$$h(y) = \sqrt{sen(y)} + \arccos\left(\frac{1}{y}\right) \Leftrightarrow h'(y) = \left((sen(y))^{1/2} + \arccos\left(\frac{1}{y}\right)\right)' =$$

$$= \underbrace{\left((sen(y))^{1/2}\right)'}_{(u^\alpha)' = \alpha \times u^{\alpha-1} \times u'} + \underbrace{\left(\arccos\left(\frac{1}{y}\right)\right)'}_{(\arccos(u))' = -\frac{u'}{\sqrt{1-u^2}}} = \frac{1}{2} \cdot (sen(y))^{\frac{1}{2}-1} \cdot (sen(y))' - \frac{\left(\frac{1}{y}\right)'}{\sqrt{1-\left(\frac{1}{y}\right)^2}} =$$

$$= \frac{(sen(y))^{-\frac{1}{2}}}{2} \cdot \cos(y) - \frac{(y^{-1})'}{\sqrt{1-\left(\frac{1}{y}\right)^2}} = \frac{\cos(y)}{2 \cdot (sen(y))^{\frac{1}{2}}} - \frac{(-1) \cdot (y^{-1-1}) \cdot (y)'}{\sqrt{1-\left(\frac{1}{y}\right)^2}} = \frac{\cos(y)}{2 \cdot (sen(y))^{\frac{1}{2}}} + \frac{\frac{1}{y^2}}{\sqrt{1-\left(\frac{1}{y}\right)^2}}$$

d) $i(x) = \cos[arctg(3x)]$

R:

$$i'(x) = \underbrace{\cos[arctg(3x)]'}_{(\cos(u))' = -sen(u)\cdot u'} = -sen[arctg(3x)] \cdot \underbrace{[arctg(3x)]'}_{(arctg(u))' = \frac{u'}{1-u^2}} = -sen[arctg(3x)] \cdot \frac{(3x)'}{1-(3x)^2} =$$

$$= -\frac{3 \cdot sen[arctg(3x)]}{1-(3x)^2}$$

e) $j(t) = 3t \cdot arcsen\left(\sqrt{t^2-1}\right)$

R:

$$j'(t) = \left(3t \cdot arcsen\left(\sqrt{t^2-1}\right)\right)' = (3t)' \cdot arcsen\left(\sqrt{t^2-1}\right) + 3t \cdot \underbrace{\left(arcsen\left(\sqrt{t^2-1}\right)\right)'}_{(arcsen(u))' = \frac{u'}{\sqrt{1-u^2}}} =$$

$$= 3 \cdot arcsen\left(\sqrt{t^2-1}\right) + 3t \cdot \frac{\left(\sqrt{t^2-1}\right)'}{\left(\sqrt{t^2-1}\right)^2} = 3 \cdot arcsen\left(\sqrt{t^2-1}\right) + 3t \cdot \frac{\left((t^2-1)^{\frac{1}{2}}\right)'}{\left(\sqrt{t^2-1}\right)^2} =$$

$$= 3 \cdot arcsen\left(\sqrt{t^2-1}\right) + 3t \cdot \frac{\frac{1}{2} \cdot (t^2-1)^{\frac{1}{2}-1} \cdot (t^2-1)'}{\left(\sqrt{t^2-1}\right)^2} = 3 \cdot arcsen\left(\sqrt{t^2-1}\right) + 3t \cdot \frac{\frac{1}{2} \cdot (t^2-1)^{-\frac{1}{2}} \cdot 2t}{\left(\sqrt{t^2-1}\right)^2} =$$

$$= 3 \cdot arcsen\left(\sqrt{t^2-1}\right) + \frac{6t^2 \cdot (t^2-1)^{-\frac{1}{2}}}{2 \cdot \left(\sqrt{t^2-1}\right)^2}$$

f) $m(y)=\dfrac{1}{\cos(y)}-arctg\left(\dfrac{y}{2}\right)$

R:

$$m'(y)=\underbrace{\left((\cos(y))^{-1}\right)'}_{(u^{\alpha})'=\alpha\times u^{\alpha-1}\times u'}-\underbrace{\left(arctg\left(\frac{y}{2}\right)\right)'}_{(arctg(u))'=\frac{u'}{1+u^2}}=-1\cdot(\cos(y))^{-1-1}\cdot\underbrace{(\cos(y))'}_{(\cos(u))'=-sen(u)\cdot u'}-\frac{\left(\frac{y}{2}\right)'}{1+\left(\frac{y}{2}\right)^2}=$$

$$=\frac{sen(y)}{(\cos(y))^2}-\frac{\overbrace{\left(\frac{1}{2}\right)'}^{=0}\cdot y+\frac{1}{2}\cdot\underbrace{(y)'}_{=1}}{1+\left(\frac{y}{2}\right)^2}=\frac{sen(y)}{(\cos(y))^2}-\frac{\frac{1}{2}}{1+\left(\frac{y}{2}\right)^2}$$

6. Considere a função real de variável real definida por:

$$t(x)=\frac{\pi}{4}+arctg\left(\frac{1}{x+1}\right)$$

a) Calcule: $t(0)+t(-2)$

R:

$$t(0)=\frac{\pi}{4}+arctg\left(\frac{1}{0+1}\right)\Leftrightarrow t(0)=\frac{\pi}{4}+\underbrace{arctg(1)}_{=\pi/4}\Leftrightarrow t(0)=\frac{\pi}{4}+\frac{\pi}{4}\Leftrightarrow t(0)=\frac{2\pi}{4}\Leftrightarrow t(0)=\frac{\pi}{2}$$

$$t(-2)=\frac{\pi}{4}+arctg\left(\frac{1}{-2+1}\right)\Leftrightarrow t(-2)=\frac{\pi}{4}+\underbrace{arctg(-1)}_{=-\pi/4}\Leftrightarrow t(-2)=\frac{\pi}{4}+\left(-\frac{\pi}{4}\right)\Leftrightarrow t(-2)=0$$

Então: $t(0)+t(-2)=\frac{\pi}{2}+0=\frac{\pi}{2}$

b) Determine o domínio e o contradomínio de t.

R:

Sabendo que para: $f^{-1}(x)=arctg(x)\Rightarrow\left\{\begin{matrix}D=\Re\\ D'=\left]-\frac{\pi}{2};\frac{\pi}{2}\right[\Rightarrow-\frac{\pi}{2}<arctg(x)<\frac{\pi}{2}\end{matrix}\right\}$, então:

- **Domínio:** $D=\{x\in\Re:x+1\neq0\}$

  $x+1\neq0\Leftrightarrow x\neq-1\Rightarrow D=\Re\setminus\{-1\}$

- **Contradomínio:**

$$-\frac{\pi}{2}<arctg\left(\frac{1}{x+1}\right)<\frac{\pi}{2}\Leftrightarrow\frac{\pi}{4}-\frac{\pi}{2}<\frac{\pi}{4}+arctg\left(\frac{1}{x+1}\right)<\frac{\pi}{4}+\frac{\pi}{2}\Leftrightarrow$$

$$\Leftrightarrow-\frac{\pi}{4}<\frac{\pi}{4}+arctg\left(\frac{1}{x+1}\right)<\frac{3\pi}{4}\Rightarrow D'=\left]-\frac{\pi}{4};\frac{3\pi}{4}\right[\setminus\{????\}$$

Para se determinar a restrição normalmente associada ao contradomínio, teremos que começar por determinar a inversa da função, para em seguida determinar o seu domínio. Será então o domínio da inversa a restrição a colocar no contradomínio.

Assim sendo teremos então que:

$$y=\frac{\pi}{4}+arctg\left(\frac{1}{x+1}\right)\Leftrightarrow y-\frac{\pi}{4}=arctg\left(\frac{1}{x+1}\right)\Leftrightarrow tg\left(y-\frac{\pi}{4}\right)=tg\left(arctg\left(\frac{1}{x+1}\right)\right)\Leftrightarrow$$

$$\Leftrightarrow tg\left(y-\frac{\pi}{4}\right)=\frac{1}{x+1}\Leftrightarrow x+1=\frac{1}{tg\left(y-\frac{\pi}{4}\right)}\Leftrightarrow x=\frac{1}{tg\left(y-\frac{\pi}{4}\right)}-1\Rightarrow D=\left\{x\in\Re:tg\left(y-\frac{\pi}{4}\right)\neq 0\right\}$$

- $tg\left(y-\frac{\pi}{4}\right)\neq 0\Leftrightarrow y-\frac{\pi}{4}\neq \underbrace{arctg(0)}_{=0}\Leftrightarrow y-\frac{\pi}{4}\neq 0\Leftrightarrow y\neq\frac{\pi}{4}\Rightarrow D_{inversa}=\Re\setminus\left\{\frac{\pi}{4}\right\}$

Como: $D_{inversa}=\Re\setminus\left\{\frac{\pi}{4}\right\}$, então a restrição ao contradomínio será: $D^{'}=\left]-\frac{\pi}{4};\frac{3\pi}{4}\right[\setminus\left\{\frac{\pi}{4}\right\}$

c) Determine o conjunto solução de: $A=\{x\in\Re:t(x)>0\}$**.**

R:

$$t(x)>0\Leftrightarrow\frac{\pi}{4}+arctg\left(\frac{1}{x+1}\right)>0\Leftrightarrow arctg\left(\frac{1}{x+1}\right)>-\frac{\pi}{4}\Leftrightarrow tg\left[arctg\left(\frac{1}{x+1}\right)\right]>\underbrace{tg\left(-\frac{\pi}{4}\right)}_{=-1}{}^{8}\Leftrightarrow$$

$$\Leftrightarrow\frac{1}{x+1}>-1\Leftrightarrow\frac{1}{x+1}+1>0\Leftrightarrow\frac{1+x+1}{x+1}>0\Leftrightarrow\frac{x+2}{x+1}>0$$

	$-\infty$	-2		-1	$+\infty$
$x+2$	-	0	+	+	+
$x+1$	-	-	-	0	+
$\frac{x+2}{x+1}$	+	0	-	s.s.	+

$$\frac{x+2}{x+1}>0\Rightarrow D=\left]-\infty;-2\right[\cup\left]-1;+\infty\right[$$

[8] $tg(-\alpha)=-tg(\alpha)\Rightarrow tg\left(-\frac{\pi}{4}\right)=-tg\left(\frac{\pi}{4}\right)=-1$

d) Caracterize a função inversa de t.

R:

$$t^{-1}: \left]-\frac{\pi}{4};\frac{3\pi}{4}\right[\setminus \left\{\frac{\pi}{4}\right\} \to \Re\setminus\{-1\}$$

$$x \to \frac{1}{tg\left(x-\frac{\pi}{4}\right)}-1$$

e) Escreva a equação da recta tangente de t, no ponto de abcissa 0 (zero).

R:

Sabendo que a equação geral da recta é dada por: $y = m\cdot x + b$, onde: $m = t'(x)$

E que um ponto é definido por: $P(x;y)$, onde: $y = t(x) \Rightarrow P[x;t(x)]$

Sabendo ainda que: $Abcissa = 0 \Rightarrow x = 0$, então teremos que:

$P[x;t(x)] \Leftrightarrow P[0;t(0)]$ e $m = t'(x) \Leftrightarrow m = t'(0)$

- $t(x)=\frac{\pi}{4}+arctg\left(\frac{1}{x+1}\right) \Rightarrow t(0)=\frac{\pi}{4}+arctg\left(\frac{1}{0+1}\right) \Leftrightarrow t(0)=\frac{\pi}{4}+\underbrace{arctg(1)}_{=\pi/4} \Leftrightarrow t(0)=\frac{\pi}{4}+\frac{\pi}{4} \Leftrightarrow$

  $\Leftrightarrow t(0)=\frac{2\pi}{4} \Leftrightarrow t(0)=\frac{\pi}{2}$

  Daqui resulta então que: $P[0;t(0)] \Leftrightarrow P\left(0;\frac{\pi}{2}\right)$

- $t'(x)=\left[\frac{\pi}{4}+arctg\left(\frac{1}{x+1}\right)\right]' =$[9] $0+\frac{\left(\frac{1}{x+1}\right)'}{1+\left(\frac{1}{x+1}\right)^2} = \frac{\frac{\overbrace{(1)'}^{=0}\cdot(x+1)-1\cdot\overbrace{(x+1)'}^{=1}}{(x+1)^2}}{1+\left(\frac{1}{x+1}\right)^2} = \frac{-1}{(x+1)^2\cdot\left(1+\left(\frac{1}{x+1}\right)^2\right)}$

[9] A derivada a aplicar aqui é: $(arctg(u))' = \frac{u'}{1+u^2}$

$$t^{'}(x)=\frac{-1}{(x+1)^2\cdot\left(1+\left(\frac{1}{x+1}\right)^2\right)}\Rightarrow t^{'}(0)=\frac{-1}{(0+1)^2\cdot\left(1+\left(\frac{1}{0+1}\right)^2\right)}\Leftrightarrow t^{'}(0)=\frac{-1}{1\cdot(1+1)}\Leftrightarrow t^{'}(0)=-\frac{1}{2}$$

Daqui resulta que: $m=t^{'}(0)\Leftrightarrow m=-\frac{1}{2}$

Então finalmente teremos que:

$$\left\{\begin{array}{l} y=m\cdot x+b \\ (x;y)=\left(0;\frac{\pi}{2}\right) \\ m=-\frac{1}{2} \end{array}\right\}\Rightarrow \frac{\pi}{2}=-\frac{1}{2}\cdot 0+b\Leftrightarrow b=\frac{\pi}{2}\Rightarrow y=-\frac{1}{2}\cdot x+\frac{\pi}{2}$$ →recta tangente.

f) Que pode concluir acerca da continuidade de t **no ponto de abcissa 0 (zero). Justifique a resposta.**

R:

Uma vez que a função admite derivada no ponto 0 (zero) então podemos dizer que ela é contínua, pois toda a função que admite derivada num ponto é continua nesse ponto.

7. Considere a função real de variável real definida por:

$$g(x)=\frac{\pi}{3}+2\cdot arcsen\left(\frac{1}{x}\right)$$

a) Calcule: $g(1)+g(-2)$

R:

$$g(1)=\frac{\pi}{3}+2\cdot arcsen\left(\frac{1}{1}\right)\Leftrightarrow g(1)=\frac{\pi}{3}+2\cdot \underbrace{arcsen(1)}_{=\pi/2}\Leftrightarrow g(1)=\frac{\pi}{3}+2\cdot\frac{\pi}{2}\Leftrightarrow g(1)=\frac{4\pi}{3}$$

$$g(-2)=\frac{\pi}{3}+2\cdot arcsen\left(\frac{1}{-2}\right)\Leftrightarrow g(-2)=\frac{\pi}{3}+2\cdot \underbrace{arcsen\left(-\frac{1}{2}\right)}_{=-arcsen(1/2)=-\pi/6}\Leftrightarrow g(-2)=\frac{\pi}{3}+2\cdot\left(-\frac{\pi}{6}\right)\Leftrightarrow g(-2)=0$$

Então: $g(1)+g(-2)=\dfrac{4\pi}{3}+0=\dfrac{4\pi}{3}$

b) Determine o domínio e o contradomínio de g **.**

R:

Sabendo que para: $f^{-1}(x)=arcsen(x)\Rightarrow\left\{\begin{array}{l} D=[-1;1]\Rightarrow -1\leq x\leq 1 \\ D'=\left[-\frac{\pi}{2};\frac{\pi}{2}\right]\Rightarrow -\frac{\pi}{2}\leq arcsen(x)\leq\frac{\pi}{2}\end{array}\right\}$, então:

- **Domínio:** $D=\left\{x\in\Re:-1\leq\frac{1}{x}\leq 1\right\}$

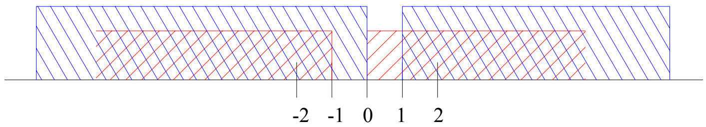
$$-1\leq\frac{1}{x}\leq 1\Leftrightarrow\left\{\begin{array}{l}\frac{1}{x}\geq -1\\ \frac{1}{x}\leq 1\end{array}\right\}\Leftrightarrow\left\{\begin{array}{l}\frac{1}{x}+1\geq 0\\ \frac{1}{x}-1\leq 0\end{array}\right\}\Leftrightarrow\left\{\begin{array}{l}\frac{1+x}{x}\geq 0\\ \frac{1-x}{x}\leq 0\end{array}\right\}$$

	$-\infty$	-1		0	$+\infty$
$1+x$	-	0	+	+	+
x	-	-	-	0	+
$\frac{1+x}{x}$	+	0	-	s.s.	+

	$-\infty$	0		1	$+\infty$
$1-x$	+	+	+	0	-
x	-	0	+	+	+
$\frac{1-x}{x}$	-	s.s.	+	0	-

$$\frac{1+x}{x} \geq 0 \Rightarrow D_A = \left]-\infty;-1\right] \cup \left]0;+\infty\right[$$

$$\frac{1-x}{x} \leq 0 \Rightarrow D_B = \left]-\infty;0\right[\cup \left[1;+\infty\right[$$



$$\Rightarrow D = D_A \cap D_B \Leftrightarrow D = \left]-\infty;-1\right] \cup \left[1;+\infty\right[$$

- **Contradomínio:**

$$-\frac{\pi}{2} \leq arcsen\left(\frac{1}{x}\right) \leq \frac{\pi}{2} \Leftrightarrow -2\cdot\frac{\pi}{2} \leq 2\cdot arcsen\left(\frac{1}{x}\right) \leq 2\cdot\frac{\pi}{2} \Leftrightarrow$$

$$\Leftrightarrow -2\cdot\frac{\pi}{2}+\frac{\pi}{3} \leq \frac{\pi}{3}+2\cdot arcsen\left(\frac{1}{x}\right) \leq \frac{\pi}{3}+2\cdot\frac{\pi}{2} \Leftrightarrow$$

$$\Leftrightarrow -\frac{2\pi}{3} \leq \frac{\pi}{3}+2\cdot arcsen\left(\frac{1}{x}\right) \leq \frac{4\pi}{3} \Rightarrow D' = \left]-\frac{2\pi}{3};\frac{4\pi}{3}\right[\setminus \{????\}$$

Para se determinar a restrição normalmente associada ao contradomínio, teremos que começar por determinar a inversa da função, para em seguida determinar o seu domínio. Será então o domínio da inversa a restrição a colocar no contradomínio.

Assim sendo teremos então que:

$$y = \frac{\pi}{3}+2\cdot arcsen\left(\frac{1}{x}\right) \Leftrightarrow y-\frac{\pi}{3} = 2\cdot arcsen\left(\frac{1}{x}\right) \Leftrightarrow \frac{y-\frac{\pi}{3}}{2} = arcsen\left(\frac{1}{x}\right) \Leftrightarrow \frac{y}{2}-\frac{\pi}{6} = arcsen\left(\frac{1}{x}\right) \Leftrightarrow$$

$$\Leftrightarrow sen\left(\frac{y}{2}-\frac{\pi}{6}\right)=sen\left(arcsen\left(\frac{1}{x}\right)\right)\Leftrightarrow sen\left(\frac{y}{2}-\frac{\pi}{6}\right)=\frac{1}{x}\Leftrightarrow x=\frac{1}{sen\left(\frac{y}{2}-\frac{\pi}{6}\right)}\Rightarrow$$

$$\Rightarrow D=\left\{x\in\Re : sen\left(\frac{y}{2}-\frac{\pi}{6}\right)\neq 0\right\}$$

- $sen\left(\frac{y}{2}-\frac{\pi}{6}\right)\neq 0\Leftrightarrow \frac{y}{2}-\frac{\pi}{6}\neq \underbrace{arcsen(0)}_{=0}\Leftrightarrow \frac{y}{2}-\frac{\pi}{6}\neq 0\Leftrightarrow \frac{y}{2}\neq\frac{\pi}{6}\Leftrightarrow y\neq\frac{2\pi}{6}\Leftrightarrow y\neq\frac{\pi}{3}\Rightarrow$

  $\Rightarrow D_{inversa}=\Re\setminus\left\{\frac{\pi}{3}\right\}$

  Como: $D_{inversa}=\Re\setminus\left\{\frac{\pi}{3}\right\}$, então a restrição ao contradomínio será: $D^{'}=\left]-\frac{2\pi}{3};\frac{4\pi}{3}\right[\setminus\left\{\frac{\pi}{3}\right\}$

c) Determine o conjunto solução de: $A=\left\{x\in\Re : g(x)\leq\frac{2\pi}{3}\right\}$**.**

R:

$$g(x)\leq\frac{2\pi}{3}\Leftrightarrow\frac{\pi}{3}+2\cdot arcsen\left(\frac{1}{x}\right)\leq\frac{2\pi}{3}\Leftrightarrow 2\cdot arcsen\left(\frac{1}{x}\right)\leq\frac{2\pi}{3}-\frac{\pi}{3}\Leftrightarrow arcsen\left(\frac{1}{x}\right)\leq\frac{\frac{\pi}{3}}{2}\Leftrightarrow$$

$$\Leftrightarrow arcsen\left(\frac{1}{x}\right)\leq\frac{\pi}{6}\Leftrightarrow sen\left(arcsen\left(\frac{1}{x}\right)\right)\leq\underbrace{sen\left(\frac{\pi}{6}\right)}_{=1/2}\Leftrightarrow\frac{1}{x}\leq\frac{1}{2}\Leftrightarrow\frac{1}{x}-\frac{1}{2}\leq 0\Leftrightarrow\frac{2-x}{2x}\leq 0$$

	$-\infty$	0		2	$+\infty$
$2-x$	+	+	+	0	-
$2x$	-	0	+	+	+
$\frac{2-x}{2x}$	-	s.s.	+	0	-

$$\frac{2-x}{2x}\leq 0\Rightarrow D=\left]-\infty;0\right[\cup\left[2;+\infty\right[$$

d) Caracterize a função inversa de g.

R:

$$g^{-1}: \left]-\frac{2\pi}{3};\frac{4\pi}{3}\right[\setminus \left\{\frac{\pi}{3}\right\} \to \left]-\infty;-1\right]\cup\left[1;+\infty\right[$$

$$x \to \frac{1}{sen\left(\frac{x}{2}-\frac{\pi}{6}\right)}$$

e) Escreva a equação da recta tangente de g, no ponto de abcissa -2.

R:

Sabendo que a equação geral da recta é dada por: $y = m\cdot x + b$, onde: $m = g^{'}(x)$

E que um ponto é definido por: $P(x;y)$, onde: $y = g(x) \Rightarrow P[x;g(x)]$

Sabendo ainda que: $Abcissa = -2 \Rightarrow x = -2$, então teremos que:

$P[x;g(x)] \Leftrightarrow P[-2;g(-2)]$ e $m = g^{'}(x) \Leftrightarrow m = g^{'}(-2)$

- $$g(x) = \frac{\pi}{3} + 2\cdot arcsen\left(\frac{1}{x}\right) \Rightarrow g(-2) = \frac{\pi}{3} + 2\cdot \underbrace{arcsen\left(\frac{1}{-2}\right)}_{=-arcsen(1/2)=-\pi/6} \Leftrightarrow g(-2) = \frac{\pi}{3} + 2\cdot\left(-\frac{\pi}{6}\right) \Leftrightarrow$$

$$\Leftrightarrow g(-2) = 0$$

Daqui resulta então que: $P[x;g(x)] \Leftrightarrow P(-2;0)$

- $$g^{'}(x) = \left(\frac{\pi}{3} + 2\cdot \underbrace{arcsen\left(\frac{1}{x}\right)}_{(arcsen(u))'=\frac{u'}{\sqrt{1-u^2}}}\right)^{'} = 0 + 2.\frac{\left(\frac{1}{x}\right)^{'}}{\sqrt{1-\left(\frac{1}{x}\right)^2}} = 2\cdot\frac{\frac{\overbrace{(1)^{'}}^{=0}\cdot(x) - 1\cdot\overbrace{(x)^{'}}^{=1}}{(x)^2}}{\sqrt{1-\left(\frac{1}{x}\right)^2}} = 2\cdot\frac{-\frac{1}{x^2}}{\sqrt{1-\left(\frac{1}{x}\right)^2}}$$

$$g'(x)=2\cdot\frac{-\frac{1}{x^2}}{\sqrt{1-\left(\frac{1}{x}\right)^2}}\Rightarrow g'(-2)=2\cdot\frac{-\frac{1}{(-2)^2}}{\sqrt{1-\left(\frac{1}{-2}\right)^2}}\Leftrightarrow g'(-2)=2\cdot\frac{-\frac{1}{4}}{\sqrt{1-\frac{1}{4}}}\Leftrightarrow g'(-2)=-\frac{2}{4\cdot\sqrt{\frac{3}{4}}}\Leftrightarrow$$

$$\Leftrightarrow g'(-2)=-\frac{1}{2\cdot\sqrt{\frac{3}{4}}}$$

Daqui resulta que: $m=g'(-2)\Leftrightarrow m=-\dfrac{1}{2\cdot\sqrt{\frac{3}{4}}}$

Então finalmente teremos que:

$$\left\{\begin{array}{l} y=m\cdot x+b \\ (x;y)=(-2;0) \\ m=-\dfrac{1}{2\cdot\sqrt{\frac{3}{4}}} \end{array}\right\}\Rightarrow 0=-\frac{1}{2\cdot\sqrt{\frac{3}{4}}}\cdot(-2)+b\Leftrightarrow b=-\frac{1}{\sqrt{\frac{3}{4}}}\Rightarrow y=-\frac{1}{2\cdot\sqrt{\frac{3}{4}}}\cdot x-\frac{1}{\sqrt{\frac{3}{4}}}$$