Chapter 12

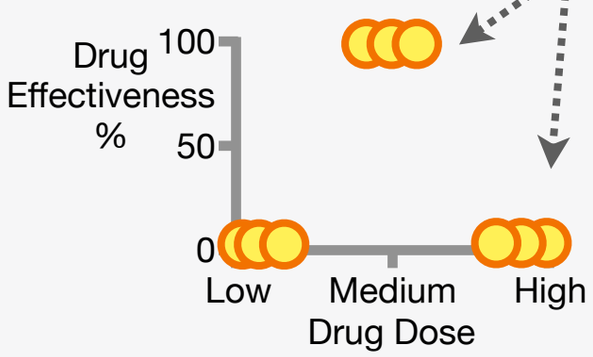
Neural Networks!!!

Neural Networks
Part One:

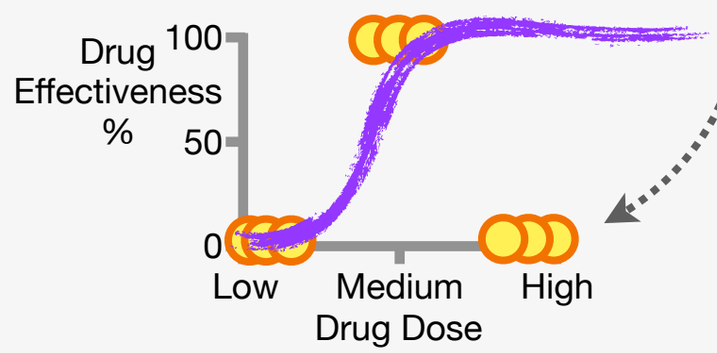
Understanding How Neural Networks Work

Neural Networks: Main Ideas

1 **The Problem:** We have data that show the Effectiveness of a drug for different Doses….



…and the **s-shaped squiggle** that **Logistic Regression** uses would not make a good fit. In this example, it would misclassify the high Doses.

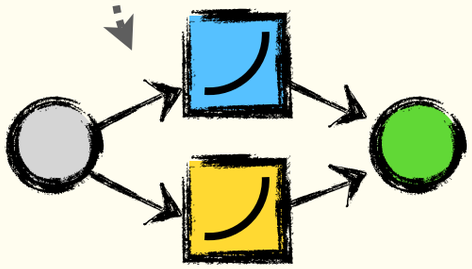


Hey! Can't we solve this problem with a **Decision Tree** or a **Support Vector Machine**?

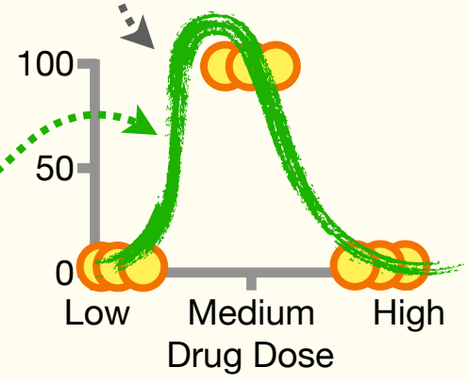
2 **A Solution:** Although they sound super intimidating, all **Neural Networks** do is fit **fancy squiggles** or **bent shapes** to data. And like **Decision Trees** and **SVMs**, **Neural Networks** do fine with any relationship among the variables.

Yes!!! But they will probably not all perform the same, so it's a good idea to try each one to see which works best.

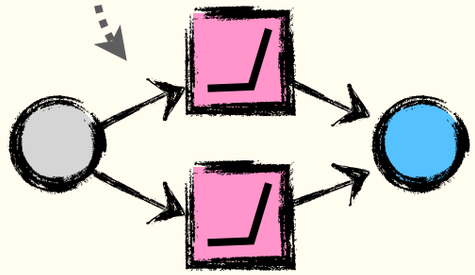
For example, we could get this **Neural Network**…



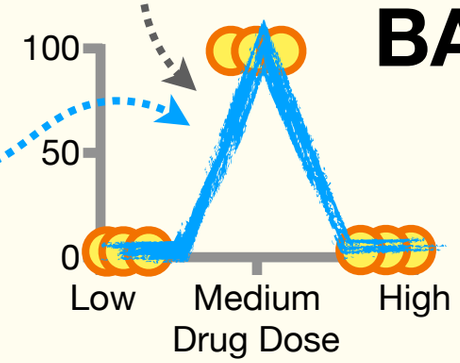
…to fit a **fancy squiggle** to the data…



…or we could get this **Neural Network**…



…to fit a **bent shape** to the data.

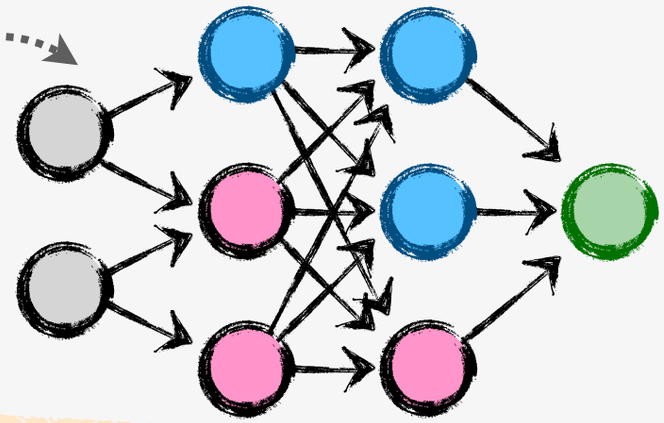


BAM!!!

Terminology Alert!!! Anatomy of a Neural Network

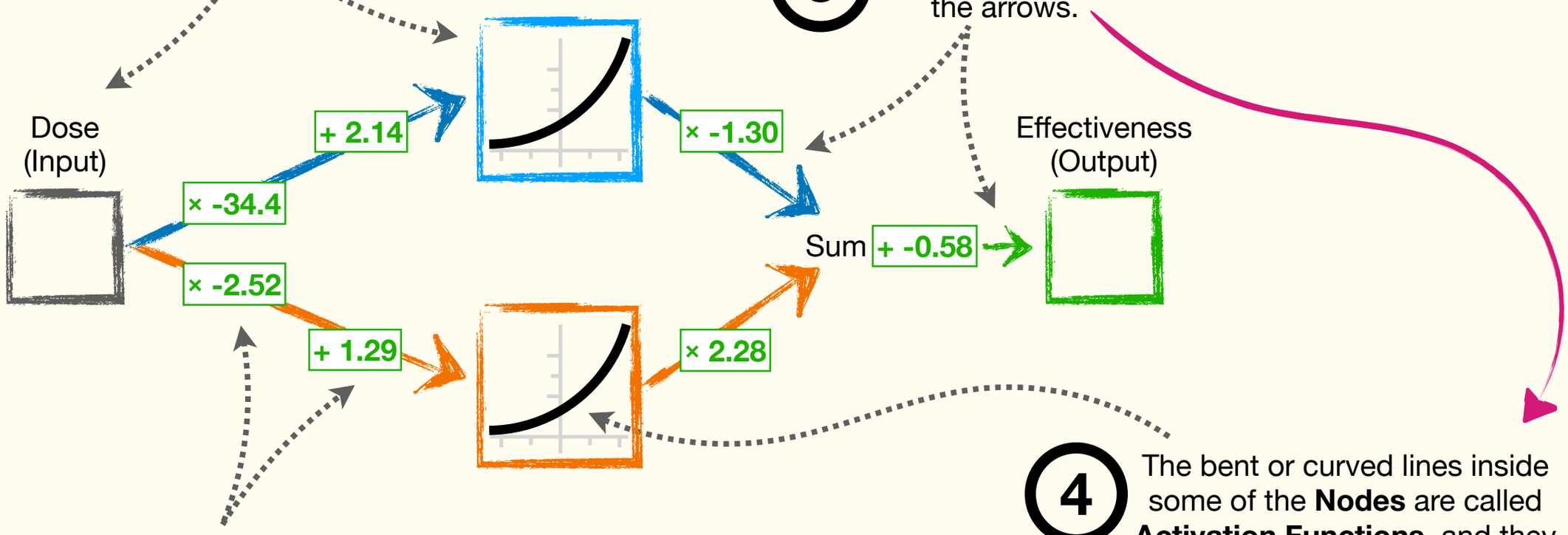
(Oh no! It's the dreaded **Terminology Alert!!!**)

1 Although **Neural Networks** usually look like super complicated groups of neurons connected by synapses, which is where the name **Neural Network** comes from, they're all made from the same simple parts.



2 **Neural Networks** consist of **Nodes**, the square boxes…

3 …and connections between **Nodes**, the arrows.



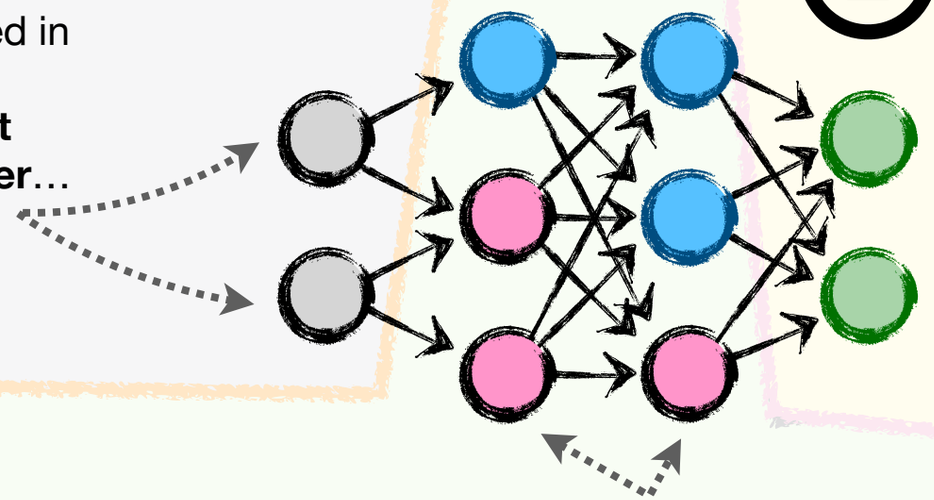
4 The bent or curved lines inside some of the **Nodes** are called **Activation Functions**, and they make **Neural Networks** flexible and able to fit just about any data.

5 The numbers along the connections represent parameter values that were estimated when this **Neural Network** was fit to data using a process called **Backpropagation**. In this chapter, we'll see exactly what the parameters do and how they're estimated, step-by-step.

Terminology Alert!!! Layers

(Oh no! *A Double* **Terminology Alert!!!**)

1 **Neural Network**s are organized in **Layers**. Usually, a **Neural Network** has multiple **Input Nodes** that form an **Input Layer**…



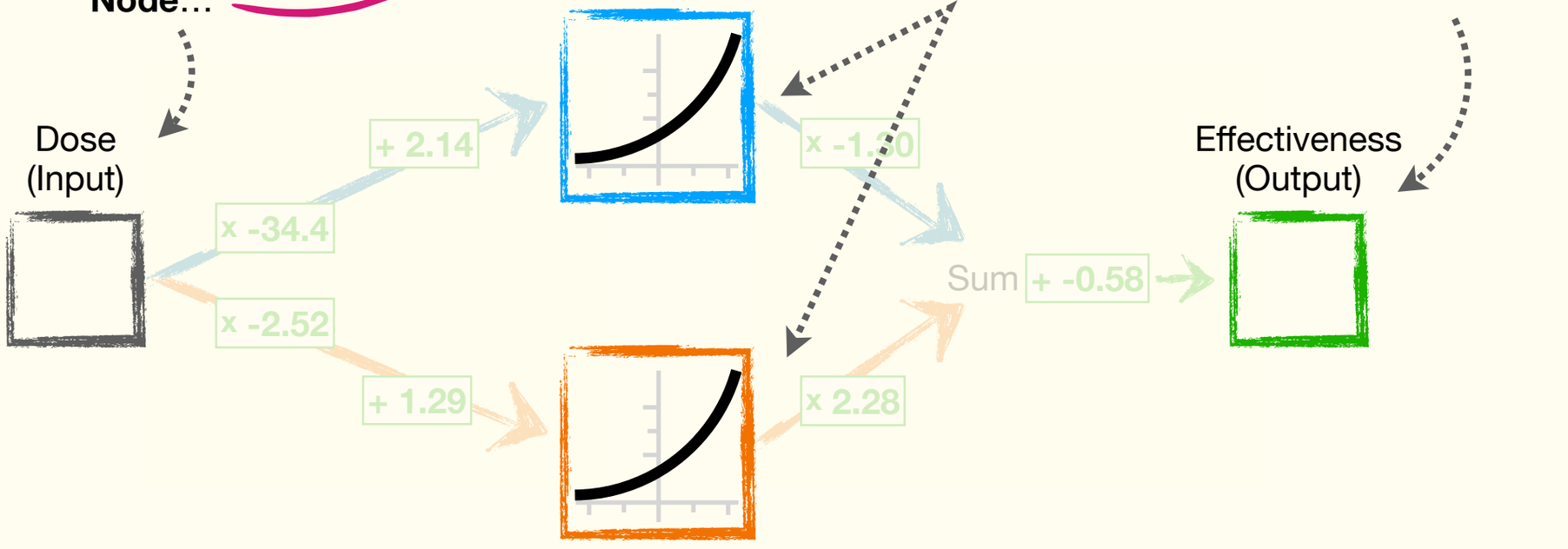
2 …and usually there are multiple **Output Nodes** that form an **Output Layer**…

3 …and the **Layers** of **Nodes** between the **Input** and **Output Layers** are called **Hidden Layers**. Part of the *art* of **Neural Networks** is deciding how many **Hidden Layers** to use and how many **Nodes** should be in each one. Generally speaking, the more **Layers** and **Nodes**, the more complicated the shape that can be fit to the data.

4 In the example we'll use, we have a single **Input Node**…

…a single **Hidden Layer** with **2 Nodes** in it…

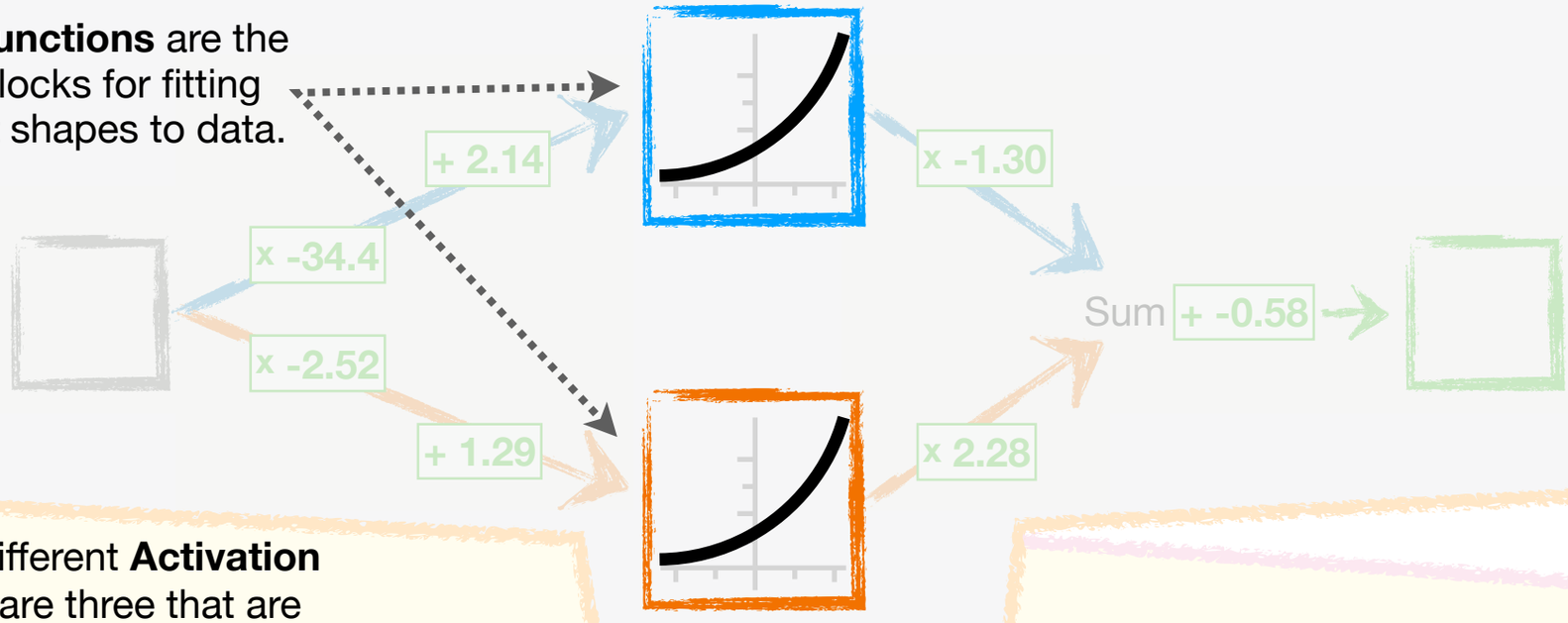
…and a single **Output Node**.



Terminology Alert!!! Activation Functions

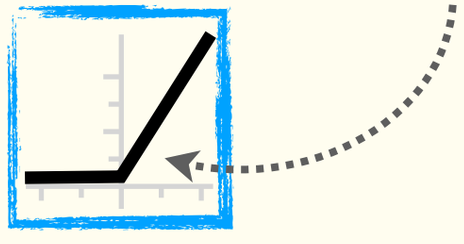
(Oh no! *A TRIPLE* **TERMINOLOGY ALERT!!!**)

1 The **Activation Functions** are the basic building blocks for fitting squiggles or bent shapes to data.

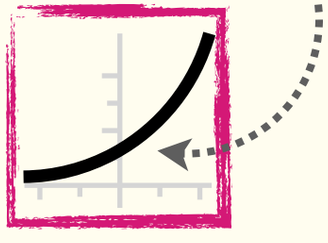


2 There are lots of different **Activation Functions**. Here are three that are commonly used:

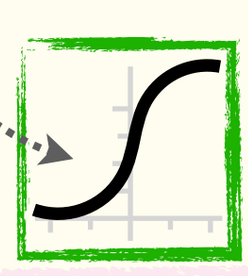
ReLU, which is short for **Rectified Linear Unit** and sounds like the name of a robot, is probably the most commonly used **Activation Function** with large **Neural Networks**. It's a **Bent Line**, and the bend is at $x = 0$.



SoftPlus, which sounds like a brand of toilet paper, is a modified form of the **ReLU Activation Function**. The big difference is that instead of the line being bent at **0**, we get a nice **Curve**.



Lastly, the **Sigmoid Activation Function** is an **s-shaped squiggle** that's frequently used when people teach **Neural Networks** but is rarely used in practice.



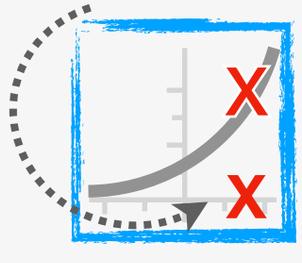
3 Although they might sound fancy, **Activation Functions** are just like the mathematical functions you learned when you were a teenager: you plug in an x-axis coordinate, do the math, and the output is a y-axis coordinate.

For example, the **SoftPlus** function is:

$$\textbf{SoftPlus}(x) = \textbf{log}(1 + e^{x})$$

…where **the log()** function is the natural log, or **log base *e***, and ***e*** is **Euler's Number**, which is roughly **2.72**.

So, if we plug in an x-axis value, $x = 2.14$…

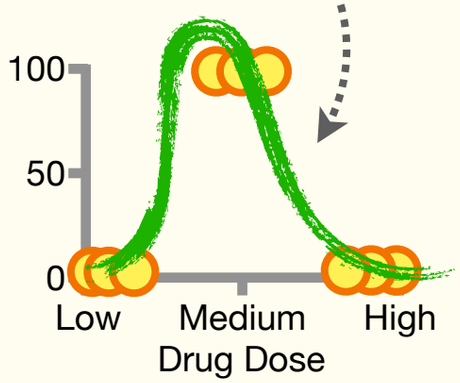


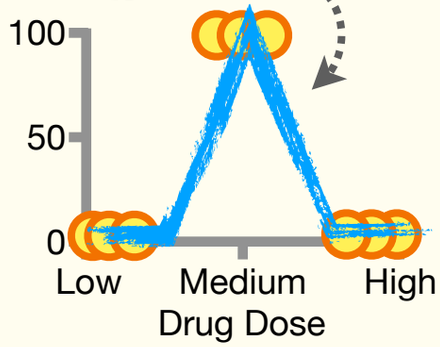
…then the **SoftPlus** will tell us the y-axis coordinate is **2.25**, because $\textbf{log}(1 + e^{2.14}) = \textbf{2.25}$.

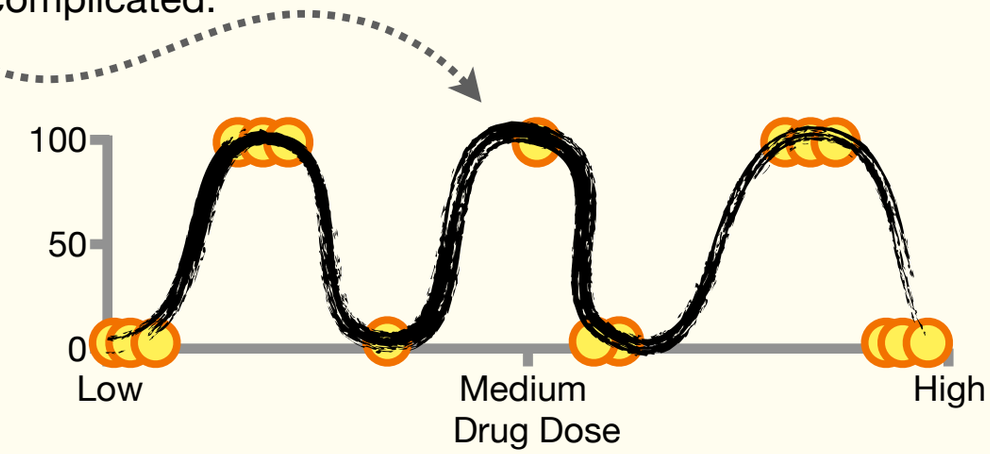
Now let's talk about the main ideas of how **Activations Functions** work.

Activation Functions: Main Ideas

1 **The Problem:** We need to create new, exciting shapes that can fit any dataset with a **squiggle** or **bent lines**, even when the dataset is super complicated.





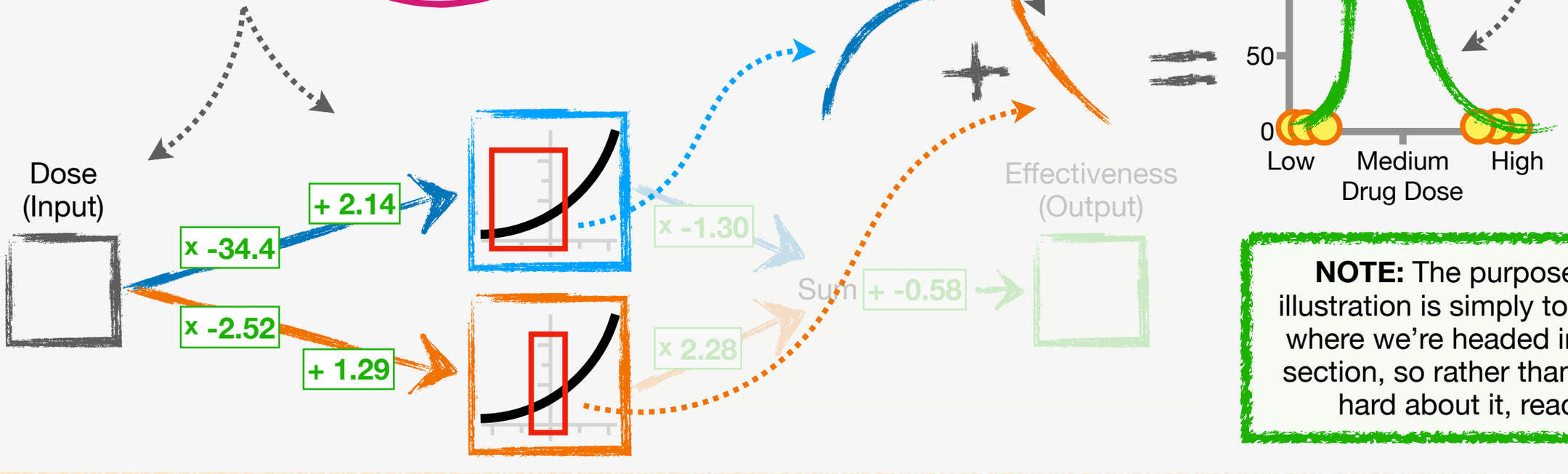


2 **The Solution: Neural Networks** stretch, rotate, crop, and combine **Activation Functions** to create new, exciting shapes that can fit anything!!!

In the next section, **A Neural Network in Action: Step-by-Step**, we'll show you how this **Neural Network**…

…stretches, rotates, crops, and combines **SoftPlus Activation Functions**…

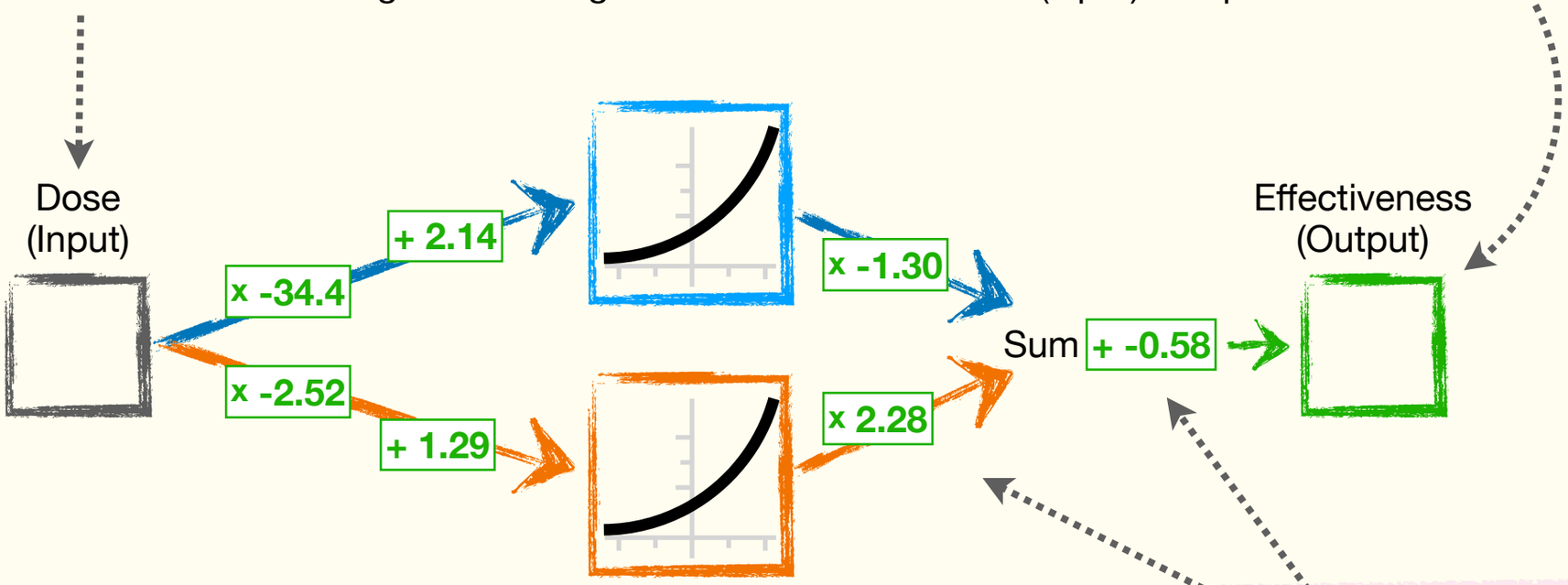
…to create this **squiggle** that fits the **Training Data**.



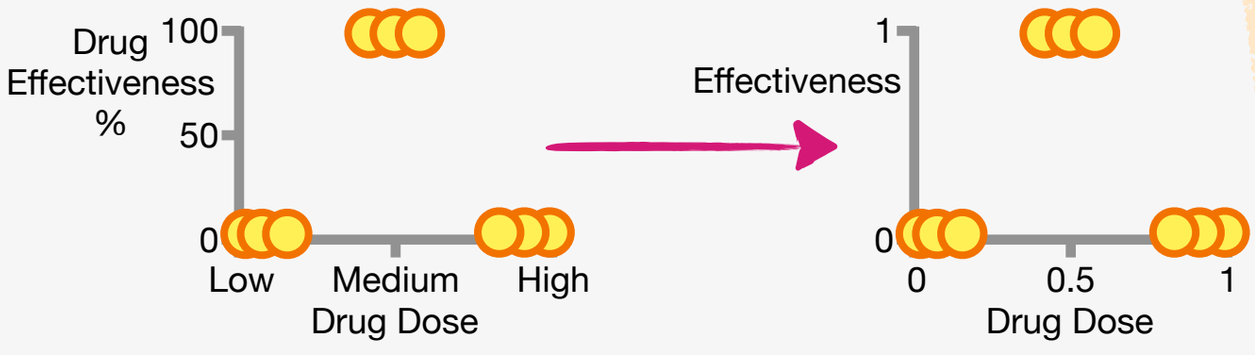
NOTE: The purpose of this illustration is simply to show you where we're headed in the next section, so rather than think too hard about it, read on!!!

A Neural Network in Action: Step-by-Step

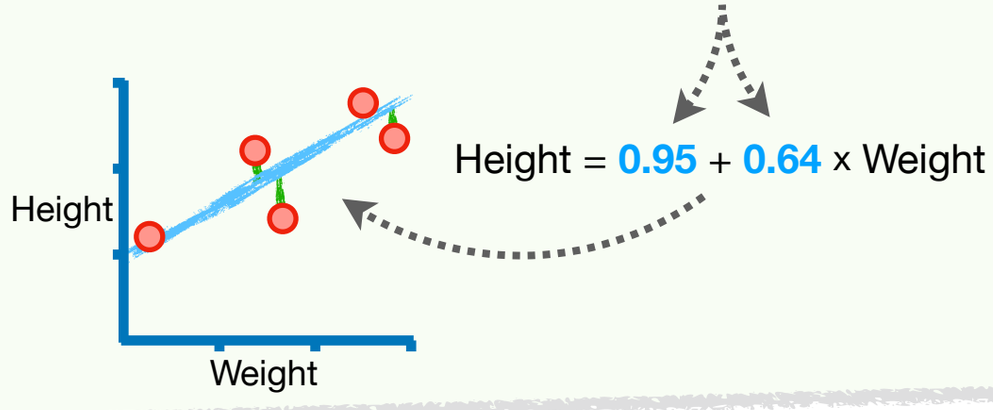
1 A lot of people say that **Neural Networks** are black boxes and that it's difficult to understand what they're doing. Unfortunately, this is true for big, fancy **Neural Networks**. However, the good news is that it's *not* true for simple ones. So, let's walk through how this simple **Neural Network** works, one step at a time, by plugging in Dose values from low to high and seeing how it converts the Dose (input) into predicted Effectiveness (output).



2 **NOTE:** To keep the math in this section simple, let's scale both the x- and y-axes so that they go from **0**, for *low* values, to **1**, for *high* values.

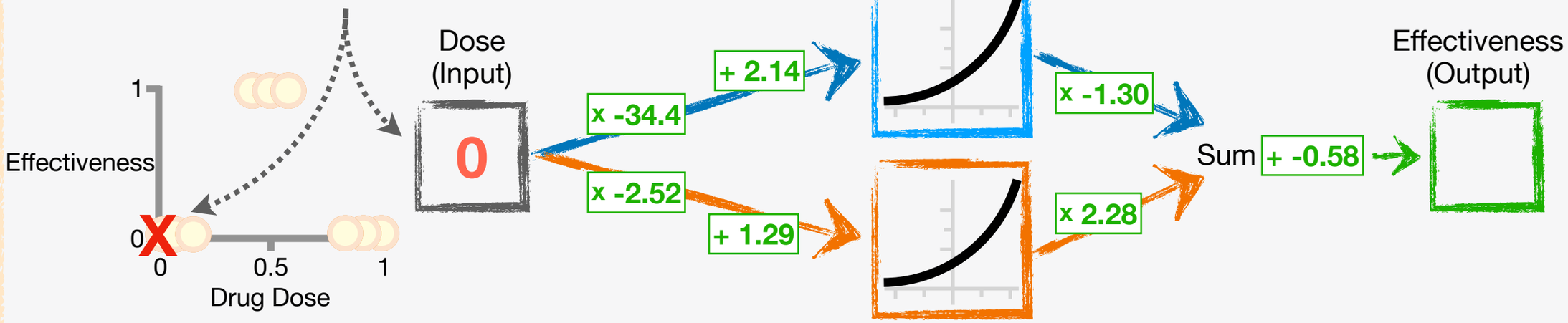


3 **ALSO NOTE:** These numbers are *parameters* that are estimated using a method called **Backpropagation**, and we'll talk about how that works, step-by-step, later. For now, just assume that these values have already been optimized, much like the **slope** and **intercept** are optimized when we fit a line to data.



A Neural Network in Action: Step-by-Step

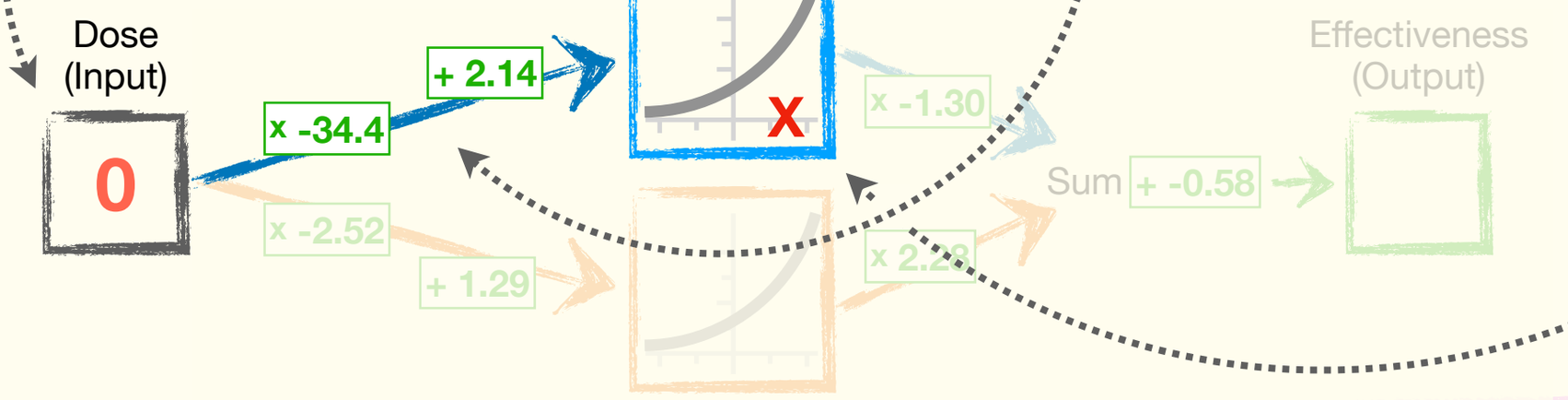
4 The first thing we do is plug the lowest Dose, **0**, into the **Neural Network**.



5 Now, the connection from the **Input Node**… …to the *top* **Node** in the **Hidden Layer**… …multiplies the Dose by **-34.4** and then adds **2.14** … …to get a new x-axis coordinate for the **Activation Function**, **2.14**.

(**Dose** x -34.4) + 2.14 = x-axis coordinate

(**0** x -34.4) + 2.14 = x-axis coordinate = 2.14



A Neural Network in Action: Step-by-Step

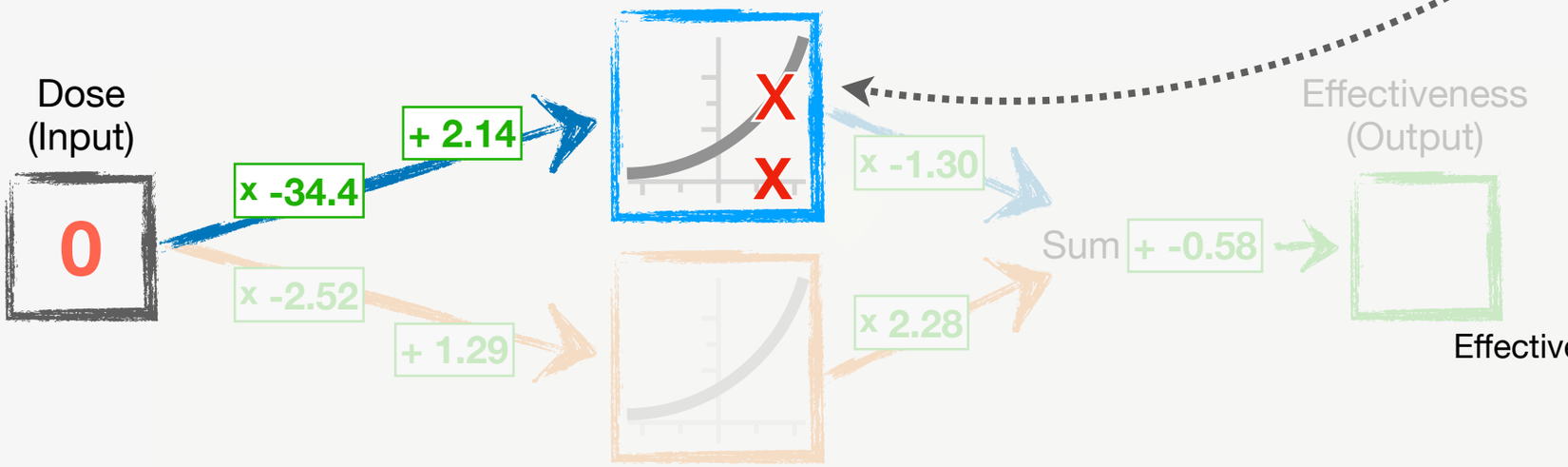
6 Then we plug the x-axis coordinate, **2.14** into the **SoftPlus Activation Function**…

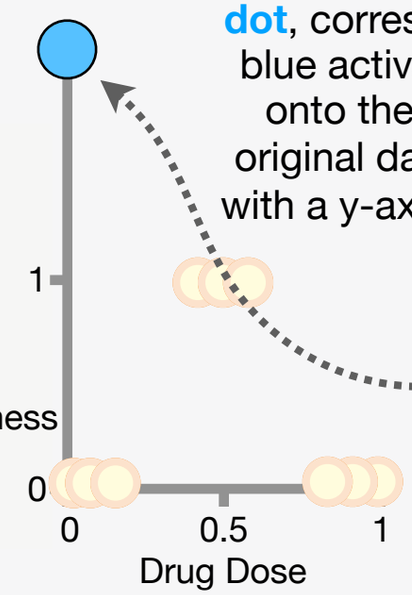
…and do the math, just like we saw earlier…

…and we get **2.25**. So, when the x-axis coordinate is **2.14**, the corresponding y-axis value is **2.25**…

…thus, when Dose **= 0**, the output for the **SoftPlus** is **2.25**. So, let's put a **blue dot**, corresponding to the blue activation function, onto the graph of the original data at Dose **= 0** with a y-axis value = **2.25**.

SoftPlus(x) = **SoftPlus**(2.14) = **log**(1 + $e^{2.14}$) = y-axis coordinate = **2.25**





7 Now let's increase the Dose to **0.2**…

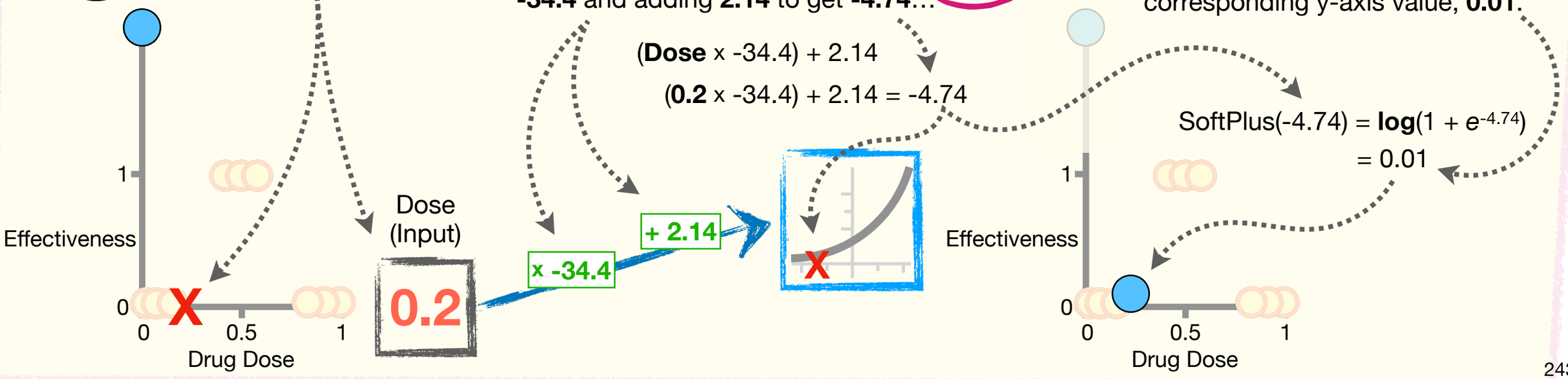
…and calculate the new x-axis coordinate by multiplying the Dose by **-34.4** and adding **2.14** to get **-4.74**…

(**Dose** x -34.4) + 2.14

(**0.2** x -34.4) + 2.14 = -4.74

…and plug **-4.74** into the **SoftPlus Activation Function** to get the corresponding y-axis value, **0.01**.

SoftPlus(-4.74) = **log**(1 + $e^{-4.74}$) = 0.01



A Neural Network in Action: Step-by-Step

8 Now let's increase the Dose to **0.4**…

…and calculate the new x-axis coordinate by multiplying the Dose by **-34.4** and adding **2.14** to get **-11.6**…

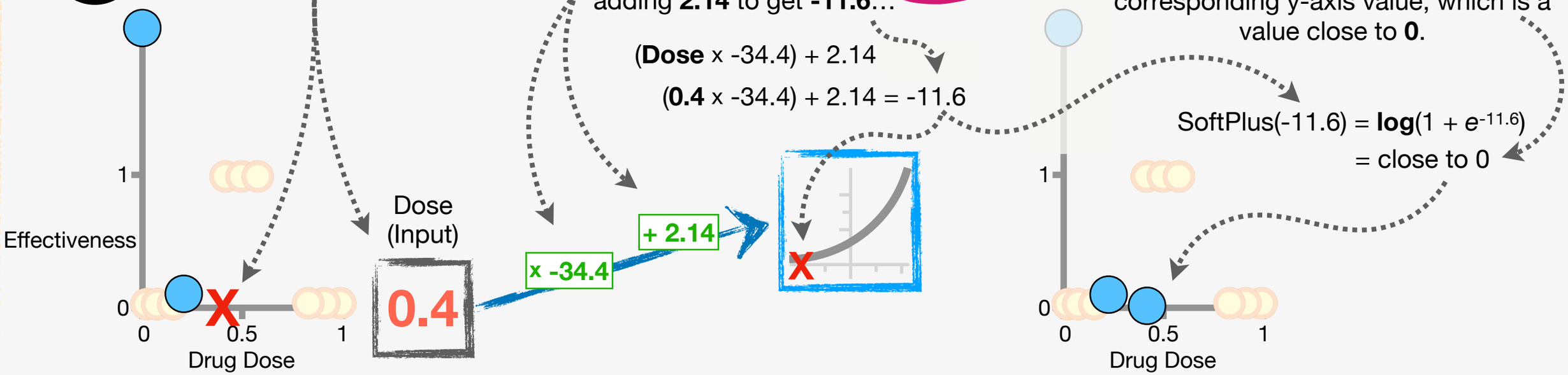
(**Dose** x -34.4) + 2.14

(**0.4** x -34.4) + 2.14 = -11.6

…and plug **-11.6** into the **SoftPlus Activation Function** to get the corresponding y-axis value, which is a value close to **0**.

SoftPlus(-11.6) = **log**(1 + $e^{-11.6}$)

= close to 0



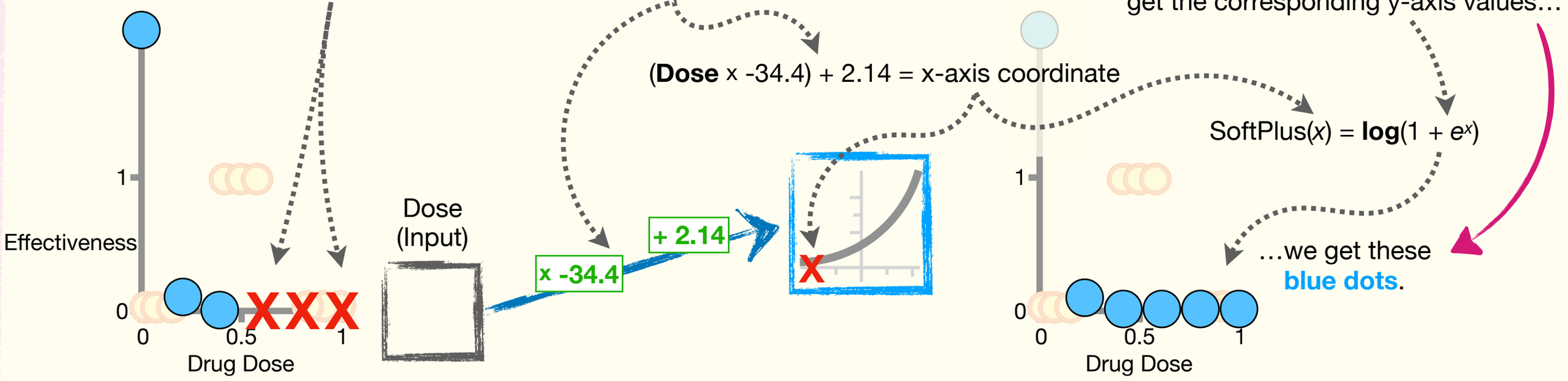
9 And if we continue to increase the Dose all the way to **1** (the maximum Dose)…

…and calculate the new x-axis coordinates by multiplying the Dose by **-34.4** and adding **2.14**…

…and plug the x-axis coordinates into the **SoftPlus Activation Function** to get the corresponding y-axis values…

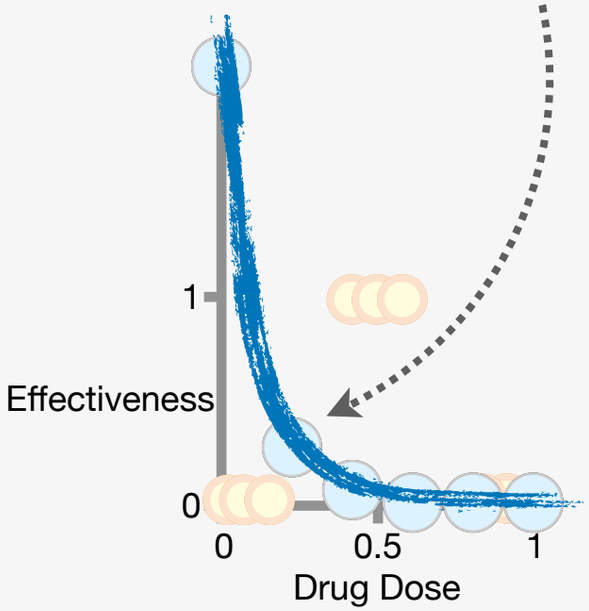
(**Dose** x -34.4) + 2.14 = x-axis coordinate

SoftPlus(x) = **log**(1 + e^{x})

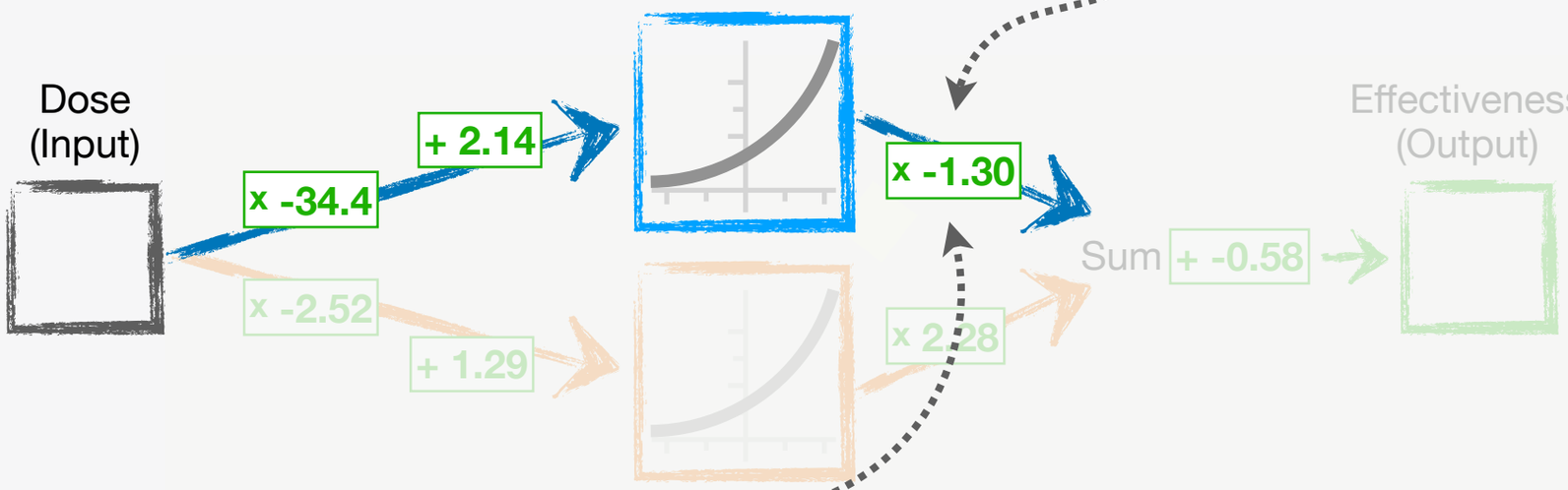


A Neural Network in Action: Step-by-Step

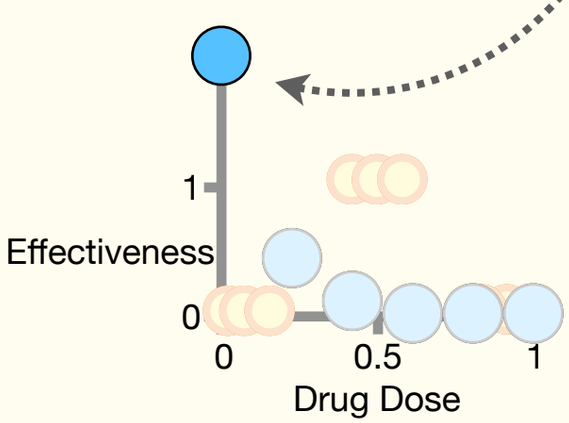
10 Ultimately, the **blue dots** result in this **blue curve**…



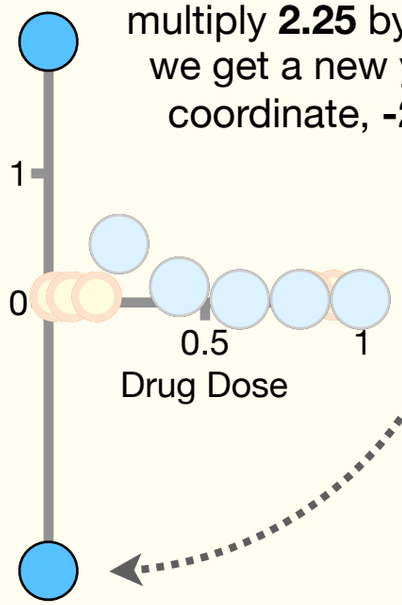
…and the next step in the **Neural Network** tells us to multiply the y-axis coordinates on the **blue curve** by **-1.30**.



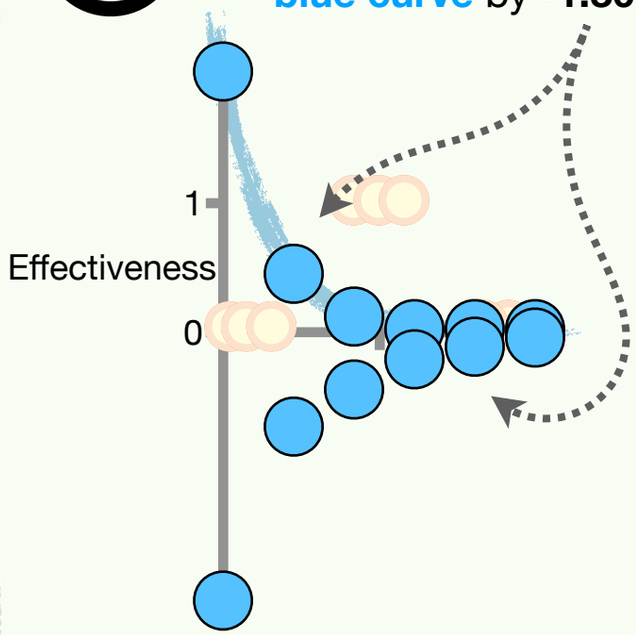
11 For example, when Dose = **0**, the current y-axis coordinate on the **blue curve** is **2.25**…



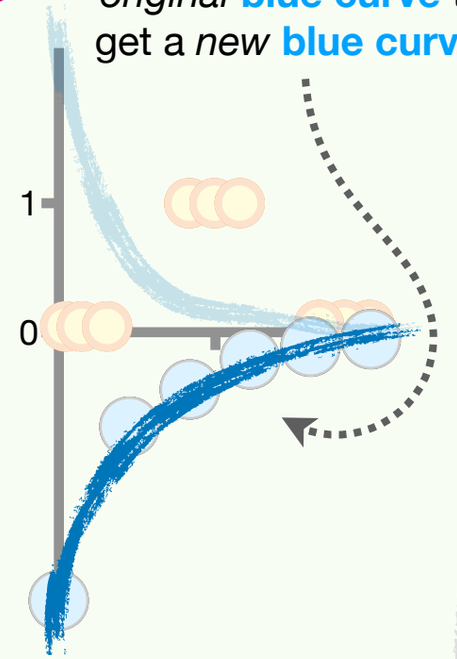
…and when we multiply **2.25** by **-1.30**, we get a new y-axis coordinate, **-2.93**.



12 Likewise, when we multiply all of the y-axis coordinates on the **blue curve** by **-1.30**…



…we end up flipping and stretching the *original* **blue curve** to get a *new* **blue curve**.



A Neural Network in Action: Step-by-Step

13 Okay, we just finished a major step, so this is a good time to review what we've done so far.

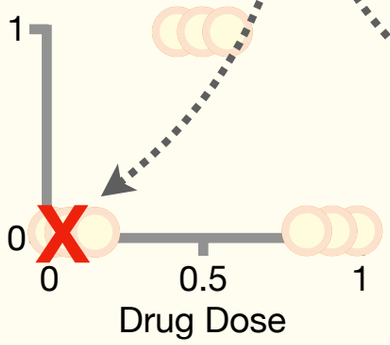
First, we plugged Dose values, ranging from **0** to **1**, into the input…

…then we transformed the Doses by multiplying them by **-34.4** and adding **2.14**…

…then the **SoftPlus Activation Function** converted the transformed Doses into y-axis coordinates…

…and lastly, we transformed the y-axis coordinates by multiplying them by **-1.30**.

BAM!!!



Dose (Input)

x -34.4

+ 2.14

x -2.52

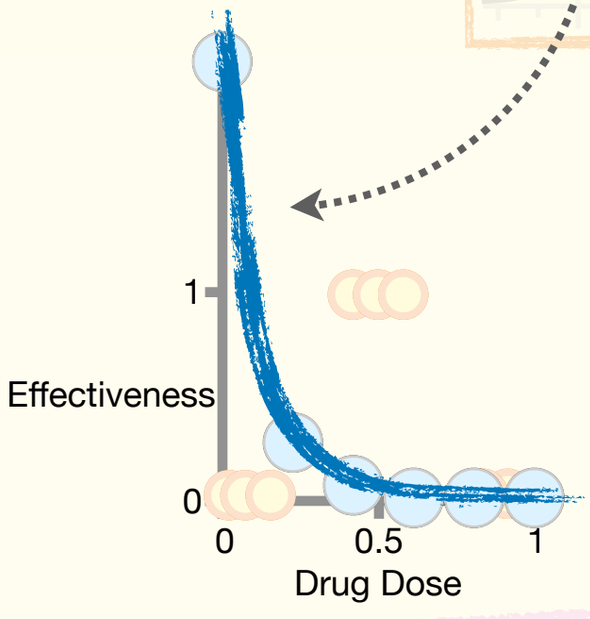
+ 1.29

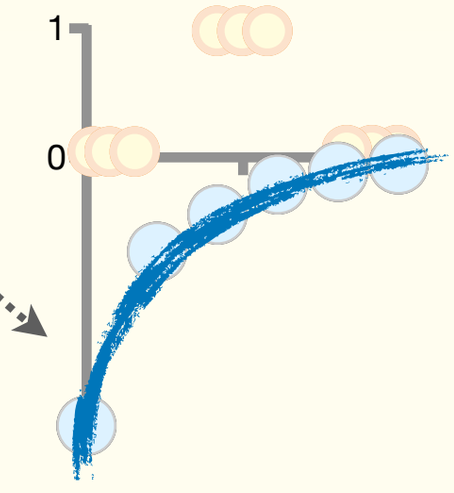
x -1.30

x 2.28

Sum + -0.58

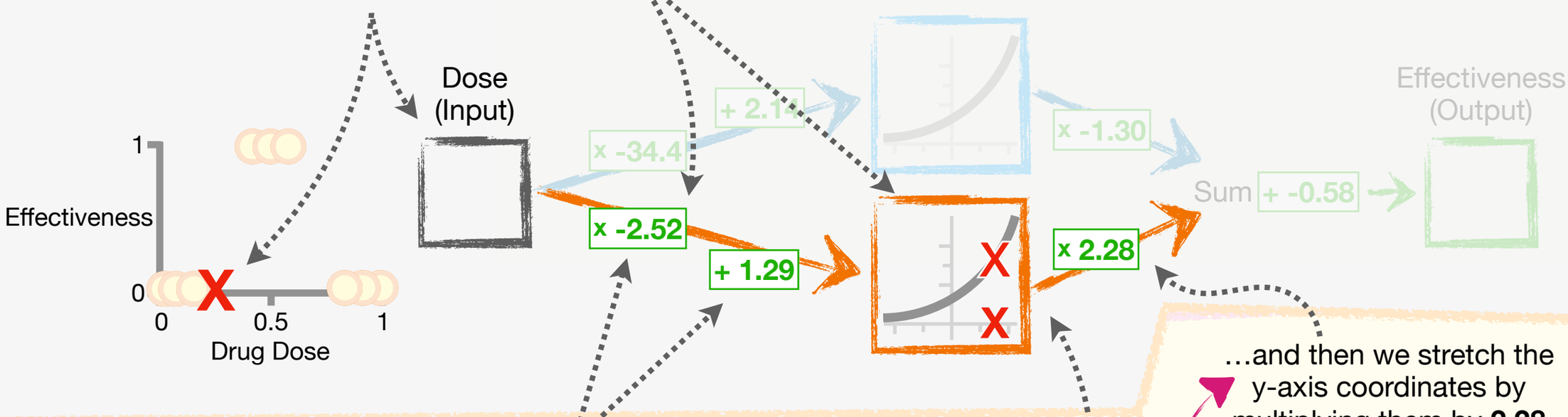
Effectiveness (Output)



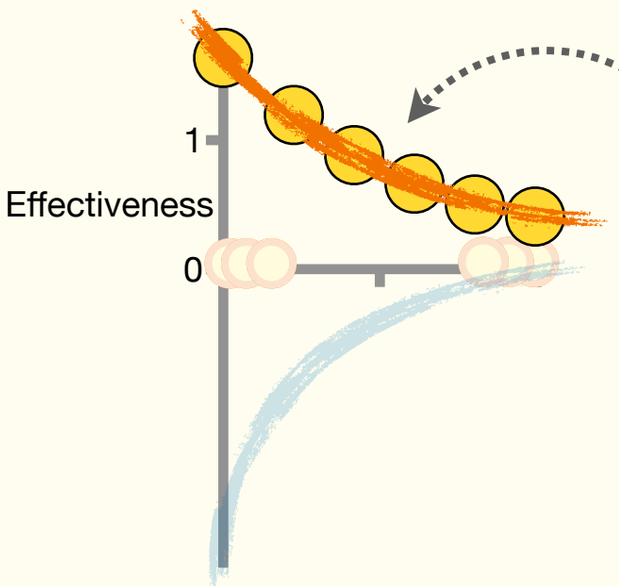


A Neural Network in Action: Step-by-Step

14 Now we run Dose values through the connection to the *bottom* **Node** in the **Hidden Layer**.

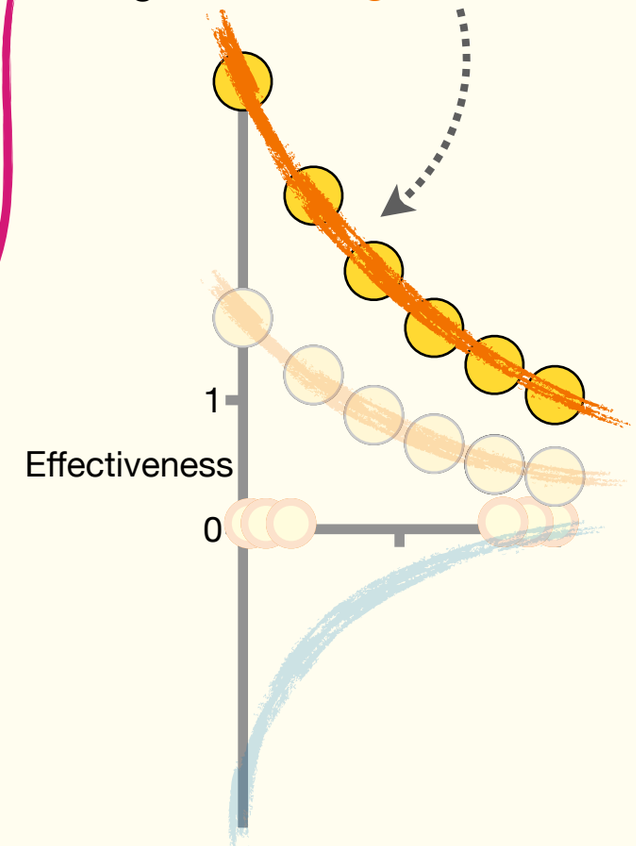


15 The good news is that the only difference between what we just did and what we're doing now is that now we multiply the Doses by **-2.52** and add **1.29**…



…before using the **SoftPlus Activation Function** to convert the transformed Doses into y-axis coordinates…

…and then we stretch the y-axis coordinates by multiplying them by **2.28** to get this **orange curve**.

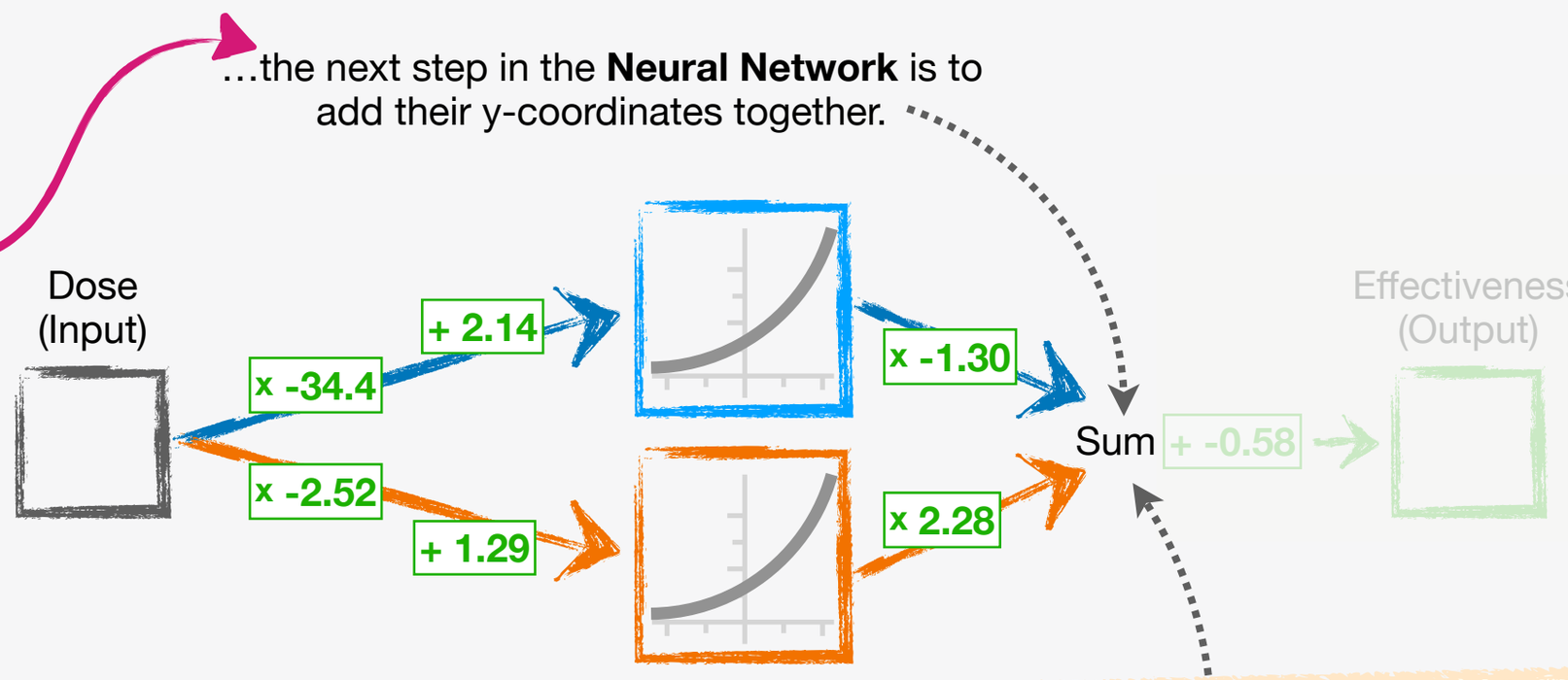


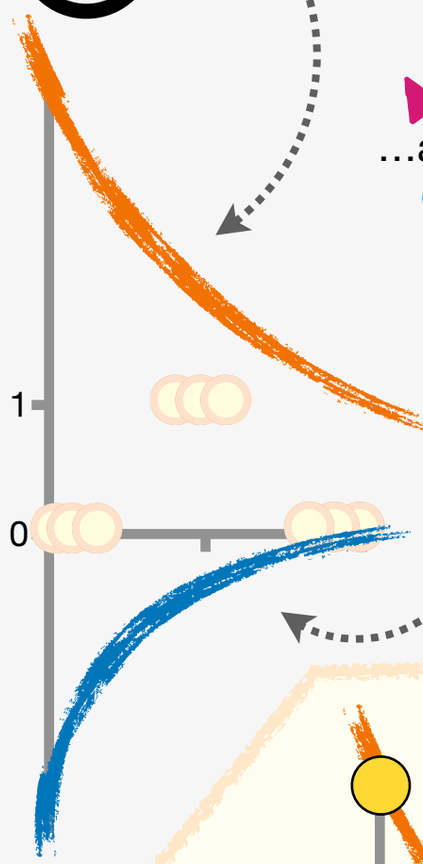
A Neural Network in Action: Step-by-Step

16 So, now that we have an **orange curve**…

…and a **blue curve**…

…the next step in the **Neural Network** is to add their y-coordinates together.

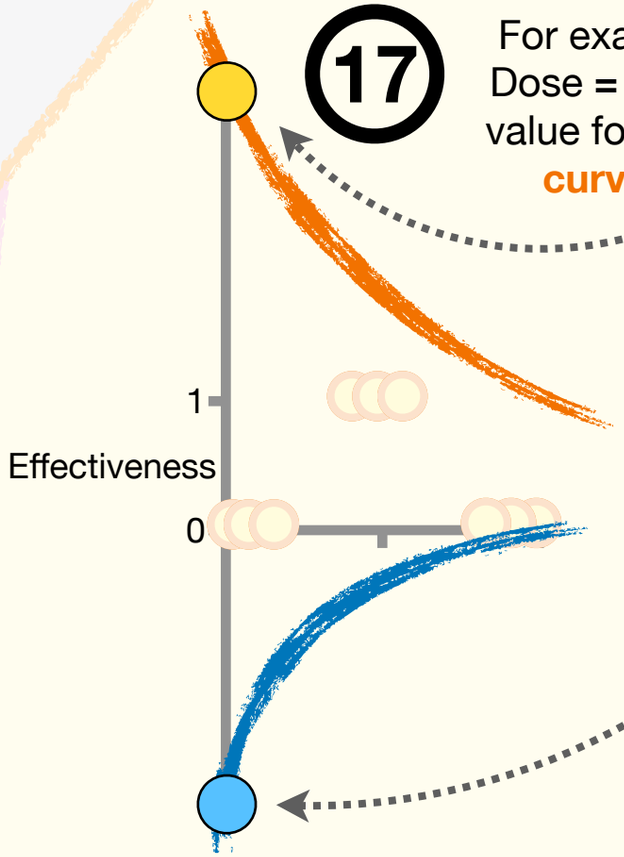


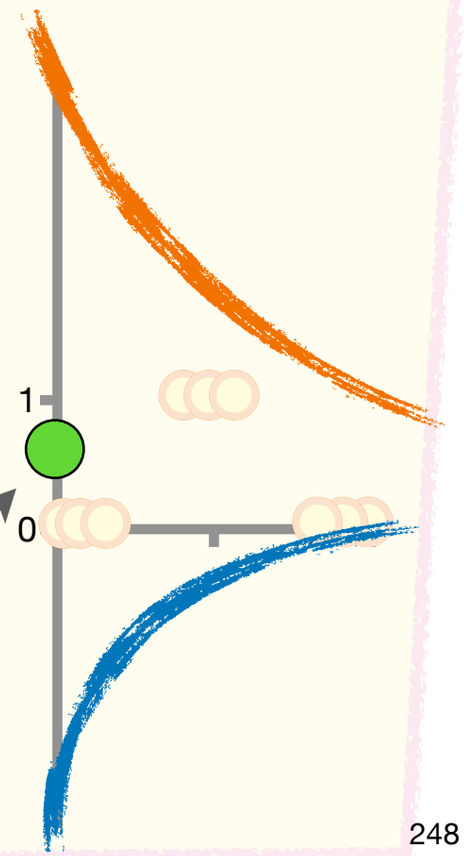


17 For example, when Dose = **0**, the y-axis value for the **orange curve** is **3.5**…

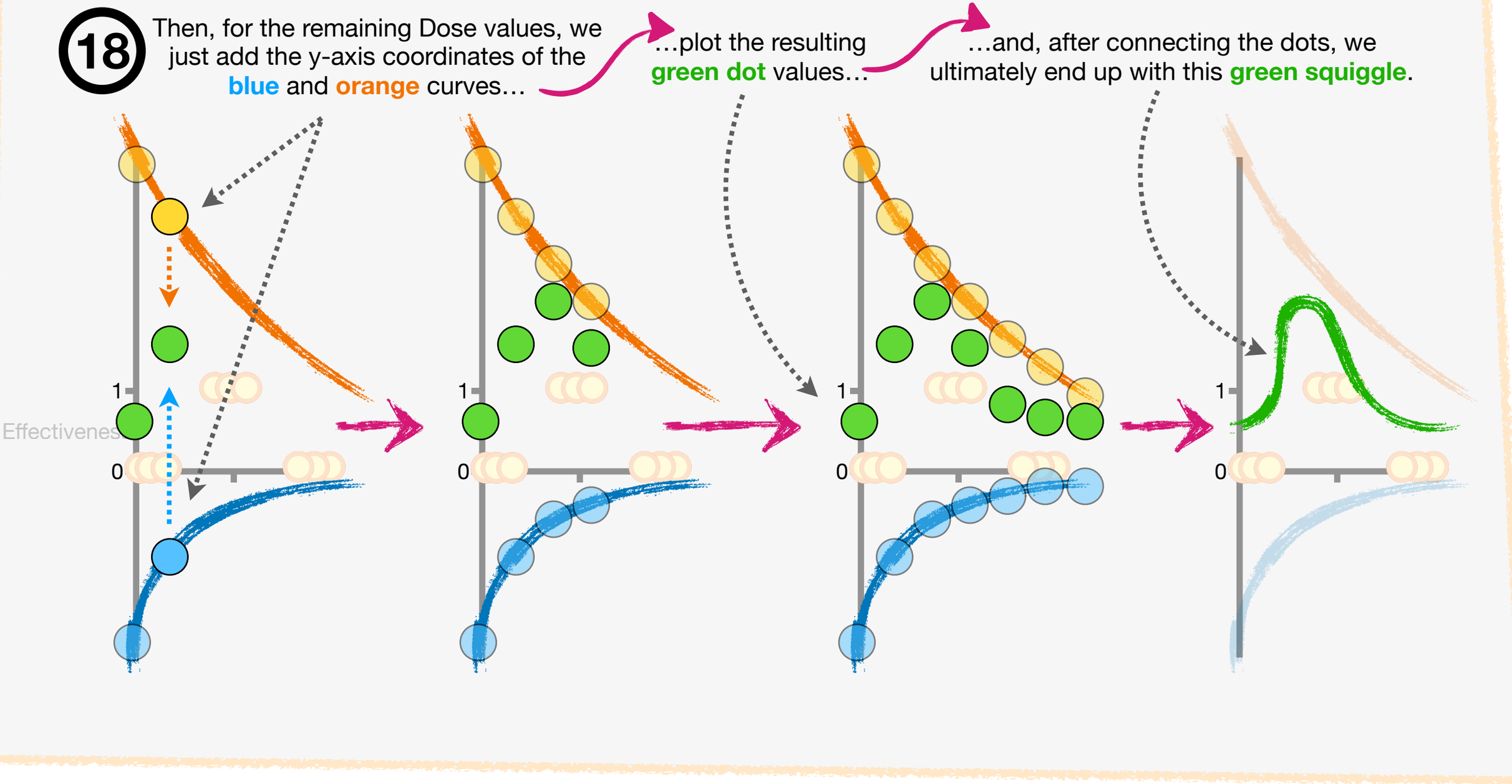
…and the y-axis value for **blue curve** is **-2.9**…

…thus, when Dose = **0**, the new y-axis value is the sum of the **orange** and **blue** y-axis values, **3.5 + -2.9 = 0.6**. We'll keep track of that **0.6** value by putting a **green dot** here.



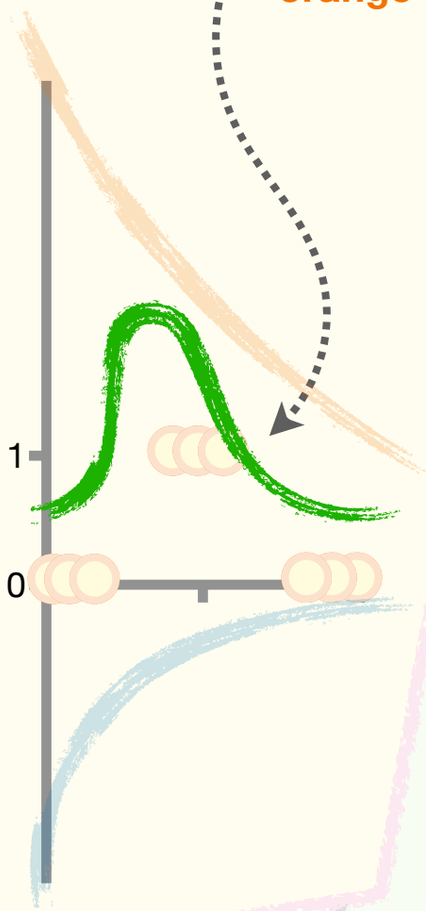


A Neural Network in Action: Step-by-Step

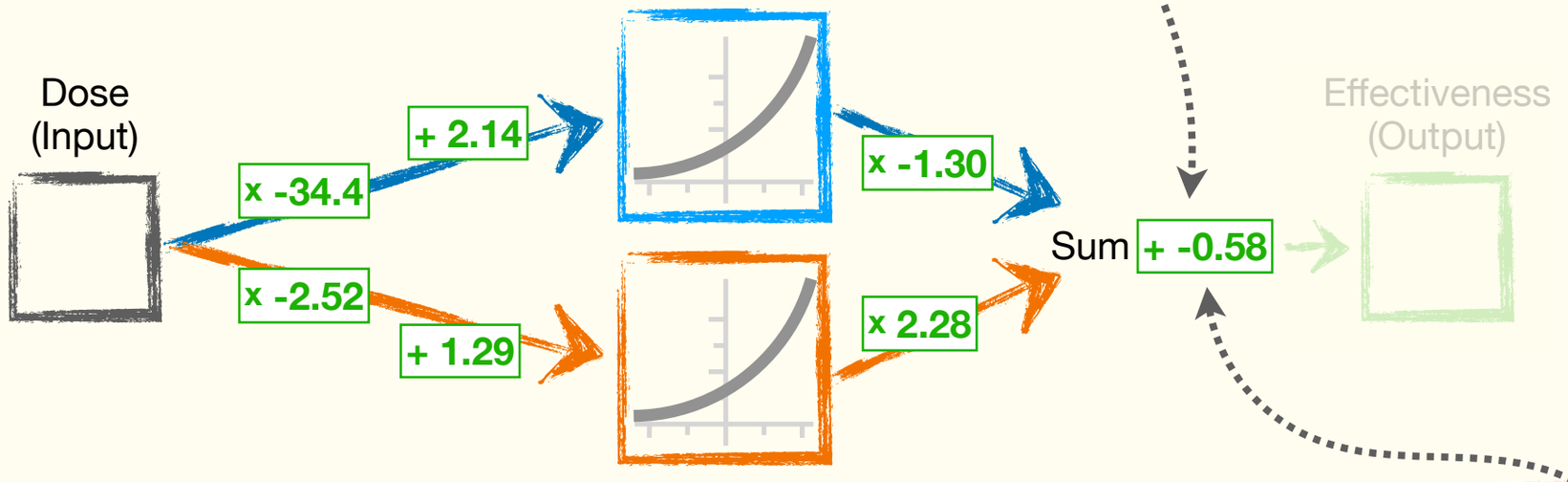


A Neural Network in Action: Step-by-Step

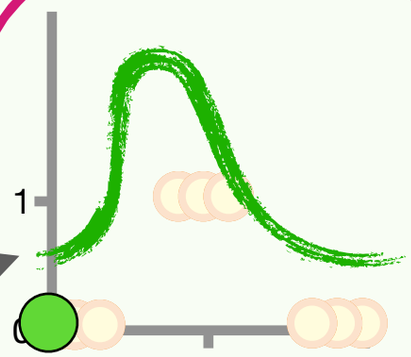
19 Now that we have this **green squiggle** from the **blue** and **orange** curves…



…we're ready for the final step, which is to add **-0.58** to each y-axis value on the **green squiggle**.

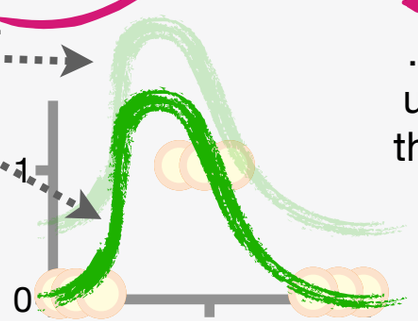


20 For example, when Dose **= 0**, the y-axis coordinate on the **green squiggle** starts out at **0.6**…

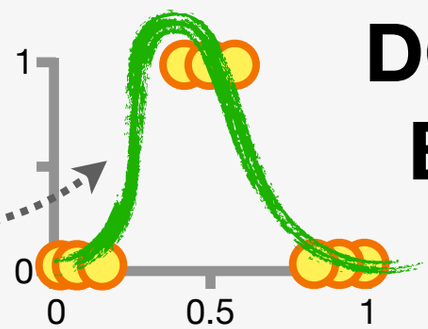


…but after subtracting **0.58**, the new y-axis coordinate for when Dose **= 0** is **0.0** (rounded to the nearest **tenth**).

21 Likewise, subtracting **0.58** from all of the other y-axis coordinates on the **green squiggle** shifts it down…



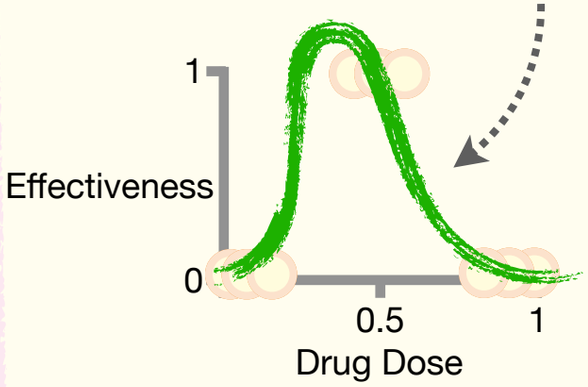
…and, ultimately, we end up with a **green squiggle** that fits the **Training Data**.



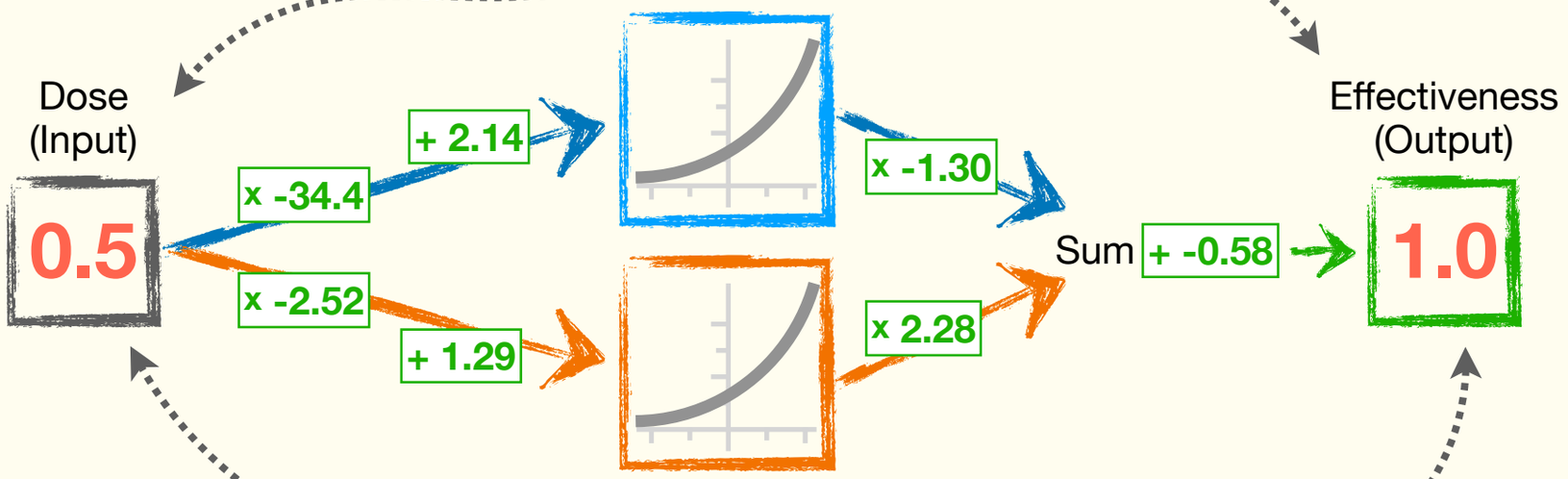
DOUBLE BAM!!!

A Neural Network in Action: Step-by-Step

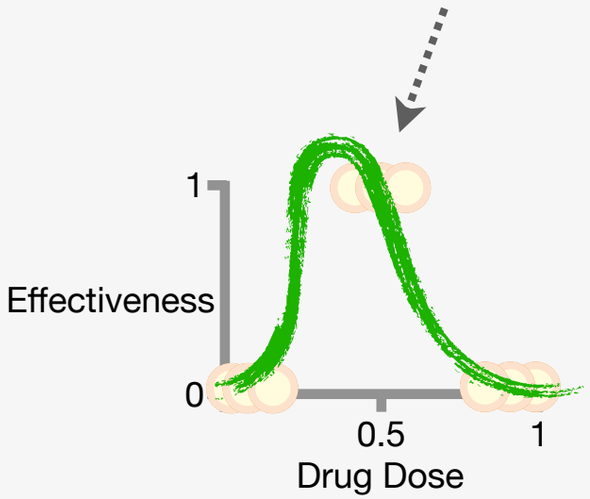
22 Hooray!!! At long last, we see the final **green squiggle**…



…that this **Neural Network** uses to predict **Effectiveness** from Doses.

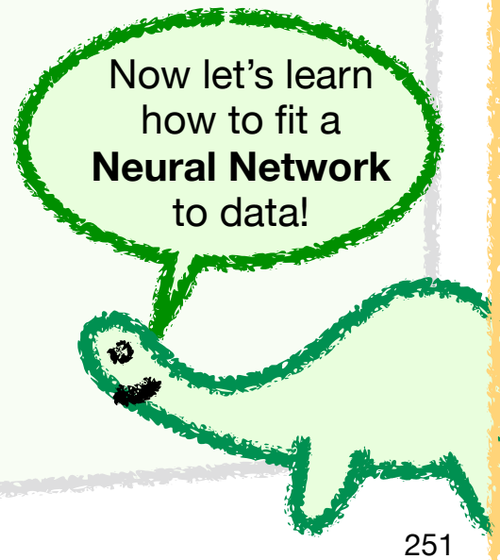


23 Now, if we're wondering if a medium Dose of **0.5** will be effective, we can look at the **green squiggle** and see that the output from the **Neural Network** will be **1**, and thus, Dose **= 0.5** will be effective.



24 Alternatively, if we plug **Dose = 0.5** into the **Neural Network** and do the math, we get **1**, and thus, Dose **= 0.5** will be effective.

TRIPLE BAM!!!



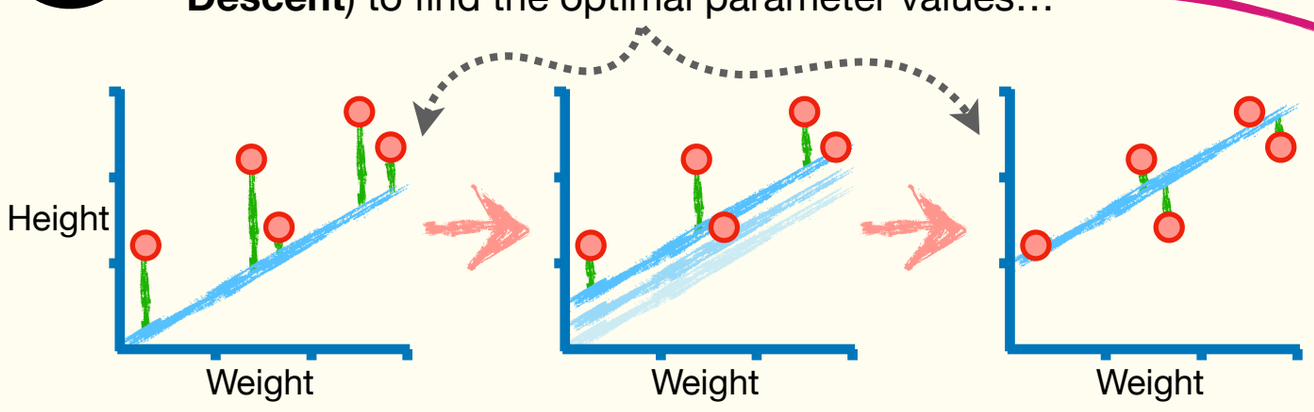
Neural Networks
Part Deux:

Fitting a Neural Network to Data with Backpropagation

Backpropagation: Main Ideas

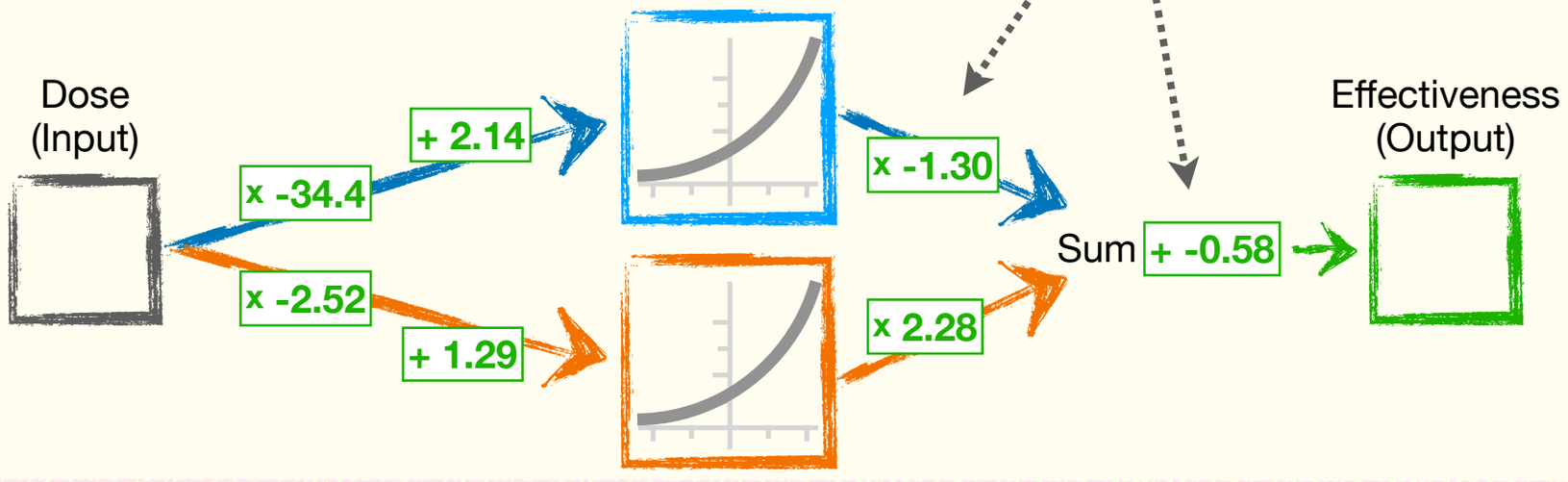
1 **The Problem:** Just like for **Linear Regression**, **Neural Networks** have parameters that we need to optimize to fit a squiggle or bent shape to data. How do we find the optimal values for these parameters?

2 **A Solution:** Just like for **Linear Regression**, we can use **Gradient Descent** (or **Stochastic Gradient Descent**) to find the optimal parameter values…

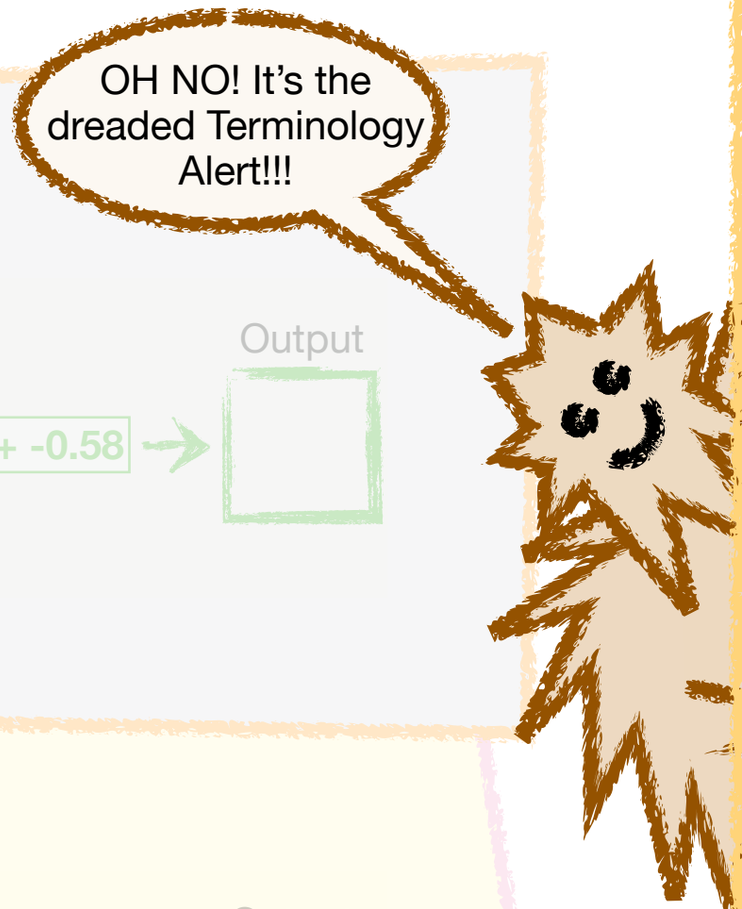


…however, we don't call it **Gradient Descent**. That would be too easy. Instead, because of how the derivatives are found for each parameter in a **Neural Network** (from the back to the front), we call it **Backpropagation**.

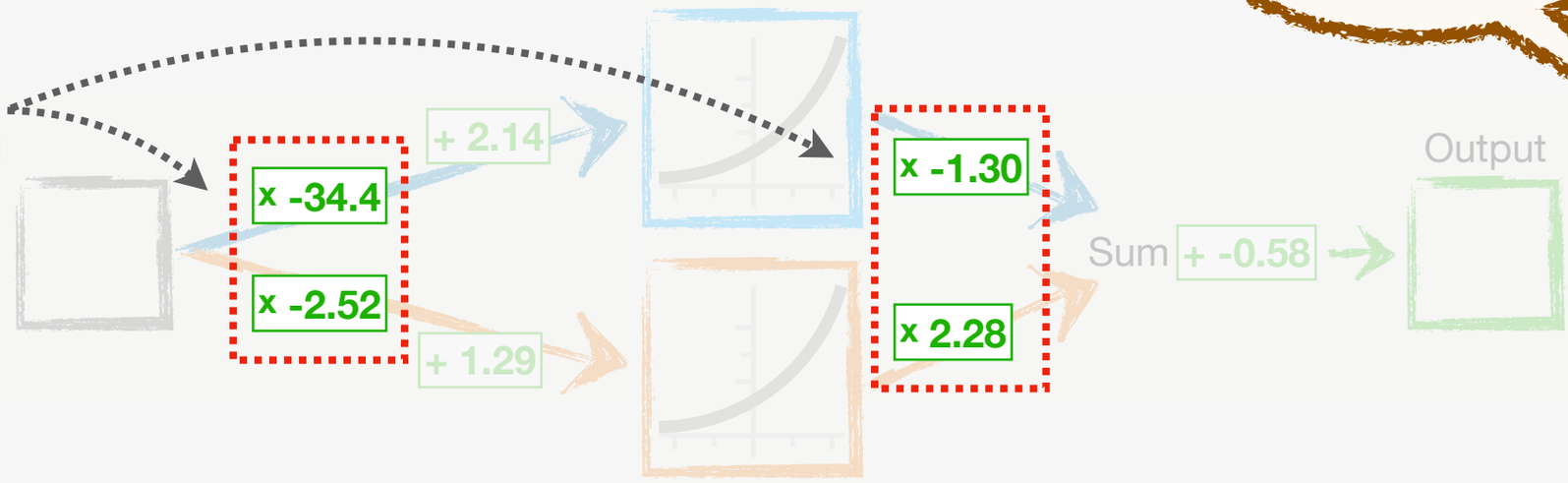
BAM!!!



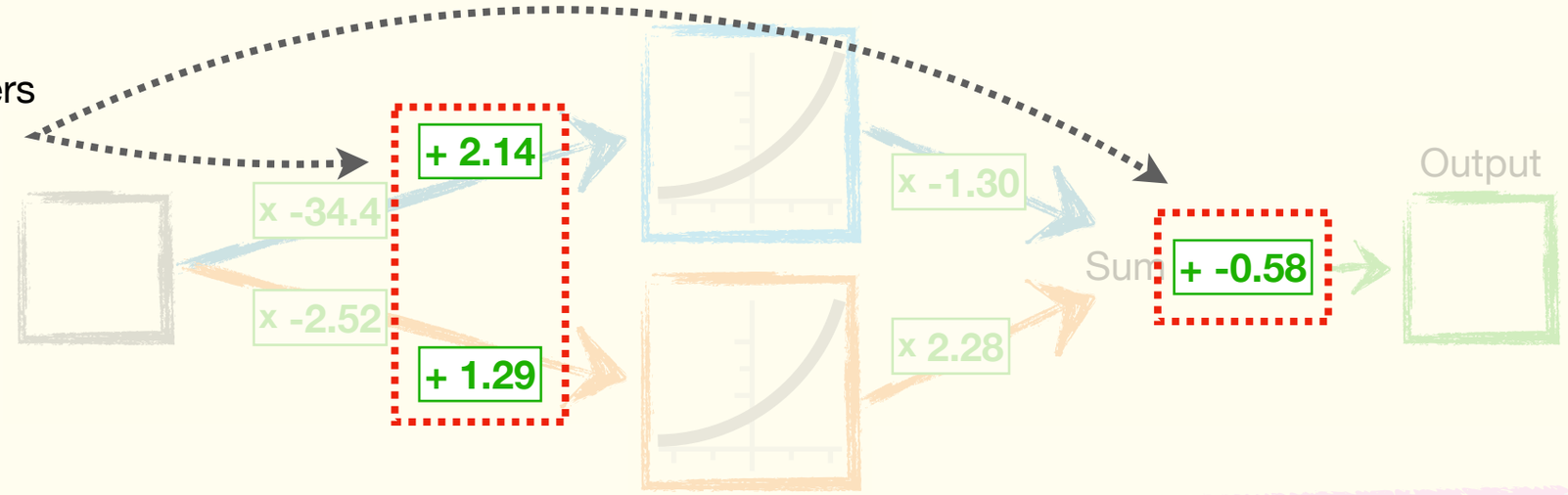
Terminology Alert!!! Weights and Biases

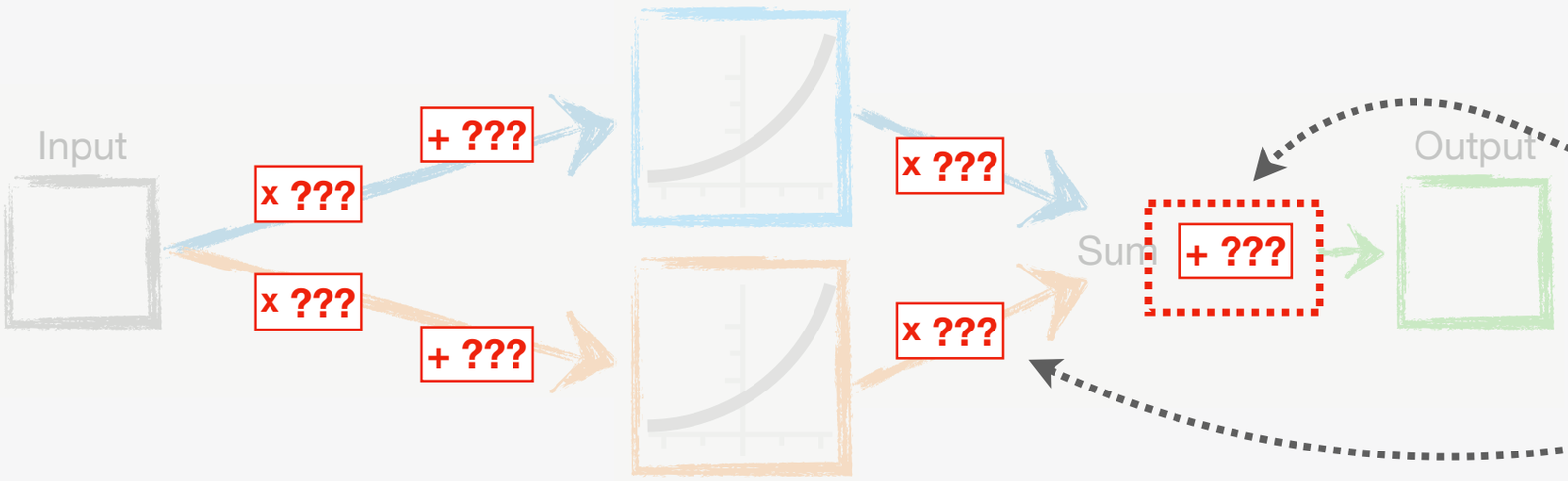


1 In **Neural Networks**, the parameters that we multiply are called **Weights**…



2 …and the parameters we add are called **Biases**.



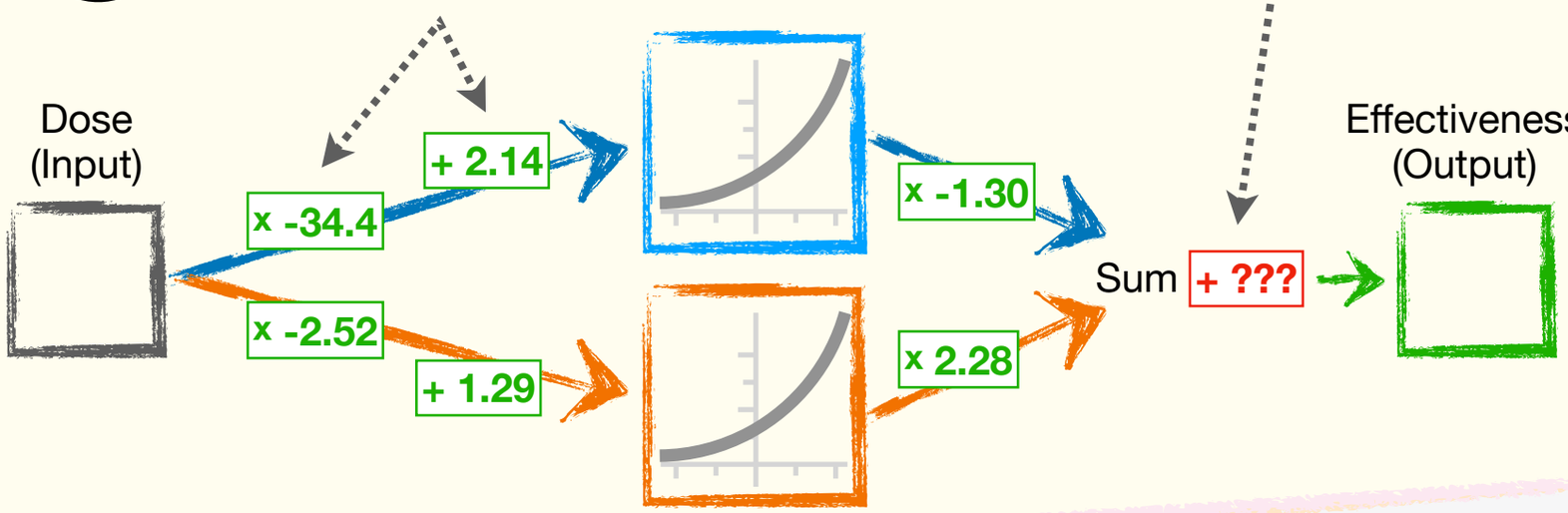


3 In the example that follows, we'll use **Backpropagation** to optimize the **Final Bias**. However, the same process and ideas apply to optimizing all of the parameters.

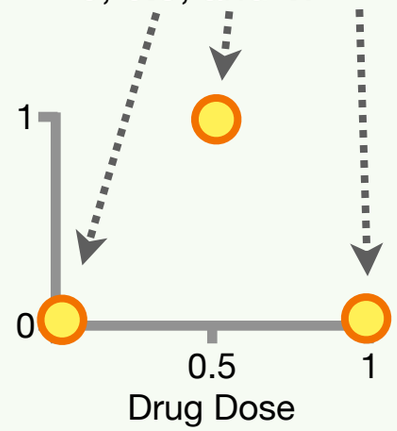
Backpropagation: Details Part 1

1 In this example, let's assume that we already have the optimal values for all of the **Weights** and **Biases**…

…except for the **Final Bias**. So the goal is to use **Backpropagation** to optimize this **Final Bias**.



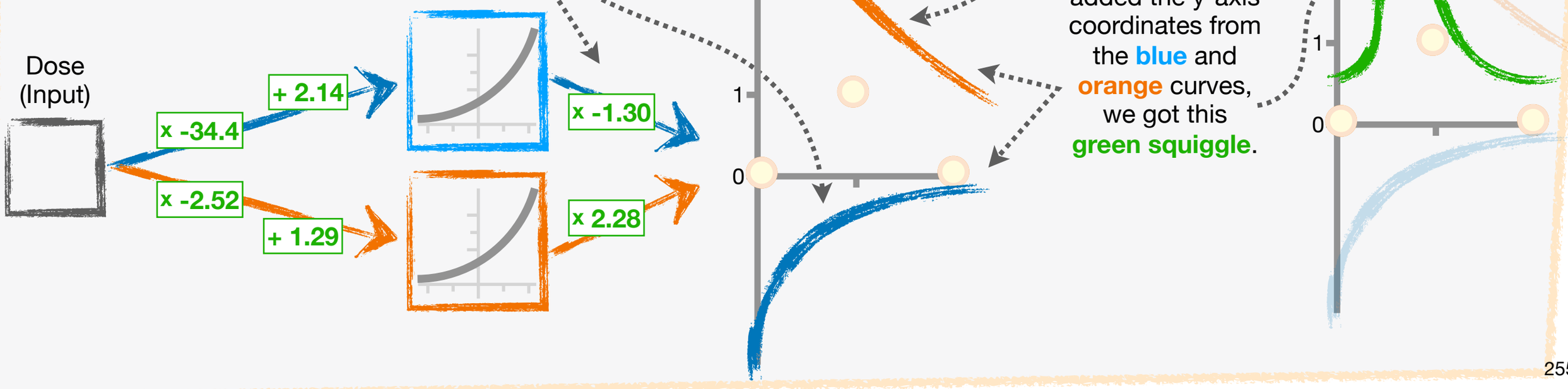
NOTE: To keep the math relatively simple, from now on, the **Training Data** will have only **3 Dose** values, **0**, **0.5**, and **1**.



2 Now let's remember that when we plugged Doses into the input, the top of the **Hidden Layer** gave us this **blue curve**…

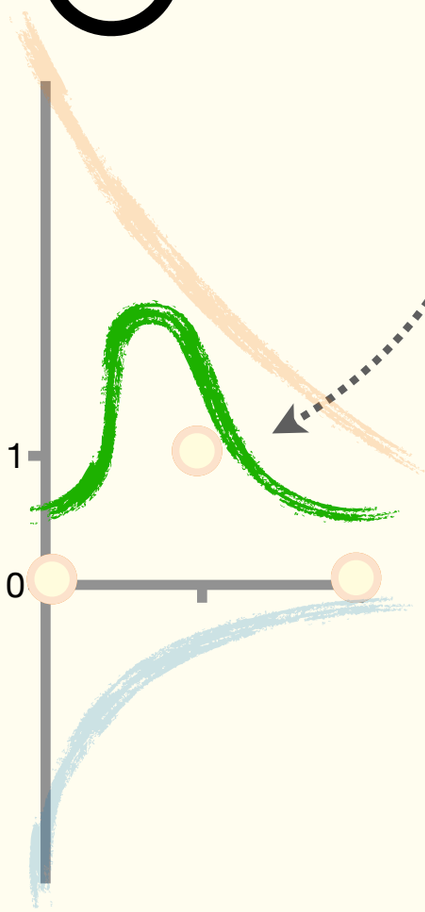
…and the bottom of the **Hidden Layer** gave us this **orange curve**…

…and when we added the y-axis coordinates from the **blue** and **orange** curves, we got this **green squiggle**.

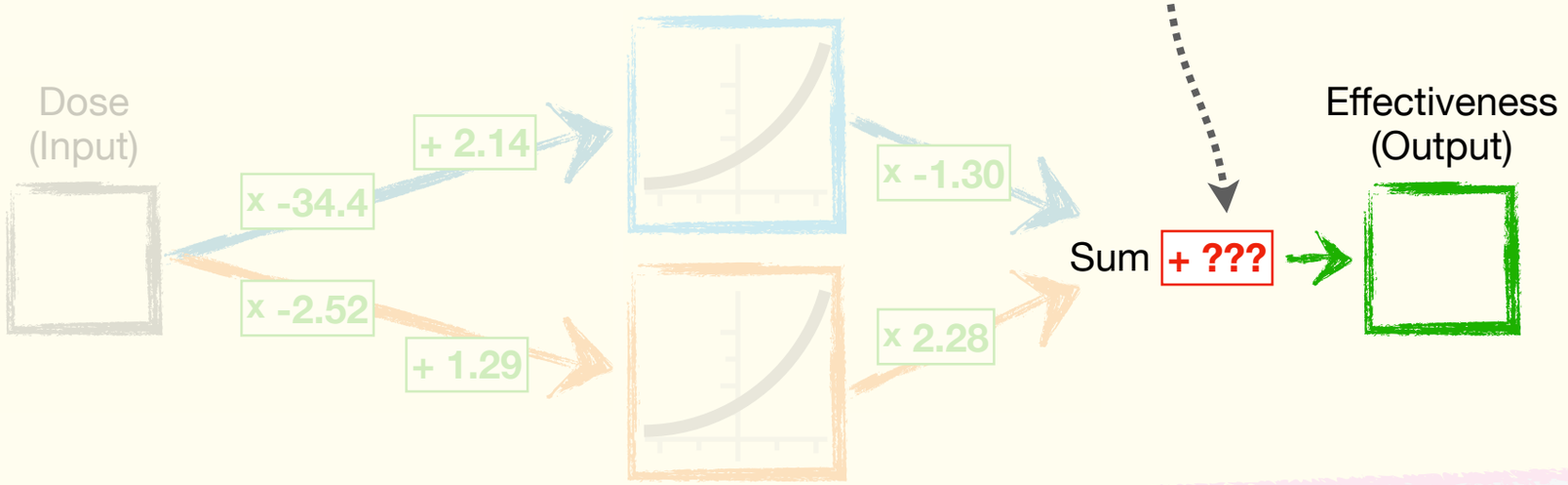


Backpropagation: Details Part 2

3 Now that the **Neural Network** has created the **green squiggle**…

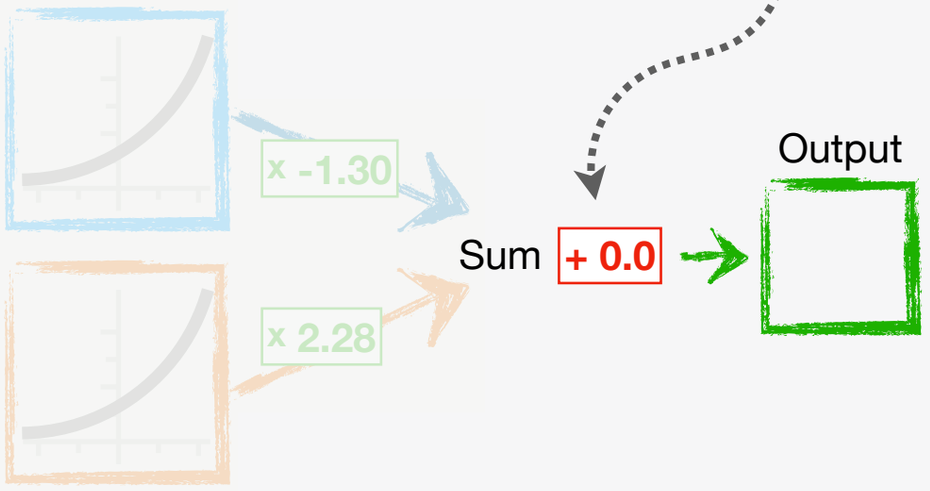


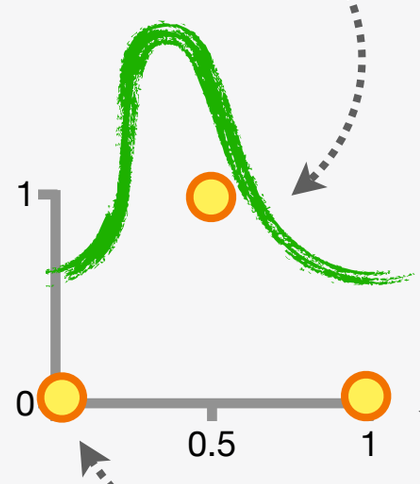
…we're ready to add the **Final Bias** to its y-axis coordinates.



4 However, because we don't yet know the optimal value for the **Final Bias**, we have to give it an initial value. Because **Bias** terms are frequently initialized to **0**, we'll set it to **0**…

…and adding **0** to all of the y-axis coordinates on the **green squiggle** leaves it right where it is…



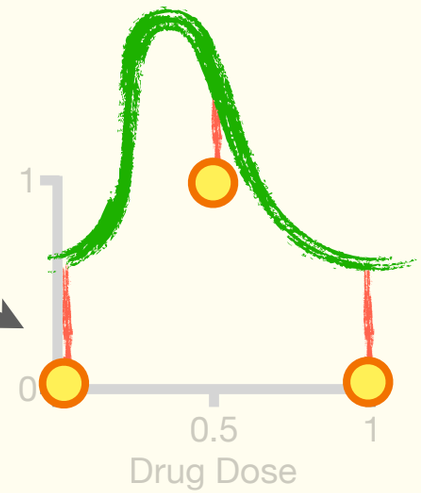


…which means the **green squiggle** doesn't fit the **Training Data** very well.

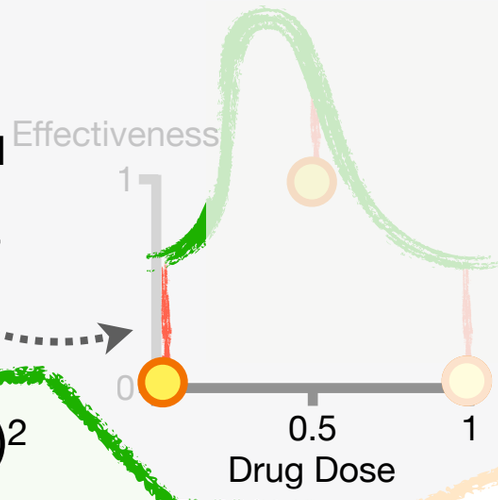
Backpropagation: Details Part 3

5 Now, just like we did for R^2, **Linear Regression**, and **Regression Trees**, we can quantify how well the **green squiggle** fits all of the **Training Data** by calculating the **Sum of the Squared Residuals** (**SSR**).

$$\text{SSR} = \sum_{i=1}^{n} (\text{Observed}_i - \text{Predicted}_i)^2$$

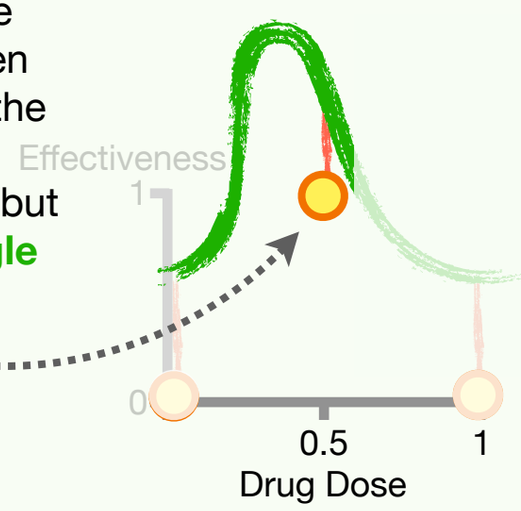


6 For example, for the first Dose, **0**, the **Observed** Effectiveness is **0** and the **green squiggle** created by the **Neural Network** predicts **0.57**, so we plug in **0** for the **Observed** value and **0.57** for the **Predicted** value into the equation for the **SSR**.



$$\text{SSR} = (0 - 0.57)^2$$

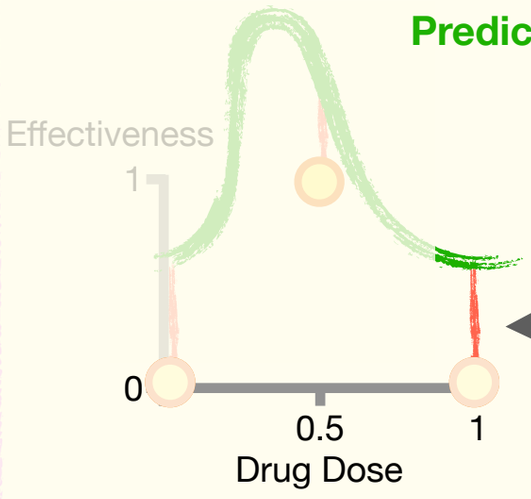
7 Then we add the **Residual** for when Dose **= 0.5**. Now the **Observed** Effectiveness is **1**, but the **green squiggle** predicts **1.61**.



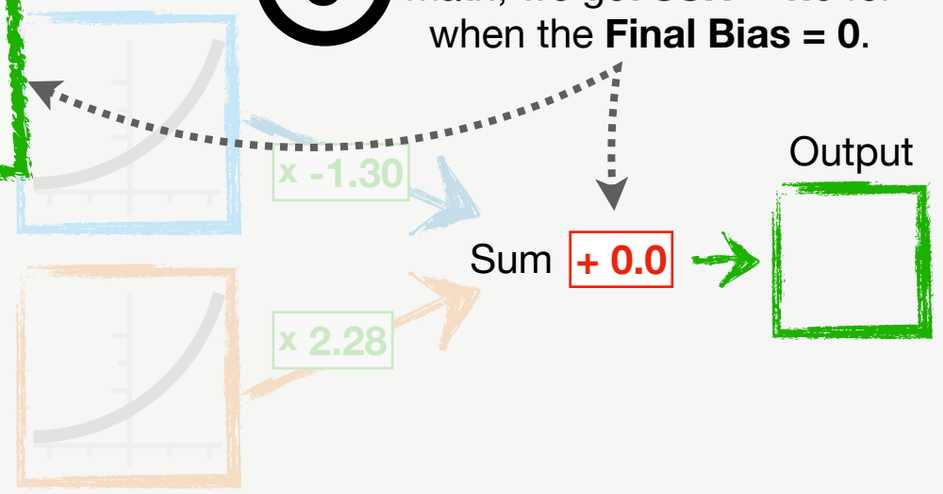
$$+ (1 - 1.61)^2$$

8 Then we add the **Residual** when Dose **= 1**, where the **Observed** value is **0** and **Predicted** value is **0.58**.

$$+ (0 - 0.58)^2 = 1.0$$

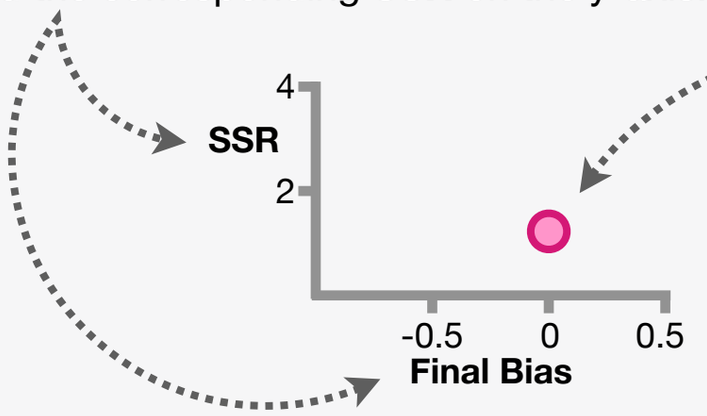


9 Lastly, when we do the math, we get **SSR = 1.0** for when the **Final Bias = 0**.

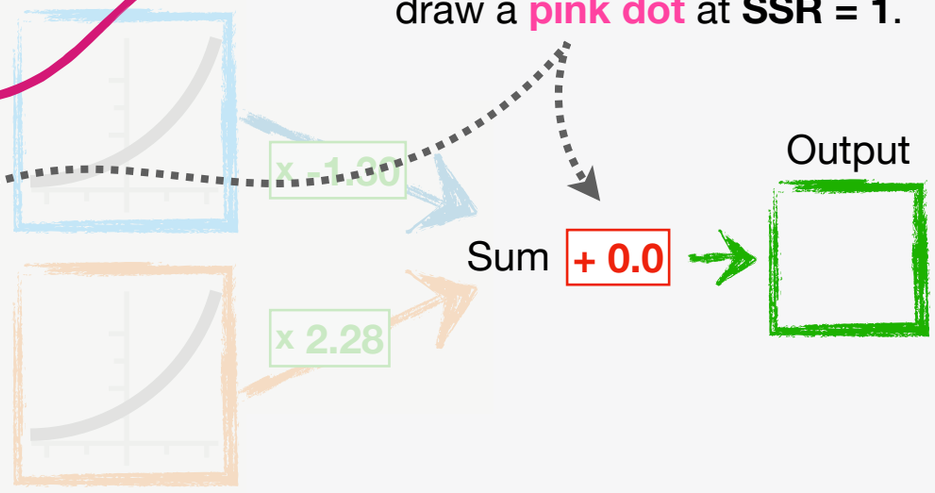


Backpropagation: Details Part 4

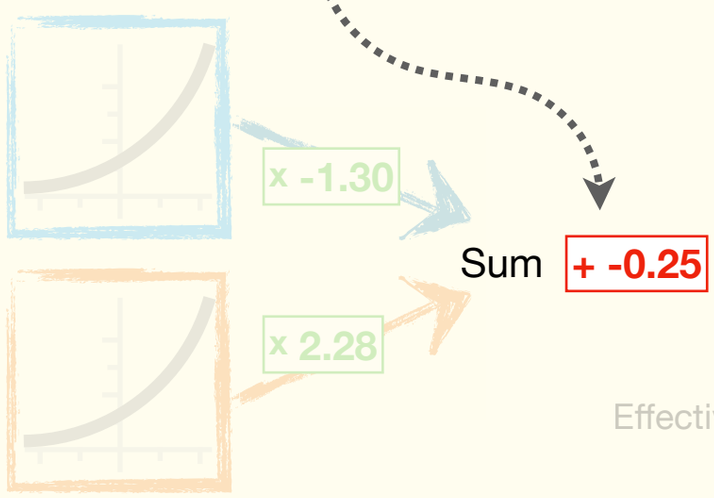
10 Now we can compare the **SSR** for different values for the **Final Bias** by plotting them on this graph that has values for the **Final Bias** on the x-axis and the corresponding **SSR** on the y-axis.



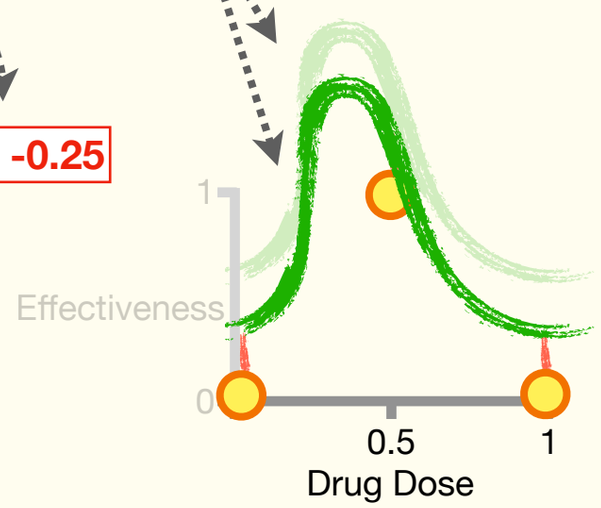
Thus, when the **Final Bias = 0**, we can draw a **pink dot** at **SSR = 1**.



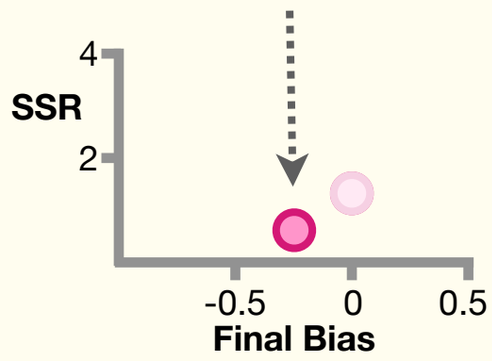
11 If we set the **Final Bias** to **-0.25**…



…then we shift the **green squiggle** down a little bit…

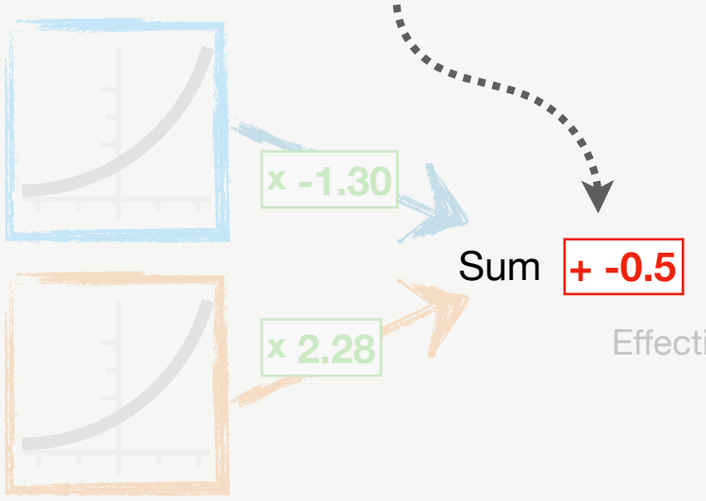


…and we can calculate the **SSR** and plot the value on our graph.

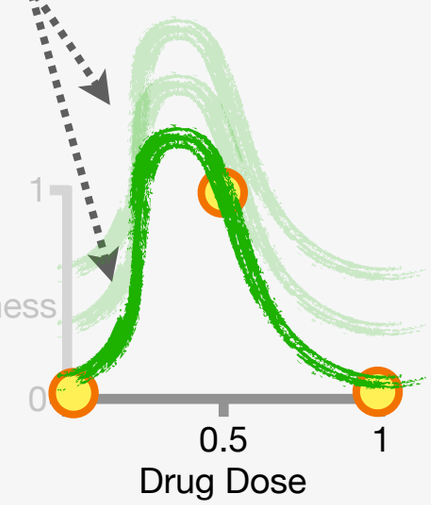


Backpropagation: Details Part 5

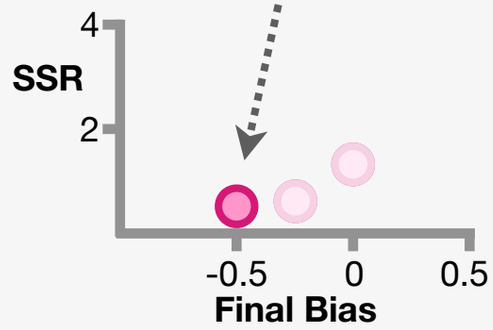
12 Setting the **Final Bias** to **-0.5**…



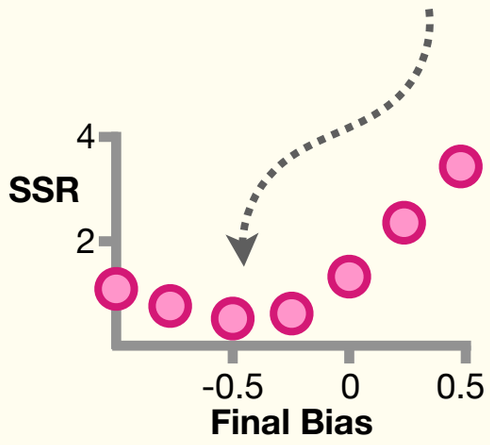
…shifts the **green squiggle** down a little bit more…



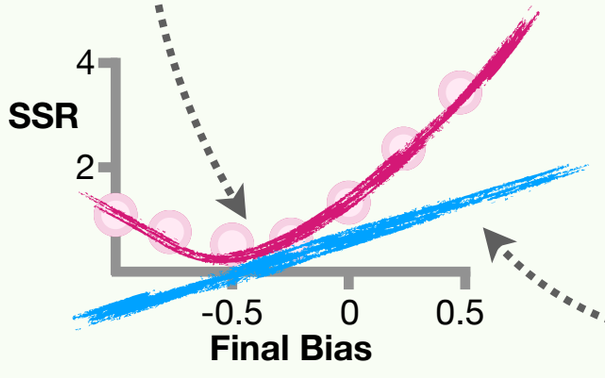
…and results in a slightly lower **SSR**.



13 And if we try a bunch of different values for the **Final Bias**, we can see that the lowest **SSR** occurs when the **Final Bias** is close to **0.5**…



14 …however, instead of just plugging in a bunch of numbers at random, we'll use **Gradient Descent** to quickly find the lowest point in the **pink curve**, which corresponds to the **Final Bias** value that gives us the minimum **SSR**…



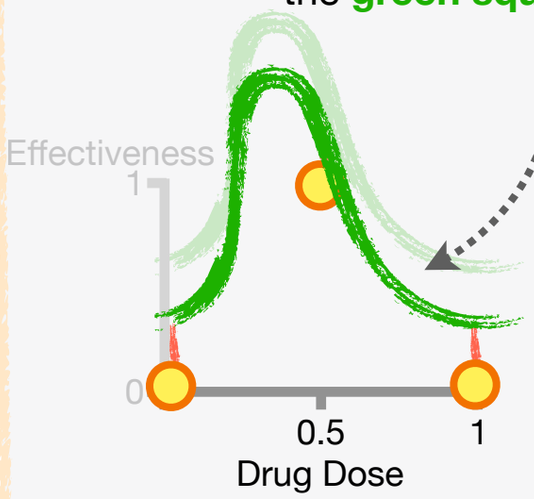
…and to use **Gradient Descent**, we need the derivative of the **SSR** with respect to the **Final Bias**.

$$\frac{d\,SSR}{d\,Final\,Bias}$$

Backpropagation: Details Part 6

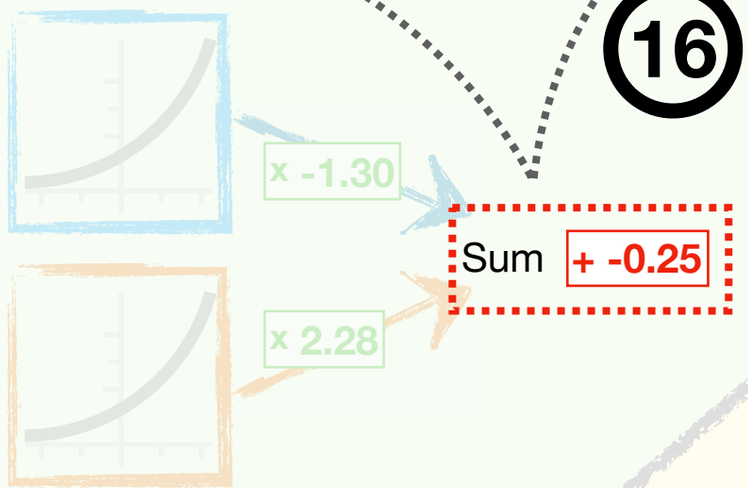
15 Remember, each **Predicted** value in the **SSR** comes from the **green squiggle**…

$$\textbf{SSR} = \sum_{i=1}^{n} (\text{Observed}_i - \text{Predicted}_i)^2$$



Predicted = **green squiggle** = **blue curve** + **orange curve** + **Final Bias**

16 …and the **green squiggle** comes from the last part of the **Neural Network**, when we add the y-axis values from the **blue** and **orange** curves to the **Final Bias**.



17 Now, because the **Predicted** values link the **SSR**…

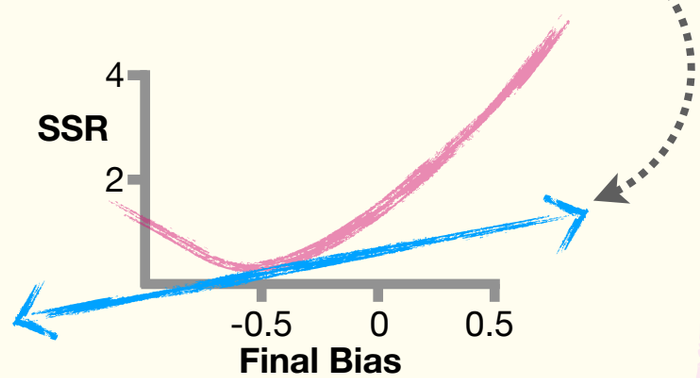
…to the **Final Bias**…

…we can use **The Chain Rule** to solve for the derivative of the **SSR** with respect to the **Final Bias**.

$$\textbf{SSR} = \sum_{i=1}^{n} (\text{Observed}_i - \text{Predicted}_i)^2$$

Predicted = **green squiggle** = **blue curve** + **orange curve** + **Final Bias**

$$\frac{d\,SSR}{d\,Final\,Bias}$$



Backpropagation: Details Part 7

18 **The Chain Rule** says that the derivative of the **SSR** with respect to the **Final Bias**…

$$\frac{d\,SSR}{d\,Final\,Bias} = \frac{d\,SSR}{d\,Predicted} \times \frac{d\,Predicted}{d\,Final\,Bias}$$

…is the derivative of the **SSR** with respect to the **Predicted** values…

$$SSR = \sum_{i=1}^{n} (Observed_i - Predicted_i)^2$$

…multiplied by the derivative of the **Predicted** values with respect to the **Final Bias**.

Predicted = green squiggle = blue curve + orange curve + **Final Bias**

BAM!!!

Psst!
If this doesn't make any sense and you need help with **The Chain Rule**, see **Appendix F**.

Backpropagation: Details Part 8

19 Now that we see that the derivative of the **SSR** with respect to the **Final Bias**… …is the derivative of the **SSR** with respect to the **Predicted** values… …multiplied by the derivative of the **Predicted** values with respect to the **Final Bias**…

$$\frac{d\,SSR}{d\,Final\,Bias} = \frac{d\,SSR}{d\,Predicted} \times \frac{d\,Predicted}{d\,Final\,Bias}$$

20 …we can solve for the *first* part, the derivative of the **SSR** with respect to the **Predicted** values… …which, in turn, can be solved using **The Chain Rule**… …by moving the square to the front… …and multiplying everything by the derivative of the stuff inside the parentheses, which is **-1**…

$$\frac{d\,SSR}{d\,Predicted} = \frac{d}{d\,Predicted} \sum_{i=1}^{n} (Observed_i - Predicted_i)^2 = \sum_{i=1}^{n} 2 \times (Observed_i - Predicted_i) \times -1$$

$$\frac{d\,SSR}{d\,Predicted} = \sum_{i=1}^{n} -2 \times (Observed_i - Predicted_i)$$

…and, lastly, we simplify by multiplying **2** by **-1**.

BAM!!!

We solved for the *first* part of the derivative. Now let's solve for the *second* part.

NOTE: For more details on how to solve for this derivative, see **Chapter 5**.

Backpropagation: Details Part 9

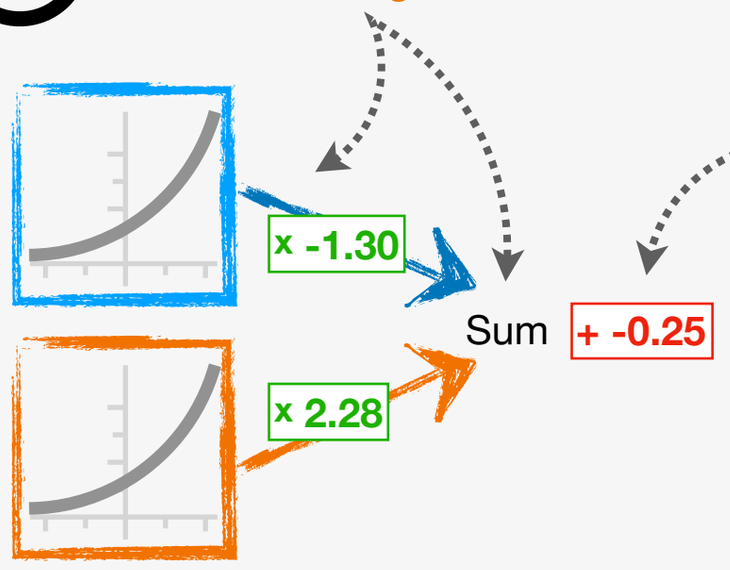
21 The *second* part, the derivative of the **Predicted** values with respect to the **Final Bias**…

…is the derivative of the **green squiggle** with respect to the **Final Bias**…

…which, in turn, is the derivative of the sum of the **blue** and **orange** curves and the **Final Bias**.

$$\frac{d\,Predicted}{d\,Final\,Bias} = \frac{d}{d\,Final\,Bias}\,\text{green squiggle} = \frac{d}{d\,Final\,Bias}(\text{blue curve} + \text{orange curve} + \text{Final Bias})$$

22 Now, remember that the **blue** and **orange** curves…



…were created before we got to the **Final Bias**…

…thus, the derivatives of the **blue** and **orange** curves with respect to the **Final Bias** are both **0** because they do not depend on the **Final Bias**…

$$\frac{d}{d\,Final\,Bias}(\text{blue curve} + \text{orange curve} + \text{Final Bias}) = 0 + 0 + 1$$

…and the derivative of the **Final Bias** with respect to the **Final Bias** is **1**. So, when we do the math, the derivative of the Predicted values with respect to the **Final Bias** is **1**. **DOUBLE BAM!!!**

$$\frac{d\,Predicted}{d\,Final\,Bias} = 1$$

We solved for the *second* part of the derivative.

Backpropagation: Details Part 10

23 Now, to get the derivative of the **SSR** with respect to the **Final Bias**, we simply plug in… …the derivative of the **SSR** with respect to the **Predicted** values… …and the derivative of the **Predicted** values with respect to the **Final Bias**.

$$\frac{d\,SSR}{d\,Predicted} = \sum_{i=1}^{n} -2 \times (Observed_i - Predicted_i)$$

$$\frac{d\,Predicted}{d\,Final\,Bias} = 1$$

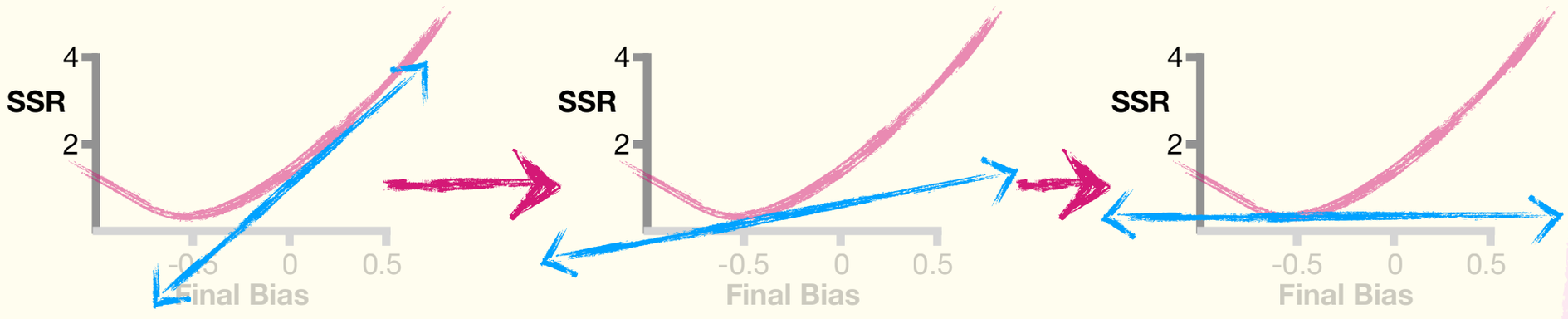
$$\frac{d\,SSR}{d\,Final\,Bias} = \frac{d\,SSR}{d\,Predicted} \times \frac{d\,Predicted}{d\,Final\,Bias}$$

$$\frac{d\,SSR}{d\,Final\,Bias} = \sum_{i=1}^{n} -2 \times (Observed_i - Predicted_i) \times 1$$

24 At long last, we have the derivative of the **SSR** with respect to the **Final Bias!!!**

TRIPLE BAM!!!

25 In the next section, we'll plug the derivative into **Gradient Descent** and solve for the optimal value for the **Final Bias**.



Backpropagation: Step-by-Step

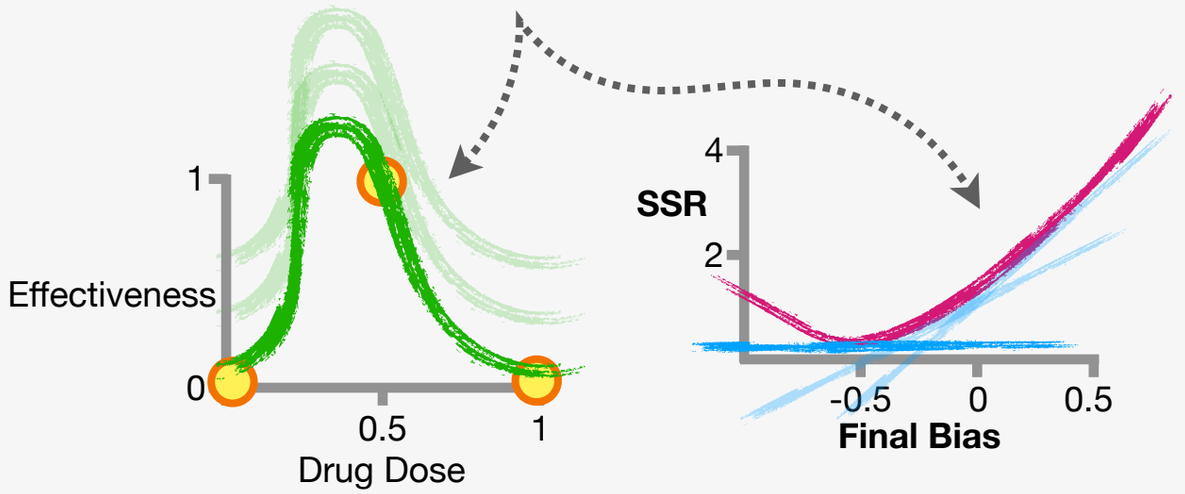
1 Now that we have the derivative of the **SSR** with respect to the **Final Bias**…

…which tells us how the **SSR** changes when we change the **Final Bias**…

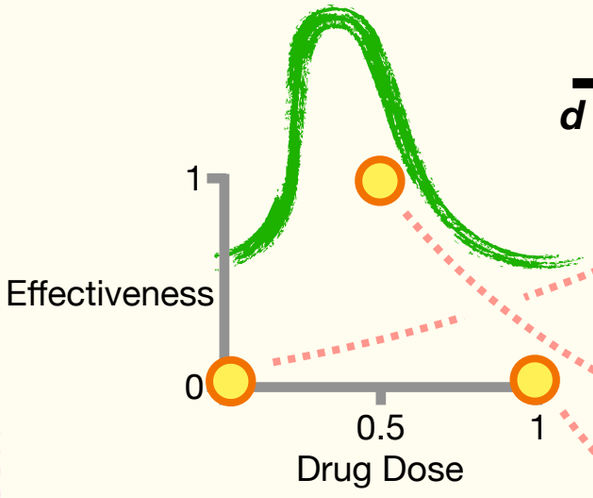
…we can optimize the **Final Bias** with **Gradient Descent**.

$$\frac{d\,SSR}{d\,Final\,Bias} = \sum_{i=1}^{n} -2 \times (Observed_i - Predicted_i) \times 1$$

NOTE: We're leaving the **"x 1"** term in the derivative to remind us that it comes from **The Chain Rule**. However, multiplying by **1** doesn't do anything, and you can omit it.



2 First, we plug in the **Observed** values from the **Training Dataset** into the derivative of the **SSR** with respect to the **Final Bias**.



$$\frac{d\,SSR}{d\,Final\,Bias} = \sum_{i=1}^{n} -2 \times (Observed_i - Predicted_i)^2 \times 1$$

$$= -2 \times (\,Observed_1 - Predicted_1\,) \times 1$$

$$+ -2 \times (\,Observed_2 - Predicted_2\,) \times 1$$

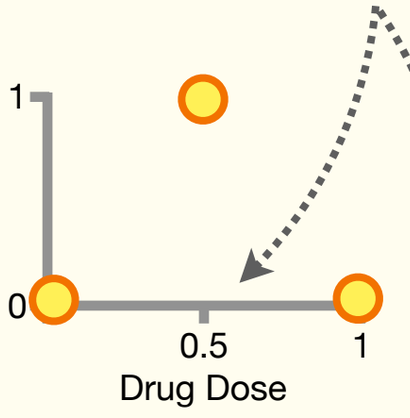
$$+ -2 \times (\,Observed_3 - Predicted_3\,) \times 1$$

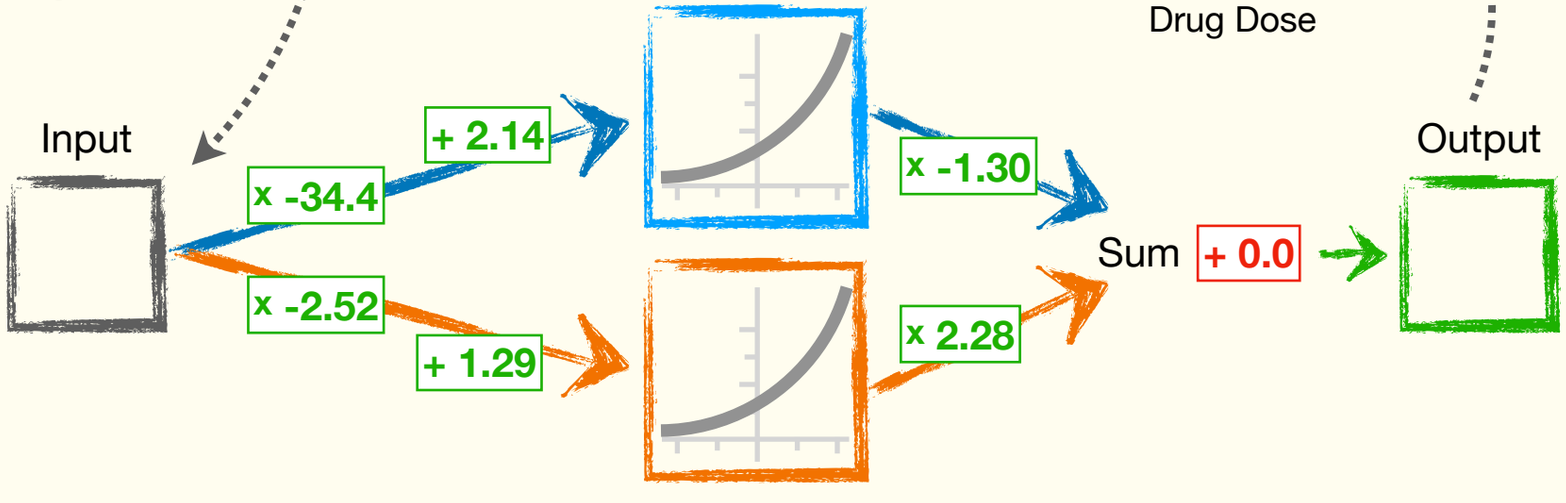
$$\frac{d\,SSR}{d\,Final\,Bias} = -2 \times (\,0 - Predicted_1\,) \times 1$$
$$+ -2 \times (\,1 - Predicted_2\,) \times 1$$
$$+ -2 \times (\,0 - Predicted_3\,) \times 1$$

Backpropagation: Step-by-Step

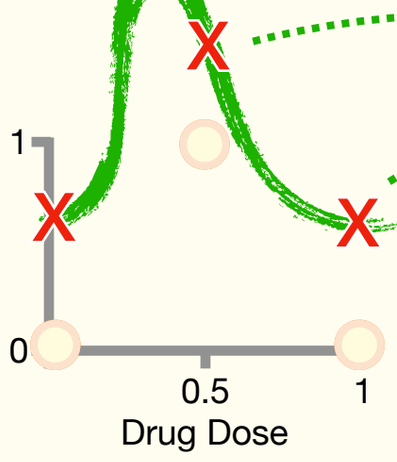
3 Then we initialize the **Final Bias** to a random value. In this case, we'll set the **Final Bias** to **0.0**.

4 Then we run the **3 Doses** from the **Training Dataset**, **0**, **0.5** and **1**, through the **Neural Network** to get the **Predicted** values….





…and we plug the **Predicted** values into the derivative.



$$\frac{d\,SSR}{d\,Final\,Bias} = -2 \times (0 - Predicted) \times 1 + -2 \times (1 - Predicted) \times 1 + -2 \times (0 - Predicted) \times 1$$

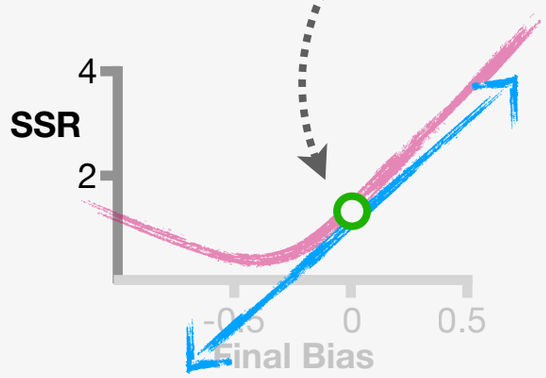
$$= -2 \times (0 - 0.57) \times 1 + -2 \times (1 - 1.61) \times 1 + -2 \times (0 - 0.58) \times 1$$

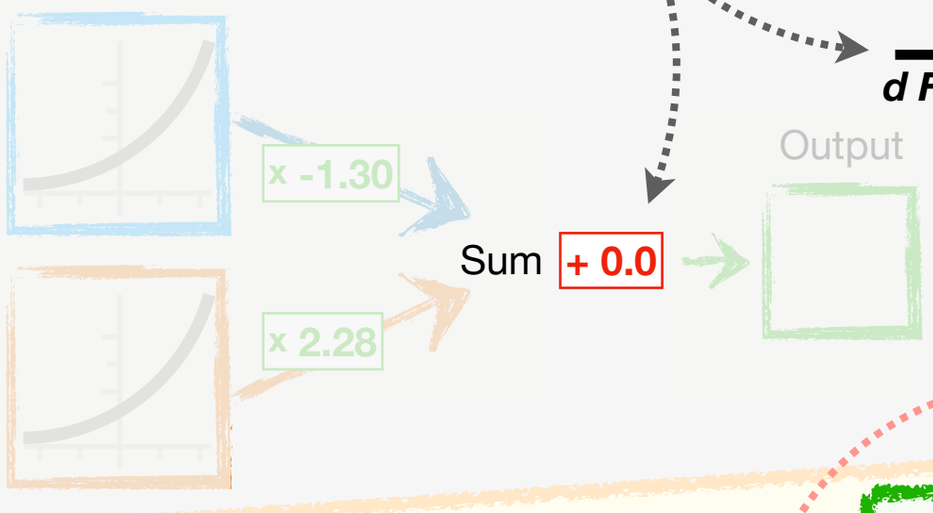
Backpropagation: Step-by-Step

5 Now we evaluate the derivative at the current value for the **Final Bias**, which, at this point, is **0.0**.

When we do the math, we get **3.5**…

…thus, when the **Final Bias = 0**, the slope of this **tangent line** is **3.5**.





$$\frac{d\,SSR}{d\,Final\,Bias} = -2 \times (0 - 0.57) \times 1 + -2 \times (1 - 1.61) \times 1 + -2 \times (0 - 0.58) \times 1 = 3.5$$

6 Then we calculate the **Step Size** with the standard equation for **Gradient Descent** and get **0.35**.

NOTE: In this example, we've set the **Learning Rate** to **0.1**.

Step Size = **Derivative** x **Learning Rate**

= **3.5** x **0.1**

= **0.35**

Gentle Reminder: The magnitude of the derivative is proportional to how big of a step we should take toward the minimum. The sign (+/-) tells us in what direction.

7 Lastly, we calculate a *new* value for the **Final Bias** from the *current* **Final Bias**…

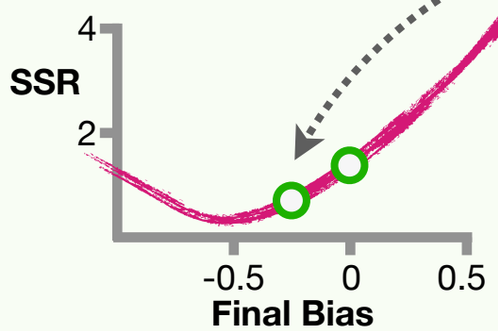
New Bias = **Current Bias** - **Step Size**

= **0.0** - **0.35**

= **-0.35**

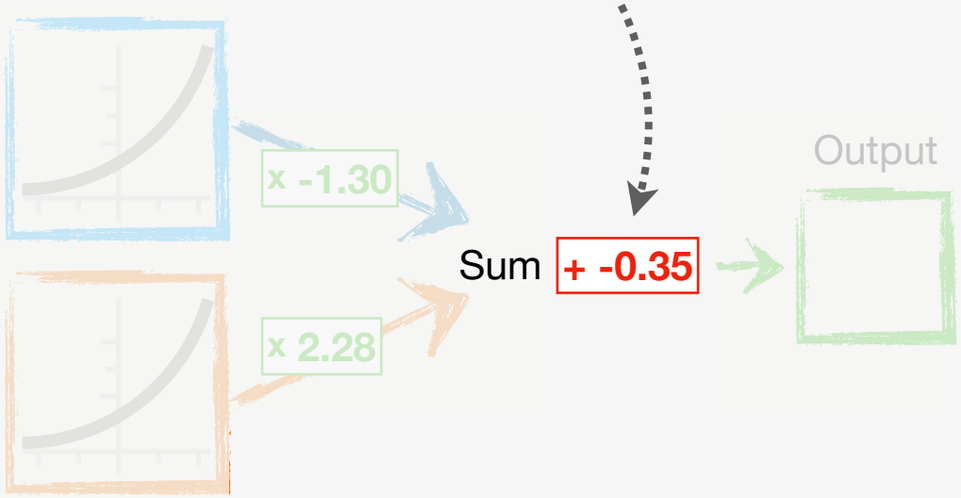
Remember, we initialized the **Final Bias** to **0.0**.

…and the *new* **Final Bias** is **-0.35**, which results in a lower **SSR**.

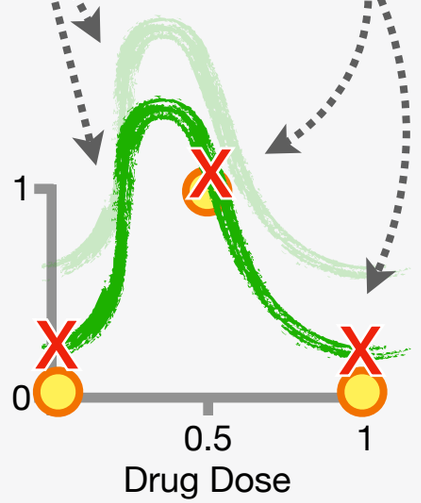


Backpropagation: Step-by-Step

8 With the new value for the **Final Bias**, **-0.35**…



…we shift the **green squiggle** down a bit, and the **Predicted** values are closer to the **Observed** values.



BAM!!!

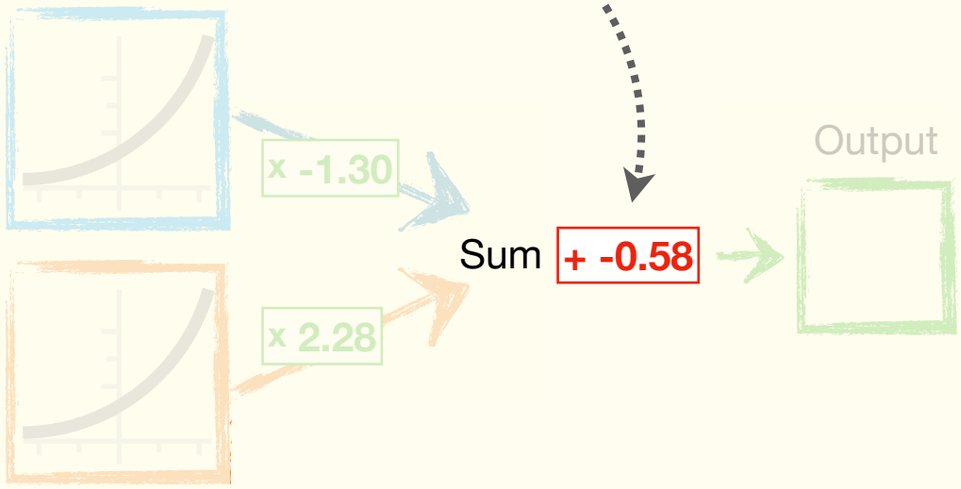
9 Now **Gradient Descent** iterates over the past three steps…

a Evaluate the derivative at the current value…

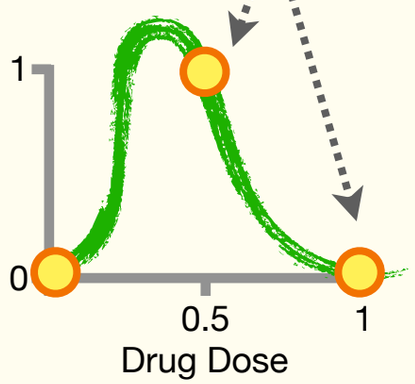
b Calculate the **Step Size**…

c Calculate the new value…

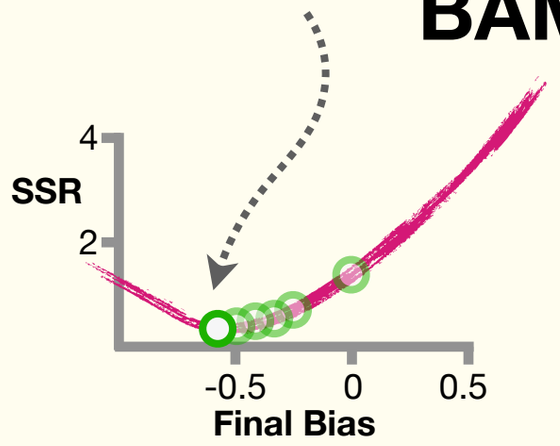
…and after **7** iterations, the **Final Bias** = **-0.58**…



…and the **green squiggle** fits the **Training Data** really well…



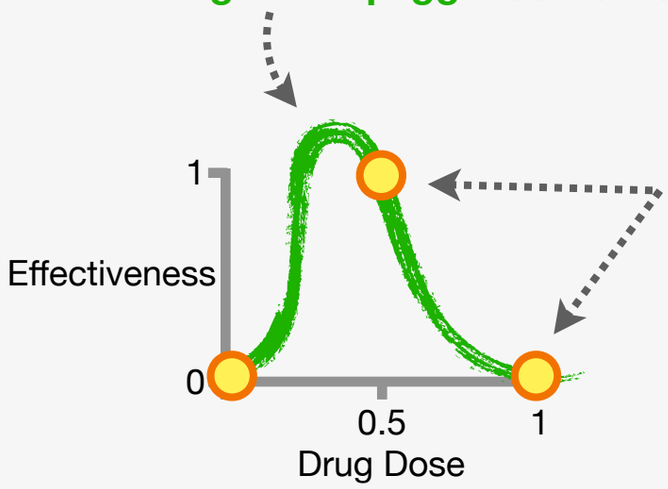
…and we've made it to the lowest **SSR**.



BAM!!!

Neural Networks: FAQ

Where the heck did this bump in the green squiggle come from?

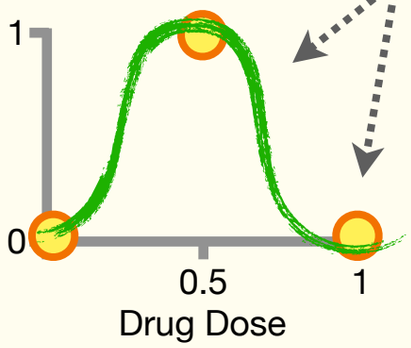


When we used **Backpropagation** to estimate the **Weights** and **Biases** in the **Neural Network**, we only calculated the **SSR** for the original **3** Doses, **0**, **0.5**, and **1**.

That means we only judge the **green squiggle** by how well it predicts Effectiveness at the original **3** Doses, **0**, **0.5**, and **1**, *and no other* Doses.

And that means the **green squiggle** can do whatever it wants in between the original **3** Doses, including making a strange bump that may or may not make good predictions. This is something I think about when people talk about using **Neural Networks** to drive cars. The **Neural Network** probably fits the **Training Data** really well, but there's no telling what it's doing between points, and that means it will be hard to predict what a self-driving car will do in new situations.

Wouldn't it be better if the green squiggle fit a bell-shaped curve to the Training Data?

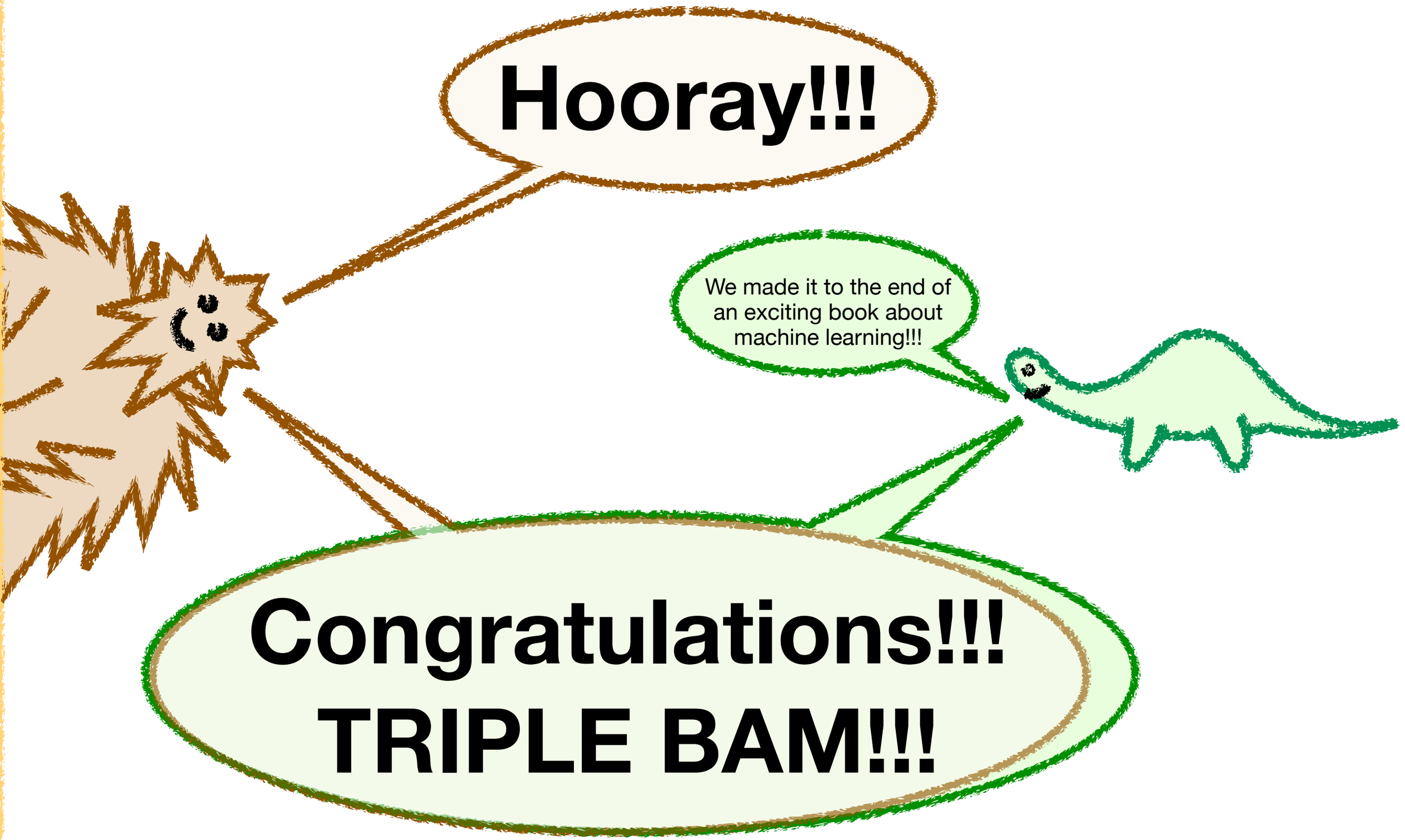


Maybe. Because we don't have any data between the **3** Doses in the **Training Data**, it's hard to say what the best fit would be.

When we have *Neural Networks*, which are cool and super flexible, why would we ever want to use *Logistic Regression*, which is a lot less flexible?

Neural Networks are cool, but deciding how many **Hidden Layers** to use and how many **Nodes** to put in each **Hidden Layer** and even picking the best **Activation Function** is a bit of an art form. In contrast, creating a model with **Logistic Regression** is a science, and there's no guesswork involved. This difference means that it can sometimes be easier to get **Logistic Regression** to make good predictions than a **Neural Network**, which might require a lot of tweaking before it performs well.

Furthermore, when we use a lot of variables to make predictions, it can be much easier to interpret a **Logistic Regression** model than a **Neural Network**. In other words, it's easy to know how **Logistic Regression** makes predictions. In contrast, it's much more difficult to understand how a **Neural Network** makes predictions.


Hooray!!!
We made it to the end of an exciting book about machine learning!!!
Congratulations!!!
TRIPLE BAM!!!