

#### An example of the Drexel beamer template

Exploring color schemes and graphics

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## Variational Enhanced Sampling



A free energy surface, F(s), associated with set of collective variables, s, is defined as

$$F(\mathbf{s}) = -(1/\beta) \log \int d\mathbf{R} \delta(\mathbf{s} - \mathbf{s}(\mathbf{R}) e^{-\beta U(\mathbf{R})}$$
 (1)

Valsson and Parrinello introduce a functional of bias potential V(s),

$$\Omega[V] = \frac{1}{\beta} \log \frac{\int d\mathbf{s} e^{-\beta [F(\mathbf{s}) + V(\mathbf{s})]}}{\int d\mathbf{s} e^{-\beta F(\mathbf{s})}} + \int d\mathbf{s} \rho(\mathbf{s}) V(\mathbf{s})$$
(2)

where  $p(\mathbf{s})$  is an arbitrary probability distribution (normalized).

- $\Rightarrow \Omega[V]$  is convex!
- The potential which renders  $\Omega[V]$  stationary is (to within a constant):

$$V(\mathbf{s}) = -F(\mathbf{s}) - (1/\beta)\log p(\mathbf{s}) \tag{3}$$

#### Variational Enhanced Sampling



- Write bias potential  $V(\mathbf{s}; \alpha)$  as function of variational parameters  $\alpha = (\alpha_1, \alpha_2, ..., \alpha_K)$
- 2 Minimize  $\Omega[V(\alpha)]$  with respect to  $\alpha$  using the gradient  $\Omega'(\alpha)$  and Hessian  $\Omega''(\alpha)$ ,

$$\frac{\partial \Omega}{\partial \alpha_i} = -\left\langle \frac{\partial V(\mathbf{s}; \boldsymbol{\alpha})}{\partial \alpha_i} \right\rangle_{V(\boldsymbol{\alpha})} + \left\langle \frac{\partial V(\mathbf{s}; \boldsymbol{\alpha})}{\partial \alpha_i} \right\rangle_p \tag{4}$$

$$\frac{\partial^{2}\Omega(\boldsymbol{\alpha})}{\partial\alpha_{j}\partial\alpha_{i}} = \beta \operatorname{Cov}\left[\frac{\partial V(\mathbf{s};\boldsymbol{\alpha})}{\partial\alpha_{j}}; \frac{\partial V(\mathbf{s};\boldsymbol{\alpha})}{\partial\alpha_{i}}\right]_{V(\boldsymbol{\alpha})} - \left\langle\frac{\partial^{2}V(\mathbf{s};\boldsymbol{\alpha})}{\partial\alpha_{j}\partial\alpha_{i}}\right\rangle_{V(\boldsymbol{\alpha})} + \left\langle\frac{\partial^{2}V(\mathbf{s};\boldsymbol{\alpha})}{\partial\alpha_{j}\partial\alpha_{i}}\right\rangle_{p}$$
(5)

where  $\langle ... \rangle_{V(\alpha)}$  and  $\langle ... \rangle_{p(\mathbf{s})}$  are the expectation value obtained in a biased simulation with potential  $V(\mathbf{s}; \alpha)$  and distribution  $p(\mathbf{s})$ , respectively

 $\hbox{\bf 3} \hbox{ Use stochastic gradient descent-based algoritm to iterate } \alpha \\ \Rightarrow \hbox{In practice, only the diagonal of the Hessian is used during optimization}$ 

### Block copolymers are awesome



They can assemble into so many different morphologies

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